NASA/GSFC Grant NSG 5306

ESTIMATION OF EFFECTIVE HYDROLOGIC PROPERTIES OF SOILS FROM OBSERVATIONS OF VEGETATION DENSITY

(NASA-CR-162797) ESTIMATION OF EFFECTIVE HYDROLOGIC PROPERTIES OF SOILS FROM OBSERVATIONS OF VEGETATION DENSITY M.S. Thesis (Massachusetts Inst. of Tech.) 125 p HC A06/MF A01 CSCL 08M G3/43 N80-18534 Unclas 47264

B.S., Loyola-Marymount University
(1978)

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
DEGREE OF

MASTER OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

February 1980

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ESTIMATION OF EFFECTIVE HYDROLOGIC PROPERTIES OF SOILS

FROM OBSERVATIONS OF VEGETATION DENSITY

by

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Submitted to the Department of Civil Engineering
on January 18, 1980, in partial fulfillment of the
requirements for the Degree of Master of Science
in Civil Engineering

ABSTRACT

An existing one-dimensional model of the annual water balance is reviewed. Slight improvements are made in the method of calculating the bare soil component of evaporation, and in the way surface retention is handled. A natural selection hypothesis, which specifies the equilibrium vegetation density for a given, water limited, climate-soil system, is verified through comparisons with observed data and is employed in the annual water balance of watersheds in Clinton, Ma., and Santa Paula, Ca., to estimate effective areal average soil properties. Comparison of CDF's of annual basin yield derived using these soil properties with observed CDF's provides excellent verification of the soil-selection procedure. This method of parameterization of the land surface should be useful with present global circulation models, enabling them to account for both the non-linearity in the relationship between soil moisture flux and soil moisture concentration, and the variability of soil properties from place to place over the earth's surface.

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ACKNOWLEDGEMENTS

This work was done under Grant NSG 5306, from the National Aeronautics and Space Administration.

This thesis is the result of many months of effort which was only possible with the help and encouragement of many wonderful people.

The supervision and guidance of my research and thesis advisor, Prof. Peter S. Eagleson, was paramount in the success and completion of this study. I am very grateful for the opportunity to work with him.

I am indebted to all my teachers at the Parsons Lab, and to the professors at Loyola-Marymount University, for their high standards of education and the individual attention given to each of their students.

Special thanks go to the Hydros, and particularly the friends in Room 320, for making the past 1 1/2 years a joyful experience. Thanks for the trips, outings, games, movies, and especially the parties. It would not have been the same without them.

Many thanks are due John and Tse-san for their wonderful hospitality and incredible ice cream. Good luck, you two, I deeply appreciate your friendship.

I also wish to thank Bernhard Metzger, friend and accomplice, for the time and help shared in research, study and many enjoyable moments.

A superb typing job was done by Anne Clee, who worked on 5 theses in 1 1/2 months, and deserves much more than these words express.

My final and most grateful thanks go to Gina DaCosta for suggesting that I apply to MIT, and for her continued support and inspiration during my time here; and to my family, for always being there when I needed them.

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LIST OF PRINCIPAL SYMBOLS

A	albedo
A_o	gravitational infiltration rate as modified by capillary rise from water table, cm/sec.
c	pore disconnectedness index
d	diffusivity index
$\bar{e}_p$	long-term average rate of potential evaporation, cm/sec.
e_T	long-term average rate of actual evaporation, cm/sec.
e_v	rate of transpiration, cm/day
E	exfiltration parameter
E_{p_A}	average annual potential evapotranspiration, cm
E_{r_A}	annual surface retention, cm
E_s	soil moisture evaporation from bare soil fraction of surface, cm
E_T	total evapotranspiration, cm
E_{T_A}	annual evapotranspiration, cm
$E^*_{T_A}$	annual evaporation from soil moisture, cm
E_v	transpiration from vegetated fraction of surface, cm
f_e	exfiltration rate, cm/day
f^*_e	exfiltration capacity, cm/day
f_i	infiltration rate, cm/day
f^*_i	infiltration capacity, cm/day
G	gravitational infiltration parameter
h	storm depth, cm
h_o	surface retention capacity, cm

H	average residual sensible heat flux, ly/min
i	precipitation rate, cm/day
$k(1)$	saturated intrinsic permeability, sq. cm
$K(1)$	saturated effective hydraulic conductivity, cm/sec.
k_v	plant coefficient
L_e	latent heat of vaporization
m	pore size distribution index
m_h	mean storm depth, cm
m_i	mean rainfall intensity, cm/day
m_{P_A}	average annual precipitation, cm
m_{t_b}	mean time between storms, days
m_{t_r}	mean storm duration, days
m_ν	mean number of storms per year
m_τ	mean length of rainy season
M	vegetal canopy density
M_o	equilibrium vegetal canopy density
n	medium effective porosity
N	percent cloud cover
P_A	annual precipitation, cm
$\bar{q}_b$	average rate of net outgoing long wave radiation, ly/min.
$\bar{q}_i$	average rate of insolation, ly/min.
R_{g_A}	annual groundwater runoff, cm
R_{s_A}	annual surface runoff, cm
$R^*_{s_A}$	annual rainfall excess, cm
s_o	time and spatial average soil moisture concentration in surface boundary layer

S	relative humidity
S_e	exfiltration sorptivity, $cm/s^{1/2}$
t_b	time between storms, days
t_e	duration of exfiltration rate at exfiltration capacity, days
t_o	duration of exfiltration at the rate $\bar{e}_p$, days. time at which surface retention reaches saturation during precipitation, days
t_r	storm duration, days
$\bar{T}_A$	normal annual temperature, °C
w	upward apparent pore fluid velocity representing capillary rise from the water table, cm/sec.
Y_A	annual yield, cm
α	reciprocal of average rainstorm intensity, days/cm
β	reciprocal of average time between storms, $days^{-1}$
γ_w	specific weight of water, $dynes/cm^3$
δ	reciprocal of average storm duration, $days^{-1}$
η	reciprocal of mean storm depth, cm^{-1}
κ	parameter of Gamma distribution of storm depth
λ	parameter of Gamma distribution of storm depth, cm^{-1}
μ_w	dynamic viscosity of water, poises
ρ_e	mass density of evaporating water, gr/cm^3
σ	capillary infiltration parameter
σ_w	surface tension of water, dynes/cm
ϕ	latitude
ϕ_e	dimensionless exfiltration diffusivity
ϕ_i	dimensionless infiltration diffusivity

$\Phi(m)$	pore shape parameter
$\Psi(1)$	saturated soil matrix potential, cm (suction)
γ/Δ	atmospheric parameter
$E[\ ]$	expected value of [ ]
$J(\ )$	evapotranspiration function
$\Gamma(\ )$	Gamma function
$\gamma(a, x)$	incomplete Gamma function

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Chapter 1

INTRODUCTION

In order to increase the accuracy of global climate models, a more sophisticated representation of the land surface boundary condition is required than that which is presently employed (GARP, 1978). The interaction, in particular the water flux, between the atmosphere and the soil-vegetation system at this boundary is highly non-linear in nature, and is not simply defined. Any attempt to satisfactorily account for this non-linearity in a model must incorporate two effects which are not included in current models:

1. variability of soil properties and soil moisture dynamics from place to place over the earth's surface, and
2. non-linearity in the relationship between soil moisture flux and soil moisture concentration.

In this work, it is intended to make use of a one-dimensional water balance at the land-air interface in order to parameterize the climate-soil-vegetation relationship in such a way as to reflect the non-linearity and areal variability.

Chapter 2

OBJECTIVES

The specific objectives of this work are twofold:

The first objective is verification of a vegetal equilibrium hypothesis developed by Eagleson (1978f). This hypothesis proposes that the natural vegetation density in a watershed will seek, through natural selection, an optimal "climax" value at which available soil moisture is a maximum. Comparison of a theoretical curve of evapotranspiration versus canopy density based on this hypothesis with observed data will provide the necessary check on the accuracy of the hypothesis.

The second objective is establishment of an algorithm for estimating effective areal soil properties from observations of vegetation density by using the natural selection hypothesis in a one-dimensional water balance model. By defining the level of evapotranspiration from soil moisture through observations of the canopy cover density, it may be possible, knowing the climate, to determine the soil properties that enable the soil-vegetation system to respond at the indicated level. The estimated values of these parameters can then be used in the water balance equation to evaluate desired components of the water flux. Verification of the desired algorithm will be sought through comparison of computed and observed statistics of annual yield.

Chapter 3

REVIEW OF LITERATURE

Past efforts to model the coupling among physical processes of the atmosphere, soil, and vegetation across the land-surface interface have been largely of two types:

1. Numerical studies which employ detailed formulations of the processes involved. Examples of such studies are those of Philip (1957), Sasamori (1970), Deardorff (1977), and Philip and de Vries (1957). Although these models simulate the system response to climatic inputs very well, they usually do so in terms of a large number of climate, soil, and vegetal parameters. Due to their complexity and the detailed data requirements for their validation, these studies have found little application in general circulation models.

2. Empirical studies which utilize validated interrelationships among the principle variables. Because of the ease of their application, and negligible programming and data requirements, most global climate models use this type of parameterization of the land-surface boundary with regards to actual evapotranspiration, average soil moisture content, and runoff.

The primary GCM's today utilize the approach first introduced by Manabe (1969) to parameterize the land surface boundary condition. In this approach, the above mentioned parameters are handled in the following way.

A. Evapotranspiration

Actual evapotranspiration is related to potential evapotranspiration linearly through the soil moisture and a single soil parameter following the work of Budyko (1956). This parameterization is

$$\frac{e_T}{e_P} = \begin{cases} s/k, & s \leq k \\ 1 \;, & k < s \leq 1 \end{cases} \tag{3.1}$$

in which

e_T = actual rate of evapotranspiration

e_P = potential rate of evapotranspiration

s = effective soil moisture concentration

k = empirical coefficient, $0 < k \leq 1$ generally assumed to be constant everywhere

As mentioned above, the only soil parameter appearing in this model is the empirical coefficient, k. This representation grossly distorts the sensitivity of e_T to s and makes no allowance for the spatial variance of this sensitivity due both to soil type and to the presence of vegetation.

More recently, Lettau (1969) and Lettau and Baradas (1973), in their "evaporation climatonomy" formulation, refine the water balance evapotranspiration term through use of an energy balance. This approach seeks theoretical solutions in the form of "response functions" (i.e. evapotranspiration cycles, temperature, etc.) as a physical consequence of a mathematically defined "forcing function" of the environmental system. However, parameterization is achieved without any explicit consideration of the soil and vegetal properties which will control the evaporation under all but the most humid conditions. The only input parameters linked to the land surface are evaporivity, e*, which is a non-dimensional measure of the capacity of land surfaces to utilize solar energy for the evaporation of rainfall received in a specified time interval, and t* which denotes a characteristic soil moisture residence time. Values for these parameters are either assumed on the basis of empirical data, or are estimated from a systematic classification of watersheds according to morphology, soil structure and permeability, vegetation cover, etc. The lumping of all these parameters into a single term in no way fully represents the complex interrelationships between the various processes involved in the water balance.

Other studies concerning the evapotranspiration term are those of Czarnowski (1964) and Ritchie (1972), and Ritchie and Burnett (1971). Czarnowski assumes that total evapotranspiration is a sum of plant transpiration, evaporation from surface retention, and evaporation from soil, and that these values are functions of vegetation density, and consequently of climatic factors. He treats the development of plant cover as a function of the form

$$M = 1 - e^{-\frac{P}{V_m}} \tag{3.2}$$

where

P = precipitation, mm

V_m = sum of mean daily deficits of air humidity, mm Hg

Finally, he concludes that evapotranspiration can be expressed as

$$V = V_m - M \left[1.17M + \frac{.4}{\sqrt{M}} \right] \tag{3.3}$$

or

$$\frac{V}{V_m} = J = M \left[1.17M + \frac{.4}{\sqrt{M}} \right] \tag{3.4}$$

where the constants, 1.17 and .4 are determined by a least squares fit to empirical data obtained primarily from cultivated agricultural lands.

Ritchie (1972) and Ritchie and Burnett (1971) develop a set of empirical functions relating leaf area index and fractional net radiation at a soil surface for a row crop to plant evaporation efficiency. These equations may be written

$$R_{ns} = e^{-.398L_{Ai}} \tag{3.5}$$

$$\frac{e_T}{e_P} = .70L_{Ai}^{1/2} - .21 \tag{3.6}$$

where

L_{Ai} = leaf area index

R_{ns} = net radiation reaching soil surface

R_{no} = net radiation above plant canopy

Relating leaf area index to canopy density using Equation (3.5) and the assumption that

$$\frac{R_{ns}}{R_{no}} = 1 - M \qquad (3.7)$$

gives

$$J = \frac{e_T}{e_P} \; .70 \left[\frac{\ln(1-M)}{-.398}\right]^{1/2} - .21 \qquad (3.8)$$

Again, the constants appearing in the above equations are determined from the method of least squares.

While both of the above formulations are attempting to relate evapotranspiration to more physically significant parameters, there is little inclusion of the actual physics. Since a linear regression is performed to obtain the above equations, there is a lack of generality and understanding of the sensitivity to other parameters besides vegetation

B. Soil Moisture

The change in the average soil moisture concentration is determined from a water balance relation written

$$nh \frac{\partial s}{\partial t} = i - e_T - Y_S - Y_G \tag{3.9}$$

in which

h = thickness of surface layer

i = intensity of rainfall

Y_S = intensity of surface runoff

Y_G = intensity of percolation of water out of the surface layer to groundwater

The product, nh, represents the maximum water content of the surface layer and is assigned a value which is common for all soil surfaces.

C. Runoff

Runoff, as written in Equation (3.9) consists of two different components. Surface runoff is regulated by the infiltration of rainfall and additions to soil moisture. Groundwater runoff is governed by the state of soil moisture concentration. All global climate models use highly simplified empirical formulae which lump these two dynamically different runoff-generating processes into total yield relations of the form

$$Y = Y(i, e_T, s)$$

These relations include one or more coefficients which may incorporate spatial variability, but there is no physical basis for their selection without natural yield measurements.

The models referred to in the preceding paragraphs include those of Arkawa (1972, U.C.L.A.); Somerville et al. (1979, G.I.S.S.); Gates and Schlesinger (1977, Rand-O.S.U.); Sellers (1973, Arizona); and Corby et al. (1978, B.M.O.). In all of these models, there is no use of the present high level of physical understanding of the natural processes involved to develop a generalized, accurate representation of the land-surface interface.

Eagleson (1978a,b,c,d,e,f,g), has developed a generalized water balance based upon simplified physics of the component processes. The development is sufficiently rigorous to capture the essential system dynamics yet simple enough to permit analytical solution. The model produces valuable insights into the interactive role of soil moisture in the determination of climate. Foremost in this development is the accounting for the areal variability of soil properties over the earth's surface and the reflection of the inherent non-linearity in the relationships between soil moisture concentration and the interfacial moisture fluxes. This model will be presented and utilized in the following chapters to attain the objectives stated in Chapter 2.

Chapter 4

THEORETICAL BACKGROUND

4.1 The Water Balance

The major source of theoretical background used here is the work of Eagleson (1978a,b,c,d,e,f,g). In these papers, a one-dimensional water balance based on soil moisture dynamics and statistics of climatic data is derived. This water balance, expressed in terms of annual expected values, may be represented as

$$E[E_{P_A}] = E[E_{T_A}] + E[R_{S_A}] + E[R_{G_A}] \tag{4.1}$$

and

$$E[Y_A] = E[R_{S_A}] + E[R_{G_A}] \tag{4.2}$$

where

$E[\ ]$ = expected value of $[\ ]$

P_A = annual precipitation

E_{T_A} = annual evapotranspiration

R_{S_A} = annual surface runoff

R_{G_A} = annual groundwater runoff

Y_A = annual yield

An analytic expression is obtained for Equation (4.1) by deriving the individual components through the use of derived probability distributions and one-dimensional dynamic equations approximating the physics of the separate soil moisture fluxes. These expressions are then introduced into Equation (4.1) to produce the equation for the (soil moisture) water balance

$$m_{P_A}(1 - e^{-G-2\sigma}\,\Gamma(\sigma + 1)\sigma^{-\sigma}) =$$

$$E[E_{P_A}]\; J(E,M,k_v,h_o) - E[E_{r_A}] + m_\tau K(1)\; s_o^{\,c} - Tw$$

for

$$E[E_{r_A}] \,/\, m_{P_A} < e^{-G-2\sigma}\,\Gamma(\sigma + 1)\sigma^{-\sigma} \qquad (4.3a)$$

(The term to the left of the equal sign is infiltration, the first term to the right, total evapotranspiration, the second is evaporation from surface retention, and the last two terms are groundwater runoff [the first is groundwater recharge and the last is groundwater loss]). Otherwise,

$$m_{P_A} = E[E_{P_A}]\; J(E,M,k_v,h_o) + m_\tau K(1) s_o^{\,c} - Tw \qquad (4.3b)$$

In the above,

E_{r_A} = annual surface retention

E_{p_A} = average annual potential evapotranspiration

J = evapotranspiration efficiency

G = gravitational infiltration parameter

σ = capillary infiltration parameter

E = evapotranspiration parameter

M = vegetation canopy density

k_v = plant transpiration coefficient

m_τ = mean length of rainy season

h_o = surface retention capacity

s_o = average annual soil moisture

$K(1)$ = saturated hydraulic conductivity

T = 1 year, seconds

w = apparent velocity of capillary rise

m_{P_A} = mean annual precipitation

c = pore disconnectedness index

It will be helpful and important to review the development of the expressions for evapotranspiration and surface runoff, and to present an alternative approach for the former, and a slightly different interpretation for the latter.

4.2 Evapotranspiration

The expected value of annual evapotranspiration is derived (Eagleson, 1978d) by calculating bare soil evaporation and vegetal transpiration for an interstorm period as functions of properties of the storm sequence, the surface, the soil, and the average rate of potential evapotranspiration, using observed distributions of the random climatic variables, and averaging over the rainy season. The bare soil evaporation and plant transpiration are determined by considering the vertical flux of moisture in a soil column. In Figure 4.1, the modeled column of soil and the different moisture fluxes are sketched. In this figure

f_e = bare soil exfiltration rate

M = vegetation canopy density

e_v = vegetation transpiration rate

$K(\theta_o)$ = effective hydraulic conductivity at long-term average soil moisture

It is assumed here that

1. Soil moisture throughout the surface boundary layer is spatially uniform at the start of each interstorm period at the long-term average value, $s = s_o$;
2. The medium is semi-infinite;

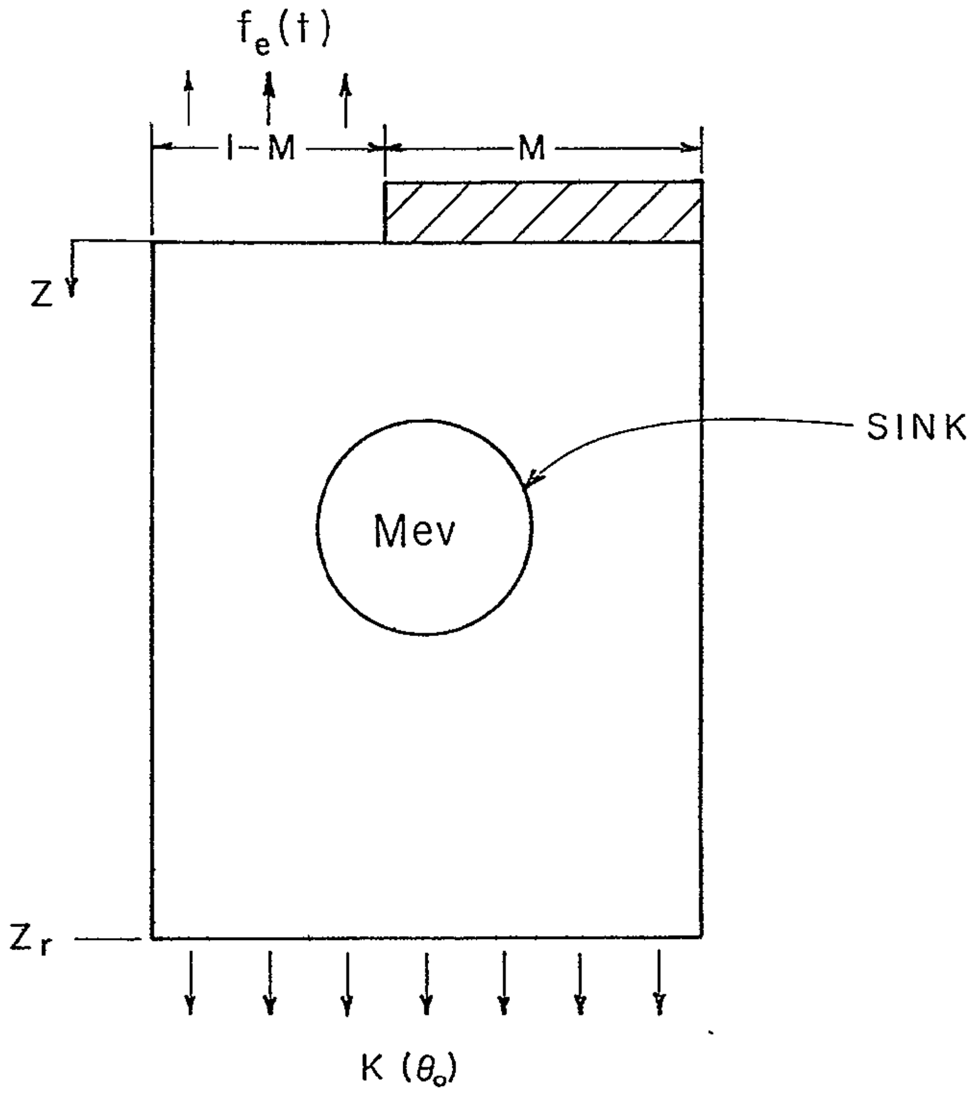


Figure 4.1

SCHEMATIC REPRESENTATION OF VEGETATED SOIL COLUMN
DURING AN INTERSTORM PERIOD

3. The vegetation is distributed uniformly, and its roots extend uniformly into the entire volume of the soil in the surface boundary layer. This implicitly assumes that the plant species have adapted by natural selection to a density and root structure which is in balance with the available soil moisture;
4. The rate of moisture extraction by the roots is in equilibrium with the transpiration rate by the leaves, thus forming a uniformly distributed sink for soil moisture of strength, Me_v.

Following the work of Philip (1969), Eagleson writes the total decrease in soil moisture during infiltration:

$$\int_0^{\infty} (\theta_o - \theta) dz = \int_{\theta_1}^{\theta_o} z d\theta = F_e(t) + [K(\theta_o) + Me_v] \, t \tag{4.4}$$

where $F_e(t)$ is the cumulative exfiltration in centimeters.

The integral on the left-hand side is evaluated in the manner of Philip (1960). Assuming a vertical flow passage of constant cross-section, the exfiltration rate is found to be

$$f_e(t) = \frac{1}{2} S_e t^{-\frac{1}{2}} - \frac{1}{2}[K(\theta_1) - K(\theta_o)] - Me_v \tag{4.5}$$

Note that this neglects the restriction, by vegetation canopy density, of the bare soil area through which exfiltration occurs. Further simplification and analysis result in the exfiltration capacity:

$$f_e^* \approx \frac{1}{2} S_e t^{-\frac{1}{2}} - Me_v \qquad (4.6)$$

where S_e is the exfiltration sorptivity.

A typical interstorm period, and the relationship between the various rates of evaporation and time for bare soil is illustrated in Figure 4.2. In this figure, $\overline{e}_p$ is the potential rate of bare soil evaporation, which is considered a constant. The times, t_o and t_e, are evaluated by assuming

$$\overline{e}_p = f_e^* \qquad (4.7a)$$

when

$$f_e^*(\forall e) = f_e^*(\overline{e}_p t_o) \qquad (4.7b)$$

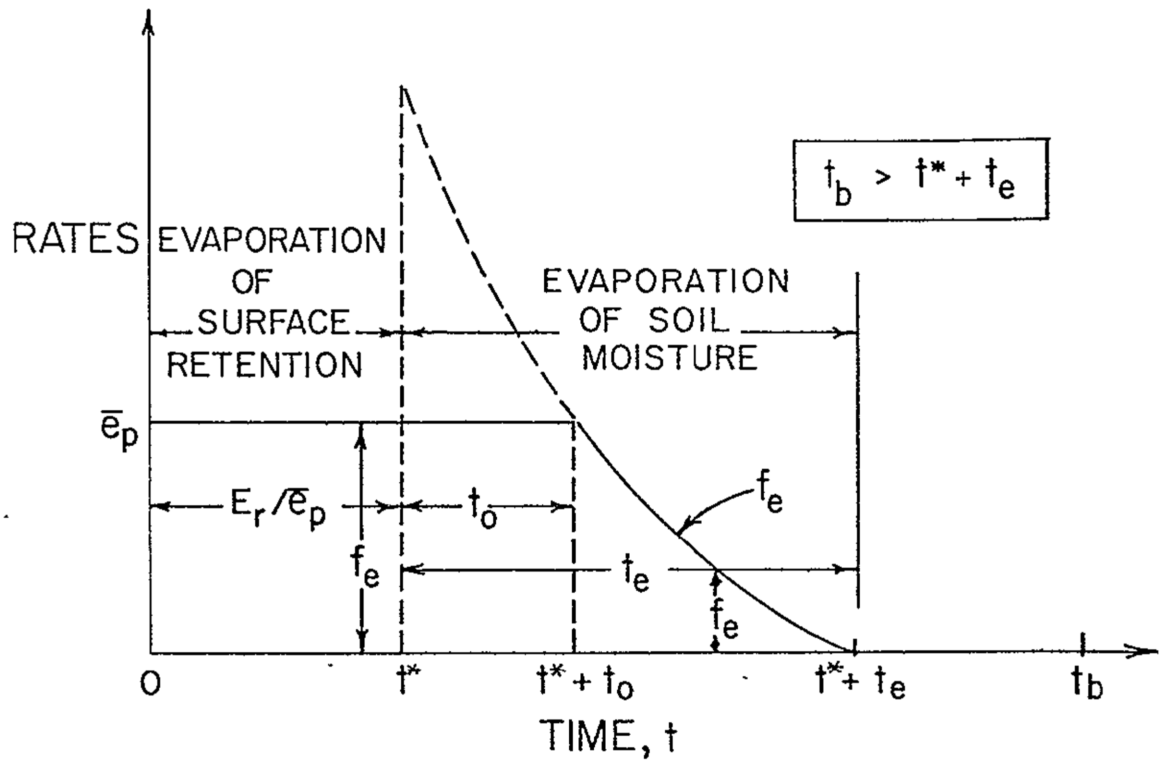
and that

$$f_e^* = 0 \qquad (4.8a)$$

when

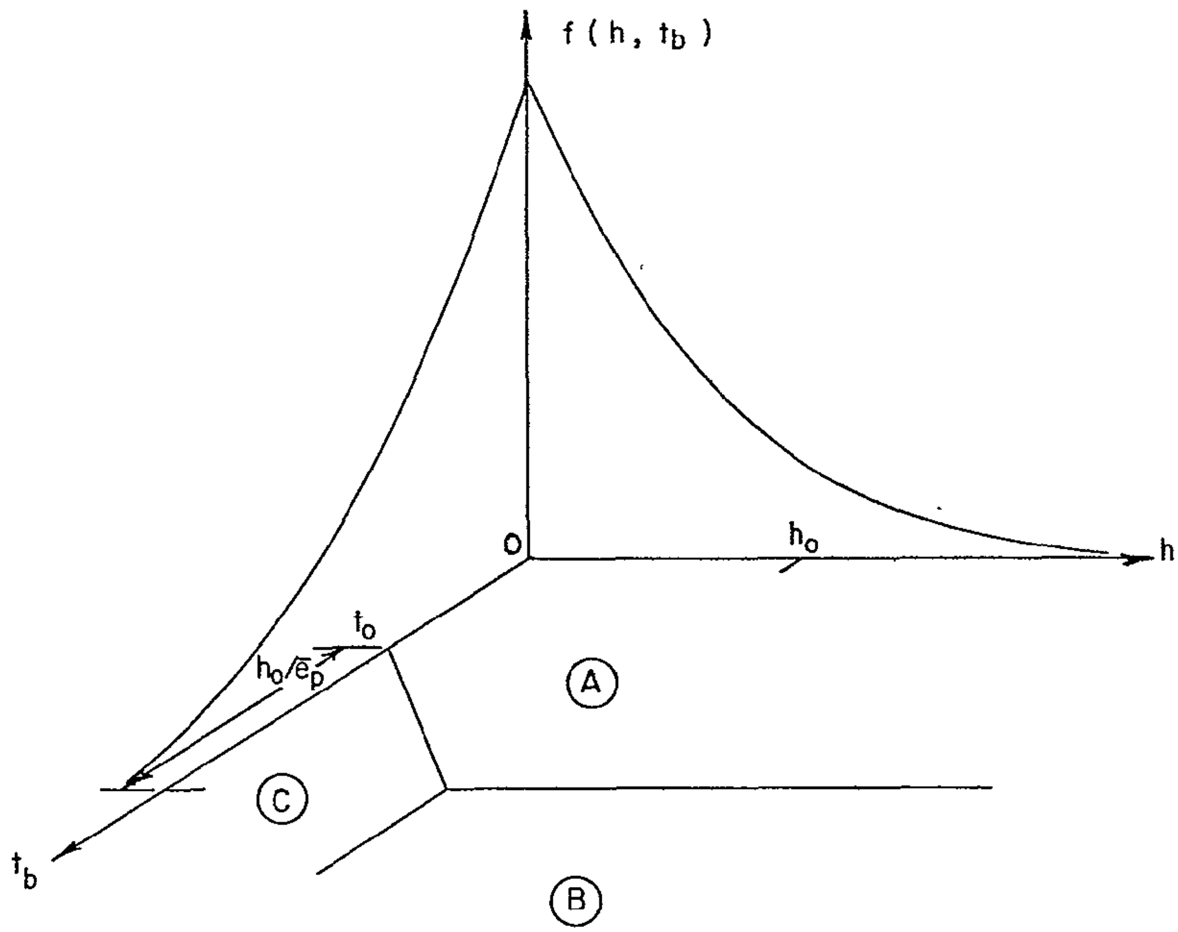
$$t = t_e \qquad (4.8b)$$

respectively. Exfiltration capacity, and the times t_e and t_o, are then used by Eagleson along with the relationships represented in Figure 4.2 to determine total evapotranspiration, E_T. To do this,



INTERSTORM EVAPORATION FROM BARE SOIL

Figure 4.2



INTEGRATION REGIONS FOR CALCULATION OF EXPECTED VALUE OF INTERSTORM EVAPOTRANSPIRATION

Figure 4.3

E_T from a unit land surface is proportioned according to

$$E_T = (1-M)E_s + ME_v \tag{4.9}$$

in which

E_s = bare soil evaporation from soil moisture plus evaporation from soil surface retention

E_v = evapotranspiration from vegetation plus evaporation from plant surface retention.

It is not necessary to present the development of $E[E_s]$ here, which is done by calculating the volume under the solid line in Figure 4.2, multiplying by the joint probability distribution of storm depth and time between storms, and integrating over the regions shown in Figure 4.3. What is important to note is the previously mentioned approximation made in the development of the bare soil exfiltration capacity. The expression obtained for $E[E_{s_j}]$, bare soil evapotranspiration for one interstorm period, from the above procedure is

$$E[E_{s_j}] = \frac{\bar{e}_p}{\beta}\left\{\frac{\gamma[\kappa,\lambda h_o]}{\Gamma(\kappa)} - \left[\frac{1+\beta h_o/\bar{e}_p}{\lambda h_o}\right]^{-\kappa} \frac{\gamma[\kappa,\lambda h_o+\beta h_o/\bar{e}_p]}{\Gamma(\kappa)} e^{-BE}\right.$$

$$+ \left\{\frac{1-\gamma[\kappa,\lambda h_o]}{\Gamma(\kappa)}\right\}\left\{1-e^{-BE-\beta h_o/\bar{e}_p} \cdot [1+Mk_v + (2B)^{1/2} E-w/\bar{e}_p]\right.$$

$$+ e^{-CE-\beta h_o/\overline{e}_p} [Mk_v + (2C)^{1/2} E - w/\overline{e}_p] +$$

$$(2E)^{1/2} e^{-\beta h_o/\overline{e}_p} [\gamma(\tfrac{3}{2},CE) - \gamma(\tfrac{3}{2},BE)\Big\}$$

$$+ \left[1 + \frac{\beta h_o/\overline{e}_p}{\lambda h_o}\right]^{-\kappa} \frac{\gamma[\kappa, \lambda h_o + \beta h_o/\overline{e}_p]}{\Gamma(\kappa)} \Big\{(2E)^{1/2} [\gamma(\tfrac{3}{2},CE) - \gamma(\tfrac{3}{2},BE)]$$

$$+ e^{-CE} [Mk_v + (2C)^{1/2} E - w/\overline{e}_p] - e^{-BE} [Mk_v + (2B)^{1/2} E - w/\overline{e}_p] \Big\} \Bigg\}$$

(4.10)

Here

$$B = \frac{1-M}{1+Mk_v - w/\overline{e}_p} + \frac{M^2 k_v + (1-M) w/\overline{e}_p}{2(1+Mk_v - w/e_p)^2} \qquad (4.11)$$

and

$$C = \frac{1}{2} (Mk_v - w/e_p)^{-2} \qquad (4.12)$$

Also,

β = reciprocal of mean time between storms

λ = parameter of Gamma distribution of storm depth

κ = paramater of Gamma distribution of storm depth

h_o = surface retention capacity

Upon studying Figure 4.1 and Equations (4.4) and (4.6) it can be seen that the term, f_e, is defined as the exfiltration rate for bare soil, and F_e as the total volume of moisture exfiltrated from the soil column across the bare soil surface. The rate, f_e, is obtained by differentiating the volume, F_e, with respect to time. The result of the differentiation leaves f_e multiplied by the area of bare soil. Thus, in the two-dimensional problem which includes the presence of vegetation, f_e should be multiplied by the term, 1-M, to account for the fact that only a fraction of the land surface, the bare soil fraction, is exfiltrating at this rate. Equation (4.6) should be rewritten as

$$(1-M) f_e^* = \frac{1}{2} S_e t^{-1/2} - M e_v \tag{4.13}$$

The new form for the expected value of bare soil evaporation, E_s, may then be evaluated in terms of this altered expression for exfiltration. The new expression for $E[E_{s_j}]$ is

$$E[E_{s_j}] = \frac{\bar{e}_p}{\beta} \left\{ \frac{\gamma[\kappa, \lambda h_o]}{\Gamma(\kappa)} - e^{-BE} \left[1 + \frac{\beta h_o / \bar{e}_p}{\lambda h_o} \right]^{-\kappa} \cdot \frac{\gamma[\kappa, \lambda h_o + \beta h_o / \bar{e}_p]}{\Gamma(\kappa)} \right.$$

$$+ \left[1 - \frac{\gamma[\kappa, \lambda h_o]}{\Gamma(\kappa)} \right] \left\{ 1 - e^{-BE - \beta h_o / \bar{e}_p} \left[1 + \frac{1}{1-M} \left[(2B)^{1/2} E - \frac{w}{\bar{e}_p} + M k_v \right] \right] \right.$$

$$- e^{-CE - \beta h_o / \bar{e}_p} \left[\frac{1}{1-M} \left[\frac{w}{\bar{e}_p} - M k_v - (2C)^{1/2} E \right] \right]$$

$$\left. \left. + \frac{1}{1-M} \left[(2E)^{1/2} e^{-\beta h_o / \bar{e}_p} \left[\gamma(\tfrac{3}{2}, CE) - \gamma(\tfrac{3}{2}, BE) \right] \right] \right\} \right.$$

$$+ \left[1 + \frac{\beta h_o/\bar{e}_p}{\lambda h_o}\right]^{\kappa} \frac{\gamma[\kappa, \lambda h_o + \beta h_o/\bar{e}_p]}{\Gamma(\kappa)} \left\{ e^{-BE} \left[\frac{1}{1-M}\left(\frac{w}{\bar{e}_p} - Mk_v - (2B)^{1/2}E\right)\right]\right.$$

$$\left. - e^{-CE}\left[\frac{1}{1-M}\left(\frac{w}{\bar{e}_p} - Mk_v - (2C)^{1/2}E\right)\right] + \frac{1}{1-M}\left[(2E)^{1/2}\left[\gamma(\tfrac{3}{2}, CE) - \gamma(\tfrac{3}{2}, BE)\right]\right]\right\}\Bigg\}$$

(4.14)

where

$$B = 1/(1+Mk_v) \qquad (4.15)$$

$$C + 1/2(Mk_v - w/\bar{e}_p)^{-2} \qquad (4.16)$$

E_T is obtained in the same way as before; by multiplying the bare soil term by (1-M), and the vegetation term by M. The result of this alteration on the expected value of annual basin evapotranspiration will be presented in a later section. Although this approach may seem more accurate than the original, its use will create other, and possibly greater problems. Attempting to expand the problem into two dimensions at this point will cause some inconsistencies concerning evaluation of the Philip exfiltration equation. This equation is

$$\int_{\theta_1}^{\theta_o} z d\theta = A_1 t^{1/2} + A_2 t + A_3 t^{3/2} + \ldots \qquad (4.17)$$

Since this was developed for a one-dimensional formulation, the expressions obtained for the constants, A_i, on the right hand side will not necessarily apply to the two-dimensional situation. This can be seen by noting that in Equation (4.6), as M approaches the value of 1, the right hand side does not go to zero, as it should for a fully vegetated surface where there would be no bare soil exfiltration of soil moisture. So, although there are certain misgivings about Eagleson's original derivation, the alternate approach presented above may involve more serious inaccuracies. However, for areas with a large vegetal canopy density, where the effect of the vegetation on bare soil exfiltration is large, this approach may come closer to reality than the previous one.

4.3 Surface Runoff

To derive the probability of storm surface runoff, Eagleson (1978) integrates the difference between rainfall intensity and the Philip infiltration equation over the duration of a rainstorm. Infiltration is assumed to occur uniformly over both bare soil and vegetated portions of the surface. Illustrated in Figure 4.4 is a sequence of surface states beginning from t = 0 at the start of the rainfall period. In this figure

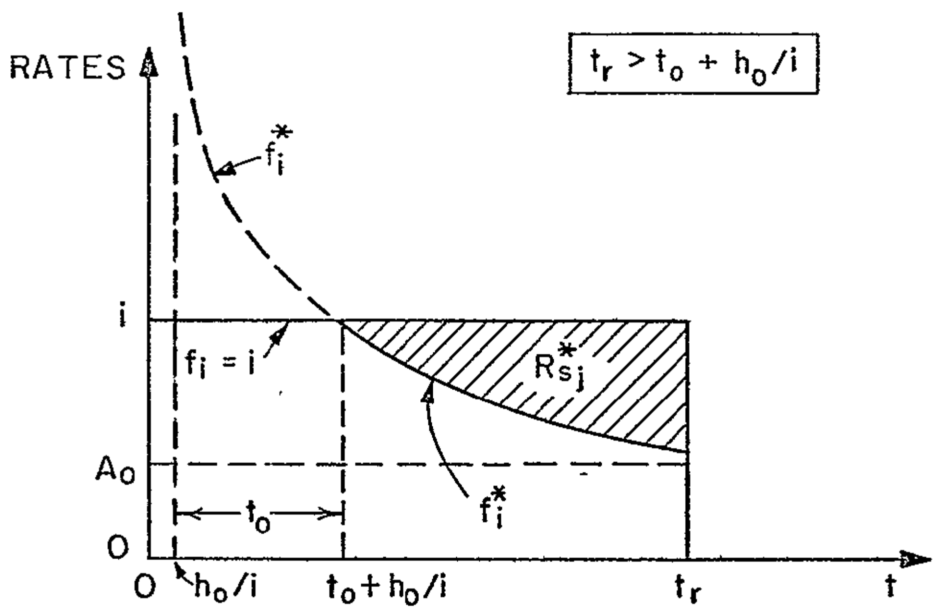
h_o = surface retention capacity

i = rainfall intensity

f_i^* = infiltration capacity

t_r = storm duration

A_o = gravitational infiltration rate as modified by capillary rise from the water table



SURFACE RUNOFF GENERATION DURING TYPICAL STORM ($t_r > t_o$)

Figure 4.4

Initially, there is a withdrawal of rainfall to satisfy surface retention. If t_r is greater than h_o/i, as shown, this surface retention reaches its capacity, h_o. If $t_r \leq h_o/i$, there would be no infiltration or runoff, and the surface retention would equal the storm depth, h. For the case illustrated in Figure 4.4, however, infiltration will begin at time $t = h_o/i$. From this time until $t = h_o/i + t_o$, when $f_i^* = i$, infiltration will take place at the rainfall rate, i. After this time, the capacity of the soil to infiltrate moisture is no longer larger than the rainfall intensity, and the excess is represented by the shaded area of the figure. Rainfall excess, $R_{s_j}^*$, is then generated until time $t = t_r$. The expected value of the rainfall excess is obtained in a manner similar to that of the evapotranspiration.

A question may be raised relating to the handling of the surface retention. In his development, Eagleson argues that the surface retention must be subtracted from the rainfall excess, since it is moisture that is not infiltrated into the soil. The expression he obtained for the expected value of annual rainfall excess is then

$$E[R_{s_A}^*] = m_{P_A}[e^{-G-2\sigma}\,\Gamma(\sigma + 1)\sigma^{-\sigma}] \qquad (4.18)$$

in which

$R_{s_A}^*$ = annual rainfall excess

The expected value of annual surface retention, E_{r_A}, is then subtracted from this to get the annual surface runoff. This charges the entire annual surface retention against those events producing rainfall excess, however.

	CLAY	CLAY-LOAM	SILTY-LOAM	SANDY-LOAM
k(1)	$1.0x10^{-10}$	$2.8x10^{-10}$	$1.2x10^{-9}$	$2.5x10^{-9}$
n	.45	.35	.35	.25
c	12	10	6	4

Table 4.1

REPRESENTATIVE SOIL PROPERTIES

A slightly different interpretation of this rainfall excess results from a closer examination of Figure 4.4 and the shaded area therein. It is known, and mentioned above, that surface retention must be satisfied before any infiltration can occur. From this, it seems necessary to subtract the surface retention from the beginning of the rainfall period, as indicated in the figure, rather than from the rainfall excess at the end. The volume represented by the shaded area would then be equal to the surface runoff, and not surface runoff plus surface retention. The resulting water balance equation then becomes

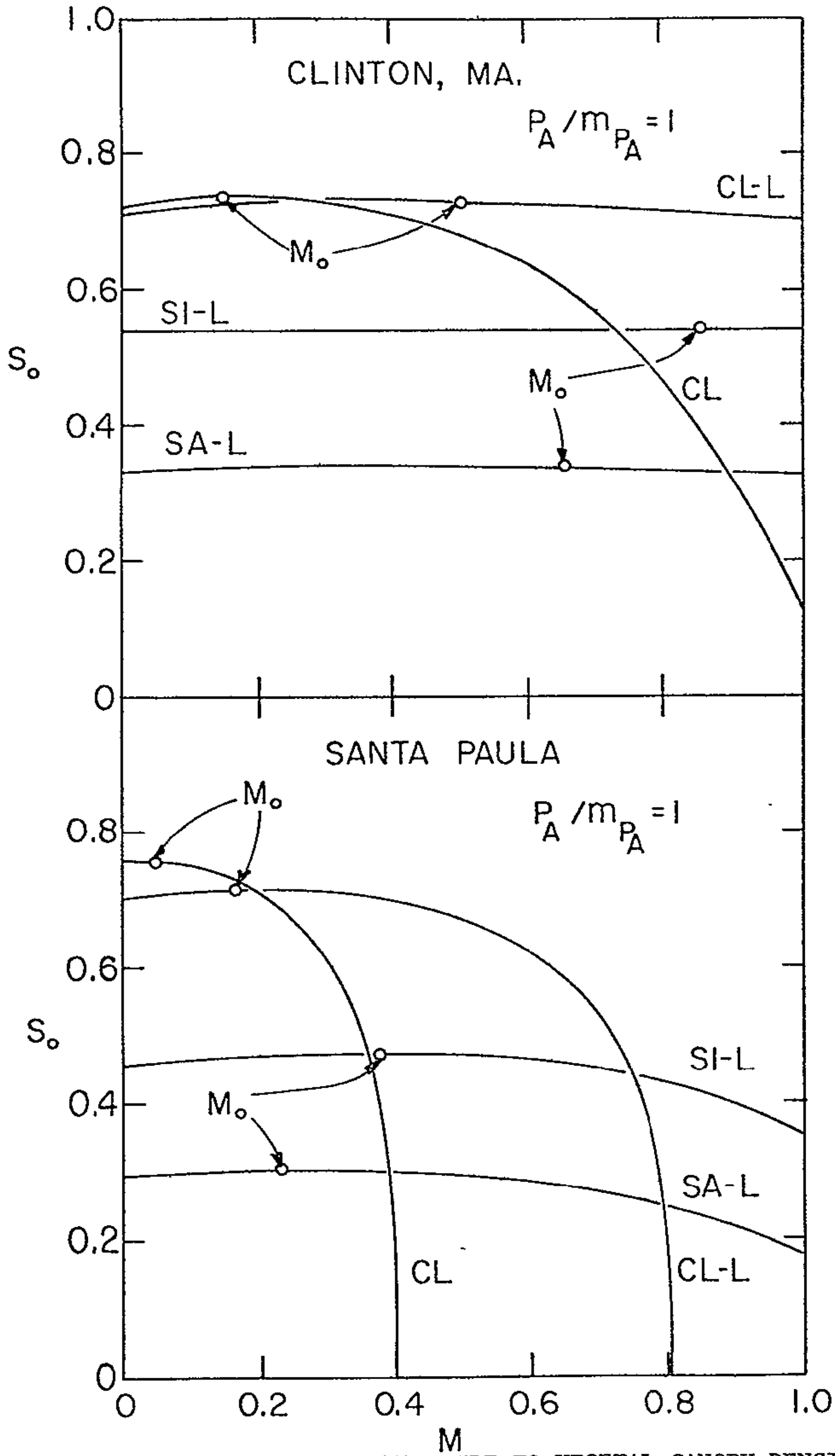
$$m_{P_A}(1 - e^{-G-2\sigma}\Gamma(\sigma+1)\sigma^{-\sigma}) = E[E_{P_A}]\ J(E,M,k_v h_o) + m_\tau\ K(1)s_o^c - \tau w \quad (4.19)$$

This alternative procedure will increase the calculated value of surface runoff, and decrease the amount of moisture calculated as infiltration. The effect of this difference on the CDF of annual yield will be discussed in Chapter 6.

4.4 Vegetal Equilibrium Hypothesis

From examining the role of vegetation canopy density in the average annual water balance, Eagleson (1978f) observed that for a given set of climate and soil parameters and for a given k_v, Equation (4.3) defines s_o as a function of M. This relationship is illustrated in Figure 4.5 by using four sets of representative soil properties, listed in Table 4.1, and the conditions $P_A = m_{P_A}$ and $k_v = 1$, for the climates of Clinton, Mass. and Santa Paula, Calif. It can be seen that there exists a particular value of $M = M_o$ for each climate-soil combination at

Figure 4.5



SENSITIVITY OF MEAN ANNUAL SOIL MOISTURE TO VEGETAL CANOPY DENSITY FOR TYPICAL SOILS ($k_v = 1$ AND $w/\bar{e}_p << 1$)

which s_o is a maximum. This point of maximum s_o corresponds to maximum surface and groundwater runoff, which means, for fixed precipitation, that there is minimum evapotranspiration from soil moisture. Thus, at $M = M_o$, it is expected that the term representing evapotranspiration from soil moisture

$$E[E_{p_A}]J(E,M,k_v,h_o) - E[E_{r_A}] \qquad (4.20)$$

will be a minimum for a given climate-soil combination. This minimization is seen in Figure 4.6 for the same information as that used in Figure 4.5. Note that in Santa Paula, the clay and clay loam soils cannot absorb enough water to produce canopy densities greater than 0.4 and 0.8, respectively, as long as $k_v = 1$.

The numerical value of k_v is a matter of some controversy. Linacre, et al. (1970), report values of k_v for water plants which range from .6 to 2.5 depending upon species. Slatyer (1967, p. 53) states that the value of k_v can be greater than one since total evapotranspiration from a plant community, per unit land area, may exceed that from a similar area of bare wet soil due to the larger actual evaporating surface area. Kramer (1969, p. 338) however, states that evaporation from a plant community never exceeds that from a similar area of wet soil. For the present, k_v will not be allowed to exceed one.

From observations of the relationships presented above, Eagleson (1978f) develops the vegetal equilibrium hypothesis mentioned in Chapter 2. In the light of the above arguments, this hypothesis says that natural vegetal systems of given species will develop a canopy density

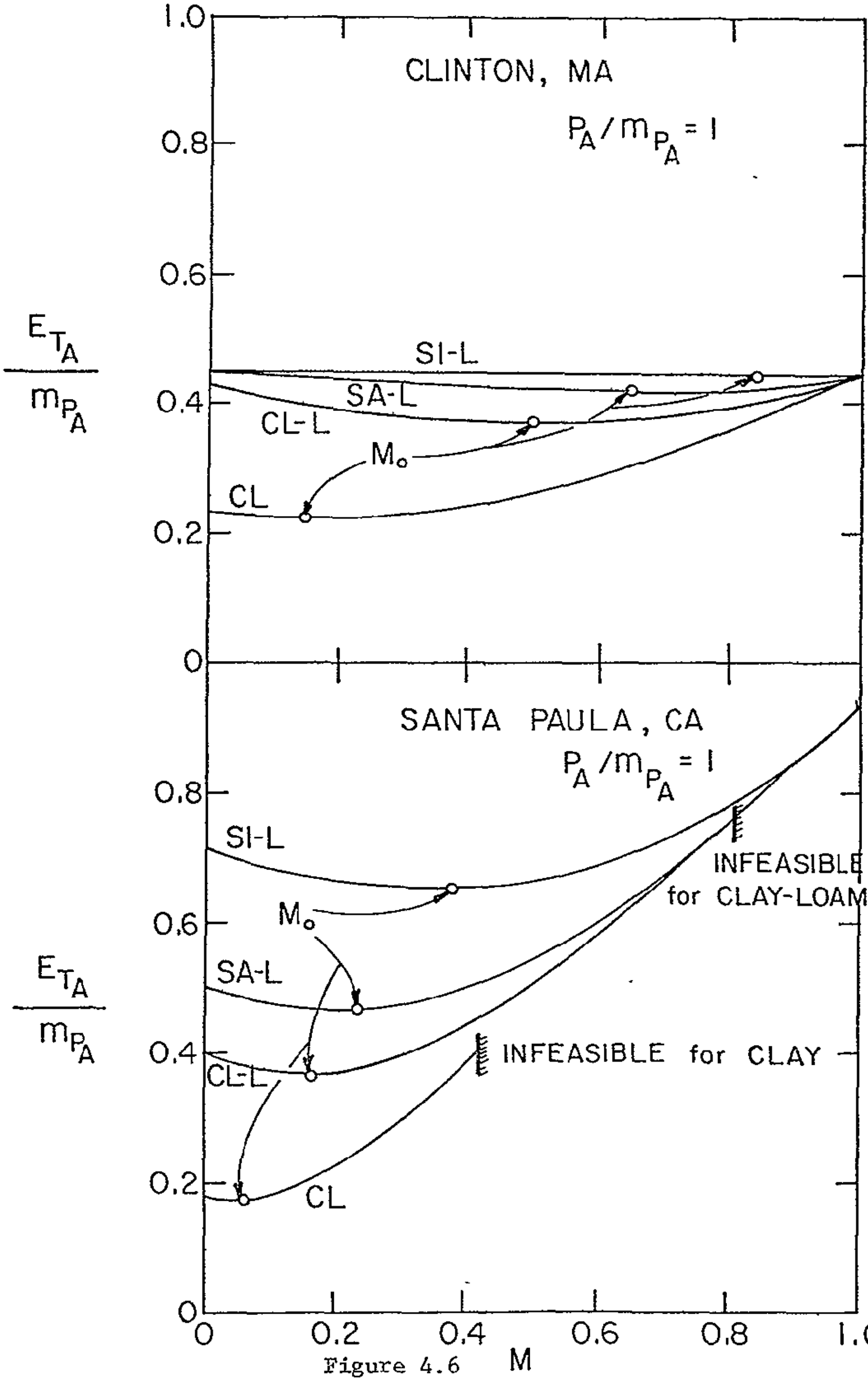


Figure 4.6

SENSITIVITY OF MEAN ANNUAL EVAPOTRANSPIRATION TO VEGETAL CANOPY FOR TYPICAL SOILS (k_v = 1 AND $w/\bar{e}_p$ << 1) which produces minimum stress under local climatic conditions. A necessary condition for minimum stress is that soil moisture take on the largest possible value. Thus, by using this hypothesis, the given climate, soil, and plant coefficient determine the equilibrium canopy density, $M = M_o$, through the water balance equation, where the soil moisture is maximum or, equivalently, where the soil moisture evapotranspiration is a minimum. Figure 4.7 illustrates the relationship between the dimensionless evapotranspiration parameter, E, and the dimensionless evapotranspiration function, J(E, kv), for the equilibrium condition, $M = M_o$. This plot is obtained by minimizing evapotranspiration from soil moisture for a given k_v, ($k_v = 1$ in this case), and E using Eagleson's constant soil column cross-section assumption. The expression for E is

$$E = \frac{2\beta n\ K(1)\ \Psi(1)\ \phi_e}{\pi m e_p^{-2}}\ s_o^{d+2} \tag{4.21}$$

in which

β = reciprocal of mean time between storms

n = porosity

$\psi(1)$ = saturated soil matrix potential

m = soil pore size distribution index

d = soil diffusivity index

ϕ_e = dimensionless desorption diffusivity

The other terms have been previously defined.

Also shown in Figure 4.7 is the M_o vs. E relationship for the equilibrium condition, $P_A = m_{P_A}$. As P_A varies from m_{P_A}, E and thus,

$J(E, M_o, k_v)$ will change accordingly, while to the first approximation, M will remain constant at M_o.

Eagleson (1978f) performs an asymptotic behavior analysis of the evapotranspiration function to gain insight into the meaning of the parameter, E. The evapotranspiration asymptotes shown in Figure 4.7 are thereby determined. The intersection of these two asymptotes occurs at $E = 2/\pi$, which separates soil controlled from climate-controlled evapotranspiration (Eagleson, 1978d). Thus, low values of E correspond to relatively dry, warm climates, while larger values indicate humid climates. As can be seen from Figure 4.7, low values of M_o occur for low E values, and vegetation densities approaching 1 correspond to a large E.

It can now be seen that observations of canopy cover will provide a key to determining the effective properties of a soil for a given climate. By using the vegetal equilibrium hypothesis in reverse, observations of M_o may be used in the water balance to obtain information about the soil if the climatic variables are known.

Figure 4.8, which is a plot of J vs. M_o, can be obtained directly from the information in Figure 4.7. Thus, from observations of vegetation density, the evapotranspiration efficiency, J, can be determined. To assure the generality of this relationship, the sensitivity of J to its independent parameters is studied. From the expression obtained by Eagleson (1978d), the primary parameters other than E and M are:

k_v = plant coefficient

λh_o accounts for storms which do not fill retention capacity

$\beta h_o/\bar{e}_p$ measures effect of surface retention on exfiltration

κ = parameter of Gamma distribution of storm depth

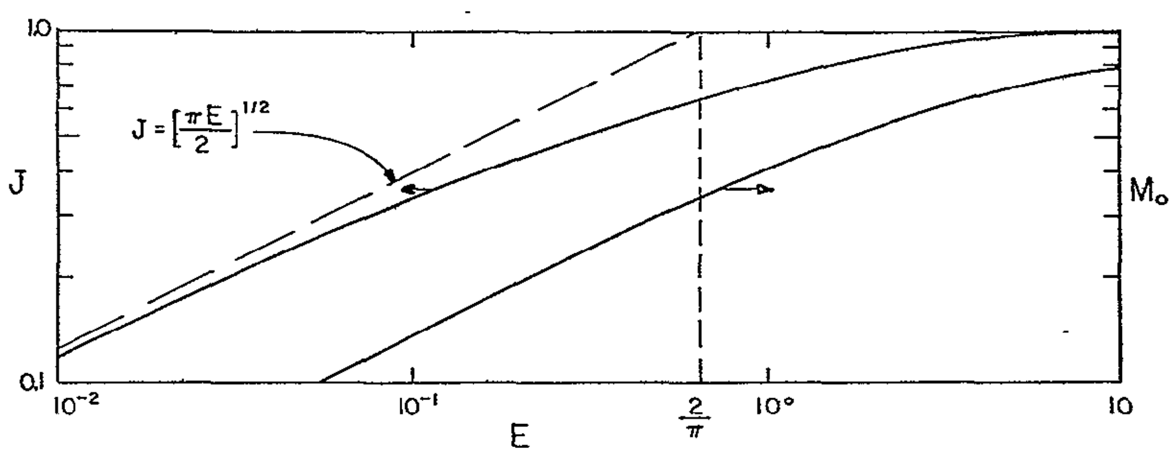


Figure 4.7

EVAPOTRANSPIRATION FUNCTION FOR NATURAL SYSTEMS AND EQUILIBRIUM VEGETAL CANOPY DENSITY ($k_v = 1$, $h_o = 0.0$, $\kappa = .5$, $w/\bar{e}_p << 1$)

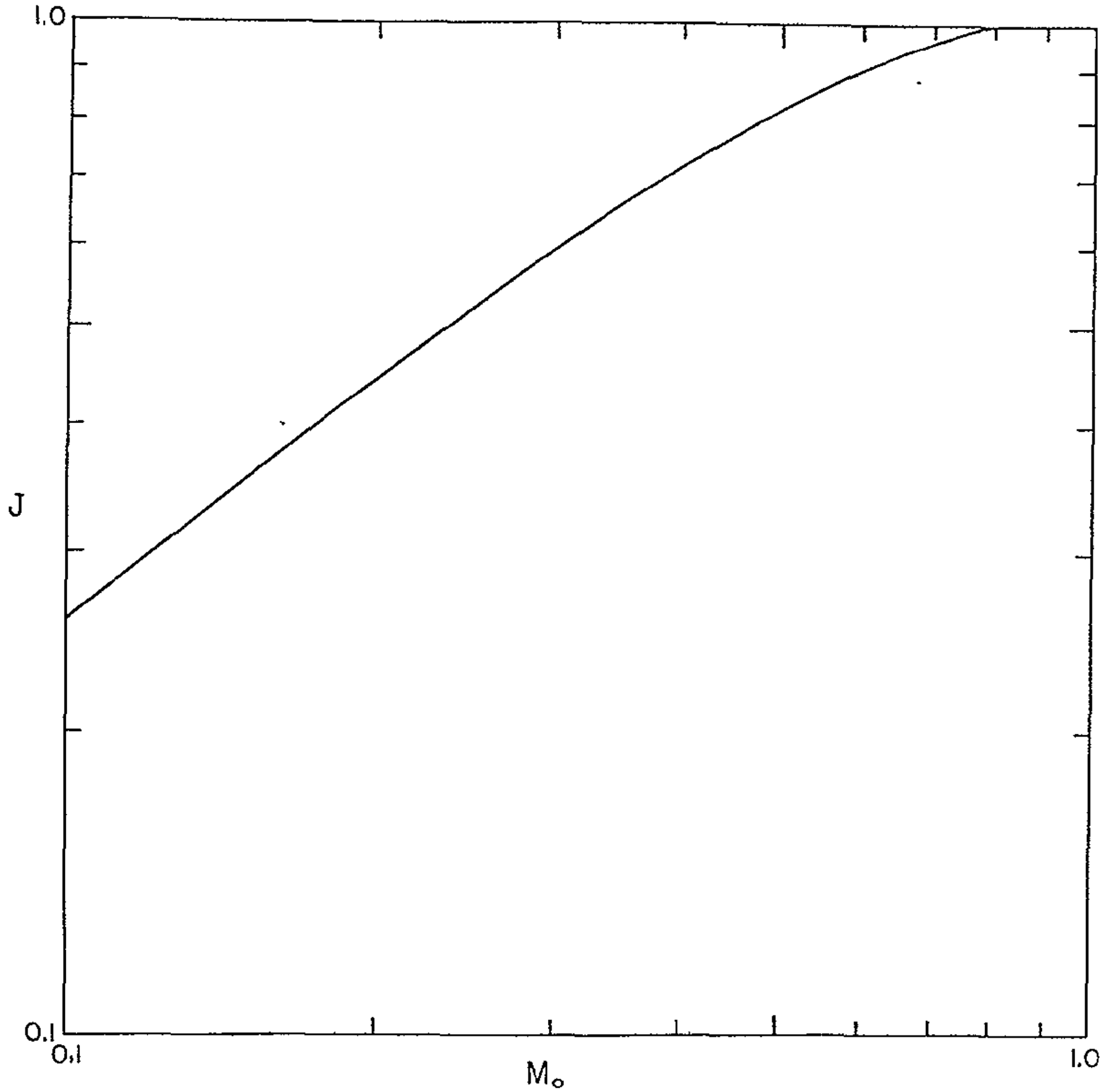


Figure 4.8

EVAPOTRANSPIRATION FUNCTION FOR NATURAL SYSTEMS
(k_v = 1, h_o = 0.0, κ = .5, $w/\bar{e}_p$ << 1)

The plots obtained by varying these parameters over their reported ranges are presented in Figures 4.9, 4.10 and 4.11. Figure 4.9, which holds κ and k_v constant at .5 and 1, respectively, illustrates the insensitivity of the evapotranspiration function to changes in λh_o as compared to $\beta h_o/\bar{e}_p$. By holding $\beta h_o/\bar{e}_p$ equal to .1 and k_v equal to 1, κ is varied in Figure 4.10. As can be seen, the function changes infinitesimally with changes in κ. In Figure 4.11, the two variables κ and λh_o are held constant at median values, and $\beta h_o/\bar{e}_p$ and k_v are allowed to vary. From this analysis, the evapotranspiration function is shown to be most sensitive to the two parameters, $\beta h_o/\bar{e}_p$ and k_v. Also shown in Figure 4.11 as dashed lines are the curves obtained using the alternate formulation of evapotranspiration, Equation (4.14), developed in Section 4.2. In review, this expression was developed by accounting for the effect of the vegetated fraction of the soil column surface on the vertical flux of the exfiltrating soil moisture in Equation (4.6). Expanding the Philip exfiltration equation, which was developed for the one-dimensional case of a constant cross-section, to two dimensions introduced an inconsistency with the results Philip obtained as explained in Section 4.2. By multiplying the term f_e^* in Equation (4.6) by (1-M), and not adjusting the terms on the right hand side of the expression, an infinite exfiltration capacity is obtained for the case when M = 1. Although the term (1-M) appears in the denominator of several components of the equation (4.14) for bare soil storm exfiltration volume, an infinite result is not obtained since the total volume of bare soil exfiltration, E_s, is weighted by (1-M) in

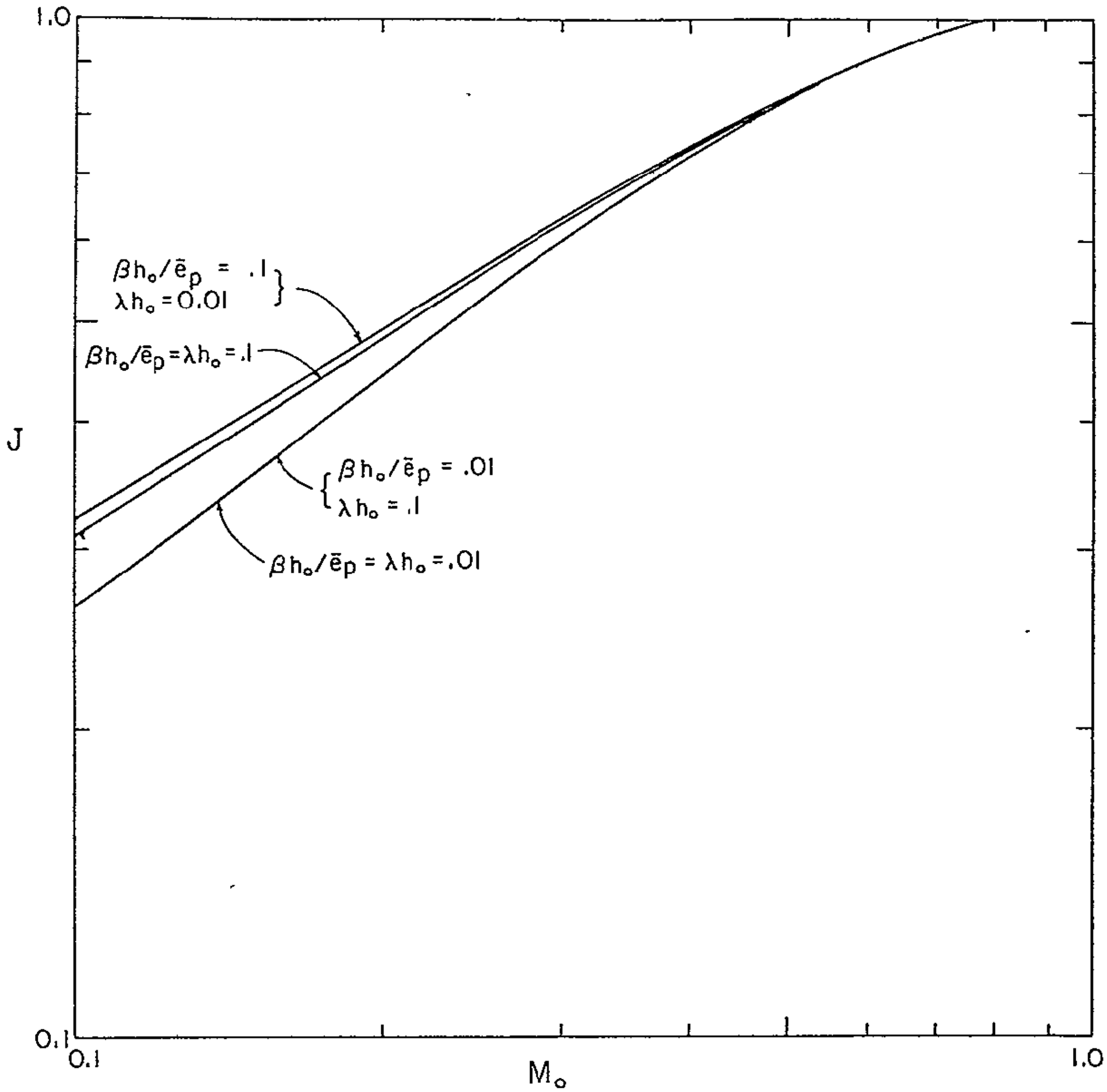


Figure 4.9

SENSITIVITY OF EVAPOTRANSPIRATION FUNCTION TO $\beta h_o/\bar{e}_p$ AND λh_o

($k_v = 1$, $\kappa = .5$, $w/\bar{e}_p << 1$)

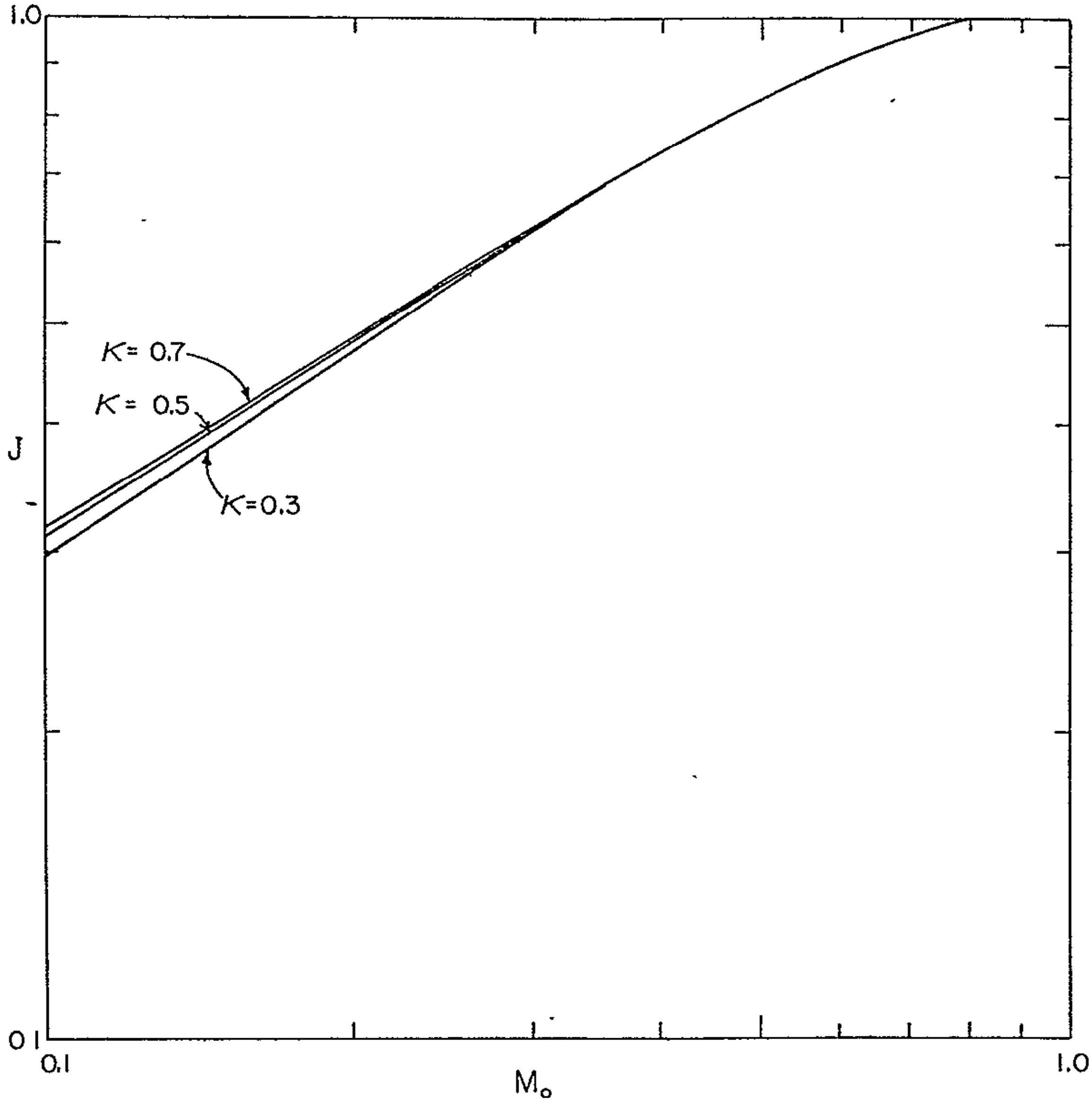


Figure 4.10

SENSITIVITY OF EVAPOTRANSPIRATION FUNCTION TO κ

($\beta h_o/\bar{e}_p$ = .1, k_v = 1, $w/\bar{e}_p$ << 1)

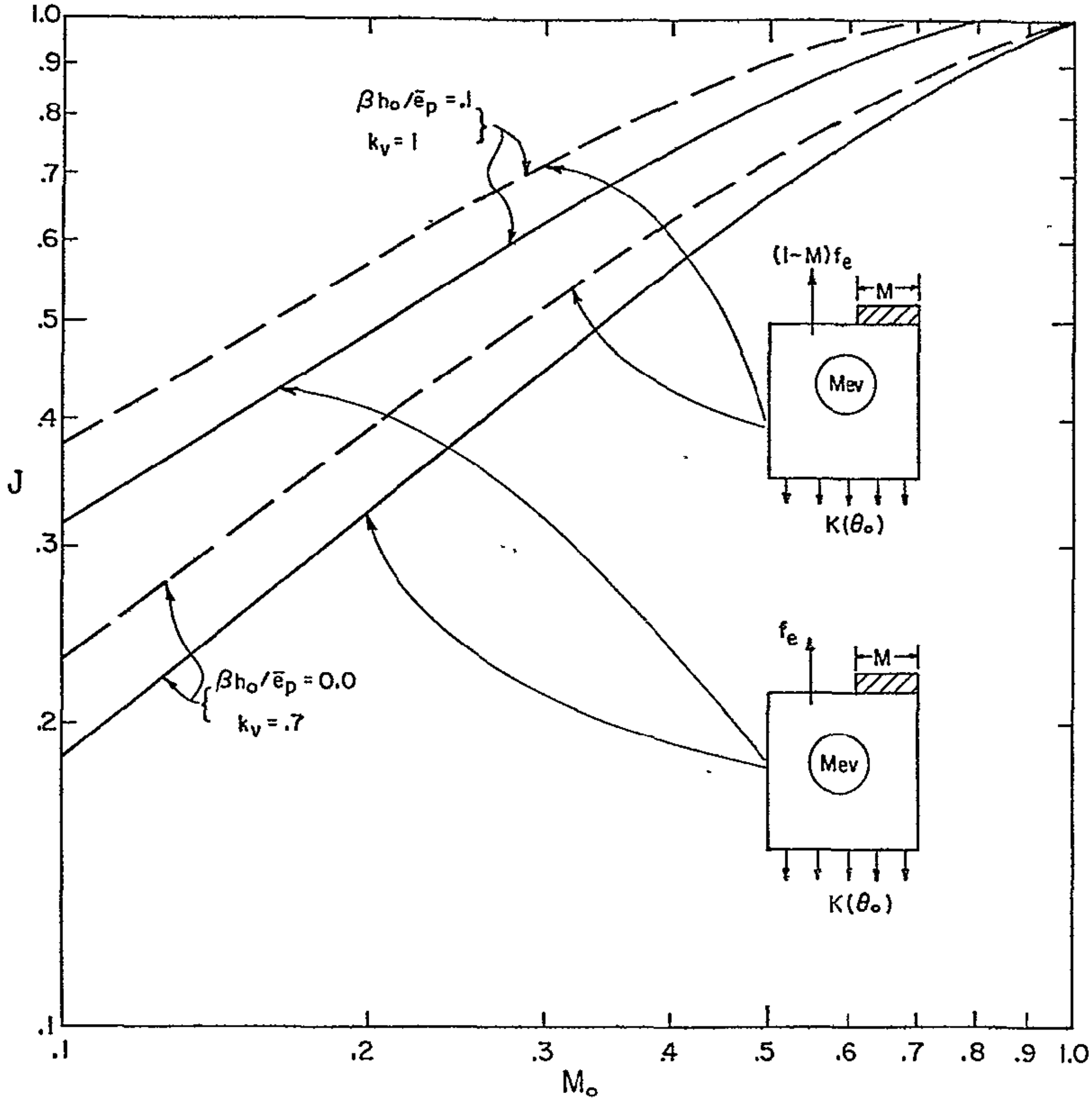


Figure 4.11

SENSITIVITY OF EVAPOTRANSPIRATION FUNCTION OF $\beta h_o/\bar{e}_p$ = .1 AND k_v

(κ = .5, λh_o = 0.5, $w/\bar{e}_p \ll 1$)

Equation (4.9). The net result of this modification is to raise the total evapotranspiration for a given value of M since the rate has been increased. This is seen in Figure 4.11 where the dashed lines are plotted above the corresponding solid lines for the same vegatation density. The main problem with this approach, as mentioned above, is that the terms on the right hand side of Equation (4.6) do not identically go to zero as M approaches 1. If the necessary corrections were known, the result would be a reduction in the bare soil exfiltration capacity for each value of M. This would lower the dashed lines of Figure 4.11. The actual function may therefore lie somewhere between these two sets of curves. With this in mind, these plots will be used in the following chapters to study the validity of the vegetal equilibrium hypothesis, and to determine its utility in estimating the effective average areal soil properties of a natural watershed.

Chapter 5

METHOD OF APPROACH

5.1 Vegetal Equilibrium Hypothesis

Verification of the vegetal equilibrium hypothesis presented in Chapter 4 is the first objective of this work. This can be accomplished through comparisons of actual data (from watersheds representing various types of climates) with the hypothesized relationship illustrated in Figure 4.11. In review, the hypothesis states that the vegetation denisty seeks that value, M_o, which maximizes soil moisture. This value maximizes water yield and thus, for a given climate and soil, minimizes evapotranspiration from soil moisture. Minimum evapotranspiration can be translated into a value of evaporation efficiency, J. (i.e. the ratio of actual to potential evapotranspiration) leading to the relationships previously presented in Figure 4.11.

The average annual water balance is presented by Eagleson (1978a) as

$$E[P_A]-E[E_{T_A}]=E[R_{S_A}]+E[R_{G_A}]=E[Y_A] \qquad (5.1)$$

which states that average annual precipitation minus average annual evapotranspiration will equal the average annual basin yield which is composed of surface runoff plus groundwater runoff. When analyzing catchment data to calculate average annual actual evapotranspiration, mean annual basin yield is subtracted from mean annual precipitation.

Although relatively accurate annual precipitation data are readily obtained from station records, yield information is only available in the form of streamflow records. Therefore, it is necessary that the total yield of the catchment studied appear as streamflow. This means that the entire groundwater component of yield must be influent to the stream channel upstream of the basin mouth. Under such conditions, most closely approached in humid climates, the total evapotranspiration is equal to precipitation minus streamflow. This restriction may lead to overestimating actual evapotranspiration if there are losses of yield to ungaged groundwater, or underestimation if there is contribution to streamflow of groundwater from adjacent watersheds.

Potential evapotranspiration is estimated by using the modified Penman equation (Penman, 1948). The form used here is the combination form as presented by Eagleson (1977)

$$\bar{e}_p = \frac{\bar{q}_i(1-A) - \bar{q}_b + H}{\rho_e L_e (1 + \gamma/\Delta)} \tag{5.2}$$

in which

$\bar{q}_i$ = average rate of insolation

$\bar{q}_b$ = average rate of net outgoing long wave radiation

H = average residual sensible heat flux

A = shortwave albedo of surface

ρ_e = mass density of evaporating water

L_e = latent heat of vaporization

γ/Δ = atmospheric parameter, a function of atmospheric temperature

The above parameters are calculated or estimated as follows:

$\bar{q}_i = \bar{q}_i(\phi)$; from Figure 5.1, where ϕ = latitude

$A = A$ (surface structure); from Table 5.1

$1/(1 + \gamma/\Delta) = f(\bar{T}_A)$; from Figure 5.2, where $\bar{T}_A$ = average annual temperature

$L_e = 597$ cal/gr

$\rho_e = 1$ gr/cm^3

$\bar{q}_b = (1 - .8N)[.245 - .45 \times 10^{-10}\bar{T}_A^4]$

$H = q_b(.25 + 1/(1 - S))$

The necessary climatic variables are available from U.S. Weather Bureau publications. They must be averaged over the rainy season which is assumed to be identical with the vegetation growing season.

Equation (5.2) gives the average potential evapotranspiration rate. The total potential volume is obtained by multiplying $\bar{e}_p$ by the season length as determined from monthly rainfall records.

With actual and potential evapotranspiration known, the only remaining variables needed for comparison with the hypothesis are the vegetation species (to obtain k_v) and the canopy density. The canopy density is estimated either from aerial photographs, from personal observation, or from literature available for the catchment studied. In this work, no photographs were available, and it was possible to estimate only ranges of density from the information in the literature, depending upon each author's method of measurement and interpretation. As a result of this uncertainty regarding the type and canopy density of

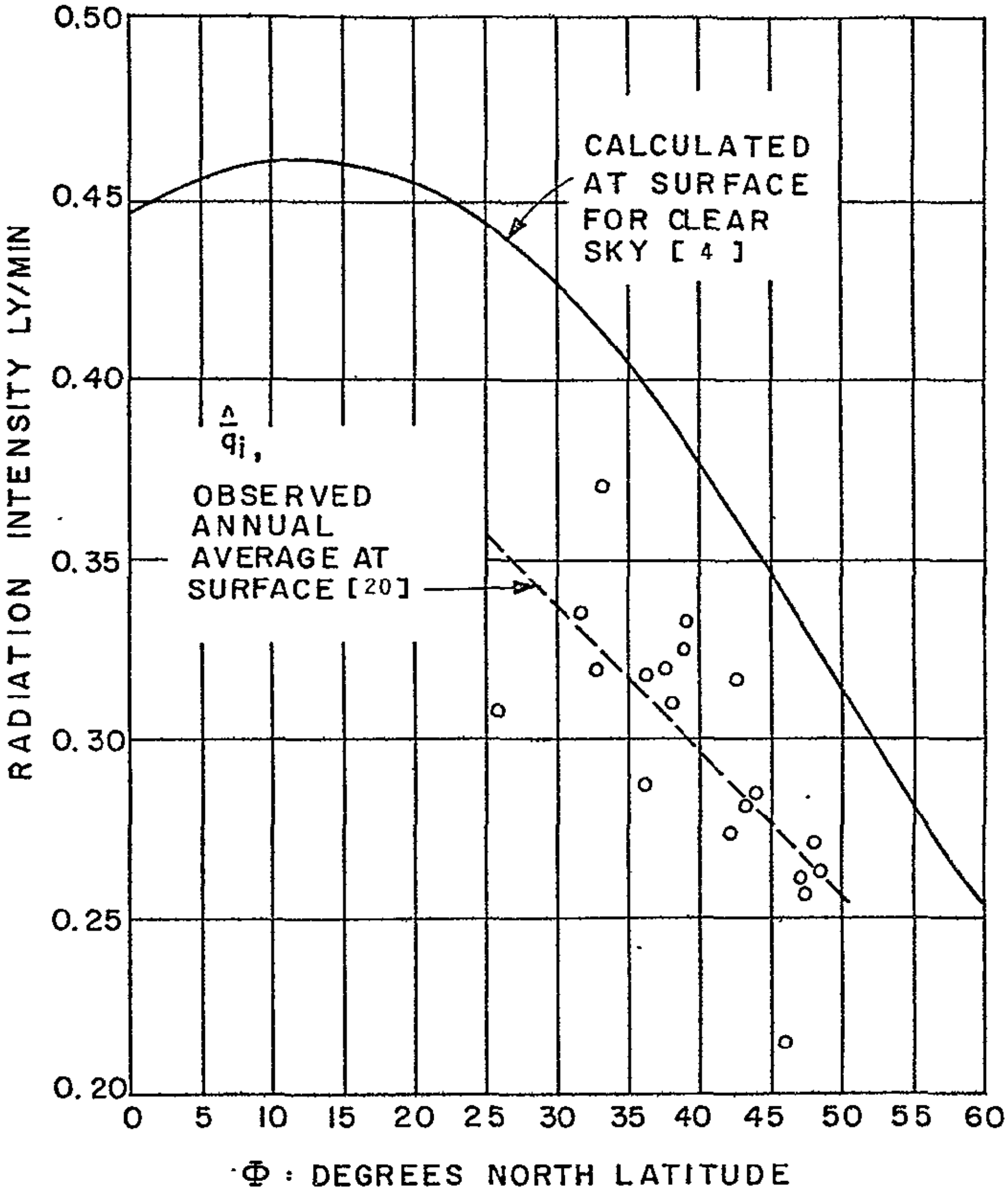


Figure 5.1

INSOLATION AT EARTH'S SURFACE

(from Ref. (15))

Table 5-1

Albedo of Natural Surfaces
(from Ref. [17])

Surface	Albedo, A	Surface	Albedo, A
Water	0.03-0.40	Spring wheat	0.10-0.25
Black, dry soil	0.14	Winter wheat	0.16-0.23
Black, moist soil	0.08	Winter rye	0.18-0.23
Gray, dry soil	0.25-0.30	High, dense grass	0.18-0.20
Gray, moist soil	0.10-0.12	Green grass	0.26
Blue, dry loam	0.23	Grass dried in sun	0.19
Blue, moist loam	0.16	Tops of oak	0.18
Desert loam	0.29-0.31	Tops of pine	0.14
Yellow sand	0.35	Tops of fir	0.10
White sand	0.34-0.40	Cotton	0.20-0.22
River sand	0.43	Rice field	0.12
Bright, fine sand	0.37	Lettuce	0.22
Rock	0.12-0.15	Beets	0.18
Densely urbanized areas	0.15-0.25	Potatoes	0.19
Snow	0.40-0.85	Heather	0.10
Sea ice	0.36-0.50		

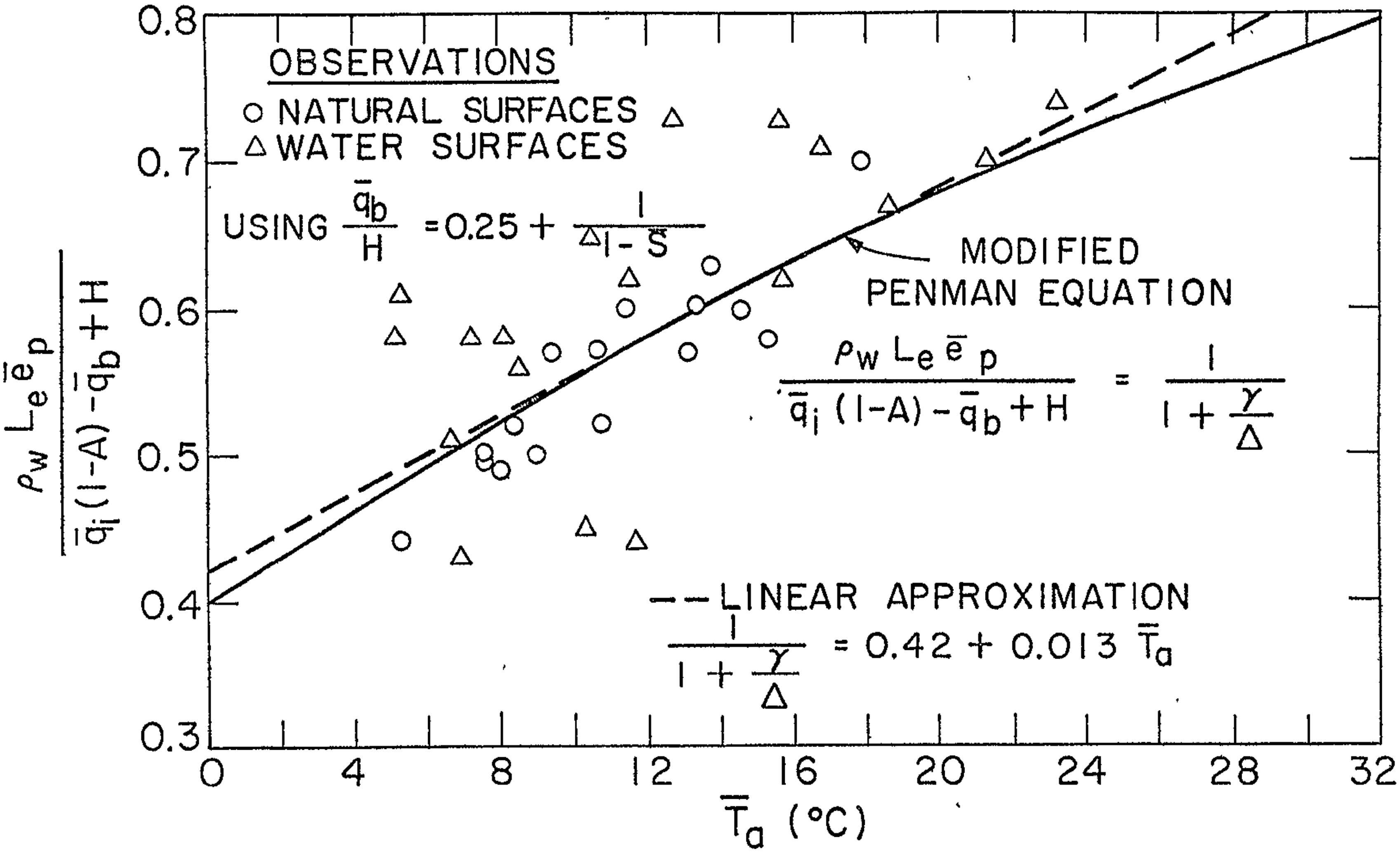


Figure 5.2

LONG-TERM AVERAGE POTENTIAL EVAPOTRANSPIRATION

(from Ref. (15))

the vegetation, the other variable which is a function of the surface structure, albedo, is subject to error as well. Therefore, the catchments studied will be plotted on Figure 4.11 in terms of the expected range of both J and M_o.

5.2 Estimation of Effective Soil Properties

The second goal of this work is to use the hypothesized relationship between vegetation canopy density and evapotranspiration to estimate effective average areal properties of the soil.

Three types of parameters are considered: climate, soil and vegetation. The climatic and vegetal properties are easily obtained from observations; this leaves the four soil parameters, s_o, k(1), n, and c to be determined from the derived relationships between climate, soil and vegetation.

The range of values of the porosity, n, is known to be quite small, from .25 to about .45, and does not have a large effect on solutions of the water balance equation. Assuming a value for n leaves the soil moisture, intrinsic permeability, and pore disconnectedness index as unknowns. To solve for these variables, three equations or relationships are needed which incorporate the vegetation and climate as well. The first relationship is the water balance, Equation (4.3), which expresses the soil moisture, s_o, as an implicit function of the climate, vegetation and soil. The vegetal equilibrium hypothesis provides the second relationship between the same three parameters. The third expression used is a rather weakly correlated regression between the

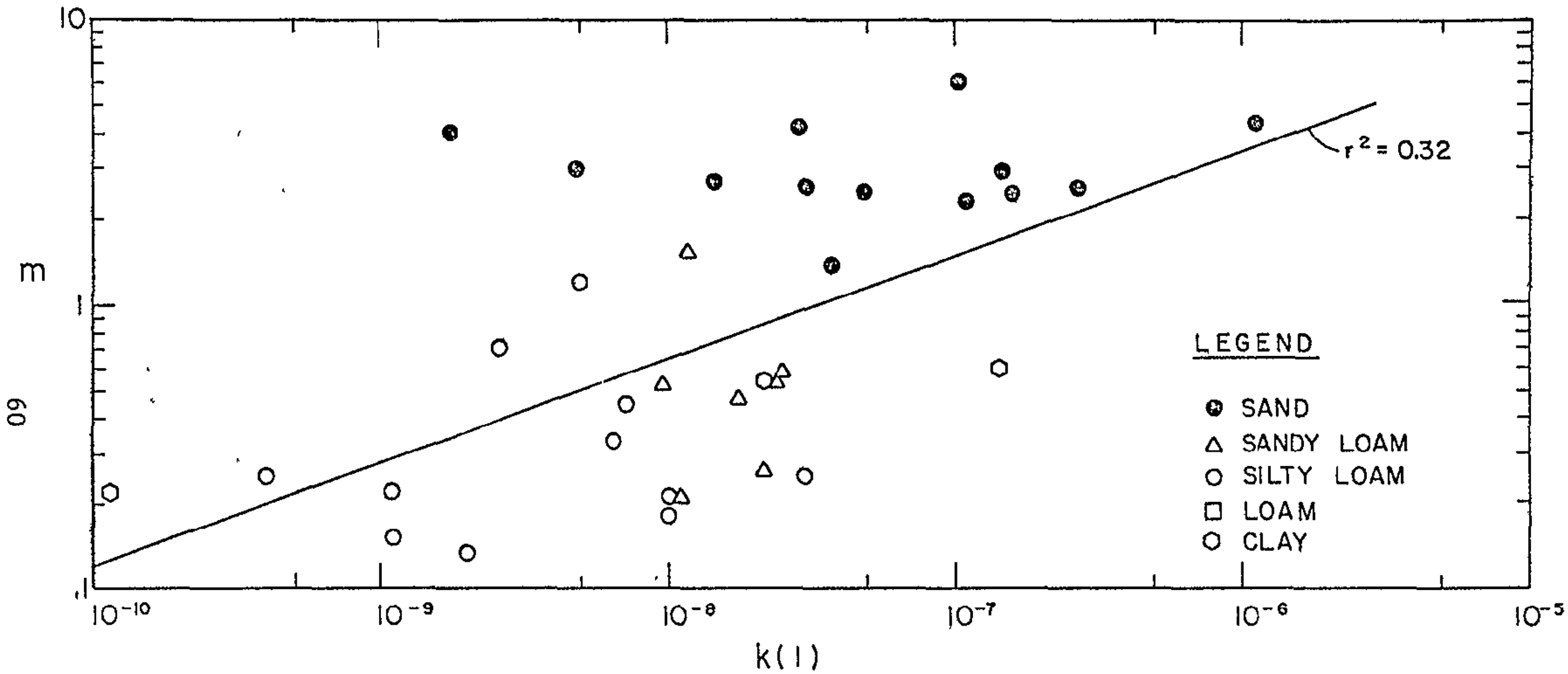


Figure 5.3

SATURATED PERMEABILITY VS. PORE SIZE DISTRIBUTION INDEX

(from Eagleson, Personal Communication)

intrinsic permeability of the soil, k(1), and its pore size distribution index, m, presented in Figure 5.3. This expression is (Eagleson, Personal Communication)

$$k(1) = \left(\frac{m}{512.7}\right)^{2.75} \tag{5.3}$$

where

$$m = 2/(c-3) \tag{5.4}$$

The coefficient of determination of this regression is small due to the extreme variability of these parameters in nature. The effect of this regression equation on the derived CDF of annual yield will be observed in Chapter 6.

In order to explain the procedure followed in the estimation of soil properties, it is necessary to present mathematically the water balance and the vegetal equilibrium hypothesis. The mean annual water balance, Equation (4.3), is again

$$m_{P_A}(1 - e^{-G-2\sigma}\Gamma(\sigma+1)\sigma^{-\sigma}) = E[E_{p_A}]\ J(E,M,k_v,h_o) - E[E_{r_A}] + m_\tau\ K(1)s_o^c - Tw$$

for

$$E[E_{r_A}]/m_{P_A} < e^{-G-2\sigma}\ \Gamma(\sigma + 1)\sigma^{-\sigma} \tag{5.5a}$$

Otherwise,

$$m_{P_A} = E[E_{p_A}]\ J(E,\ M,\ k_v,\ h_o) + m_\tau\ K(1)\ s_o^c - Tw \tag{5.5b}$$

If the interpretation of surface runoff developed in Section 4.3 is used, the above equation becomes

$$m_{p_A}(1 - e^{-G-2\sigma}\,\Gamma(\sigma + 1)\sigma^{-\sigma}) = E[E_{p_A}]\,J(E,M,k_v,h_o) + m_\tau K(1)s_o^c - Tw \tag{5.6}$$

Although the components and symbols have been defined earlier, their full expressions have not all been stated. In the above equations

$$G = \alpha K(1)[(1 + s_o)^c/2 - w/K(1)] \tag{5.7}$$

$$\sigma = \left[\frac{5n\eta^2\,K(1)\,\Psi(1)(1 - s_o)^2\,\phi_i(d,\,s_o)}{6\,\pi\,\delta\,m}\right]^{1/3} \tag{5.8}$$

$$E[E_{p_A}] = m_\nu\,m_{t_b}\,[1 - M(1 - k_v)]\,\bar{e}_p \tag{5.9}$$

$$K(1) = k(1)\,\gamma_w/\mu_w \tag{5.10}$$

where

$$\Psi(1) = \frac{\sigma_w}{\gamma_w}\left[\frac{n}{K(1)\,\phi(m)}\right]^{1/2} \tag{5.11}$$

in which

α = reciprocal of mean storm intensity

η = reciprocal of mean storm depth

$d = 2 + 1/m$

δ = reciprocal of mean storm duration

m_ν = mean number of storms

m_{t_b} = mean time between storms

γ_w = specific weight of water

σ_w = surface tension of water

μ_w = viscosity of water

$\phi(m)$ = pore shape parameter = $10^{0.66 + 0.55/m + 0.14/m^2}$

The vegetal equilibrium hypothesis states that

$$\frac{\partial E[E^*_{T_A}]}{\partial M} = 0 \qquad \text{at} \qquad M = M_o \tag{5.12}$$

where, as mentioned before,

$$E[E^*_{T_A}] = \text{evapotranspiration from soil moisture;}$$

which is

$$E[E^*_{T_A}] = J(E,\ M,\ k_v,\ h_o)\ E[E_{p_A}] - E[E_{r_A}] \tag{5.13}$$

with

$$\begin{aligned} E[E_{r_A}] = \frac{\bar{e}_p}{\beta} \cdot m_\nu \Bigg[(1 - M)\Bigg\{ 1 - e^{-\beta h_o/\bar{e}_p} \frac{\Gamma[\kappa,\ \lambda h_o]}{\Gamma(\kappa)} \\ - \left[1 + \frac{\beta h_o/\bar{e}_p}{\lambda h_o}\right]^{-\kappa} \frac{\gamma[\kappa,\ (\lambda h_o + \beta h_o/\bar{e}_p)]}{\Gamma(\kappa)} \Bigg\} \\ + k_v M \Bigg\{ 1 - e^{-\beta h_o/\bar{e}_p} \frac{\Gamma[\kappa,\ \lambda k_v h_o}{\Gamma(\kappa)} \\ - \left[1 + \frac{\beta h_o/\bar{e}_p}{\lambda k_v h_o}\right]^{-\kappa} \frac{\gamma[\kappa,\ (\lambda k_v h_o + \beta h_o/\bar{e}_p)]}{\Gamma(\kappa)} \Bigg\} \Bigg] \end{aligned} \tag{5.14}$$

Therefore,

$$\frac{\partial E[E^*_{T_A}]}{\partial M} = J(E,\ M,\ k_v,\ h_o)\ \frac{\partial E[E_{p_A}]}{\partial M} + E[E_{p_A}]\ \frac{\partial J(E,\ M,\ k_v,\ h_o)}{\partial M} - \frac{\partial E[E_{r_A}]}{\partial M} \tag{5.15}$$

For $k_v = 1$, all M sensitivity comes from $J(E, M, k_v, h_o)$, which will then have a minimum at M_o, in which special case, according to Equations (5.11) and (5.12),

$$\frac{\partial J(E, M, k_v, h_o)}{\partial M} = 0 \qquad \text{at } M = M_o \tag{5.16}$$

As discussed in Section 4.4, evapotranspiration efficiency, J, can be determined, for a given climate, from observations of vegetation density and species by using the vegetal equilibrium hypothesis, Eq. (5.14). The actual procedure for doing this is to pick a value for the evaporation parameter, E, and calculate J for different values of M until evapotranspiration from soil moisture, Eq. (5.13), is minimized. If the vegetation density obtained which minimizes E_{T_A} is not equal to the observed value, E is incremented and a new M_o is found. For a fixed climate, variations in E correspond to variations in the soil properties k(1), c, n, and s_o. Therefore, what is actually done is seeking the soil which produces the observed vegetation canopy density for a specific climate. Once this value of evapotranspiration is found, the value of E is also known, which is a function only of the soil parameters for a given climate.

With this information in mind, the following procedure is used to estimate the average areal effective soil parameters for a given set of climatic and vegetal parameters:

1. A value for n is assumed and k_v is set = 1

2. The above procedure is followed to determine E

3. The lowest possible value for c, approximately 3.1, is picked as an initial value

4. k(1) is calculated from Equation (5.3)

5. With these values for the three soil parameters, n, k(1), and c, it can be seen from Eq. (4.21) that s_o remains as the only variable needed for determining E. With E known from step 2, s_o is calculated

6. Annual precipitation is calculated via Equations (5.5) through (5.10)

7a. If the annual precipitation from Step 6 is not equal to the actual mean rainfall, c is incremented upward from its initially low value and Steps 4-6 are repeated.

7b. Due to the approximation introduced by using Equation (5.3), the precipitation, P_A, calculated in Step 6 may never exactly equal the actual mean value, m_{P_A}, for any value of c. P_A will approach m_{P_A} as c is increased, coming to within ΔP_A of equality at intermediate c before diverging again for large c. For low values of c, the calculated k(1) is large, representing a soil with high permeability and well connected pores. With evapotranspiration specified at the optimum (i.e., minimum) value, a large precipitation is therefore calculated in order to produce the inevitably large groundwater yield of the highly porous soil. For large c and small k(1), the soil is extremely impervious and the surface yield will be high. With minimum evapotranspiration, a large value for precipitation is again needed. Somewhere between these two extremes, a set of suitable soil parameters is obtained which gives an annual precipitation, P_A, which is closest to the actual mean, m_{P_A}. This relationship is illustrated in Figure 5.4. Holding c constant at the value which gives the minimum ΔP_A, k(1) is then deviated from regression equation (5.3) until another minimum in calculated precipitation is reached. If this value is above the mean precipitation, c is decreased, if it is below the mean, c is increased. Another search is done on k(1) until the minimum precipitation is found. This step is repeated until the minimum calculated precipitation is equal to the mean

8. If the values obtained for k(1) and c are not consistent with the assumed porosity, n is adjusted to a more appropriate value corresponding to a more pervious or impervious soil type depending on the values of k(1) and c. Steps 1 through 7 are repeated.

The soil parameters obtained from Steps 1-9 are used to construct the CDF of annual yield in the same manner as Eagleson (1978g). In this paper, the annual water balance is written as

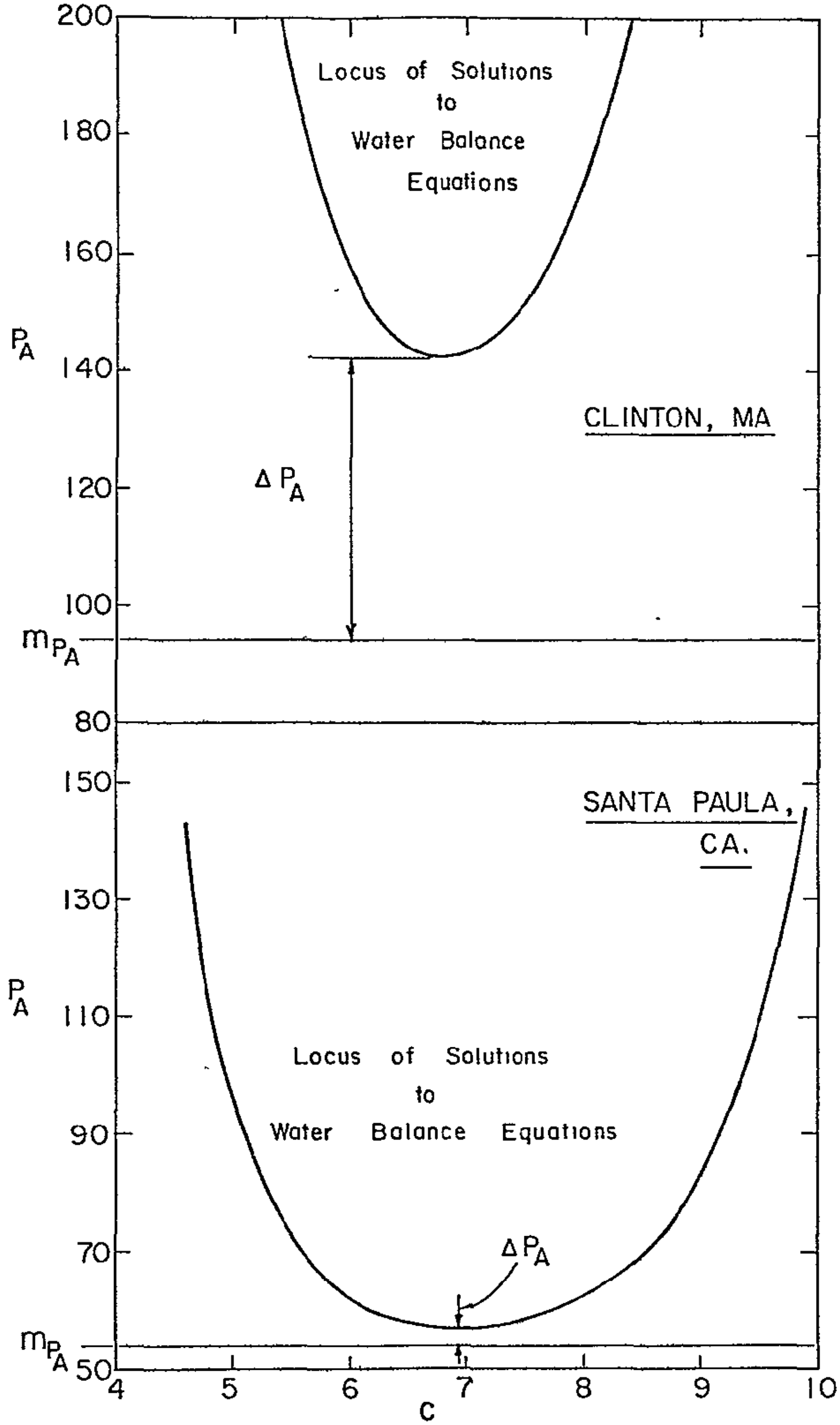


Figure 5.4

WATER BALANCE SOLUTIONS USING SOIL PROPERTIES FROM EQUATION (5.3)

$$Y_A = P_A - E_{T_A} = R_{s_A} + R_{g_A} \qquad (5.17)$$

In order to relate the annual water balance, (5.17), to the mean annual water balance, (5.1), Eagleson defines a climatic mean, m_c, where $P_A = m_{P_A}$ and $E_{T_A} = E[E_{T_A}]$, and expands (5.17) about this point in a multidimensional Taylor expansion [Hildebrand, 1959, p. 353]. By taking expected values of this expansion term by term, neglecting higher order terms, and assuming all variances, covariances, and curvatures are small, the "first order approximation" of $E[Y_A]$ is obtained:

$$E[Y_A] = P_A - E_{T_A} = R_{s_A} + R_{g_A} \qquad (5.18)$$

This allows the use of the mean annual water balance equation to calculate annual values by letting the annual precipitation, and thus the average annual soil moisture, vary. The CDF of annual yield can then be calculated. Comparison of this CDF with that obtained from observations of annual streamflow provides the test for the accuracy of the estimated parameters, n, k(1) and c.

Chapter 6 will present the results of this procedure in the form of annual CDF's of basin yield, in addition to verification of the equilibrium vegetation density hypothesis.

Chapter 6

PRESENTATION OF RESULTS

6.1 Verification of Vegetal Equilibrium Hypothesis

The results of the applied methods of analysis explained in Chapter 5 are presented in this chapter. The vegetal equilibrium hypothesis is verified first in order to assure its validity for use in the estimation of soil parameters.

Appendix A presents the individual catchments studied, the data used, location of the catchment, the values obtained for potential and actual evapotranspiration, vegetation density, and the estimated value of J.

Figure 6.1 presents the agreement of these experimental data with the hypothesized theoretical curves of Figure 4.11. As can be seen, the dashed curves, which represent the derivation accounting for the presence of vegetation at the surface of the soil column in the exfiltration equation, provides a better fit for catchments with a vegetal canopy density greater than 0.2. This may mean that the presence of vegetation has a much greater effect on soil moisture exfiltration than previously believed. Although the equation used has serious flaws, they may be negligible compared to the possible importance of the presence of vegetation.

Possible reasons for catchments W-4, W-5, and part of W-8 lying above the curve may be ungaged yield which escapes through

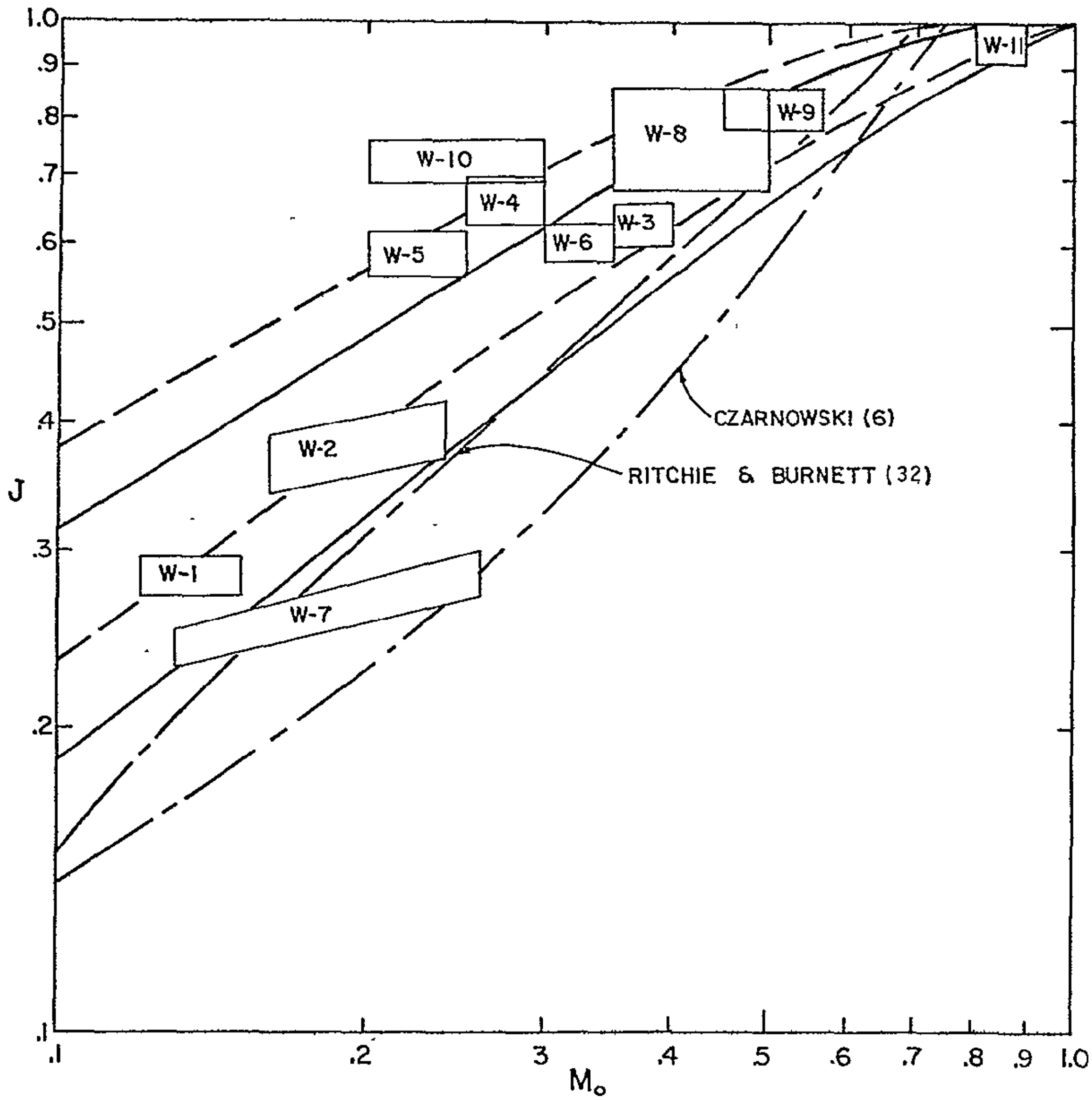


Figure 6.1

VERIFICATION OF VEGETAL EQUILIBRIUM HYPOTHESIS groundwater aquifers, or flaws in the vegetal equilibrium hypothesis. Until it can be determined if all these yields are present in the observed streamflows, the vegetal equilibrium hypothesis would seem to give a reasonably accurate relationship between vegetation density and evapotranspiration.

Also shown on Figure 6.1 are the results obtained from the empirical formulas developed by Czarnowski (1964) and Ritchie and Barnett (1971). These functions exhibit the same type of relationship between evaporation efficiency and vegetation density, but do not fit the observed data quite so well as Equations (4.10) or (4.14). The fact that the data for these studies are primarily from agricultural areas, where cultivation and irrigation significantly violate the assumption of natural watersheds, is a likely reason for the poor observed fit.

6.2 Estimation of Effective Average Areal Soil Properties

To determine the accuracy of the procedure described in Section 5.2, the two catchments studied here will be those studied by Eagleson (1978f, g); Clinton, Ma. and Santa Paula, Ca. Table 6.1 presents the list of necessary input variables (Eagleson, 1978g) and the computer program employed is listed in Appendix B. Tables 6.2 and 6.4 list the results obtained for the inputs given in Table 6.1. Listed probabilities are calculated for given P_A/m_{P_A} using the Poisson model of Eagleson (1978b). In Clinton, the value of M_o is held constant for the entire range of soil moistures, while in Santa Paula, the vegetation density is allowed to vary with annual precipitation, as explained by Eagleson

(1978g). These results are obtained using the form of surface runoff developed in Section 4.3 and presented in Section 5.2, (Equation 5.6). Figures 6.2 and 6.3 present the results in the form of CDF's of annual yield.

Figure 6.2, which represents Santa Paula, also shows the CDF obtained by Eagleson (1978g) for his silty-loam soil, which is listed in Table 6.3. The soil properties estimated by the algorithm explained in Section 5.2 indicated a slightly less permeable soil than the silty loam. This soil gives an improved fit over the entire range of CDF values, especially in the critical lower tail.

The results for Clinton are illustrated in Figure 6.3. The soil properties obtained in this case indicate, again, a more impermeable soil than the silty-loam employed by Eagleson. Although these values for k(1) and c are quite different, the resulting CDF of annual yield is indistinguishable from that obtained for silty-loam. To facilitate the comparison between the two results, Table 6.5 lists the annual water balance components for Clinton, using the silty-loam soil properties. Since the estimated soil properties represent a tighter soil which reduces the mobility of moisture, the soil moisture values are higher than for the silty-loam. The other major differences between the two soils are the values for surface and groundwater runoff. The more permeable silty-loam yields a large groundwater component, and a surface runoff component which seems unrealistically low for all values of annual precipitation. In the case of the estimated soil properties, the surface runoff is

Clinton, Mass.		Santa Paula, Ca.
.15	$\bar{e}_p$ cm/day	.273
3	m_{t_b} days	10.4
.32	m_{t_r} days	1.43
365	m_τ days	212
.5	κ	.25
.1	h_o cm	.1
0	$w/\bar{e}_p$	0
8.4	T_A °C	13.8
94	m_{P_A} cm	54
1	k_v	1
.8	M_o	.4

Table 6.1

INPUT CLIMATE AND VEGETATION PARAMETERS

EFFECTIVE AREAL AVERAGE SOIL PROPERTIES

k(1) = 0.2815708E-09 n=0.35 c= 6.204

so	E	Mo	Pa/mpa	Ya/mpa	RSA	RGA	ETA	PROB
0.30	0.027	0.068	0.2108	0.0158	0.6264	0.225	10.533	0.0216
0.32	0.038	0.082	0.2454	0.0202	0.7565	0.335	12.158	0.0314
0.34	0.054	0.099	0.2833	0.0258	0.9071	0.488	13.901	0.0446
0.36	0.074	0.117	0.3246	0.0329	1.0807	0.696	15.752	0.0619
0.38	0.101	0.138	0.3694	0.0417	1.2798	0.974	17.694	0.0840
0.40	0.134	0.160	0.4177	0.0527	1.5077	1.339	19.712	0.1114
0.42	0.176	0.183	0.4697	0.0663	1.7679	1.812	21.786	0.1446
0.44	0.228	0.209	0.5255	0.0830	2.0648	2.418	23.896	0.1839
0.46	0.293	0.236	0.5854	0.1035	2.4036	3.186	26.021	0.2294
0.48	0.372	0.264	0.6496	0.1285	2.7909	4.148	28.141	0.2809
0.50	0.468	0.294	0.7187	0.1589	3.2345	5.344	30.233	0.3380
0.52	0.582	0.324	0.7933	0.1956	3.7443	6.816	32.277	0.4001
0.54	0.720	0.355	0.8741	0.2397	4.3323	8.614	34.253	0.4662
0.56	0.882	0.387	0.9621	0.2927	5.0138	10.794	36.144	0.5352
0.58	1.074	0.419	1.0585	0.3561	5.8080	13.420	37.934	0.6056
0.60	1.298	0.451	1.1650	0.4315	6.7388	16.561	39.610	0.6753
0.62	1.560	0.483	1.2833	0.5210	7.8362	20.297	41.163	0.7424
0.64	1.864	0.514	1.4156	0.6269	9.1381	24.716	42.586	0.8046
0.66	2.215	0.544	1.5645	0.7520	10.6921	29.914	43.874	0.8595
0.68	2.618	0.574	1.7331	0.8992	12.5580	36.001	45.027	0.9054
0.70	3.079	0.602	1.9250	1.0723	14.8116	43.094	46.046	0.9412
0.72	3.606	0.629	2.1446	1.2754	17.5485	51.324	46.936	0.9668
0.74	4.204	0.655	2.3968	1.5134	20.8905	60.833	47.703	0.9833

Table 6.2

ANNUAL WATER BALANCE COMPONENTS, SANTA PAULA, CA. ESTIMATED SOIL PROPERTIES. EQUATION (5.6), M_o = .4

EFFECTIVE AREAL AVERAGE SOIL PROPERTIES

k(1) = 0.1200000E-08 n=0.35 c= 6.000

so	E	Mo	Pa/mpa	Ya/mpa	RSA	RGA	ETA	PROB
0.26	0.031	0.073	0.2174	0.0106	0.0538	0.519	11.165	0.0233
0.28	0.047	0.091	0.2592	0.0162	0.0673	0.809	13.120	0.0360
0.30	0.068	0.112	0.3062	0.0242	0.0835	1.224	15.225	0.0539
0.32	0.097	0.135	0.3586	0.0353	0.1027	1.803	17.456	0.0784
0.34	0.135	0.161	0.4168	0.0504	0.1256	2.593	19.789	0.1109
0.36	0.185	0.188	0.4815	0.0705	0.1527	3.654	22.194	0.1526
0.38	0.250	0.218	0.5533	0.0970	0.1850	5.055	24.639	0.2046
0.40	0.331	0.250	0.6332	0.1315	0.2234	6.876	27.092	0.2675
0.42	0.433	0.284	0.7223	0.1756	0.2691	9.215	29.521	0.3410
0.44	0.559	0.318	0.8222	0.2316	0.3238	12.182	31.892	0.4240
0.46	0.714	0.354	0.9347	0.3018	0.3896	15.906	34.177	0.5142
0.48	0.902	0.391	1.0620	0.3889	0.4688	20.533	36.349	0.6080
0.50	1.129	0.427	1.2070	0.4962	0.5647	26.232	38.383	0.7004
0.52	1.401	0.464	1.3729	0.6273	0.6813	33.191	40.262	0.7859
0.54	1.724	0.500	1.5634	0.7861	0.8233	41.626	41.973	0.8592
0.56	2.106	0.535	1.7830	0.9773	0.9968	51.777	43.508	0.9162
0.58	2.554	0.569	2.0367	1.2059	1.2092	63.911	44.864	0.9559
0.60	3.078	0.602	2.3304	1.4777	1.4692	78.327	46.043	0.9799

Table 6.3

ANNUAL WATER BALANCE COMPONENTS, SANTA PAULA, CA.
SILTY-LOAM SOIL PROPERTIES. EQUATION (5.6), $M_o = .4$

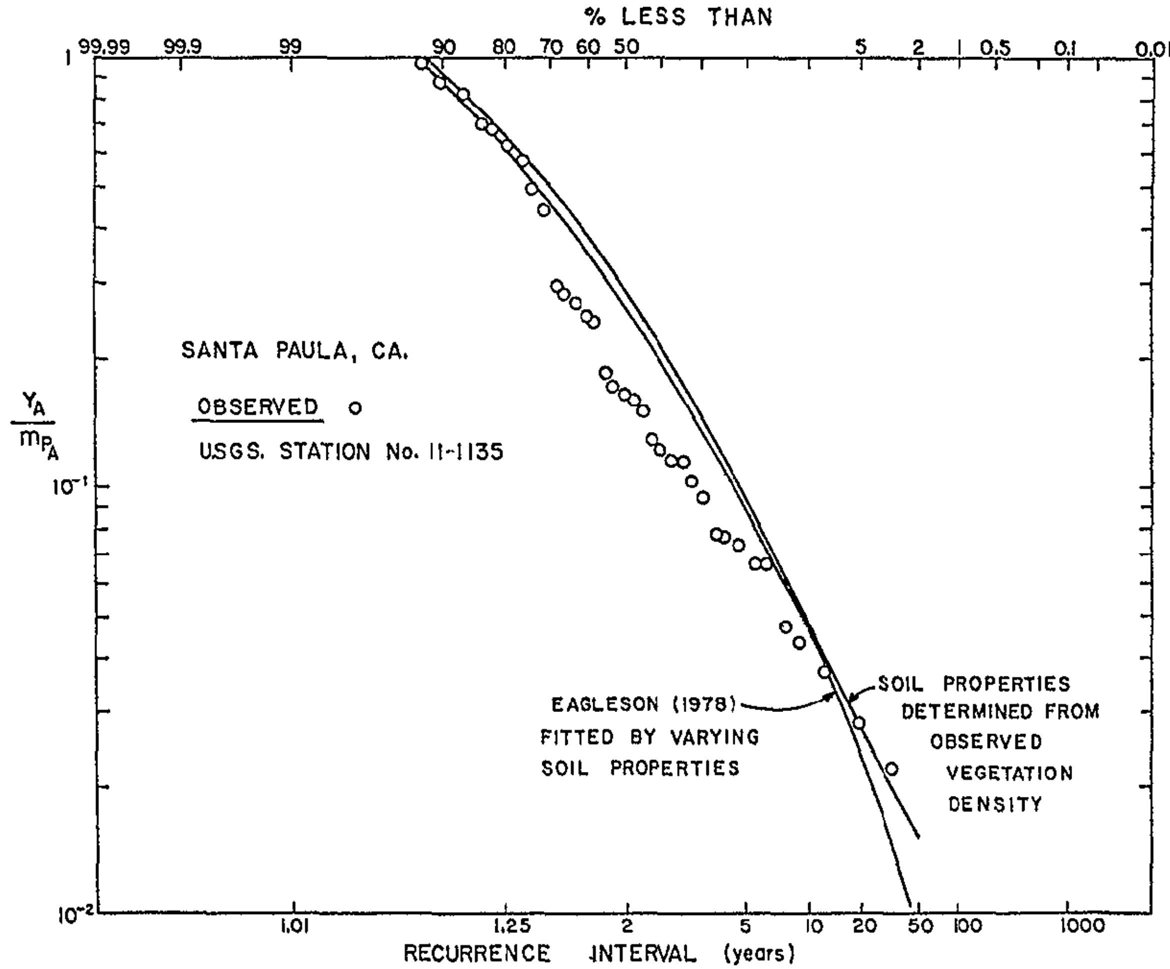


Figure 6.2

FREQUENCY OF ANNUAL BASIN YIELD, SANTA PAULA, CA.

EFFECTIVE AREAL AVERAGE SOIL PROPERTIES

k(1)= 0.1947604E-09 n=0.35 c= 7.399

so	E	Mo	Pa/mpa	Ya/mpa	RSA	RGA	ETA	PROB
0.60	4.187	0.800	0.6801	0.1776	7.5385	9.153	47.241	0.0180
0.62	5.131	0.800	0.7225	0.2145	8.5004	11.666	47.752	0.0373
0.64	6.247	0.800	0.7725	0.2598	9.6665	14.755	48.195	0.0766
0.66	7.560	0.800	0.8317	0.3151	11.0936	18.528	48.562	0.1531
0.68	9.097	0.800	0.9023	0.3826	12.8580	23.108	48.852	0.2873
0.70	10.888	0.800	0.9869	0.4649	15.0614	28.636	49.071	0.4866
0.72	12.966	0.800	1.0887	0.5650	17.8406	35.272	49.227	0.7155
0.74	15.367	0.800	1.2118	0.6870	21.3803	43.200	49.332	0.8959
0.76	18.130	0.800	1.3612	0.8357	25.9318	52.623	49.398	0.9795

Table 6.4

ANNUAL WATER BALANCE COMPONENTS, CLINTON, MA.
ESTIMATED SOIL PROPERTIES. EQUATION (5.6), M_o = .8

EFFECTIVE AREAL AVERAGE SOIL PROPERTIES

k (1) = 0.1200000E-08 n=0.35 c= 6.000

so	E	Mo	Pa/mpa	Ya/mpa	RSA	RGA	ETA	PROB
0.44	5.540	0.800	0.7032	0.1933	0.2482	17.925	47.932	0.0272
0.46	7.074	0.800	0.7674	0.2521	0.2936	23.404	48.441	0.0716
0.48	8.940	0.800	0.8446	0.3251	0.3509	30.213	48.827	0.1742
0.50	11.190	0.800	0.9375	0.4151	0.4239	38.598	49.099	0.3673
0.52	13.884	0.800	1.0493	0.5251	0.5174	48.839	49.274	0.6330
0.54	17.087	0.800	1.1837	0.6584	0.6376	61.250	49.378	0.8653
0.56	20.871	0.800	1.3448	0.8189	0.7930	76.185	49.432	0.9750

Table 6.5

ANNUAL WATER BALANCE COMPONENTS, CLINTON, MA.
SILTY-LOAM SOIL PROPERTIES. EQUATION (5.6), M_o = .8

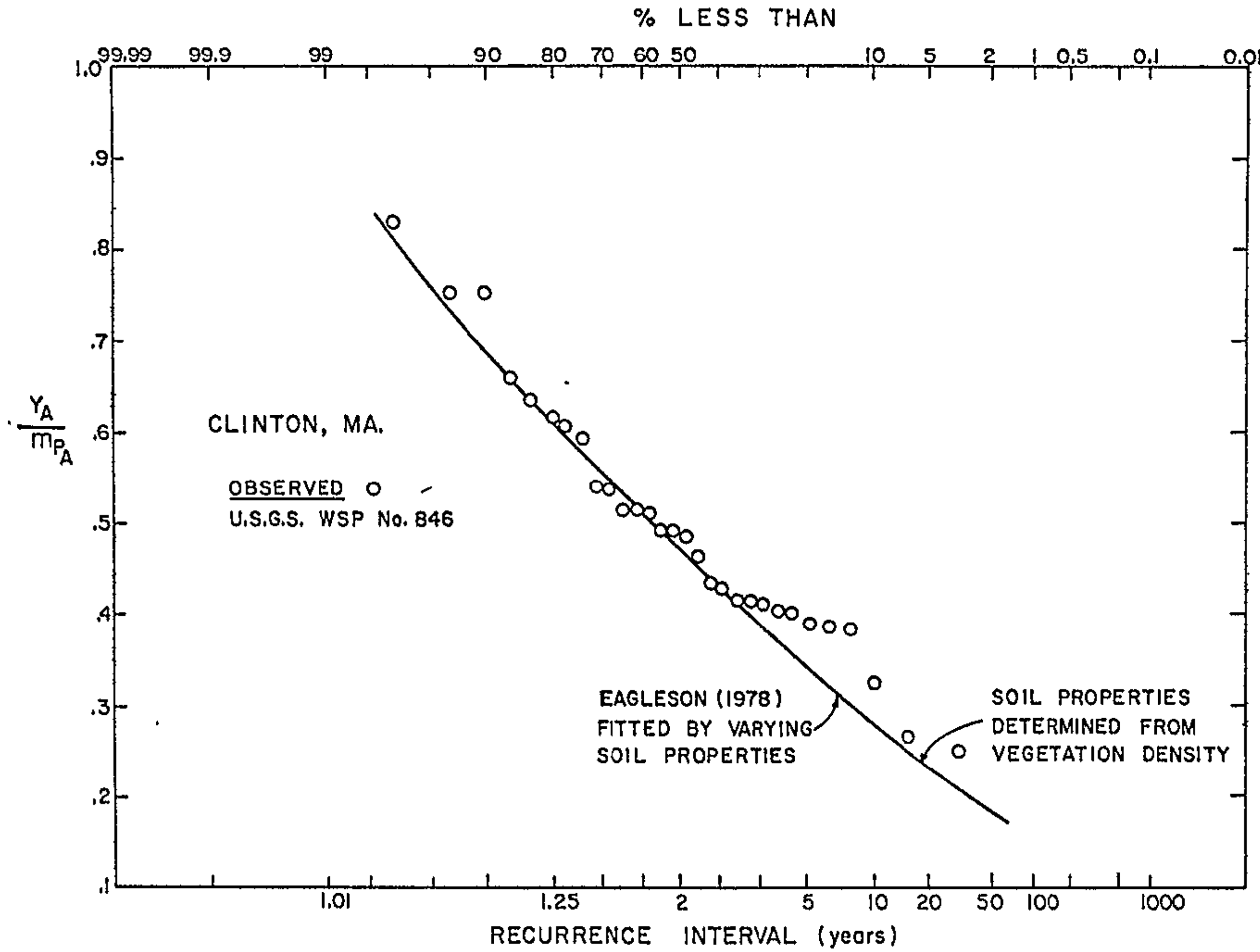


Figure 6.3

FREQUENCY OF ANNUAL BASIN YIELD, CLINTON, MA.

greater and the groundwater runoff lower than for the silty-loam soil. The identical CDF's of yield for the two soils can be explained in the following manner:

In this development, the storage of moisture is not taken into account, therefore, yield is equal to precipitation minus evapotranspiration. In the Clinton system, evapotranspiration is controlled primarily by the climate (Eagleson, 1978d), and is relatively insensitive to the soil properties except for extreme cases. Thus, for a given precipitation, evapotranspiration and hence yield, will be the same for different types of soil. The only variations occur in the proportioning of yield between surface and groundwater runoff. The permeable soil encourages gravitational percolation and hence groundwater, while the impermeable soil rejects precipitation as surface runoff.

In Santa Paula, where evapotranspiration is primarily soil controlled, the yield is more sensitive to changes in the soil properties, and thus there is a difference in the CDF's for the two different soils.

In Figure 6.4, the estimated soil properties are used to show the effect on the yield CDF of the two methods of handling surface retention in calculating surface runoff. As expected, the values obtained for yield, using Eq. (5.5) are reduced from those calculated by Eq. (5.6) due to the reduction of rainfall excess in favor of surface retention. Although the difference between the two equations is not large, Equation (5.6) still fits the observed data better in the lower tail.

Tables 6.6 and 6.7 list the CDF's obtained for Clinton and Santa Paula, using Eq. (5.5). Again, in the case of Clinton, the CDF is

EFFECTIVE AREAL AVERAGE SOIL PROPERTIES

k(1) = 0.2964968E-09 n=0.35 c= 6.423

so	E	Mo	Pa/mpa	Ya/mpa	RSA	RGA	ETA	PROB
0.32	0.031	0.074	0.2208	0.0128	0.4166	0.275	11.233	0.0243
0.34	0.044	0.089	0.2556	0.0170	0.5129	0.406	12.885	0.0348
0.36	0.062	0.106	0.2937	0.0225	0.6267	0.586	14.649	0.0487
0.38	0.084	0.125	0.3353	0.0295	0.7608	0.830	16.513	0.0669
0.40	0.112	0.146	0.3803	0.0384	0.9181	1.153	18.463	0.0898
0.42	0.149	0.168	0.4289	0.0496	1.1018	1.578	20.481	0.1182
0.44	0.194	0.193	0.4813	0.0638	1.3161	2.127	22.548	0.1525
0.46	0.250	0.218	0.5378	0.0814	1.5658	2.830	24.646	0.1930
0.48	0.319	0.246	0.5987	0.1033	1.8567	3.720	26.754	0.2399
0.50	0.402	0.274	0.6645	0.1302	2.1960	4.835	28.849	0.2930
0.52	0.503	0.304	0.7357	0.1632	2.5923	6.221	30.913	0.3521
0.54	0.624	0.334	0.8131	0.2034	3.0565	7.927	32.924	0.4165
0.56	0.768	0.366	0.8977	0.2521	3.6023	10.013	34.863	0.4852
0.58	0.939	0.397	0.9908	0.3109	4.2466	12.544	36.715	0.5569
0.60	1.139	0.429	1.0939	0.3816	5.0109	15.595	38.463	0.6297
0.62	1.374	0.461	1.2087	0.4662	5.9222	19.251	40.097	0.7014
0.64	1.647	0.492	1.3375	0.5670	7.0147	23.605	41.606	0.7695
0.66	1.964	0.523	1.4830	0.6869	8.3314	28.763	42.986	0.8313
0.68	2.329	0.553	1.6482	0.8291	9.9270	34.842	44.231	0.8842
0.70	2.748	0.582	1.8368	0.9971	11.8709	41.972	45.342	0.9266
0.72	3.228	0.610	2.0531	1.1953	14.2509	50.297	46.322	0.9577
0.74	3.775	0.637	2.3024	1.4288	17.1789	59.974	47.175	0.9783

Table 6.6

ANNUAL WATER BALANCE COMPONENTS, SANTA PAULA, CA.
ESTIMATED SOIL PROPERTIES. EQUATION (5.5), M_o = .4

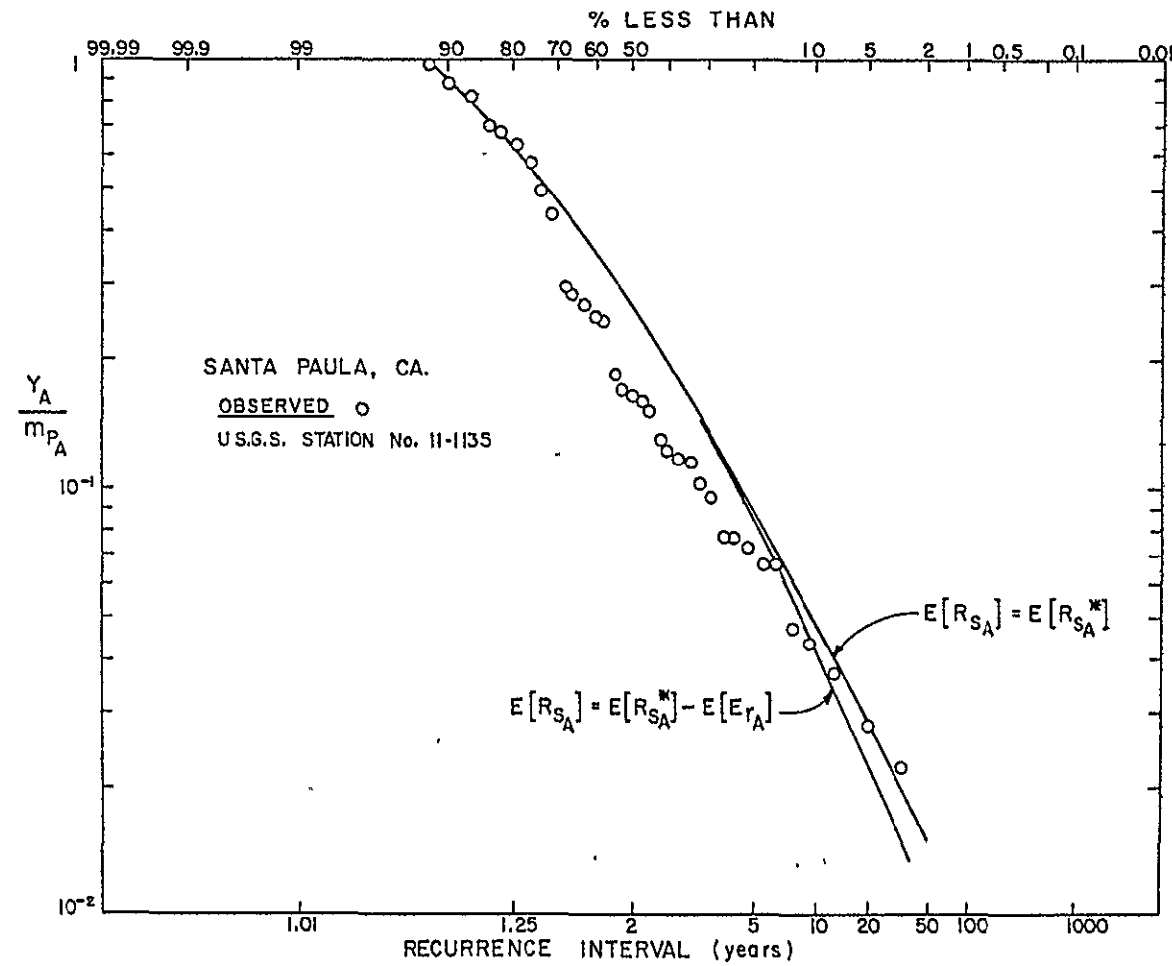


Figure 6.4

SENSITIVITY OF ANNUAL BASIN YIELD TO TWO METHODS OF HANDLING SURFACE RETENTION, SANTA PAULA, CA.

EFFECTIVE AREAL AVERAGE SOIL PROPERTIES

k(1)= 0.1916398E-09 n=0.35 c= 7.664

so	E	Mo	Pa/mpa	Ya/mpa	RSA	RGA	ETA	PROB
0.64	4.956	0.800	0.6876	0.1805	4.0637	12.900	47.669	0.0207
0.66	6.023	0.800	0.7389	0.2270	5.0093	16.331	48.117	0.0479
0.68	7.276	0.800	0.8001	0.2843	6.1903	20.530	48.493	0.1080
0.70	8.742	0.800	0.8736	0.3545	7.6819	25.637	48.795	0.2275
0.72	10.449	0.800	0.9620	0.4404	9.5866	31.815	49.026	0.4261
0.74	12.428	0.800	1.0690	0.5457	12.0451	39.249	49.194	0.6756
0.76	14.715	0.800	1.1990	0.6745	15.2515	48.150	49.309	0.8827
0.78	17.345	0.800	1.3576	0.8322	19.4752	58.756	49.383	0.9786

Table 6.7

ANNUAL WATER BALANCE COMPONENTS, CLINTON, MA.
ESTIMATED SOIL PROPERTIES. EQUATION (5.5), M_o = .8

identical to that obtained from Eq. (5.6). This can again be attributed to the fact that Clinton is primarily climate controlled, and evapotranspiration is held almost constant near the potential regardless of the amount of water that is infiltrated or removed as surface runoff.

In order to study the sensitivity of the results presented here to the vegetation density, values of M_o that bracket the observed values are used in the soil property estimation program. Figure 6.5 illustrates the results obtained for Santa Paula, which are listed in Table 6.8. Inputing an M_o of 0.2 generates a set of soil properties that produces more yield and less evapotranspiration than the soil obtained using an M_o of 0.4. By specifying such a low vegetation density, the vegetal equilibrium hypothesis used in the water balance produces a low value of evaporation efficiency, J (Figure 4.8). This corresponds to an annual evapotranspiration considerably below the potential. By reducing the evapotranspiration, the yield must be increased for a given precipitation, as can be seen by Eq. (5.1).

On the other hand, attempting to input an M_o which is greater than 0.41 does not give a solution. That is, no soil can be found for the Santa Paula climate which will produce a vegetation density much larger than the observed value of .4. The climatic variables, $\bar{e}_p$, m_{P_A}, and m_{t_b}, at Santa Paula prohibit the system from sustaining a larger vegetation density, and thus a higher evaporation efficiency. If annual precipitation is increased, or $\bar{e}_p$ decreased, the resulting increased availability of moisture would allow a greater M_o.

The same type of results are seen in Figure 6.6 and Table 6.9 for Clinton. Even though the vegetation density is already large, and

EFFECTIVE AREAL AVERAGE SOIL PROPERTIES

k (1) = 0.9733559E-09 n=0.35 c= 9.696

so	E	Mo	Pa/mpa	Ya/mpa	RSA	RGA	ETA	PROB
0.48	0.019	0.056	0.2057	0.0349	0.7780	1.105	9.226	0.0204
0.50	0.026	0.067	0.2401	0.0476	0.9311	1.642	10.390	0.0298
0.52	0.034	0.078	0.2809	0.0652	1.1181	2.401	11.649	0.0437
0.54	0.045	0.090	0.3298	0.0891	1.3481	3.462	13.000	0.0643
0.56	0.059	0.104	0.3889	0.1215	1.6328	4.926	14.441	0.0946
0.58	0.077	0.120	0.4607	0.1650	1.9877	6.923	15.965	0.1386
0.60	0.099	0.136	0.5485	0.2231	2.4329	9.617	17.567	0.2010
0.62	0.125	0.154	0.6564	0.3002	2.9938	13.216	19.238	0.2864
0.64	0.158	0.174	0.7899	0.4016	3.7033	17.980	20.968	0.3972
0.66	0.199	0.195	0.9552	0.5340	4.6027	24.231	22.747	0.5300
0.68	0.247	0.217	1.1606	0.7057	5.7433	32.366	24.563	0.6726
0.70	0.306	0.241	1.4159	0.9270	7.1878	42.870	26.402	0.8047
0.72	0.376	0.266	1.7333	1.2101	9.0110	56.335	28.249	0.9054
0.74	0.460	0.292	2.1272	1.5699	11.2994	73.478	30.091	0.9652

Table 6.8

ANNUAL WATER BALANCE COMPONENTS, SANTA PAULA, CA.
ESTIMATED SOIL PROPERTIES. EQUATION (5.6), M_o = .2

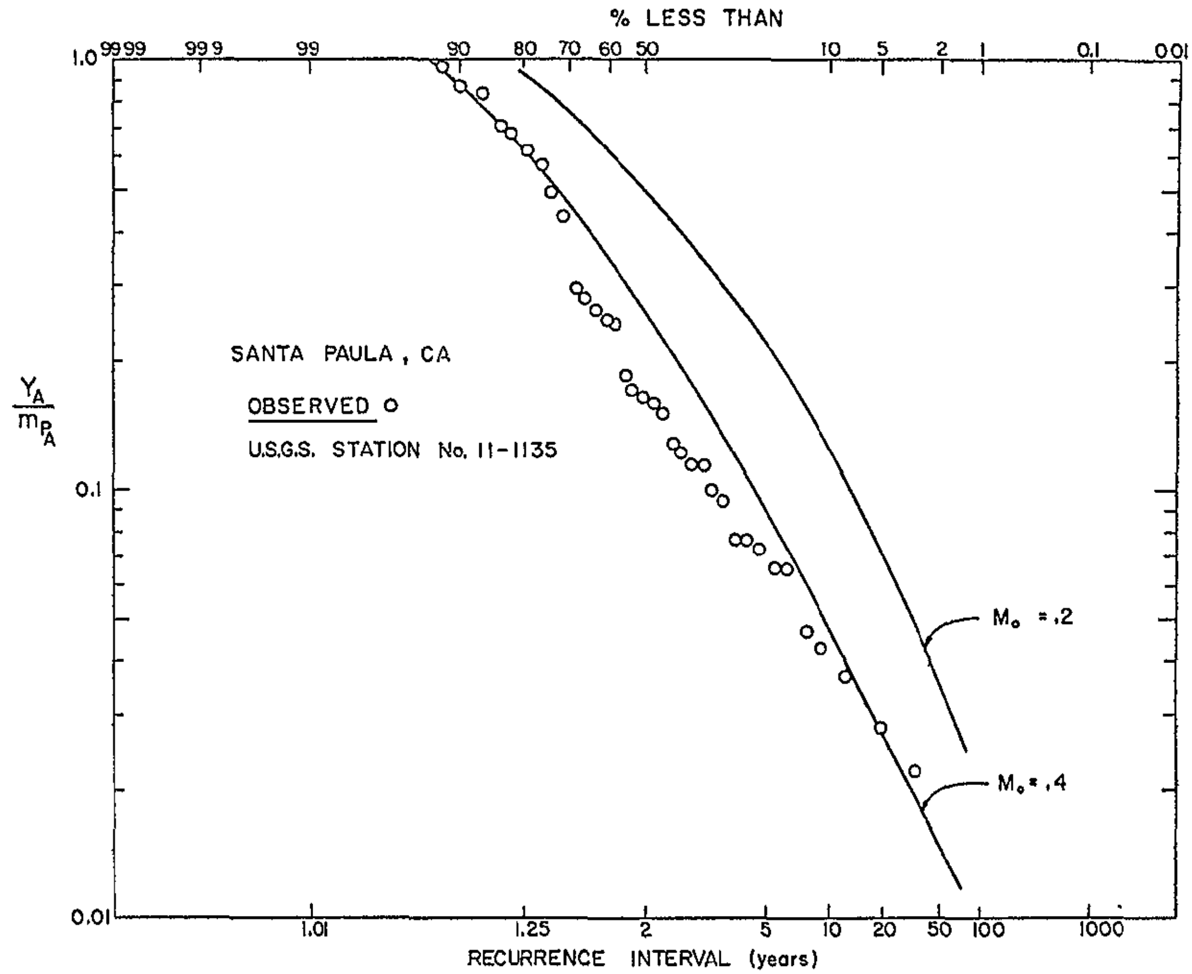


Figure 6.5

SENSITIVITY OF ANNUAL BASIN YIELD TO VEGETAL CANOPY DENSITY, SANTA PAULA, CA.

EFFECTIVE AREAL AVERAGE SOIL PROPERTIES

k(1)= 0.7135189E-09 n=0.35 c= 9.247

so	E	Mo	Pa/mpa	Ya/mpa	RSA	RGA	ETA	PROB
0.62	1.700	0.600	0.7109	0.2612	6.8828	17.667	42.272	0.0309
0.64	2.132	0.600	0.8015	0.3386	8.1368	23.695	43.507	0.1097
0.66	2.655	0.600	0.9140	0.4387	9.7412	31.495	44.682	0.3132
0.68	3.284	0.600	1.0539	0.5672	11.8042	41.508	45.758	0.6433
0.70	4.037	0.600	1.2281	0.7313	14.4697	54.268	46.703	0.9109

Table 6.9

ANNUAL WATER BALANCE COMPONENTS, CLINTON, MA.
ESTIMATED SOIL PROPERTIES. EQUATION (5.6), $M_o = .6$

EFFECTIVE AREAL AVERAGE SOIL PROPERTIES

k(1)= 0.7220880E-10 n=0.35 c= 5.449

so	E	Mo	Pa/mpa	Ya/mpa	RSA	RGA	ETA	PROB
0.56	7.268	0.900	0.6513	0.1321	6.1125	6.309	48.805	0.0103
0.58	8.731	0.900	0.6741	0.1530	6.7426	7.639	48.986	0.0161
0.60	10.422	0.900	0.7000	0.1773	7.4778	9.189	49.131	0.0257
0.62	12.370	0.900	0.7295	0.2056	8.3421	10.986	49.242	0.0415
0.64	14.602	0.900	0.7633	0.2386	9.3664	13.061	49.324	0.0678
0.66	17.148	0.900	0.8023	0.2770	10.5902	15.446	49.382	0.1108
0.68	20.043	0.900	0.8474	0.3217	12.0647	18.174	49.420	0.1791
0.70	23.320	0.900	0.8998	0.3738	13.8567	21.284	49.444	0.2819
0.72	27.018	0.900	0.9609	0.4348	16.0541	24.816	49.458	0.4235
0.74	31.176	0.900	1.0325	0.5062	18.7745	28.812	49.466	0.5950
0.76	35.837	0.900	1.1167	0.5904	22.1777	33.318	49.470	0.7672
0.78	41.046	0.900	1.2164	0.6901	26.4843	38.384	49.472	0.9003
0.80	46.851	0.900	1.3355	0.8092	32.0064	44.062	49.472	0.9721

Table 6.10

ANNUAL WATER BALANCE COMPONENTS, CLINTON, MA.
ESTIMATED SOIL PROPERTIES. EQUATION (5.6), M_o = .9

evapotranspiration is near the potential, it is still impossible to find a soil which allows an M_o much larger than the observed value of 0.8. Again, reduction of M_o produces a soil which generates a larger amount of annual yield for the same reasons mentioned for Santa Paula.

On the basis of these comparisons we see the soil properties determined from the estimation algorithm describe the behavior of these two systems very well through the water balance model. A brief summary, and conclusions drawn from these results will be presented in Chapter 7.

Although the yield CDF's for Clinton derived from varying soil properties are identical, the values obtained for the average annual soil moisture vary significantly between the silty-loam soil and the soil found from the algorithm. Since soil moisture is a state variable, it is desirable to be able to verify the accuracy of its prediction. One possible method for doing this would be to compare the CDF's of surface runoff, rather than total yield. It has been noticed that the surface runoff components of the annual water balance are much more sensitive to changes in soil properties than is the total yield. One problem with this, however, is the lack of measurements of surface runoff, although streamflow in arid climates may actually be composed totally of surface runoff.

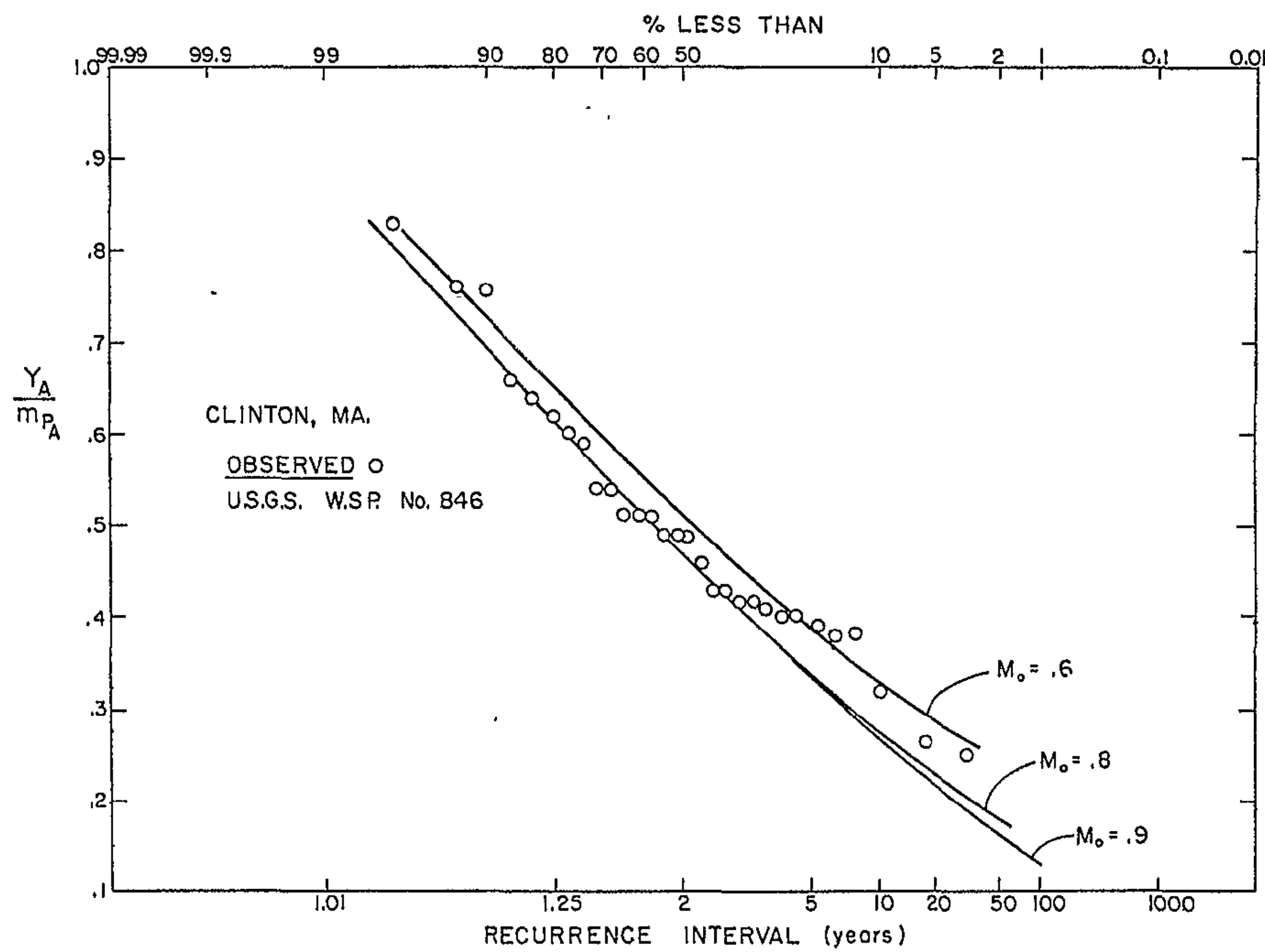


Figure 6.6

SENSITIVITY OF ANNUAL BASIN YIELD TO VEGETAL CANOPY DENSITY, CLINTON, MA.

Chapter 7

SUMMARY AND CONCLUSIONS

A one-dimensional water balance model (Eagleson, 1978a,b,c,d, e,f) is employed to parameterize the climate-soil-vegetation relationship at the land-air interface. A vegetal equilibrium hypothesis proposed by Eagleson (1978f) provides a second relationship between the climate, soil and vegetation.

Improvements are made in the method of calculating the bare soil component of evaporation, and in the way surface retention is handled.

The vegetal equilibrium hypothesis is developed, and its use in the water balance is explained. The sensitivity of this hypothesis to various parameters of the evapotranspiration function is explored. It is found that the two parameters to which the system is most sensitive are $\beta h_o/\bar{e}_p$, which can be readily evaluated, and k_v, whose value is uncertain. It is believed that k_v is usually equal to one, except in very dry climates, where the plants transpire at a rate less than an equivalent area of bare wet soil. In this work, k_v is held at its nominal value, 1.

Reasonable verification of the vegetal equilibrium hypothesis is obtained through comparisons of the theoretical relationship between density of canopy cover and the evapotranspiration efficiency to data obtained from observations in watersheds representing various types of climates.

An algorithm is derived which searches for the soil properties that produce, in a given climate, the level of evapotranspiration determined through observations of vegetation density. By using the vegetal equilibrium hypothesis, the water balance, and a regression equation relating the soil's intrinsic permeability and pore size distribution index, a consistent set of soil properties is found which generates the implied evapotranspiration and also satisfies the mean annual water balance.

This estimation of soil properties produces results, through the water balance, in the form of CDF's of annual basin yield, that describe the observed behavior of the Clinton and Santa Paula systems very well. In both Clinton and Santa Paula, the soils determined were slightly less permeable than the silty-loam which Eagleson (1978g) used as his best-fitting soil. These soils also produce a more realistic (although unverified) surface runoff component than those used by Eagleson.

A remaining important question is the sensitivity of the water balance model to the vegetation parameters, M_o and k_v. Inclusion of this analysis was beyond the scope of this study, and it is left as an important subject of future work.

From this summary, the following conclusions may be drawn:

1. The vegetal equilibrium hypothesis is sufficiently valid to justify its use as a supplementary water balance relationship between the soil, climate, and the vegetation.

2. The algorithm for estimating the effective areal soil properties works well, producing CDF's of annual yield which fit the observed CDF's closely.
3. It is more accurate to subtract surface retention from the volume of infiltrated precipitation at the beginning of the rainfall period than from the rainfall excess.
4. Use of the vegetal equilibrium hypothesis and the soil estimation algorithm should facilitate the incorporation of the areal variability of soil properties and soil moisture dynamics into global climate models.

Chapter 8

RECOMMENDATIONS FOR FUTURE WORK

Questions remaining and subjects for future study are:

1. Evaluation of the Philip exfiltration equation for a varying soil column cross-section.
2. Sensitivity of the water balance to vegetation through the parameters, M_o and k_v.
3. Development of a procedure for determining the accuracy of predicted values of average annual soil moisture.

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Appendix A

DATA FOR CATCHMENTS STUDIED IN VERIFICATION OF VEGETAL EQUILIBRIUM HYPOTHESIS

e-2

Location: Albuquerque, New Mexico

Latitude: ϕ = 35°N

Rainfall: P = 4.37 in.

Streamflow: Q = .2 in.

E_{T_A} = 4.37 - .2
= 4.17 in.

Season Length = 4 mos., July - Oct.

Cloud Cover: N = .37

Humidity: S = 39.97%

Temperature: $\bar{T}$ = 69.61°F

Vegetation Density: M_o = .12 to .15

Albedo: A =	.25	to	.3
$\bar{e}_p$ =	15.47 in/season	to	14.19 in/season
J =	.27	to	.294

Watershed Conditions: Rough broken rangeland. About 85% is bare. Sparse vegetation consists of short grasses, shrubs, and a few small juniper and pinion trees.

Comments: The value for M_o is estimated directly from the percent bare ground, and taking into account the crown spread of the trees.

Source*: Hydrologic Data for Experimental Watersheds in the United States, 1967. U.S.D.A.

* Indicates reference from which vegetation density values are obtained, and in some cases, precipitation and streamflow data as well. All other data is obtained from U.S. Weather Bureau publications and U.S.G.S. reports of surface water resources.

Location: Cornfield Wash, New Mexico

Latitude: $\phi = 35°N$

Rainfall: $P = 6.29$ in.

Streamflow: Q = function of M_o

Season Length = 4 mos., July - Oct.

Cloud Cover: $N = .37$

Humidity: $S = 39.97\%$

Temperature: $T = 69.61°F$

Vegetation Density:	$M_o = .16$		$Q = 1.07$ in.		$E_{T_A} = 5.22$ in.
	$M_o = .24$		$Q = .28$ in.		$E_{T_A} = 6.01$ in.
Albedo:	$A =$	.25		to	.30
	$\bar{e}_p =$	15.47 in/season		to	14.19 in/season
$M_o = .16$	$J =$	.34		to	.37
$M_o = .24$	$J =$	.39		to	.42

Watershed Conditions: The dominant vegetation is galleta grass. Remaining areas have a mixture of other grasses, Russian thistle, and big sagebrush in small upland drainages.

Comments: Runoff data was recorded as a function of percent bare soil in the paper used as the source, therefore, the calculation of E_{T_A} gives two values, one for each M_o and Q data pair. Vegetation density values were recorded for each value of percent bare soil, and the two extreme values were used here.

W-2 (continued)

Source: F. A. Branson and J. B. Owen, "Plant Cover, Runoff and Sediment Yield Relationships on Mancos Shale in Western Colorado," W.R.R., 6(3), 1979.

W-3, W-4, W-5

Location: Tombstone, Ariz.

Latitude: $\phi = 32°N$

Rainfall: $P_{W-3} = 8.65$ in $\quad E_{T_{A_{W-3}}} = 8.01$ in.

Streamflow: $Q_{W-3} = .64$ in.

$P_{W-4} = 8.65$ in. $\quad E_{T_{A_{W-4}}} = 8.44$ in.

$Q_{W-4} = .21$ in.

$P_{W-5} = 8.65$ in. $\quad E_{T_{A_{W-5}}} = 7.56$ in.

$Q_{W-5} = 1.09$ in.

Season Length = 3 mos., July - Sept.

Cloud Cover: $N = .35$

Humidity: $S = .4467$

Temperature: $T = 82.17°F$

Vegetation Density: $M_{o_{W-3}} = .35$ to $.4$

$M_{o_{W-4}} = .25$ to $.3$

$M_{o_{W-5}} = .2$ to $.25$

Albedo:

$A =$	.24	to	.30
$\bar{e}_p =$	13.45 in/season	to	12.14 in/season
$J_{W-3} =$	.60	to	.66
$J_{W-4} =$	.63	to	.70
$J_{W-5} =$	.56	to	.62

Watershed Conditions: All watersheds have cover of desert shrubs (whitehorn, creosote bush, tarbush) with an understory of grass (black grama, tobosa grass, blue grama, sideoats grama, and curly mesquite grass).

W-3: Entire area covered by shrubs with 38% crown spread. $M_o \approx .35$ to .40.

W-4: 78% of area covered by shrubs with crown spread of 30%. Remaining 22% covered with grass with .2% basal area. $M_o \approx .25$ to .3.

W-5: Shrub canopy approximately 20%. Remaining area covered by grass with .2% basal area. $M_o \approx .2$ to .25.

Comments: The three watersheds are all sub-catchments of a larger catchment. Therefore, while vegetation densities and streamflow vary slightly, the annual climatic properties are all the same.

Location:	Flagstaff, Arizona			
Latitude	$\phi = 35°N$			
Rainfall:	$P = 12.38$ in.		$E_{T_A} = 11.98$ in.	
Streamflow:	$Q = .4$ in.			
Season Length	= 7 mos., July - Jan.			
Cloud Cover:	$N = .4$			
Humidity:	$S = .52$			
Temperature:	$T = 47.13°F$			

Vegetation Density: $M_o = .3$ to $.35$

Albedo:	$A =$	.2	to	.25
	$\bar{e}_p =$	20.63 in/season	to	18.95 in/season
	$J =$	.58	to	.63

Watershed Conditions: The terrain is undulating uplands dissected by many small drainages. The vegetation is mainly upper piñion juniper woodland with a sparse understory of grasses.

Source: Brown, H.W., "Characteristics of Recession Flows from Small Watersheds in a Semiarid Region," W.R.R., 1(4), 1965.

Location:	Badger Wash, Colorado
Latitude:	ϕ = 38°N
Rainfall:	P = 4.69 in.
Streamflow:	Q = function of M_o
Season Length	= 6 mos., August. - Jan.
Cloud Cover:	N = .5
Humidity:	S = .4817
Temperature:	T = 47.8°F

Vegetation Density:	M_o = .13,	Q = .96 in.,		E_{T_A} = 3.73 in.	
	M_o = .26,	Q = .35 in.,		E_{T_A} = 4.34 in.	
Albedo:	A =	.25	to	.30	
	$\bar{e}_p$ =	16.04 in/season	to	14.66 in/season	
	M_o = .13:	J = .23	to	.25	
	M_o = .26:	J = .27	to	.30	

Watershed Conditions: The catchment is in a semiarid area with predominantly desert-type shrubs.

Comments: This data was obtained in the same way as that for W-2. Thus, the values for J are presented in the same way.

This watershed is located in an area where there is considerable snowfall. The model used in this work does not account for snowmelt in any way, and only works with yield resulting from precipitation in the form of rainfall. Therefore, if the yield measurement includes runoff from snowmelt, the value of precipitation used here is not large enough to account

W-7 (continued)

for that much streamflow, and the resulting calculated value of actual evapotranspiration is too small. It would not be surprising then if the value plotted for J vs. M_o is below the hypothesized curve.

Source: Branson, F. A. and J. B. Owen, "Plant Cover, Runoff, and Sediment Yield Relationships on Mancos Shale in Western Colorado," W.R.R., 6(3), 1979.

Location:	Santa Paula, California			
Latitude:	= 34.4°N			
Rainfall:	P = 21.26 in.	E_{T_A} = 14.41 in.		
Streamflow:	Q = 6.85 in.			
Season Length:	= 7 mos., Oct. - Apr.			
Cloud Cover:	N = .37			
Humidity:	S = .6897			
Temperature:	T = 53.06°F			
Vegetation Density:	M_o =	.35	to	.5
Albedo:	A =	.2	to	.32
	$\bar{e}_p$ =	21.23 in/season	to	16.73 in/season
	J =	.68	to	.86

Watershed Conditions: Fairly rugged terrain with wide variation of vegetation type. Dominant species are desert-type shrubs which are common in Southern California mountain ranges.

Source: 1) Eagleson, P. S., "Climate, Soil and Vegetation," Parts 1-7, W.R.R., 14(5), Oct. 1978.

2) On-site observations.

W-9, W-10

Location:	Chickasha, Oklahoma			
Latitude:	$\phi = 35°N$			
Rainfall:	$P_{W-9} = 23.52$ in.		$E_{T_{A_{W-9}}} = 22.40$ in.	
Streamflow:	$Q_{W-9} = 1.12$ in.			
	$P_{W-10} = 23.52$ in.		$E_{T_{A_{W-10}}} = 19.75$ in.	
	$Q_{W-10} = 3.77$ in.			
Season Length:	= 7 mos., Apr. - Oct.			
Cloud Cover:	$N = .47$			
Humidity:	$S = 67\%$			
Temperature:	$T = 70.61°F$			

Vegetation Density:	$M_{o_{W-9}}$	=	.45	to	.57
	$M_{o_{W-10}}$	≈	.2	to	.3
Albedo:	A	≈	.18	to	.24
	e_p	≈	28.80 in/season	to	26.09 in/season
W-9:	J	≈	.78	to	.86
W-10:	J	≈	.69	to	.76

Watershed Conditions: The vegetation of both catchments consists of native grasses (buffalo grass, blue grama, little bluestem). Values of M_o are interpreted from radiation shielding values obtained from average values of leaf area index and percent mulch cover. The equation used is (2)

$$M_o \approx 1 - \frac{R_{ns}}{R_{no}} = e^{-.4(L_{Ai}+2.5M)}$$

where

R_{ns} = net radiation reaching the soil surface

R_{no} = net radiation above plant canopy

L_{Ai} = leaf area index

M = fraction of surface covered by Mulch

Comments: It is reported in the source paper that W-10 is constantly overgrazed, thus, it is likely that the value obtained for M_o is unnaturally small, and the plotted position of this catchment will be above the hypothesized curve.

Source: 1) Hydrologic Data for Experimental Agricultural Watersheds in the U.S. 1976. U.S.D.A.

2) J. T. Ritchie, E. D. Rhoades and C. W. Richardson, "Calculating Evaporation from Native Grassland Watersheds," Transactions of the A.S.C.E., Aug. 1976.

Location:	Clinton, Massachusetts	
Latitude:	ϕ = 42.50°N	
Rainfall:	P = 43.82 in.	E_{T_A} = 22.01 in.
Streamflow:	Q = 21.81 in.	
Season Length:	= 12 mos.	
Cloud Cover:	N = .35	
Humidity:	S = .70	
Temperature:	T = 47.12°F	

Vegetation Density:	M_o =	.8	to	.9
Albedo:	A =	.25	to	.30
	$\bar{e}_p$ =	24.25 in/season	to	21.64 in/season
	J =	.91	to	1.02

Watershed Conditions: No specific conditions are available, only the range of vegetation density.

Source: 1) Eagleson, P. S., "Climate, Soil and Vegetation," Parts 1-7, W.R.R., 14(5), Oct. 1975.

2) Visual observations of nearby watersheds.

Appendix B

FORTRAN PROGRAM FOR ESTIMATION OF SOIL PROPERTIES

```
c THIS PROGRAM CALCULATES EFFECTIVE AREAL AVERAGE SOIL PROPERTIES. WHEN
c THE SOIL PROPERTIES ARE VARIED USING A REGRESSION EQUATION, CALCULATED
c PRECIPITATION, Pa, REACHES A MINIMUM AT A MEDIAN VALUE OF c. THE PARA-
c METER, k(1), IS THEN DEVIATED FROM THE REGRESSION UNTIL ANOTHER MINI-
c MUM Pa IS FOUND. DEPENDING ON WHETHER THIS VALUE FOR Pa IS ABOVE OR
c BELOW THE KNOWN VALUE OF mpa, THE PARAMETER c, IS INCREMENTED UP OR
c DOWN, AND k(1) IS SEARCHED AGAIN UNTIL ANOTHER MINIMUM IS REACHED.
c THIS INCREMENTATION AND SEARCHING IS CONTINUED UNTIL THE MIMIMUN Pa
c FOUND IS EQUAL TO mpa.

        integer change,ftm,cfbl,runs,number,mon,iter
        real*8 mnu,p1
        real mtb,mtr,mh,mpa
        real mi,mo,m,n,nu,k1,k2

c DIMENSIONLESS INFILTRATION DIFFUSIVITY
        fii(d,so)=1./(d*(1.-so)**(1.45-.0375*d)+5./3.)

c PORE SHAPE PARAMETER
        fi(em)=10.**(.66+.55/em+.14/em**2.)

        print,'Input parameters in the correct units.'
        print,'ep,cm/day mtb,days mtr,days tau,days kappa,-.'
        print,'ho,cm w/ep,- ta,degrees C.'
        input,epr,mtb,mtr,tau,ak,ho,wep,ta

        pistol=1
c IF pistol=1, THE ARRAY OF FACTORIALS IN THE CDF SUBROUTINE HAS NOT
c BEEN CALCULATED YET. ONCE pistol=2, THE FACTORIALS HAVE BEEN STORED
c AND THE LINES WHICH DO THIS CALCULATION ARE THEN SKIPPED.

  5     print,'Input mpa,cm kv,- Mo,- n,- .'
        input,mpa,akv,mo,n
        print,'For annually varying Mo,type 1, for constant Mo,type 2'
        input,mon
        if(mon.eq.0)stop
```

```
          change=1
c IF change=1, SOIL PROPERTIES ARE NOT YET DETERMINED. IF change=2, THE
c SOIL PROPERTIES HAVE BEEN DETERMINED AND ONLY THOSE STEPS NEEDED FOR
c DERIVING THE CDF OF THE WATER BALANCE COMPONENTS ARE USED.

          runs=1
c IF runs=1, THIS IS THE FIRST SET OF SOIL PROPERTIES USED, AND NO COM-
c PARISON OF CALCULATED Pa IS POSSIBLE. IF runs=2, THE NEW VALUE OF Pa
c IS COMPARED TO THE OLD VALUE TO SEE IF A MINIMUM HAS BEEN REACHED.

          cfbl=1
c IF cfbl=1, THIS IS THE FIRST DEVIATION OF k(1) FROM THE REGRESSION
c AND NO COMPARISON OF Pa IS DONE. IF cfbl=2, THE Pa CALULATED WITH
c THIS k(1) IS COMPARED TO THAT CALCULATED USING THE PREVIOUS k(1) TO
c SEE IF THE SECOND MINIMUN HAS BEEN REACHED.

          ftm=1
c IF ftm=1, THE SECOND MIMIMUM HAS JUST BEEN FOUND, BUT IF THIS MINIMUM
c Pa≠mpa, c MUST BE CHANGED AND THE ENTIRE PROCESS MUST BE REPEATED.
c THE VALUE OF THE DIFFERENCE BETWEEN THE MINIMUM Pa AND mpa, awbal, IS
c PRESERVED AND COMPARED TO THE NEXT ONE OBTAINED. THIS COMPARISON IS
c SIGNALED WHEN ftm=2. WHEN awbal < .001, THE SOIL PROPERTIES HAVE BEEN
c FOUND.

c SET INITIAL VALUES
          p1=0.0
          so=0.0
          dcs=.1
          dics=.1

c************************************************************************
c    COMPUTE WATER CONSTANTS
c     sut=SURFACE TENSION
c     nu=VISCOSITY
c     gamsw=SPECIFIC WEIGHT
```

```
      call WATCN(ta,sut,nu,gamsw)

c*********************************************************************

c     COMPUTE CLIMATIC PARAMETERS

      delta=1./mtr
      mh=mpa/(tau/(mtb+mtr))
      mnu=tau/(mtb+mtr)
      mi=mh/mtr
      eta=1./mh
      alpha=1./mi
       pi=3.14159
      beta=1./mtb
      epa=epr*tau*mtb/(mtb+mtr)
      al=ak/mh
      alh=al*ho
      bhe=beta*ho/epr
      if(ho.eq.0.0)goto 10
      ble=beta/(al*epr)
      goto 20
   10 ble=0.0
   20 alkh=alh*akv
      blke=ble/akv
   30 if(change.eq.1)goto 40

c*********************************************************************
      print
      print
      print,' so       E       Mo       Pa/mpa     Ya/mpa     RSA

      do 400 i=1,45
      so=so+.02
      e=ecnst*so**d2
      fiid=fii(di,so)

c*********************************************************************
```

```
40       if(change.eq.2)goto 45
         goto 50
45       if(mon.eq.2)goto 60
         goto 55

c TO SPEED UP THE SEARCH FOR THE VALUE OF e THAT MINIMIZES ETA AT THE
c OBSERVED Mo, e AND m ARE GIVEN INITIAL VALUES DEPENDING ON THE VAL-
c UE OF Mo. BY PICKING A VALUE FOR e, THE m THAT MINIMIZES ETA CAN BE
c FOUND. IF THIS m≠Mo, ANOTHER e IS PICKED UNTIL m=Mo.

50       if(mo.ge..2)e=.3
         if(mo.ge..3)e=.5
         if(mo.ge..4)e=1.
         if(mo.ge..6)e= 3.
         if(mo.ge..7)e=6.
         if(mo.ge..8)e=10.
         if(mo.ge..9)e=20.
55       if(e.ge..01)bm=.1
         if(e.ge..1)bm=.4
         if(e.ge.1.)bm=.6
         if(e.ge.10.)bm=.9
         if(mo.lt..4)de=.01
         if(mo.ge..4)de=.1
         number=1
60       iter=1
         dm=.01
70       bmkv=bm*akv

c*********************************************************************
c COMPUTE EVAPOTRANSPIRATION PARAMETERS, B & C.
         b=((1.-bm)/(1.+bmkv)+(bmkv*bm)/(2.*(1.+bmkv)**2.))
         if(bmkv.eq.0.0)goto 80
         c=1./(2.*(bmkv*bmkv))
         goto 90
80       c=1.e10
90       be=b*e
```

```
      ce=amin1(c*e,80.)

c************************************************************************
      gamk=gamt(ak,alh)/gamma(ak)
      gamkl=gamt(ak,alh+bhe)/gamma(ak)
      gambe=gamt(1.5,be)
      gamce=gamt(1.5,ce)
      gamkv=gamt(ak,alkh)/gamma(ak)
      gamkvl=gamt(ak,(alkh+bhe))/gamma(ak)

c************************************************************************
c COMPUTE ANNUAL EVAPORATION FROM SURFACE RETENTION
      era=epr/beta*((1.-bm)*(1.-exp(-bhe)*(1.-gamk)-(1.+ble)**(-ak)
   &   *gamkl)
   &   +bmkv*(1.-exp(-bhe)*(1.-gamkv)-(1.+ble)**(-ak)*gamkvl))*mnu
      eram=era

c************************************************************************
c COMPUTE INTERSTORM BARE SOIL EVAPORATION
    esj=gamk-(1.+ble)**(-ak)*gamkl*exp(-be)+
   & (1.-gamk)*(1.-exp(-be-bhe)*(1.+bmkv+sqrt(2.*b)*e-wep)
   & +exp(-ce-bhe)*(bmkv+sqrt(2.*c)*e)
   & +sqrt(2.*e)*exp(-bhe)*(gamce-gambe))
   & +(1.+ble)**(-ak)*gamkl*(sqrt(2.*e)*(gamce-gambe)
   & +exp(-ce)*(bmkv+sqrt(2.*c)*e)
   & -exp(-be)*(bmkv+sqrt(2.*b)*e-wep))


c COMPUTE EVAPOTRANSPIRATION FUNCTION
      hj=1./(1-bm+bmkv)*((1-bm)*esj+bmkv)
      ETN=hj*(1.-bm+bmkv)


      if(change.eq.2)goto 95
      goto 100
   95 if(mon.eq.2)goto 160
```

```
c***********************************************************************
c THESE LINES FIND THE M THAT MINIMIZES ETA.
c IF iter=1, IT IS THE FIRST TIME THROUGH AND NO COMPARISON IS MADE
  100     if(iter.eq.1)goto 120
          if(abs(dm).lt..000001)goto 150
          if(ETN.gt.ETMIN)goto 110
          goto 120
  110     bm=bm-1.5*dm
          dm=-.5*dm
          goto 130
  120     ETMIN=ETN
          bmin=bm
          iter=2
          bm=bm+dm
  130     if(bm)140,70,70
  140     bm=.1*(bm-dm)
          q=q+1
          if(q.lt.4)goto 70

c AT THIS POINT, NO Mo CAN BE FOUND THAT IS GREATER THAN 0,AND NEW PAR-
c AMETERS MUST BE INPUT.
             goto 395

  150     bm=bmin
          ETN=ETMIN
  160     if(change.eq.2)goto 230

c THESE LINES FIND THE E CORRESPONDING TO THE GIVEN Mo.
c IF number=1, IT IS THE FIRST TIME THROUGH AND NO COMPARISON IS MADE.
          diff=mo-bm
          if(abs(diff).lt..0001)goto 200
          if(number.eq.1)goto 170
          if(diff*diffold.le.0.0)goto 190
          if(number.eq.2)goto 180
  170     if(diff.lt.0.0)de=-1.*de
```

```
      number=2
180   diffold=diff
      e=e+de
      goto 60
190   de=-de*.5
      diffold=diff
      e=e+de
      goto 60
200   continue

c***********************************************************************
c AT THIS POINT, THE VALUE OF e HAS BEEN DETERMINED, AND SOIL PROPER-
c TIES ARE NOW SEARCHED.

      cs=4.
210   m=2./(cs-3.)
      fic=fi(m)
      dE=2.+1./m
      di=cs-1./m-1.
      d2=dE+2.
      fied=fie(dE)

c REGRESSION EQUATION
      k1=(m/512.7)**2.75

      k2=k1
      dk1=k1/10.
220   continue
      bk1=k1*gamsw/nu
      si1=sqrt(n/(k1*fic))*sut/gamsw
      sigc=n*eta**2.*bk1*si1/(pi*m*delta)*72000.
      ecnst=2.*beta*n*bk1*si1*fied/(pi*m*epr**2.)*86400.

c SOIL MOISTURE IS CALCULATED.
      so=(e/ecnst)**(1./d2)
```

```
         fiid=fii(di,so)

c*************************************************************************
c     COMPUTE WATER BALANCE

c     COMPUTE ANNUAL EVAPOTRANSPIRATION
  230    ETA=ETN*epa

c*************************************************************************
c     COMPUTE ANNUAL GROUNDWATER RUNOFF
         RGA=tau*bk1*so**cs*86400

         sigrf=(sigc*fiid*(1.-so)**2.)**.33333
         g=alpha*bk1*86400*.5*(1.+so**cs)
         blop=g+2.*sigrf
         if(blop.gt.85.)blop=85.
         blip=exp(-blop)*gamma(sigrf+1.)*sigrf**(-sigrf)
         if(blip.gt..95)blip=.95

c*************************************************************************
c      COMPUTE PRECIPITATION,YIELD,RUNOFF
         Pa=(ETA+RGA)/(1.-blip)
         RSA=blip*Pa
         Ya=RSA+RGA
         if(change.eq.2)goto 380
         awbal=Pa-mpa

c NOTE-awbal IS THE DIFFERENCE BETWEEN CALCULATED Pa AND KNOWN mpa. THE
c FOLLOWING LINES WILL PERFORM THE SEARCH FOR SOIL PROPERTIES THAT PRO-
c DUCE Pa=mpa.
         if(cfbl.eq.2)goto 260
         if(ftm.eq.2)goto 280
         if(runs.eq.1)goto 250

c THESE LINES PERFORM THE FIRST MINIMIZATION WHICH ADHERES TO THE RE-
c GRESSION EQUATION.
```

```
        if(abs(dcs).lt..001)goto 260
        if(awbal.gt.awbol)goto 240
        goto 250
  240   cs=cs-1.5*dcs
        dcs=-.5*dcs
        goto 210
  250   awbol=awbal
        cs=cs+dcs
        runs=2
        goto 210

c THESE LINES PERFORM THE SECOND MINIMIZATION WHICH HOLDS c CONSTANT
c AND DEVIATES k(1) FROM THE REGRESSION.

  260   if(cfbl.eq.2)goto 270
        if(abs(awbal).lt..001)goto 320
        if(cfbl.eq.1)goto 280
  270   if(abs(dk1).lt.k2/1000.)goto 320
        if(awbal.gt.awbol)goto 290
  280   awbol=awbal
        k1=k1-dk1

c SINCE k(1) VARIES BY ORDERS OF MAGNITUDE, dk1 MUST BE REDEFINED IF
c k(1) GETS TOO BIG OR SMALL.

        if(k1.lt.k2/9)goto 300
        if(k1.gt.k2*9)goto 310
        cfbl=2
        goto 220
  290   k1=k1+1.5*dk1
        dk1=-.5*dk1
        goto 220
  300   dk1=dk1/10.
        k2=k1
        goto 220
```

```
  310     dk1=dk1*10
          k2=k1
          goto 220

c THESE LINES PERFORM THE SEARCH ON c WHICH LOCATES THE MINIMUN Pa FROM
c THE ABOVE PROCEDURE WHICH EQUALS mpa.

  320     cfbl=1
          if(abs(awbal).lt..01)goto 360
          if(ftm.eq.1)goto 330
          if(awbal*awbold.lt.0.0)goto 350
          goto 340
  330     if(awbal.gt.0.0)dics=-1.*dics
          ftm=2
  340     awbold=awbal
          cs=cs+dics
          goto 210
  350     dics=-dics*.5
          awbold=awbal
          cs=cs+dics
          goto 210

c***************************************************************************
  360     print,'AVERAGE EFFECTIVE PARAMETERS'
          print 370,e,so,k1,cs
  370     format('E=',f6.3,2x,'so=',f5.3,2x,'k(1)=',e16.7,2x,'c=',f6.3)
          change=2
          so=0.0
          goto 30
  380     y1=Ya/mpa
          p1=Pa/mpa
          if(p1.lt..2)goto 395

c    COMPUTE CDF OF PRECIPITATION
          call PROBZ(mnu,p1,prob,ak)
          if(prob.lt..009)goto 395
```

```
      if(prob.gt..99)goto 410
      print 390,so,e,bm,p1,y1,RSA,RGA,ETA,prob
390   format(f4.2,3x,f6.3,3x,f5.3,3x,f7.4,3x,f7.4,3x,f7.4,3x,f7.3,3x
 &          ,f6.3,3x,f7.4)
395   continue
400   continue
410   goto 5
      end

c*************************************************************************
c*************************************************************************
c THIS FUNCTION COMPUTES THE INCOMPLETE GAMMA FUNCTION.
      function gamt(a,x)
      if(x.eq.0)goto 40
      if(x.gt.100)goto 50
      sum=1./a
      an=1.0
      old=sum
33    old=old*x/(a+an)
      if(old/sum-1.e-6)20,10,10
10    an=an+1.
      sum=sum+old
      if(an-300.)33,33,12
12    continue
20    xxx=(a*alog(x)+alog(sum)-x)
      if(xxx.lt.-80.)goto 40
      gamt=(exp(xxx))
      goto 60
40     gamt=0.0
      goto 60
50     gamt=gamma(a)
60    return
      end

c*************************************************************************
c  This function computes the gamma function by a Stirling approx.
```

```
      function gamma(y)
      x=y+1.
      pi=3.14159
      stir1=1./(12.*x)
      stir2=1./(288.*x**2.)
      stir3=-139./(51840.*x**3.)
      stir4=-571./(2488320.*x**4.)
      stir=1+stir1+stir2+stir3+stir4
      gamma=exp(-x)*x**(x-.5)*sqrt(2.*pi)*stir/y
     end

c***********************************************************************
        subroutine WATCN(ta,sut,nu,gamsw)
        real nu,nut
        dimension sutt(11),nut(11),gamst(11)
        data sutt/75.6,74.9,74.2,73.5,72.0,72.1,71.4,70.7,70.0,
     &        69.3,68.6/
        data nut/17.93e-3,15.18e-3,13.09e-3,11.44e-3,10.08e-3,8.94e-3,
     &       8.e-3,7.2e-3,6.53e-3,5.97e-3,5.94e-3/
        data gamst/0.99987,0.9999999,0.99973,0.99913,0.99823,0.99708
     &       ,0.99568,0.99406,0.99225,0.99025,0.98807/
        if(ta.gt.50.)go to 10
        ita=ifix(ta*.2)+1
        frac=ta-float(ifix(ta))
        ita1=ita+1
        sut=(sutt(ita1)-sutt(ita))*0.2*frac+sutt(ita)
        nu=(nut(ita1)-nut(ita))*0.2*frac+nut(ita)
        gamsw=((gamst(ita1)-gamst(ita))*.2*frac+gamst(ita))*980.
        return
   10    sut=sutt(11)
        nu=nut(11)
        gamsw=gamst(11)
        return
        end

c***********************************************************************
```

```
c DIMENSIONLESS EXFILTRATION DIFFUSIVITY
        function fie(d)
        dimension y(6)
        data y/0.18,0.11,0.077,0.056,0.044,0.034/
        if(d.gt.7.) goto 10
        x=d-1.
        if(x.lt.1.)x=1.
        i=ifix(x)
        frac=x-float(i)
        y1=alog(y(i))
        y2=alog(y(i+1))
        fie=exp((y2-y1)*frac+y1)
        return
 10     fie=.034
        return
        end

c**********************************************************************
        subroutine PROBZ(mnu,p1,prob,ak)
c   THIS PROGRAM COMPUTES THE CDF OF NORMALIZED PRECIPITATION.
        real*8 fac(500)
        real*8 x,a,dlog,gama,gamlid,eps
        real*8 m,k,w,t,z,zl,zu,inz
        real*8 p1,mnu
        real*8 xold,xsum,sum1,sum2,tot,vtot,vold,vnew
        integer v,vm,vmax

        if(pistol.eq.2) goto 301
        do 300 j=1,500
        vtot=0.0d0
        do 700 iv=1,j
  700   vtot=vtot+dlog(dble(float(iv)))
        fac(j)=vtot
  300   continue
  301   continue
        eps=1.e-5
```

```
      pistol=2
      w=mnu
      t=1.
      k=ak
c   INITIALIZING VALUES
c
      m=w*t
      z=p1
      vm=ifix(sngl(m))
      vmax=ifix(sngl(3.*m))
 3    x=m*k*z
      ii=0
      jj=1
      sum1=0.0d0
      sum2=0.0d0
 13   v=vm-ii
      if(v.eq.0)goto 500
 23   if(v.eq.vmax)goto 600
c
c
c      COMPUTE LOG INCOMPLETE GAMMA DISTRUBITION
      a=dble(float(v))*k
      xold=1.0d0/a
      xsum=1.0d0/a
      i=1
 100  xold=(xold/(a+i))*x
      xsum=xsum+xold
      if((xold/xsum).le.eps)goto 200
      i=i+1
      goto 100
 200  continue
      call mlgama(a,gamm,ier)
      gamlid=a*dlog(x)-x+dlog(xsum)-dble(gamm)
c
c      COMPUTE THE SUMMATION OF ALL V TERMS
c
```

```
      vold=dble(float(v))*dlog(m)-fac(v)+gamlid-m
      if(vold.le.-85.)vold=-85.
      vnew=dexp(vold)
      if(v.gt.vm)goto 800
      sum1=sum1+vnew
      if((vnew/sum1).le.eps)goto 500
      ii=ii+1
      goto 13
500   v=vm+jj
      goto 23
800   sum2=sum2+vnew
      if((vnew/sum2).le.eps)goto 600
      jj=jj+1
      goto 500
c
c    COMPUTE CDF OF NORMALIZED PRECIPITATION
c
600   if(m.gt.85.)m=85.
      prob=sum1+sum2+dexp(-m)
      return
      end
```