N O T I C E

THIS DOCUMENT HAS BEEN REPRODUCED FROM MICROFICHE. ALTHOUGH IT IS RECOGNIZED THAT CERTAIN PORTIONS ARE ILLEGIBLE, IT IS BEING RELEASED IN THE INTEREST OF MAKING AVAILABLE AS MUCH INFORMATION AS POSSIBLE

986-??

(NASA-CR-162847) THE GEOMETRY OF THE 37-TILE MICROWAVE ANTENNA SUPPORT STRUCTURE (Astro Research Corp.) 24 p HC A02/MF A01 CSCL 20N

N80-19321

Unclas

G3/32 47477

ASTRO
RESEARCH
CORPORATION

THE GEOMETRY OF THE 37-TILE MICROWAVE ANTENNA SUPPORT STRUCTURE

by
Laurence A. Finley

ARC-TN-1083

5 March 1980

Prepared for
Jet Propulsion Laboratory
under Contract No. 955543

Prepared by
Astro Research Corporation
6390 Cindy Lane
Carpinteria, California 93013

TABLE OF CONTENTS

LIST OF TABLES AND FIGURES

THE GEOMETRY OF THE 37-TILE MICROWAVE ANTENNA SUPPORT STRUCTURE

FOREWORD

The purpose of this report is to specify the geometry of the support structure for a proposed parabolic-shaped microwave antenna. The surface of the antenna is comprised of 37 hexagonal-shaped tiles, each connected to a truss module. These units are joined together to form a rigidized, faceted, concave parabolic surface (see Figure 1). (A 37-unit array is discussed here, but the results may be easily extended to larger structures.)

Although the antenna and its related supporting structure form a complex system of surfaces, points, and lines, a few simplifying assumptions regarding their relative locations make the entire problem of description much more straightforward. In particular, if certain restrictions are placed on the locations of the truss members relative to the hexagonal grid formed by the tiles, the geometry of the structure becomes very manageable. Specifying this geometry requires an explanation of the structural components which make up the antenna, a description of the coordinate system devised to identify the structure, and a presentation of the nondimensional results.

STRUCTURAL COMPONENTS

The structure is composed of two types of members: repeating hexagonal tiles, and repeating skewed triangular-truss modules. Since this work deals principally with the support structure, all that we are interested in concerning the tiles is their size and arrangement relative to the support structure.

The tiles form a large hexagonal array comprised of three concentric hexagonal rings around a central tile. The outer edge of this array is a sawtooth pattern. It can be stipulated that the tiles form a regular array of identical hexagons when projected into a plane perpendicular to the axis of revolution of the paraboloic surface (see Figure 2). Thus, the tiles we are considering are only approximate hexagons, the actual lengths of each edge varying over the surface of the antenna. Nothing is assumed about the surface curvature of the individual tiles. Only the vertices of each one are restricted to lie within the theoretical parabolic surface of the antenna. (For future reference, this theoretical surface is designated as the "upper" surface.)

Attached to each of the tiles just described is a skewed triangular-truss module. Just as the basic shape of the hexagonal tile is continuously altered to form a paraboloidal surface, so are the dimensions of a simple skewed module altered throughout the structure to form a rigid support for the tiles (see Figure 1). To describe this structure accurately, it is first necessary to describe the necessary modifications of its dimensions as its location in the structure changes.

The basic truss module (see Figure 3) is made up of nine struts arranged in the following manner: six struts of equal length ℓ make up two equilateral triangles. The vertices of these triangles are connected by the remaining three struts. The triangles are positioned such that they are in parallel planes a distance of 0.885 ℓ apart. They are clocked 60 degrees from one another when viewed along their line of centers. The skewed connecting struts are oriented in the sense of a right-hand screw. The length of the connecting strut in this basic configuration is 1.057 ℓ.

In order to generate a paraboloidal surface with this module, it is necessary to vary the length of all the members so as to produce the correct curvature when they are joined together. To do this in a regular manner, use is made of the fact that the vertices of the two triangles also form a hexagon when they are viewed from along their line of centers. The geometry of the entire structure becomes remarkably simple if the hexagons formed by the projection of these vertices into the plane, perpendicular to the axis of rotation of the paraboloid, coincide with the similar projections of the vertices of the hexagonal tiles used to make up the surface of the antenna. (This also assumes that the line of centers of all triangle pairs is parallel to the axis of revolution of the paraboloid.) Now the structure is almost determined, except for the description of the manner in which the tiles are joined to the truss. Simply stated, the vertices of the upper triangle of each truss module are attached to alternating vertices of each hexagonal tile. All of these points are assumed to lie within the upper surface.

There is a further geometric restriction of the vertices of the unattached lower triangles of the truss modules. They must also form a paraboloid of revolution with the same curvature as the upper surface, but displaced along the axis of rotation by an amount equal to the depth of the truss, 0.885 ℓ. In addition, the projections of all the intersections of all the truss members in the lower plane all lie at the locations of the projections of the vertices of the hexagonal tiles in the perpendicular plane. Now that a general description of the structure has been completed, it is possible to specify the location of all points exactly by using a convenient coordinate system.

COORDINATE SYSTEM FOR POINT DESIGNATION

A right-handed Cartesian coordinate system (x,y,z) will be used to specify the structure. The positive z axis points towards the center of the earth and the positive x axis lies along the major axis of the antenna. At a point d on the z axis (0,0,d), a paraboloid of revolution with focal length F intersects the +z axis and is concave towards the xy plane. The axis of revolution is coincident with the z axis. The portion of the paraboloid lying between the xy plane and the point (0,0,d) contains 37 nonuniform hexagonal figures with the following characteristics:

- The projections of these 37 hexagonal figures into the xy plane form a regular array of 37 uniform hexagons each with side length $\ell/\sqrt{3}$.
- They are arranged such that their outer periphery forms another large hexagon with a sawtooth edge.
- ℓ is the length of the side of an inscribed equilateral triangle and is the principal characteristic dimension of the support structure struts as shown in Figure 3. (The 37 hexagonal figures in the paraboloid represent the 37 tiles forming the surface of the antenna.)
- The +x axis lies along a major diameter of the large hexagonal array.

In addition to being the "vertical" projection of the tiles, the hexagonal array shown in Figure 2 also has another important characteristic: each vertex of each hexagon defines the location of the projection of a joint between members of the support structure and, in fact, represent a one-to-one mapping of all such joints into the xy plane. For this reason, it is necessary to specify completely the xy coordinates of these vertices. Once this is done, the geometric constraints of the support structure will determine uniquely the z values for each joint. Once these are given, the purpose of this work will be accomplished.

The points in the structure are represented by their projected position in the projected hexagonal tile array for convenience. To easily identify each point, it will be useful to label each hexagon. To do this, consider the hexagons forming a grid centered around the origin. The centers of each hexagon form an array of points which can be described by an ordered couple of numbers (m,n). If x and y are the coordinates of each point in question, the quantities $x\sqrt{3}/\ell$ and $y\sqrt{3}/\ell$ are two nondimensional integers which identify each point by its position relative to the origin. All the numbered pairs and the associated hexagons may be seen in Figure 2.

If the alternate vertices of each hexagon are all connected, two overlapping triangular grids are formed. In Figure 4a, we see the grid which has vertices pointing along the +y axis. This grid is the xy-plane projection of all the struts forming the upper surface of the support structure. It is possible to identify each point in this grid by using the hexagon number pairs (m,n). By moving along the +y axis a distance $\ell/\sqrt{3}$ from each hexagon center, a point of the grid is reached. This point in the upper surface is designated by the numbered pair which describes the associated hexagon center. The same number pair also labels the triangle which is centered on the hexagon. This is shown in the small figure shown in Figure 4a. (Notice that some points must be named by use of imaginary hexagons outside the array mentioned, but which are a continuation of the existing pattern.)

To specify the rest of the structure, it is necessary to describe the lower surface which is similar to the upper surface but translated a distance H along the +z axis. The projections of the members of this portion of the structure form the second triangular grid obtained from the original hexagonal grid. The vertices of these triangles point in the -y axis direction and are named by moving a distance $\ell/\sqrt{3}$ in the -y direction from the identifying hexagon center (see Figure 4b). The triangles of the lower surface are also labeled with the (m,n) designation of the hexagon they are centered on in the xy-plane projection. Thus, for each hexagon number pair (m,n), there are two associated triangles: the (m,n) triangle in the upper surface, and the (m,n) triangle in the lower surface. These triangle pairs form the upper and lower surface of each module previously described.

Thus far, we have described pairs of triangles separated from each other by a distance H with the vertices of each triangle lying in an upper or lower parabolic surface. To complete the structure, it is necessary to connect the points of the triangles in the lower surface to those in the upper surface by connecting each vertex in the lower triangle with a vertex in the upper triangle following a right-handed screw direction (see Figure 3). Thus, if we are considering the triangle pair (m,n) in the upper and lower surfaces, the following points would be connected:

From (lower surface)	To (upper surface)
(m,n)	(m-1,n-1)
(m-1,n+1)	(m,n)
(m+1,n+1)	(m+1,n-1)

The only other fact necessary to completely characterize the structure is the depth H of the truss. It is important to note that this is determined by a nondimensional ratio characteristic of the truss module, and not contained in the parabolic specifications of the antenna. For this study, a value of $0.885 = H/\ell$ is used. This number arises because of the retraction/deployment requirements of the module configuration and is independent of the antenna size or shape.

An assumption in this paper is that all the struts intersect at common points which lie exactly in the upper and lower paraboloidal surfaces. No corrections are made for hinges, joint thicknesses, or actual lengths or cross-sectional areas of struts.

NONDIMENSIONAL FORMULAS AND RESULTS

Beginning with the basic cell designations shown in Figure 2, the x coordinates of all truss points in the upper and lower surfaces can be simply calculated by

$$\frac{x(m,n)}{\ell} = +\frac{m}{2} \tag{1}$$

Similarly, for the y coordinates we get

$$\frac{y(m,n)}{\ell} = \frac{\sqrt{3}}{2}\left(n \pm \frac{2}{3}\right) \tag{2}$$

Here the ± refers to the values of the y coordinate in the upper and lower surfaces, respectively. The values of these points are calculated and shown in Tables I and II.

It now remains to calculate the z coordinate of all the points in the upper and lower surfaces. The values of z for each point are

$$z = (-1/4F)\ (x^2+y^2) + d \tag{3}$$

in the upper surface and

$$z = (-1/4F)\ (x^2+y^2) + H + d \tag{4}$$

in the lower surface.

We recognize that the surface has the uppermost point at (7,-1) on the upper surface, so this is the point used to evaluate the parabola parameter. From Eqs. (1) and (2), the coordinates of this point are $x = 3.5\ \ell$, and $y = -\ell/2\sqrt{3}$. If these values are substituted into Eq. (3), it is possible to solve for d by

$$z = 0 = -1/4F\ (3.5^2\ \ell^2 + \ell^2/4(3)) + d$$

or

$$d = (\ell^2/4F)\ (12.333333)$$

The expression for z becomes

$$z = (\ell^2/4F)\ (12.333333 - (x/\ell)^2 - (y/\ell)^2)$$

or

$$z(4F/\ell^2) = 12.333333 - (x/\ell)^2 - (y/\ell)^2 \tag{5}$$

For the lower surface, this becomes

$$(z-H)(4F/\ell^2) = 12.333333 - (x/\ell)^2 - (y/\ell)^2 \qquad (6)$$

These expressions containing z in the upper and lower surface have been calculated and are shown in Tables III and IV.

These results conclude the specification of the location of the important points in the support structure. The results are presented in terms of nondimensional parameters. Tables III and IV reveal some extremely unusual information regarding the values of the z coordinate of the points of the upper and lower surface. The nondimensional quantity $4zF/\ell^2$ is an integer for all points of the structure. Since F and ℓ are fixed quantities, it means that the z coordinate is directly proportional to an integer multiple of a set quantity for all points. This in turn has a great effect on the length of the struts in the structure. This can be seen by considering the expression used to calculate the length of a line segment between two points:

$$\left[(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2\right]^{1/2}$$

It may be seen by inspection that the difference in the x and y coordinates for all points of intersection in the upper and lower surfaces are equal. Thus, the length of any strut will vary only as the value of the differences between the z coordinates and its end points change. Because these are all integers, their differences will also be small integers. In fact, for the structure under consideration, there are only seven different values for z_1-z_2 in the entire structure. Hence, there are only seven lengths of struts necessary to fabricate the entire upper and lower surfaces. A similar result can be found regarding the lengths of the skewed connecting struts. (When the necessary calculations are performed, it may be determined that only four different lengths of skewed struts are necessary.)

To show how these results may be used to size a structure, the lengths of all the struts in the upper and lower surfaces have been calculated and are shown in Figures 5 and 6. For this example, ℓ = 1.5 meters and F = 4.2 meters.

TABLE I. VALUES OF $\frac{x(m,n)}{\ell}$ AND $\frac{y(m,n)}{\ell}$ FOR THE UPPER SURFACE*

m \ n	-4	-3	-2	-1	0	1	2	3	
7				3.500000 -0.288675					
6			3.000000 -1.154701		3.000000 0.577350				
5		2.500000 -2.020726		2.500000 -0.288675		2.500000 1.443376			
4	2.000000 -2.886751		2.000000 -1.154701		2.000000 0.577350		2.000000 2.309401		
3		1.500000 -2.020726		1.500000 -0.288675		1.500000 1.443376		1.500000 3.175426	
2	1.000000 -2.886751		1.000000 -1.154701		1.000000 0.577350		1.000000 2.309401		
1		0.500000 -2.020726		0.500000 -0.288675		0.500000 1.443376		0.500000 3.175426	
0	0.000000 -2.886751		0.000000 -1.154701		0.000000 0.577350		0.000000 2.309401		℄

Symmetric about m = 0.

*The top entry of the number pairs is the x coordinate and the bottom entry is the y coordinate.

TABLE II. VALUES OF $\frac{x(m,n)}{\ell}$ AND $\frac{y(m,n)}{\ell}$ FOR THE LOWER SURFACE*

m \ n	-3	-2	-1	0	1	2	3	4
7					3.500000 0.288675			
6				3.000000 -0.577350		3.000000 1.154701		
5			2.500000 -1.443376		2.500000 0.288675		2.500000 2.020726	
4		2.000000 -2.309401		2.000000 -0.577350		2.000000 1.154701		2.000000 2.886751
3	1.500000 -3.175426		1.500000 -1.443376		1.500000 0.288675		1.500000 2.020726	
2		1.000000 -2.309401		1.000000 -0.577350		1.000000 1.154701		1.000000 2.886751
1	0.500000 -3.175426		0.500000 -1.443376		0.500000 0.288675		0.500000 2.020726	
0		0.000000 -2.309401		0.000000 -0.577350		0.000000 1.154701		0.000000 2.886751 ℄

Symmetric about m = 0.

*The top entry of the number pairs is the x coordinate and the bottom entry is the y coordinate.

TABLE III. VALUES OF $z(4F/\ell^2)$ FOR POINTS ON THE UPPER SURFACE

m \ n	-4	-3	-2	-1	0	1	2	3	
7				0					
6			2		3				
5		2		6		4			
4	0		7		8		3		
3		6		10		8		0	
2	3		10		11		6		
1		8		12		10		2	
0	4		11		12		7		℄

Symmetric about m = 0.

TABLE IV. VALUES OF $(z-H)(4F/\ell^2)$ FOR POINTS ON THE LOWER SURFACE

m \ n	-3	-2	-1	0	1	2	3	4	
7					0				
6				3		2			
5			4		6		2		
4		3		8		7		0	
3	0		8		10		6		
2		6		11		10		3	
1	2		10		12		8		
0		7		12		11		4	℄

Symmetric about m = 0.

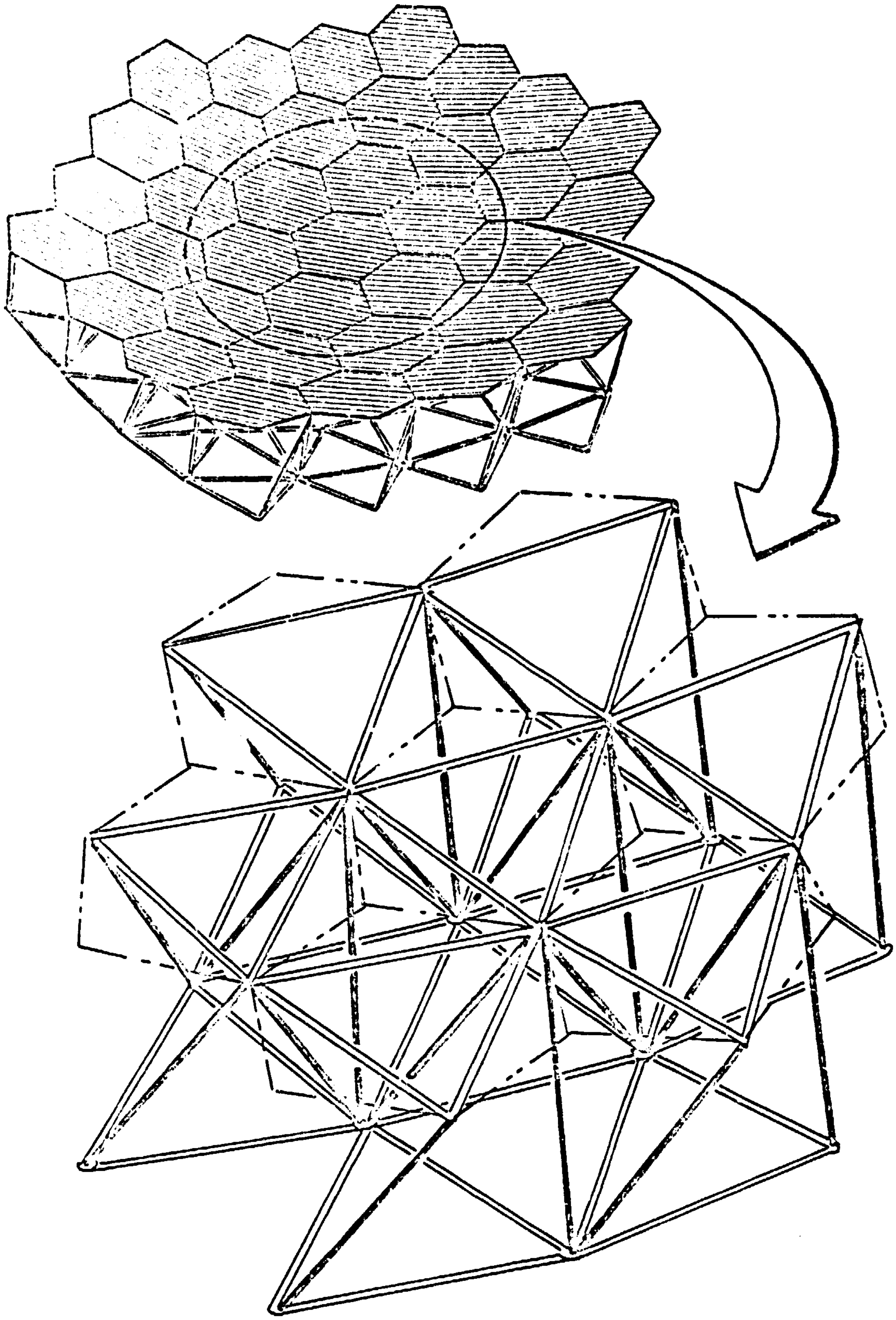


Figure 1. The 37-tile parabolic microwave antenna showing detail of the supporting structure (skewed connecting struts are darkened).

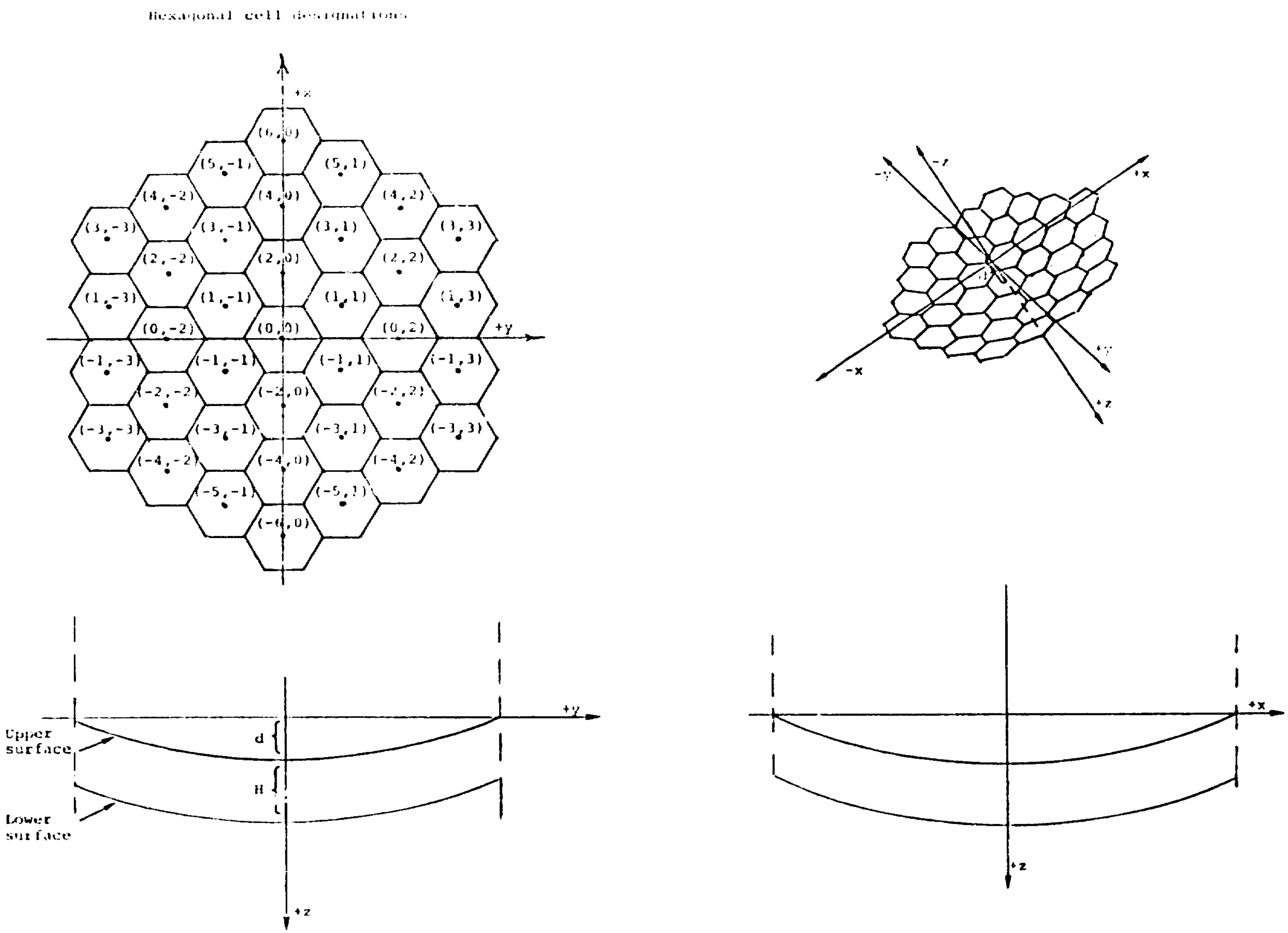


Figure 2. Axis orientation and cell designation.

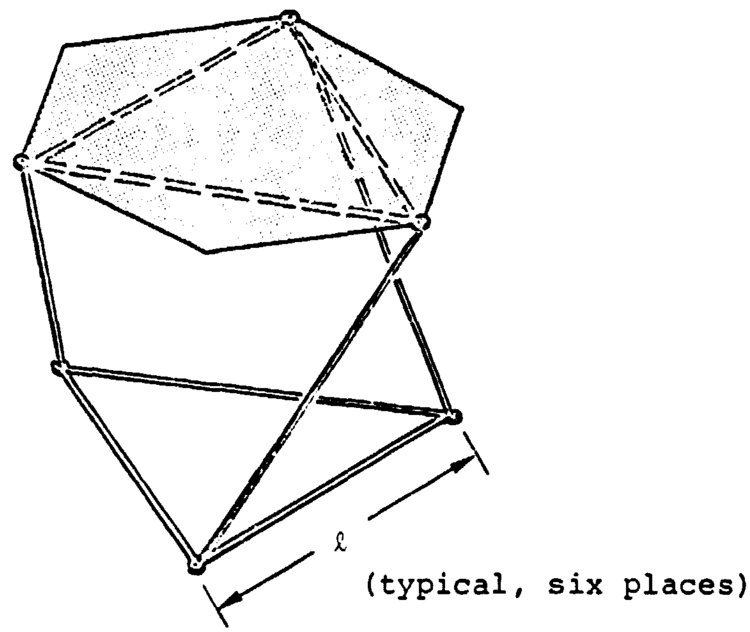


Figure 3. The basic truss module.

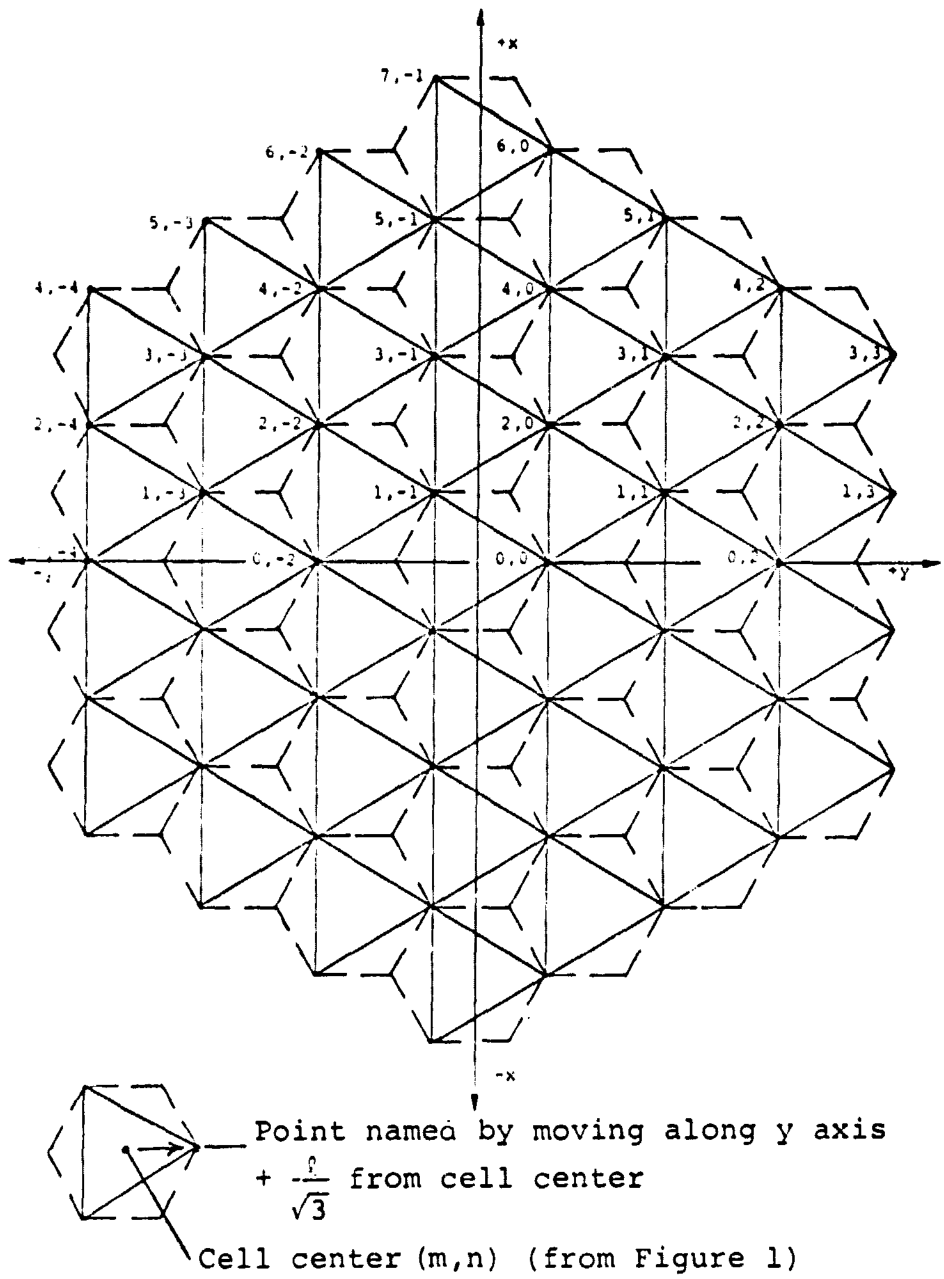


Figure 4a. Upper surface point designations.

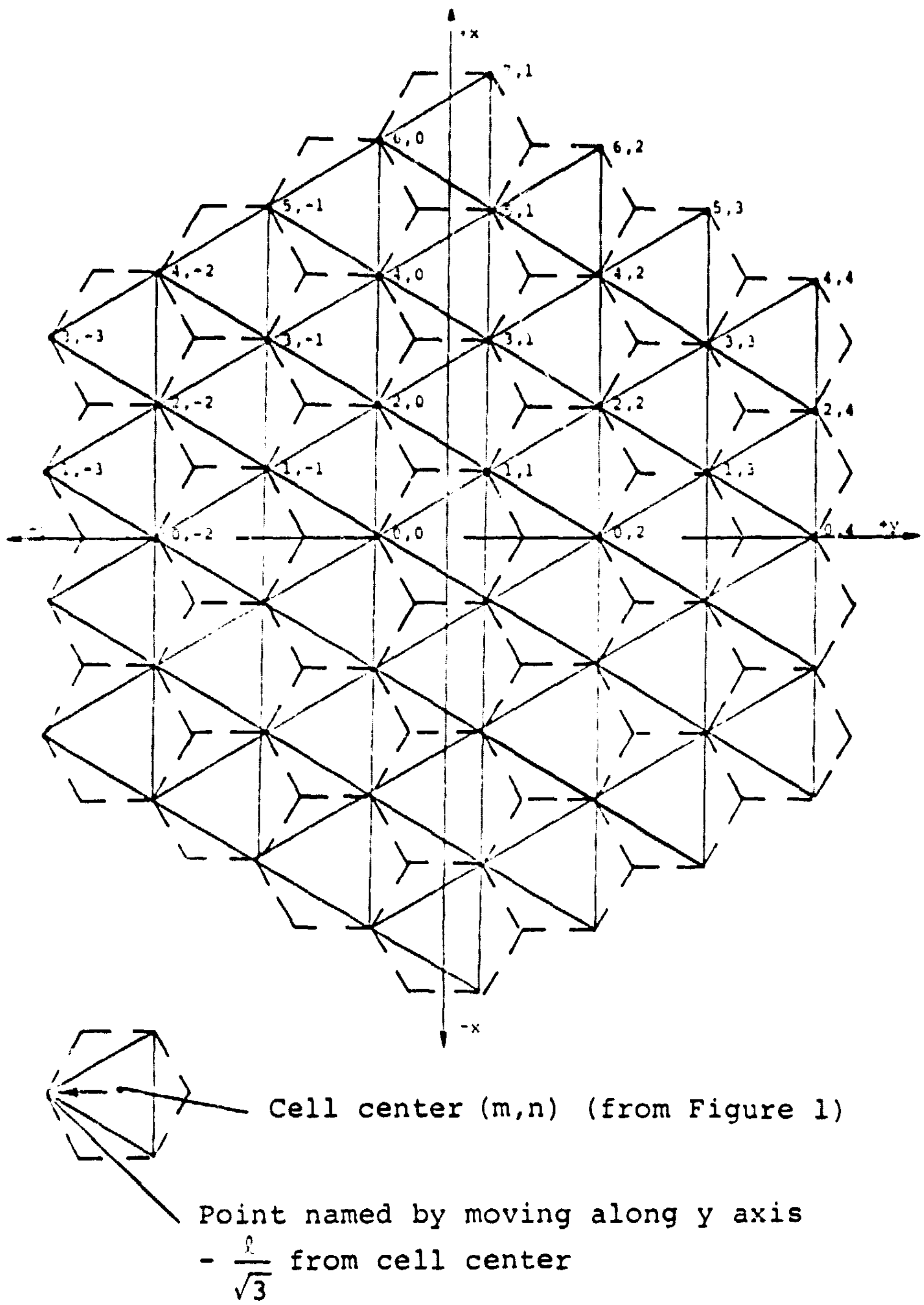
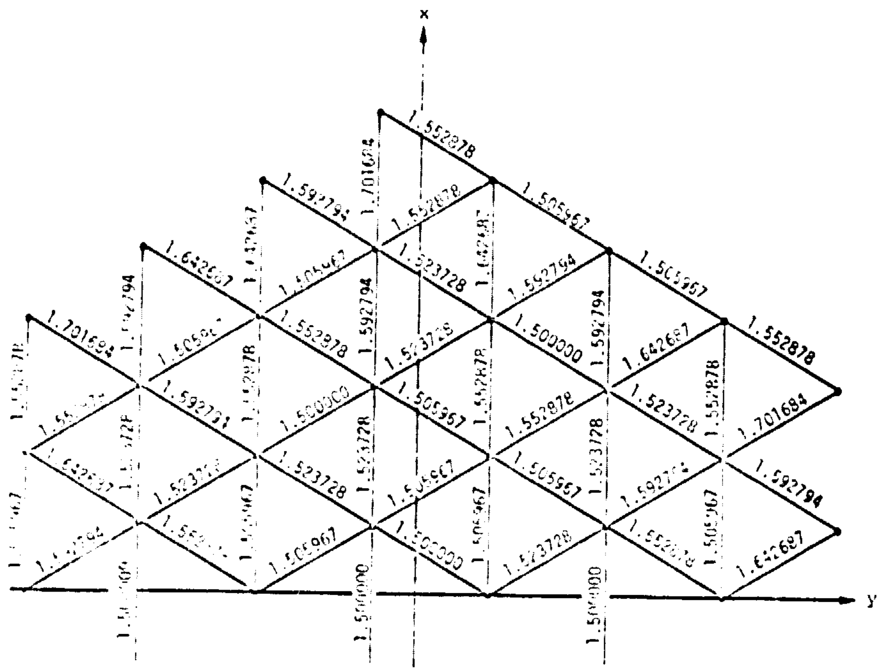


Figure 4b. Lower surface point designations.

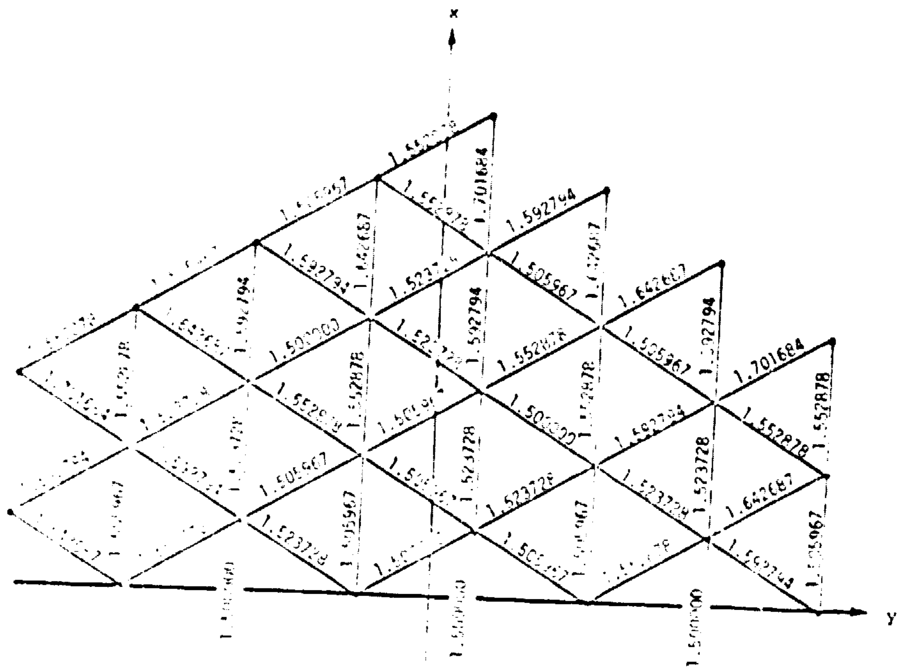


Symmetric about y axis.

ℓ = 1.5 m

F = 4.2 m

Figure 5. Lengths of members between points of upper surface (shown in meters).



Symmetric about y axis.

= 1.5 m

F = 4.2 m

Figure 6. Lengths of members between points of lower surface (shown in meters).