TENSIONING OF A BELT AROUND A DRUM USING MEMBRANE ELEMENT*

C. H. S. Chen
The BFGoodrich Company

SUMMARY

A class of problem which can be solved by using the membrane element approach is that in which the membrane surface is either unchanged or allowed to undergo a given amount of transverse displacement. A problem belonging to the latter case is the tensioning of a belt around the drum. In this paper the belt tension increase due to drum edge wear or material trapped between the drum and the belt is investigated and some interesting results are obtained. In both cases the increase in belt tension is due to the additional stretching of the belt resulting from the drum radius change rather than from the transverse deflection of the belt.

INTRODUCTION

One shortcoming of the NASTRAN membrane elements is that in their for-mulations the coupling between bending and stretching is neglected (Ref. 1). In other words, the in-plane strains are independent of the transverse dis-placements. As a result, the use of NASTRAN membrane elements is very restricted. One class of problems which permits the use of the membrane elements is that in which the membrane surface is either unchanged (such as in a plane stress elasticity problem) or is allowed to undergo given amounts of transverse displacement. In either case, the applied load vectors should always be in the plane of the membrane element. This means that in this class of problems the membrane elements can not take either a normal pressure load or a concentrated load. The problem of belt tensioning around a drum can be classified into that particular class of problem and is used here as an example.

ANALYSIS

This paper demonstrates an application of the membrane element to the problem of the tensioning of a conveyer belt which wraps around a drum. The conveyer belt to be considered has one row of longitudinal wire cable reinforcement placed in a thin sheet of rubber. Conventionally the wire cables are equally spaced across the width of the belt. The dimensions of the belt cross-section and the drum diameter are such that the assumption of the

---

belt as a membrane structure in the conventional engineering sense can be justified. The belt is assumed to be orthotropic linear elastic and its in-plane material properties are derived using composite material theory. In particular the well known Halpin-Tsai equations are used by knowing the constituent properties and the wire volume ratio (Ref. 2). The tensioning of the belt considered here comes from the counterweight which is needed to develop sufficient friction between the belt and drum so that the belt can be driven when the driving pulley turns.

We are particularly interested in investigating the stresses developed in the belt under the following situations:

(1) When the drum edges wear out,

(2) When there is material trapped between the belt and the drum,

(3) When both the drum edge wearing and the material trapping occur.

In all three cases there are dangers of an increase in cable tension which affects the belt safety.

The trapped material considered here is in the form of a narrow strip wrapping around the drum for the entire region of belt-drum contact. Figure 1 shows the modeling of a belt using isoparametric quadralateral membrane elements. This belt has 180 degree contact with the drum (called 180 degree wrap angle), but because of symmetry only one half of the wrap angle is included in the model. The drum is not modeled in the problem since it is considered to be rigid. The belt-drum contact region extends from grid point 1 to grid point 7 as shown in Figure 1. The fine grids are used at the edge regions of the belt for convenience in examining different degree of the drum edge wear. The width of the edge regions is arbitrarily assumed.

The tensioning of the belt is simulated by imposing at the straight end of the belt (marked by grid points 11 and 187) a specified amount of uniform displacement while retaning the wrapped end at the half-wrap angle (marked by grid points 1 and 177) unmoved. The drum edge wearing is simulated by allowing the grid points in the belt edge regions to be unrestrained in the radial direction. The trapped materials between the belt and the drum is simulated by specifying at the appropriate grid points within the belt-drum contact a specified amount of radial displacement. For all three cases just mentioned, the loaded belt can be said to have well defined surfaces and thus the membrane elements can be satisfactorily applied.

NUMERICAL RESULTS

The belt under consideration has the following dimensional and material property characteristics:

34

Ratio of edge region width ($2d_e$) to center region width ($2d_c$) = $2d_e/2d_c$
= 0.35

Ratio of belt thickness (t) to drum radius (R) = t/R = 1/22.529

Ratio of belt thickness (t) to belt total width ($2d_e$+ $2d_c$) = $t/2(d_e$+ $2d_c$)
= 1/78.488

Ratio of longitudinal Young's modulus (E_1) to transverse Young's
modulus (E_2^1) = E_1/E_2 = 160.417

Ratio of the height of trapped material (h) to drum radius (R) = h/R
=1/31.

The specific amount of displacement required at the straight end (or far
end) of the belt to produce a given amount of belt tension is obtained by
interpolation between several solutions. Once the element stress (σ_e) is
known the cable force (F) can be calculated thus,

$$F = \sigma_e EA/E_1$$

where E and A are the Young's modulus and the cross sectional area of the
reinforcing cables respectively.

Figure 2 shows the results of the cable force when there is no drum edge
wear, whereas Figure 3 shows that when there is complete drum edge wear. They
are all normalized with respect to the ideal uniform cable force (F_0) when the
drum has no edge wear and there is no trapped material. F_0 is obtained either
by dividing the total belt tension by the number of cables or using the
equation after the NASTRAN run. The force profiles of Figures 2 and 3 are
those along the line of the half-wrap angle where the peaks are always found
to be maximum. Five locations of the trapped material are examined. These
locations are the belt centerline and four other locations at distances of
$d_c/4$, $d_c/2$, $3d_c/4$ and d_c away from the belt centerline. Here d_c is the half-
width of the belt center region.

It can be seen from Figures 2 and 3 that, for this particular case, the
cable force (or stress) increases by about 3.3 times due to material trapping
alone, about 1.3 times due to drum edge wear alone and about 4 times due to
the combination of the two. These clearly indicate the danger of material
trapping and excessive drum edge wear. Figures 2 and 3 also reveal that the
force or stress concentration is highly local in nature and that the peak
values appear to be independent of the location of the trapped material.
These imply that the interactions between the cables are not very strong and
they decay fast. This fact suggests that in the simplified analysis the cable
interaction can be neglected.

CONCLUDING REMARKS

Using the problem of tensioning of a belt as an example, the present

paper has pointed out a class of problems which can be adequately solved by using NASTRAN membrane elements. This class of problems are such that the middle surface of the membrane is either undeformed or undergoes a specified amount of deflection. We have studied the effects of material trapping and drum edge wear on the cable tension in a belt. It should be noted here that the material trapped between the drum and belt is of a special form that has the equvalent effect of increasing the drum radius. The increase of the belt tension really comes from the additional stretching of the belt resulting from the growth of the drum radius rather than due to the transverse deflection of the membrane.

REFERENCES

1. The NASTRAN Theoretical Manul, NASA SP-221(04), Dec. 1977.

2. Jones,R.M., Mechanics of Composite Materials, McGraw-Hill Book Co., 1975.
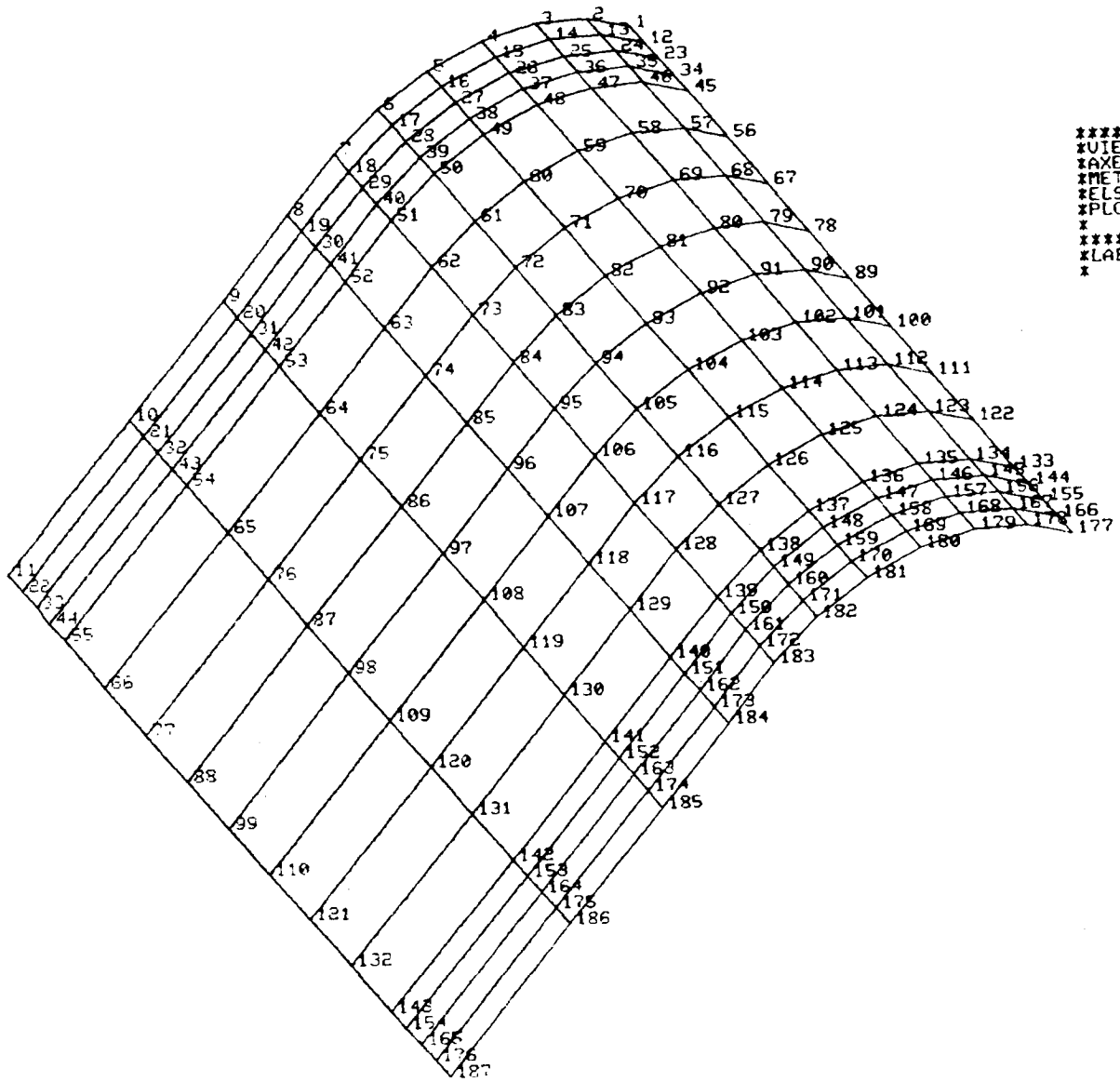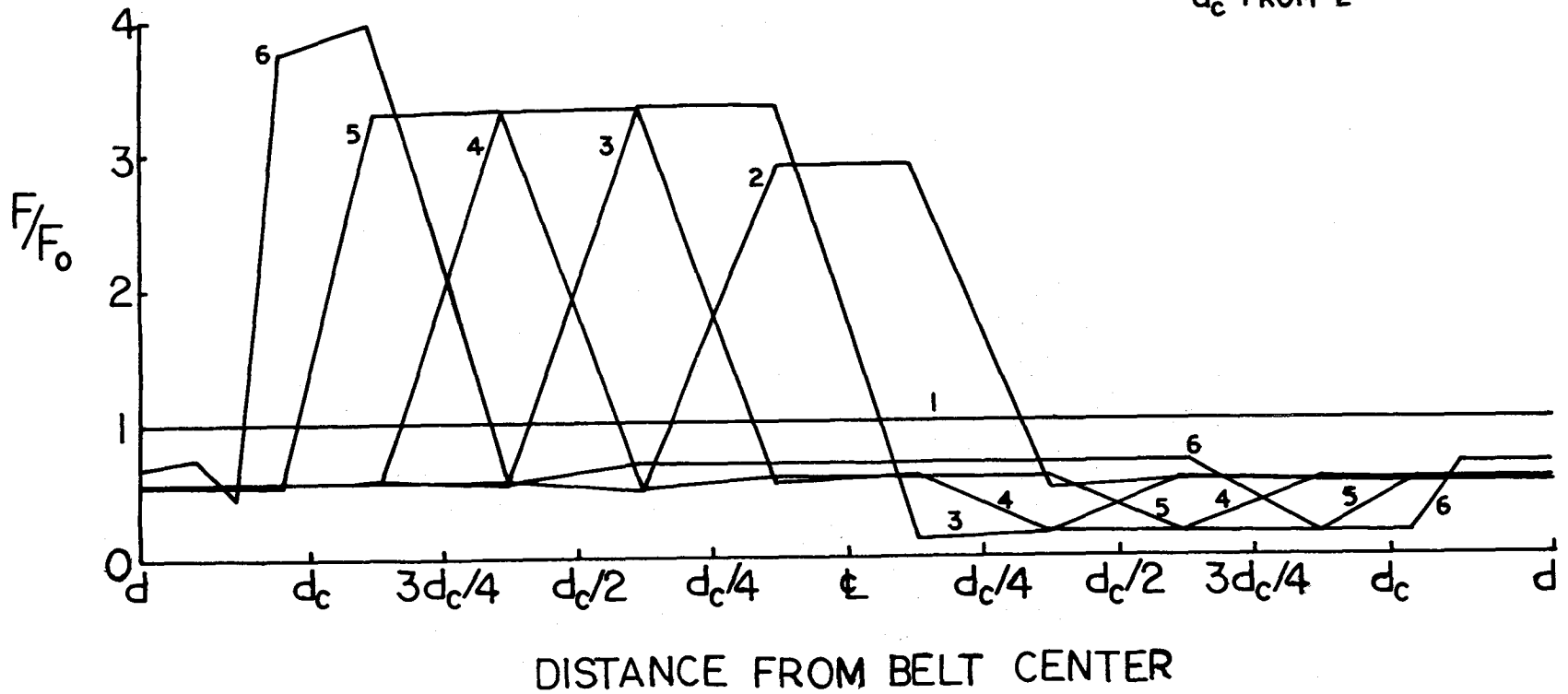
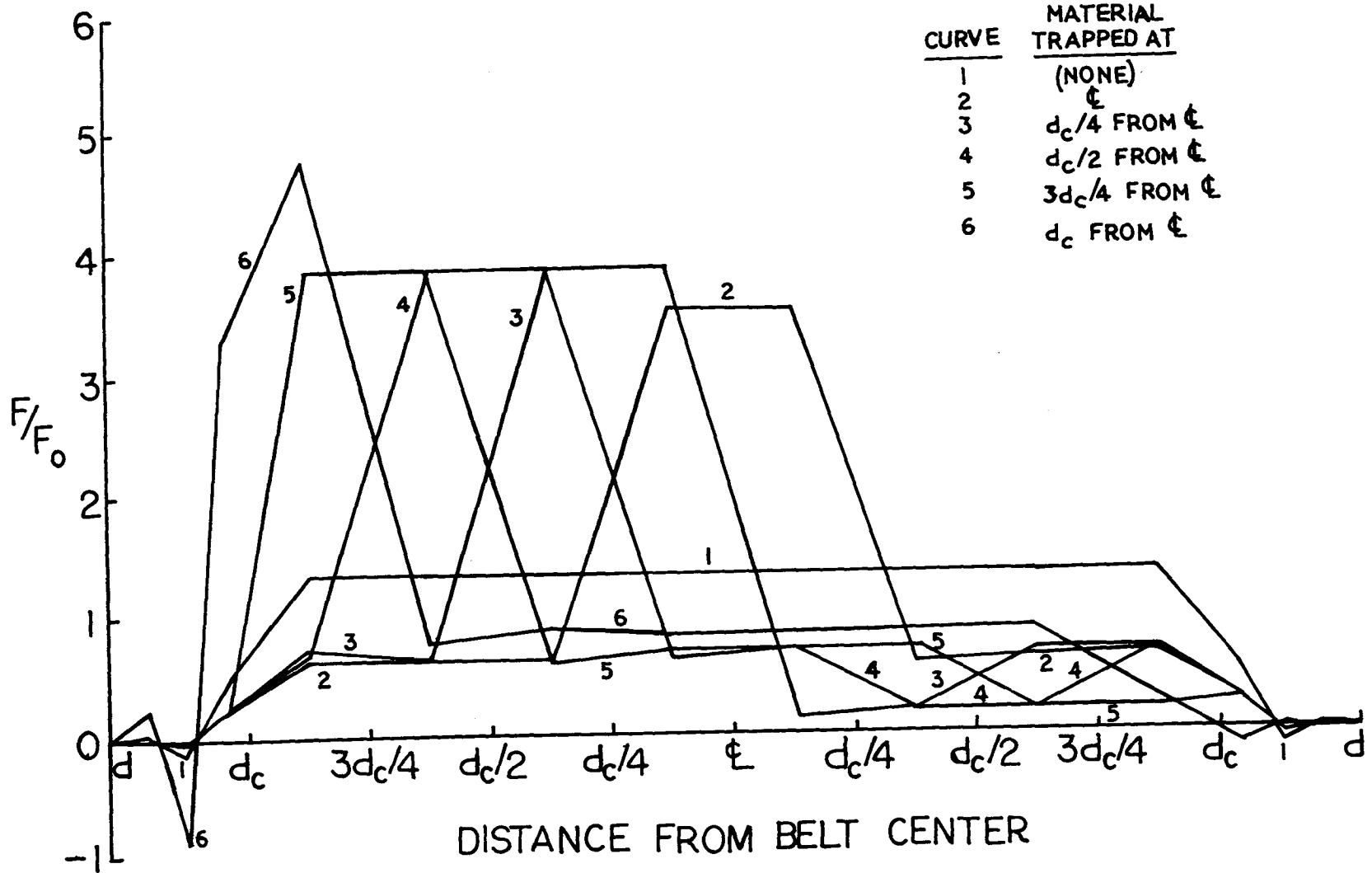FIGURE 1 - BELT MODEL

FIGURE 2 - CABLE FORCE PROFILE - NO EDGE WEAR OF DRUM

FIGURE 3 - CABLE FORCE PROFILE - COMPLETE EDGE WEAR OF DRUM