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ALTITUDE RESPONSE OF SEVERAL AIRPLANES
DURING LANDING APPROACH

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SUMMARY

Calculations made by the Laplace Transform method are presented showing the response to abrupt longitudinal control motion of the Shuttle and of four other airplanes, two of conventional and two of delta-wing configuration. Of these airplanes, three have been used in power-off landing experiments at the Dryden Flight Research Center. The effects of airplane type, center-of-gravity location, pitch damping, lift coefficient, and position of the cockpit on the response in height and pitching velocity are presented.

Of the airplanes studied, only the Shuttle has been found to experience difficulties due to lack of precision in controlling the flight path or a tendency for longitudinal pilot-induced oscillations. The results show that the conventional airplanes experience relatively small initial reversal of response in altitude of the center of gravity compared to the delta-wing airplanes, and of these, the Shuttle has considerably larger reversed response. Factors contributing to this difference are discussed. The control difficulties of the Shuttle cannot necessarily be attributed to the response reversal, however, because of the presence of other adverse effects due to time delay and rate limiting in the control system.

INTRODUCTION

In the past, designers of unusually large or heavy airplanes have been concerned with the lag in response of these airplanes during the landing approach and flare. A number of studies of this problem have been made, such as those given in references 1-4. In reference 1, which considered transport airplanes of conventional configuration up to a gross weight of about 1.33×10^6 N (300 000 lb), the increased lag was predicted to be not of serious concern. Later reports, references 2-4, considered airplanes of larger gross weight and slender delta-wing designs with elevon control. Longitudinal control of the delta-wing configuration was predicted to be a cause of concern because of the initially incorrect response in height due to lift produced by the elevons. In addition, the effect of the large distance of the pilot from the landing wheels in very large airplanes was thought likely to cause increased dispersion in touchdown point and sink rate in landing. The use of direct lift control was suggested as a means of alleviating some of these problems (ref. 5).

Since the publication of these reports, numerous large transport airplanes, including a delta-wing configuration, have been placed in service without encountering any serious operational difficulties. In some cases, direct lift control was considered in the design but was found to be unnecessary in practice.

The problem of the adverse initial response of a delta-wing configuration has received increased interest in the case of the Shuttle because it was thought to contribute to a pilot-induced oscillation problem encountered in the final landing of five landings made in the approach and landing tests. In discussion of this problem, data on the height response characteristics of different types of airplanes did not appear to be readily available. Perhaps one reason for this lack of data is that most flight measurements of transient responses are made with internal instrumentation which record linear accelerations and angular rates, but not changes in altitude. Changes in altitude are difficult to obtain directly with sufficient accuracy and sensitivity during rapid maneuvers. In addition, such changes in altitude are not readily apparent to the pilot except possibly during the landing flare where the fixed ground reference is available.

The purpose of this report is to present calculated altitude variations and other longitudinal response quantities during abrupt control maneuvers for the Shuttle and for a number of types of airplanes. The effect of certain changes in stability characteristics, such as those due to center-of-gravity position and pitch damping, are also presented.

All except one of the airplanes taken as examples have been used in power-off landing experiments at the Dryden Flight Research Center. These calculations, therefore, serve as a basis of comparison with the Shuttle results. The one which has not been tested, a delta-wing bomber, was selected because it is a delta-wing airplane with weight similar to the Shuttle.

The data presented may be of value for correlation with the known characteristics of airplanes similar to those analyzed. In addition, they may give a general appreciation of the range of altitude response characteristics encountered with different airplane types. A complete study of the effect of these characteristics on the longitudinal control problems during landing would require man-in-the-loop studies either by means of pilot modeling or by manned simulation investigations using ground-based or flight simulators.

SYMBOLS

A_1, A_2, A_3, A_4	terms in formula (26)
$a_0, a_1, b_0, b_1, a_{oD}$	coefficients in transfer functions
B_0, B_1, B_2	terms in formula (29)
$C_1, C_2, C_3, C_4, C_5, C_6$	constants of integration
c	wing chord
D	differential operator, $\frac{d}{ds}$
h	altitude, positive up
K	gain of pitch damper, $\frac{d\delta_e}{d(qc/2V)}$
K_y	ratio of radius of gyration in pitch to wing chord
K_Z	coefficient multiplying transfer function for Z/δ_e
K_D	coefficient multiplying transfer function for $D\theta/\delta_e$
k_y	radius of gyration in pitch
l	ratio of tail length to wing chord
l_p	ratio of distance between center of gravity and cockpit to chord length
l_x	ratio of distance between center of gravity and instantaneous center of rotation to chord length
M	pitching moment
m	mass
S	wing area
s	nondimensional distance, chord lengths

t	time
U	term in formula (25)
V	airspeed
w	vertical velocity, positive down
Z	vertical displacement or vertical force, positive down
α	angle of attack
α, γ	real roots of characteristic equation
$\alpha \pm i\beta$	complex roots of characteristic equation
Δ_o	characteristic determinant
δ_e	elevator angle
ϵ	downwash angle
θ	pitch angle, positive up
μ	relative density factor, $\frac{m}{\rho S c}$
ρ	air density
τ	elevator effectiveness factor $C_{Z\delta_e} / C_{Z\alpha_t}$
ψ_1	phase angle (formula 25)
ψ_2	phase angle (formula 40)

Subscripts

w	wing or wing-body
t	tail

dot over a quantity indicates differentiation with respect to time

Stability derivatives are defined in accordance with the following examples.

$$C_Z = \frac{\partial C_Z}{\partial \alpha}$$

$$C_{m_q} = \frac{\partial C_m}{\partial \left(\frac{qc}{2V}\right)}$$

$$C_{m_D \alpha} = \frac{\partial C_m}{\partial \left(\frac{\dot{\alpha}c}{2V}\right)}$$

ANALYSIS

Derivation of Equations of Motion

Because this analysis is concerned with rapid longitudinal maneuvers, the assumption of constant airspeed is made. The derivation of the short-period longitudinal equations has been given in numerous previous reports and text-books, for example in reference 6. The derivation is repeated here for completeness, and to provide a clear presentation of the manner in which the results depend on the separate effects of the wing-body combination and the tail. This approach was also used in reference 7.

The longitudinal equations in dimensional form are as follows:

$$\begin{aligned} m(\dot{w} - V\dot{\theta}) &= \alpha_w \frac{\partial Z}{\partial \alpha_w} + \alpha_t \frac{\partial Z}{\partial \alpha_t} + \delta_e \frac{\partial Z}{\partial \delta_e} \\ m k_y \ddot{\theta} &= \alpha_w \frac{\partial M}{\partial \alpha_w} + \alpha_t \frac{\partial M}{\partial \alpha_t} + \delta_e \frac{\partial M}{\partial \delta_e} \end{aligned} \quad (1)$$

These equations are non-dimensionalized by making the following substitutions:

$$\begin{aligned} s &= \frac{tV}{c} \\ D &= \frac{d}{ds} = \frac{c}{V} \frac{d}{dt} \\ K_y &= k_y/c \\ \mu &= \frac{m}{\rho S c} \end{aligned} \quad (2)$$

In addition, the vertical force and moment are expressed in coefficient form in accordance with the usual conventions.

$$C_z = \frac{Z}{\frac{\rho}{2} V^2 S} \quad C_m = \frac{M}{\frac{\rho}{2} V^2 S c}$$

The equations then become

$$2\mu \left(D \frac{W}{V} - D\theta \right) = \alpha_w C_{z_{\alpha_w}} + \alpha_t C_{z_{\alpha_t}} + S_e C_{z_{\alpha_e}} \quad (3)$$

$$2\mu K_y^2 D^2 \theta = \alpha_w C_{m_{\alpha_w}} + \alpha_t C_{m_{\alpha_t}} + S_e C_{m_{\alpha_e}}$$

Note that in these equations the coefficients of tail moment and vertical force, $C_{m_{\alpha_t}}$, $C_{z_{\alpha_t}}$, and so forth, are based on wing area and wing chord.

The following approximations are made in calculating the angle of attack at the wing and the tail.

$$\alpha_w = \frac{W}{V} = \alpha \quad (4)$$

$$\alpha_t = \alpha - \alpha \frac{\partial \epsilon}{\partial \alpha} e^{-lD} + lD\theta$$

Furthermore, the lag operation e^{-lD} is expanded in a power series, and only the first-order term is retained.

$$e^{-lD} = 1 - lD$$

The expression for the angle of attack at the tail then becomes:

$$\alpha_t = \alpha - \alpha \frac{\partial \epsilon}{\partial \alpha} (1 - lD) + lD\theta \quad (5)$$

If the expressions (4) and (5) are substituted in (3) and coefficients of α , $D\alpha$, and so forth are collected, the equations may be written in the following form:

$$2\mu D(\alpha - \theta) - \alpha C_{Z\alpha} - \frac{1}{2} D\alpha C_{ZD\alpha} - \frac{1}{2} D\theta C_{Z\theta} = \delta_e C_{Z\delta_e} \quad (6)$$

$$2\mu K_y^2 D^2\theta - \alpha C_{m\alpha} - \frac{1}{2} D\alpha C_{mD\alpha} - \frac{1}{2} D\theta C_{m\theta} = \delta_e C_{m\delta_e}$$

where

$$C_{Z\alpha} = C_{Z\alpha_w} + (1 - \frac{\partial \epsilon}{\partial \alpha}) C_{Z\alpha_t} \quad C_{m\alpha} = C_{m\alpha_w} + (1 - \frac{\partial \epsilon}{\partial \alpha}) C_{m\alpha_t}$$

$$C_{ZD\alpha} = C_{ZD\alpha_w} + 2l \frac{\partial \epsilon}{\partial \alpha} C_{Z\alpha_t} \quad C_{mD\alpha} = C_{mD\alpha_w} + 2l \frac{\partial \epsilon}{\partial \alpha} C_{m\alpha_t} \quad (7)$$

$$C_{Z\theta} = C_{Z\theta_w} + 2l C_{Z\theta_t} \quad C_{m\theta} = C_{m\theta_w} + 2l C_{m\theta_t}$$

The stability derivatives of the basic airplane are considered to be composed of contributions from the wing-body combination and from the tail. This separation of effects makes it possible to estimate the derivatives for either a conventional or a tailless configuration. In the case of a tailless design, the effect of the tail contributions are omitted. Theories and empirical data such as those summarized in reference 8 may be used to estimate the values of the derivatives. With a conventional configuration, the tail contribution to the rotary derivatives may greatly exceed the contribution of the wing-body combination. In this case, the effect of the wing-body combination on the rotary derivatives may be estimated roughly or omitted. Formula (7) may be used, along with the expression $C_{m\alpha_t} = l C_{Z\alpha_t}$,

to estimate the effects of the tail. The effect of the elevator may be calculated from the equations:

$$C_{Z\delta_e} = T C_{Z\alpha_t} \quad C_{m\delta_e} = T C_{m\alpha_t}$$

Transfer Functions for Longitudinal Response

The transfer functions relating various non-dimensional response quantities to elevator motion may be found from formula (6). The transfer function for angle of attack, α , is:

$$\frac{\alpha}{s_e} = \frac{D(2\mu K_y^2 C_{z_{\delta e}}) + 2\mu C_{m_{\delta e}} + \frac{1}{2}C_{z_q} C_{m_{\delta e}} - \frac{1}{2}C_{m_q} C_{z_{\delta e}}}{\Delta_0} \quad (8)$$

The transfer function for pitching velocity, $D\theta$, is

$$\frac{D\theta}{s_e} = \frac{D(2\mu C_{m_{\delta e}} - \frac{1}{2}C_{z_{D\alpha}} C_{m_{\delta e}} + \frac{1}{2}C_{m_{D\alpha}} C_{z_{\delta e}}) + C_{z_{\delta e}} C_{m_{\alpha}} - C_{m_{\delta e}} C_{z_{\alpha}}}{\Delta_0} \quad (9)$$

The transfer function for non-dimensional normal acceleration, $D(\alpha - \theta)$ is

$$\frac{D(\alpha - \theta)}{s_e} = \frac{1}{\Delta_0} \left\{ D^2(2\mu K_y^2 C_{z_{\delta e}}) + D \left[C_{m_{\delta e}} \left(\frac{1}{2}C_{z_q} + \frac{1}{2}C_{z_{D\alpha}} \right) - C_{z_{\delta e}} \left(\frac{1}{2}C_{m_q} + \frac{1}{2}C_{m_{D\alpha}} \right) \right] - C_{z_{\delta e}} C_{m_{\alpha}} + C_{m_{\delta e}} C_{z_{\alpha}} \right\} \quad (10)$$

The characteristic determinant, Δ_0 , is

$$\begin{aligned} \Delta_0 = & D^2(2\mu - \frac{1}{2}C_{z_{D\alpha}}) 2\mu K_y^2 + D \left[-2\mu \left(\frac{1}{2}C_{m_q} + \frac{1}{2}C_{m_{D\alpha}} \right) \right. \\ & \left. + \frac{1}{4}C_{z_{D\alpha}} C_{m_q} - \frac{1}{4}C_{z_q} C_{m_{D\alpha}} - 2\mu K_y^2 C_{z_{\alpha}} \right] \\ & - 2\mu C_{m_{\alpha}} - \frac{1}{2}C_{z_q} C_{m_{\alpha}} + \frac{1}{2}C_{m_q} C_{z_{\alpha}} \end{aligned} \quad (11)$$

The non-dimensional vertical velocity and change in altitude may be obtained by integrating the non-dimensional normal acceleration $D(\alpha - \theta)$ once and twice respectively. Thus

$$\frac{w}{\delta e} = \frac{1}{D} \int \left[\frac{D(\alpha - \theta)}{\delta e} \right] \quad (12)$$

$$\frac{z}{\delta e} = \frac{1}{D^2} \int \left[\frac{D(\alpha - \theta)}{\delta e} \right]$$

The vertical motion of a point in the fuselage other than the center of gravity may be obtained by combining the effects of the vertical motion of the center of gravity and the pitching of the aircraft. For example, the non-dimensional normal acceleration at a point located at l_p chord-lengths ahead of the center of gravity is given by the expression:

$$\left[\frac{D(\alpha - \theta)}{\delta e} \right]_p = \frac{D(\alpha - \theta)}{\delta e} + l_p \frac{D^2 \theta}{\delta e} \quad (12a)$$

In order to obtain dimensional values of the response quantities, the following relations may be used:

$$\begin{aligned} q &= \frac{V}{c} \dot{\theta} \\ a_z &= - \frac{V^2}{c} D(\alpha - \theta) \\ \dot{h} &= -V w \\ h &= -c z \\ t &= \frac{c}{V} s \end{aligned} \quad (13)$$

Effect of Pitch Damper

Most modern airplanes have longitudinal stability augmentation in the form of pitch damping. A complete representation of such systems requires the consideration of instrument and servo dynamics, washout filters, structural filters, etc. As a first approximation, however, the effect of a pitch damper may be considered as a change in the values of the stability derivatives C_{Z_q} and C_{m_q} . The augmented value of these derivatives may be determined from the formulas

$$C_{Z_q} = C_{Z_{qA}} + K C_{Z_{\dot{\delta}_e}} \quad (14)$$

$$C_{m_q} = C_{m_{qA}} + K C_{m_{\dot{\delta}_e}}$$

where

$$K = \frac{d\delta_e}{dq/c/2V} = \frac{2V}{c} \left(\frac{d\delta_e}{dq} \right)$$

The use of pitch rate feedback affects only denominator terms in the transfer functions relating airplane response to control input. As shown by formula (9), the values of C_{m_q} and C_{Z_q} do not occur in the numerator of the transfer function for $D\theta/\delta$. These terms do occur in the numerator of the transfer function for α/δ , formula (8), and hence in $D(\alpha - \theta)/\delta_e$, formula 10. The combination of terms in which these quantities occur is:

$$\frac{1}{2} C_{Z_q} C_{m_{\dot{\delta}_e}} - \frac{1}{2} C_{m_q} C_{Z_{\dot{\delta}_e}}$$

If the modified derivatives given by (14) are substituted in this expression, the result is

$$\begin{aligned} & \frac{1}{2}(C_{zq_A} + K C_{z\delta_e}) C_{m\delta_e} - \frac{1}{2}(C_{mq_A} + K C_{m\delta_e}) C_{z\delta_e} \\ & = \frac{1}{2} C_{zq_A} C_{m\delta_e} - \frac{1}{2} C_{mq_A} C_{z\delta_e} \end{aligned}$$

This result shows that the effect of rate feedback disappears from the numerator terms. Physically, this result can be understood by realizing that the relation between airplane response and elevator motion is the same whether the elevator is moved by the pilot or by a feedback signal. All response quantities, such as pitch rate and normal acceleration, therefore, have exactly the same relative values whether or not feedback is used.

Calculation of Response to a Longitudinal Control Input

The response to an arbitrary longitudinal control input may be obtained by numerical integration of the equations of motion, or, in the case of simple inputs, by the use of Laplace transforms. In the present study, the response following an abrupt control motion is of interest. For this purpose, response to impulsive, step, or ramp inputs are examined. These responses may be obtained by the Laplace transform method. An application of the response to a ramp input is the initial motion which results if the elevator control encounters rate limiting. The effect of such rate limiting, however, is to nullify the effect of any pitching velocity feedback. The stability derivatives of the basic airplane should, therefore, be used under conditions of rate limiting.

Laplace transform tables usually present the transfer functions in a form with the coefficients of the highest power of the operator factored out. The transfer function of interest for the response of vertical position, Z , to control command takes the form

$$\frac{Z}{\delta_e} = K_Z \frac{D^2 + a_1 D + a_0}{D^2 (D^2 + b_1 D + b_0)} \quad (16)$$

Factoring out the coefficients of the higher powers of D in equations (10) and (11) gives for the constants in equation (16):

$$K_z = \frac{C_{zse}}{(2\mu - \frac{1}{2}C_{zDx})} \quad (17)$$

$$a_1 = \frac{C_{mse}(\frac{1}{2}C_{zq} + \frac{1}{2}C_{zDx}) - C_{zse}(\frac{1}{2}C_{mq} + \frac{1}{2}C_{mDx})}{2\mu K_y^2 C_{zse}} \quad (18)$$

$$a_0 = \frac{-C_{zse} C_{m\alpha} + C_{mse} C_{z\alpha}}{2\mu K_y^2 C_{zse}} \quad (19)$$

$$b_1 = \frac{-2\mu(\frac{1}{2}C_{mq} + \frac{1}{2}C_{m\alpha}) + \frac{1}{4}C_{Dx} C_{mq} - \frac{1}{4}C_{zq} C_{mDx} - 2\mu K_y^2 C_{z\alpha}}{2\mu K_y^2 (2\mu - \frac{1}{2}C_{zDx})} \quad (20)$$

$$b_0 = \frac{-2\mu C_{m\alpha} - \frac{1}{2}C_{zq} C_{m\alpha} + \frac{1}{2}C_{mq} C_{z\alpha}}{2\mu K_y^2 (2\mu - \frac{1}{2}C_{zDx})} \quad (21)$$

The denominator of equation (16) may have real or complex roots. In the case of real roots, equation (16) may be written

$$\frac{Z}{s_e} = K_Z \frac{D^2 + a_1 D + a_0}{D^2(D + \alpha)(D + \gamma)} \quad (22)$$

The inverse transform of this expression, which may be found, for example, in reference 9 (page 347, entry 2.249) is

$$Z = K_Z \left[\frac{\alpha^2 - a_1 \alpha + a_0}{\alpha^2(\gamma - \alpha)} e^{-\alpha t} + \frac{\gamma^2 - a_1 \gamma + a_0}{\gamma^2(\alpha - \gamma)} e^{-\gamma t} + \frac{a_0}{\alpha \gamma} t + \frac{a_1 \alpha \gamma - a_0(\alpha + \gamma)}{\alpha^2 \gamma^2} \right] \quad (23)$$

The expression gives the time response of Z to a unit impulsive longitudinal control input, because the inverse transform of an impulse is unity.

In case the denominator of equation (16) has complex roots, equation (16) may be written

$$\frac{Z}{s_e} = K_Z \frac{D^2 + a_1 D + a_0}{D^2[(D + \alpha)^2 + \beta^2]} \quad (24)$$

The inverse transform of this expression (ref. 10, pg. 237, entry 02.201) is

$$Z = \frac{K_Z}{U^2 \beta} \left\{ \left[(a_0 - a_1 \alpha + \alpha^2 - \beta^2)^2 + \beta^2 (a_1 - 2\alpha)^2 \right]^{\frac{1}{2}} e^{-\alpha t} \sin(\beta t - \gamma_1) + a_0 \beta t + \left(a_1 \beta - \frac{2\alpha a_0 \beta}{U^2} \right) \right\} \quad (25)$$

where

$$U^2 = \alpha^2 + \beta^2$$

$$\gamma_1 = 2 \tan^{-1} \frac{\beta}{\alpha} + \tan^{-1} \frac{\beta(2\alpha - a_1)}{a_0 - a_1 \alpha + \alpha^2 - \beta^2}$$

The response to a step input is obtained by multiplying the transfer function by the Laplace transform of a step, $1/D$, and taking the inverse transform of the result. Similarly, the response to a ramp is obtained by multiplying the transfer function by $1/D^2$ and taking the inverse transform of the result. The time response of a system to a step input is, therefore, the integral of the response to an impulse, and the response to a ramp input is the second integral of the response to an impulse. The expressions for these transforms are of too high order to be found in most tables of Laplace transforms. As a result, the inverse transforms must be obtained from basic principles. In these cases, the simplest procedure for obtaining these time responses appears to be to take the first and second integrals, respectively, of the expressions for the time response to an impulse.

For example, in the case of real roots, the expression for the response of the vertical position to an impulsive longitudinal input, equation (23), has the form

$$Z = K_Z (A_1 e^{-\alpha t} + A_2 e^{-\gamma t} + A_3 t + A_4) \quad (26)$$

The response to a step longitudinal input, obtained by integrating equation (26), is

$$Z = K_Z \left(-\frac{A_1}{\alpha} e^{-\alpha t} - \frac{A_2}{\gamma} e^{-\gamma t} + \frac{A_3}{2} t^2 + A_4 t + C_1 \right) \quad (27)$$

The constant of integration, C_1 , is obtained from the condition that $Z = 0$ when $t = 0$. Hence,

$$C_1 = \frac{A_1}{\alpha} + \frac{A_2}{\gamma}$$

In the same manner, equation (27) may be integrated to obtain the response of vertical position to a ramp command. The result is:

$$Z = K_Z \left(\frac{A_1}{\alpha^2} e^{-\alpha t} + \frac{A_2}{\gamma^2} e^{-\gamma t} + \frac{A_3}{6} t^3 + \frac{A_4}{2} t^2 + C_1 t + C_2 \right) \quad (28)$$

Where

$$C_2 = -\frac{A_1}{\alpha^2} - \frac{A_2}{\gamma^2}$$

In the case of complex roots, the equation for the response to an impulse, equation 25, has the form

$$Z = K_Z \left[-B_2 e^{-\alpha t} \sin(\beta t - \psi) + B_1 t + B_0 \right] \quad (29)$$

The response to a step longitudinal input is:

$$Z = K_Z \left[B_2 \frac{e^{-\alpha t}}{\alpha^2 + \beta^2} (\alpha \sin(\beta t - \psi) + \beta \cos(\beta t - \psi)) + \frac{B_1 t^2}{2} + B_0 t + C_3 \right] \quad (30)$$

The response to a ramp is:

$$Z = K_Z \left\{ \frac{-B_2}{(\alpha^2 + \beta^2)^2} e^{-\alpha t} \left[(\alpha^2 - \beta^2) \sin(\beta t - \psi) + 2\alpha\beta \cos(\beta t - \psi) \right] + \frac{B_1 t^3}{6} + \frac{B_0 t^2}{2} + C_3 t + C_4 \right\} \quad (31)$$

The expressions for C_3 and C_4 are not written out because their values may be more readily obtained numerically in an actual example.

The equations for the response in pitching velocity are as follows. As shown by equation (9), the transfer function has the form

$$\frac{D\theta}{s_t} = K_{D\theta} \left(\frac{D + a_0 D}{D^2 + b_1 D + b_0} \right) \quad (32)$$

where

$$K_{D\theta} = \frac{2\mu C_{m_{se}} - \frac{1}{2} C_{z_{D\alpha}} C_{m_{se}} + \frac{1}{2} C_{m_{D\alpha}} C_{z_{se}}}{2\mu K_y^2 (2\mu - \frac{1}{2} C_{z_{D\alpha}})} \quad (33)$$

and

$$a_{D\theta} = \frac{C_{z_{se}} C_{m_{\alpha}} - C_{m_{se}} C_{z_{\alpha}}}{2\mu C_{m_{se}} - \frac{1}{2} C_{z_{D\alpha}} C_{m_{se}} + \frac{1}{2} C_{m_{D\alpha}} C_{z_{se}}} \quad (34)$$

In the case that the denominator has real roots, equation (32) may be written

$$\frac{D\theta}{s_e} = K_{D\theta} \left(\frac{D + a_{D\theta}}{(D + \alpha)(D + \gamma)} \right) \quad (35)$$

The inverse transform of this expression, which may be found, for example, in reference 9 (pg. 338, entry 1.107) is:

$$D\theta = K_{D\theta} \left[\frac{(a_{D\theta} - \alpha) e^{-\alpha t} - (a_{D\theta} - \gamma) e^{-\gamma t}}{\gamma - \alpha} \right] \quad (36)$$

This expression gives the response of $D\theta$ to an impulsive control input. Again, the response to a step and to a ramp control input may be obtained by integrating this expression. The response to a step is

$$D\theta = K_{D\theta} \left[-\frac{1}{\alpha} \left(\frac{a_{D\theta} - \alpha}{\gamma - \alpha} \right) e^{-\alpha t} + \frac{1}{\gamma} \left(\frac{a_{D\theta} - \gamma}{\gamma - \alpha} \right) e^{-\gamma t} + \frac{a_{D\theta}}{\gamma \alpha} \right] \quad (37)$$

The response to a ramp is

$$D\theta = K_{DE} \left[\frac{1}{\alpha^2} \left(\frac{a_{0D} - \alpha}{\gamma - \alpha} \right) e^{-\alpha t} - \frac{1}{\gamma^2} \left(\frac{a_{0D} - \gamma}{\gamma - \alpha} \right) + \frac{a_{0D}}{\gamma \alpha} t + C_5 \right] \quad (38)$$

If pitching acceleration is wanted, it may be obtained by differentiating these equations.

In the case the denominator has complex roots equation (29) may be written

$$\frac{D\theta}{s_e} = K_{DE} \left[\frac{D + a_{0D}}{(D + \alpha)^2 + \beta^2} \right] \quad (39)$$

The inverse transform of this expression is (ref. 9, pg. 342, entry 1.303)

$$D\theta = K_{DE} \left\{ \frac{1}{\beta} \left[(a_{0D} - \alpha)^2 + \beta^2 \right]^{\frac{1}{2}} e^{-\alpha t} \sin(\beta t + \psi_2) \right\} \quad (40)$$

where

$$\psi_2 = \tan^{-1} \frac{\beta}{a_{0D} - \alpha}$$

Again, the response to a step or ramp motion of the control may be obtained by integrating these expressions. The response to a step is:

$$D\theta = K_{DE} \left\{ \frac{1}{\beta} \left[(a_{0D} - \alpha)^2 + \beta^2 \right]^{\frac{1}{2}} \frac{e^{-\alpha t}}{(\alpha^2 + \beta^2)^2} \left[-\alpha \sin(\beta t + \psi_2) - \beta \cos(\beta t + \psi_2) \right] + \frac{a_{0D}}{\alpha^2 + \beta^2} \right\} \quad (41)$$

The response to a ramp is:

$$D\theta = K_{D\theta} \left\{ \frac{1}{\beta} [(\alpha_0 - \alpha)^2 + \beta^2]^{1/2} \frac{e^{-\alpha t}}{(\alpha^2 + \beta^2)^2} \left[(\alpha^2 - \beta^2) \sin(\beta t + \psi) \right. \right. \\ \left. \left. + 2\alpha\beta \cos(\beta t + \psi) \right] + \frac{\alpha_0}{\alpha^2 + \beta^2} t + C_6 \right\} \quad (42)$$

Examples Used in Calculations

Sketches of the airplanes used in the analysis are given in figure 1. The dimensions, weights, inertias, and longitudinal stability derivatives are given in table I. These derivatives were obtained partly from existing reports and partly from estimation by methods given in reference 8 or by use of equations (2) presented previously. All data for airplane 2 and part of the data for airplane 4 were obtained from reference 11. In any case, the data should not be considered as design values approved by the manufacturer, but merely as rough estimates adequate for the type of comparisons made in this report.

In all cases, the stability derivatives are considered to be constant, independent of angle of attack, lift coefficient, or dynamic pressure. In addition, ground effects on the derivatives are neglected. As a result of these assumptions, the flight path through the atmosphere following a disturbance is independent of airspeed, dynamic pressure, or ground proximity. Changes in time histories of plotted quantities at different values of lift coefficient are simply the result of the change in time scale as a function of airspeed.

RESULTS AND DISCUSSION

Simplifications Used in Analysis

In presentation of the calculated data, the simplifying assumptions used should be kept in mind. In an actual airplane, true impulsive, step or ramp inputs cannot be obtained because of lags and rate limiting in the control system and control actuators. For a more realistic representation of an actual control system a complete digital or analog simulation would be required. The data presented, however, can be used to survey some of the fundamental differences in response resulting from airplane configuration and from the other variables studied.

Presentation of Calculated Results

Response of the Shuttle to impulsive, step, and ramp inputs of the elevator are shown in figure 2(a). Similar results with stability augmentation in the form of rate damping are shown in figure 2(b). In order to obtain a basis of comparison of the magnitude of the three types of inputs, the impulse was assumed to have a magnitude of 1 radian second, or the area under the impulse is 1 radian times 1 second. The step input has a magnitude of 1 radian, and the ramp input a slope of 1 radian per second. As a result, the area under the curve of elevator angle versus time for the step input equals that under the impulse after 1 second, and the area under this curve for the ramp input equals that under the impulse after 2 seconds. These inputs have a very large magnitude, greater than that attainable in practice. Because of linearity of the calculated responses, however, the curves may readily be scaled down to correspond to inputs of any desired magnitude, such as, for example, 0.1 or 0.01 times those plotted.

The curves of figure 2 and subsequent figures are plotted in dimensional form for a lift coefficient typical of that used in the landing approach prior to flare. In the case of the Shuttle, this lift coefficient was taken as 0.6. The results for a pull-up of the Shuttle show a marked downward movement of the center of gravity prior to the subsequent response in the upward direction. This initial reversal of response occurs whether the input has the form of an impulse, step, or ramp. The main difference between these three types of inputs is a progressively increasing lag in obtaining the desired altitude responses in going from the impulse to the ramp.

The effect of stability augmentation in the form of rate damping is to reduce the magnitude of the initial reversed response and to reduce slightly the time to obtain the desired response. The rate feedback also reduces the steady-state pitching velocity resulting from a step command. In practice, if rate feedback were used, the sensitivity of the controller might be increased to retain the same steady-state response. In this case, the main effect of stability augmentation on the initial altitude response would be a slight decrease in the time required for the altitude to start to respond in the desired direction.

The response of airplane 1, a heavy bomber airplane of conventional configuration, is shown in figure 3. The most obvious difference between this case and the previous one is the great reduction in the initial reversed response of the motion of the center of gravity.

The altitude responses of the center of gravity for a step elevator (or elevon) motion for all of the airplanes studied are compared in figure 4. No stability augmentation is assumed. The two airplanes of conventional configuration, airplanes 1 and 2, show very little reversed response at the start of the motion. The three delta-wing airplanes show reversed responses, but that of the Shuttle is much greater than those of the other two. The reason for this difference is discussed subsequently.

A more detailed survey of the effects of stability augmentation on the Shuttle is given in figure 5. The pitching velocity feedback is varied from 0 to 1.241 degrees elevon per degree per second pitching velocity. This value is comparable to the feedback gain (GDQ in Shuttle nomenclature) used in the actual Shuttle, though the stability augmentation system of the Shuttle contains additional features not simulated herein. As shown by this figure, the pitching velocity feedback causes the elevon angle to decrease following the initial step input. As a result, the response is more nearly similar to that for an impulsive input, and the time for the correct altitude response of the center of gravity is slightly reduced.

The effect of lift coefficient on the altitude response of the Shuttle and of airplane 1 are shown in figure 6. As mentioned previously, the effects of lift coefficient are entirely the result of the change in time scale. In other words, the airplane flying more slowly takes longer to traverse the same path.

The effects of shifting the center of gravity rearward from its assumed initial location are shown in figure 7 for the Shuttle and for airplane 2, both without stability augmentation. Larger increments of center of gravity movement are shown for airplane 2 because this airplane had more static stability to begin with. In either case, the effects of center of gravity position on the response during the first few seconds are practically negligible. This result is true even though the Shuttle was longitudinally unstable (center of gravity aft of the maneuver point) for the rearmost center of gravity location used.

A comparison of the vertical motion at the cockpit location with that at the center of gravity for three airplanes is shown in figure 8. The effect of vertical location of the cockpit was neglected in making these calculations. In the case of the Shuttle, the instantaneous center of rotation following an impulsive elevon motion is somewhat ahead of the cockpit. The center of rotation is estimated to be 55.7 feet ahead of the center of gravity, whereas the cockpit is 49.5 feet ahead of the center of gravity. Despite this difference, the vertical motion of the cockpit for the first 0.7 second of the motion is practically negligible, probably because the response to lift due to angle of attack of the wing is starting to take effect during this period. The difference in vertical motion between the cockpit and the center of gravity resulting from the pitch angle of the airplane is shown as a dashed line.

In the case of airplane 3, which is also a delta-wing configuration, the instantaneous center of rotation is aft of the cockpit and the cockpit responds in the correct direction from the start of the motion. The same result is shown for airplane 1, a conventional airplane configuration. In all three cases, the effect of the cockpit location ahead of the center of gravity is to reduce the lag in reaching a given change in altitude by about 0.5 second.

REASONS FOR DIFFERENCES IN TRANSIENT RESPONSE

As shown by the preceding calculations, the airplanes with conventional configuration exhibit a relatively small initially reversed motion of the center of gravity compared to the delta-wing configuration, and of these, the Shuttle has considerably larger reversed response. As noted in reference 2, one factor influencing this behavior is the position of the instantaneous center of rotation of the aircraft following an abrupt elevator movement. If this center of rotation is far ahead of the center of gravity, initial reversed motion of the center of gravity is increased.

The position of the instantaneous center of rotation may be obtained from formula 12a, which gives the transfer function for vertical acceleration at any point along the fuselage. The initial acceleration, as shown by the initial value theorem (ref. 9, pg. 267) may be obtained by considering only the terms with the highest powers of the operator D in the transfer function. If these terms are substituted in formula 12a and the expression equated to zero, the distance in chord lengths l_x to the instantaneous center of rotation may be obtained. The result is

$$l_x = \frac{-2\mu K_y^2 C_{Z\delta e}}{2\mu C_{m\delta e} - \frac{1}{2} C_{ZD\alpha} C_{m\delta e} + \frac{1}{2} C_{mD\alpha} C_{Z\delta e}}$$

In the last two terms of the denominator of this equation, the values $C_{ZD\alpha}$ and $C_{mD\alpha}$ are frequently assumed equal to zero for delta-wing configurations.

This assumption is perhaps made for lack of better information. In the case of a conventional configuration, the combination of terms $\frac{1}{2} C_{ZD\alpha} C_{m\delta e} + \frac{1}{2} C_{mD\alpha} C_{Z\delta e}$ may be shown by substituting values from formula (7) to go to zero if only the tail contributions to the derivatives are considered.

In either case, then, these terms may be neglected. The expression for l_x then reduces to

$$l_x = - \frac{K_y^2 C_{z\delta e}}{C_{m\delta e}}$$

The values of l_x for the airplanes studied are listed in table I. The distance in meters (ft) from the center of gravity to the cockpit and to the instantaneous center of rotation for these airplanes are given in table II.

If the values for the variables from table I are examined, the ratio $C_{z\delta e} / C_{m\delta e}$ is seen to be quite similar for the three delta-wing configurations. The value of K_y^2 , however, is much greater for the Shuttle. The greater radius of gyration in pitch is therefore an important reason for the increased reversed response of the Shuttle. This difference in inertia characteristics probably results from the unusual design layout of the Shuttle, which incorporates a large central cargo bay with most of the vehicle equipment in the nose and tail sections. The cargo bay is mostly empty in the loading condition considered.

The instantaneous center of rotation determines the initial motion of the airplane, but the subsequent response cannot be expressed so simply. Nevertheless, a large radius of gyration in pitch would be expected to reduce the rate of increase of angle of attack following a control deflection and, therefore, delay the buildup of lift in the desired direction.

DISCUSSION OF FLIGHT EXPERIENCE

As mentioned previously, airplanes 1-3 have been used in power-off landing studies at the NASA Dryden Flight Research Center to study the power-off landing problems of airplanes such as the Shuttle and NASA research airplanes. In no case did these airplanes experience difficulties in obtaining the desired precision of touchdown conditions. Airplane 4 has not been tested in the power-off condition, but has been used in service with no complaints about its longitudinal characteristics in landing. In the case of the Shuttle, however, the control of altitude during the approach and flare has been described as lacking in predictability. A pilot-induced oscillation was experienced in the fifth landing of the approach and landing tests when an attempt was made to touch down at a specific point on the runway. Pilot-induced oscillations have also been experienced in flight simulation studies of a variable-stability airplane with characteristics adjusted to represent the Shuttle (ref. 12).

The Shuttle control system in the approach and landing tests had other response characteristics which might adversely affect the precision of control. These characteristics include a time delay between controller motion and elevon response which varied from 0.3 to 0.5 sec, and rate limiting of the elevons to 20 degrees per second. Thus, the control difficulties experienced cannot necessarily be attributed to the delay in altitude response caused by the lift due to elevon deflection. In unpublished simulation studies improvements in all three factors have been shown to be required to obtain desirable control characteristics.

The presence of these additional adverse factors in the actual Shuttle control system prevents the use of the calculated response of the various airplanes (fig. 4) to devise any simple criterion for allowable magnitude of the reversed response in altitude. Obviously, the path response of the Shuttle exhibits a more severe reversed response than any of the other airplanes studied. In the case of the Shuttle, analytical studies of the control characteristics using a pilot model, or manned simulation studies of the complete system using ground-based or flight simulators have proved promising in analyzing the cause of the difficulties and suggesting improvements. Such studies are beyond the scope of the present report. Some comments are offered, however, on the relative importance to the pilot of motion of the cockpit and motion of the center of gravity.

The primary objective of a satisfactory landing is to touch down on the main landing gear with a desirably low sink rate. Inasmuch as the main landing gear is usually at about the same horizontal location as the center of gravity, the vertical motion of the center of gravity can be considered as representative of that of the main landing gear.

In some fixed-base simulation studies, a question has arisen as to whether the pilot should be given a simplified visual display representing the altitude of the cockpit or the altitude of the center of gravity. In cases in which the response has considerable lag, the reduction in lag associated with the forward location of the cockpit reduces the tendency for pilot-induced oscillations. Inasmuch as the pilot in the actual airplane is in a position to sense the vertical motion of the cockpit, the argument has been advanced that the simulation should represent the motion of the cockpit. In effect, this viewpoint holds that if the pilot can control the cockpit along a smooth flare path to the desired position at touchdown, the rest of the airplane will follow along behind and the wheels will touch down with a low sink rate.

In actual flight, of course, the pilot can sense both normal acceleration and pitching acceleration, and the resulting velocities and positions in altitude and pitch. For this reason, the pilot can learn to sense the position of the wheels with respect to the ground whether he is in the nose of the airplane or in some other location. It is known that some fighter or racing type airplanes have been controlled successfully from a cockpit aft of the center of gravity. Thus, the position of the cockpit in the nose of an airplane such as the Shuttle may not alleviate all the control problems associated with lag and reversal of response at the center of gravity. If any control

corrections are required, the motion of the wheels will initially be incorrect, resulting in possibly increased vertical velocity at touchdown. The capability of the pilot to avoid pilot-induced oscillations may be alleviated by his ability to sense pitching acceleration, which is the same regardless of his location in the airplane. These arguments indicate that further research is required to determine the effect of pilot location on his ability to make satisfactory landings. In analytical and simulation studies of landing control problems, every effort should be made to include the effects of all cues which can be sensed by the pilot. The data of the present report may aid in evaluating the results of such studies and in comparing the results of tests on different types of airplanes.

CONCLUDING REMARKS

Results of calculations are presented showing the response to abrupt longitudinal control motion of the Shuttle and of four other airplanes, three of which have been used in power-off landing experiments at the Dryden Flight Research Center. The effects of airplane type, pitch damping, center-of-gravity location, lift coefficient, and position of the cockpit are presented. These results are given mainly to provide a survey of response characteristics for different types of airplanes, including conventional and delta-wing configurations.

Airplanes with the control surface aft of the center of gravity exhibit an initially reversed response in altitude caused by the lift due to control deflections. The airplanes with conventional configuration exhibit a relatively small initially reversed motion of the center of gravity compared to the delta-wing configurations, and of these the Shuttle has considerably larger reversed response. Because of the presence of unusually large effects of time lag and elevon rate limiting in the Shuttle control system, however, control difficulties experienced by the Shuttle cannot necessarily be attributed to the delay in altitude response caused by the lift due to elevon deflection.

REFERENCES

1. Adamson, D.: Effect of Aircraft Size on the Difficulties Involved in Landing. RAE Report No. Aero 2202, 1947 (also ARC R&M 2567).
2. Pinsker, W. J. G.: Features of Large Transport Aircraft Affecting Control During Approach and Landing. AGARD Report 421, January 1963.
3. Flora, C. C.: Dynamic Motions of Aircraft--Survey and Introduction. AGARD Flight Mechanics Panel Specialists Meeting--Stability and Control. Cambridge, England, September 20-23, 1966.
4. Pinsker, W. J. G.: The Landing Flare of Large Transport Aircraft. ARC R&M No. 3602. 1969.
5. Pinsker, W. J. G.: The Control Characteristics of Aircraft Employing Direct Lift Control. RAE Technical Report 68140, May 1968.
6. Etkin, Bernard: Dynamics of Flight. John Wiley & Sons, Inc., New York, Third Printing, March 1963.
7. Phillips, W. H.; and Kraft, C. C., Jr.: Theoretical Study of Some Methods for Increasing the Smoothness of Flight Through Rough Air. NACA TN 2416 July 1951.
8. Hoak, E. E.; et.al.: USAF Stability and Control DATCOM, Flight Control Division, Air Force Flight Dynamics Lab., WPAFB, Ohio, Sept. 1970.
9. Gardner, M. F.; and Barnes, J. L.: Transients in Linear Systems. John Wiley & Sons, Inc., New York, Second Printing, December 1945.
10. Nixon, Floyd E.: Handbook of Laplace Transformation, Fundamentals, Applications, Tables, and Examples. Second Edition. Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1965.
11. Teper, Gary L.: Aircraft Stability and Control Data. STI Technical Report 176-1. Systems Technology, Inc., Hawthorne, CA, April 1969.
12. Weingarten, Norman C.: In-flight Simulation of the Space Shuttle Orbiter During Landing Approach and Touchdown in the Total In-flight Simulator (TIFS). Calspan Report No. 6339-F-1, Spetember 1978.

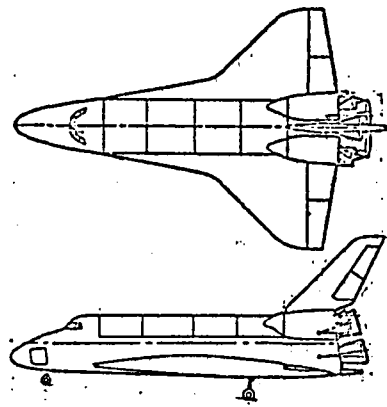
TABLE I
CHARACTERISTICS OF AIRPLANES USED AS EXAMPLES

Airplane	Shuttle	1	2	3	4
m, kg (slug)	82309 (5640)	1.183×10^5 (8103)	10885 (745.9)	34602 (2371)	68037 (4662)
ρ , kg/m ³ (slug/ft ³)	1.139 (.00221)	1.139 (.00221)	1.139 (.00221)	1.139 (.00221)	1.139 (.00221)
S, m ² (ft ²)	249.9 (2690)	371.6 (4000)	18.22 (196.1)	149.1 (1605)	143.2 (1542)
c, m (ft)	12.06 (39.57)	6.995 (22.95)	2.904 (9.53)	11.49 (37.70)	11.03 (36.20)
I_y , kgm ² (slug ft ²)	8.729×10^6 (6.438×10^6)	1.302×10^7 (9.60×10^6)	8.813×10^4 (6.50×10^4)	1.600×10^6 (1.18×10^6)	1.417×10^6 (1.045×10^6)
K_y	0.8539	1.500	0.9795	0.5917	0.4136
μ	23.97	39.94	180.6	17.73	37.79
C_{Z_α} , per rad	-2.70	-3.75	-4.44	-3.15	-2.84
C_{m_α} , per rad	-0.029	-0.187	-1.496	-0.089	-0.142
C_{Z_q} , per rad	0.0	-3.66	0.0	0.0	-1.80
C_{m_q} , per rad	-2.778	-10.76	-5.61	-1.40	-5.91
$C_{Z_{D\alpha}}$, per rad	0.0	-1.28	0.0	0.0	-1.41
$C_{m_{D\alpha}}$, per rad	0.0	-3.77	-3.44	0.0	0.0
$C_{Z_{\delta e}}$, per rad	-0.956	-0.249	-0.762	-0.630	-1.136
$C_{m_{\delta e}}$, per rad	-0.495	-0.732	-1.50	-0.351	-0.437
l_x	1.41	0.765	0.487	0.628	0.445

TABLE II

COMPARISON OF DISTANCE FROM CENTER OF GRAVITY TO COCKPIT WITH
DISTANCE FROM CENTER OF GRAVITY TO INSTANTANEOUS CENTER OF ROTATION

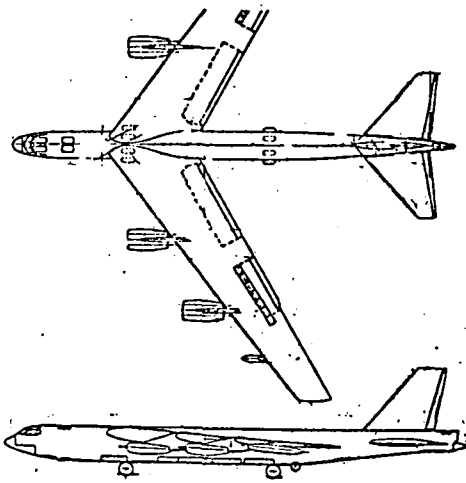
Airplane	c.g. to Cockpit m (ft)	c.g. to Center of Rotation m (ft)
Shuttle	15.1 (49.5)	17.0 (55.7)
1	17.7 (58.1)	5.4 (17.6)
2	6.3 (20.6)	1.4 (4.6)
3	15.2 (49.8)	7.2 (23.7)
4	11.6 (38.2)	4.9 (16.1)



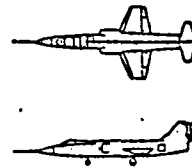
Shuttle

Scale, m 0 20 40

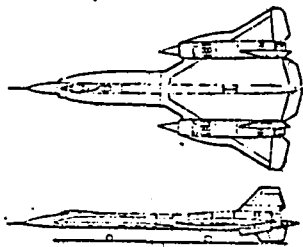
Scale, ft 0 50 100



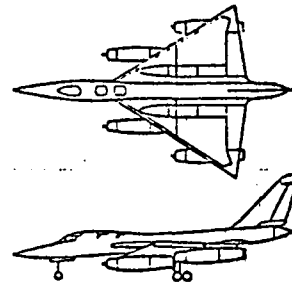
Airplane 1



Airplane 2

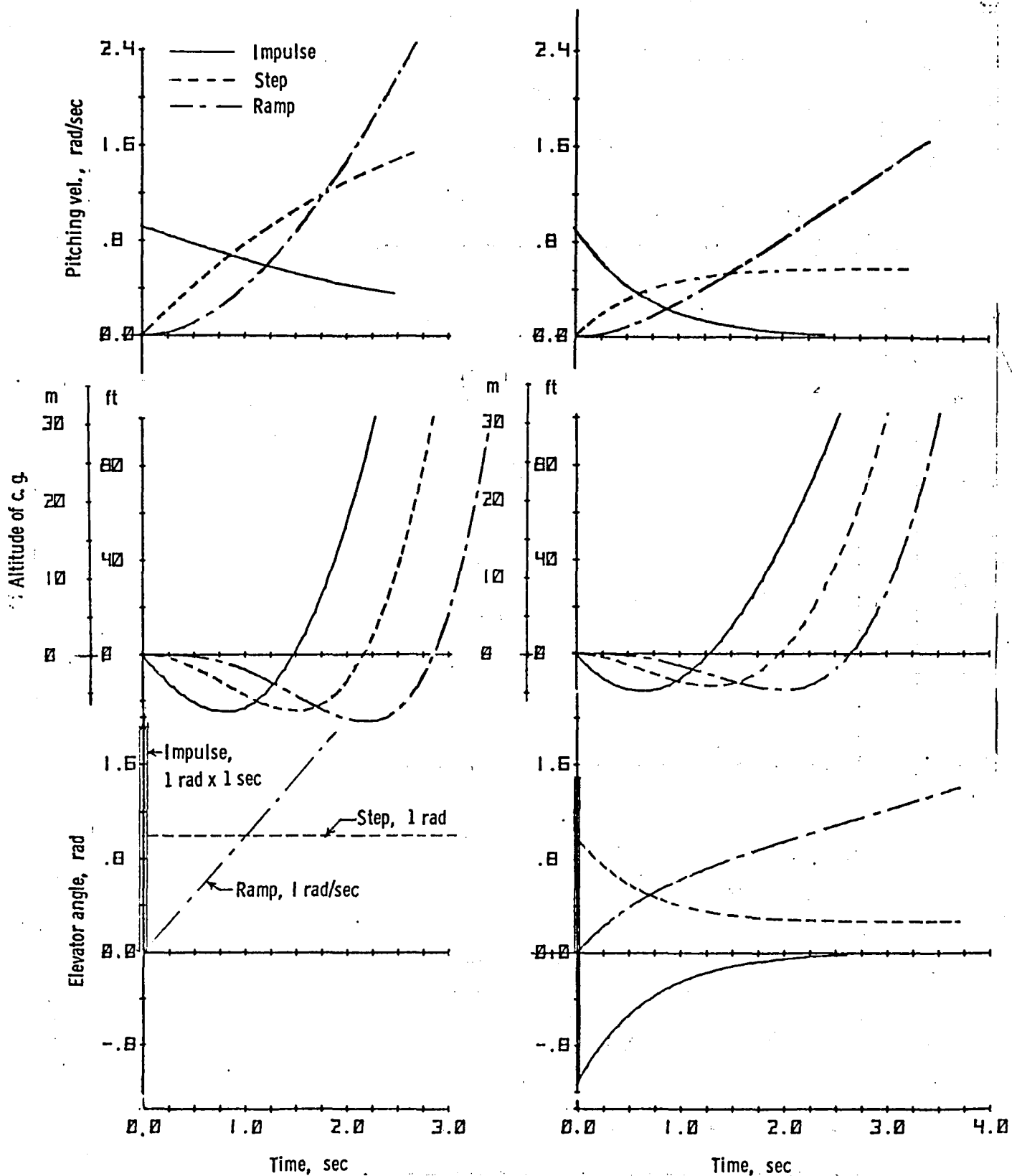


Airplane 3



Airplane 4

Figure 1.- Sketches of airplanes used as examples, drawn to same scale.



(a).- Unaugmented.

(b).- Augmented, $d\delta_e/dq = 1.24$

Figure 2.- Response of Shuttle to impulse, step, and ramp inputs. $C_L = 0.6$.

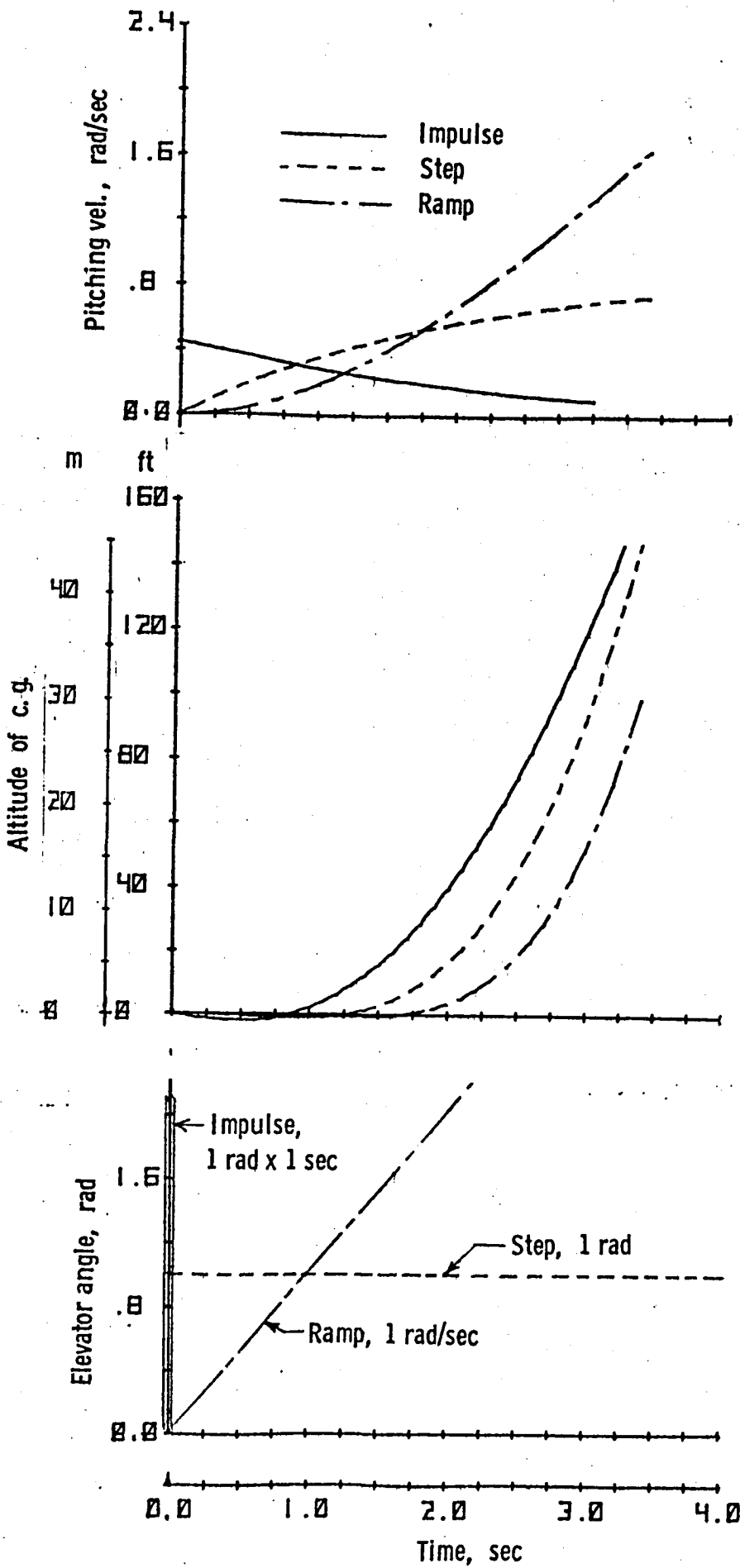


Figure 3.- Response of airplane 1 to impulse, step, and ramp inputs. $C_L = 1.0$, unaugmented.

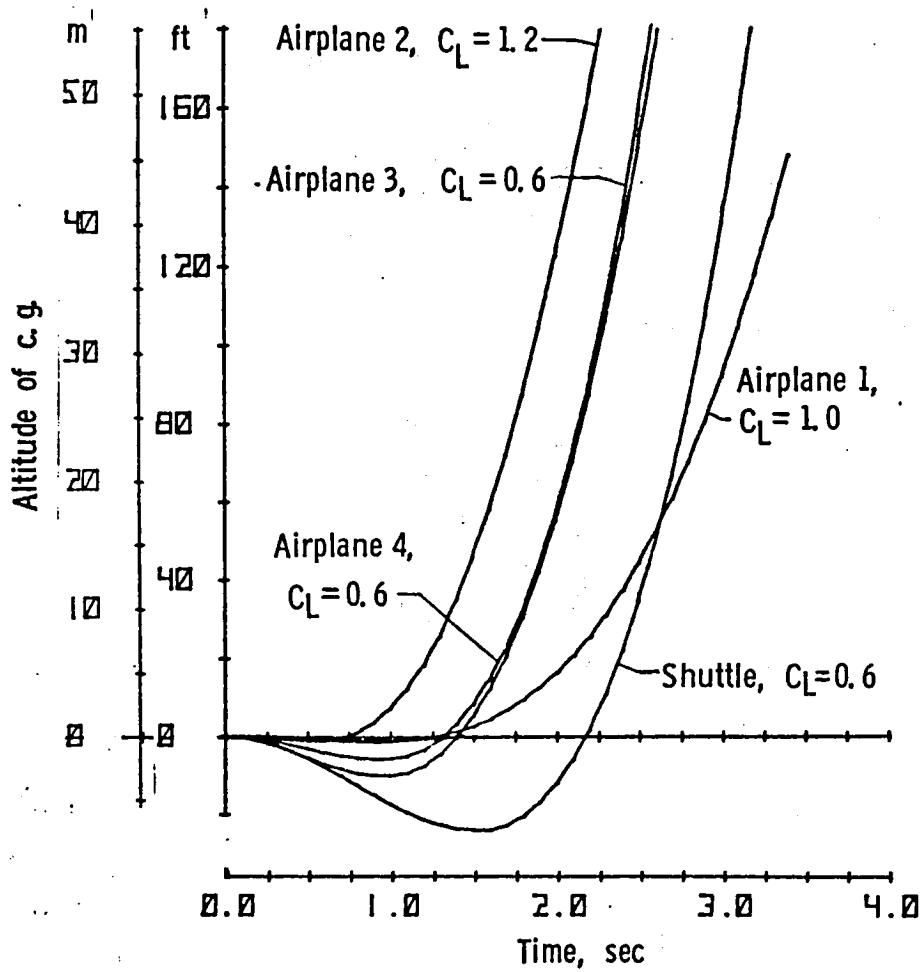


Figure 4.- Comparison of vertical motion of the center of gravity following a step elevator input of 1 radian for all airplanes studied. No stability augmentation.

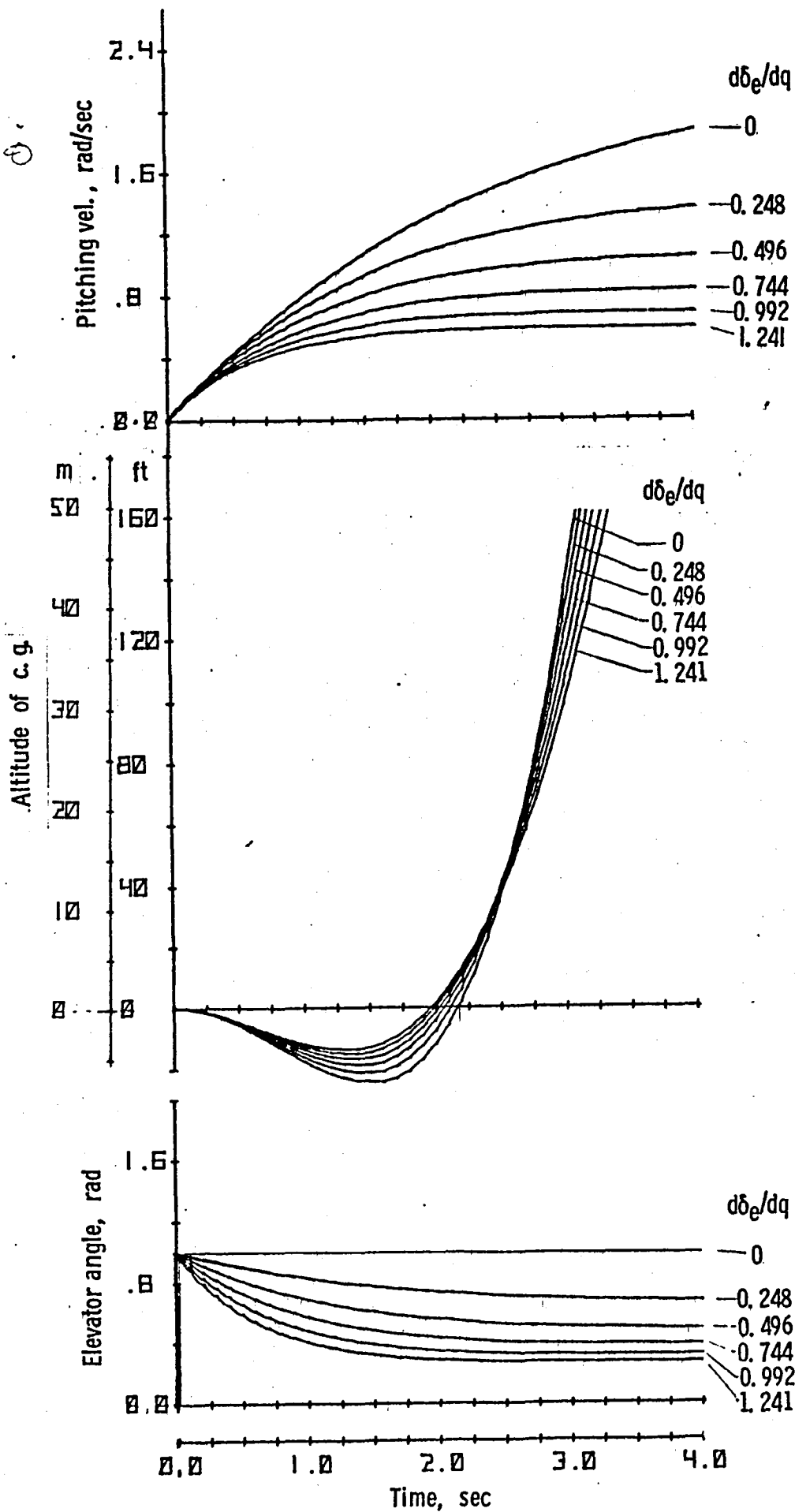


Figure 5.- Effect of varying pitching velocity feedback on response of Shuttle to a step control input of 1 radian. $C_L = 0.6$.

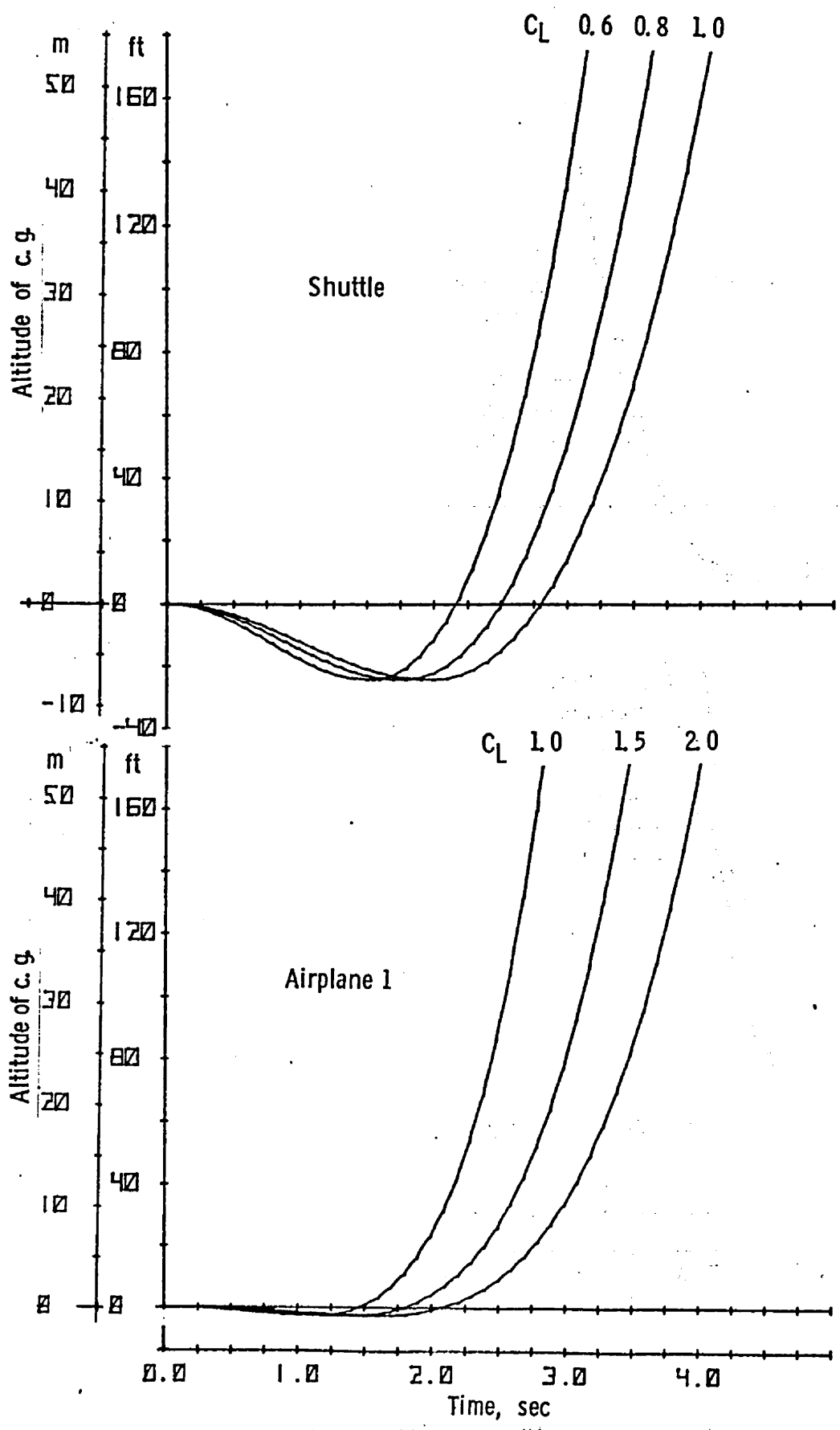


Figure 6.- Effect of lift coefficient on the altitude response to a step elevator input of 1 radian for two airplanes.

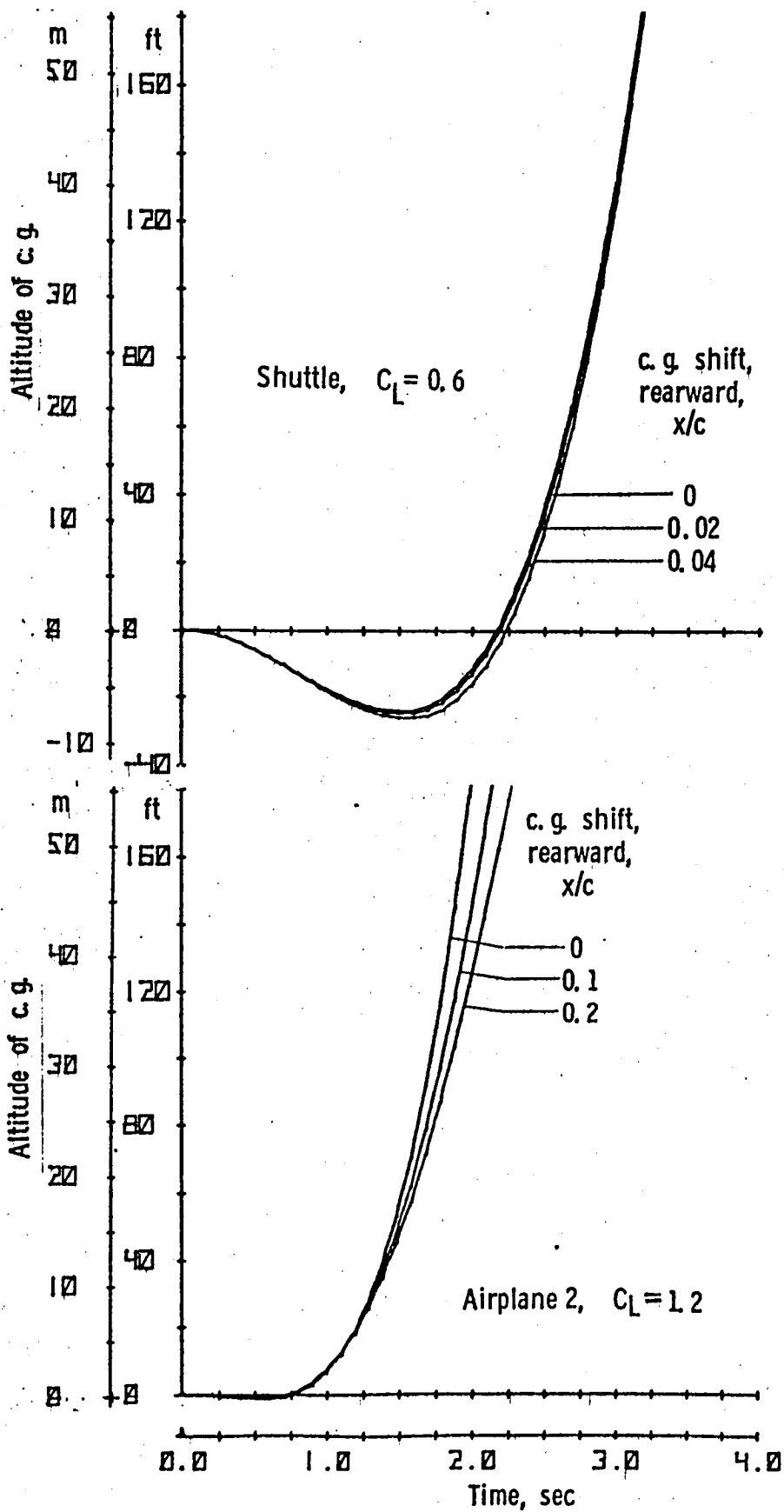


Figure 7.- Effect of shift of center of gravity on altitude response to a step elevator input of 1 radian for two airplanes.

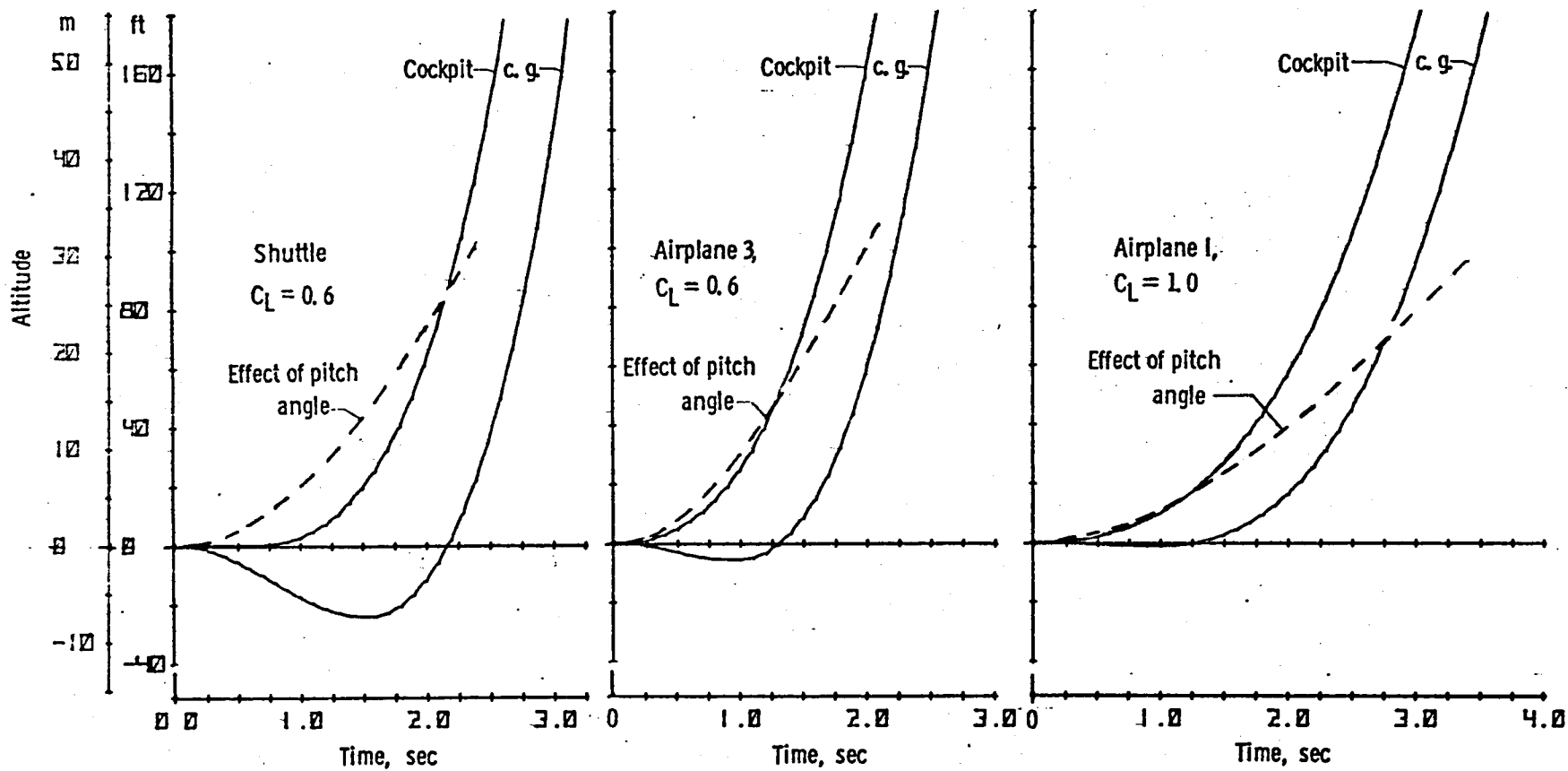
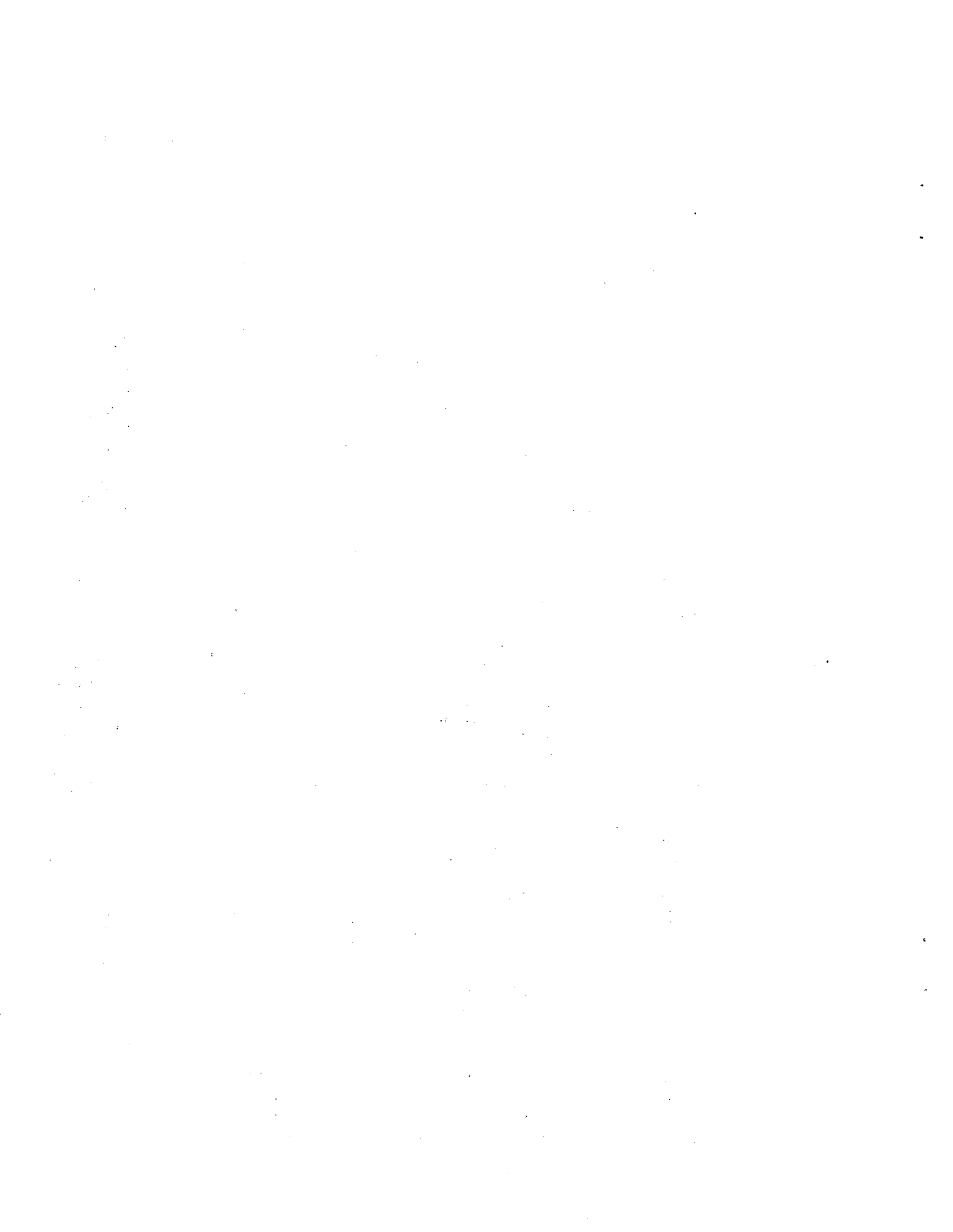


Figure 8.- Comparison of altitude response at cockpit to altitude response at c.g. following a step input of the elevator for three airplanes. The effect of pitch angle on the difference in altitude between the cockpit and c.g. is also shown.



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