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Theory for Computing the Size and Shape of a Region of Influence Associated With a Maneuvering Vehicle

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SUMMARY

A general procedure for computing the region of influence of a maneuvering vehicle is described. Basic differential geometric relations, including the use of a general trajectory parameter and the introduction of auxiliary variables in the envelope theory are presented. To illustrate the application of the method, the destruct region for a maneuvering fighter firing missiles is computed.

INTRODUCTION

In the course of the flight of an airplane, a missile, or even a space vehicle, certain regions in the neighborhood of the flight path are of interest, usually because of the potential influence that the vehicle has on the space in its vicinity. This influence may be of a passive nature, such as noise, engine exhaust, or trailing vortices. It may, on the other hand, be deliberate in character. The vehicle may, for example, emit missiles or other projectiles, radar seeking or jamming signals, and laser guidance signals.

In such a situation it is useful to know the extent of the region of influence as a function of the vehicle's path and its speed. The shape of the region is generally known for constant conditions, that is, for the vehicle at rest or in straight steady flight. The total region of influence for a general flight trajectory is the interior of the envelope of all the instantaneous regions. It would be possible to calculate each region separately and then determine the envelope numerically by interpolation. However, it is much more efficient to calculate the envelope directly by determining the characteristic lines (ref. 1) as described in the present paper. Another advantage of this method is that the analytical nature of the envelope equations provides a qualitative insight into the effects of various flight parameters on the size and shape of the region of influence.

In the present paper the general procedure is described. The basic equations are given, together with some specialized analytical results that are useful in the application of the method. Two illustrative examples, both dealing with fighter maneuvers, are given in some detail. These examples treat the intercept region as a function of the trajectory of the attacking airplane only.

The basic technique has already been applied specifically to the sonic boom problem (ref. 2), but the generality of the method and the wide range of its potential applications in aerospace problems have apparently not been recognized. Some of these applications include the calculation of a missile intercept region, a jet noise constant-decibel region, and a region limited by the lens resolution of a camera carried by a spaceship.

SYMBOLS

a	speed of sound, m/sec
$\bar{b}$	unit vector perpendicular to both $\bar{T}$ and $\bar{n}$, $\bar{T} \times \bar{n}$
c	constant of proportionality (used in eq. (8)), 1/m
c_1	constant relating missile range increment to vehicle velocity (used in eq. (25)), 1/sec
d	total maximum range, m
d_o	basic range for projectile fired from rest, m
$\bar{i},\bar{j},\bar{k}$	unit normal base vectors fixed in space
l	local torsion of flight path, 1/m
M	Mach number
n	function shape parameter (used in eq. (24))
$\bar{n}$	unit normal vector (used in eq. (3))
p	general trajectory parameter
R	radial variable in spherical coordinate system, m
$\bar{r}$	position vector of a point on the periphery of the region of influence of a maneuvering vehicle, m
$\bar{r}_t$	position vector of vehicle, m
s	arc distance along trajectory, m
t	time, sec
$\bar{T}$	unit vector locally tangent to flight path at vehicle location
V	velocity, m/sec
V_T	total velocity of projectile, m/sec
u,v	general variables
x,y,z	components of $\bar{r}$
θ	angle coordinate relative to $\bar{T}$ vector, deg
θ^*	radar semicone angle, deg

κ local trajectory curvature, 1/m

ρ local trajectory radius of curvature, $1/\kappa$, m

τ time at which vehicle is at point $\bar{r}_t$, sec

ϕ angle coordinate, deg

Superscripts:

$'$ derivative with respect to s

$\cdot$ derivative with respect to time

Subscripts:

o conditions when a projectile is fired from rest

p,u,v,θ,τ derivative with respect to the variable

ANALYSIS AND EXAMPLES

Review of Basic Equations

In this section, mathematical expressions are given for the moving trihedral coordinate system and the trajectory curvature and torsion. The position vector of the vehicle at time τ is

$$\bar{r}_t(\tau) = x\bar{i} + y\bar{j} + z\bar{k} \tag{1}$$

The moving trihedral base vectors are determined as follows:

If s denotes the length along the trajectory path, the vector $\bar{T}$ is given by

$$\bar{T} = \frac{dx}{ds}\,\bar{i} + \frac{dy}{ds}\,\bar{j} + \frac{dz}{ds}\,\bar{k} \tag{2}$$

which represents a unit vector tangent to the flight path. Therefore, it is perpendicular to its derivative, which can be written in the form

$$\bar{n} = \frac{1}{\kappa}\frac{d\bar{T}}{ds} \tag{3}$$

where $\bar{n}$ is a unit vector, and κ is the magnitude of $d\bar{T}/ds$ and is termed the trajectory curvature (ref. 1, p. 13). A third unit vector perpendicular to both $\bar{T}$ and $\bar{n}$ is defined by

$$\bar{b} = \bar{T} \times \bar{n} \tag{4}$$

The three mutually perpendicular unit vectors $\bar{T}$, $\bar{n}$, and $\bar{b}$ define the moving trihedral coordinate system (fig. 1). The vector $\bar{T}$ points in the direction of flight, and $\bar{n}$ points in the direction of the local center of curvature of the trajectory. Other quantities which are used later are the derivatives of $\bar{n}$ and $\bar{b}$ which are given by

$$\frac{d\bar{n}}{ds} = -\kappa\bar{T} + \imath\bar{b} \tag{5}$$

$$\frac{d\bar{b}}{ds} = -\imath\bar{n} \tag{6}$$

In equations (5) and (6), which together with equation (3) are known as the Frenet-Serret formulas, $\imath$ is the local trajectory torsion (ref. 1, pp. 15-18).

The trajectory parameters s and τ are related through the velocity which is assumed to be known either as a function of s or τ:

$$\frac{ds}{d\tau} = V \tag{7}$$

Example: Fighter Firing Cannon Shells

To illustrate the application of these equations within a physical context, consider a fighter that is firing projectiles from a cannon while maneuvering. The projectiles are fired in the (local) direction of flight, that is, in the direction of $\bar{T}$. Their deceleration due to aerodynamic drag is proportional to the square of the speed:

$$\dot{V}_T = -cV_T{}^2 \tag{8}$$

Thus,

$$\int_{V+V_o}^{V_T} \frac{dV_T}{V_T^2} = \int_{\tau}^{t} - c\ dt$$

which gives for the projectile velocity at time t,

$$V_T = \frac{V + V_o}{1 + c(V + V_o)(t - \tau)} \tag{8a}$$

and for the distance traveled from the point of firing,

$$d = \frac{1}{c} \ln \left[1 + c(V + V_o)(t - \tau)\right] \tag{9}$$

If the projectiles are fired over the period $t_1 < \tau < t$, then the distribution of the projectiles in space at time t is given by the vector expression

$$\bar{r} = \bar{r}_t(\tau) + d(t,\tau)\ \bar{T}(\tau) \qquad (t_1 < \tau < t) \tag{10}$$

where d is given by equation (9). Figure 2 shows an example of such a distribution for an airplane performing a turning maneuver. (In the example, $c = 0.0676/m$, $V = 183$ m/sec, $V_o = 183$ m/sec, and $\rho = 1219$ m.)

If it is assumed that the projectile loses its effectiveness after its velocity decreases to a certain value, then by inserting this value in the left side of equation (8a) and eliminating $(t - \tau)$ from equations (8a) and (9), an effective range can be calculated. With this range, an alternate problem would be to compute the entire "destruct" surface swept out by the shells fired over a period of time, allowing for each shell its full range as limited by effective velocity. For this problem, the surface is bounded on one edge by the flight trajectory $\bar{r}_t(\tau)$ and on the other by the line

$$\bar{r} = \bar{r}_t(\tau) + d(V)\bar{T} \tag{11}$$

An example of this type of problem is shown in figure 3 for the same type of maneuver used for the example of figure 2. In this example, the projectile effective range is taken to be $d = 1830$ m.

In these examples, simplified mathematical models have been assumed for purposes of illustration. They could be made more sophisticated with some increase in mathematical complexity. For example, the velocity equation (8a) could be written as a vector equation with a component in the $\bar{k}$-direction resulting from gravitational attraction, and components taken in the $\bar{T},\bar{n}$ plane because, in performing a turning maneuver, the direction of firing is not exactly in the $\bar{T}$-direction.

Form of Trajectory Equations

The preceding examples require the differentiation of the flight path vector $\bar{r}_t$ in order to compute the unit tangent vector $\bar{T}$ (eq. (2)). More complex problems require higher derivatives of $\bar{r}_t$. It is generally advantageous to express the flight trajectory in analytic form so that the derivatives can be calculated analytically. Analytic expressions would normally be used in any hypothetical situation, but even if the trajectory represented an actual flight, with locations determined at specific times, it would be preferable to fit the trajectory with an analytic curve.

However, it is not a simple matter to express most curves in terms of a specific parameter, such as the arc-length parameter s. Circular and elliptic arcs, for example, are more naturally expressed in terms of an angle variable. Thus, if the trajectory coordinates are expressed in terms of a general parameter p, then the trajectory vector is of the form

$$\bar{r}_t(p) = x(p)\bar{i} + y(p)\bar{j} + z(p)\bar{k}$$

and

$$\bar{T} = p'(x_p\bar{i} + y_p\bar{j} + z_p\bar{k}) \tag{12}$$

where

$$p' = \frac{1}{\sqrt{x_p{}^2 + y_p{}^2 + z_p{}^2}} \tag{13}$$

Also

$$\frac{d\bar{T}}{ds} = p''\bar{r}_{tp} + p'^2\bar{r}_{tpp}$$

which, when written in component form, is

$$\frac{d\bar{T}}{ds} = (p''x_p + p'^2x_{pp})\bar{i} + (p''y_p + p'^2y_{pp})\bar{j} + (p''z_p + p'^2z_{pp})\bar{k} \tag{14}$$

where

$$p'' = p'\frac{dp'}{dp}$$

or

$$p'' = -\frac{x_px_{pp} + y_py_{pp} + z_pz_{pp}}{(x_p^2 + y_p^2 + z_p^2)^2} \tag{15}$$

Thus, performing all calculations in terms of the parameter p, one can calculate $\bar{T}$ from equations (12) and (13), and $\bar{n}$ and κ from equations (3), (14), (13), and (15).

Envelope Theory

The region of interest that is associated with a maneuvering vehicle is usually not a simple set of vector lines, as in the example of the cannon shells fired from a fighter, but rather is a three-dimensional region. Suppose, for example, the fighters were firing missiles rather than cannon shells. Since the missiles can be fired over a certain cone angle range, there is, associated with each point of the trajectory, a destruct region within which a target can be destroyed. Then, to determine the boundary of the destruct region associated with the entire trajectory, one must determine the envelope of all the surfaces that bound these individual regions. Somewhat similar problems involving the envelope of a family of surfaces arise in calculations of subsonic and supersonic airplane noise.

The theory of calculating the envelope of a one-parameter family of surfaces is given in reference 1 (pp. 162-168). If the parameter is p and the family of surfaces is given in the form

$$F(x,y,z,p) = 0 \tag{16}$$

then, when the equation

$$F_p(x,y,z,p) = 0 \tag{17}$$

is solved simultaneously with equation (16) at a given value of p, the resulting solution determines a characteristic line. The set of these characteristic lines, determined as p varies, defines the envelope of the surfaces described by equation (16).

A simple example is the family of spheres representing the periphery of the pressure disturbance generated by a moving point source (see ref. 2):

$$F(x,y,z,\tau) = (\bar{r} - \bar{r}_t) \cdot (\bar{r} - \bar{r}_t) - a^2(t - \tau)^2 = 0 \tag{18}$$

Differentiating with respect to the parameter τ yields

$$-2V\bar{T} \cdot (\bar{r} - \bar{r}_t) + 2a^2(t - \tau) = 0$$

or,

$$(\bar{r} - \bar{r}_t) \cdot \bar{T} = \frac{a(t - \tau)}{M}$$

This is the equation of a plane which, together with the sphere equation (18), determines a circle (at supersonic speeds) for any given value of τ. At time t, the set of these circles formed as τ varies from 0 to t comprise the Mach surface of the moving source.

For this illustration, the basic region, being spherical, is readily described in the form of equation (16). But other problems, as for example those involving symmetry about just one axis, involve surfaces that are not easily expressed analytically in this form. A more versatile expression for the equation of the basic surface is

$$\bar{r} = \bar{r}(u,v,p) \tag{19}$$

where u and v represent auxiliary variables or new coordinates. With this formulation the characteristic line is obtained by solving equation (19) simultaneously with the equation

$$\bar{r}_u \cdot \bar{r}_v \times \bar{r}_p = 0 \tag{20}$$

(See ref. 1, p. 168.)

A common case of interest in trajectory problems is that of a region that is symmetric about the direction of motion, that is, about the $\bar{T}$ axis. The geometry for this situation is depicted in figure 4. According to the figure

$$\bar{r} = \bar{r}_t + R \cos \theta \, \bar{T} + R \sin \theta \cos \phi \, \bar{n} + R \sin \theta \sin \phi \, \bar{b} \tag{21}$$

where, for the axisymmetric region, R is independent of ϕ:

$$R = R(\theta,\tau) \tag{21a}$$

Thus, if the trajectory parameter is taken to be τ,

$$\bar{r} = \bar{r}(\theta,\phi,\tau)$$

which, if (θ,ϕ,τ) are substituted respectively for (u,v,p), is in the form of equation (19). Thus equation (20) yields

$$\begin{vmatrix} R_\theta \cos \theta - R \sin \theta & (R_\theta \sin \theta + R \cos \theta) \cos \phi & (R_\theta \sin \theta + R \cos \theta) \sin \phi \\ 0 & -R \sin \theta \sin \phi & R \sin \theta \cos \phi \\ V(1 - \kappa R \sin \theta \cos \phi) + R_\tau \cos \theta & VR(\kappa \cos \theta - \imath \sin \theta \sin \phi) + R_\tau \sin \theta \cos \phi & (VR\imath \cos \phi + R_\tau \sin \phi) \sin \theta \end{vmatrix} = 0$$

Expanding this determinant and collecting terms yields the equation

$$RV \sin \theta \left(-RR_\theta \kappa \cos \phi + R_\theta \sin \theta + R \cos \theta + \frac{RR_\tau}{V} \right) = 0 \tag{22}$$

It is interesting that, in evaluating the determinant, the torsion terms subtract out.

For a straight trajectory $\kappa = 0$, and the first term in equation (22) drops out. The resulting equation is independent of ϕ, and consequently the characteristic line is a circle. When the trajectory is curved, equation (22) can be written in the form

$$R \sin\theta \cos\phi = \rho \sin\theta \left(\sin\theta + \frac{R}{R_\theta}\cos\theta + \frac{\dot{R}}{R_\theta}\frac{R_\tau}{V}\right) \qquad (23)$$

unless the range of θ includes values for which $R_\theta = 0$. The left side of equation (23) is the component of the characteristic vector in the direction of $\bar{n}$ (eq. (21)). The corresponding component in the $\bar{T}$-direction is $R(\theta)\cos\theta$. These two components and the vector length R, obtained from equation (21a), are sufficient to determine the component in the $\bar{b}$-direction. Thus, in this case, the characteristic line can be plotted as a function of θ without computing the corresponding ϕ values explicitly.

Example: Missile Destruct Region

An illustration of the use of these equations is provided by the previously mentioned problem of determining the region within which a target can be destroyed by air-to-air missiles fired by a maneuvering airplane. For this example the basic region is considered to be determined by the radar visibility region and the missile range.

The radar visibility region is a function of the radar sweep azimuth and elevation angles and may be asymmetric and time-varying in form. However, for simplicity in the illustration, it is approximated by a fixed-angle cone. The missile range is determined as a basic range d_o, when fired from rest, together with a component Δd imparted to it by the forward motion of the airplane. The total range is therefore a maximum in the direction of flight and falls off somewhat if the missile is fired at an angle to this direction.

The total destruct region, then, is limited laterally by the radar scan and longitudinally by the missile range, as depicted diagrammatically in figure 5(a). This region will be approximated by the region represented by the analytic expression

$$R = d(\tau)\cos n\theta = [d_o + \Delta d(\tau)]\cos n\theta \qquad (24)$$

where d is the maximum range and the parameter n is chosen so that the ray to the point of maximum diameter matches the radar cone angle. (See fig. 5(b).) The point of maximum diameter is determined by differentiating the expression for the diameter

$$R \sin\theta = d \cos n\theta \sin\theta$$

and setting the derivative equal to zero:

$$d(\cos\theta \cos n\theta - n \sin n\theta \sin\theta) = 0$$

Thus, if θ^* is the radar semicone angle, the value of n satisfying

$$n = \cot \theta^* \cot n\theta^*$$

is to be inserted in the approximating equation (24). If the simple linear estimate

$$\Delta d(\tau) = c_1 V \tag{25}$$

is assumed, then equations (23) and (24) yield

$$R \sin \theta \cos \phi = \rho \sin \theta \left[\sin \theta - \frac{\cot n\theta}{n} \left(\cos \theta + \frac{c_1 V_\tau \cos n\theta}{V} \right) \right] \tag{26}$$

The remaining components of the characteristic vector can be obtained as explained following equation (23).

Figure 6 shows, in perspective view, an example for a radar angle of 16° and a maximum range of 3.11 km. The vehicle trajectory has an increasing curvature, giving rise to an increasing elongation of the characteristic curves and, consequently, of the envelope.

Thus the intercept region for a missile in straight flight is confined to a relatively narrow cylinder, but the lateral extent of the intercept region for an airplane on a helical trajectory is significantly greater.

The shape of the characteristic lines is strongly sensitive to the trajectory curvature when the basic region given by equation (24) is highly elongated. This dependence can be observed qualitatively by studying the envelopes for various values of the parameter n in equation (24). For $n = 1$, the region is a sphere and the characteristic lines are circles.

Figure 7 shows, for a slightly curved trajectory (ρ = 3.05 km), the envelope that results for values of n = 1.5, 2.0, and 3.0 corresponding, respectively, to radar angles of 31.5°, 24.1°, and 16.3°. It is seen that, as the radar angle θ^* decreases, the characteristic lines and the envelope become highly elongated.

CONCLUDING REMARKS

A general procedure for computing a region of influence associated with a maneuvering vehicle has been described. The method described for calculating the envelope of the individual constant-state regions directly from the analytic expression for the characteristic lines has significant advantages over the strictly numerical approach. It is more efficient computationally, and the analytic form of the characteristic equation provides a qualitative insight into the effect of the various flight parameters on the size and shape of the region of influence. Application of the theory was illustrated by computation of the destruct region for a fighter firing cannon shells, as an example of a simple vector line surface, and the destruct region for a fighter firing missiles, as an example of the application of the envelope theory.

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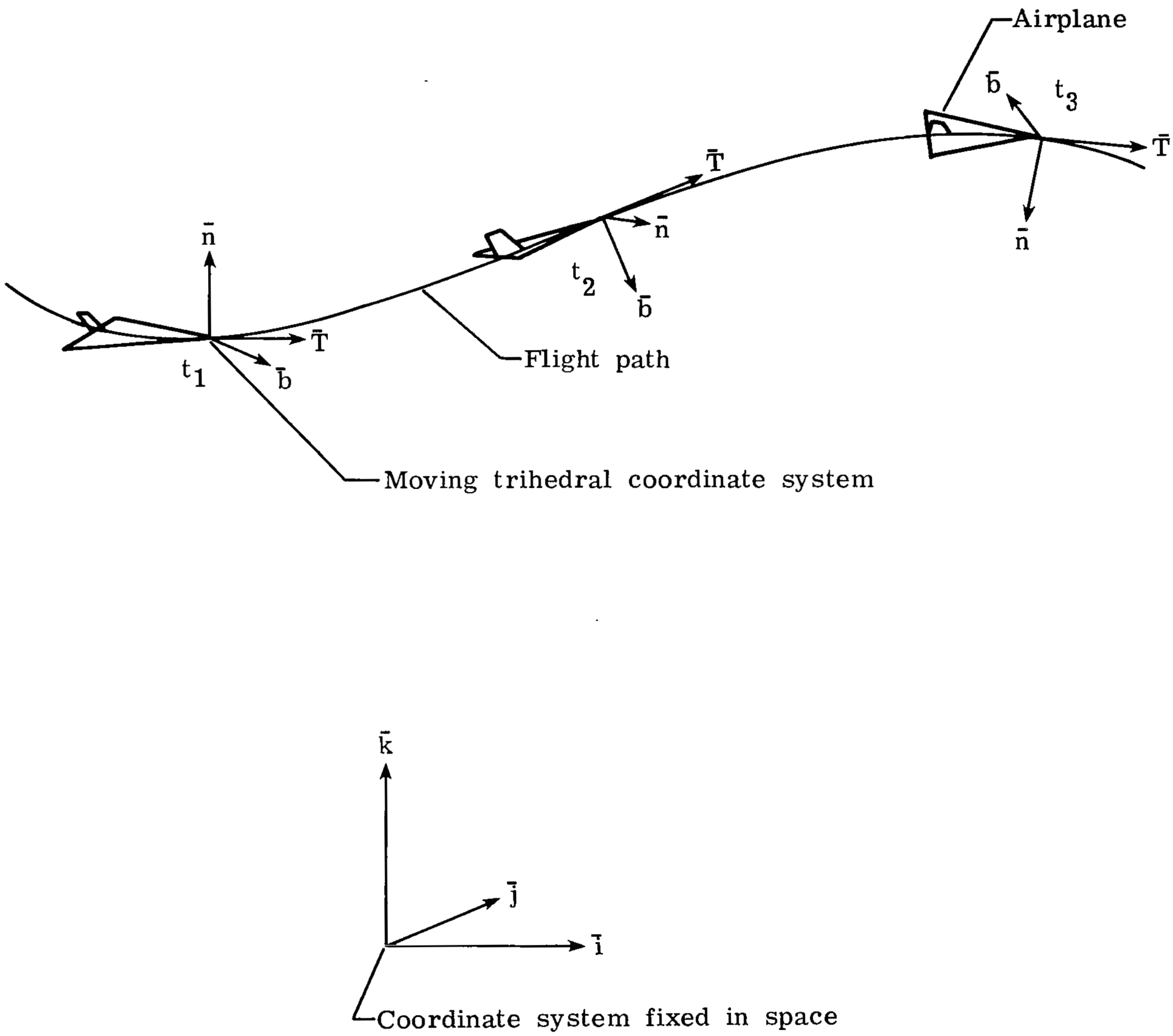


Figure 1.- Fixed and moving coordinate systems.

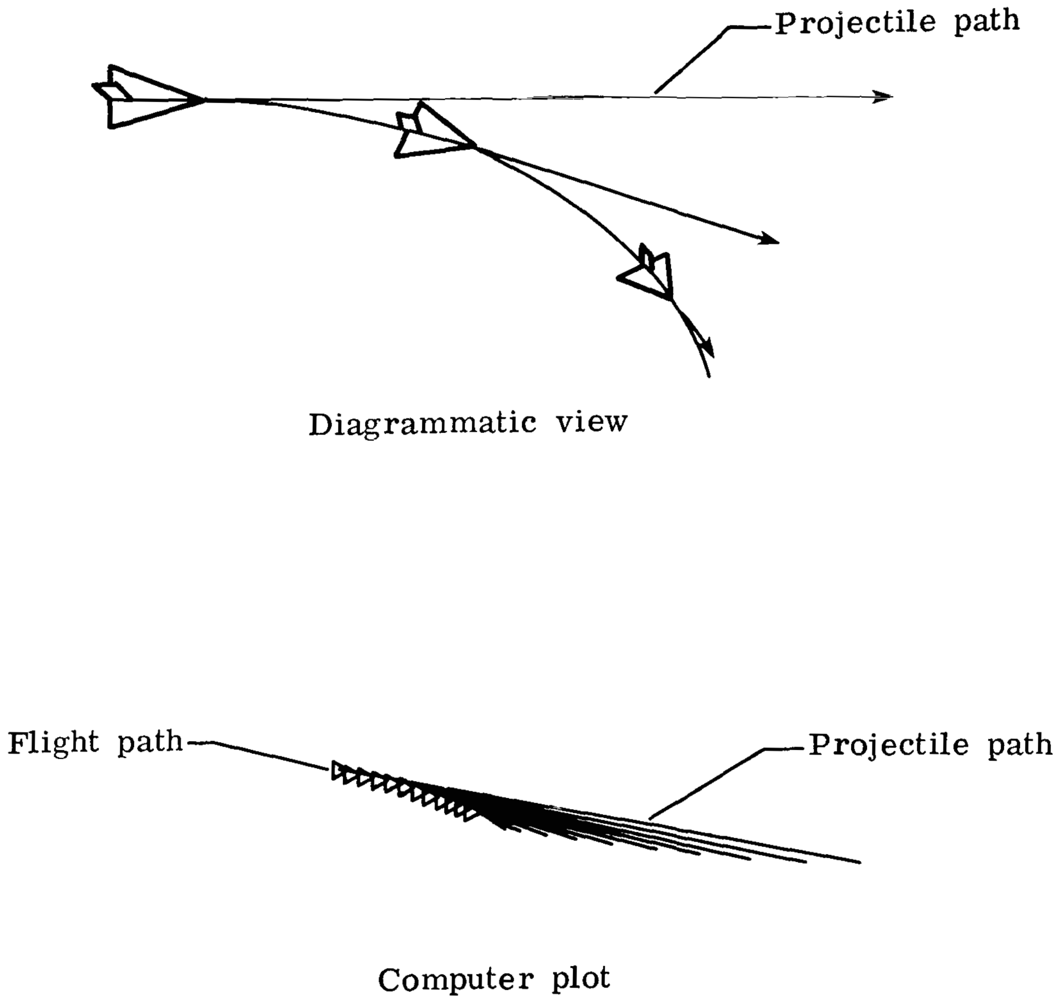
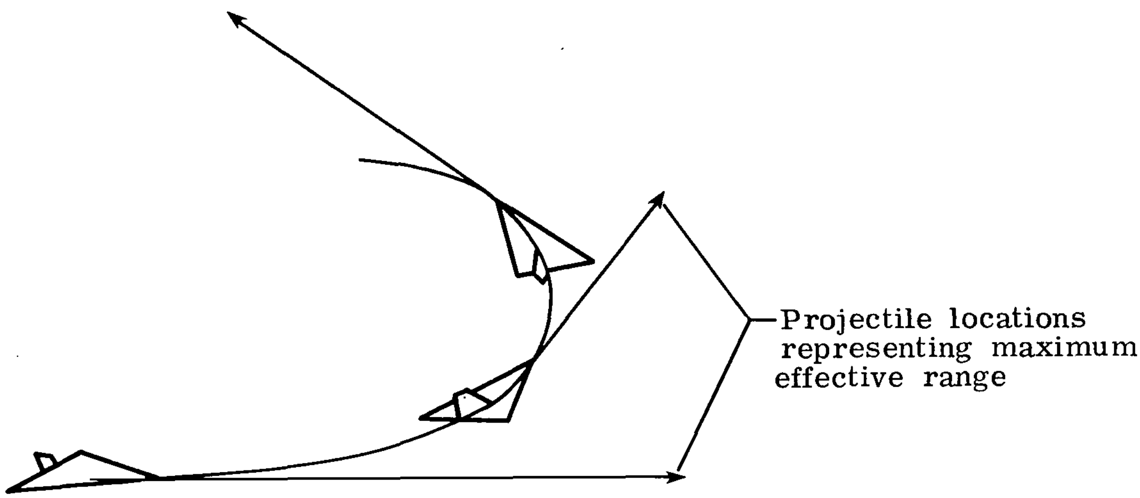
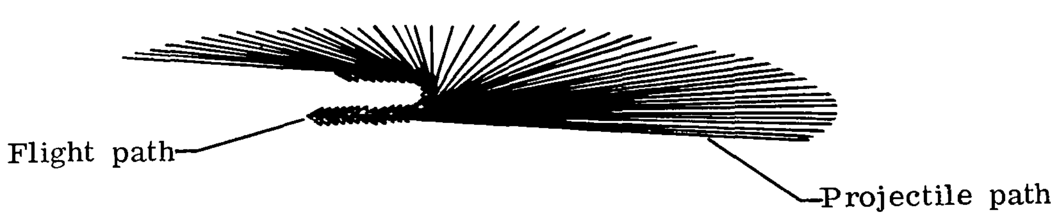


Figure 2.- Example of computing distribution at fixed time of projectiles fired from turning airplane.



Diagrammatic view



Computer plot

Figure 3.- Destruct surface swept out by projectiles fired from turning airplane (perspective view).

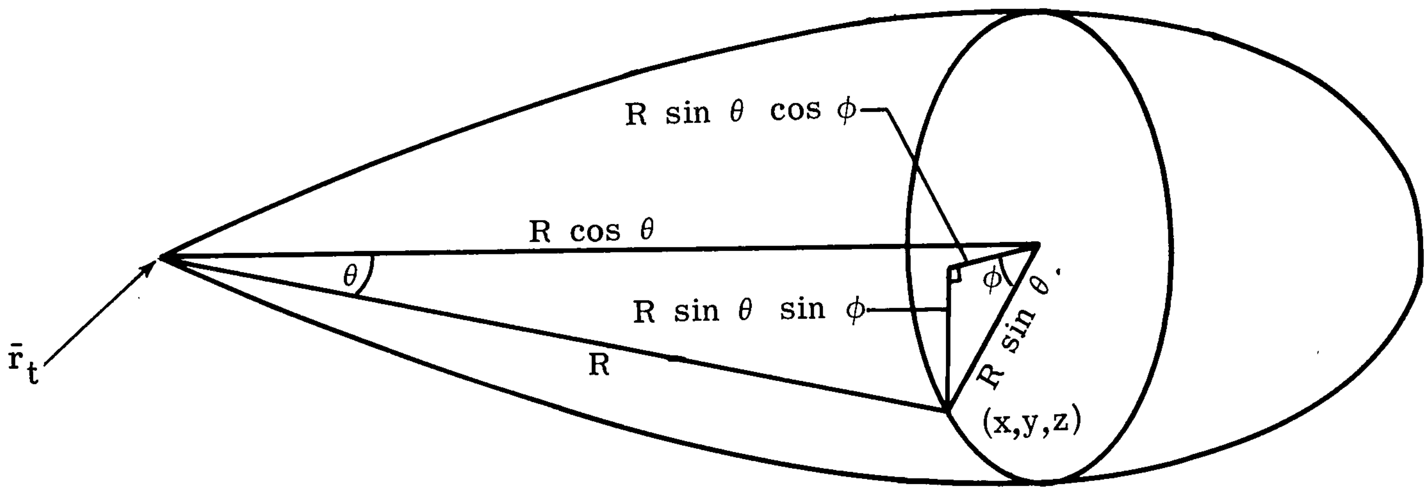
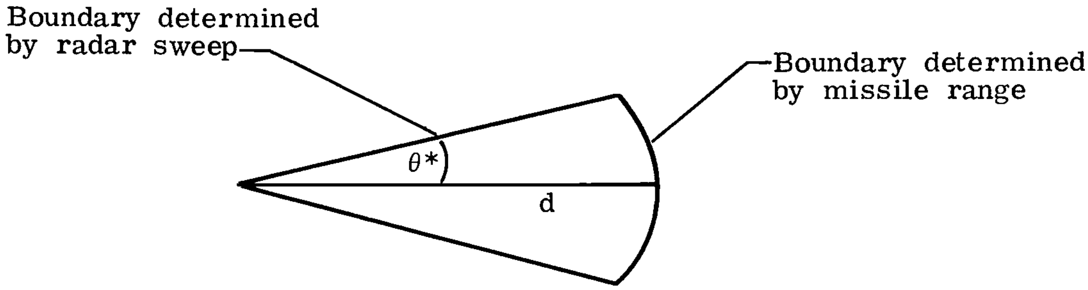
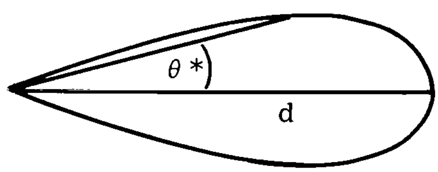


Figure 4.- Geometry defining general point on axisymmetric surface.

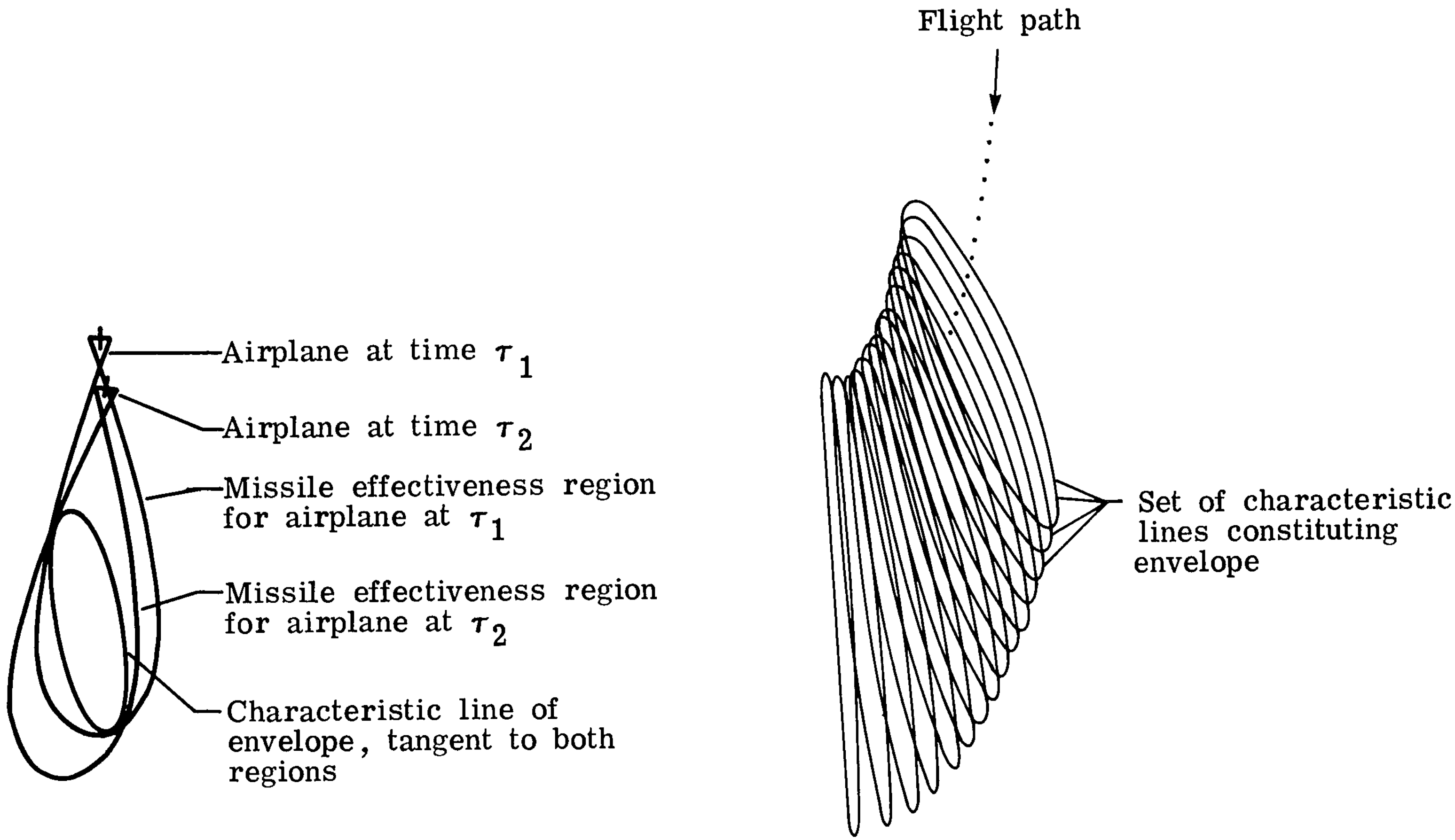


(a) Potential missile effectiveness region as determined by radar sweep angle and missile range.



(b) Analytic approximation to missile effectiveness region in the form $d \cos n\theta$.

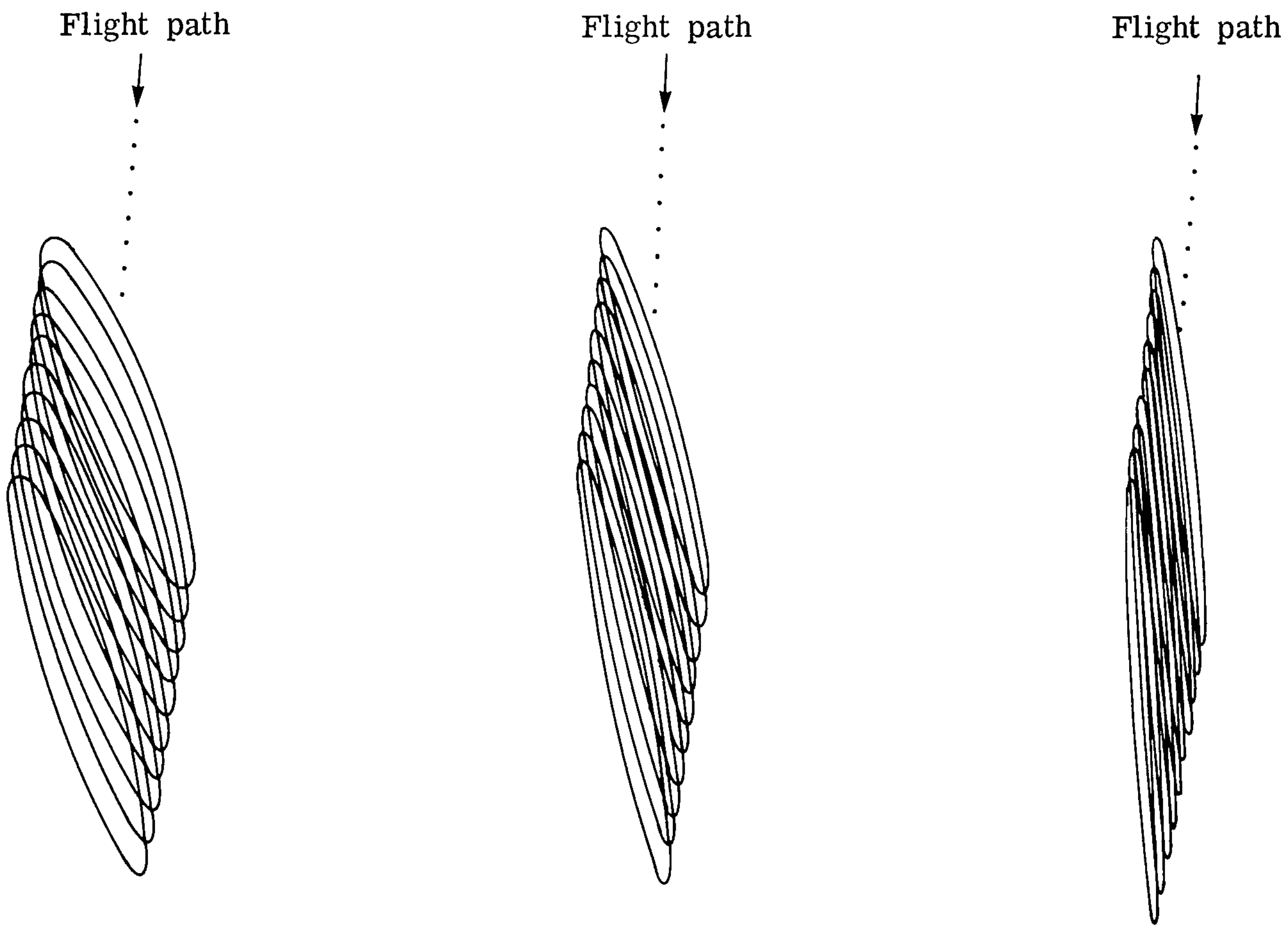
Figure 5.- Physical and analytic approximations to missile effectiveness region.



(a) Concept schematic.

(b) Computed example.

Figure 6.- Section of missile effectiveness envelope for airplane performing turn of increasing curvature.



(a) $\theta^* = 31.5^\circ$. (b) $\theta^* = 24.1^\circ$. (c) $\theta^* = 16.3^\circ$.

Figure 7.- Comparison of envelope shapes for increasing elongation of basic effectiveness region (decreasing radar cone angle), for slightly curved flight path.

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