NASA Contractor Report 159245

NASA-CR-159245

1980 00 13869

# EFFECT OF A FLEXIBLY MOUNTED STORE ON THE FLUTTER SPEED OF A WING

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NASA Grant NSG-1438
April 1980

NASA
National Aeronautics and
Space Administration

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N80-22356 #

# INTRODUCTION

Many aircraft carry heavy concentrated weights attached to the wings, in the form of engines, auxiliary fuel tanks or weapons. These heavy weights can produce deliterious effects on the flutter speed of the wing. If they are not located at a propitious position, an undesirable lowering of the flutter speed results. Recently attempts have been made to adapt the concepts of active control to increase the flutter speed, but this method requires a complex system of electronics, sensors, controls, and activators. A passive method of achieving the desired effect would be more desirable from several standpoints, namely, less complex, less weight and more dependable. A passive system proposed by Mr. Wilmer Reed of NASA, Langley Research Center involves the concept of mounting the store on a pitch pivot having a very low pitch stiffness relative to the wing stiffness. It is the purpose of this research grant to analytically study the proposed concept.

The first task was to investigate the concept utilizing a two-dimensional approach involving 4 degrees of freedom, namely, wing bending, wing torsion, store pitch and store vertical translation. This preliminary analysis was very encouraging and the results demonstrated that if the uncoupled store pitch frequency was below the wing bending frequency that the flutter speed was greatly increased. A second more complete analysis was developed utilizing a three dimensional structure, but retaining the two-dimensional, incompressible unsteady airforces of Theodorsen. The details of the analysis are given in the Appendix.

## SYMBOLS

| | |
|---|---|
| $a$ | location of wing elastic axes, measured from wing midchord, based on wing half chord, positive aft |
| $b$ | wing half chord |
| EI | wing bending stiffness |
| EMF | $-m_F/\ell_m$ |
| EMS | $m_S/\ell_m$ |
| F,G | real and imaginary parts of Theodorsen Function C(k) |
| g(1),g(2), g(3),g(4) | structural damping coefficients for first four uncoupled bending modes |
| g(5),g(6), g(7) | structural damping coefficients for first three uncoupled torsional modes |
| g(8),g(9), g(10) | structural damping coefficients for store vertical translation, pitch, and fore and aft, respectively |
| GJ | wing torsional stiffness |
| $I_F$ | mass moment of inertia of fixed weight, about CG |
| $I_S$ | mass moment of inertia of store about pivot point |
| $I_\alpha$ | mass moment of inertia of wing about E.A., per unit length |
| $k$ | reduced frequency bw/U |
| $\ell$ | wing semi-span |
| $\ell_w$ | spanwise location of store, based on $\ell$ |
| $\ell_F$ | chordwise location of fixed weight c.g., measured from EA, based on b, and positive aft |
| $\ell_S$ | chordwise location of store pitch pivot, measured from wing mid-chord, positive aft, based on b |
| $\ell_{F_1}$ | vertical distance of fixed weight, measured from wing mean chord line, based on b, positive down |
| $\ell_1$ | vertical distance of store pitch pivot point, measured from mean chord line, based on b, positive down |

| | |
|---|---|
| $m$ | mass of wing per unit length |
| $m_F$ | mass of fixed weight |
| $m_S$ | mass of store |
| $r_\alpha$ | wing radius of gyration based on wing half chord and referred to wing EA |
| $r_S$ | store radius of gyration, based on wing half chord, referred to store pitch axis location |
| $r_F$ | radius of gyration of fixed weight, based on half chord, referred to wing E.A. $(-m_F b^2/I_{\alpha\ell})$ |
| $U$ | forward velocity |
| $x_\alpha$ | wing center of gravity location, based on wing half chord, positive aft |
| $x_s$ | store center of gravity location, based on wing half chord, positive aft |
| $x$ | spanwise coordinate, based on $\ell$ |
| $\kappa$ | virtual mass of wing $\pi\rho b^2/m$ |
| $\rho$ | air density |
| $\omega_h$ | fundamental uncoupled banding frequency of wing with rigid mass |
| $\omega_h(N)$ | bare wing uncoupling bending frequency in $N^{th}$ mode |
| $\omega_\alpha(M)$ | bare wing uncoupled torsion frequency in $M^{th}$ mode |
| $\omega_{hs}$ | uncoupled store vertical frequency |
| $\omega_{ts}$ | uncoupled store fore and aft frequency |
| $\omega_\theta$ | uncoupled store pitch frequency |
| $\omega$ | flutter frequency |

## Concept Description

The concept of installing a flexibly mounted store on an aircraft to increase the flutter speed is based mainly on the knowledge that flutter is a function of the torsional frequency. In general, the higher the torsional frequency, the higher the flutter speed. When a large store is rigidly fastened to a wing, the torsional frequency is reduced which results in a reduction in flutter speed. If a system of store suspension could be devised such that the store inertia effect was essentially removed from the system, it was presumed that the flutter speed should show a marked increase since the torsional frequency would not suffer a large reduction. A system to accomplish this is mechanically very easy, for it is only necessary to hinge the store on a pivot with low flexibility. Fig. 1 illustrates the mechanical arrangement of such a system.

From practical considerations, it would not be advisable to maintain a store in a completely free condition, so some spring stiffness is required to maintain the store in its proper relationship to the wing. Also some system for maintaining the alignment of the store with the wing may be necessary. Such a system should have a very low frequency in order to maintain the low pitch frequency of the store. The present report analytically investigates this concept and presents some of the results .

## Analysis

The analysis was specialized to the case of a structurally uniform wing, having a rectangular aerodynamic planform of relatively high aspect ratio. Three degrees of freedom in wing torsion, and four degrees of freedom in wing bending were utilized in a modal type analysis. A single, spring mounted weight was added to the system which could be located at any spanwise or chordwise position. The spring-mass was restricted to three degrees-of-freedom, namely, pitch, vertical translation, and fore-and aft motion. Two-dimensional incompressible, unsteady aerodynamic theory was used, ref. 1. Details of the analysis are given in the Appendix. The next section contains a discussion of the results.

## Results and Discussion

The analysis technique presented was applied to a rectangular wing whose properties were held constant throughout the study. The wing characteristics are given in Table 1. A small store support bracket was assumed to be rigidly attached to the wing and its properties are given in Table 2. The store properties are given in Table 3.

Of primary importance is the store pitch stiffness and this property was first considered, the results of which are given in Figure 2. The variation of center of gravity position is shown in Figure 3, and finally the effect of moving the store spanwise is given in Figure 4.

## Effect of Store Pitch Frequency on Flutter

On Figure 2, the ratio of the flutter dynamic pressure for the wing with a flexible store to the flutter dynamic pressure of the wing with no store attached is plotted against the ratio of the store uncoupled pitch frequency to the wing bending frequency with the rigid store attached. Starting at the left of the figure, $\omega_\theta/\omega_h=0$, it is seen that for a completely free store in pitch, the flutter speed is well above the bare wing flutter speed. As the store pitch spring stiffness is increased, the flutter speed rapidly increases to until $\omega_\theta/\omega_h \simeq 1$; for larger values of $\omega_\theta/\omega_h$, a rapid decrease is found which has a minimum at $\omega_\theta/\omega_h \simeq 1.5$, where the flutter speed begins to slowly increase and approaches an asymptotic value for $\omega_\theta/\omega_h \simeq \infty$, which represents a rigid pylon. Therefore, for this configuration, if the ratio $\omega_\theta/\omega_h$ is maintained below unity, substantial benefits arise from the use of a low store pitch frequency. It should be pointed out that it was found that the pylon stiffness in the vertical direction must be stiff, since some calculations made with a reduced vertical translation stiffness reduced the flutter speed well below the base wing flutter speed.

## Effect of Store Center of Gravity Location

In Figure 3 is shown the results of calculations for a rigid and a reduced stiffness case shown in Figure 2, except that the center of gravity of the store was moved fore and aft. Examining the rigid pylon case first, it is seen that as the C. G. is moved forward from an aft position, the flutter speed is essentially constant in the range of $q/q_{base\ wing}$ of from .3 to .5, which is well below the flutter speed of the bare wing. As the center of gravity approaches a more forward position, the flutter speed increases rapidly when $x_s \simeq -.1$.

For the case of reduced stiffness in the store pitch mode ($\omega_\theta/\omega_h = 0.54$ when $x_s = 0$) the flutter speed trend is virtually independent of C. G. location, ranging from $q/q_{bare\ wing} = 2.3$ for $x_s = .4$ to $q/q_{bare\ wing} = 1.7$ for $x_s = -.4$.

Effect of Spanwise Store Location

Calculations were made to determine the effect of the store spanwise location for both the rigid and flexible pylon. These results are given in Figure 4. For the rigid pylon, the flutter q drops as the store is moved from the base to the wing midspan, then increases as the wing tip is approached and for the store located at the wing tip, the bare wing flutter speed is approached. For the reduced stiffness pitch case, ($\omega_\theta/\omega_h = 0.54$) the flutter speed gradually increases as the store is moved towards the wing tip where the flutter q is about 2.4 times the bare wing flutter q.

## Concluding Remarks

A calculation procedure has been presented for a uniform cantilever wing on which is attached a flexibly mounted store.

Results of calculations based on the procedure indicated the following results:

1) The flutter speed is well above the bare wing flutter speed for frequency ratio $\omega_\theta/\omega_h$ less than unity.

2) Over the range of pylon stiffnesses, representative of production aircraft, the flutter speed was considerably less than that of the bare wing.

3) Large variation in some of the more important store parameters showed the flutter speed to be virtually independent of the variations and the speed remained well above the bare wing flutter speed, if the ratio $\omega_\theta/\omega_h$ is maintained below unity.

# APPENDIX A

## Equations of Motion

The equations of motion for flutter were derived on the basis of Lagrange's equations. In matrix form, the equations appear as $A = \lambda B$ where the coefficients A & B are given in detail subsequently, and $\lambda=(\frac{\omega}{\omega_\alpha})^2(1+ig)$. The equations were specialized for the case of a uniform cantilever wing carrying a fixed and sprung mass at an arbitrary spanwise and chordwise location (figure 1). The equations were linearized and the unsteady aerodynamics were limited to the incompressible, two dimensional case. For the structural part, ten degrees of freedom were utilized, namely, four uncoupled uniform beam bending modes, three uncoupled uniform beam torsion modes, and three degrees of freedom for the sprung mass comprising pitch, heave, and fore-and aft degrees of freedom. No aerodynamic forces were applied to the store.

Since the wing mass, inertial and unsteady aerodynamic forces are constant along the span, and theoretical uncoupled beam modes were assumed, it was convenient to calculate the generalized mass, inertial and aerodynamic terms in closed form. The expressions used for the uniform cantilever modes were slightly different from those found in textbooks on vibration. It has been found that the forms given below provide more accurate results for the higher modes.

For odd modes, the bending modal function is

$$f_w(N) = \frac{1}{2}\left[\frac{\cosh(\beta_n/2)(2x-1)}{\cosh(\beta_n/2)} + \frac{\sin(\beta_n/2)(2x-1)}{\sin(\beta_n/2)}\right]$$

For even modes, the modal function is

$$f_w(N) = \frac{1}{2}\left[\frac{\sinh(\beta_n/2)(2x-1)}{\sinh \beta_n/2} + \frac{\cos(\beta_n/2)(2x-1)}{\cos \beta_n/2}\right]$$

where $\beta_n$ must satisfy the following relation

$$\cosh \beta_n \cos \beta_n + 1 = 0 \qquad n=1,2,3,4...$$

and where x is the spanwise coordinate, normalized by the span length, $\ell$.

For the torsion mode, the modal function was selected to be

$$f_\theta(M) = (-1)^{m+1} \sin \pi(m-\tfrac{1}{2})\, x \qquad m=1,2,3...$$

Several integrals of these functions are required as follows:

For N odd

$$FW(N) = \int_0^1 f_w(N)^2\, dx$$

$$= \frac{(\sinh \beta_n/2) \cosh \beta_n/2+\beta_n/2}{4\, \beta_n \cosh^2 \beta_n/2}$$

$$+ \frac{\beta_n/2-\frac{1}{2} \sin \beta_n}{4\beta_n \sin^2 \beta_n/2}$$

For N even

$$FW(N) = \int_0^1 f_w(N)^2 \, dx$$

$$= \frac{\sinh(\beta_n/2)\cosh(\beta_n/2) - \beta_n/2}{4\beta_n \sinh^2 \beta_n/2}$$

$$+ \frac{\frac{1}{2}\sin \beta_n}{4\beta_n \cos^2 \beta_n/2}$$

For N odd

$$F(N,M) = \int_0^1 f_w(N) f_\alpha(M) \, dx$$

$$= \frac{\beta_n \mathrm{cotan}(\beta_n/2) - (-1)^{m+1}\pi(m-1/2)}{-2\beta_n^2 + \pi(m-1/2)^2}$$

$$+ \frac{\beta_n \mathrm{tank}(\beta_n/2) + \pi(m-2/2)(-1)^{m+1}}{2\beta_n^2 + 2\pi(m-1/2)^2}$$

For N even

$$F(N,M) = \int_0^1 f_w(N) \; f_\alpha \; (M) \, dx$$

$$= \frac{\beta_n \mathrm{cotanh} \; (\beta_n/2) - (-1)^{(m+1)} (m-1/2)\pi}{2(\beta_n^2 + \pi(m-1/2)^2)}$$

$$+ \frac{1 - \beta_n \tan\beta_n/2 + (-1)^{m+1}\pi(m-1/2)}{2(-\beta_n^2 + \pi(m-1/2)^2)}$$

The order of the modes in the matrix is as follows - the first four rows are the bending modes, the 5-7 rows are the torsion modes, and the 8-10 rows are store vertical translation, store pitch, and store fore and aft respectively.

The definition of the elements of the matrices A and B follows where the symbols are defined after the definition:

MATRIX DIAGONAL ELEMENTS

N=M N= 1,2,3,4

$$A(N,M) = -FW(N)(H1) - (m_s/\ell_m)(FWL(N))^2 + (-m_F/\ell_m)(FWL(N))^2 + i(2KF(FW(N))/k)$$

\- - - - - - - -

N=M N= 5,6,7

$$A(N,M) = -FA(N-4)(H12) - (m_s/\ell_m)\ R_B(FAL(N-N))^2/r_\alpha^2 + (FI)(FAL(N-4))^2 + i(FA(N-4)(H13))$$

\- - - - - - - -

$$A(8,8) = -1$$

\- - - - - - - -

$$A(9,9) = \ell_1\ FAL(1)$$

\- - - - - - - -

$$A(10,10) = -1$$

OFF DIAGONAL TERMS

N= 1,2,3,4 M=1,2,3,4 M≠N

$$A(N,M) = -((m_F+m_s)/\ell_m)(FWL(N))(FWL(M))$$

\- - - - - - - -

N=5,6,7 M=5,6,7 M≠N

$$A(N,M) = (-(m_s/\ell_m)r_B + FI)(FAL(N-4)(FAL(m-4))$$

\- - - - - - - - -

N=5,6,7 M=1,2,3,4

$A(N,M) = (F(M,N-4))(H10)$

$-(H7)(FAL(N-4))(FWL(M))/r_\alpha^2$

$+r_F^2 \ \ell_F \ FWL(M) - FAL(M-4)$

$+i(-(F(M,N-y))(H11)$

- - - - - - - -

N=1,2,3,4 M=5,6,7

$A(N,M)=F(N,M-4)(H5)-(H7)(FAL(1))(FWL(M-4)$

$-(M_F/\ell_F(FAL(N))(FWL(M-4))$

$+ I(F(N,M-4))(H6)$

- - - - - - - -

N=1,2,3,4 M=8

$A(N,M) = -(M_S/\ell_M)(FWL(N))$

- - - - - - - -

N=1,2,3,4 M=9

$A(N,M) = -(M_S/\ell_M)x_S(FWL(N))$

- - - - - - - -

A(1,10) = A(2,10) = A(3,10) = A(4,10) = 0

- - - - - - - -

N=5,6,7 M=8

$A(N,8) = -(H7)(FAL(N-4)/r_\alpha^2$

- - - - - - - -

N=5,6,7 M=9

$A(N,9) = (H16)(FAL(N-4))$

- - - - - - - -

N=5,6,7 M=10

$A(N,10) = (m_s/\ell_M)\ \ell_1\ (FAL(N-4))/r_\alpha^2$

- - - - - - - -

N=8 M=1,2,3,4

A(8,M) = FWL(M)

- - - - - - - -

N=8 M=5,6,7

$A(8,M) = -(\ell_s + x_s)(FAL(M-4))$

$A/8,9 = -X_s$

- - - - - - - -

N=9 M=1,2,3,4

A(9,M) = (H14) (FWL(N))

- - - - - - - -

N=9 M=5,6,7

A(9,M) = (H15) (FAL(M-4))

- - - - - - - -

A(9,8) = H14

- - - - - - - -

A(9,9) = -1

- - - - - - - -

A(10,1) = A(10,2) = A(10,3) = A(10,4) = 0

- - - - - - -

N=10 M=5,6,7

$A(10,M) = \ell_1(FAL(M-1))$

- - - - - - - -

A(10,10) = -1

- - - - - - -

N=1,2,3,4

$$B(N,M) = -\left(\frac{\omega_h(N)}{\omega_{\alpha_1}}\right)^2 (FW(N))(1 + ig(N))$$

- - - - - - - -

N=5

$$B(5,5) = -FA(1)(1 + ig(5))$$

- - - - - - - -

N=6,7

$$B(N,N) = -\left(\frac{\omega_{\alpha_N}}{\omega_{\alpha_1}}\right)^2 (FA(N-4))(1 + ig(N))$$

- - - - - - - -

$$B(8,8) = -\left(\frac{\omega_{h_s}}{\omega_{\alpha_1}}\right)^2 (1 + ig(8))$$

- - - - - - - -

$$B(9,9) = -\left(\frac{\omega_\theta}{\omega_{\alpha_1}}\right)^2 (1 + ig(9))$$

- - - - - - - -

$$B(10,10) = -\left(\frac{\omega_{t_s}}{\omega_{\alpha_1}}\right)^2 (1 + ig(10))$$

$$H1 = 1 + \kappa + \frac{2\kappa}{k} G$$

$$H2 = \frac{2\kappa}{k} F$$

$$H3 = m_s/\ell_m$$

$$H5 = -x_\alpha + \kappa a + \frac{2\kappa F}{k^2} - 2\kappa G(\tfrac{1}{2} - a)/k$$

$$H6 = \kappa/k + 2\kappa G/k^2 + 2\kappa F(\tfrac{1}{2} - a)/k$$

$$H7 = (\ell_s + x_s)\ m_s/\ell_m$$

$$H8 = x_s/r_s^2$$

$$H9 = x_s^2/r_s^2$$

$$H10 = -x_\alpha/r_\alpha^2 + \kappa a/r_\alpha^2 + 2\kappa(a+\tfrac{1}{2})\ G/(kr_\alpha^2)$$

$$H11 = 2\kappa(a+\tfrac{1}{2})\ F/(kr_\alpha^2)$$

$$H12 = 1 + \kappa(\tfrac{1}{8}+a^2)/r_\alpha^2 + 2\ (9+\tfrac{1}{2})F/(k^2 r_\alpha^2)$$

$$- 2\kappa(a+\tfrac{1}{2})(a-\tfrac{1}{2})G/(kr_\alpha^2)$$

$$H13 = (\kappa(\tfrac{1}{2} - a)/k - 2\kappa(a + \tfrac{1}{2})G/k^2 - 2\kappa F(a + \tfrac{1}{2})(a - \tfrac{1}{2})/k)/r_\alpha^2$$

$$H14 = -x_s/r_s^2$$

$$H15 = -(\ell_s x_s/r_s^2 + 1)$$

$$H16 = -(I_\theta/I_\alpha \ell + (\ell_s x_x m_s/\ell_m)/r_\alpha^2)$$

$$r_B^2 = \ell_1^2 + x_s^2 + \ell_s^2 + \ell_s x_s + r_s^2$$

$$FI = -((\ell_F^2 + \ell_{F_1}^2)m_F b^2/(I_\alpha \ell) + I_F/(I_\alpha \ell))$$

TABLE 1. - WING CHARACTERISTICS

$x_\alpha$ = .15754

$\kappa$ = .06204

EI = $2.756 \times 10^{10}$ N/m$^2$

GJ = $2.407 \times 10^{9}$ N/m$^2$

m = 59.13 Kg/m

I = .00282 Kg-m$^2$

$r^2$ = .1549

a = -.3

TABLE 2. - FIXED WEIGHT CHARACTERISTICS

$m_F$ = .5922 Kg

$\ell_F$ = .1473

$\ell_{F_1}$ = 0

$I_F$ = .00727 Kg-m$^2$

TABLE 3. - STORE CHARACTERISTICS

$x_s$ = -.01

$r_s^2$ = .4162

$m_s$ = .0382 Kg

I = .2877 Kg-m$^2$

## References

1. Theodorsen, Theodore: General Theory of Aerodynamic Instability and the Mechanism of Flutter. NACA Rep. 496, 1935.

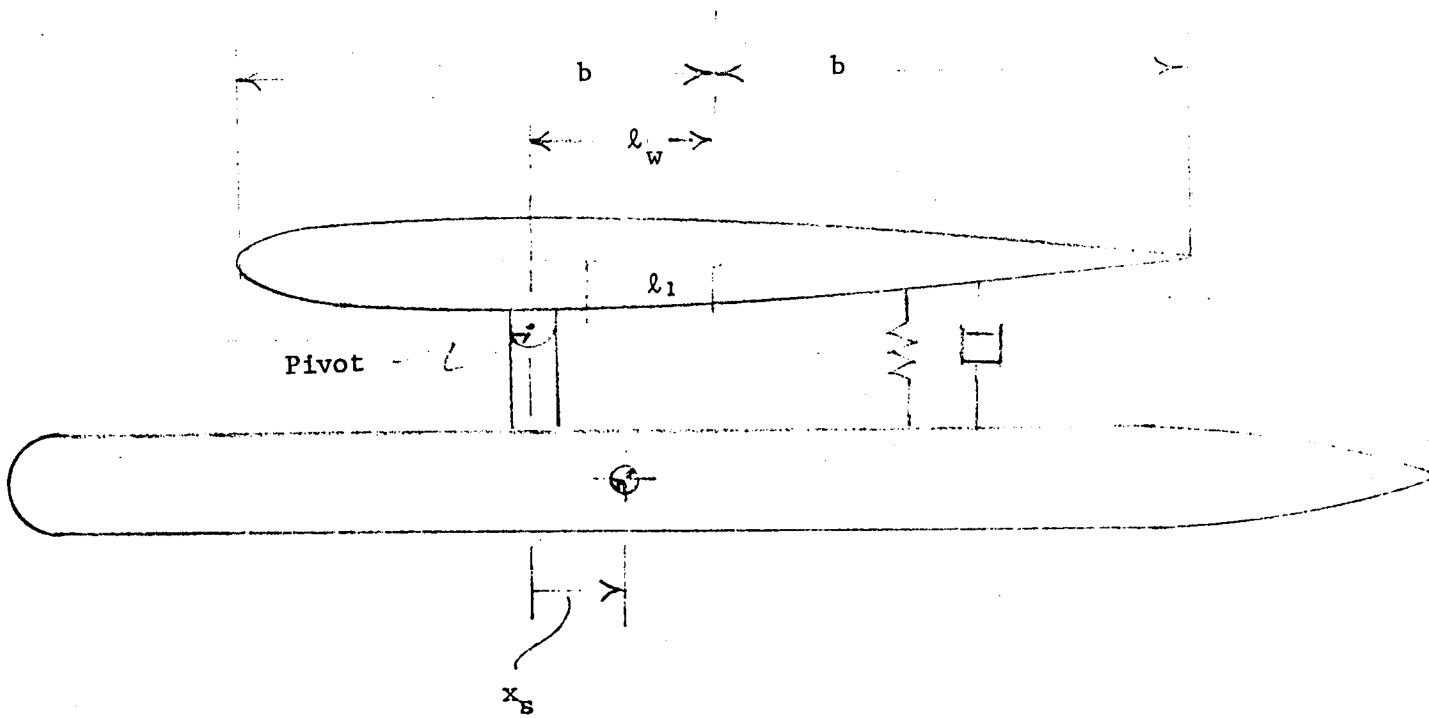


Figure 1.- Schematic of flexible store-wing system.

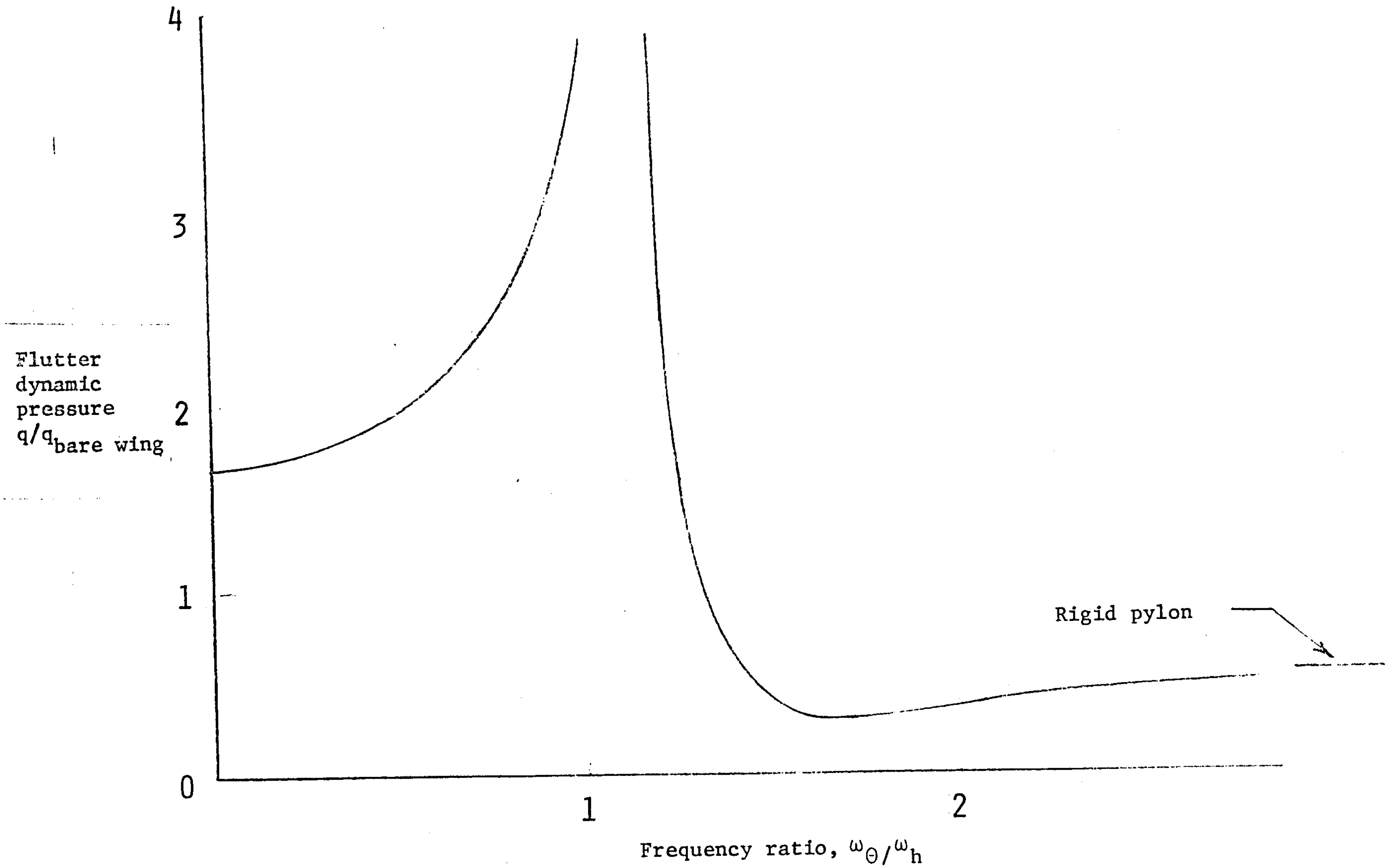


Figure 2.- Effect of store pitch frequency on wing-store flutter.

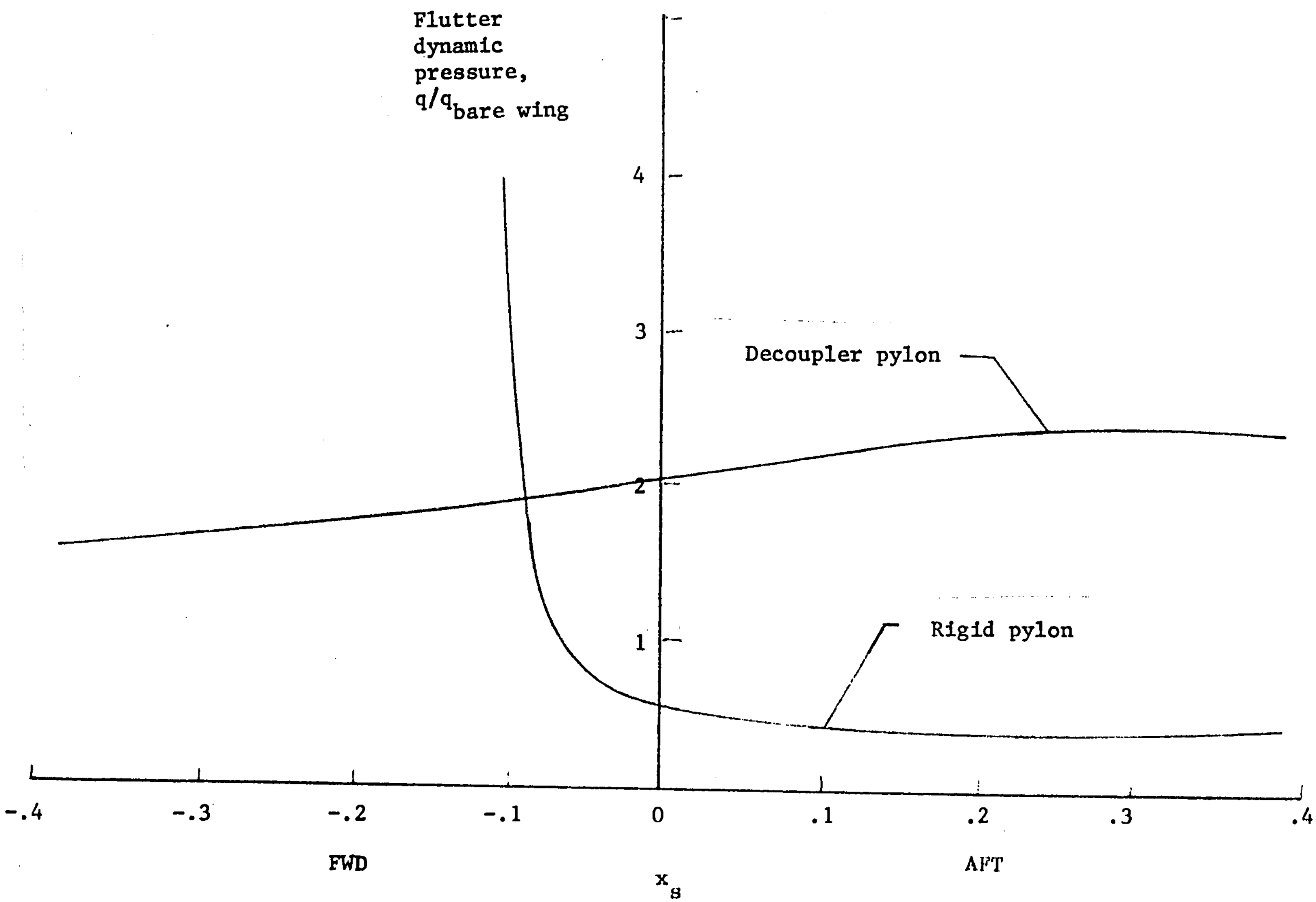


Figure 3.- Effect of store c.g. location on wing/store flutter.

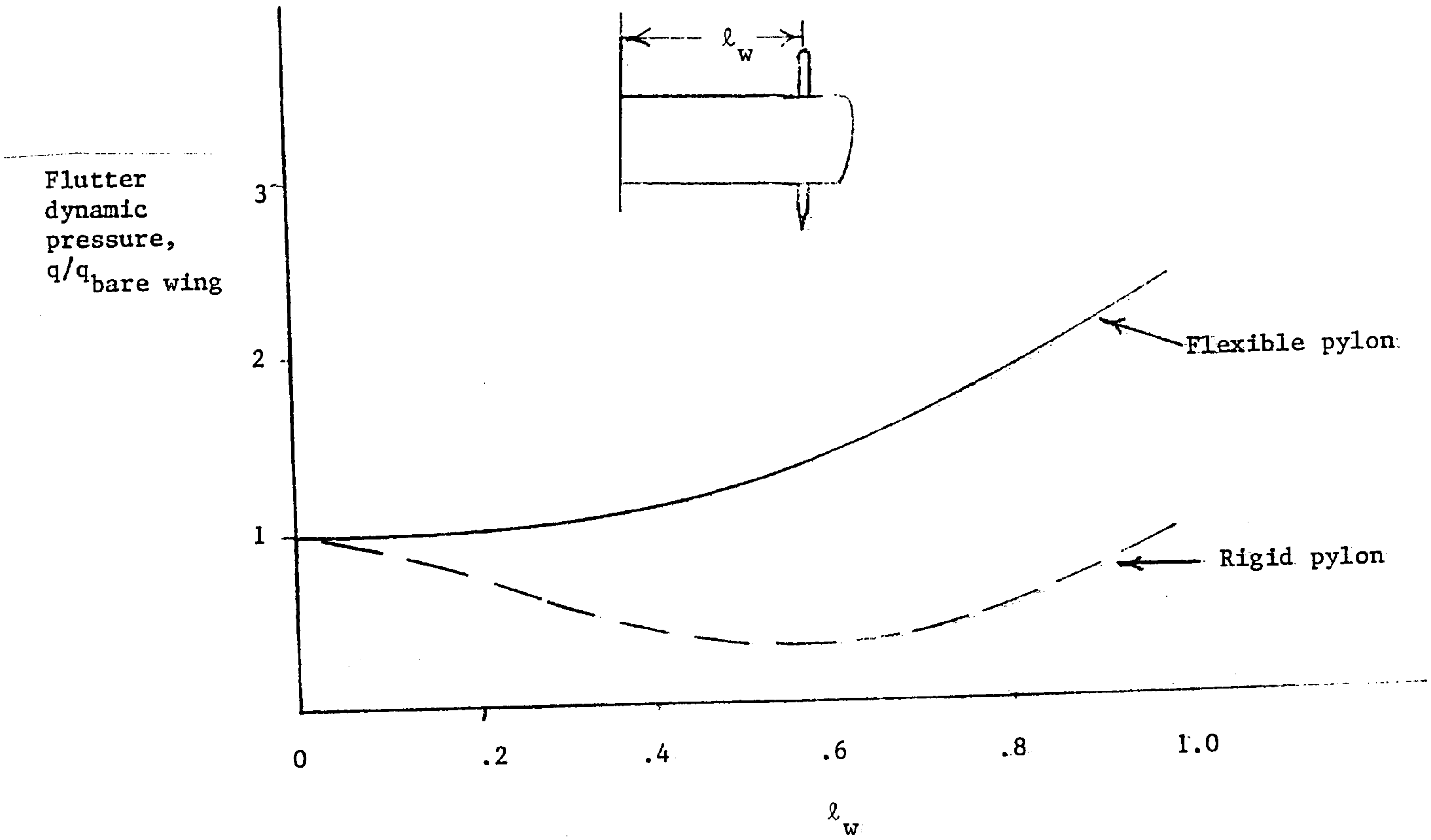


Figure 4.- Effect of spanwise store location.