

**ON THE ORIGIN OF PERIOD CHANGES IN RR LYRAE STARS**

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## I. INTRODUCTION

Since the work of Martin (1938, 1942), it has been known that the pulsation periods of many RR Lyrae stars vary at a typical rate of a few parts in  $10^5$  every 100 yrs. It was originally hoped that the observed period changes would provide information about the rate and direction of the evolution of RR Lyrae stars in the HR diagram and thus of horizontal-branch stars in general, particularly those in globular clusters. However, the comparison of the observed rates of period change with the rates predicted from evolutionary models did not substantiate this expectation in two major respects: i) in several globular clusters increasing and decreasing periods occur with nearly equal frequency, while the models generally spend much of their horizontal-branch lifetime evolving in one direction, and ii) the observed rates of period change typically exceed the evolutionary ones by roughly an order of magnitude (cf. Iben and Rood 1970). For these reasons many authors have suggested that most period changes are due to some sort of "noise" or random fluctuations superimposed on the mean evolutionary changes in the stellar structure (Ledoux and Walraven 1958; Iben and Rood 1970; Schwarzschild 1970, 1973; Szeidl 1975). A fundamental problem therefore is to determine the origin of this noise.

The available observational material on period changes sets several constraints on the characteristics of the noise. First, the typical size of the period changes requires that the noise should be able to produce variations in the stellar radius of a few parts in  $10^5$  in both directions. Second, such fluctuations should occur approximately every 100 yrs. Third, the noise should produce both relatively slow and fast changes in the stellar radius, since both continuous and abrupt period changes apparently occur in RR Lyrae stars.

The last point is somewhat controversial and deserves a little more attention. In most cases, phase-shift diagrams can be fitted both with parabolas (implying a continuous period change) or with straight lines (implying an abrupt period change) as shown, for instance, by Goranskij, Kukarkin and Samus (1973). It is worth emphasizing that the terms "slow" (continuous) and "fast" (abrupt) are relative to the time interval spanned by the observations  $\Delta t_{\text{obs}}$ , which is typically around 50 to 75 yrs. A fluctuation in the stellar structure is characterized by its time duration  $\tau$ . If  $\tau \geq \Delta t_{\text{obs}}$ , a "continuous" period change is produced, while if  $\tau \ll \Delta t_{\text{obs}}$ , one has an "abrupt" period change. Among the best observed cases (Szeidl 1975), RR Leo has a "continuous" period change, implying  $\tau \geq \sim 60$  yrs, while RR Gem has suffered an "abrupt" period change. Only an upper limit,  $\tau < \sim 4$  yrs, can be set for the duration of the period change in RR Gem because of the limited time resolution of the observations. From the existing observations it is difficult to decide whether "continuous" and "abrupt" period changes are two entirely different kinds of phenomena or whether there is a continuous spectrum of  $\tau$  values from, say, a few years to a few decades. In the following discussion we will assume the second point of view.

Some period changes may be related to the Blazhko effect (variable pulsation amplitude generally suspected to originate from some sort of modal interference). RR Lyrae stars exhibiting the Blazhko effect have on average larger period changes than non-Blazhko variables. For instance, RR Gem passed through a Blazhko phase just around the epoch of its abrupt period change (Szeidl 1975). In a non-linear oscillating system the period marginally depends on the amplitude of the oscillation. This dependence is a second order effect and is shown, for example, by the ordinary pendulum. Consequently amplitude fluctuations in RR Lyrae stars might also lead to

period fluctuations whose cumulative effects might produce observable period changes. A quantitative check of this possibility is extremely difficult. In principle, one could look at the behavior of the period during the growth to full amplitude pulsation of non-linear models, but, since we are looking for an effect in the fifth decimal place, such models are currently far from being sufficiently accurate. However, it is unlikely that the Blazhko effect can account for the bulk of the observed period changes. RR Lyrae stars with the Blazhko effect are normally excluded when deriving period changes for the variables in a globular cluster. We must thus look elsewhere for the main source of the required noise.

The most obviously "noisy" process occurring in the stellar interior is convection, and in this respect RR Lyrae stars present quite a complex picture: i) RR Lyrae stars possess a convective helium-burning core, ii) they have a convective zone just below the photosphere due to hydrogen and helium ionization, and, finally, iii) around the fully convective core most RR Lyrae stars have a semiconvective zone (SCZ) (Castellani, Giannone and Renzini 1971a,b; Renzini 1977). Although semiconvection has been included here under the general category of convection, we emphasize that there are fundamental differences between semiconvection and ordinary convection. For example, the composition within a SCZ is not uniform. Furthermore, the mass motions needed to maintain the proper composition distribution within a SCZ are always assumed to carry a negligible amount of the outward flux.

There is no doubt that the structure of a convective region can fluctuate in time because of the inherent nature of the convective process. Such fluctuations can be described in terms of a statistical fluctuation in the number of "convective elements" present at a given time in the

convective region or in terms of a fluctuation in the convective pattern, i.e., in the geometry of the convective cells, columns, currents and so on. Variations of this kind might be reasonably expected to produce slight variations in the convective efficiency and therefore in the average temperature gradient. This in turn would affect the structure of the convective zone and hence of the whole star. In the framework of the mixing-length theory a fluctuation in the convective efficiency can be treated as a fluctuation in the mixing length, but the mixing-length theory itself can say nothing about the magnitude or the characteristic time scale of such fluctuations. However, in spite of these limitations, both the convective core and the outer convective zone can be ruled out as the region where the noise causing the observed period changes originates.

Convection within the core is so efficient (Schwarzschild 1958) that even a huge fluctuation in the mixing length does not significantly perturb the stellar structure. In other words, varying the mixing length within a range much wider than the "canonical" one (which is between  $1/2$  and  $2$  pressure-scale heights) has a completely negligible effect on the actual temperature gradient within the convective core. Since the actual temperature gradient always remains very close to the adiabatic-temperature gradient, it would be difficult to understand how core convection could generate the required noise.

This kind of argument does not, however, apply to the outer convective zone, where sizable departures from the adiabatic-temperature gradient can occur and where fluctuations in the convective efficiency might therefore affect the stellar radius. Such fluctuations in the convective efficiency could, in principle, produce period changes. However, the characteristic time scale of convection in the outer layers is only a few hours, and consequently it is not clear how slow changes in the period could be produced.

As we have seen, period changes with a duration on the order of several decades or more are observed. Furthermore, the structure (thickness, degree of superadiabaticity, etc.) of the outer convective zone in RR Lyrae stars changes dramatically in going from the blue to the red edge of the instability strip. If the observed period changes were due to fluctuations in the outer convective zone, the typical period change should increase dramatically from the blue to the red edge, i.e., with decreasing effective temperature or increasing period. This trend is not in fact observed. There is no obvious correlation between the period changes and the location of the RR Lyrae stars within the instability strip. For these reasons the outer convective zone would also seem to be an unlikely source of the required noise.

There remains the possibility that the noise is associated with the SCZ surrounding the fully convective core. We have analyzed this possibility in detail (Sweigart and Renzini 1978, hereafter SR), and in the following section we will summarize the most important concepts and results.

## II. SEMICONVECTION AND PERIOD CHANGES

During the initial horizontal-branch evolution the convective core grows in mass as a result of convective overshooting. (Castellani et al. 1971a; Renzini 1977). Once the helium abundance  $Y_c$  within the convective core falls below  $\approx 0.7$  (Sweigart and Gross 1974, 1976), a SCZ develops around the convective core in order to maintain convective neutrality (i.e., equality of the radiative- and adiabatic-temperature gradients) at the convective-core edge (Castellani et al. 1971b). By definition a SCZ is a region where the chemical composition varies from point to point in a way that ensures convective neutrality. As a horizontal-branch star

evolves, the SCZ becomes more extensive until it typically contains  $\sim 0.1 M_{\odot}$  shortly before the end of core-helium burning. Some mixing process is therefore necessary in order to adjust properly the abundances of helium, carbon and oxygen throughout the SCZ at each stage of the evolution.

It is universally known that stellar convection still awaits a physically satisfactory treatment, and the case of semiconvection is even worse. Most efforts in this field pertain either to the formulation of algorithms suitable for treating SCZ's in stellar-model calculations (Robertson and Faulkner 1972) or to the proper definition of "convective neutrality", i.e., whether the Schwarzschild or Ledoux criterion is more appropriate. There is unfortunately very little theoretical understanding of how a SCZ actually manages to maintain convective neutrality. However, the composition distribution within a SCZ would not be expected to change continuously during the evolution. Rather, the composition changes probably result from many discrete mixing events occurring randomly throughout the SCZ (see SR). The typical size of such mixing events would have to be substantially smaller than the whole SCZ. Otherwise a large part of the SCZ would be mixed up, and the composition gradient would disappear, thus leading to a major departure from convective neutrality. During the evolution one would expect small parts of the SCZ to become occasionally unstable due to the changes in the interior structure caused by the central helium burning. The resulting convection within these small parts of the SCZ would persist until the composition distribution has been changed sufficiently to restore convective neutrality. These considerations suggest that the SCZ manages to remain close to neutrality through a series of relatively small mixing events, each of which perturbs the interior structure by slightly altering the composition distribution. Semiconvection should therefore be an inherently noisy phenomenon.

The mixing events can be characterized by their size  $\Delta M_r$ , duration  $\tau_{ME}$  and the time interval  $\Delta t_{ME}$  between them. In principle, a spectrum of  $\Delta M_r$ ,  $\tau_{ME}$  and  $\Delta t_{ME}$  values would be expected. The typical values of these quantities may vary during the evolution and thus may depend on  $Y_c$ . Without a theory of the semiconvective process there is no way to predict these typical values. However, one can use the observed properties of the period changes to obtain some insight into the characteristics of the mixing events and hence into the nature of the semiconvective process itself. The period changes could thus be one of the few available tools for probing the stellar interior.

By imposing canonical semiconvection (i.e., the requirement of convective neutrality both at the convective-core edge and within the SCZ), one can compute horizontal-branch sequences without a detailed knowledge of how the composition redistribution actually takes place within the SCZ. In these sequences the composition redistribution is usually treated as a continuous process, not as the result of discrete mixing events (Robertson and Faulkner 1972). However, the computed evolution is not significantly affected as long as neutrality is maintained on average throughout the SCZ. Sequences obtained in this manner provide information on the mean evolution of horizontal-branch stars and determine the average rate at which the composition distribution within the core is changing.

The composition redistribution within the core leads to the mixing of a substantial amount of helium (and therefore carbon and oxygen) through the SCZ and between the SCZ and the convective core. This mixing probably takes place in the following manner. As the helium burning gradually reduces  $Y_c$ , the opacity and hence the radiative-temperature gradient increase within the convective core (Castellani et al. 1971a). The resulting superadiabaticity at the convective-core edge leads to convective overshooting. Each overshooting event captures a small amount of helium from the inner layers of



the SCZ. In this way the convective core is able to restore convective neutrality at its edge. However, the carbon and oxygen which have been deposited into the inner layers of the SCZ cause these layers to become convectively unstable, and consequently the whole SCZ must then readjust. This readjustment is probably accomplished through a series of mixing events which have the cumulative effect of transferring the excess carbon and oxygen outward through the SCZ or, equivalently, of bringing additional helium into the SCZ. Castellani et al. (1971b) have referred to this process as "induced semiconvection" to emphasize how overshooting at the convective-core edge fundamentally drives the development of the SCZ. Each mixing event produces an irreversible change in the stellar structure, since helium is always carried inward and carbon and oxygen are always carried outward. During the horizontal-branch lifetime ( $\sim 10^8$  yrs), approximately  $0.1 M_{\odot}$  of helium are transferred inward through the SCZ and into the convective core. Therefore the number of mixing events of a given size that a star can have during its entire core-helium-burning phase is prescribed by standard horizontal-branch sequences if canonical semiconvection is on average fulfilled.

In view of this description of the mixing process we can distinguish three different types of mixing events, hereafter denoted as types A, B and C (see Fig. 4 of SR). An overshooting event at the convective-core edge will be referred to as a type A event. Mixing events confined to the SCZ will be labeled type B, while those responsible for bringing helium from the outer radiative region into the top of the SCZ will be termed type C.

The change in the composition distribution caused by a mixing event forces a readjustment in both the hydrostatic and thermal structure of a star. The hydrostatic readjustment, occurring on a dynamical time scale, leads to a concomitant change in the radius and hence to a change in the

pulsation period. However, the thermal readjustment proceeds on the much longer Kelvin time scale and hence would be unlikely to cause a detectable period change.

Along an evolutionary sequence the interior composition distribution changes as a result of i) the various types of mixing events and ii) the nuclear burning in the convective core and in the hydrogen shell. Each of these separate effects would by itself alter the pulsation period. According to our previous discussion, the period changes associated with the mixing events occur discretely in time. Whether or not these period changes appear to be continuous or abrupt depends on how  $\tau_{ME}$  compares with  $\Delta t_{obs}$ . Between mixing events the nuclear burning gradually alters the interior composition, an effect that can only give rise to a continuous period change. In general, the evolutionary period change  $\Delta P_{ev}$  between two consecutive horizontal-branch models can be written in terms of four components

$$\Delta P_{ev} = \Delta P_n + \Delta P_A + \Delta P_B + \Delta P_C \quad (1)$$

where  $\Delta P_n$  is the period change due to the nuclear burning alone and  $\Delta P_A$ ,  $\Delta P_B$  and  $\Delta P_C$  are, respectively, the cumulative period changes due to the type A, B and C mixing events.

In evolutionary sequences constructed according to canonical semi-convection the composition redistribution due to the mixing events is assumed to be tightly coupled to the composition changes due to the nuclear burning. Over long time intervals such a coupling must on average exist, since it is the nuclear composition changes which basically determine the growth of the SCZ. However, this coupling need not necessarily hold at every moment. In actual stars one might expect to observe the individual

period changes resulting from the separate mixing events and from the nuclear burning, provided that these individual period changes are large enough to be detected. If these individual period changes do not all have the same sign, then the sum  $|\Delta P|$  of all period changes observable over a time interval, as given by

$$|\Delta P| = |\Delta P_n| + |\Delta P_A| + |\Delta P_B| + |\Delta P_C| \quad , \quad (2)$$

will exceed  $|\Delta P_{ev}|$ . Because of this decoupling between the mixing events and the nuclear burning, one could have relatively large fluctuations in the period superimposed on the small evolutionary period changes. The only requirement is that the individual period changes must on average combine to give  $\Delta P_{ev}$ .

Using a stellar-evolution program, we have artificially imposed mixing events of types A, B and C on the composition profile of several suitably selected horizontal-branch models and in this way have determined how the period changes depend on the mixing events (see SR for details). It turns out that type A events produce negative period changes while type B and C events produce positive period changes. For the representative sequence studied by SR the net period change  $\Delta P_{ME} (= \Delta P_A + \Delta P_B + \Delta P_C)$  due to all types of mixing events differs in sign from  $\Delta P_{ev}$  during much of the evolution. According to Equation (1),  $\Delta P_{ME}$  must then also differ in sign from  $\Delta P_n$ . The observational requirement that both positive and negative period changes be produced is therefore satisfied.

The results of SR show that only small mixing events are needed to produce a typical period change  $|\Delta P/P|$  of  $3 \times 10^{-5}$ . At  $Y_c = 0.37$ , for example, a type A event involving overshooting by only  $0.008 M_\odot$  in  $\Delta M_r$  and 0.02 in  $\Delta \log$  pressure is sufficient to cause this period change. The

amount of helium captured by the convective core during such an event is only  $5 \times 10^{-6} M_{\odot}$ . As  $Y_c$  decreases, the pulsation period becomes appreciably more sensitive to the type A events. For the type B events the required width of the mixed region is also about  $0.008 M_{\odot}$  in  $\Delta M_r$  and 0.02 in  $\Delta \log$  pressure over a wide range in  $Y_c$ . Since the SCZ contains  $\sim 0.1 M_{\odot}$ , the condition that the type B events be small compared with the SCZ is amply fulfilled. The type C events turn out to be relatively unimportant. The sensitivity of the period changes to the mixing events can vary by a factor of 2, depending on the horizontal-branch parameters (i.e., the mass and composition).

The frequency of a typical period change  $|\Delta P/P|$  of  $3 \times 10^{-5}$  can be obtained from the following procedure. The period-change testing of horizontal-branch models outlined above determines the amount of helium mixed inward during a typical mixing event. At each point along an evolutionary sequence the average rate at which helium is mixed through the SCZ and into the convective core can be derived from the change in the composition distribution between consecutive models. Knowing this rate and the amount of helium involved in a single mixing event, one can compute the frequency of each type of mixing event and hence the period changes  $\Delta P_A$ ,  $\Delta P_B$  and  $\Delta P_C$ . The evolutionary period change  $\Delta P_{ev}$  can be readily obtained from the models. The value of  $\Delta P_n$  then follows from Equation (1). We find that the mixing events and the nuclear burning make roughly equal contributions to the observed period changes. The results for the sequence studied by SR indicate that a typical period change should occur roughly every 300 yrs while the observed value is roughly 100 yrs. In view of the sensitivity of the period changes to the model parameters and in view of the substantial theoretical and observational uncertainties, we consider the agreement between the present theory and the observations to be quite

satisfactory. For a more detailed discussion, see SR.

### III. CONCLUSIONS

Our proposed explanation for the observed period changes is based on the behavior of the SCZ within the core of an RR Lyrae star. General physical considerations suggest that the composition changes occurring within the SCZ during the horizontal-branch evolution result from many small mixing events, each of which slightly perturbs the pulsation period. Between mixing events the interior structure of an RR Lyrae star gradually changes because of the nuclear burning, and this effect should also contribute substantially to the observed period changes. A more detailed examination of this theory would require an improved theoretical understanding of the semiconvective process as well as the availability of better observational data on the sizes and frequencies of the period changes.

Our main conclusions may be summarized as follows:

1. Small mixing events within the core of an RR Lyrae star can produce changes in the pulsation period comparable with those typically observed.
2. These mixing events together with the nuclear burning between them can produce period changes of both signs.
3. The theoretically predicted frequency of the period changes is in satisfactory agreement with the observed value, although the uncertainties involved are substantial.

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## Discussion

Stellingwerf: If you look at calculations of linear pulsation analysis, you find that the period is very insensitive to actual conditions deep in the model.

Sweigart: That was the initial suspicion that one would have.

Stellingwerf: The period changes must reflect an overall redistribution in the structure of the model. Would this then produce a change in the luminosity or effective temperature?

Sweigart: That is basically what happens. When you change the interior composition by a small amount, you cause a small fluctuation in the location of the model in the H-R diagram and hence a small change in the radius. The result is a change in the period.

Stellingwerf: Can you see that?

Sweigart: No, you are talking about changes of a few parts in  $10^5$ .

Belserene: How quickly does the event take place?

Sweigart: That's an important question, because it relates to the question of whether or not the period changes are continuous or abrupt. There is unfortunately little theoretical understanding of how the semiconvection zone actually readjusts during the horizontal-branch evolution.

Belserene: Does your scenario give you some feeling for it?



Sweigart: We impose the mixing events on the models without assuming a time scale for these events, so we cannot say. Nevertheless, from the general requirement of convective neutrality within the semiconvective zone, we do know the overall extent of the composition redistribution that occurs within the core during the evolution. Since our calculations provide an estimate for the size of a typical mixing event, we can therefore predict the frequency of these events. However, the duration of a particular mixing event cannot be theoretically predicted at the present time. One might speculate offhand that, since a mixing event involves convective motions, the time scale of a mixing event might be relatively short, thus leading to an abrupt period change. However, the composition changes due to the nuclear burning between the mixing events are also very important in producing period changes, and these composition changes would only lead to continuous period changes. So, it may in fact be possible to produce both abrupt and continuous period changes, as are observed among the field RR Lyrae stars.

Aizenman: Wouldn't you expect that those changes are happening where some are giving positive and some negative contributions, such that the net effect comes back to the change in evolution?

Sweigart: That is a basic condition. When you construct an evolutionary sequence, you are simultaneously changing both the composition distribution and the total amount of helium in the core. Standard model computations only determine the net effect on the stellar structure and hence only determine the small evolutionary changes in the period. What we are trying to do here is to separate the effects of the composition redistribution due to the mixing events from the effects of the composition changes due to the nuclear burning.

These separate effects can each produce relatively large period changes, even though they add up to a small evolutionary rate of period change. Essentially we have examined the question of how does a horizontal-branch star actually evolve from one point to another along an evolutionary sequence. We are suggesting that this evolution is inherently noisy because of the basic characteristics of the semiconvective process and that the observed period changes are in fact a measure of this noise.

Sreenivasan: What kind of numerical mixing scheme do you use? And is there a difference whether you use the Schwarzschild criterion or the Ledoux criterion?

Sweigart: All of our models have been computed with the Schwarzschild criterion. The numerical technique for treating semiconvection was that described by Robertson and Faulkner (1972, Ap. J., 171, 309). When studying the effects of the individual mixing events, we first mixed small regions of the semiconvective zone and then computed the resulting change in the stellar structure and hence in the period.

Sreenivasan: Would you see a difference if you used a diffusive type of mixing?

Sweigart: We have not investigated different numerical techniques for determining the structure of the semiconvective zone. If exact neutrality is assumed to exist within the semiconvective zone, one would not expect the structure of the semiconvective zone to depend on the adopted numerical technique, once the stability criterion is specified.

Sreenivasan: We found that the type of mixing would make a change in the fluctuations.

Sweigart: The thing which is of basic importance as far as our calculations are concerned is how extensive the semiconvective zone becomes or, equivalently, how much helium flows through it.

Baker: I have several comments. First, whether a process like this takes place or not is something that has to do with the hydrodynamics. It is not sufficient to do it in a completely local way. It is a global question, and you have to see what the eigenfunctions look like throughout the whole star. Second, if you look at the observations of RR Lyrae stars, what you see is not only changes in period, but also changes in amplitude and almost discontinuous changes in phase. Certainly phase changes occur, as well as period changes. Nonperiodic behavior is also seen in Population II Cepheids. It would be helpful to have more observations of such objects. The third point is that it is not at all clear that it is necessary to invoke major changes in the structure of the star in order to find abrupt changes in period or phase. It is pervasive characteristic of coupled nonlinear oscillators or other nonlinear systems with enough degrees of freedom, that in certain regions of the parameter space, it is quite easy to find nonperiodic behavior -- sudden changes in amplitude, jumps in period, changes in phase, and so on. These things deserve to be looked into in efforts to explain these stars.

Wesselink: I have a question with regard to the period changes in the RR Lyrae stars in  $\omega$  Cen. There are a large number of them, and a dominance of one sign -- more positive changes than negative changes. Does that agree with your theory?

Sweigart: Our theory would predict that the mean rate of period change for the RR Lyrae stars in a globular cluster should correspond to the average evolutionary period change. The fact that the mean period change in  $\omega$  Cen is positive would therefore indicate that the RR Lyrae stars in  $\omega$  Cen are on average evolving redward in the H-R diagram.

Wesselink: It seems to me that the mean value should be determined rather well. Does that agree with your theory?

Sweigart: It depends upon precisely where the RR Lyrae stars in  $\omega$  Cen are located along their horizontal-branch tracks. If they are returning to the asymptotic branch following the end of the horizontal-branch phase, the rate of evolution could be relatively rapid. The evolution could then produce a relatively large positive period change, as observed.

Wesselink: Inversely, you may be able to identify what position the RR Lyrae's are in.

Sweigart: There is a consistency check which one can make: does the morphology of the horizontal branch agree with the mean rate of period change one observes? If the morphology is such that you have a predominance of blue horizontal-branch stars, then you would expect to see, if anything, a positive mean rate of period change. That is in fact the case in  $\omega$  Cen.

A. Cox: I want to drop a name. What seems to happen is, you want the period to change, but as Norman says, the period isn't going to change right away because the bell rings for a while in its original frequency.

Sweigart: There will be a hydrostatic readjustment of the star immediately following the adjustment of the composition.

A. Cox: But that will create a pulsation itself. The name I wanted to drop is the Blazhko effect. Maybe this is a beating of the period it had before, with the period it wants to go to. What do you think?

Sweigart: You are the expert on that! [Laughter] Normally, when people report changes in period, they try to exclude variables showing the Blazhko effect. If the Blazhko effect results from a beating between two periods, I would suspect that the required difference between these periods would considerable exceed the period change associated with a typical mixing event.