THE RADII OF SU CAS AND TU CAS

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It is possible to obtain the masses of Cepheid variables by several methods involving the pulsation theory. However, these masses are frequently smaller than those indicated by the theory of stellar evolution. The cause of this discrepancy is not fully understood although a number of possibilities have been suggested. Since the pulsation theory indicates that there is a relation among the mass, the radius and the period, the discrepancy also manifests itself in the radii of these stars (Cogan 1978). With this in mind, we have undertaken radius determinations for two Cepheids which are of special interest in connection with pulsation theory.

It has been suggested that the short period Cepheid SU Cas is pulsating in the first overtone mode (Iben and Tuggle 1975, Gieren 1976). If this is correct the radius should be larger than a fundamental pulsator of the same period. Thus, the radius can be used to test the pulsation mode. Since Stellingwerf (1975) found from pulsational calculations that the hottest short period Cepheids should be overtone pulsators, this provides an opportunity to test a prediction of the theory.

TU Cas is a beat Cepheid with a fundamental period of 2.14 days. Several investigators (Schmidt 1972c, Petersen 1973, King et al. 1975, Stobie 1977) have shown that it is possible to determine the radii and masses of such stars from the periods which are observed. The beat masses so determined are only a little greater than half the evolutionary mass and the radii are much smaller than those calculated from the period-Luminosity-color relation. On the other hand, the PLC relation radii are consistent with evolutionary masses. Thus, an independent determination of the radius will help decide if the beat mass discrepancy is due to the pulsation theory or the evolution theory.

We have employed a modified version of the Wesselink method proposed by Barnes et al. (1977) in which surface brightnesses are determined from the (V-R) color. This has the advantage, especially in the analysis of TU Cas, of allowing a radius to be determined without complete light and velocity curves. There are a number of problems connected with the application of any version of the Wesselink method. The phase matching between the velocity and light curves must be very good. In the case of a beat Cepheid, where the curves do not repeat, simultaneous observations are necessary. We have obtained these by the use of the coudé feed telescope and one of the 0.9 meter telescopes at Kitt Peak National Observatory. A further difficulty is that any version of the Wesselink method is sensitive to small photometric errors. We have dealt with this by using two comparison stars for each variable and observing the variable and comparison stars repeatedly. The standard deviations in the photometry averaged $0^{m}_{.}004$ for both V and (V-R). Finally radii determined in this fashion are subject to a variety of systematic errors. By adopting the method of Barnes et al. we can compare our stars directly with the stars they have analyzed. Thus, we can determine how the radii of SU Cas and TU Cas compare with other Cepheids even in the presence of systematic errors.

The data were obtained over a period of 11 days in September 1977. Figure 1 shows the data for TU Cas. The top two panels contain the magnitudes and colors while the radial velocities are in panel c. A third order Fourier fit was made to the velocities following the methods described by Faulkner (1977). Using a systemic velocity of -14.5 km/sec and a foreshortening factor of 24/17, this fitted curve was integrated to produce the displacement curve shown in the lower panel. It can be seen that the gaps which occur in the observations during each day have produced apparently spurious bumps in the fitted curve. Without more velocity measurements, we can not improve this situation. For SU Cas we have combined the velocity measurements of Abt (1959) and Gieren (1976) with our own to produce the velocity curve. Their data were adjusted in phase and zero point to fit ours.

In Figure 2 we plot the relative radius, R/R_0, determined from the photometry against the displacement, $R-R_0$, obtained from the velocity integration. The value R_0 refers to an arbitrarily chosen initial epoch. The slope of the relation in this diagram determines the value of R_0 and a suitable constant is added to this to convert it to the mean radius. The sensitivity of this method to photometric errors can be seen from the error bars in the upper left corners of the diagrams. These represent the effect of an error of $0\overset{m}{.}01$ in (V-R). The scatter can be seen to be consistent with small photometric errors. In the lower panel the data for TU Cas are plotted with a different symbol for each of the nights on which it was observed. Significant shifts are evident from night to night. We attribute these to our inability to define the velocity curve continuously over the entire interval. We have therefore determined the radius of TU Cas on each night and have averaged over all the nights.

The measured radii for SU Cas and TU Cas are listed in the fourth column of Table 1. The radii in the fifth column were calculated from the luminosities obtained from the period-luminosity-color relation. For SU Cas we have done the calculation under assumptions of fundamental and first overtone pulsation. The radii in column six are from a period-radius relation which is a straight line fit to the radii listed by Barnes et al. These were derived in the same fashion as the present radii and should be directly comparable. For TU Cas we also list the radius determined from the period ratios (King et al. 1975). The effective temperatures listed in the last column are the averages of temperatures determined for these stars from continuum scans and Hα profiles (Schmidt 1972a, 1972b, 1972c).

Looking first at the results for SU Cas we see that the present radius agrees with that obtained from the PLC relation if we assume fundamental pulsation. However, the radii determined by Barnes et al. are much smaller than the PLC radii at these short periods. Whether this reflects an error in the calibration of the PLC relation or some systematic effect in the radii, it is clear that the present value should be compared with the Barnes et al. values. Figure 3 shows a plot of the Barnes et al. radii and the present determinations against period. From the diagram it is clear that the radius of SU Cas is too large if it assumed to be pulsating in the fundamental. However, for overtone pulsation the present radius in only slightly more than one standard error larger than the value indicated by the period-radius line. This evidence therefore supports the previous contention that this star is an overtone pulsator.

The original suggestion that SU Cas pulsates in the first overtone was made by Iben and Tuggle (1975) on the basis of a distance modulus determined by Racine (1968). Racine had noted that the field around SU Cas contains several stars which illuminate reflection nebulae as does the Cepheid. He then obtained spectroscopic parallaxes for the field stars. Within the accuracy of these parallaxes it appeared that the field stars defined a group at a true distance modulus of 7.5. He took this to be the distance of SU Cas. Because spectroscopic parallaxes are relatively inaccurate the distance moduli of the stars in this region have been redetermined using uvbyβ photometry (Schmidt 1978). Two of the stars are at a distance modulus of 7.2 while a third is at 8.2. If SU Cas is at the nearer distance, the implied radius is 19.1 $R_\odot$. Referring to Table 1 we see that the smaller value is consistent with the radius for overtone pulsation if the Barnes _et al_. relation is correct while the larger value is consistent if the PLC relation is valid. Neither agrees with fundamental pulsation under any assumption. While this gives some indication that SU Cas is an overtone pulsator, it should be viewed with caution. There are field stars at three distinct distances in this region which are associated with reflection nebulae. It is possible that SU Cas is associated with none of them. Its pulsation mode must ultimately be decided on other grounds.

A further point which bears on the pulsation mode of SU Cas is its temperature. Stellingwerf (1970) studied the mode behavior of a model representing U TrA. He found that models between T_{eff} = 6350K and 6675K should be unstable in the first overtone only while those between 6200K and 6350K should be unstable both to the fundamental and the first overtone. At cooler temperatures there is a region with only fundamental pulsation which extends to 5840K. The temperature given in Table 1 for SU Cas clearly places it in the first overtone region. Thus, there are three pieces of evidence favoring the identification of SU Cas as an overtone pulsator and the observational data support Stellingwerf's calculations.

Turning now to TU Cas we see that the present radius is much smaller than the PLC radius. However, the agreement with the period-radius relation and the beat radius is good. As stated above, our determination should be consistent with the period-radius relation and we thus conclude that TU Cas has a radius appropriate to its fundamental period. Also, the agreement between the Wesselink radii and the beat radius suggests that the beat radius is correct and that the PLC radius is too large.

We note that the temperature for TU Cas given in the table is at the edge of the range Stellingwerf predicted for stars unstable in both the fundamental and the first overtone. This suggests that while SU Cas is an overtone pulsator due to its high temperature, TU Cas is switching from one mode to the other at the present time.

We conclude, because of the agreement between the present radius and the beat radius of TU Cas, that the pulsation theory is giving us correct information about the radii of beat Cepheids. This implies that the luminosities of short period Cepheids have been overestimated. It further implies that the masses are significantly less than evolutionary masses. Thus, the solution to the mass discrepancy should perhaps be sought in the theory of stellar evolution or in the possibility of mass loss.

Table 1

RADII OF SU CAS AND TU CAS

		$R/R_\odot$					
		P_0	Wess.	PLC	P - R	Beat	T_{eff}
SU Cas	F	1.95	25.2±4	24.1	14.5	--	6530
	1H	2.75	25.2±4	30.7	19.9	--	6530
TU Cas		2.14	17.4±4	25.6	15.8	15.7	6360

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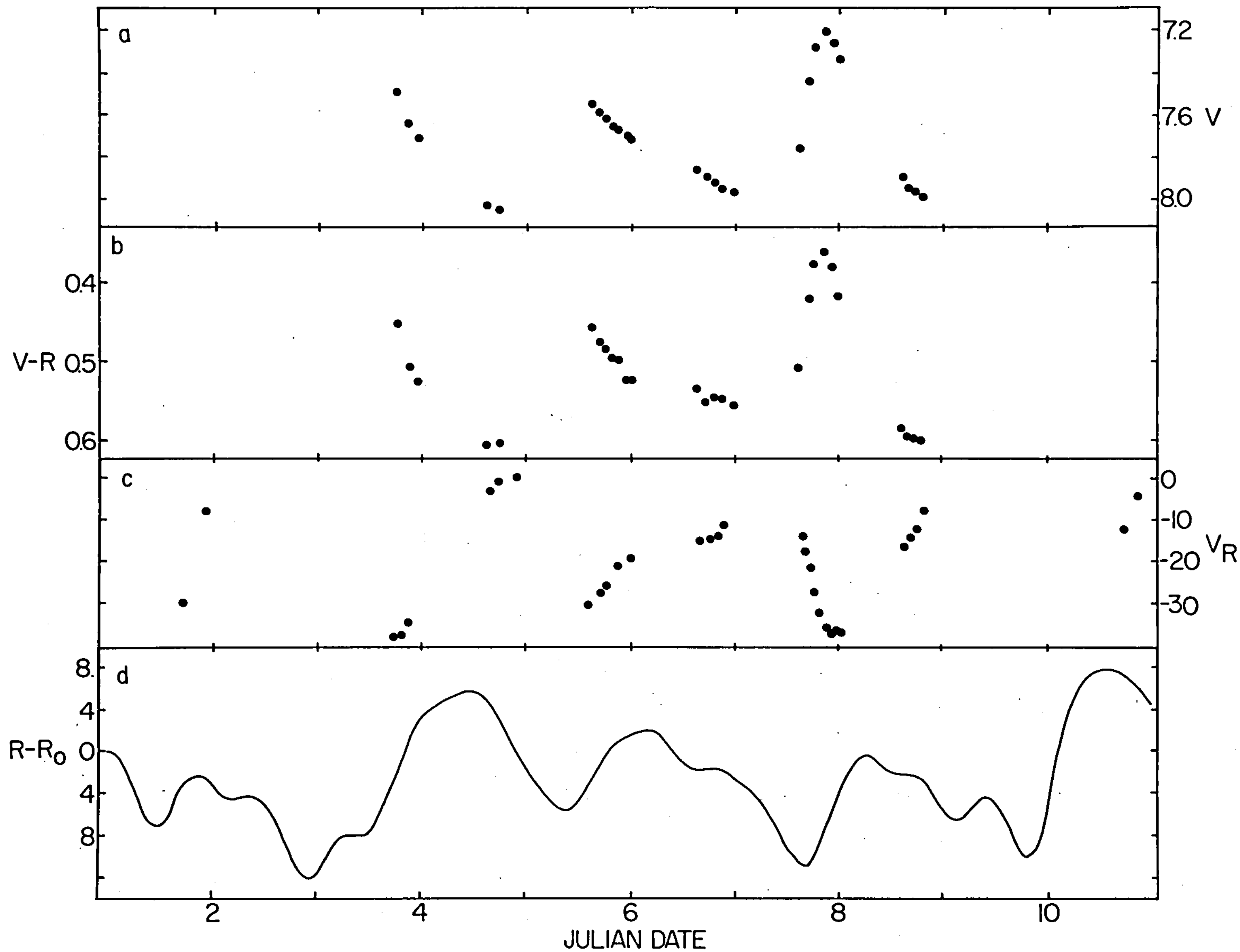

Figure 1. The observational data for TU Cas. The Julian Date -2443400 is plotted as the abscissa. a). V magnitudes. b) V-R colors. c) radial velocity in km/sec. d) a radius displacement curve generated from a third order fit to the velocities.

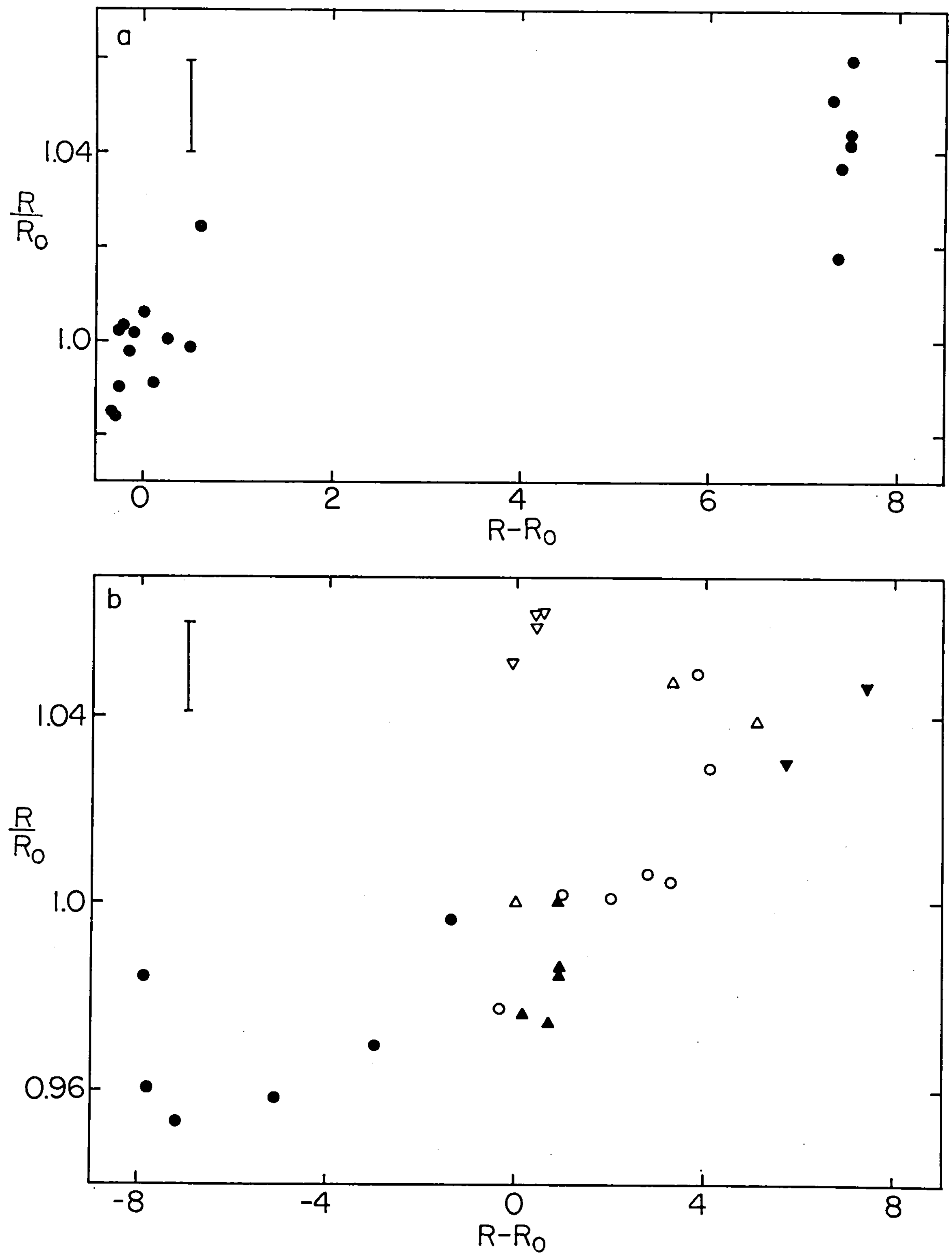

Figure 2. A plot of relative radius versus displacement. The radius displacement is in units of 10^5km. a) SU Cas. b) TU Cas. Different symbols denote different nights.

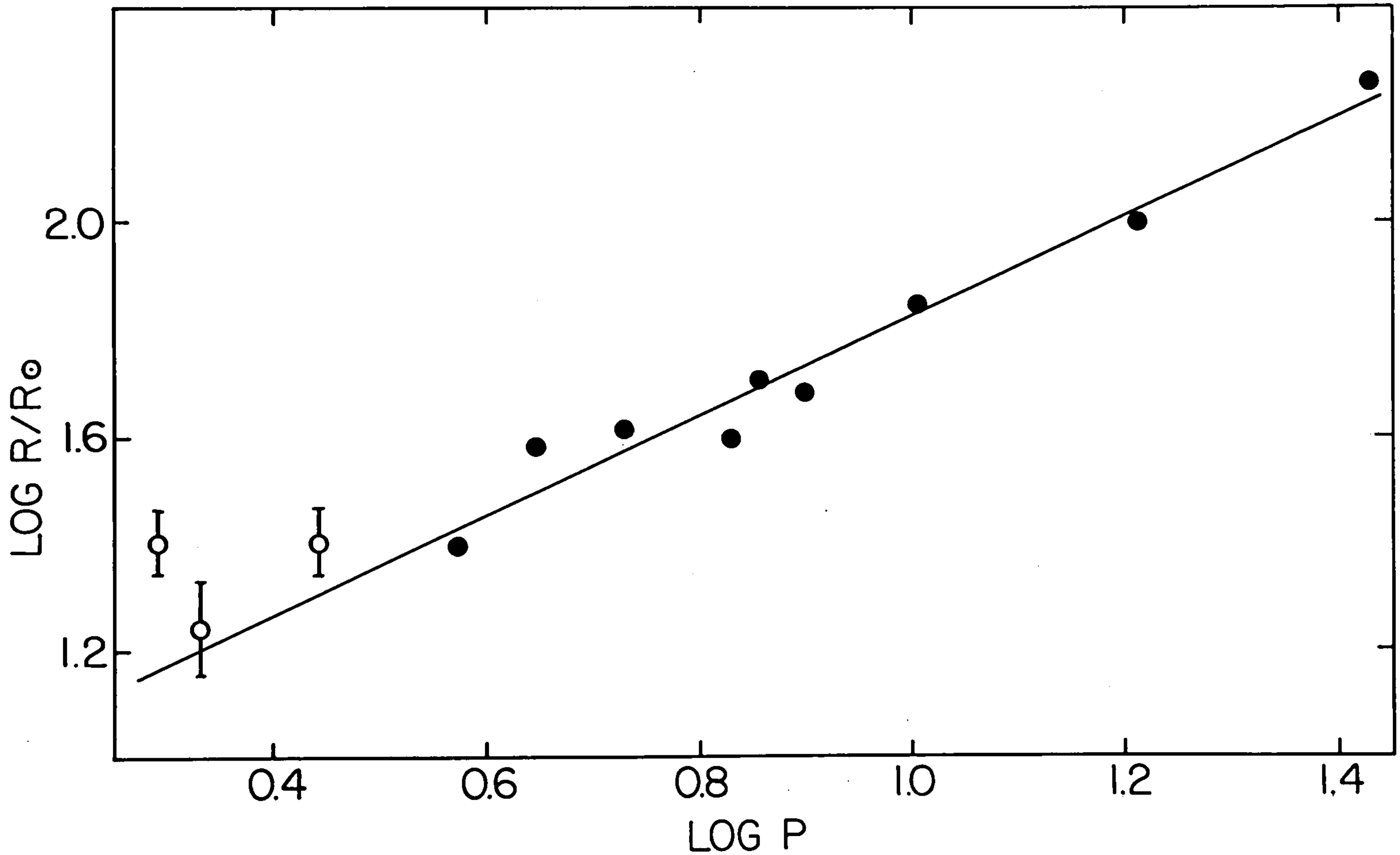

Figure 3. The radius-period relation. The filled circles represent radii obtained by Barnes *et al*. and the solid line is fitted to them. The two open circles with error bars at log $R/R_{\odot}$= 1.4 represent the present radius for SU Cas assuming fundamental and first overtone pulsation. The lower circle with error bars is the radius of TU Cas.

Discussion

A. Cox: It seems to me Opolski had observations of TU Cas which were reported at the Budapest meeting, but I haven't seen the proceedings yet.

Rosendhal: We earlier saw divergent temperature scales. How sensitive are the results you obtained to the use of different temperature scales?

Schmidt: The various temperature scales are reasonably consistent at the hot end of the range. This star is quite hot so it doesn't matter which scale is used. I used my own scale.