GRAPHIC CONSTRUCTION OF JOUKOWSKI WINGS.*

By E. Trefftz.

In plotting the cross-sectional outline (or profile) of a Joukowski wing, we proceed as follows (Fig. 6).

We first plot an xy system of coordinates with the origin O such that the x axis forms the angle β with the horizontal direction of the wing and mark on the x axis the point L, for which $x = -l$, and on the y axis the point F, for which $y = f$.

We now describe two circles and label them K_1 and K_2. The center M_1 of the first circle is situated on the straight line LF at a distance 2δ from the point F (beyond the section LF). The circle, moreover, passes through the point L. The second circle likewise passes through the point L and its center M_2 is likewise on LF, the position of M on LF being determined by the following condition. If OV_1 is the portion of the positive x axis cut off by the circle K_1 and OV_2 the portion cut off by the circle K_2, then $OV_1 \times OV_2 = l^2$.

We now draw, from the point O, the two lines OA_1 and OA_2, so as to form equal angles with the x axis, A_1 being the point of intersection of the first line with the circle K_1 and A_2 the intersection of the second line with the circle

* From "Zeitschrift für Flugtechnik und Motorluftschiffahrt," May 31, 1913, pp. 130 and 131.

K_2. Then the center P of the line $A_1 A_2$ is the point sought on the Joukowski wing profile.

In plotting the preceding figures, 24 points were found in this manner for each one, by shifting the first line from the point L 15° each time and drawing the second line symmetrically with reference to the x axis.

In order to determine the pressure on each point of the profile, when the wing is exposed to a horizontal wind having the velocity V, we must know the velocity q at which the air flows by each point of the profile. The pressure on each unit area of the wing surface is then proportional to q^2.

We can now find the values of q in a very simple manner. For this purpose, we draw a horizontal line through the point L. If we designate by h the distance of the point A_1 (of the circle K_1) from this horizontal line, we obtain, for any desired point P of the figure, the corresponding value of q in the following manner. We take from the diagram the distance between the points A_1 and A_2, at the middle of which we had found the point P, and also the distances of the point A_1 from the origin O, from the center M_1 of the circle K_1 and from the horizontal line passing through L. We then have

$$q = V \frac{O\,A_1}{A_1\,A_2} \quad \frac{2\,h}{M_1\,A_1}$$

The mathematical proof for the given constructions is simple. As already mentioned, the profile of a Joukowski wing can be constructed by describing on the z plane, with the aid of the formula $z = \zeta + \frac{l^2}{4\zeta}$, the circle K, determined by the camber and radii difference. This circle passes through the point $\zeta = -\frac{l}{2}$.

The systems of coordinates are plotted both in the ζ plane and in the z plane in such manner that the ξ axis and the x axis form the angle β with the horizontal wind direction.

If we now describe, in the z plane, both circles, which we obtain from the given circle K in the ζ plane by employing the two conversion formulas

$$z_1 = 2\zeta \quad \text{and} \quad z_2 = \frac{l^2}{2\zeta}$$

then these are the same two circles we designated above by K_1 and K_2.

The point A_1 has the coordinate z_1 and the point A_2 has the coordinate z_2, hence the center of $A_1 A_2$ has the coordinate $z = \frac{1}{2}(z_1 + z_2) = \zeta + \frac{l^2}{4\zeta}$, as desired. P is therefore an actual point on the Joukowski curve.

The following formula holds good for the velocity q at which the air flows by every point on the Joukowski figure.

$$q = \frac{\kappa(\xi, \eta)}{\left|\frac{dz}{d\zeta}\right|}$$

From $z = \zeta + \frac{l^2}{4\zeta}$ it follows that

$$\frac{dz}{d\zeta} = 1 - \frac{l^2}{4\,\zeta^2} = \frac{1}{2\zeta}\left(2\zeta - \frac{l^2}{2\zeta}\right) = \frac{z_1 - z_2}{z_1}$$

whence we obtain

$$\left|\frac{dz}{d\zeta}\right| = \frac{A_1\,A_2}{O\,A_1}$$

since the absolute value of $z_1 - z_2$ equals the distance $A_1\,A_2$ and the absolute value of z_1 = the distance OA_1.

For $\kappa\,(\xi,\eta)$, we obtain, from formula 2 of the preceding article, $\kappa = \frac{2Vh}{M_1 A_1}$, in which h is the distance of the point A_1 from the horizontal line passing through L. In the expression there given for the numerator, it is equal to h and the denominator is equal to $\frac{1}{2}(M_1 A_1)$, as may be easily verified. We thus obtain

$$q = V\,\frac{2\;h}{M_1\,A_1}\;\frac{O\;A_1}{A_1\,A_2}$$

which is just the formula given above for q.

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.

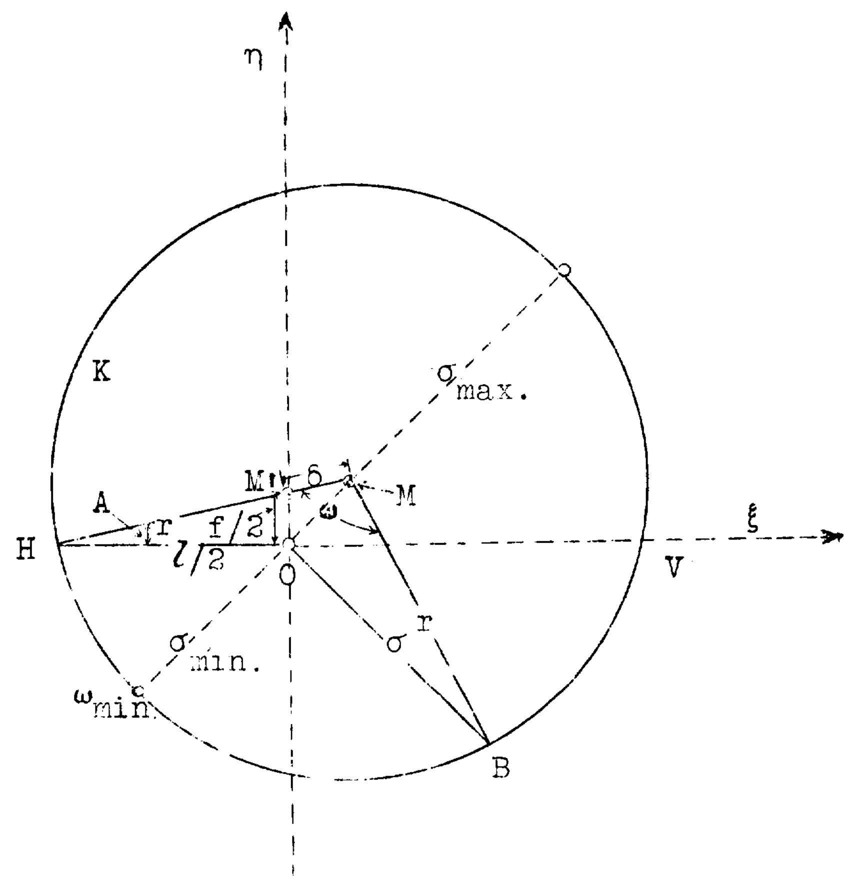

Fig.1

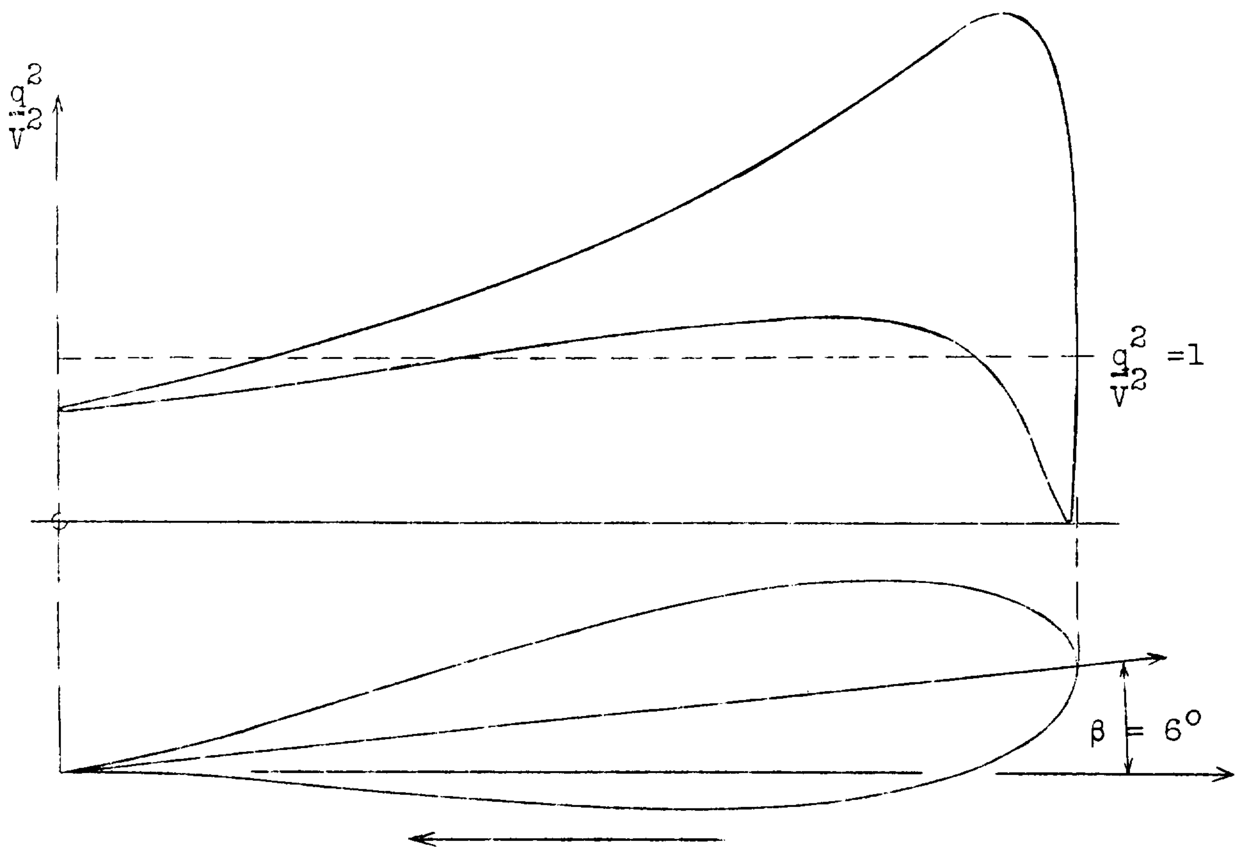

Fig.2: $f = 0, \delta/l = \frac{1}{10}$

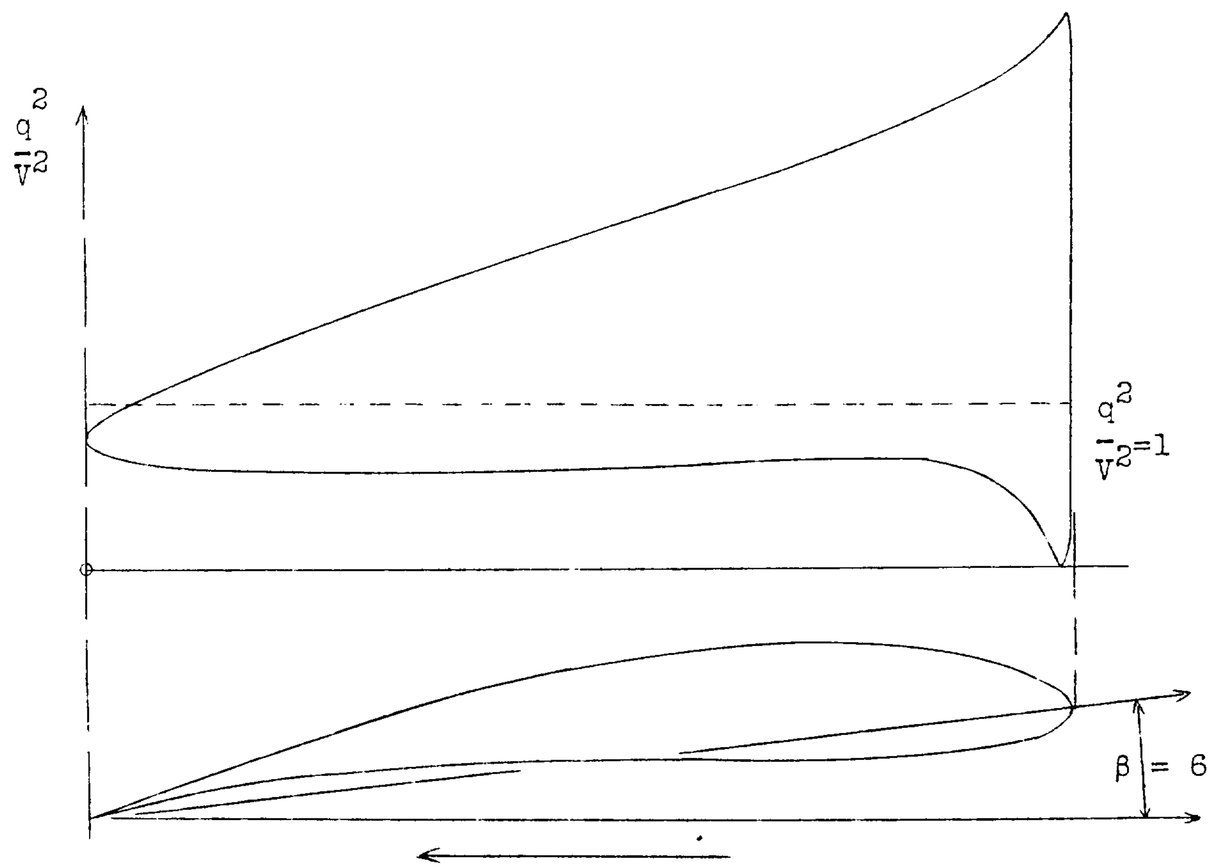

Fig.3: $f/l = \frac{1}{10}$, $\delta/l = \frac{1}{20}$

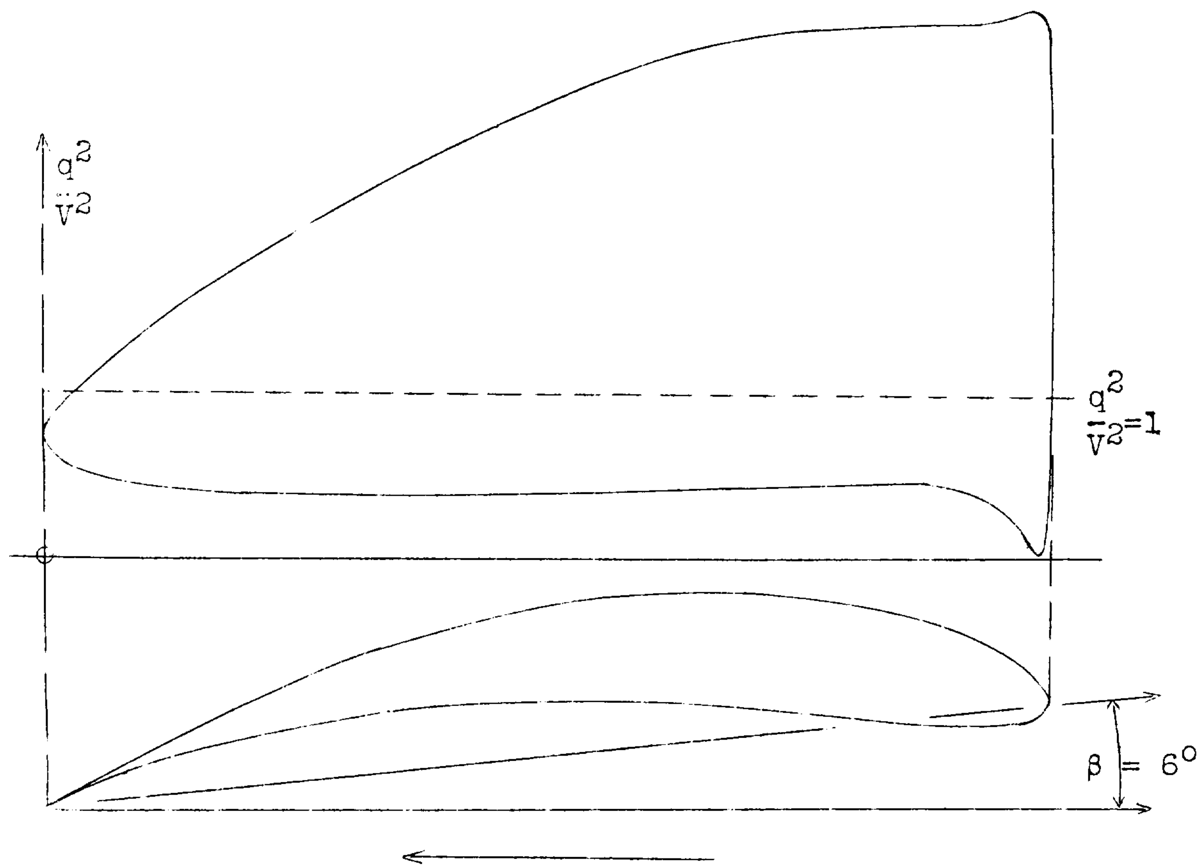

Fig.4: $f/l = \frac{1}{5}$, $\delta/l = \frac{1}{20}$

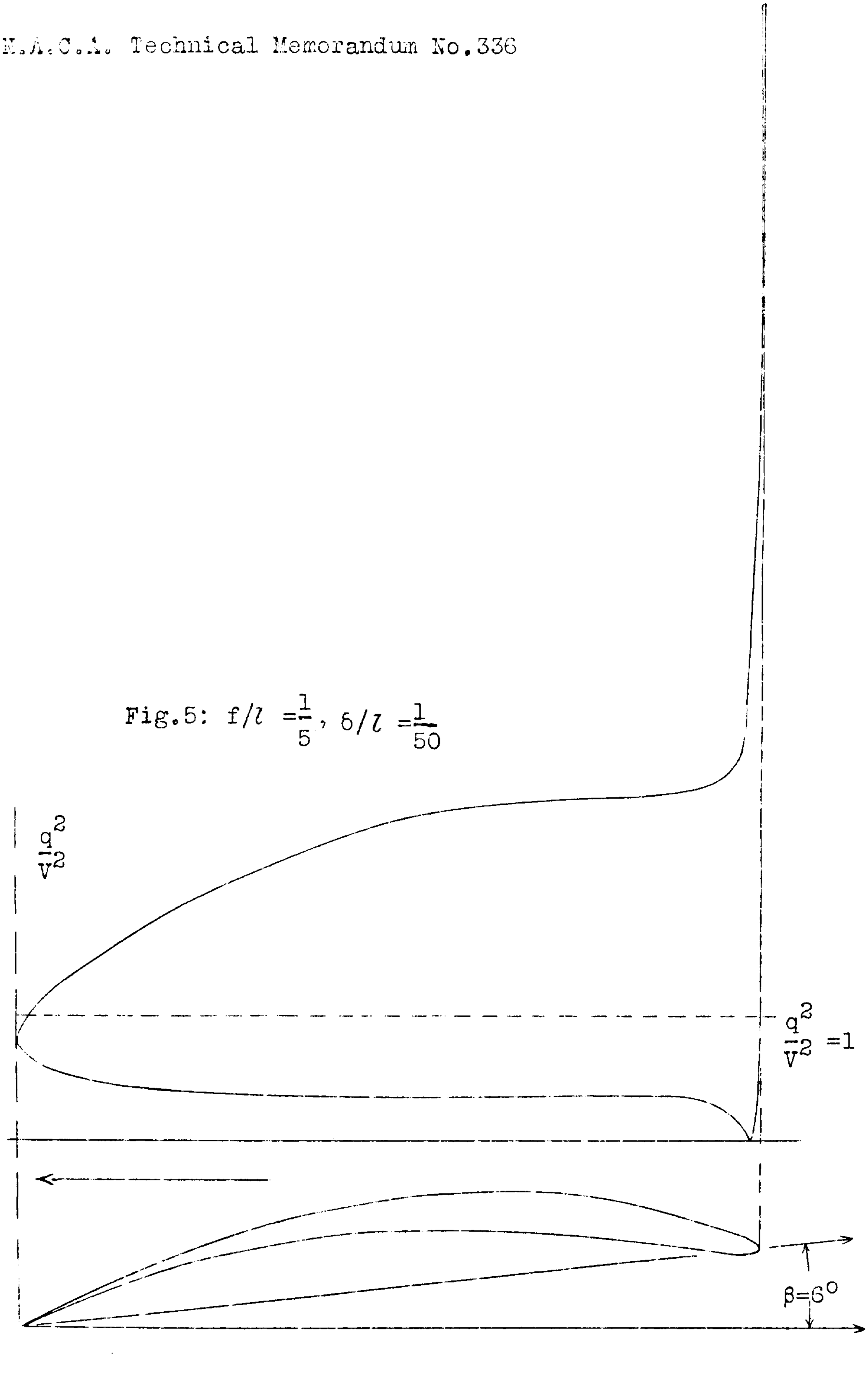

Fig.5: $f/l = \frac{1}{5}$, $\delta/l = \frac{1}{50}$

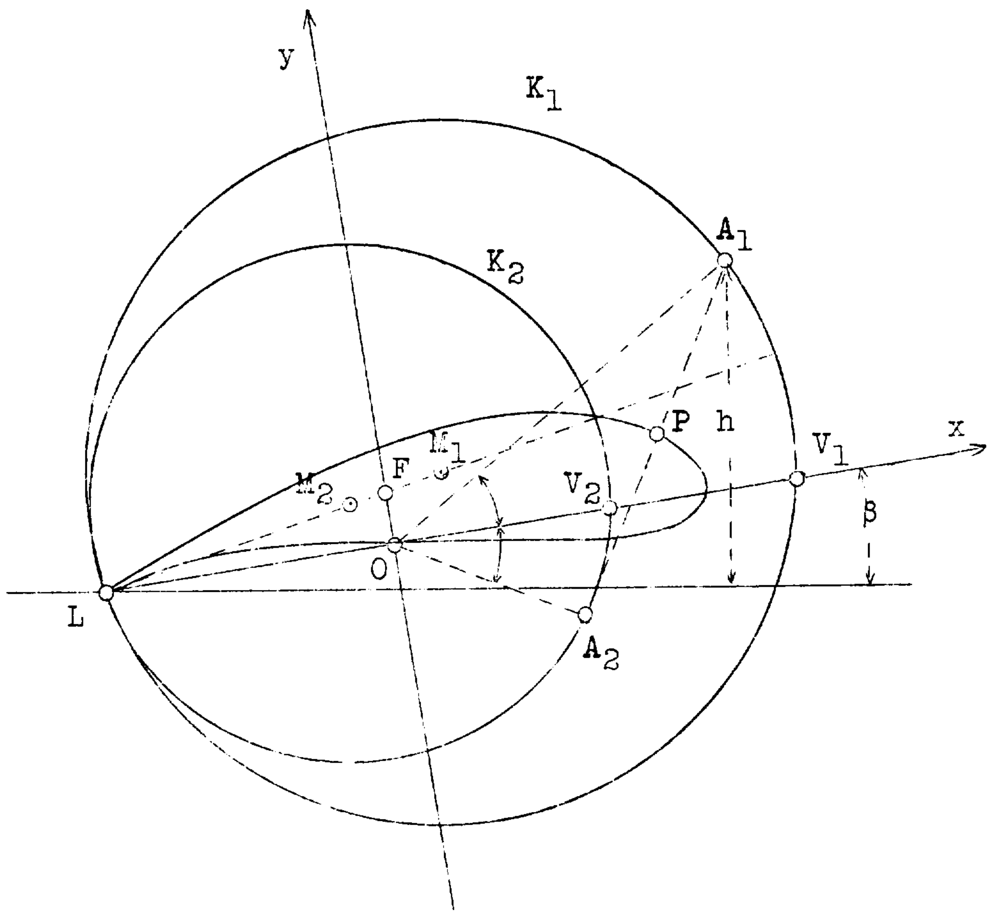

Fig.6