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LARGE SPACE SYSTEM CONTROL TECHNOLOGY

MODEL ORDER REDUCTION STUDY

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This is a report on the model order reduction for large space structures performed under contract to JPL by Purdue University with R. E. Skelton as Principal Investigator. The main objective of the contract is to find the best pre-flight dynamical models for large structures and thereby retain the most significant vehicle dynamics in the controller designs.

MISSION CONTROL REQUIREMENTS

Large structures have to satisfy a wide range of mission control requirements including pointing and shape control. Specific missions with these requirements may include platforms and large antennas.

MISSION CONTROL REQUIREMENTS

- POINTING CONTROL
- SHAPE CONTROL
  - ANTENNAS
  - PLATFORMS

PROBLEMS OF ACTIVE CONTROL OF
LARGE STRUCTURES IN SPACE

One of the main problems in active control of large structures in space is that the controller designs require accurate models to represent the structure dynamic response. Present models obtained typically by means of a finite-element analysis of the structural dynamics are not sufficiently accurate as a result of four classes of errors: parameter uncertainties, nonlinearities, neglected variables and external disturbances. In addition the process of model order reduction must be carried out in order to satisfy on-board memory and speed computational requirements.

PROBLEMS OF ACTIVE CONTROL OF LARGE STRUCTURES IN SPACE

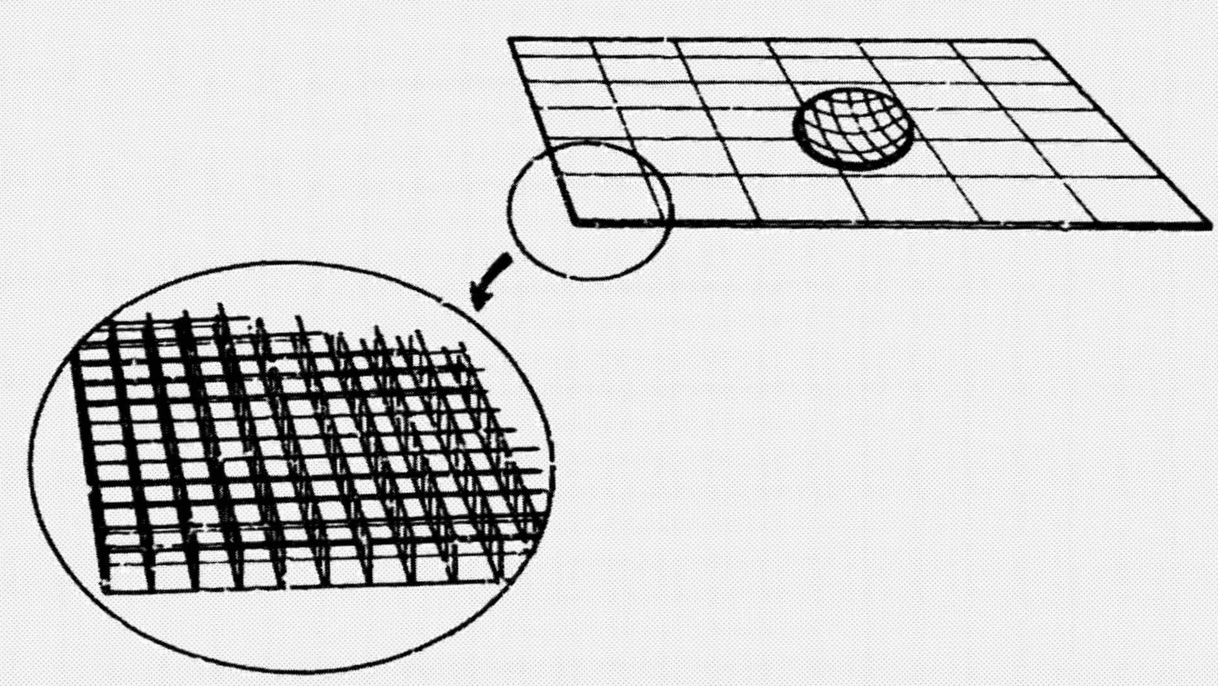
- CONTROL DESIGN REQUIRES ACCURATE MODEL
- PRESENT MATHEMATICAL MODELS NOT ACCURATE ENOUGH
    - PARAMETER ERROR
    - NONLINEARITIES
    - NEGLECTED VARIABLES
- LIMITATIONS OF ON-BOARD DIGITAL COMPUTERS
    - MEMORY
    - SPEED

PURDUE MODEL

An Equivalent Distributed Parameter Model of Truss Structure

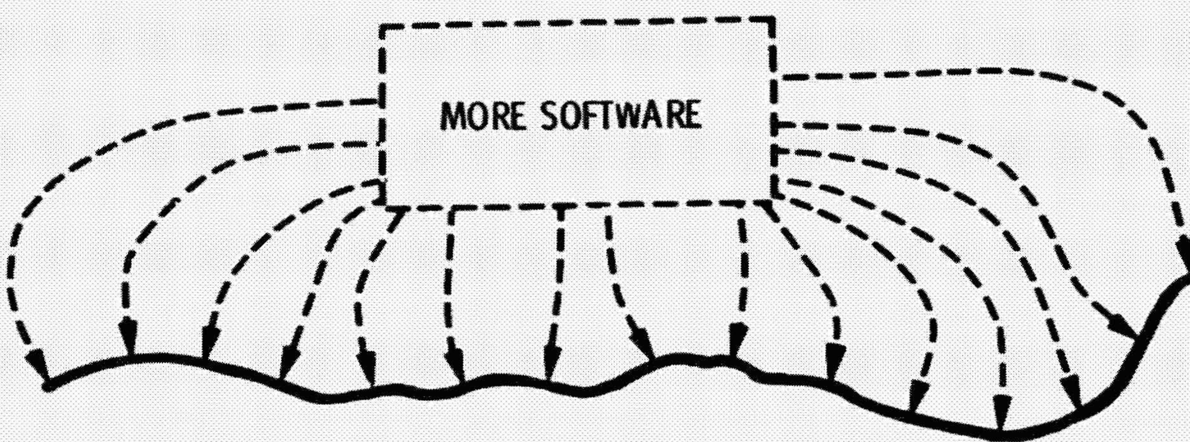
In order to study the process of model order reduction a generic model has been selected that contains most of the features important in the modeling and control problem for large structures. The model consists of a rectangular array that is an equivalent distributed model of a truss structure. The continuum model is easier to work with as it does not require the complexities of a more detailed numerical model that may tend to conceal the control-related problems. In the center of the model is an articulated rigid body representing possibly a payload that must be pointed to a higher degree of precision than the remainder of the structure. This model has been used to study the problem of model order reduction for large structures. The results obtained, however, are applicable to a wide range of large structures and are not limited to the selected model.

PURDUE MODEL
AN EQUIVALENT DISTRIBUTED PARAMETER MODEL OF TRUSS STRUCTURE

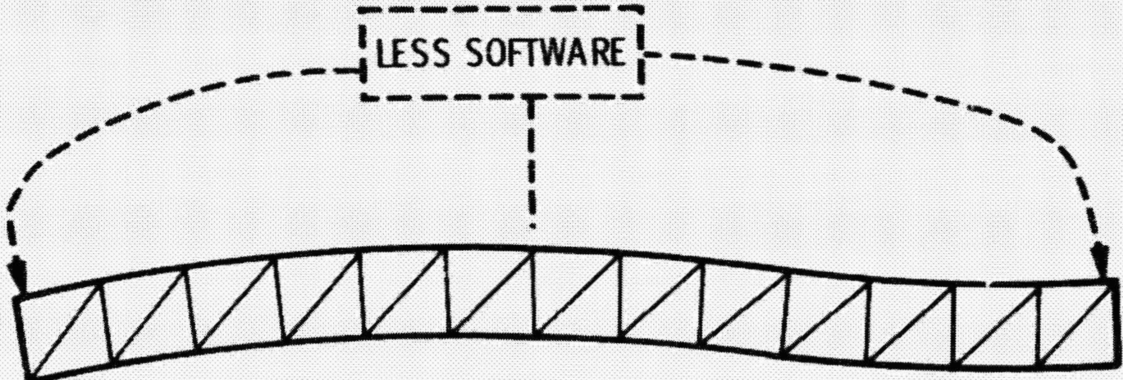


A fundamental tradeoff must be made in developing an integrated control/structure design. Control can either be achieved by means of more software resulting in a lightweight structure or with less software and a heavier structure. One of the results of this study is to provide the necessary methods to perform this tradeoff systematically.

CONTROL/STRUCTURE TRADEOFF



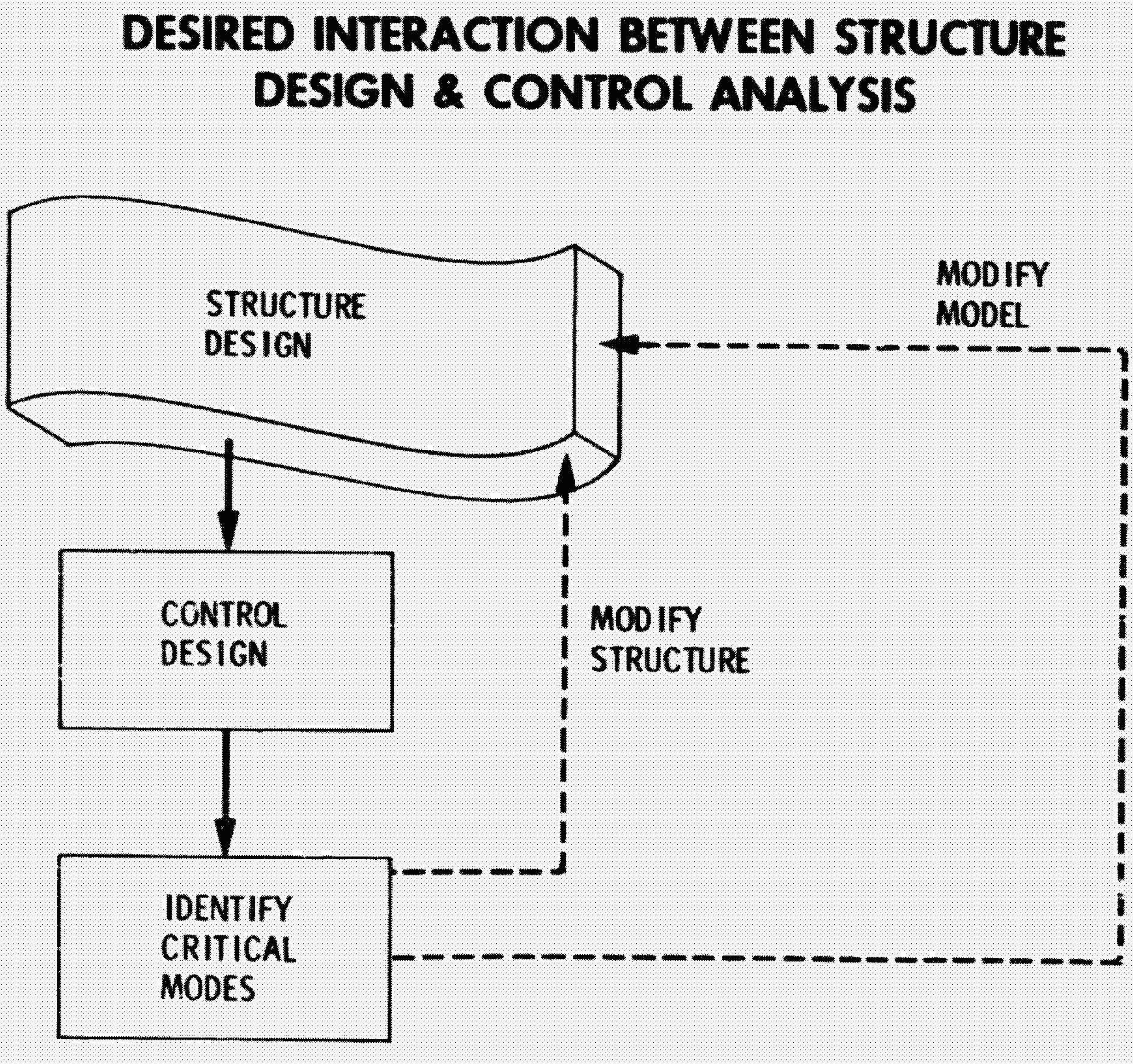
- MORE CONTROL HARDWARE, SOFTWARE (LESS ROBUST)
- LIGHTWEIGHT STRUCTURE



- LITTLE CONTROL HARDWARE, SOFTWARE (ROBUST)
- HEAVY STRUCTURE

DESIRED INTERACTION BETWEEN STRUCTURE DESIGN & CONTROL ANALYSIS

The desired interaction between structure and control design is displayed in the viewgraph. The objective is to select the critical modes in order to modify the model and the structure and thereby achieve integrated design.



CONTROL STUDY OBJECTIVES

The control study objectives are to suggest desired dynamic structure properties for improved performance and reduced weight. An example of these properties may be the selection of modes that may be damped by introducing passive damping in certain parts of the structure.

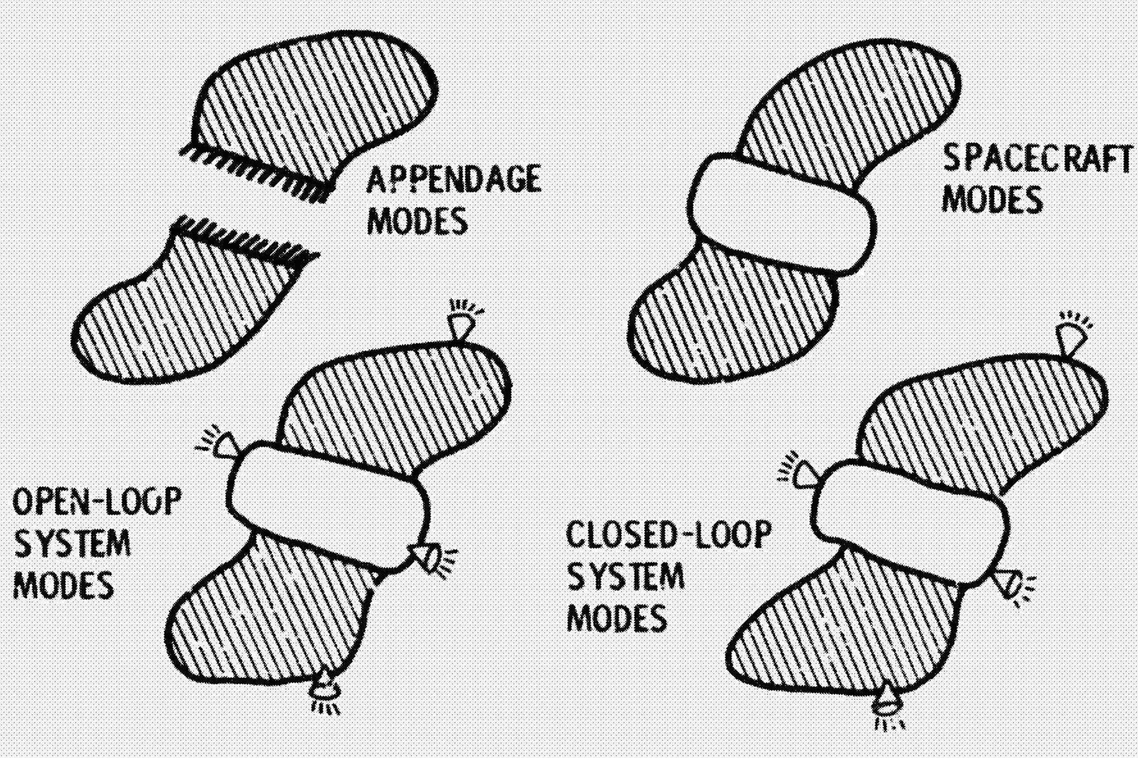
CONTROL STUDY OBJECTIVES

- SUGGEST DESIRED DYNAMIC STRUCTURE PROPERTIES FOR
  - IMPROVED PERFORMANCE
  - REDUCED WEIGHT

SELECTION OF DYNAMICAL MODEL

A number of options exist for dynamical modeling of large structures. The modes selected can be: 1) appendage modes corresponding to the dynamics of individual appendages, 2) spacecraft modes for the overall vehicle dynamic response, 3) open-loop system modes that include also the dynamics of certain elements in the control system, and 4) closed-loop system modes that model the overall system including structure, control and the dynamics of the feedback controllers. The selection is very much dependent upon the configuration. The closed-loop system modes tend to retain the most significant dynamics when the structure is in its operational state.

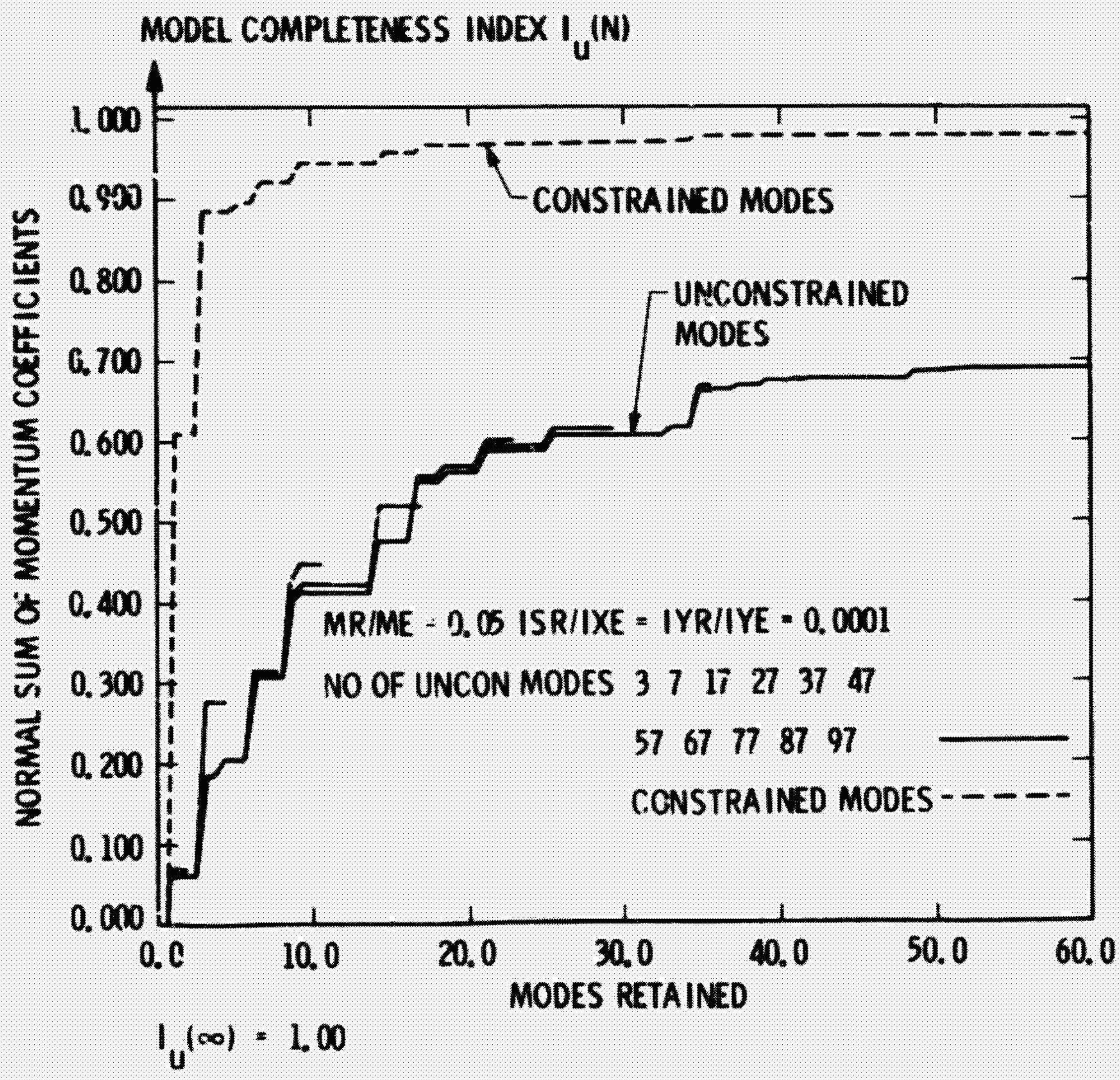
SELECTION OF DYNAMICAL MODEL



PURDUE MODEL

The figure shows a plot of model completeness index versus the number of modes retained for the Purdue model. The model completeness index gives an indication of the degree of fidelity of a particular modal model with respect to the continuum model which is assumed to be the exact representation of the vehicle dynamics. As more modes are retained, the models become more accurate. A comparison between constrained and unconstrained modes reveals that fewer constrained modes have to be retained in order to obtain a specified model completeness.

PURDUE MODEL



MODAL COST ANALYSIS

COMPONENT COSTS V_i

The method currently under study for model order reduction is modal cost analysis where a so-called cost is associated to each mode. This cost reflects the contribution of each mode to the overall system performance. The component value criterion indicates that the most critical modes are the ones with the largest cost. Conversely, the least critical components have the lowest modal cost.

MODAL COST ANALYSIS
COMPONENT COSTS V_i

$$V = \sum_{i=1}^{M} V_i$$

$$V_i = \frac{1}{2} \sum_{j=1}^{M} (V_{ij} + V_{ji})$$

$$V_{ij} = \mathcal{E}\left[W_i^* K_{ij} W_j\right]_{ZSR} = \mathcal{E}\left[X_i^*(0) K_{ij} X_j(0)\right]_{ZIR}$$

WHERE K SATISFIES

$$0 = KA + A^T K + C^T QC$$

COMPONENT VALUE CRITERION:

$$V_1 \geq V_2 \geq \ldots \geq V_{M-1} \geq V_M$$

MOST CRITICAL COMPONENT (V_1)

LEAST CRITICAL COMPONENT (V_M)

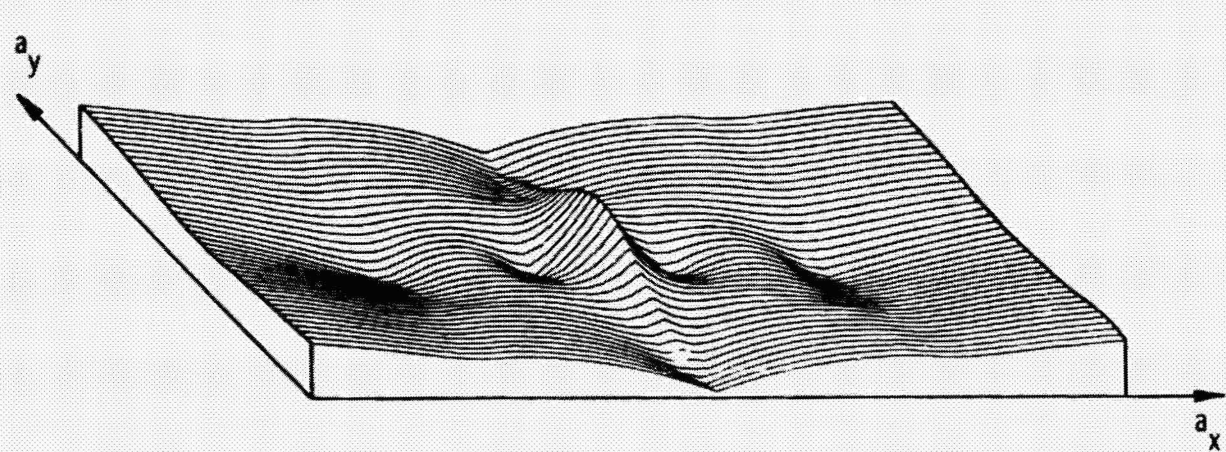
ACTUATOR LOCATIONS CONTROLLABILITY SURFACE

In a distributed system, actuator locations are an additional degree-of-freedom in the controller design. A good selection of actuator number and location can lead to improved system performance. A poor selection can at worst lead to overall system instability. The model order selection study also provides criteria for actuator placemen s. The viewgraph shows a so-called controllability surface for the seventh mode of the Purdue model. These actuator locations where the surface is a maximum are where that particular mode is most strongly affected by the control inputs. The points where the surface is zero correspond to locations where the mode is uncontrollable.

ACTUATOR LOCATIONS CONTROLLABILITY SURFACE

SEVENTH MODE

[TWO TORQUE ACTUATORS COINCIDENT AT (a_x, a_y)]



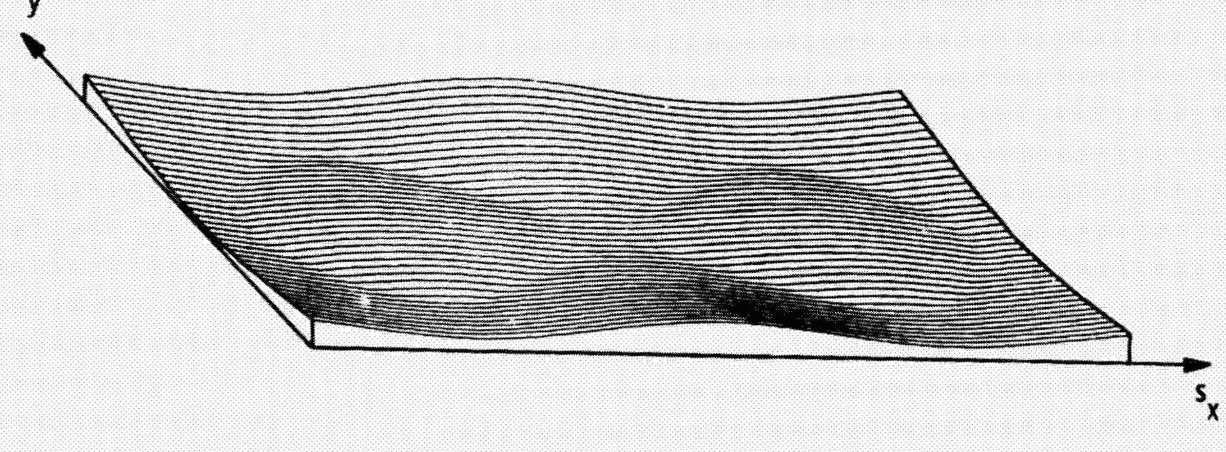
SENSOR LOCATIONS OBSERVABILITY SURFACE

A similar surface can be used for sensor location. The points where the surface is a maximum correspond to sensor locations where the particular mode is most observable by the sensors. Similarly, the points where the surface is zero are those where the mode cannot be observed by means of the sensor measurements.

SENSOR LOCATIONS OBSERVABILITY SURFACE

EIGHTH MODE

[TWO ORTHOGONAL ATTITUDE SENSORS ON R, PLUS ELASTIC DEFLECTION AT (s_x, s_y)]

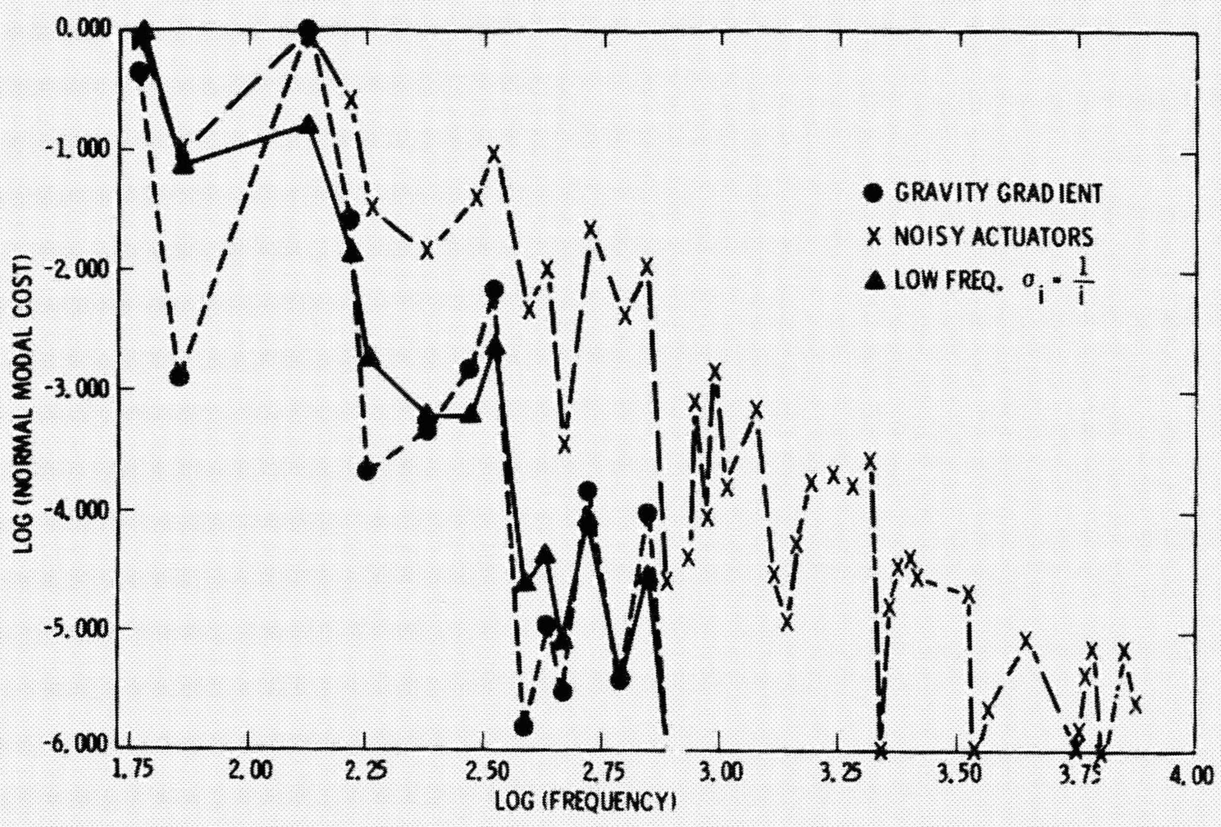


ATTITUDE CONTROL

Modal Cost Analysis

The viewgraph displays results of the application of the technique of modal cost analysis to the problem of attitude control to the Purdue model. By assigning a cost to each individual mode, those modes that have the most effect on the control/structure performance can be selected. The modes with the highest cost are the most important in the control system design and should be retained. Those modes with the lowest cost have little influence on the control performance and are therefore the likeliest candidates for elimination in the controller design.

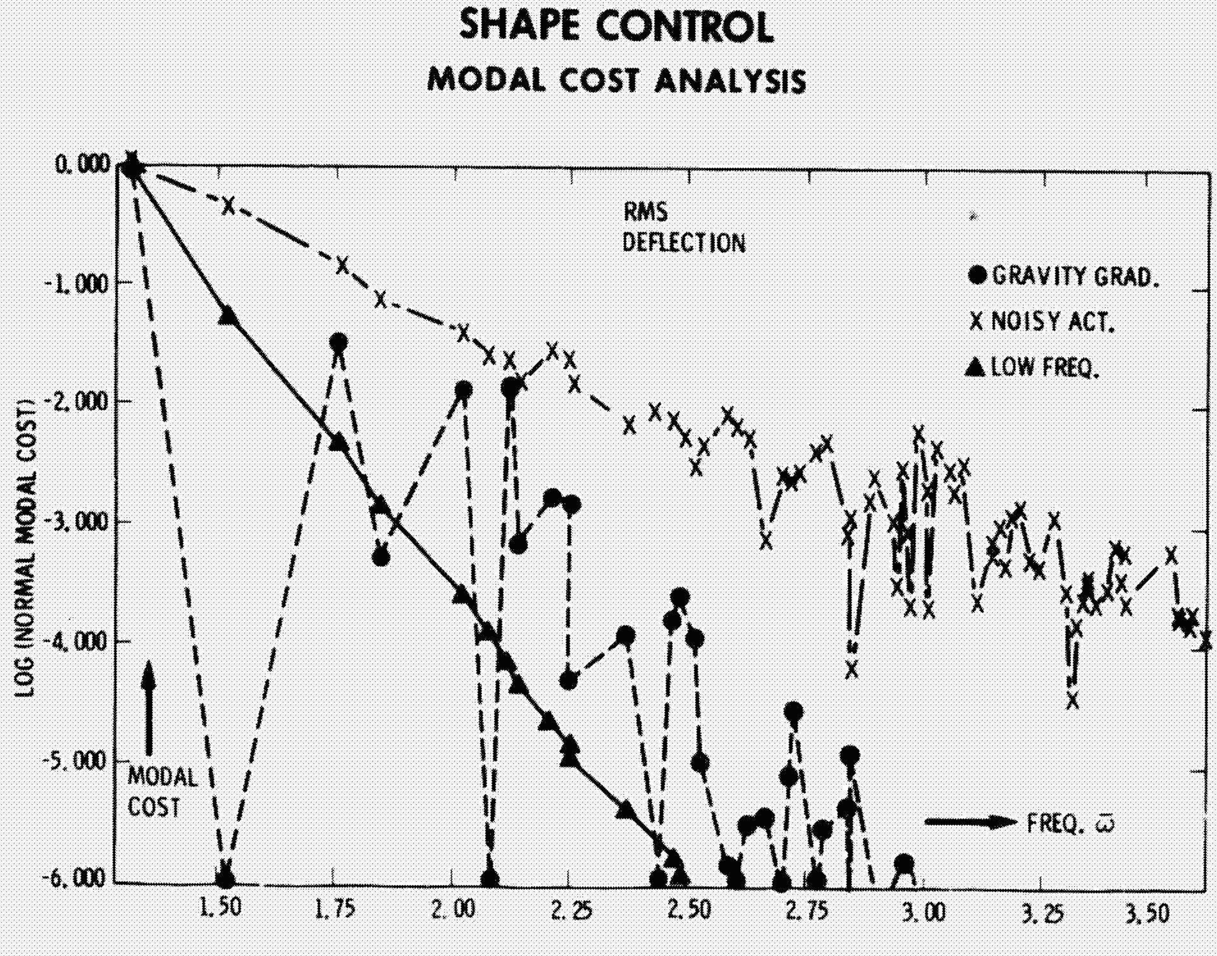
ATTITUDE CONTROL
MODAL COST ANALYSIS



SHAPE CONTROL

Modal Cost Analysis

The viewgraph shows results of the application of the modal cost analysis to the problem of shape control for the Purdue model. By establishing a relative ranking of the modes in terms of their effect on control performance, the most significant modes can be extracted and used in the controller design.



MODEL ERROR VS. CONTROL OBJECTIVE

The plot shows the model error as a result of using an approximate model. As the model order is increased, the model error is redu d in all cases. The plot corresponding to shape control has a large lue for a given model order. This indicates that more modes have to be retained in order to achieve a prescribed fidelity for the shape control problem. Fewer modes have to be retained if attitude control is the desired objective.

MODEL ERROR VS. CONTROL OBJECTIVE