FINAL PROJECT REPORT
PRECISE RELATIVE LOCATION ESTIMATION FROM SATELLITE LASER OBSERVATIONS

CONTRACT NO. NAS 5–25028

Prepared For:

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Planetary Sciences Department Report No. 003–79

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TABLE OF CONTENTS

INTRODUCTION

The field of satellite geodesy has greatly benefited from recent advances in the development of precise tracking systems and techniques for data analysis. Measurements of the relative locations of tracking stations with sub-meter accuracy can be used to supplement standard surveying methods. Interstation distance measurement at the decimeter level will also improve the accuracy of regional models of the motions and distortions in the earth's crust, particularly in areas of high seismic activity.

Since the beginning of this decade, precise laser range measurements to satellites carrying retro-reflector arrays have been employed in station position estimation (Gaposchkin et al., 1970; Marsh et al., 1971). In these early applications, the laser observations were combined with optical camera data to achieve the required orbital accuracy. As the precision of the laser systems improved (Plotkin et al., 1973), the capability of a single instrument to define subtle variations in station and satellite position was demonstrated (Smith et al., 1972; 1973b), using geometry-dependent data analysis techniques (Dunn et al., 1973).

In late 1973, improved signal detection techniques reduced the noise level of the Goddard Space Flight Center (GSFC) laser systems from more than 50 cm. to less than 10 cm. A two station network of mobile lasers was deployed in 1974 in the western United States to exploit this added capability in the early phases of the San Andreas Fault Experiment (Smith et al., 1974). Observations from the stationary system at Greenbelt, Maryland were also collected during this period, although the long (3,700 km.) inter-continental station separation did not allow mutual visibility from all three stations to the Beacon Explorer-3 satellite employed in the experiment.

All the above work relied heavily on orbit dynamics to establish a stable reference frame in which to define the station positions. Observations from at least three consecutive satellite revolutions were required, and the resulting dependence on the geopotential model helped to motivate some significant improvements in its definition (Smith et al., 1973a).

During the first year's operation of the GEOS-3 geodetic satellite mission, laser systems with 7 cm. ranging precision were located at Greenbelt, in Bermuda and on Grand Turk Island in the Bahamas, to assist in the calibration of the satellite's radar altimeter. The close spacing of the stations in this network allowed an unprecedented level of mutual visibility to the 850 km. altitude satellite, starting in July 1975. Added network strength was provided by observations from a site at Patrick Air Force Base in Florida, which became operational in early 1976.

We describe here an experiment in which the laser data from these four stations have been employed to determine precise inter-station baselines and relative heights in short orbital arcs of no more than 12 minutes duration. The results are shown to be relatively insensitive to errors in reasonably accurate gravity models and can provide an important supplement to those produced by dynamic techniques.

METHOD OF DATA ANALYSIS

Each day, the near-polar (115^{o}) inclination of GEOS-3's orbit gives rise to a pair of two (or occasionally three) consecutive passes over the Western Atlantic laser network shown in Figure 1. The ground trace shown in Figure 1 corresponds to the orbit path observed in a twelve hour period by the Greenbelt laser on August 28, 1975. The first two passes were observed with predominantly south-going orbital motion and are separated in time by the orbital period of 102 minutes. Observations on the third pass commence less than ten hours from the beginning of the first, by which time the earth's rotation gives rise to a largely north-going ground trace.

During the period of the experiment, between July 2, 1975 and February 29, 1976, the Greenbelt, Bermuda and Grand Turk lasers collected observations from the same satellite revolution on twenty separate occasions. On the last five of these simultaneously observed arcs, the Patrick site also provided measurements. The total time intervals between acquisition by the first station and loss of sight by the last station in each arc ranged from 7 minutes to 12 minutes, and no pass from a single laser lasted more than 7 minutes.

In the technique adopted to resolve the relative station locations from these observations we assume a fixed position for the Greenbelt laser. The common latitude, longitude and spheroid heights of each of the other stations are adjusted, simultaneously with six orbital elements for the multiple arcs of data. The orbital estimation scheme employs a conventional batched weighted least squares approach with no constraint on either the orbit or on the estimated station locations. The estimated orbit thus acts as a convenient interpolation scheme to provide accurate time synchronization between the measurements, as long as a sufficiently accurate gravity field is employed to represent the dynamic perturbations to the 7 cm. level for up to 12 minutes, the longest arc length used.

The strength of the relative location solutions lies primarily in the geometry of the three or four station observing network. No significant orbit accuracy or relative station orientation information is available due to the weak dynamic characteristics of the very short arcs. However, very precise inter-station baselines (chord distances) and relative height locations can be provided by this configuration, if its important geometrical properties are carefully considered.

BASELINE AND RELATIVE HEIGHT RESULTS

The strongest solution for relative locations was provided by a simultaneous estimate using all of the 20 arcs of data. We will refer to this as the base solution, and in this case, six unconstrained orbital parameters were adjusted for each arc and the three coordinates of the Bermuda, Grand Turk and Patrick laser positions were allowed to adjust as a set common to all arcs.

The perturbation model for the satellite's motion in the short arc study included only the effects of the geopotential and the direct gravitational forces of the sun and moon, as atmospheric drag, solar radiation pressure and earth tide perturbations have insignificant effects over a 10 minute time period. The effects of solid earth tides on the station were also neglected, as their influence on the relative station locations is below the sensitivity of the experiment.

When the Goddard Earth Model-9 (GEM-9) gravity field (Lerch et al., 1977) was employed in the perturbation model the resulting baseline and relative height estimates were those presented in Table I. The formal standard deviations (sigmas) on the estimated locations amounted to less than 6 cm. in baseline and 7 cm. in relative height for ranges of 7 cm. noise precision. The root mean square (R.M.S.) fit of the refraction-corrected (Marini et al., 1973) range observations to the adjusted orbits based on the station locations of Table I was 6.7 cm. for a total of 8,882 measurements. The reduction of the orbital fit to the noise level of the range data indicates that any measurement or dynamic model error was absorbed by the station location adjustment.

The fixed station location chosen for Greenbelt was that presented with the GEM-9 gravity model (Lerch et al., 1977), and was derived assuming a value of 398,600.64 km.^{3}/sec.^{2} for the earth's gravitational constant (GM) and 6,378.145 km. for the earth's semi-major axis. The values of latitude and longitude adopted for the fixed station are not critical to baseline and relative height estimation using single passes of data, but an error in the assumed station height of 14.60 meters gives rise to a scaling error in the solution. For example, if the height of the Greenbelt station is decreased by 5m., the resulting baseline estimates decrease by approximately 15 cm. (.1 part per million) with an increase in relative station heights of less than 5 cm. Baseline contractions of a similar magnitude are also produced by an increase in GM to 398,601.30 km.^{3}/sec.^{2} (1.6 parts per million).

To examine the extent to which the station location estimates are affected by geopotential model error and data distribution, a sequence of consistency tests was conducted. The weakest solution for relative station location is provided by the estimate from a single arc of data. Examination of the baseline results given by twenty such solutions employing Greenbelt, Bermuda and Grand Turk data, revealed a clear geometrical effect due to the relative orientation of the estimated baseline and the satellite ground trace. The baseline resolution, indicated by its formal standard deviation for 7 cm. range observations, is strongest when the satellite ground trace lies more nearly parallel to the baseline direction and, consequently, relatively low error in the Bermuda-Grand Turk estimate from single south-going passes was observed (see Figure 1). Improvement in baseline precision is achieved when a number of data arcs are used to estimate a common set of station positions in a multiple arc configuration, and all results presented here employed this technique. However, the geometrical weakness of baselines lying more perpendicular to the satellite ground trace are also evident when multiple arcs of only north-going or only south-going passes are considered.

In order to test the accuracy of the base solution, the consistencies of baseline and relative height estimates using subsets of the data were examined. In Tables 2 and 3 the results from two disjoint sets of five arcs from the Greenbelt, Bermuda and Grand Turk sites, each reduced in a multiple arc configuration, are presented, and are compared with those obtained with twenty arcs of data from these three sites (excluding Patrick). Each set contains data collected during a fifteen day period and includes a balance of north-going and south-going ground traces. The formal standard deviations (sigmas) for 7 cm. ranging data of the results shown in Tables 2 and 3 are less than 15 cm. in baseline and less than 35 cm. in relative height. The largest difference from the base solution of Table 1 in the baseline estimates is 29 cm. and the largest relative height difference is 22 cm. (each for Greenbelt-Grand Turk).

GRAVITY MODEL DEPENDENCE

The R.M.S. residual fits of range data from the Greenbelt laser to the twelve hour orbit for which the ground trace is given in Figure 1 were used as a separate test of the overall accuracy of several different gravity fields. In this test, the orbit perturbation model was expanded to include the effects of atmospheric drag, solar radiation pressure and earth tides. When GEM-8 (Wagner et al., 1976) was used in the perturbation model for this multiple-revolution arc, it was found to produce inferior results (1.6m.) to GEM-9 (27 cm.) and GEM-10 (21 cm.), both of which had included GEOS-3 observations in their estimation schemes.

The sensitivity to the geopotential of the multiple arc solution using 20 arcs from four stations was investigated by repeating the solution but employing each of these three gravity fields in the perturbation model. The baseline sensitivity to this variation in gravity field is shown in Table 4 to remain below 15 cm. and the relative height sensitivity below 35 cm. Their formal standard deviations are identical to those tabulated in Table 1 as the same data set was employed in each of the gravity model tests.

In a further test of the baseline estimates' sensitivity to gravity model, the 20 arc solution was repeated using the Standard Earth-4 (SE-4, Gaposchkin et al., 1975) and the GRIM-2 (Balmino et al., 1976) fields. The preliminary tests of data fit to the twelve hour orbit yielded disappointingly large range R.M.S. values of 9.40m. for SE-4 and 2.62m. for GRIM-2, which were not improved by simultaneously estimating the Greenbelt station height with the orbital elements. This apparent inability to model GEOS-3's motion was reflected in differences from the GEM-9 solution of up to 70 cm. in baseline estimates from the 20 single pass arcs, and up to 2m. in relative station height. However, the baselines estimated with each of these two fields were all consistently shorter than those given by GEM-9, by a factor of approximately .3 parts per million, suggesting that there is a common local gravity difference from the GEM fields in this area of the globe.

CONCLUSIONS

Laser ranging observations to low altitude geodetic satellites can be employed to measure precise relative locations from short arcs of data, when three or more stations track the same satellite pass. If orbital geometry with respect to the station configuration is carefully considered, no more than five arcs of data are required to define the inter-station baselines to 30 cm. precision. Baselines running parallel to the orbital motion can be defined to sub-meter precision from a single short arc of data. Combining arcs of differing orbital geometry in a common adjustment of two or more stations relative to the base station helps to compensate for weak baseline definition in any single arc.

The results of a combined solution including fifteen short arcs of observations from three stations and five arcs using added measurements from a fourth station have been described. The relative locations thus obtained are shown to be sensitive to reasonable variations in the gravity field to less than 15 cm. in inter-station baseline and less than 35 cm. in relative height. The formal standard deviations for range measurements at the observed 7 cm. noise level are less than 10 cm. for either baseline or relative height determinations from the twenty-arc combination solution.

The technique described here offers an important supplement to conventionally employed methods for station estimation, which require orbital fits to arcs of data collected over several consecutive satellite revolutions. The GSFC mobile lasers will be deployed in several different networks to support future missions during which they will track a variety of spacecraft, including Lageos, a high-altitude retro-reflector-carrying satellite designed for precise laser ranging studies. The number of chance (or scheduled) occurrences of simultaneous visibility of a satellite by three or more lasers can therefore be expected to greatly increase. The ability to exploit these occurrences will allow accurate triangulation of the networks when orientation information can be provided by dynamic methods.

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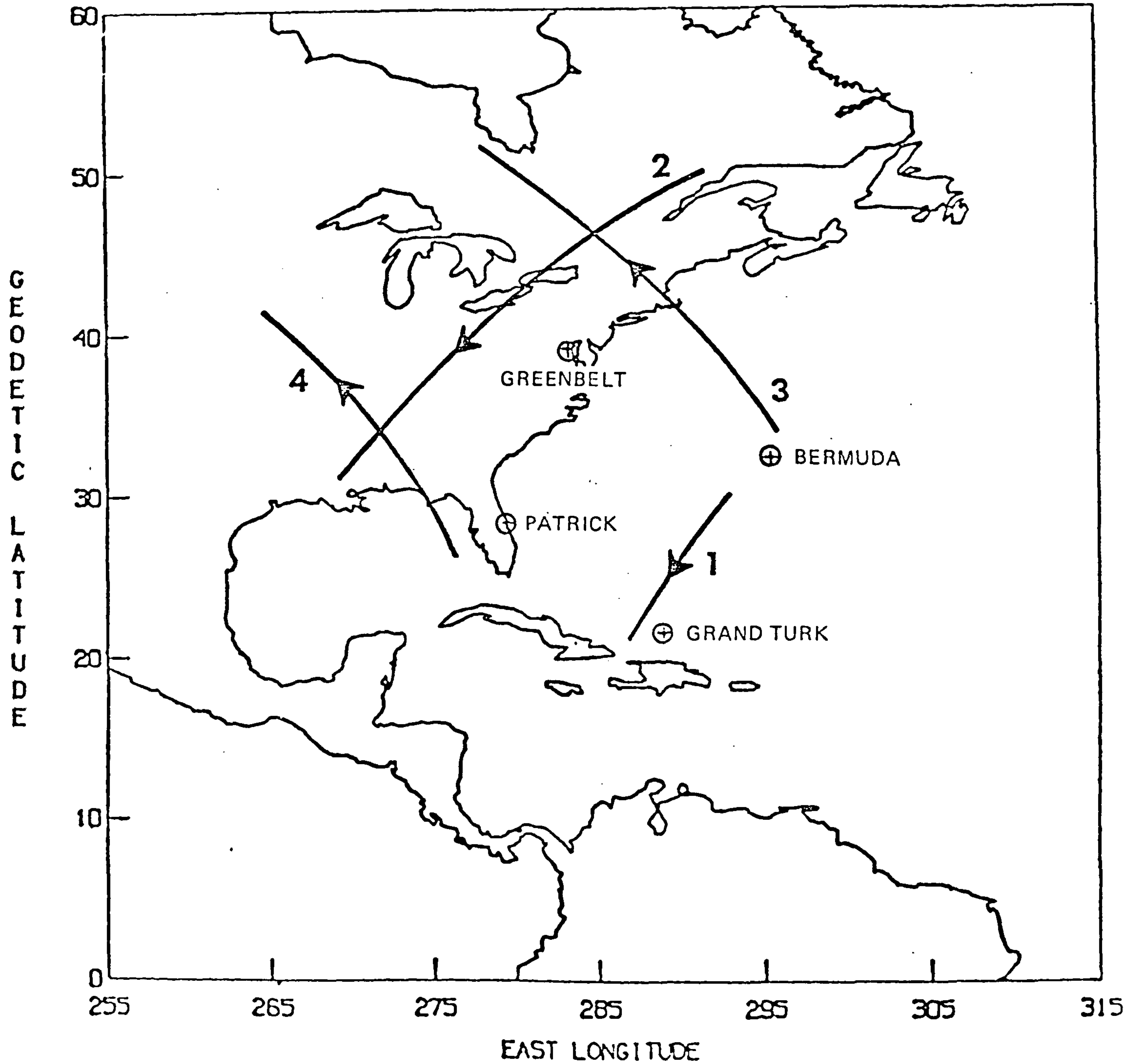

Figure 1. The locations of the four laser stations participating in the experiment and the sub-satellite ground trace of the GEOS-3 satellite during four consecutive passes observed by the Greenbelt laser. The observations were taken on August 28th, 1975 and the first north-going pass (3) was acquired ten hours later than the first south-going pass (1). Each pass represents a single opportunity for simultaneous observability from 3 stations.

TABLE I. Estimates of Baseline and Relative Height Between Pairs of Stations in the Network

Station Pair	Baseline Value (m.)	Baseline Sigma (m.)	Relative Height Value (m.)	Relative Height Sigma (m.)
Greenbelt - Bermuda	1,322,742.10	0.03	41.52	0.06
Greenbelt - Grand Turk	2,012,724.63	0.03	37.55	0.07
Bermuda - Grand Turk	1,364,265.20	0.02	-3.97*	----
Greenbelt - Patrick	1, 244, 991.04	0.02	42.76	0.05
Patrick - Bermuda	1,595,083.50	0.06	1.24*	----
Patrick - Grand Turk	1,213,393.83	0.03	5.21*	----

*The heights of stations relative to those other than the fixed station at Greenbelt were computed from the corresponding heights relative to Greenbelt.

TABLE 2: Inter-Station Baselines Estimated from Subsets of Available Data

Data Employed in Estimation		Greenbelt-Bermuda		Greenbelt-Grand Turk		Bermuda-Grand Turk	
No. of Arcs	Time of Arcs	Baseline (m.)	Sigma (m.)	Baseline (m.)	Sigma (m.)	Baseline (m.)	Sigma (m.)
20	July 1975 to Feb. 1976	1,322,741.99	0.03	2,012,724.60	0.03	1,364,265.24	0.02
5	Aug. 1975	742.01	0.04	724.92	0.05	265.09	0.03
5	Feb. 1976	741.92	0.10	724.42	0.13	265.17	0.09

TABLE 3. Inter-Station Heights Estimated from Subsets of Available Data

Data Employed in Estimation		Greenbelt-Bermuda		Greenbelt-Grand Turk	
No. of Arcs	Time of Arcs	Relative Height (m.)	Sigma (m.)	Relative Height (m.)	Sigma (m.)
20	July 1975 to Feb. 1976	41.36	0.06	37.46	0.07
5	Aug. 1975	41.67	0.08	37.38	0.09
5	Feb. 1976	41.45	0.19	37.77	0.32

TABLE 4: Baselines and Relative Heights Estimated Using Three Different Gravity Models

Gravity Model	Greenbelt-Bermuda Baseline (m.)	Greenbelt-Bermuda Relative Height (m.)	Greenbelt-Grand Turk Baseline (m.)	Greenbelt-Grand Turk Relative Height (m.)	Bermuda-Grand Turk Baseline (m.)	Bermuda-Grand Turk Relative Height (m.)
GEM 8	1,322,742.20	41.48	2,012,724.60	37.85	1,364,265.34	-3.63
GEM 9	742.10	41.52	724.63	37.55	265.20	-3.97
GEM 10	742.05	41.53	724.60	37.69	265.20	-3.84

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