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Wilhelm Kinner

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16. Abstract

Prandtl's method of sources and sinks for aerofoils is used to investigate the aerodynamics of circular wings in constant flow. Lift distribution, total lift, and pitching moment are investigated, as well as the influence of changes in the angle of attack.

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ON WINGS OF CIRCULAR DESIGN[1]

W. Kinner
Göttingen

In this article, it will be shown how Prandtl's method of sources and sinks for aerofoils[2] can be used to investigate wings of circular design in constant flow. The equations are made linear in the usual fashion; in particular, we move the reference point of the acceleration potential Φ from the wing to the surface of the circular base. The center of the circle is made to lie at the origin, its surface in the xy-plane; the radius is c. Let the dimensionless Cartesian coordinates be /349*

$$\bar{x}=\frac{x}{c},\quad \bar{y}=\frac{y}{c},\quad \bar{z}=\frac{z}{c}.$$

The x-axis is directed to the rear, the y-axis to the right, and the z-axis is directed vertically. The flow velocity in the x direction is v. The liquid is assumed to be weightless, frictionless, and of constant volume.

Since the calculation of the potential for the given lift distribution leads to difficulties in integration, we work backwards, i.e. we use the "potential function of the first kind" (2) from potential theory, determine the appropriate aerodynamics, and attempt to satisfy the kinematic condition by linear combinations. For this we use the elliptical coordinates φ, μ, η which are associated with the Cartesian coordinates by the following orthogonal transformations:

$$x=c\sqrt{1-\mu^2}\sqrt{1+\eta^2}\cos\varphi,\qquad y=c\sqrt{1-\mu^2}\sqrt{1+\eta^2}\sin\varphi,\qquad z=c\mu\eta. \tag{1}$$

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1. Excerpt from the author's dissertation. The investigation of circular wings is important for the theory of the autogiro.
2. L. Prandtl, this volume, p. 360. The author's oral presentation followed immediately after that of Prandtl.
* Numbers in the margin indicate pagination in the foreign text.

The reverse transformation becomes single-valued when one allows the hyperbolic coordinate μ to change from positive to negative values on going from the upper half-space to the lower. In elliptical coordinates, the well-known[3] solutions of $\Delta\Phi = 0$ have the following real, normalized form: /350

$$\Phi_n^m(\mu, \eta, \varphi) = \frac{1}{i^{n-m+1}} \frac{(n-m)!}{(n+m)!} C_n^m P_n^m(\mu) Q_n^m(i\eta) \cos m\varphi . \tag{2}$$

C_n^m is a constant, and P_n^m and are the associated spherical functions of the first and second kinds, respectively. We are only interested in the functions Φ_n^m with odd n + m, since those with n + m even are constant on the base surface. The lift density at a point (μ, φ) on the base surface is proportional to the potential differencQ_n^m

$$p_u - p_{ob} = \varrho (\Phi_{ob} - \Phi_u) \tag{3}$$

The total lift $A = \iint (p_u - p_{ob})\, dF$ is:

$$A_1^0 = \frac{4}{3} \pi \varrho c^3 C_1^0, \quad A_n^m = 0 \text{ for all other } (n,m) \tag{4}$$

The moment about the y-axis $M = \iint (p_u - p_{ob})\, x\, dF$ is:

$$M_2^1 = \frac{4}{15} \pi \varrho c^3 C_2^1, \quad M_n^m = 0 \text{ for all other } (n,m) \tag{5}$$

The downdraught on the wing follows from $w(x,y) = \frac{1}{v} \int_{-\infty}^{x} \frac{\partial \Phi}{\partial z} dx$ by integrating in the plane of the wing parallel to the direction of flow. One obtains:

$$w_1^0(\mu, \varphi) = \frac{C_1^0}{v} \left[Q_1(\overline{y}) - \frac{\pi}{2} P_1^1(\mu) \cos\varphi \right], \quad w_2^1(\mu, \varphi) = \frac{\pi}{4} \frac{C_2^1}{v} P_2(\mu) \tag{6}$$

In general, $w_n^m(\mu, \varphi)$ may be represented by tesseral spherical functions, to which for odd n the factor $(-1)^{\frac{n-1}{2}} \frac{C_n^m}{v} Q_n(\overline{y})$ is attached.

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3. M. J. O. Strutt, Lamé, Mathieu, and Related Functions in Physics and Technology, Ergebnisse der Mathematik und ihrer Grenzgebiete, Vol. 1, 1932, p. 59.

In this manner, the downdraught for the functions concerned is made to change logarithmically.

The shape of the wing is given by $z(x, y) = \frac{1}{v}\int^{x} w\,dx$.

The drag W_n^m, according to the separation law of Munk, corresponds to its one dimensional analog. For odd m, the lift density $a(y) = \int_{-x_0}^{+x_0} (p_u - p_{ob})\,dx$ (with $x_0 = \sqrt{c^2 - y^2}$) vanishes identically; for even m one obtains:

$$a_{2\lambda+1}^{2\varkappa}(\bar{y}) = (-1)^{\lambda+1} 2\pi \varrho c\, C_{2\lambda+1}^{2\varkappa} \int_{-1}^{\bar{y}} P_{2\lambda+1}(\bar{y})\,d\bar{y}, \qquad w_{2\lambda+1}^{2\varkappa}(\bar{y}) = (-1)^{\lambda} \frac{C_{2\lambda+1}^{2\varkappa}}{v} Q_{2\lambda+1}(\bar{y}) \tag{7}$$

$$W_1^0 = \pi \frac{\varrho c^2 C_1^{0\,2}}{v^2}, \qquad W_3^0 = \frac{\pi}{6} \frac{\varrho c^2 C_3^{0\,2}}{v^2} \quad \ldots\ldots\ldots\ldots \tag{8}$$

For the function Φ_n^m, there is no suction pressure at the leading edge since the induced speeds (except at the wing ends) are finite. It follows that these functions alone are insufficient to permit investigation of the influence of changes in the angle of attack.

The "potential function of the second kind," Φ_n necessary for this is obtained from the Φ_{n+1}^n (n = 0, 1, 2,....) by the limiting process

$$\Phi_n(\mu, \eta, \varphi) = \lim_{\Delta c \to 0} \frac{\left[1 + (n+1)\frac{\Delta c}{c}\right] \Phi_{n+1}^n(\mu + \Delta\mu, \eta + \Delta\eta, \varphi) - \Phi_{n+1}^n(\mu, \eta, \varphi)}{\Delta c} \tag{9}$$

from which

$$\Phi_n = C_n \frac{\mu (1 - \mu^2)^{n/2}}{(\mu^2 + \eta^2)(1 + \eta^2)^{n/2}} \cos n\varphi \quad \ldots\ldots\ldots \tag{10}$$

The Φ_n become infinite at the edges of the circle; they have no /351
physical meaning in and of themselves since the outflow condition at the rear edge (that the lift density approach zero) is not fulfilled.

The lift is

$$A_0 = 4\pi \varrho c^2 C_0, \qquad A_n = 0 \text{ for } n = 1, 2, 3, \ldots \tag{11}$$

The moment about the y-axis is

$$M_1 = \frac{4}{3}\pi\varrho c^3 C_1, \qquad M_n = 0 \text{ for } n = 0, 2, 3, \ldots. \tag{12}$$

To calculate the downdraught on the wing, one integrates to a line parallel to the x-axis outside the plane of the wing base, and, after the integration has been carried out, shifts the terminus to the base surface. The downdraught is independent of inside the base circle:

$$w_{2\varkappa}(\bar{y}) = (-1)^{\varkappa+1}\frac{C_{2\varkappa}}{v}\frac{d\,Q_{2\varkappa}(\bar{y})}{d\,\bar{y}}, \qquad w_{2\lambda+1}(\bar{y}) = (-1)^{\lambda}\frac{\pi}{2}\frac{C_{2\lambda+1}}{v}\frac{d\,P_{2\lambda+1}(\bar{y})}{d\,\bar{y}} \quad . \tag{13}$$

Thus the wing surfaces are ruled surfaces.

The drag vanishes for odd n and becomes infinite for even n; for a curved section of the front edge the suction pressure becomes infinite for even n if the integration is extended to the ends of the wing.

For the corresponding one dimensional equation, one has

$$a_{2\varkappa}(\bar{y}) = (-1)^{\varkappa}\, 2\pi\varrho c\, C_{2\varkappa} P_{2\varkappa}(\bar{y}), \qquad a_{2\lambda+1}(\bar{y}) = 0 \quad \ldots \tag{14}$$

$$w_{2\varkappa}(\bar{y}) = (-1)^{\varkappa+1}\frac{C_{2\varkappa}}{v}\frac{d\,Q_{2\varkappa}(\bar{y})}{d\,\bar{y}}, \qquad w_{2\lambda+1}(\bar{y}) = 0 \quad \ldots \tag{15}$$

With the mathematical apparatus we have developed the second basic problem can be solved in the following manner. From the given wing form $z(\bar{x},\bar{y})$ one calculates the downdraught $w(\bar{x},\bar{y}) = v\frac{\partial z(\bar{x},\bar{y})}{\partial \bar{x}}$ and separates from the result the downdraught function of the first kind that is contained in it. The remaining function $w(\bar{y})$ is a function of $\bar{y}$ alone; one manipulates this as a downdraught function of the second kind so that the outflow condition at the rear edge is fulfilled. The potential coefficients $C_{2\varkappa}$ are determined by an infinite system of linear equations; from these follow the $C_{2\lambda+1}$ by means of the outflow condition.

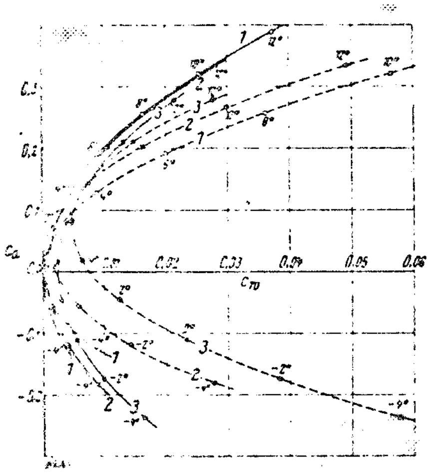

Fig. 1. Polar Curve of the Theoretical Drag.

1. Planar circular disc.
2. Surface with fixed aerodynamic center $\varphi_1^0 + \varphi_3^0 + \varphi_2^1$ with $C_1^0 = C_3^0$, $C_2^1 = -2.58\, C_1^0$ and $\frac{C_1^0}{r^2} = 0.025$
3. As in 2. with $\frac{C_1^0}{r^2} = 0.050$.

solid curves: line drag
stippled curves: surface drag

The distribution of lift, the total lift, and the pitching moment are thus known. The influences of changes in the angle of attack can be investigated by the linear superposition of a planar circular disc. The "surface drag" $W_F = -\frac{1}{v}\iint (p_u - p_{ob})\, w(x,y)\, dF$ is greater than the actual drag by the amount of the suction pressure. The actual drag is identical with the "line drag" $W_L = -\frac{1}{v}\int_{-c}^{+c} a_L(y)\, w_L(y)\, dy$, since the outflowing vortex sheet, and thus also its kinetic energy, the equivalent of the drag work, is not changed by displacement of the lift elements. According to their analytical form, the lift and its moment appear as linear expressions, the two drags and the suction pressure as quadratic expressions in the two parameters $\frac{H}{c}$ (camber parameter) and tg a. /352

According to the procedure outlined, the planar circular disc, the calotte, a wing with fixed aerodynamic center, and S-flap (?) were investigated (see illustration); also, the rolling moment of a warped wing was calculated, using the appropriate antisymmetric potential function (trigonometric factor sin ϕ).

Experiments to check the theory are in preparation at the Institute for Aerodynamic Research in Göttingen. A theoretical investigation of elliptically shaped wings in a constant stream, using Lamé functions, is underway.