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Simulation of a Navigator Algorithm for a Low-Cost GPS Receiver

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SIMULATION OF A NAVIGATOR ALGORITHM FOR A LOW-COST GPS RECEIVER

by

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SUMMARY

The analytical structure of an existing navigator algorithm for a low cost GPS receiver is described in detail to facilitate its implementation on in-house digital computers and real-time simulators. The material presented includes a simulation of GPS pseudorange measurements, based on a two-body representation of the NAVSTAR spacecraft orbits and a four component model of the receiver bias errors. A simpler test for loss of pseudorange measurements due to spacecraft shielding is also noted.

INTRODUCTION

The global positioning system (GPS) is a worldwide navigation network, being developed by the Department of Defense, which eventually will comprise a constellation of 24 NAVSTAR earth satellite spacecraft for transmitting navigation information to system users. Its obvious potential for additional utilization by a large number of non-military users has attracted attention in fields as diverse as general aviation (GA) and shipping. The cost of the GPS receiver for such users, which measures pseudorange from the user craft to four of the NAVSTAR spacecraft simultaneously, has been recognized as one of its major design factors (reference 1). The fact that simultaneous measurement of the pseudorange data requires a minimum of four receiving channels represents a significant cost consideration.

Interest in developing a practical low-cost GPS receiver for civilian users has led to the formulation of position fixing and navigation schemes based on the use of a single channel instrument. As pseudorange data must then be received sequentially rather than simultaneously, there is a need for investigating the resulting navigational accuracy and the effect of user craft motion between measurements. In this connection, the effects of intentional degradation of the GPS signals, loss of pseudorange data due to spacecraft shielding, and the time between updating all require additional study. The purpose in this paper is to describe one such low-cost navigator algorithm, as devised by the Mitre Corporation (reference 2), in sufficient detail to facilitate its implementation on digital computers and real-time simulators at LRC. The block diagram in figure 1 depicts the overall simulation structure, which comprises two main elements that respectively define the pseudorange measurements model and the navigator algorithm. Additionally, the FORTRAN names of principal program variables and calls to subroutines are indicated at appropriate places on figure 1 for convenience of reference to the current in-house version of Mitre's programing (see reference 2), which is included in the APPENDIX.

SYMBOLS

A	spacecraft azimuth, deg
a	semimajor axis of spacecraft orbit, n. mi.
$a_{\oplus}$	equatorial radius of the geoid, n. mi.
b	GPS pseudorange bias error, equivalent n. mi.
b_I	ionospheric delay component of b, n. mi.
b_{MPR}	multipath and receiver noise component of b, n. mi.
b_c	user clock bias component of b, n. mi.
b_{ID}	intentional degradation component of b, n. mi.
$b_{\oplus}$	polar radius of the geoid, n. mi.
E	eccentric anomaly of spacecraft orbit, deg
e	eccentricity of spacecraft orbit
$e_{\oplus}$	eccentricity of the geoid
f	spacecraft true anomaly, deg
H	pseudorange partial derivative matrix
h	local topocentric altitude, ft
h_k	k^{th} row of H matrix
i	inclination of spacecraft orbit, deg
M	mean anomaly of spacecraft orbit, deg
m	number of Δt updating periods (see eqn. (4))
n	spacecraft mean motion, deg/sec (eqn. (4))
R	local topocentric position, n. mi.
r	pseudorange measurements vector, n. mi.
r_k	k^{th} pseudorange measurement, n. mi.
T	time since pericenter passage in spacecraft orbit, hrs
ℓ	geographic or geodetic East longitude, deg

t	true time, hrs
t_o	GMT at start of simulation, hrs
U	local topocentric position of user craft and pseudorange bias error, n. mi. (see equation 9 (a))
V	user craft velocity vector, n. mi/hr
X, Y, Z	rectangular components of R, n. mi.
x, y, z	rectangular components of ρ, n. mi.
α,β	smoothing coefficients (see eqn. (11))
ε	apparent spacecraft elevation, deg
θ	geocentric right ascension, deg
μ	universal gravitational constant, $(\text{n. mi})^3/\text{sec}^2$
ρ	geocentric distance, n. mi.
ϕ	geodetic latitude, deg
ϕ'	geocentric latitude, deg
φ	user craft roll or bank angle, deg
ψ	user craft true velocity heading, deg
Ω	right ascension of ascending mode of spacecraft orbit, deg
ω	argument of pericenter of spacecraft orbit, deg
$\omega_\oplus$	axial rotation rate of the earth, 15^o/hr

Subscripts:

CT	cross track
G	Greenwich meridian
k	k^{th} spacecraft
p	GPS constellation index for spacecraft phase angle
s	GPS constellation index for spacecraft orbit plane phase angle or local reference site

Notation:

$\Delta(\ )$ incremental value or dwell time interval

$(\hat{\ })$ estimated value

$(\ )^T$ matrix transpose

$(\ )^{-1}$ matrix inverse

$\{\ \}$ vector

PSEUDORANGE MEASUREMENTS MODEL

The flow diagram in Figure 1 shows the simulation employed for the pseudorange measurements to incorporate models for the respective motions and positions of the NAVSTAR spacecraft and the user craft relative to a local topocentric reference site, and for the measurement errors associated with the GPS receiver. The first of these is ionospheric delay, which is modeled as a deterministic error in terms of an ion density scale factor and pseudorange path length. Multipath error and receiver noise is modeled next, and is represented simply as random white noise. The third type of measurement error simulated is the user clock bias, which includes random white noise and a starting offset in addition to drift and aging terms that increase with time and are associated with the assumption that the clock is driven by a crystal oscillator. The remaining error source is the intentional degradation bias, which is generated separately for each spacecraft by independently passing uniform random numbers through an exponential filter. Scaling of the filter's Gaussian output is then adjusted to give a standard deviation on the pseudorange for each spacecraft such that the 2σ user position error resulting from all four spacecraft is 500 meters.

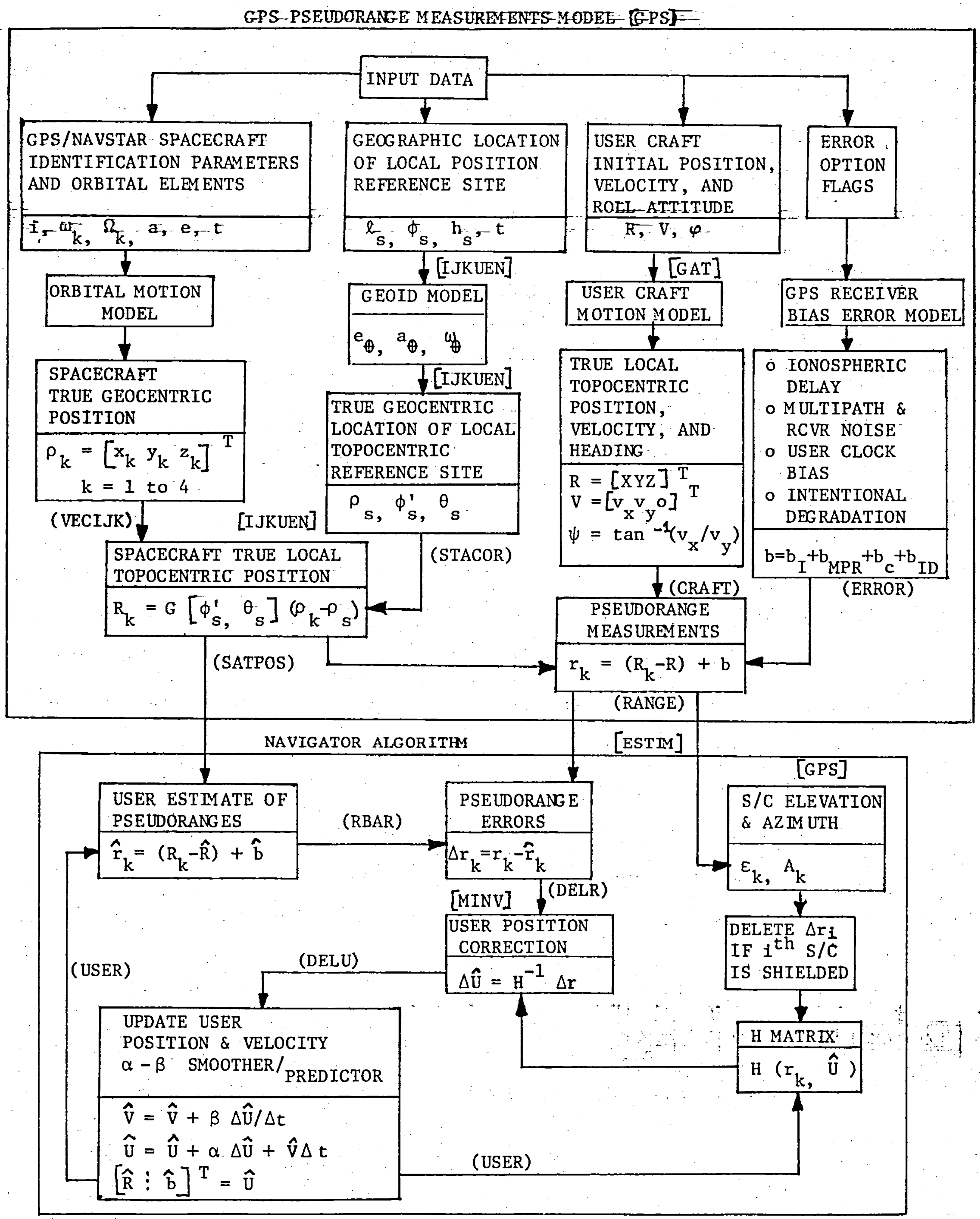

Figure 1. SIMULATION BLOCK DIAGRAM

GPS/NAVSTAR Spacecraft Position. - As indicated in Figure 1, there are several steps in determining the required true topocentric positions of the four NAVSTAR spacecraft. This process begins with the inertial position of the spacecraft orbit relative to geocentric equatorial axes I, J, K having I toward the vernal equinox and K along the earth's spin axis as illustrated in Figure 2. The orbit orientation relative to these axes is defined by the three angles Ω, i, and ω as indicated. Let PQW be a set of perifocal axes aligned with the orbit plane such that W coincides with the normal to the orbit plane, and P is along the line of apsides toward perigee. The position of the spacecraft in its orbital plane is given by its geocentric distance ρ, and true anomaly f. The transformation to perifocal coordinates P, Q, W is given by

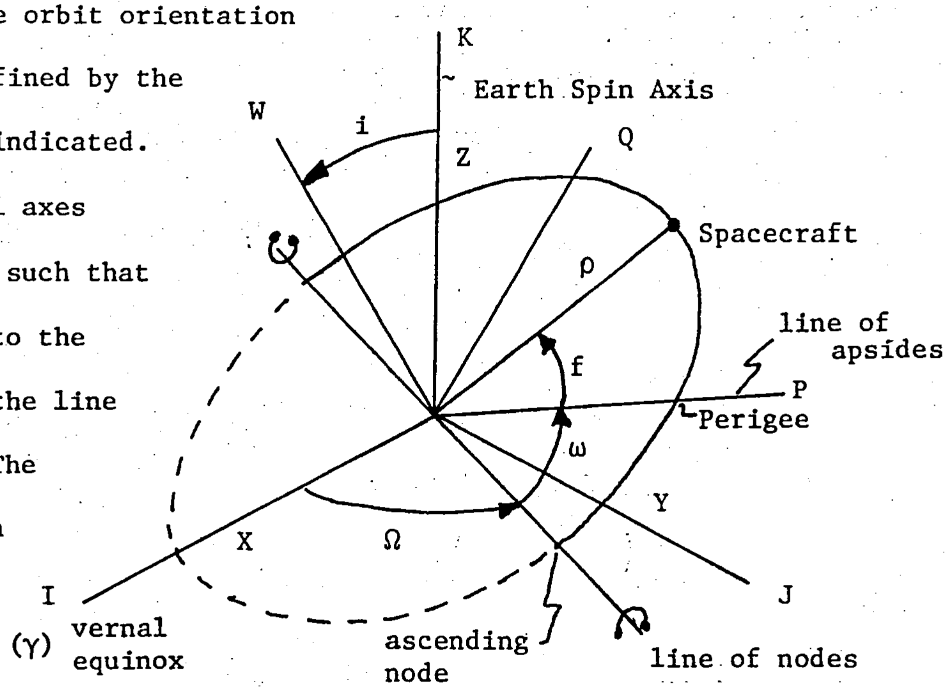

Figure 2. Spacecraft Orbit Orientation

$$\left.\begin{aligned} P &= \rho \cos f \\ Q &= \rho \sin f \\ W &= 0 \end{aligned}\right\} \qquad (1)$$

which are then transformed to I, J, K inertial coordinates

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = W\,(\Omega,\ i,\ \omega)^{-1} \begin{Bmatrix} P \\ Q \\ W \end{Bmatrix} \qquad (2)$$

where,

$$W^{-1} = \begin{pmatrix} \cos\omega\cos\Omega - \cos i\sin\omega\sin\Omega & -\sin\omega\cos\Omega - \cos i\cos\omega\sin\Omega & \sin i\sin\Omega \\ \cos\omega\sin\Omega + \cos i\sin\omega\cos\Omega & -\sin\omega\sin\Omega + \cos i\cos\omega\cos\Omega & -\sin i\cos\Omega \\ \sin\omega\sin i & \cos\omega\sin i & \cos i \end{pmatrix}$$

The task of obtaining ρ and f, for evaluating equations (1) and (2) for each of the four spacecraft, is considerably simplified by the assumption of two-body circular orbits ($e = 0$). In this case the position of the apsidal line in the orbit plane and the time of perigee passage are arbitrary, and ω and T may be set to zero so that the apsidal and nodal lines coincide and equation (2) reduces to

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{pmatrix} \cos\Omega & -\cos i\sin\Omega \\ \sin\Omega & \cos i\cos\Omega \\ 0 & \sin i \end{pmatrix} \begin{pmatrix} \rho\cos f \\ \rho\sin f \end{pmatrix} \tag{2a}$$

and usual two-body orbital equations

$$\left.\begin{aligned} \rho &= \frac{a(1-e^2)}{1+e\cos f} = a\,(1-e\cos E) \\ \tan\frac{f}{2} &= \sqrt{\frac{1+e}{1-e}}\,\tan\frac{E}{2} \\ M &= n(t-T) = E - e\sin E \end{aligned}\right\} \tag{3}$$

become

$$\begin{aligned} \rho &= a \\ f &= M = E = nt, \end{aligned} \qquad \left\{\begin{aligned} T &= 0 \\ n &= \sqrt{\frac{\mu}{a^3}} \end{aligned}\right. \tag{3a}$$

The next step is to calculate the unique values of f_k for each of the four spacecraft, which specify their locations in the GPS constellation. As the planned configuration is for eight spacecraft equally spaced in each of three orbit planes, inclined 63^o to the equator with their nodal lines equally spaced 120^o apart, each spacecraft will have a unique combination of Ω_p and an in-plane angle f_s. The arrangement assumed for the present simulation is illustrated in figure 3 for one such orbiting ring of eight spacecraft. In addition to Ω_p, each of the three spacecraft rings has an initial rotation f_p from the line of apsides as indicated. Taking f_p and f_s into account, the true anomaly for any of the spacecraft is given by

$$f_k(t) = f_p + f_s + nt \quad \} \quad (4)$$

in which,

$$f_s = \frac{\pi}{4}(s-1), \quad 1 \leq s \leq 8$$

$$f_p = \begin{cases} -\pi/12 & p = 1 \\ \pi/12 & p = 2 \\ 0 & p = 3 \end{cases}$$

$$t = t_o + m\Delta t$$

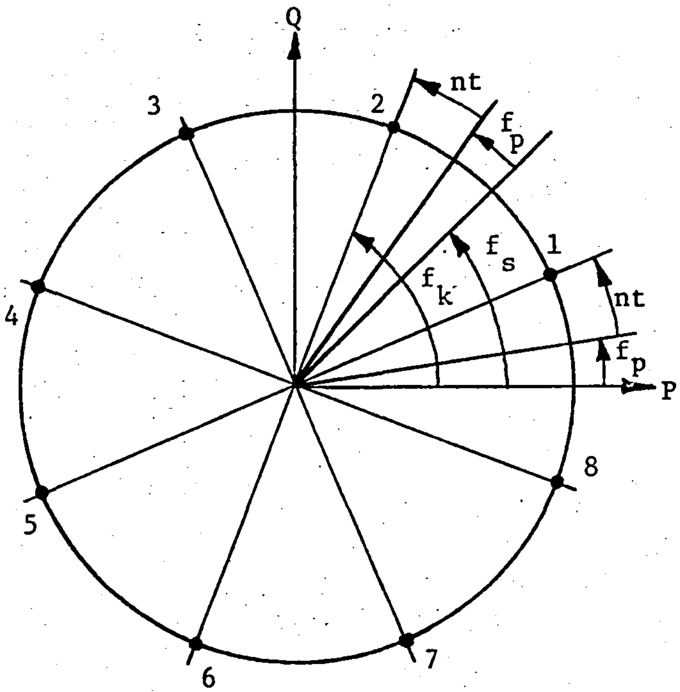

Figure 3. Spacecraft Orbital Spacing Geometry.

To illustrate these calculations, suppose the third spacecraft (k=3) happens to be the fourth one in the first ring so that p = 1 and s = 4. The true anomaly is then given by $f_3(t) = {}^{-\pi}/12 + {}^{3\pi}/4 + nt$, and $\Omega_1 = 0$. Thus, the ~~four sets of x_{k}, y_{k}, z_{k} may be readily obtained from equations (2a) and (4)~~ at the required times.

Geocentric Location of Local Reference Site. - The required spacecraft position vectors R_k relative to the user's local reference frame are obtained first in inertial coordinates as $(\rho_k - \rho_s)$, then transformed to topocentric coordinates with origin at the local reference site. The true geocentric location of the local reference site origin is given by the vector ρ_s, which is calculated from its geographic coordinates ℓ_s, ϕ_s, h_s using the geoid model and rotational transformation respectively illustrated in Figures 4 and 5. The sketch in Figure 4 represents an x-z meridian plane view of the earth in cross-section that shows the relationships between geodetic and geocentric latitude and radial distance, which are defined by

$$\left.\begin{aligned}
S &= (1-e_\oplus^2)C, \qquad e_\oplus = \sqrt{1 - \frac{b_\oplus^2}{a_\oplus^2}} \\
C &= \frac{a_\oplus}{\sqrt{1-e_\oplus^2 \sin^2 \phi_s}} \\
\cos \phi'_s &= \frac{(h_s + C) \cos \phi_s}{\rho_s} \\
\sin \phi'_s &= \frac{(h_s + S) \sin \phi_s}{\rho_s} \\
\rho_s &= \sqrt{(h_s+C)^2 \cos^2 \phi_s + (h_s + S)^2 \sin^2 \phi_s}
\end{aligned}\right\} \quad (5)$$

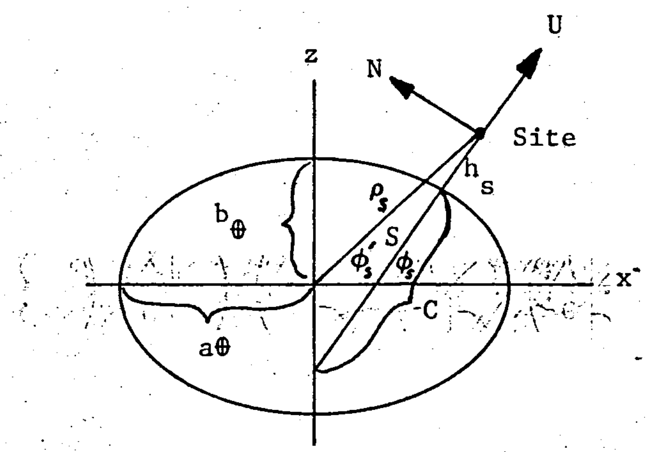

Figure 4. Geoid Geometry.

With reference to Figure 5, the geocentric right ascension of the user's local reference site at time t is

$$\theta_s(t) = \theta_G(t) + \ell_s$$

or

$$\theta_s(t) = \omega_{\oplus} t + \ell_s \quad (6)$$

so that its geocentric coordinates are

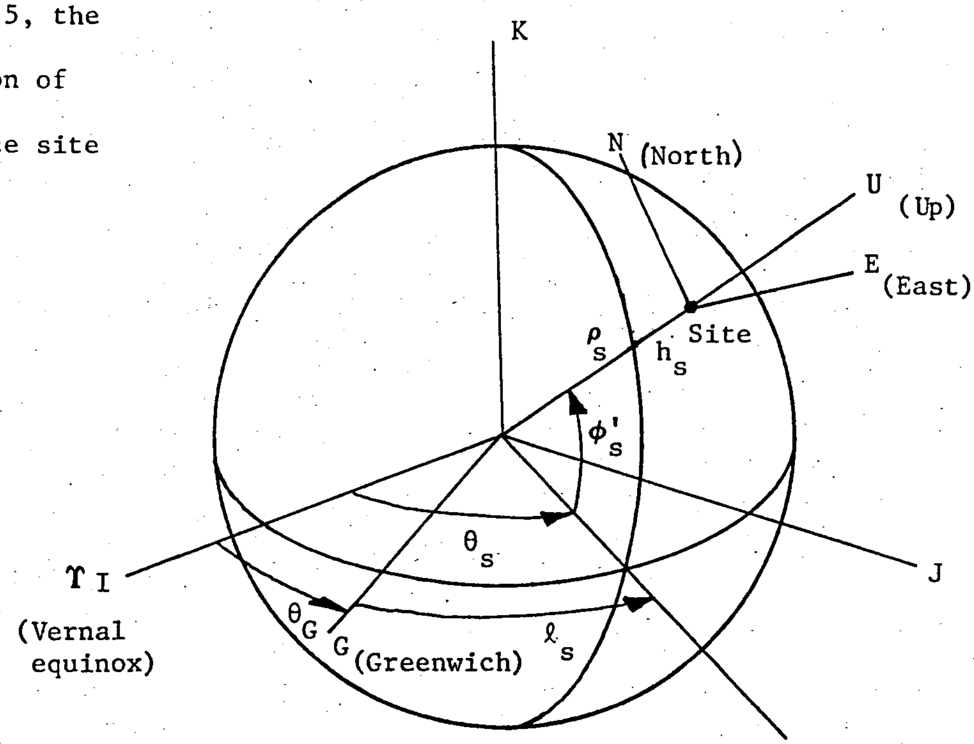

Figure 5. Transformation between local Topocentric and geocentric coordinates.

$$\left.\begin{aligned} x_s &= (h_s+C)\cos\phi_s\cos\theta_s \\ y_s &= (h_s+C)\cos\phi_s\sin\theta_s \\ z_s &= (h_s+S)\sin\phi_s \end{aligned}\right\} \quad (7)$$

As $(\rho_s, \theta_s, \phi'_s)$ also define the origin of the local topographic axes (U, E, N), with respect to which the user craft motion is referred, the transformation of $(\rho_k - \rho_s)$ to these coordinates is

$$R_k = \begin{Bmatrix} Z_k \\ X_k \\ Y_k \end{Bmatrix} = G\,(\phi'_s,\ \theta_s)\begin{Bmatrix} x_k - x_s \\ y_k - y_s \\ z_k - z_s \end{Bmatrix} \qquad (8)$$

where

$$G(\phi_s', \theta_s) = \begin{pmatrix} \cos\phi_s' \cos\theta_s & \cos\phi_s' \sin\theta_s & \sin\phi_s' \\ -\sin\theta_s & \cos\theta_s & 0 \\ -\sin\phi_s' \cos\theta_s & -\sin\phi_s' \sin\theta_s & \cos\phi_s' \end{pmatrix}$$

Pseudorange Measurements. - The remaining step in simulating the pseudorange measurements is to express the R_k in terms of range to the user craft, then corrupting the resulting range vectors (R_k-R) by adding the simulated GPS receiver bias errors as indicated in Figure 1. This procedure is illustrated by the sketch in Figure 6, and the resulting pseudoranges are given by

$$r_k = (R_k-R) + b \tag{9}$$

where R is the user's assumed true position in (U, E, N) coordinates as furnished by a user craft motion simulator such as a general aviation trainer (GAT).

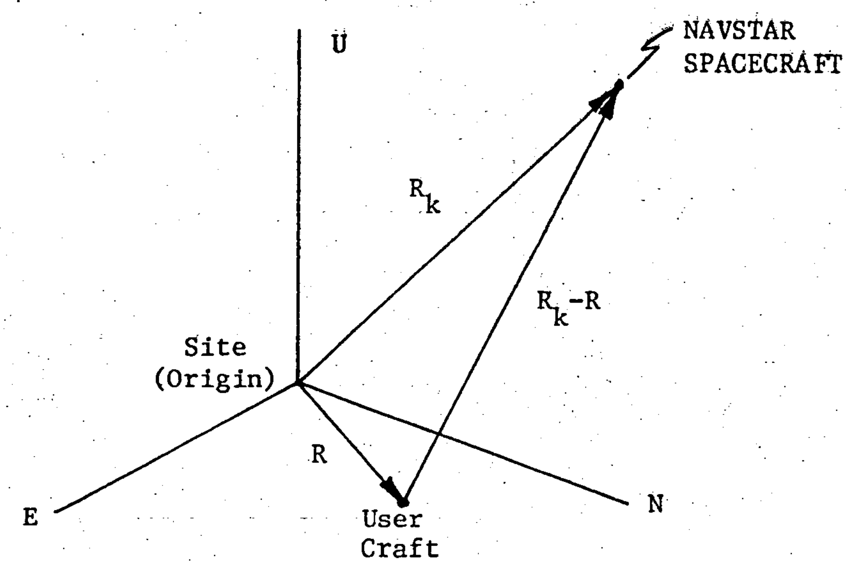

Figure 6. - USER/NAVSTAR Range Geometry.

GPS LOW-COST NAVIGATOR ALGORITHM

The lower portion of Figure 1 depicts the general structure of Mitre's GPS low-cost navigator algorithm. There are three main computational tasks associated with the algorithm operation. These are determining if any of the four pseudorange measurements are lost due to shielding of the GPS receiver, estimation of position and receiver bias corrections, and computing the position and velocity updates by means of an α-β smoother.

<u>GPS/NAVSTAR Shielding</u>. In order for the GPS receiver to acquire a pseudorange measurement, the apparent elevation of the NAVSTAR spacecraft relative to the receiver antenna on top of the user craft must be greater than 10^o. The spacecraft is considered to be shielded, so that the pseudorange measurement to it is lost, if either its orbital motion or user craft maneuvering cause this condition not to be met. The testing procedure employed by Mitre for determining whether any of the four spacecraft are shielded is a rather complex scheme, based on

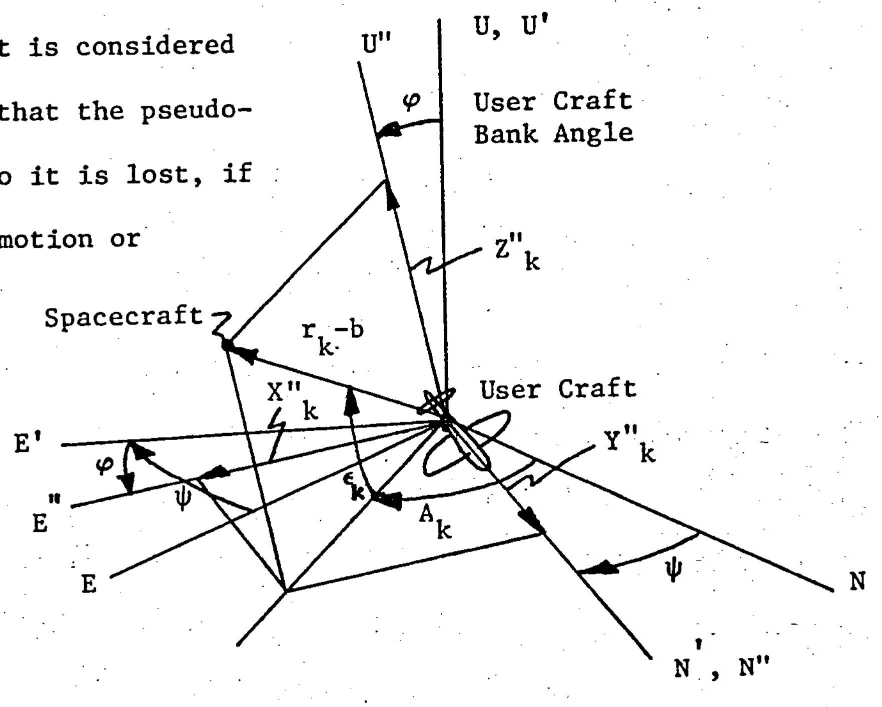

Figure 7. - Spacecraft Shielding Geometry.

their azimuthal positions and apparent elevations relative to the user craft. A much simpler test is illustrated in Figure 7. The only requirement is to determine whether the spacecraft in question is above the E"-N" plane, which coincides with that of the user craft's wings. Thus, the k^{th} spacecraft will not be shielded as long as the Z''_k component of $r_k - b$

$$\begin{Bmatrix} X''_k \\ Y''_k \\ Z''_k \end{Bmatrix} = \begin{pmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & 0 & \cos\varphi \end{pmatrix} \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{Bmatrix} X_k \\ Y_k \\ Z_k \end{Bmatrix} \tag{10}$$

remains positive.

The apparent spacecraft elevation

$$\varepsilon_k = \tan^{-1}\left(\frac{Z''_k}{\sqrt{X''^2_k + Y''^2_k}}\right) \tag{11}$$

still must be tested if the receiver antenna coverage is assumed to be limited to a minimum value of ε_k. However, the fact that ε_k is defined relative to the E"-N" plane, rather than the local horizon (E-N plane), still permits avoiding the need to evaluate complicated conditions on the spacecraft azimuthal position and the user craft bank angle.

User Craft Position and Bias Error Corrections. - The procedure used for obtaining these quantities is based on a linearized Taylor's series expansion of equation (9) about the current estimate of user craft position and GPS receiver bias (see reference 1). This expansion is

$$r_k = \hat{r}_k + \left.\frac{\partial r_k}{\partial U}\right|_{\hat{U}} \Delta\hat{U} + - - - \tag{9a}$$

where $U = \left[R \vdots b\right]^T = \left[XYZb\right]^T$ and $\Delta\hat{U} = U - \hat{U}$. By expressing equation (9) in rectangular form and differentiating,

$$\left.\frac{\partial r_k}{\partial U}\right|_{\hat{U}} \Delta\hat{U} = \left.\frac{\partial r_k}{\partial X}\right|_{\hat{U}} \Delta\hat{X} + \left.\frac{\partial r_k}{\partial Y}\right|_{\hat{U}} \Delta\hat{Y} + \left.\frac{\partial r_k}{\partial Z}\right|_{\hat{U}} \Delta\hat{Z} + \left.\frac{\partial r_k}{\partial b}\right|_{\hat{U}} \Delta\hat{b}$$

$$= \left(\frac{\hat{X}-X_k}{r_k-\hat{b}}\right) \Delta\hat{X} + \left(\frac{\hat{Y}-Y_k}{r_k-\hat{b}}\right) \Delta\hat{Y} + \left(\frac{\hat{Z}-Z_k}{r_k-\hat{b}}\right) \Delta\hat{Z} + \Delta\hat{b}$$

$$= h_k \, \Delta\hat{U}$$

Rearranging equation (9a) and solving for ΔU gives

$$\Delta\hat{U} = H^{-1} \Delta r \tag{12}$$

where $\Delta r = \left\{r_k - \hat{r}_k\right\}$ and $H = \left[h_1 \; h_2 \; h_3 \; h_4\right]^T$, in which the estimated pseudorange measurements $\hat{r}_k$ may be calculated by evaluating equation (9) in the form

$$\hat{r}_k = \sqrt{(X_k - \hat{X})^2 + (Y_k - \hat{Y})^2 + (Z_k - \hat{Z})^2} + \hat{b}$$

using the spacecraft ephemeris data R_k and the current estimate of $\hat{U}$.

User Craft Position and Velocity Update. - The Mitre GPS navigator algorithm is formulated to provide smoothed estimates of updated position and velocity, by approximate smoothing backwards in time over the four r_k measurements. The procedure is to calculate $\Delta\hat{U}$ after each new pseudorange measurement is received, by processing it with the most recent values for the other three elements of r_k. As the dwell time to receive a pseudorange measurement is Δt, new values of $\Delta\hat{U}$ are generated every 1.2 sec. The corresponding position and velocity updating is accomplished by an α-β smoother/predictor of the form

$$\left.\begin{aligned} \hat{V}(t+\Delta t) &= \hat{V}(t) + \beta\Delta\hat{U}/\Delta t \\ \hat{U}(t+\Delta t) &= \hat{U}(t) + \alpha\Delta\hat{U} + \hat{V}(t+\Delta t)\ \Delta t \end{aligned}\right\} \qquad (13)$$

These quantities are also used to estimate cross-track error

$$\Delta\hat{R}_{CT} = (X-\hat{X}) \cos\psi - (Y-\hat{Y}) \sin\psi \qquad (14)$$

where

$$\psi = \tan^{-1} (Vx/Vy)$$

in which X, Y, V_x, and V_y are outputs from the user craft motion simulator.

CONCLUDING REMARKS

The simulator structure described herein provides a useful analytical tool for conducting further research and evaluations of navigator algorithms based on the use of a low-cost GPS receiver. A simpler test for loss of pseudorange measurements due to spacecraft shielding is noted. This test eliminates the need for the relatively complex one contained in Mitre's programing (see reference 2).

REFERENCES

1. Noe, Philip S.; and Myers, Kenneth A.: A Position Fixing Algorithm for the Low-Cost GPS Receiver, IEEE Trans. on Aerospace and Electronic Systems, Vol. AES-12, March 1976.

2. Shively, Curtis A.: A Real-Time Simulation for Evaluating a Low-Cost GPS Navigator. TR-80W00081, The Mitre Corporation, 1980.

LRC Implementation of MITRE's Low-Cost GPS Navigator Simulation

```
      PROGRAM GPSNAV(INPUT,OUTPUT,TAPE6,TAPE5=INPUT)
C     W. F. HOOGE    FEO
C     GPS PARAMETER DEFINITIONS FOR BADER FIELD SCENARIO (SIMULATED)
C     (ORIGIN AT ACY VOR)    (ATLANTIC CITY)
C     WITHOUT INTENTIONAL DEGRADATION (JBIAS=0)
C     SET JBIAS=1 FOR INTENTIONAL  GPS  DEGRADATION
      DIMENSION FP(3),OMEGA(3),VT(5),PX(800),PY(800)
      DIMENSION PHASE(3),CRAFT(3),IX(4),NJSH(4),OJSH(4)
      DIMENSION RANGE(4),EL(4),AZ(4),SATPOS(4,3)
      DIMENSION VECPOW(3),VECIJK(3),VECUEN(3),STACOR(3)
      DIMENSION IR(2)
      COMMON/NAVER/ERROR(4),BIAS(4),EION(4),EMPE(4)
      COMMON/INPAR/TSTRT,DELT,ITER,PSIM
      COMMON/NVFLG/JMPE,JIONE,JSSE,JCLKE,JBIAS
      COMMON/NVPAR/ALPHA,BETA,BETA1,ADAPTA,ADAPTB
      COMMON/SATS/JORB(4),JSAT(4),JROLO(4),JSHD(4)
      COMMON/NVEST/USER(4),VEL(4),USERS(4),VELS(4)
      CALL SEFOMP
      JMPE=1 $ JIONE=1 $ JSSE= 1 $ JCLKE=1 $ JBIAS=1
      ALPHA=.2 $ BETA=.01 $ BETA1=.01 $ ADAPTA=.2 $ ADAPTB=.01
      TSTRT=13.90 $ DELT=1.2 $ ALT=0.  $RLAT=.688629 $ PLONG=-1.301608
      JORB(1)=2 $ JORB(2)=1 $ JORB(3)=1 $ JORB(4)=2
      JSAT(1)=8 $ JSAT(2)=2 $ JSAT(3)=4 $ JSAT(4)=7
C     INPUT DATA FOR SIMULATED LANDING APPROACH COURSE
      PI=3.14159265358979 $ RPD=PI/180. $ DPR=180./PI
      CALL PSEUDO
      DO 7 JTH=1,4 $ OJSH(JTH)=0.
    7 JSHD(JTH)=0.
C     SET UP  GPS  CONSTELLATION PARAMETERS ( A IS IN NAUTICAL MI.)
      A=1.436826 E+4 $ OMEGA(1)=0. $ OMEGA(2)=120. $ OMEGA(3)=240.
      FP(1)=-15. $ FP(2)=15. $ FP(3)=0.
      IX(1)=7329 $ IX(2)=1641 $ IX(3)=6753 $ IX(4)=4159
C     INITIALIZE INTENTIONAL DEGRADATION ERROR BIAS
      ZGAIN=EXP(-4.*DELT/1800.)
C     TAU = 30 MIN
      CONG=528./6080.
C     SAT 1 SIGMA = 500M2DRMS/2*HDOP1.537
      CONU=2.*SQRT(3.)*CONG
      CONZ=CONU*SQRT(2.*4.*DELT/1800.)
C     TAU = 30 MIN
      IR(1)=3117 $ IR(2) = 1379  $ IN=1
      DO 802 J=1,4
      CALL GETRAN(IR,IN,2,RN,Y1,Y2)
      IN = 2
  802 BIAS(J) = RN * CONG
C     CLOCK ERROR MODEL PARAMETERS
      SO=1000./6080.
C     1 USEC INITIAL CLOCK OFFSET
      FO=DELT/60.8
C     FRACT FREQ ERR*DELT*10**9/6080
      FD=DELT**2/1.216 E+7
C     FRACT FREQ DRIFT/SEC*DELT**2*10**9/6080
      SS=50./6080.
C     FRACT SHORT TERM STABILITY*10**9/6080
C     SET INITIAL CRAFT POSITION AND VELOCITY
      DELT=DELT/3600. $ CRAFT(1)=Z/6080. $ CRAFT(2)=X $ CRAFT(3)=Y
      DO 5 I=1,3 $ VEL(I)=0.
    5 USER(I)=CRAFT(I)+0.
      USER(4)=VEL(4)=0.
      VEL(1)=ZDT $ VEL(2)=XDT $ VEL(3)=YDT
C     ITER IS THE ITERATION FOR WHICH PSEUDORANGE AND
C     ELEVATION OF BEST  4  SPACECRAFT ARE COMPUTED
      ITER= TIM=0
C
C     MAIN LOOP STARTS HERE
   33 ITER=ITER+1
C     KTH IS THE SPACECRAFT NUMBER
      DO 17 KTH=1,4 $ NJSH(KTH)=0 $ ERROR(KTH)=0.
      TIM=TIM+1
C     ELAPSED TIME PERIODS (UPDATE INTERVALS)
      TIME=TSTRT+TIM*DELT
C     TRUE TIME (IN HOURS)
C     SIMULATE TRUE USER CRAFT POSITION AND VELOCITY
      TIME=(TIME-TSTRT)*3600. $ DELT=DELT*3600.
C     CALCULATE TRUE ANOMALY  F  FOR  KTH  SPACECRAFT
      F=(FP(JORB(KTH))+ (JSAT(KTH)-1)*45.+30.*TIME)*RPD
C     CALCULATE SPACECRAFT  POW  COORDINATES
C     CALCULATE  IJK COORDINATES OF  KTH  SPACECRAFT
      COSNDE=COS(OMEGA(JORB(KTH))*RPD)
      SINNDE=SIN(OMEGA(JORB(KTH))*RPD)
      VECIJK(1)=A*(COSNDE*COS(F)-COS(63.*RPD)*SINNDE*SIN(F))
      VECIJK(2)=A*(SINNDE*COS(F)+COS(63.*RPD)*COSNDE*SIN(F))
      VECIJK(3)=A*SIN(63.*RPD)*SIN(F)
C     CONVERT SPACECRAFT COORDINATES FROM  IJK  TO  UEN  SYSTEM
```

```
      CALL IJKUE(TIME,ALT,RLONG,RLAT,VECIJK,VECUEN)
      CRAFT(1)=Z/6080. $ CRAFT(2)=X $ CRAFT(3)=Y
      VV1=VECUEN(1)-CRAFT(1)
      VV2=VECUEN(2)-CRAFT(2)
      VV3=VECUEN(3)-CRAFT(3)
      DEN=VV3 $ IF(ABS(VV3).LT..000001) DEN=SIGN(.000001,VV3)
      AZ(KTH)=ATAN2(VV2,DEN) $ IF(AZ(KTH).LT.0.) AZ(KTH)=AZ(KTH)+2.*PI
      DEN=YDT
C     COMPUTE  GAT  VELOCITY HEADING RELATIVE TO TRUE NORTH (DX=0)
      IF(ABS(YDT).LT.000001) DEN=SIGN(.000001,YDT)
      GCRS=ATAN2(XDT,DEN)
      IF(GCRS.LT.0.) GCRS=GCRS+2.*PI
      VVN1=VV1
      VVN2=VV2*COS(GCRS)-VV3*SIN(GCRS)
      VVN3=VV3*COS(GCRS)+VV2*SIN(GCRS)
      VVNN1=VVN1*COS(PHI)+VVN2*SIN(PHI)
      VVNN2=VVN2*COS(PHI)-VVN1*SIN(PHI)
      VVNN3=VVN3
      RADS = VV2**2 + VV3**2
      RADN=SQRT(VVNN2**2+VVNN3**2)
      EL(KTH)=ATAN2(VVNN1,RADN)
C     IF SPACECRAFT ELEVATION IS LESS THAN 10 DEG, CHECK FOR SHIELDING
      IF(EL(KTH).LE.(PI/18.)) NJSH(KTH)=1
C     GO FROM SHIELDED TO NOT, OR NOT TO SHIELDED ONLY IF TWO SAME
C     DECISIONS IN SUCCESSION
      IF(NJSH(KTH).EQ.OJSH(KTH)) JSHD(KTH)=NJSH(KTH)
      OJSH(KTH)=NJSH(KTH)
C     IF JCLKE NOT 0 INCLUDE CLOCK BIAS ERROR
C     SS= SHORT TERM STABILITY, EQUIV. NAUT. MI.
C     SO= STARTING OFFSET, EQUIV. NAUTICAL MI.
C     FO= FREQUENCY OFFSET, EQUIV. NAUTICAL MI.
C     FD= FREQUENCY DRIFT, EQUIV. NAUTICAL MI.
      IF(JCLKE.EQ.0) GOTO 470
      CALL GETRAN(IR,IN,2,RN,Y1,Y2)
      ECB = RN * SS
      CBIAS=SO+FO*TIM+FD*TIM**2+ECB
      ERROR(KTH)=ERROR(KTH)+CBIAS
C     IF JMPE NOT 0 , INCLUDE MULTIPATH ERROR
  470 IF(JMPE.EQ.0) GOTO 410
      ERROR1=0.
      CALL GETRAN(IR,IN,2,RN,Y1,Y2)
      EMPE(KTH) = RN * 35./6080.
      ERROR(KTH)=ERROR(KTH)+ERROR1+EMPE(KTH)
C     IF JBIAS NOT 0, INCLUDE CORRELATED (30 MIN) NOISE BIAS
  410 IF(JBIAS.EQ.0) GOTO 460
      CALL GETRAN(IR,IN,2,RN,RY,Y2)
      BIAS(KTH)=CONZ*(RY-.5)+ZGAIN*BIAS(KTH)
      ERROR(KTH)=ERROR(KTH)+BIAS(KTH)
C     IF JIONE NOT 0, INCLUDE IONOSPHERIC DELAY ERROR
  460 IF(JIONE.EQ.0) GOTO 203
      EPRIM=.94798*COS(EL(KTH))
      EION(KTH)=.0052433/SQRT(1.-EPRIM**2)
      ERROR(KTH)=ERROR(KTH)+EION(KTH)
  203 RANGE(KTH)=SQRT((VECUEN(1)-CRAFT(1))**2+RADS)+ERROR(KTH)
      DO 204 I=1,3
  204 SATPOS(KTH,I)=VECUEN(I)
      IF(ITER.EQ.1.AND.KTH.LT.4) GOTO 17
C     DO ESTIMATE OF USER PREDICTED POSITION (USER) AND VELOCITY (VEL)
C     AND OF SMOOTHED USER POSITION (USERS) AND VELOCITY (VELS)
C
      CALL ESTIM(RANGE,SATPOS,USER,VEL,USERS,VELS,KTH)
C     COMPUTE CROSSTRACK NAVIGATION ERROR
      XX=USER(2)-CRAFT(2) $ YY=USER(3)-CRAFT(3)
      CTE=XX*COS(GCRS)-YY*SIN(GCRS)
      ATE = XX * SIN(GCRS) + YY * COS(GCRS)
      ENV = SQRT(XX**2 + YY**2)
      ELEV=EL(KTH)*DPR $ AZIM=AZ(KTH)*DPR
      HDG=PSI*DPR $ BANK=PHI*DPR
   17 CONTINUE
C     THIS IS THE END OF ONE ITERATION
      GOTO 33
      STOP
      END
```

```
      SUBROUTINE ESTIM(RANGE,SATPOS,USER,VEL,USERS,VELS,KTH)
      DIMENSION RANGE(4),USERS(4),VELS(4),VEL(4),SATPOS(4,3),USER(4),
     1HMAT(4,4),RBAR(4),DELR(4),DELU(4),BB(4,1),IPIVOT(4),INDEX(8),
     2HRMAT(3,3)
      COMMON/TMPAR/TSTRT,DELT,ITER,PSIM
      COMMON/NVFLG/JMPE,JIONE,JSSE,JCLKE,JBIAS
      COMMON/NVPAR/ALPHA,BETA,BETA1,ADAPTA,ADAPTB
      COMMON/SATS/JORB(4),JSAT(4),DROLD(4),JSHD(4)
      IF(ITER.GT.1) GOTO 10
      DO 11 J=1,4  $  DELR(J) = 0.
   11 BB(J,1)=0.
   10 CONTINUE
C     USER ESTIMATE OF USER POSITION
C     HMAT  H  MATRIX
      RBAR(KTH)=(SATPOS(KTH,1)-USER(1))**2+(SATPOS(KTH,2)-USER(2))**2
      RBAR(KTH)=SQRT(RBAR(KTH)+(SATPOS(KTH,3)-USER(3))**2)
      DELR(KTH)=RANGE(KTH)-RBAR(KTH)-USER(4)
C     SIMULATE SHIELDING IF  JSSE  NOT  0
      IF(JSSE.EQ.0) GO TO 213
C     IF SHIELDING SIMULATED, FIND FIRST SPACECRAFT SHIELDED IF ANY
      DO 211 J=1,4
      IF(JSHD(J).NE.0) GOTO 210
  211 CONTINUE
      GOTO 213
C     NO SPACECRAFT SHIELDED
  210 NS=J
C     NUMBER OF FIRST SPACECRAFT SHIELDED
C     FORM  3X3  HRMAT FROM VISIBLE SPACECRAFT
      K=0
      DO 215 J=1,3
      K=K+1
      IF(K.EQ.NS) K=K+1
C     SKIP SHIELDED SPACECRAFT
      DO 215 JCOL=1,3
  215 HRMAT(J,JCOL)=(USER(JCOL)-SATPOS(K,JCOL))/(RANGE(K)-USER(4))
      CALL MATINV(3,3,HRMAT,1,BB,0,DET,ISCALE,IPIVOT,INDEX)
C     COAST CLOCK BIAS DURING SHIELDING
      DO 216 I=1,4
  216 DELU(I)=0.
      DO 217 I=1,3
      K=0
      DO 217 J=1,3
      K=K+1
      IF(K.EQ.NS) K=K+1
C     SKIP SHIELDED SPACECRAFT
  217 DELU(I)=DELU(I)+HRMAT(I,J)*DELR(K)
      GOTO 33
C     CALCULATE  H  MATRIX FOR ALL FOUR SPACECRAFT
  213 DO 24 J=1,4
   24 HMAT(J,4)=1.
      DO 25 J=1,4
      DO 25 JCOL=1,3
      HMAT(J,JCOL)=(USER(JCOL)-SATPOS(J,JCOL))/(RANGE(J)-USER(4))
   25 CONTINUE
      CALL MATINV(4,4,HMAT ,1,BB,0,DET,ISCALE,IPIVOT,INDEX)
C     CALCULATE DELTA-U BY MATRIX MULTIPLY
      DO 34 I=1,4
   34 DELU(I)=0.
      DO 26 I=1,4
      DO 26 K=1,4
   26 DELU(I)=HMAT(I,K)*DELR(K)+DELU(I)
C     UPDATE USER ESTIMATE BY ALPHA-BETA TRACKER
C     SMOOTHING AND PREDICTION BY ALPHA-BETA
   33 DO 61 J=1,4
      USERS(J)=USER(J)+ALPHA*DELU(J)
      IF(J.NE.1) GOTO 64
      VELS(J)=VEL(J)+.2*BETA*DELU(J)/DELT
      GOTO 65
   64 VELS(J)=VEL(J)+BETA*DELU(J)/DELT
   65 VEL(J)=VELS(J)
   61 USER(J)=USERS(J)+DELT*VELS(J)
      RETURN
      END
```

```
      SUBROUTINE IJKUE(TIME,ALT,RLONG,RLAT,VECIJK,VECUEN)
      DIMENSION VECIJK(3),VECUEN(3),VEC(3),STACOR(3),TRIJK(3,3)
C     ALT = STATION ALTITUDE OF U-E-N SYSTEM, IN NAUTICAL MI.
C     RLONG = STATION LONGITUDE OF U-E-N SYSTEM , IN RADIANS
C     RLAT = STATION LATITUDE OF U-E-N SYSTEM , IN RADIANS
C
C     IJKUE IS A SUBROUTINE FOR COORDINATE TRANSFORMATION BETWEEN
C     I-J-K (GEOCENTRIC EQUATORIAL) AND U-E-N (TOPOCENTRIC LOCAL)
C     COORDINATE FRAMES
C
C
C     0.2618 IS EARTH TURN RATE (15 DEG/HR) IN RAD/HR UNITS
      THR=.2618*TIME+RLONG $ SINTH=SIN(THR) $ COSTH=COS(THR)
      SINPHI=SIN(RLAT) $ COSPHI=COS(RLAT)
C     COMPUTE THE STATION COORDINATES OF THE U-E-N SYSTEM ORIGIN IN
C     I-J-K COORDINATE FRAME
      ECCSQ=1.-.996645**2 $ DENO=SQRT(1.-ECCSQ*SINPHI**2)
      X=(3443.936/DENO+ALT)*COSPHI
      Z=(3443.936*(1.-ECCSQ)/DENO+ALT)*SINPHI
      RHO=SQRT(X**2+Z**2)
      SINPHI=Z/RHO $ COSPHI=X/RHO
      STACOR(1)=X*COSTH $ STACOR(2)=X*SINTH $ STACOR(3)=Z
C     COMPUTE THE TRANSFORMATION MATRIX FOR I-J-K TO U-E-N SYSTEMS
C
      TRIJK(1,1)=COSPHI*COSTH
      TRIJK(1,2)=COSPHI*SINTH
      TRIJK(1,3)=SINPHI
      TRIJK(2,1)=-SINTH $ TRIJK(2,2)=COSTH $ TRIJK(2,3)=0.
      TRIJK(3,1)=-SINPHI*COSTH
      TRIJK(3,2)=-SINPHI*SINTH
      TRIJK(3,3)=COSPHI
C
C     COMPUTE TRANSFORMATION FROM I-J-K TO U-E-N FRAMES
      DO 22 I=1,3
   22 VEC(I)=VECIJK(I)-STACOR(I)
C
C     VEC STORES THE POSITION COORDINATES OF THE SPACECRAFT W.R.T. TO
C     THE U-E-N ORIGIN, BUT IN I-J-K COORDINATES
      DO 23 I=1,3
      VECUEN(I)=0.
      DO 23 J=1,3
   23 VECUEN(I)=VECUEN(I)+TRIJK(I,J)*VEC(J)
      RETURN
      END
```

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15. Supplementary Notes

16. Abstract

The analytical structure of an existing navigator algorithm for a low cost GPS receiver is described in detail to facilitate its implementation on in-house digital computers and real-time simulators. The material presented includes a simulation of GPS pseudorange measurements, based on a two-body representation of the NAVSTAR spacecraft orbits and a four component model of the receiver bias errors. A simpler test for loss of pseudorange measurements due to spacecraft shielding is also noted.

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