A DISCUSSION OF CEPHEID MASSES

Arthur N. Cox
Theoretical Division
Los Alamos Scientific Laboratory
University of California

ABSTRACT

Masses and compositions of Cepheids are essential to map the places in the Hertzsprung-Russell diagram where various radial pulsation modes occur. Luminosity observations and stellar evolution theory give masses for Cepheids which range from 10 percent to a factor of four more than those given by pulsation theory. Combining the evolution and pulsation theories, a theoretical mass can be determined using only the period and an approximate surface effective temperature, T_e. The ratio of the theoretical to evolutionary masses averages 0.99 ± 0.07 for 16 Cepheids with good data. A pulsation mass can be calculated using an observed period, luminosity, and T_e. The ratio of pulsation to evolutionary masses averages 0.70 with the old distance scale for the Hyades cluster and the older T_e values, 0.84 ± 0.17 with the accepted 13 percent distance increase of the Hyades cluster and presumably all the clusters with Cepheids, 0.97 ± 0.25 with the new distance scale and improved interstellar reddening corrections giving cooler T_e values, and 1.07 ± 0.27 using in addition surface helium enriched envelope models. These inhomogeneous models allow theoretical predictions of the correct phase of light and velocity curve bumps for evolutionary theory mass Cepheids with periods between 5.5 and 13 days. They also give the proper observed period ratios for double-mode Cepheids with evolution theory masses. Using radii measured by the Wesselink method, the ratio of this mass to the evolutionary mass is

0.93 with homogeneous and 1.02 with inhomogeneous models. Above about 10-20 days a mass loss of 20-30 percent is indicated for stars in their early B star evolution with originally 15 or more solar masses. Six Cepheids with at least four different mass determinations show that for periods below 10 days there are no longer any Cepheid mass anomalies.

A long term goal in understanding the pulsation of the classical Cepheids is to theoretically map their pulsation instability strip in the Hertzsprung-Russell diagram. Several previous attempts have been made starting with J. P. Cox (1963) and including Christy (1966), Stobie (1969ab), Iben(1971), Iben and Tuggle (1972ab, 1975), and Stellingwerf (1975), and the Los Alamos centered group who have published the papers Cox, et al. (1966), King, Cox, and Eilers (1966), King, et al. (1973), Cox, King, and Tabor (1973), and King, et al. (1975). It has been shown that instability strip blue edges are well understood, and now Deupree is completing a discussion of the Cepheid red edge. What remains to learn is what pulsation modes occur and where they are in the instability strip.

The masses and compositions of the Cepheids are essential if this further mapping in the H-R diagram is attempted. Even the blue and red edges depend on the mass and composition, but their lesser dependence has resulted in the current reasonably satisfactory agreement between theory and observation. See Cox and Hodson (1978). The persistent uncertain Cepheid masses and compositions has led Christy (1966) to what I consider a very misleading relation between luminosity and transition period for the fundamental mode:

$$\Pi_{tr} = 0.057\ (L/L_{\odot})^{0.6}\ \text{days}.$$

Stobie (1969b) in discussing modal behavior proposed an incorrect rule that if the pulsation period was between 2 and 7 days, it was a first overtone pulsator, and if less than 2 days a second overtone pulsator. Stellingwerf (1975), searching for double-mode conditions got it (wrongly we now think) at the red side of the instability strip or maybe beyond the red edge. All these studies used atrocious masses and compositions, oblivious of the results of stellar evolution theory.

Starting with Cogan (1970) several others like Rodgers (1970), Fricke, Stobie, and Strittmatter (1973), and Petersen (1973), found that masses based on observations and pulsation theory were anywhere from 10 percent to a factor of four lower than those infered from observed luminosities and evolution theory.

Iben and several of his collaborators have tried to reconcile pulsation and evolution theories and both of these with observations. Various schemes for solving the Cepheid mass anomalies have been proposed. Iben and Tuggle (1972a) wanted to move the period-luminosity calibrating Cepheids further away by 15 percent making them intrinsically brighter, bigger and more nearly the mass given by evolution theory. King, et al. (1975) proposed they were really cooler and larger than the current $(B-V)_0$ - log T_e relation gives. Both these effects have now been realized with the best distance scale to the galactic clusters containing Cepheids increased by 13 percent and a new concept that the interstellar reddening was overestimated making Cepheid surface temperatures now 100 K or so cooler with the same $(B-V)_0$ - log T_e scale.

However, the phase of the bumps in Cepheids between 5.5 and 13 days and the period ratio of the dozen or two double-mode Cepheids still indicate low masses. These bump and beat masses are independent of the distance and temperature scales. Simon and Schmidt (1976) have now showed that the bump phase was related to the ratio Π_2/Π_0, making both these classes double-mode Cepheids.

Our proposal after considerable discussion, much of it already published by Cox, et al (1977) and Cox, Michaud, and Hodson (1978), is that the Cepheids have helium enriched envelopes, caused by a helium deficient Cepheid wind, which changes their internal structure enough to change Π_1/Π_0 and Π_2/Π_0 without appreciably changing Π_0. I now believe that the masses of all Cepheids are given correctly by evolution theory. Most are in blue loops, but at the shortest periods, as for the only triple-mode Cepheid AC And, and for the masses above 13 $M_\odot$, the star is having its first instability strip crossing. Can we prove that evolution and pulsation masses are now equal?

Let me first dispose of a very poor way to get Cepheid masses - use of photometric multicolor measurements interpreted in terms of log g and T_e. If

$$g = G \frac{M}{R^2}$$

$$\left(\frac{Q}{\Pi}\right)^2 = \frac{M}{R^3}$$

and $$Q_1 = Q_1 \,(M, R, L, T_e \text{ composition}) \qquad (1)$$

using the Faulkner (1977) form for Q_1, then

$$g \sim \frac{M}{R^2} \text{ and } \frac{1}{\Pi^{1.38}} \sim \frac{M}{R^{2.44}}$$

$$\text{or } M \sim g^{5.6} \text{ for Cepheids.}$$

Thus a factor of two error in g, which apparently can be improved for earlier type stars, is nevertheless disastrous for getting Cepheid masses.

The latest evolutionary tracks for Cepheids are given by Becker, Iben, and Tuggle (1977). As noted many times before, the rather tight relation between mass and blue loop luminosity allows a determination of the evolutionary mass for Cepheids with known luminosities. The relation

$$\log L/L_\odot = 0.46 - 41(Z - 0.02) + 6.6(Y - 0.28) + [3.68 + 21(Z - 0.02) - 4.5(Y - 0.28)] \log M/M_\odot \quad (2)$$

suggested by Becker, Iben, and Tuggle applying to an evolution time weighted mean of the first, second, and third crossing is used here.

Let me describe the method of determining what I call theoretical masses. We use the four equations

$$L = 4\pi R^2 \sigma T_e^4 \quad (3)$$

$$Q_0 = \Pi_0 \sqrt{\frac{M/M_\odot}{(R/R_\odot)^3}} \quad (4)$$

together with Equations (1) and (2) above to relate four unknowns M, R, L, and Q_0. Here we know Π_0, the fundamental pulsation period, very well, and we know T_e to usually better than 10 percent. If we assume Y = 0.28 and Z = 0.02, an iterative method of solution gives a theoretical mass, radius, luminosity, and a Q_0.

Table 1 gives 20 Cepheid evolutionary, theoretical, and pulsation masses, where the pulsation masses, discussed first by Cogan (1970), need an observed period, luminosity, and color. For these pulsation masses the color gives T_e which together with the luminosity gives a radius, which further, with the period, gives a mass. The Q_0 for this table comes from the Cox, King, and Stellingwerf (1972).

TABLE 1

CALIBRATION CEPHEIDS
REVISED HYADES DISTANCE
SANDAGE AND TAMMANN OR VAN DEN BERGH PERIODS AND COLORS

Cepheid	Π_0(d)	log $L/L_\odot$	$M_{ev}/M_\odot$	T_e(K)	$R/R_\odot$	Q_Q(d)	$M_Q/M_\odot$	Q_T(d)	$M_T/M_\odot$
SU Cas	1.95	2.98	4.8	6599	23.8	.0355	4.5	.0353	4.9
EV Sct	3.09	3.06	5.1	6113	30.5	.0371	4.1	.0362	5.4
CEb Cas	4.48	3.30	5.9	6138	39.8	.0381	4.6	.0369	6.3
CF Cas	4.87	3.26	5.8	5895	41.3	.0386	4.4	.0372	6.2
CEa Cas	5.14	3.34	6.1	5943	44.5	.0384	4.9	.0373	6.4
UY Per	5.36	3.40	6.3	6088	45.4	.0386	4.8	.0373	6.8
CV Mon	5.38	3.37	6.2	5943	46.1	.0385	5.0	.0374	6.6
VY Per	5.53	3.45	6.5	6162	47.0	.0386	5.0	.0373	7.0
CS Vel	5.90	3.25	5.7	5895	40.8	.0410	3.3	.0377	6.7
V367 Sct	6.29	3.54	6.8	6313	49.7	.0394	4.8	.0375	7.6
U Sgr	6.74	3.59	7.1	6162	55.2	.0390	5.6	.0378	7.6
DL Cas	8.00	3.57	7.0	5801	60.9	.0398	5.6	.0385	7.5
S Nor	9.75	3.65	7.4	5731	68.4	.0409	5.6	.0391	8.0
TW Nor	10.79	3.46	6.5	5572	58.1	.0448	3.4	.0396	8.0
VX Per	10.89	3.77	7.9	5943	73.0	.0415	5.6	.0392	8.8
SZ Cas	13.62	3.93	8.8	6015	85.7	.0424	6.1	.0398	9.8
VY Car	18.93	4.05	9.5	5309	126.3	.0416	9.7	.0418	9.4
T Mon	27.02	4.27	10.8	5224	168.1	.0426	11.8	.0433	10.6
RS Pup	41.38	4.35	11.4	5373	174.2	.0495	7.6	.0449	13.0
SV Vul	45.04	4.48	12.4	5018	232.0	.0457	12.8	.0461	12.2

All these Cepheids which are listed by Sandage and Tammann (1969) or van den Bergh (1977) are in galactic clusters or have their distance known by some other method. At least all the cluster Cepheids need their distance modulus increased by $0\overset{m}{.}26$, and, not knowing what to do for the other six, they also have been assumed more luminous. The six not in galactic clusters are: SU Cas in front of a reflection nebula, UY Per, VY Per, VX Per, SZ Cas, in the dubious h + χ Perseus association and RS Pup discussed recently by Eggen. While the h + χ Perseus cluster may not exist, Turner (1977) now proposes that at least one of the questionable Cepheids, UY Per, may be in one of the clusters King 4 or Czernik 8. A glance at the table shows that the pulsation masses are still anomalously low even with most of the Iben suggested distance increase. Evolution and theoretical masses always agree well.

But I don't want you to dwell much on this table, because there is a further necessary change in the T_e values. Dean, Warren and Cousins (1978), in a still unpublished paper, list newly determined color excesses for 16 of these 20 Cepheids. Unfortunately, the four not listed by DWC, CEb Cas, CEa Cas, CS Vel, and V367 Sct, are all in clusters. Anyway these new color excesses, based on fitting tracks in the B-V, V-I, and B-V, R-I planes for each pulsating variable are significantly less than those older ones determined by Kraft based on photometry of the G band. Therefore, the intrinsic colors are redder and the Cepheids are cooler. Figure 1 gives these 16 Cepheid mass ratios M_T/M_{ev} with both the improved distance and T_e values. Actually, the T_e used for U Sgr and S Nor is given by Pel (1978) who obtained them by his multicolor photometry. I have also used for the Q_0 values

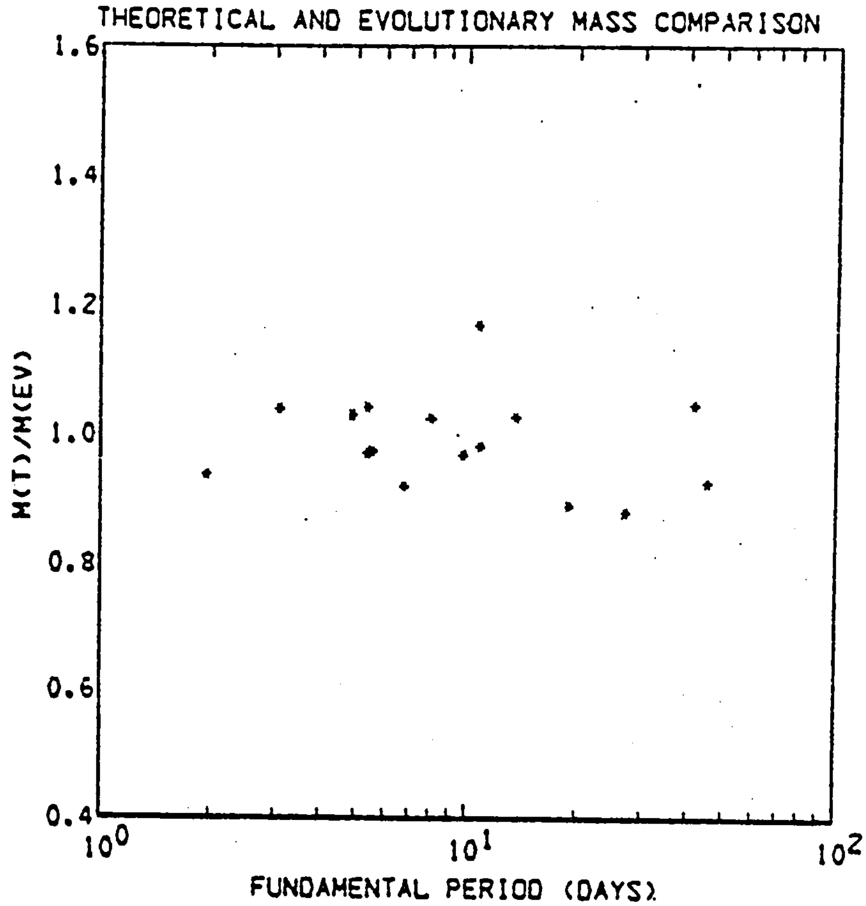

Fig. 1. Theoretical to evolutionary mass ratio versus fundamental period for 16 Cepheids. The new distance, temperature, and Q_0 values are used.

those from helium enriched models which I have advocated. One now sees again that the theoretical masses are very close to the evolutionary masses, the mean ratio being 0.99 ± 0.07.

An important point here is that one can get a theoretical mass, based on only Π_0 and an approximate T_e value, for any Cepheid if one believes the theoretical evolution mass-luminosity law and the theoretical pulsation constants. Later we will use these theoretical masses for Cepheids not in clusters to compare with masses from other methods.

Figure 2 on the same scale plots the ratio of pulsation to evolution masses for 16 Cepheids with the new observational data, but with homogeneous (not helium enriched) envelope Q_0 values. The Cogan mass anomaly has disappeared as predicted by Iben and Tuggle and Fricke, Stobie, and Strittmatter. The mean ratio M_Q/M_{ev} was 0.70 according to Stobie (1974), and it becomes 0.84 ± 0.17 if the new distance scale is used alone and here 0.97 ± 0.25 when both the new distances and T_e values are used.

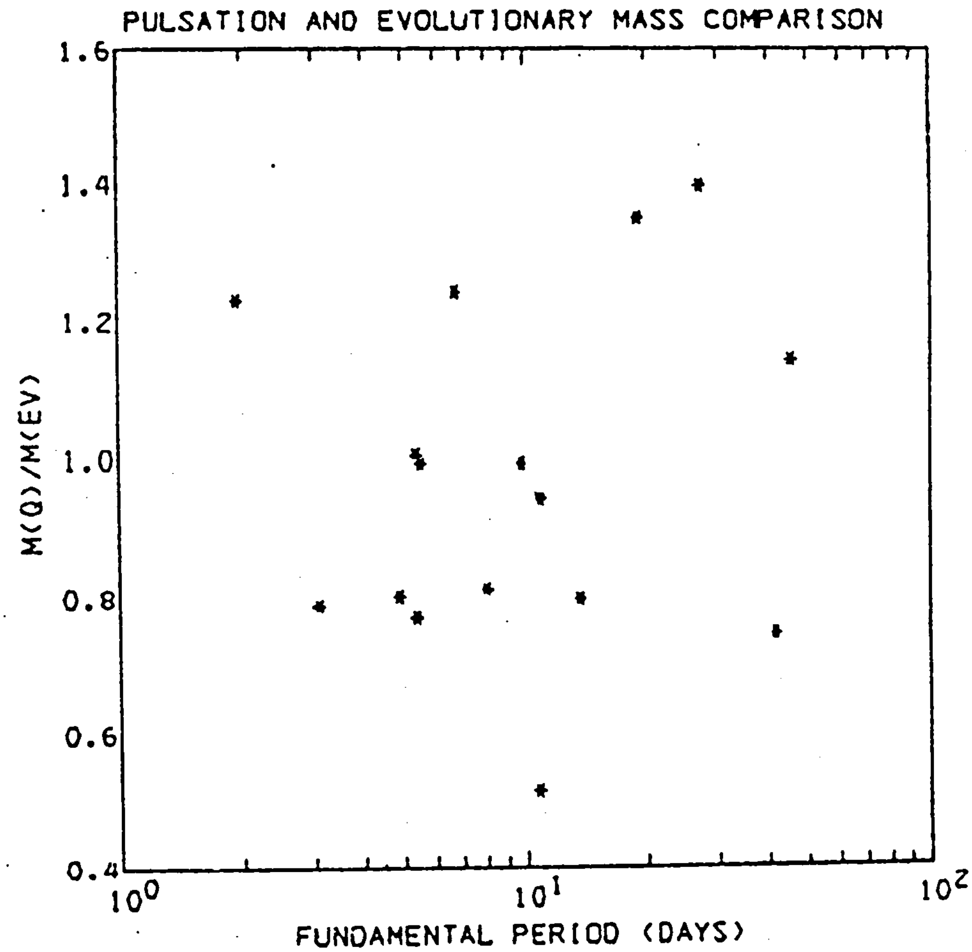

Fig. 2. Pulsation to evolutionary mass ratio versus fundamental period for 16 Cepheids. The new distances and temperatures, but homogeneous Q_0 values are used.

We must consider what happens if the inhomogeneous model Q_0 values are used, because if they solve the bump and beat mass anomalies they must not be excluded for these 16 calibrating Cepheids. Indeed, 5 of these 16 Cepheids have bumps in their light and velocity curves. The ratio M_Q/M_{ev} is given in Figure 3, and the mean ratio is 1.07 ± 0.27. Inclusion of the surface helium enriched model Q_0 values here actually reverses the pulsation mass anomaly, but maybe Schmidt's (1978) new cluster distances, based on Hβ photometry of the B stars will now decrease distances to these calibrating Cepheid clusters.

Let me briefly discuss the problem of converting mean colors to T_e. The latest word by Bell and Parsons (1974) seems to be that the venerable Kraft relation

$$\log T_e = 3.886 - 0.175 \, (B-V)$$

is still to be used. Therefore, that is the origin of T_e from unreddened DWC colors for the theoretical and pulsation masses. Note in Figure 4, however, the problem is still not solved. We plot the unreddened DWC colors versus the Pel (1978) log T_e for 44 Cepheids where both values are known. The discrepancy has been claimed by DWC to indicate that Pel has determined too little reddening from his multicolor work and obtained temperatures too low by a few percent or about 150 K. Cogan tells me that the points in this slide are better fit with the Flower (1977) formula, but it really seems that Pel's reddenings are too small by 0.05 - 0.08 in B-V. I don't worry too much about this, though we still need refinement of T_e values.

Bump masses, discussed mostly by Fricke, Stobie, and Strittmatter (1972), are based on the phase of the light curve bump for those Cepheids with periods between 5.5 and 13 days. Christy, and parti-

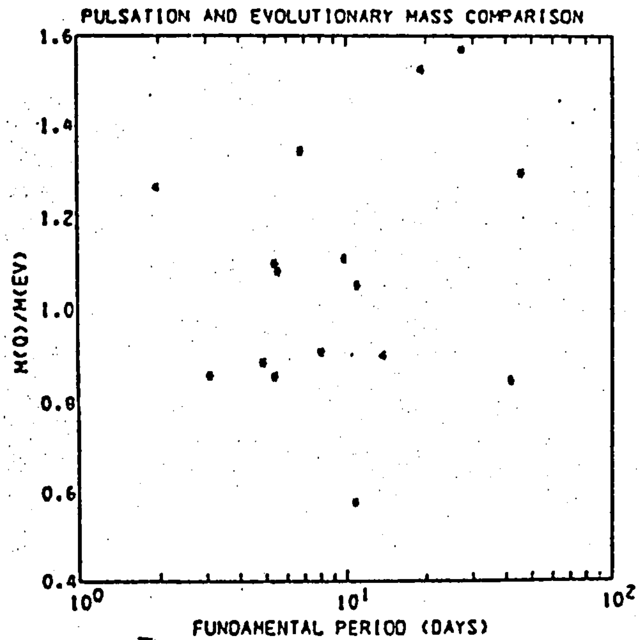

Fig. 3. Pulsation and evolutionary mass ratio versus fundamental period for 16 Cepheids. New distances, temperatures, and Q_0 values are used.

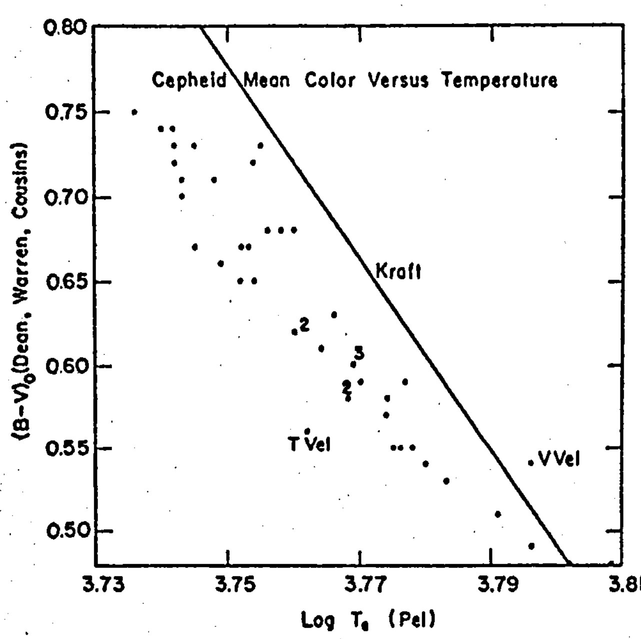

Fig. 4. Dean, Warren and Cousins unreddened colors are plotted versus Pel's log T_e values for 44 Cepheids. The Kraft relation is shown.

cularly Stobie, found that the bumps were at a phase way too late and often invisible for a 10 day Cepheid unless the mass was only like 4-5 $M_\odot$. This is about 70 percent of that given by evolution theory, and is the basis for Stobie's (1969b) different, anomalous mass luminosity law.

Fricke, Stobie, and Strittmatter (1972) related the period times the bump phase to the equilibrium radius by using Stobie's theoretical models. The radius can then be put into the period - mean density formula to get the mass. Masses averaged 0.75 the evolution masses.

Simon and Schmidt (1976) discovered that the bump phase correlated with Π_2/Π_0. If we use linear theory for Π_2/Π_0 and evaluate the sound echo time for the models we get the lines in Figure 5. Echo time is like bump phase, because if the echo time is long (in the equilibrium model) then the bump generated by nonlinear mechanisms,

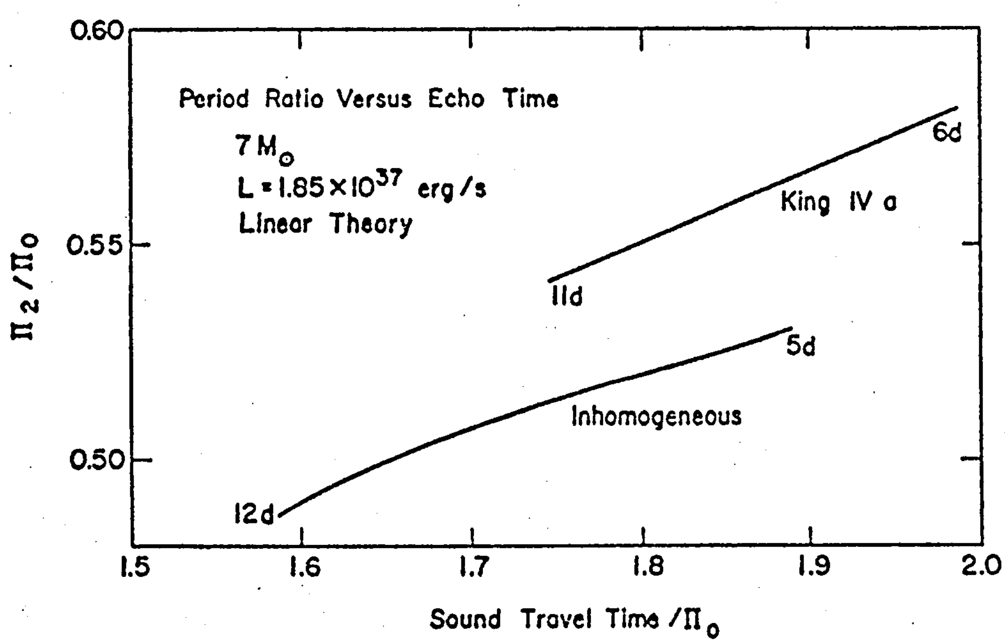

Fig. 5. Period ratio Π_2/Π_0 versus echo time from linear theory for 7 $M_\odot$ with evolution theory luminosity. Both homogeneous and inhomogeneous models are shown.

at minimum radius, will also be late in a full amplitude pulsating model. This figure shows that there is not a universal relation between echo time and Π_2/Π_0, but the relation is much as Simon and Schmidt discovered using Stobie's nonlinear models. Note that the inhomogeneous models give earlier echo times even though these linear theory times are still too late.

Figure 6 gives some results I spoke about in IAU Symposium 80. Here Π_2/Π_0 is plotted against Π_0. First look at the homogeneous King IVa 7 $M_\odot$ models at evolution luminosities. The region of Π_2/Π_0 between 0.46 and 0.53 is never reached for these linear theory models. Note the blue B, and red R edges. If we retain the luminosity but reduce the mass to 5 $M_\odot$, then bumps appear nearer the correct phase, where the Π_2/Π_0 ratio equals 0.50 at 10 days with the bump near the velocity and light curve peaks. The unpublished Carson C312 opacities give improve-

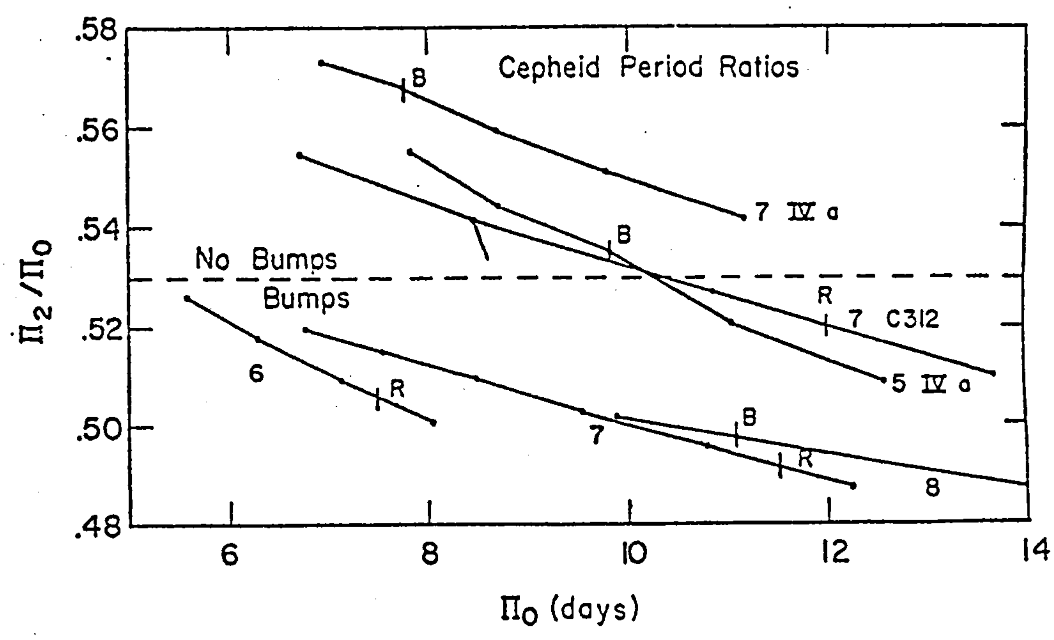

Fig. 6. The period ratio Π_2/Π_0 versus Π_0 for Cepheid models in the instability strip. Approximate blue (B) and red (R) edges are indicated. Inhomogeneous models at 6.0, 7.0, and 8.0 $M_\odot$ show the appropriate Hertzsprung relation.

ment for homogeneous compositions, but really not enough. These opacities are considered very uncertain. Finally, Y = 0.75 surface layers give the proper Π_2/Π_0 progression - that is the Hertzsprung progression - with fundamental mode period.

Above 8 or 9 $M_\odot$ there is not enough time in Cepheid blue loop evolution to have a Cepheid wind enrich the surface layers. Bumps then might be expected at long periods for these homogeneous variables, and some are actually seen. For example, bumps are on light curves for RU Sct (19.7d), UZ Pup (23.2d), and on X Pup (26.0d). If there is mass loss way back when the star is a B star, we might have a case, such as studied by Sreenivasan and Wilson (1977) of a luminosity for a 10-15 $M_\odot$ Cepheid with a mass of perhaps 25 percent less. In that case the Π_2/Π_0 would always be less than 0.46, and no bumps would occur.

Let me turn now to the double-mode Cepheids whose masses have been studied theoretically bv Petersen (1973), King, et al. (1975) and by Cox, King and Hodson (1977) and more observationally by Rodgers (1970), Fitch and Szeidl (1976), and Schmidt (1972, 1974). With only the ratio Π_1/Π_0, and we are sure that these periods have been correctly identified, the mass of a variable like U Tr A can be determined.

Figure 7 has the whole story for this Cepheid at Π_0 = 2.568 days. The homogeneous King IVa composition mass is like 1.2 $M_\odot$, but the several ways of getting enriched surface layers give larger masses. A thin layer with Y = 0.75 down to 70,000 K and the Population I King IVa composition interior gives 4 $M_\odot$. This slide has been expanded

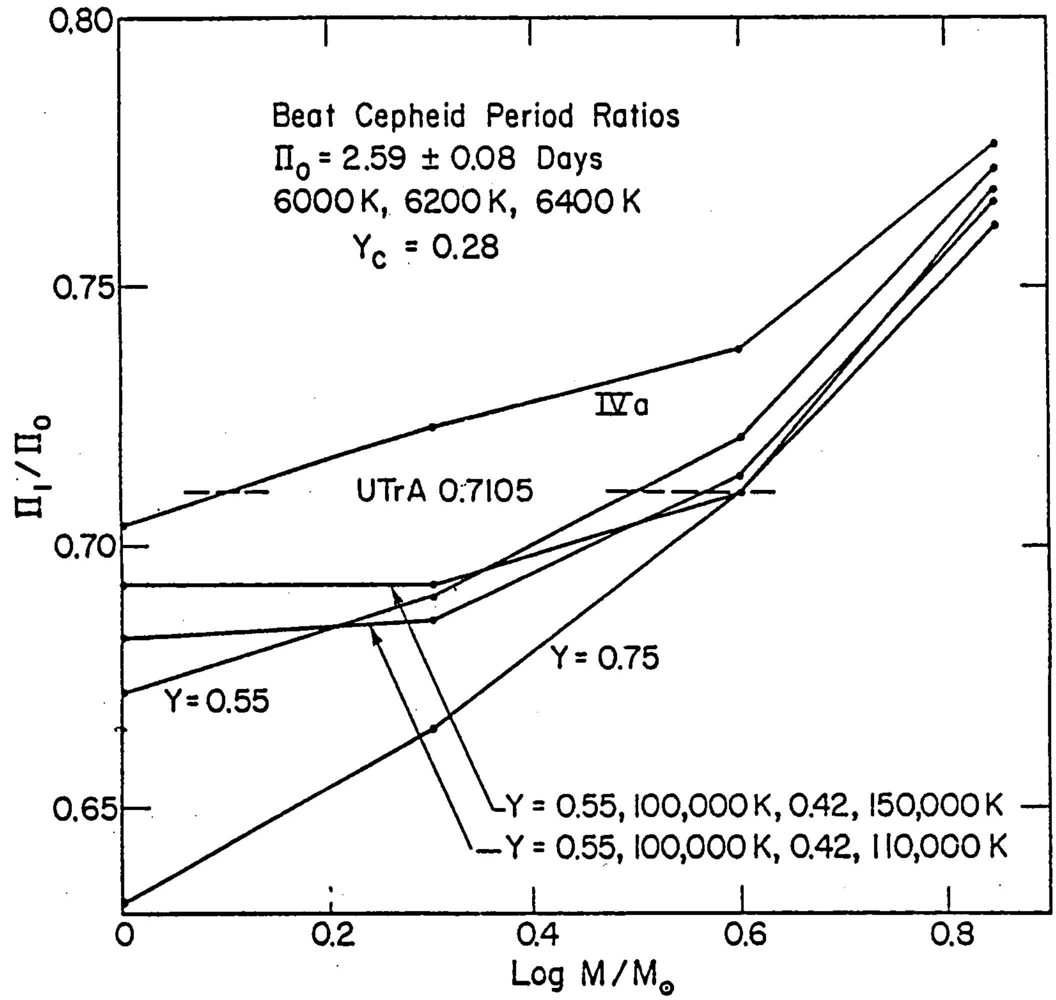

Fig. 7. The period ratio Π_1/Π_0 versus mass is shown for U Tr A models at T_e of 6000 K, 6200 K, and 6400 K. The dashed line gives the observed period ratio which indicates masses like 4 $M_\odot$ for surface Y between 0.55 and 0.75.

since I showed it at the IAU Symposium 80. I now believe that a surface Y of 0.55 is large enough if one allows the inverted μ gradient instability to mix some helium down to deeper levels like 100,000 K. These enrichments are all possible according to Cox, Michaud, and Hodson (1978) considering the age of the Cepheid and the same wind and wind composition per surface area as for the sun.

Our newest mass results are those given by Wesselink radius measurements. Balona (1977) sent me his recent tabulation, and that together with the Evans (1976) and Scarfe (1976) data, allow

69 mass determinations. The idea is simple - plug the radius and period into the period mean density relation with known Q_0 to get the mass.

Figure 8 gives the ratios of the Wesselink radius masses to theoretical masses for Q_0 values applicable for homogeneous models. The ratio is 0.93 for periods less than 10 days and 0.60 above. The tremendous scatter was not expected by me because Balona advertises errors of less than 10 percent in the radius. The Wesselink radius masses go like $R^{2.44}$. Better radii, accurate to 10 or less percent, are needed

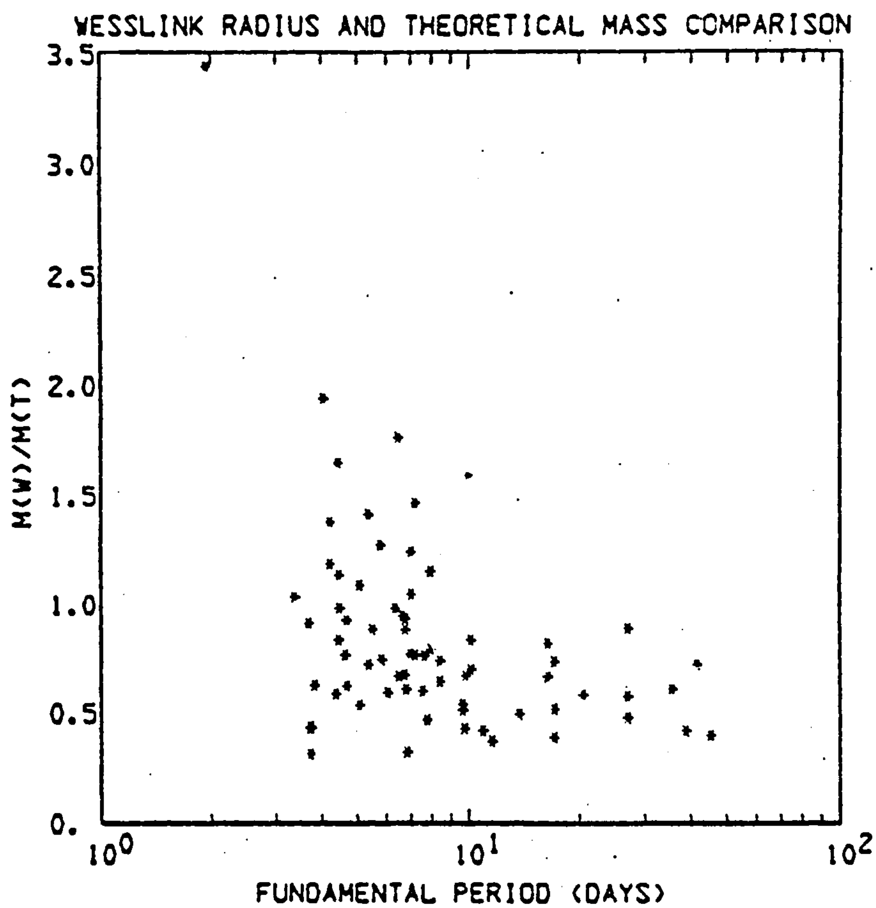

Fig. 8. The ratio of the Wesselink radius mass to theoretical mass is plotted versus period for 69 cases. Evolutionary masses use X = 0.70 and Y = 0.28, and Q_0 values are based on the homogeneous composition X = 0.602, Y = 0.354.

Figure 9 shows that the anomalous masses are partially cured with the Q_0 values for the inhomogeneous models. For these, Q_0 is up by 5-10 percent. For shorter periods, the ratio M_W/M_T is 1.02. The longer period Cepheids show a mass anomaly (ratio equal 0.70) which is not new - see Schmidt (1976) for example. I speculate that these Cepheids have had mass loss in the B star stage which does not affect their luminosity as much as their mass.

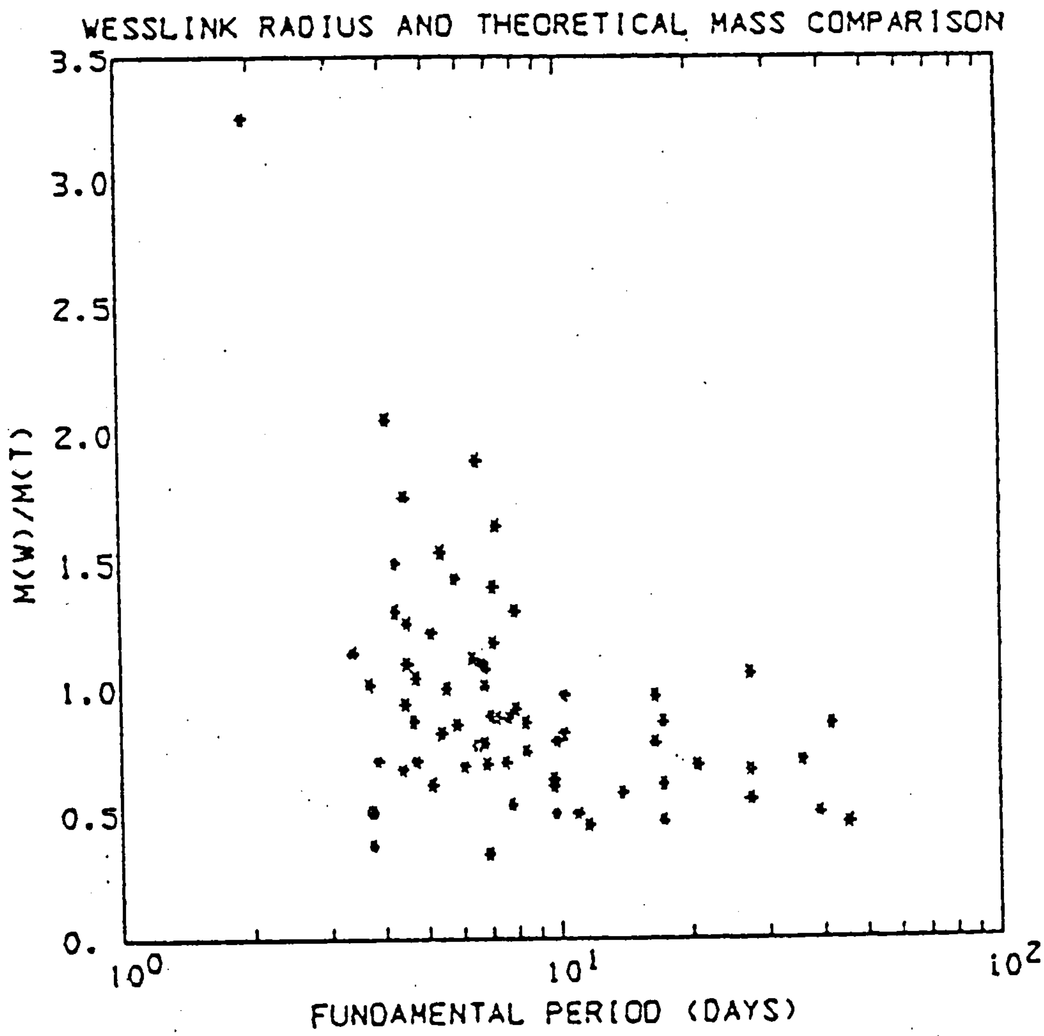

Fig. 9. The ratio of the Wesselink radius mass to theoretical mass is plotted versus period for 69 cases. Evolutionary masses use X = 0.70 and Y = 0.28, and the Q_0 values come from the inhomogeneous models with Y_S = 0.75.

Earlier I pointed out that use of a photometrically measured gravity cannot get an accurate mass. But the preceeding theoretical and evolutionary masses can give good log g values which can be compared to Pel's measurements. The difference

$$\log g_P - \left(\frac{\log g_T + \log g_W}{2}\right)$$

is + 0.02 ± 0.20 for the homogeneous models and - 0.02 ± 0.20 for the inhomogeneous models, where P, T, and W are the Pel, theoretical, and Wesselink radius values of g. The very close agreement is not unexpected since

$$\frac{M_W}{R_W{}^2} \sim \frac{R_W^{0.44}}{\Pi_0^{1.38}} \qquad \frac{M_T}{R_T{}^2} \sim \frac{T_e^{0.28}}{\Pi_0^{1.06}}$$

Evidently the comparison of log g values with theory cannot indicate whether the inhomogeneous enriched helium envelopes are more realistic than the homogeneous ones.

My Wesselink radius mass paper (Cox 1979) has more, for example the possibility of overtone pulsation and the use of the Barnes et al. radii, which I will not cover here.

As a final point let me compare the masses of U Sgr, S Nor, and V367 Sct, all cluster Cepheids which are determined by at least four ways. Table 2 gives the usual low values for the Fricke, Stobie, and Strittmatter bump, and Cox, et al. beat masses, using homogeneous models. Also in the table I have masses for T Mon, RS Pup, and SV Vul showing the Wesselink radius mass anomaly for all but RS Pup.

My conclusion is that all previous mass anomalies can be considered solved by distance, temperature, and inhomogeneous model improvements. Wesselink radii urgently need improvement. If there is no systematic radius error due to strange limb darkening effects, etc., there is a

TABLE 2

CEPHEID MASSES

Cepheid	M_{ev}	M_T	M_Q	M_Φ	M_B	M_W
U Sgr*	7.1	6.5	9.5	4.0	-	6.6
S Nor*	7.4	7.2	8.2	3.8	-	3.6
V367 Sct**	6.9	7.3	5.6	-	2.3	-
T Mon	10.8	9.6	17.0	-	-	5.3 - 10.2
RS Pup	11.4	12.0	9.6	-	-	10.3
SV Vul	12.4	11.5	16.0	-	-	5.3

* The bump masses are given by FSS.

** The V367 Sct theoretical and pulsation masses are based on the van den Bergh T_e and not the new T_e from Dean, Warren, and Cousins unreddened colors. The homogeneous model beat mass is from Cox, et al.

persistent mass anomaly for the longer period Cepheids that may really indicate early mass loss. More observations are suggested for galactic cluster distances, to improve pulsation masses, for Wesselink radii, to improve Wesselink radius masses, and for Cepheid spectra and even for the solar wind, to confirm if inhomogeneous models with surface helium enrichment really exist.

REFERENCES

Balona, L. A. 1977, Mon. Not. R.A.S., 178, 231.

Becker, S. A., Iben, I., and Tuggle, R. S. 1977, Ap. J., 218, 633.

Bell, R. A. and Parsons, S. B. 1974, Mon. Not. R.A.S., 169, 71.

Christy, R. F. 1966, Ann. Rev. Astr. Ap., 4, 353.

Cogan, B. C. 1970, Ap. J., 162, 139.

Cox, A. N. 1979, Ap. J. (in press).

Cox, A. N., Deupree, R. G., King, D. S., and Hodson, S. W. 1977, Ap. J. Lett., 214, L127.

Cox, A. N. and Hodson, S. W. 1978, Proc. IAU Symposium 80.

Cox, A. N., King, D. S., and Hodson, S. W. 1977, Ap. J., 212, 451.

Cox, A. N., King, D. S., and Tabor, J. E. 1973, Ap. J., 184, 201.

Cox, A. N., Michaud, G., and Hodson, S. W. 1978, Ap. J., 222, 621.

Cox, J. P. 1963, Ap. J., 138, 487.

Cox, J. P., Cox, A. N., Olsen, K. H., King, D. S., and Eilers, D. D. 1966, Ap. J., 144, 1038.

Cox, J. P., King, D. S., and Stellingwerf, R. F. 1972, Ap. J., 171, 93.

Dean, J. F., Warren, P. R., and Cousins, A. W. J. 1978, preprint.

Evans, N. R. 1976, Ap. J., 209, 135.

Faulkner, D. J. 1977, Ap. J., 218, 209.

Fitch, W. S. and Szeidl, B. 1976, Ap. J., 203, 616.

Flower, P. J. 1977, Astr. and Ap., 54, 31.

Fricke, K., Stobie, R. S., and Strittmatter, P. A. 1972, Ap. J., 171, 593.

Iben, I. 1971, Ap. J., 168, 225.

Iben, I. and Tuggle, R. S. 1972a, Ap. J., 173, 135.

Iben, I. and Tuggle, R. S. 1972b, Ap. J., 178, 441.

Iben, I. and Tuggle, R. S. 1975, Ap. J., 197, 39.

King, D. S., Cox, J. P., and Eilers, D. D. 1966, Ap. J., 144, 1069.

King, D. S., Cox, J. P., Eilers, D. D., and Davey, W. R. 1973, Ap. J., 182, 859.

King, D. S., Hansen, C. J., Ross, R. R., and Cox, J. P. 1975, Ap. J., 195, 467.

Pel, J. W. 1978, Astr. and Ap. Suppl., 31, 489.

Petersen, J. O. 1973, Astr. and Ap., 27, 89.

Rodgers, A. W. 1970, Mon. Not. R.A.S., 151, 133.

Sandage, A. R. and Tammann, G. A. 1969, Ap. J., 157, 683.

Scarfe, C. D. 1976, Ap. J., 209, 141.

Schmidt, E. G. 1972, Ap. J., 176, 165.

Schmidt, E. G. 1974, Mon. Not. R.A.S., 167, 613.

Schmidt, E. G. 1976, Ap. J., 203, 466.

Schmidt, E. G. 1978, Ap. J. Lett. (in press).

Simon, N. R. and Schmidt, E. G. 1976, Ap. J., 205, 162.

Sreenavasen, S. R. and Wilson, W. J. F. 1977, Astr. and Space Sci., 53, 193.

Stellingwerf, R. F. 1975, Ap. J., 199, 705.

Stobie, R. S. 1969a, Mon. Not. R.A.S., 144, 485.

——————— 1969b, Mon. Not. R.A.S., 144, 511.

Stobie, R. S. 1974, in IAU Symposium 59, Stellar Instability and Evolution.

Turner, P. G. 1977, Pub. ASP, 89, 1977.

van den Bergh, S. 1977, in IAU Colloq. 37, Decalages vers le Rouge et Expansion de l'Univers.

Discussion

Wesselink: Would there be any mass loss in the evolution of a Cepheid, and would this affect your conclusions?

A. Cox: The standard answer to that question as given by stellar evolution people is that there may be mass loss while the star is a B star, but if it loses more than 10% of its mass in the yellow giant and red giant stage, it will not go through a blue loop. This was a very amazing result that came out about 1970 by Refsdal, Roth and Weigart, and they stick to it. Even recent results for a 15 $M_\odot$ star by Sreenivasan and Wilson indicate the same thing. You might have a little bit of mass loss from the red giant as it comes back through the blue loop, but not enough to explain any of these anomalies. You can get 10%, but that's all.

Hillendahl: On the basis of believing in models (cf. PASP 82, 1231, 1970) you would predict that any time a star evolves across any instability strip in the H-R diagram -- not just the Cepheid instability strip -- there would be mass loss and helium enrichment in the atmosphere. I wonder if there is any possibility of testing that with the data?

A. Cox: There are two or three ways. You don't see the mass loss itself because it is very low, about 10^{-10} $M_\odot$/yr, and there is no way of easily measuring that low a mass loss. But there is a possibility of looking at the spectra to see if this enriched helium really exists. But it turns out that it's very hard to find helium in a yellow star, like the Sun or a Cepheid. The other thing to do is to study the solar wind to see if we understand what is happening. The solar wind is deficient in helium, so the solar atmosphere is probably being enriched. But you don't notice it because the solar atmosphere is very deep, comprising about 1% of the mass of the Sun. So it mixes in and you never notice the enrichment. For the Cepheids, we need to enrich about 10^{-4} or 10^{-5} of the mass, which is the mass of the convection zone.

J. Cox: Do you know if there is any observational evidence for a Cepheid wind?

A. Cox: No. The only reason you know about the solar wind is that you are sitting here in it. But if you take the relative size of the Cepheid and the Sun and let the solar wind blow from the Cepheid (which we call a Cepheid wind), you can enrich that very thin layer in the lifetime of the blue loops.

Scuflaire: If the external layers of the star are enriched with helium, do you get an instability?

A. Cox: Yes. Unfortunately, there is a problem, because if you have an inverted μ -gradient it is very likely to be unstable. We are working on that problem also in a two-dimensional hydrodynamical calculation to see what will happen. We fully expect that at first the layer will mix and not persist. If that is really true, there will be no explanation for the bump and the beat Cepheids. So at the moment we are trying to see if there is some way of stabilizing that layer, perhaps by pulsation or by the flow of hydrogen through that layer. Your question could be unanswered for the next 50 years. It is a question of whether you believe the period of Cepheids, indicating a helium enriched layer, or whether you believe from linear theory that the layer will mix.

Keller: You showed a diagram that put beat Cepheids and bump Cepheids in the same class by looking at the period ratio, and that all you had to do was get below a certain period ratio to get bumps. There are several ways of doing this. One is mass loss, another enrichment. It seems to me that something we should do if we don't like these two is to look for any other means for adjusting the structure of the star to get below that period ratio. Have you thought of any other scheme that might be done?

A. Cox: There are two or three points on that. Faulkner in his article in 1977 proposed that when these stars have two modes at once, the period ratio is not correctly given by linear theory. In other words, what we measure is not what we think we measure. But there hasn't been any further pursuit of that. Cogan proposed that a very deep helium convection zone changes the period. But that doesn't seem to work. In a recent letter from Cogan, he stated that he doesn't believe it will work now. Castor, in a private conversation, suggested that the opacities are wrong. After all, you can always change your opacities in astrophysics to solve your problems. That seems out of the question, because we have two widely disparate chemical compositions giving opacities which give similar results. (Dave King will talk about that.) It seems that the only thing that will significantly change the structure is a change in the equation of state.

Connolly: You said higher mass stars could experience mass loss. Could you give a range in Cepheid periods over which this might occur?

A. Cox: Sreenivasan and Wilson have done a study that shows that higher mass stars can lose 25-30% of their mass as B stars; and when they become yellow and red giants, they are undermassive. If they were to lose another

10%, then they would not blue loop. The same lack of blue loops holds for the lower mass stars that do not experience the early mass loss. The answer to your question is that there might be 25% undermassive stars for masses > 10 or $12\ M_\odot$, periods above 10 or 15 days.