

**MODE IDENTIFICATION IN BETA CEPHEI STARS**

By

**Morris L. Aizenman**

**Division of Astronomical Sciences**

**National Science Foundation**

**Washington, D. C. 20550**

and

**Janet Rountree Lesh\***

**Laboratory for Astronomy and Solar Physics**

**NASA/Goddard Space Flight Center**

**Greenbelt, Maryland 20771**

**\*NAS-NRC Senior Resident Research Associate**

## I. Introduction

The Beta Cephei stars are unique among well-known variable stars in that we still have no idea why they pulsate. Cepheids, RR Lyrae stars, Mira variables and Delta Scuti variables are now reasonably well understood; while there are still problems with discrepancies of evolutionary and pulsational masses for some of these stars, the basic physics controlling the instability seems to be known. We do not know why the Beta Cephei stars pulsate -- in fact there is still some question as to whether these stars are radial or nonradial oscillators -- and it is because such questions still are unresolved that the subject of mode identification is of high interest and significance. It is the fervent hope among workers in this field that if the pulsation mode of the Beta Cephei stars is definitely established, it will provide a clue to the instability mechanism.

## II. Observational Characteristics Related to Mode Identification

The Beta Cephei stars are short-period, small-amplitude variables of early spectral type (generally B0.5-B2). The first member of this group was discovered by Frost in 1902 (see Frost 1902, 1906). Their observational properties have been extensively reviewed and discussed, most recently by Lesh and Aizenman (1978). Here we shall summarize only the essential characteristics, which may be of importance in attempts to identify the pulsation mode:

1. Both light and velocity amplitudes in these stars are small, typically less than 0.1 mag in blue light and 50 km/sec in radial velocity. However, the light amplitudes are greater at ultra-violet wavelengths.

2. Typical periods in both light and radial velocity are 4 to 7 hours. However, about half of the stars in the Beta Cephei group exhibit more than one period. This effect is often detected as a long-period modulation of the amplitude of the principal variation, usually interpreted as an interference between two nearly equal short periods. This is known as the "beat phenomenon". It was discovered by Meyer (1934).

3. In contrast to the classical Cepheids and RR Lyrae stars, the light curve for Beta Cephei stars lags approximately 90 degrees (one-quarter period) behind the radial velocity curve. Thus if the star is pulsating radially, maximum brightness occurs at minimum radius. De Jager (1953) showed that this relationship holds for each set of light and radial velocity curves in multiperiodic stars.

4. In many Beta Cephei stars, the radial velocity curves derived from different elements are not quite in phase: the phase derived from the hydrogen lines lags behind the helium-line phase, which in turn lags behind the phase derived from the metallic lines. This is called the "van Hoof effect".

5. Although both light and radial velocity curves are usually sinusoidal for these stars, the radial velocity curve sometimes exhibits a "stillstand" -- a phase of apparently constant velocity -- on the descending branch. In extreme cases, the velocity curve may even appear discontinuous. Occasionally the light curve exhibits a stillstand as well.

6. Finally, the line profiles of many Beta Cephei stars show very complex variations when observed at high dispersion. Changes in line width,

strong asymmetry, the appearance of secondary components, and even complete splitting appear in the spectral lines of various members of this class. In multiperiodic stars, the line profiles vary with only one of the two short periods. The great detail with which this phenomenon has been observed makes it a prime candidate for the detection of potentially small differences between neighboring pulsation modes.

### III. Early Work Related to the Beat Phenomenon

It was undoubtedly the apparent presence of two oscillation frequencies in the Beta Cephei stars that generated most of the initial interest on the part of theoreticians; it led to a number of hypotheses as to their possible interpretation. Let us state at the outset that in most of these analyses, there was no attempt to identify the instability mechanism. Rather, the interest was in providing an explanation and interpretation of the two periods. Woltjer (1935, 1937, 1943, 1946) considered the case of nonlinear oscillations and analyzed the behavior of a system containing two radial frequencies  $\sigma_1$  and  $\sigma_2$  with  $\sigma_1 \approx 2\sigma_2$ . Such a situation can lead to resonance, with one of the resonance frequencies being  $\sigma = \sigma_1 - \sigma_0 \approx \sigma_0$ . It was thought that what we observe in the Beta Cephei stars is the original frequency plus the difference frequency, but the objection has been raised that it is difficult to see how this frequency could be associated with the broadening of the lines. We cannot go into the many aspects in this review but the point to be noted here is that the beat phenomenon is interpreted as being due to a resonance existing between two frequencies, one being the double of the other. In principle, one should also expect to find the phenomenon at the frequency  $2\sigma_0 = \sigma_1$ .

Struve (1955) and Odgers (1956) also tried to account for the observed splitting of the lines and van Hoof effect. They assumed radial pulsations,

and hypothesized that a thin layer of the atmosphere is ejected at regular intervals with a period  $P_0$ . The layer falls back under gravity, and if the time of fall is smaller than  $P_0$ , there is a stillstand on the velocity curve. If the time is slightly larger than  $P_0$  the interfering motions that arise between the layer and the rest of the star could give rise to a beat phenomenon.

Here we have the introduction of an underlying radial oscillation plus an atmosphere free-fall oscillation time. However, the objection has been raised by Ledoux (1958) that it is practically impossible for this free-fall period to be fixed to such an extent that regular beats could be observed over very long periods of time. Another suggestion along these lines has been that there is a strictly atmospheric oscillation, but here the difficulty is one of making this atmospheric period independent of the interior. The atmosphere would eventually experience a forced oscillation under the influence of the interior, and if nonlinear coupling were taken into account, a beat phenomenon might result. The question has never been thoroughly investigated.

With regard to all of these theories, it should be noted that Huang points out that the equivalent widths of the lines remain constant during the doubling stages. Hence the phenomenon cannot be due to superposed layers but must be due to different parts of the stellar surface. Another question is whether the broadening of some lines is simply a case of unresolved doubling.

Ledoux (1951) has also suggested that the two periods may be related to the fact that if one assumes that one is observing nonradial oscillations, then a splitting of the frequencies is possibly due to rotation, and the

amount of splitting is proportional to the rotational velocity. The interference of one of the travelling waves with the stationary oscillation would give rise to a beat frequency proportional to the angular velocity. In this theory, the broadening of the lines is attributed to the differential velocities on the surface of the star, resulting from the combination of rotation and the components of the travelling wave. The major problem is that if one of the travelling waves is chosen, the other remains unexcited. It was believed that this may be connected with the cause of the oscillation itself, possibly due to the interaction of a close companion. But this is physically unrealistic. The period of the companion would have to be almost the same as the pulsation period, which would put the companion inside the other star. Also, line duplicity and the van Hoof effect are not explained.

Other explanations of the beat phenomenon were made by Chandrasekhar and Lebovitz (1962). They criticized the Ledoux hypothesis because they felt it was unlikely that purely nonradial oscillations would be excited in preference to radial ones. Their argument was that the nonradial modes would be highly damped relative to the radial oscillation. Instead, they presented an alternative explanation of the beat phenomenon. The theory was based on the assumption that the ratio of specific heats for these stars is such that degeneracy occurs between the fundamental characteristic frequencies of the radial and the  $\ell = 2$  modes of oscillation. They showed that rotation would remove this degeneracy and lead to two normal modes characterized by slightly different frequencies. The advantage of this theory over the Ledoux theory was that a separate mechanism would not be required for the excitation of the nonradial mode. In their schema, rotation coupled the radial mode to the nonradial mode to produce two distinct normal modes, which were both nonradial.

There was an objection to this, however. On the basis of this theory, the rotational velocities that were required to give the observed beat periods were larger by factors of 3 to 4 than those observed. Clement (1965) showed that this splitting could be increased for centrally condensed models. He proceeded to make a calculation of the effects of nonradial oscillations in terms of radial velocities, variations in brightness, and periods. His calculations included broadening due to macroscopic motions and the velocity of the center of the line profile. He concluded that when beats are observed in a particular star, the angle of inclination of the rotation axis would normally have to be at an intermediate angle. At either extreme (pole-on or line-of-sight in the equatorial plane) one velocity amplitude would be much greater than the other. Clement also calculated line broadening, motion of the center of the line profile, the mean radial velocity as a function of the equatorial velocity, the angle of inclination, the ratio of the angular rotation to the oscillation frequency, and the amplitudes of the two radial velocity components. He considered the effect of the area changes and changes of magnitude with time. He found that even for polytropes with  $n = 3.0-3.5$ , the rotation velocities required to produce the beat phenomenon were considerably reduced (from  $\sim 200$  km/sec to 140 km/sec), but were still too high to match the observations. The only exception was  $\beta$  Canis Majoris, which has an observed  $\Omega R \sin \theta$  of about 60 km/sec and a very small period splitting ( $\frac{\Delta P}{P_0} = \frac{P_1 - P_2}{P_0} = 0.006$ ). Clement also calculated the broadening and radial velocity curves for three stars:  $\beta$  Canis Majoris, 12 Lacetae, and BW Vulpeculae. His results implied that  $\beta$  Canis Majoris was being viewed from a small angle of inclination (almost pole-on), and that the observed variation in broadening would vary with the longer of the two short periods.

In order to explain the observation that the line profiles of 12 Lacetae change with the shorter period, it was necessary to postulate that this star was being observed from a large angle of inclination. His final example, involving BW Vulpeculae, does predict the existence of a stillstand on the velocity curve for the profile center.

Thus, on the basis of this theory,  $\beta$  Canis Majoris and  $\sigma$  Scorpii have small angles of inclination with respect to the line of sight. Other stars exhibiting the beat phenomenon require a large angle of inclination.

Another observational point should be made. On the descending branch of the radial velocity curve of BW Vulpeculae, it is found that the absorption lines actually become double; this coincides with the phases of maximum broadening. At the points of minimum broadening, the lines are single. The theory of Clement, assuming that the line broadening arises only from the macroscopic motions of the nonradial oscillations, shows that although the profiles are asymmetrical, there is no indication of doubling.

A major problem with Clement's solution was the fact that the rotation required to produce the splitting of the modes was 3 to 4 times larger than observed. Clement (1967) considered the case of differentially rotating polytropes, with the choice of a suitable rotation law, which would be consistent with stability against local perturbations. He adapted Stockly's (1965) rotation law to polytropes. He found that with this law of rotation, the observed splitting could be produced with a considerable reduction in the equatorial velocity, and the required velocities were in agreement with observations. The subject has not been pursued beyond this point, primarily because there has been little further interest in exploring polytropic types of models. Nevertheless, this area should be examined further.



We have spent some time on this subject because it leads us naturally into the title of our review paper -- mode identification. We have discussed two types of theories: 1) Ledoux's theory, which involves nonradial modes in the presence of rotation, has the advantage that it can explain line doubling. 2) The Chandrasekhar and Lebovitz scheme, as elaborated by Clement, requires coupling of a radial mode to a nonradial mode, and leads to two distinct modes, both of which are nonradial. But this model does not seem to explain all of the variations in line widths. There is no real preference between the two schemes because the excitation mechanism is not known.

#### IV. Simple Observational Quantities Interpreted as Mode Indicators

Before approaching the question of detailed line profile variations and their theoretical interpretation, we shall review in this Section some simple, easily observed quantities that have been proposed by various authors as indicators of the pulsation mode either in individual Beta Cephei stars or in the group as a whole.

As we noted in Section II, both the light and radial velocity amplitudes of Beta Cephei stars are quite small. Leung (1968) pointed out that the ratio of these quantities

$$\frac{\Delta m \text{ (blue)}}{2K} = 0.0021 \text{ mag/km s}^{-1}$$

is also extremely small in Beta Cephei stars, compared to other types of variables that are assumed to be radial pulsators (classical Cepheids, RR Lyrae stars, and Delta Scuti stars). Leung suggested that this might indicate that Beta Cephei stars are nonradial oscillators.

However, Watson (1971) correctly pointed out that the small visible light change in Beta Cephei stars is mainly a consequence of their high temperatures, which result in most of the energy being emitted in the ultraviolet. (It has since been confirmed by satellite observations that the light amplitude of these stars does increase greatly with decreasing wavelength). Watson showed that the physically more meaningful quantity

$$\frac{(\Delta R/R)}{\Delta M_{bol}} = \frac{24 \cdot P}{17.2 \pi R} \frac{2K}{(\Delta M_v + \Delta BC)}$$

has a characteristic value of 0.12 for Beta Cephei stars; this does not differ greatly from the value computed for other types of variables.

Lesh (1976) measured values of  $(\Delta R/R)/\Delta M_{bol}$  for several Beta Cephei stars using ultraviolet data, and suggested that a comparison of the observed values of this ratio with theoretically predicted ones might serve to discriminate between radial and nonradial modes. But Stamford and Watson (1977) felt that the physical significance of  $(\Delta R/R)/\Delta M_{bol}$  would be unclear in the case of nonradial oscillations. They suggested using instead the ratio of light range to color range  $\Delta V/\Delta(U-B)$  (where the latter, of course, is indicative of temperature range), in conjunction with  $\Delta V/2K$ . Their results were ambiguous, for they found that the observed  $\Delta V/\Delta(U-B)$  suggested the presence of both radial and nonradial oscillations in the Beta Cephei stars.

Dziembowski (1977) also rejected the quantity  $(\Delta R/R)/\Delta M_{bol}$  because his theoretical calculations showed that it is insensitive to the index  $\ell$  (in the spherical harmonic  $Y_m^\ell$ ). Dziembowski carried out a general analysis of the light and radial velocity variations in a nonradially oscillating star. He computed the radial velocity averaged over the visible stellar disk, as well

as variations in the bolometric magnitude. He found that the ratio of the mean radial velocity to the magnitude variation  $V_{\text{rad}}/\Delta M_{\text{bol}}$  is independent of the angle of inclination of the rotation axis. Consequently, a straightforward generalization of the Baade-Wesselink formula for stellar radius is possible. On this basis, if radius of the star is known, it is possible in principle to determine in which spherical harmonic the star is oscillating (i.e., whether the pulsation is radial or nonradial). The quantity  $(V_{\text{rad}}/\Delta M_{\text{bol}})$  was found to have a strong dependence on  $\ell$  for models appropriate to Delta Scuti stars (with a mass of  $2M_{\odot}$ ). Dziembowski did not perform this calculation for the more massive models corresponding to Beta Cephei stars ( $10 - 15M_{\odot}$ ). But it would be of great interest to do so, for the method appears to hold out great promise.

A more "classical" approach was taken by Lesh and Aizenman (1974). They constructed period-luminosity diagrams for Beta Cephei stars in the  $(\log P, M_{\text{bol}})$  plane, using  $10$  and  $15M_{\odot}$  models. On such a diagram (shown in Figure 1), the Beta Cephei "instability strip" maps into a discrete, relatively narrow band for a given radial pulsation mode. When the observed variable stars are plotted on this diagram most of them fall in the region appropriate to the second harmonic radial mode (but for a different assumed chemical composition, the first harmonic would be preferred). A similar diagram for nonradial pulsation modes (shown in Figure 2, where only the right-hand edge of this band for each mode is drawn) has most of the observed stars falling in the region appropriate to the  $P_2$  and  $P_1$  ( $\ell = 2$ ) nonradial modes. These conclusions are reinforced by the study of the pulsation constant  $Q = P \sqrt{\bar{\rho}}$ . The mean value for the observed Beta Cephei stars is found to be  $0.025$ , a value appropriate for the first harmonic

radial mode or the  $P_1$  nonradial mode. On the basis of these data, the fundamental radial mode and nonradial f- and g- modes seem less likely than the radial harmonics and nonradial p-modes as candidates for the observed oscillation mode in Beta Cephei stars. But this method is not able to distinguish between radial and nonradial pulsation as such.

Schafgans and Tinbergen (1978) attempted to detect time variations in the linear polarization of  $\beta$  Cephei; if present, such variations might be interpreted as due to the time-varying anisotropy of the scattering geometry in nonradial pulsation. However, the authors did not detect any such variations to their level of accuracy ( $4 \times 10^{-4}$  r.m.s. in 4 minutes).

Finally, in approaching the study of the spectral lines themselves A. Karp and M. Smith (private communication) have suggested the simple criterion that if the change in line width is greater than the change in radial velocity, the pulsation is probably nonradial, and in the opposite case the pulsation is more likely to be radial. However, Karp (1978) has shown that the van Hoof effect is more strongly dependent on the ionization balance than on the velocity field in the stellar atmosphere, and hence is not a good diagnostic tool.

#### V. The Study of Line Profile Variations

By far the greatest emphasis in current work on Beta Cephei variables is on the detailed observation of the spectral line profiles, and their theoretical interpretation.

The observations have been greatly facilitated in recent years by the use of new types of detectors, which permit high time resolution and high spectral

resolution to be achieved simultaneously. One of the first such investigations was a study of  $\beta$  Cephei by Goldberg, Walker, and Odgers (1974) using image isocon and silicon diode vidicon television cameras as detectors, giving effective exposure times of 7-10 minutes. They observed Si III 4568 Å and 4553 Å, and found that the lines had an extended wing to the blue when the radial velocity was positive, indicating contraction of the star. When the radial velocity was negative, there was an extended wing to the red.

In a study of BW Vul, Goldberg, Walker and Odgers (1976) obtained a spectral resolution of 0.6 Å over a bandpass of about 140 Å with a refrigerated image isocon. The region studied included the lines He I 4471 Å, Mg II 4481 Å and Si III 4553, 4568, and 4575 Å. The effective exposure time was about 2 minutes. BW Vul has the largest light and radial velocity amplitude of any known Beta Cephei star, has a "stillstand" on the descending branch of its radial velocity curve, and exhibits dramatic line profile variations. The lines are sharpest, deepest, and essentially symmetric on the ascending branch of the velocity curve. As the cycle progresses through maximum radial velocity, they become shallower, broader, and somewhat asymmetric, with an extended wing to the red. The lines become double just before the radial velocity stillstand, are fairly sharp and deep during stillstand, and are highly asymmetric during the blueshift following the stillstand. A very detailed picture of this doubling on the descending branch of the radial velocity curve was obtained because of the high time resolution.

Similarly detailed observations of Si III 4553 Å and 4568 Å in 12 Lac were made by Allison, Glaspey and Fahlman (1977). This star also has strong,

deep, essentially symmetric lines on the rising branch of the velocity curve, and asymmetry and line splitting on the descending branch.

The U1 scanner on the Copernicus satellite was used by Lesh and Karp (1977) to obtain profiles of Si III 1294.5 Å in a number of Beta Cephei variables, with a spectral resolution of 0.05 Å and an "effective exposure time" of about 20 min. The results for  $\nu$  Eri and  $\sigma$  Sco are shown in Figures 3 and 4, respectively. There are two important points to be noted here: 1) in  $\nu$  Eri, the phase of maximum asymmetry occurs on the descending branch of the radial velocity curve, as in the stars described above, but in  $\sigma$  Sco the greatest asymmetry occurs on the ascending branch (around phase 0.75, in the notation of Lesh and Karp); 2) in both  $\nu$  Eri and  $\sigma$  Sco, the asymmetry is always in the sense of a depressed blue wing - depressed red wings do not occur.

The interpretation of the line profiles of  $\beta$  Cephei in terms of a radial oscillation - i.e., an atmosphere that alternately expands and contracts -- has been based on the early work of Wilson (1935) for uniform expansion, and Abhyankar (1965) for differential expansion following an exponential velocity law. Figure 5 shows some typical profiles for these two cases. Upon comparing their observations of  $\beta$  Cephei with these profiles, Goldberg, Walker and Odgers (1974) concluded that this star undergoes purely radial pulsation. However, Kubiak (1978) points out that stationary nonradial modes ( $m = 0$ ) produce very similar profile variations.

The modelling of line profile variations in terms of nonradial oscillations was pioneered by Osaki (1971). He considered the case of quadrupole oscillations taking place in a rotating star, and, in order to calculate the line profiles, assumed that the line broadening was produced by the Doppler shift due to the mass motion of the surface elements. He assumed

that variations in surface area and brightness due to the oscillation are negligible. The equivalent widths of the lines would then remain constant throughout the oscillation, but the detailed structure of the line profiles would be affected by brightness variations due to the oscillation itself. His calculations included the effects of both rotation and the oscillation itself. The visible hemisphere was divided into about 5000 surface elements, and their radial velocities were computed and added to construct the line profiles.

In the case of the mode  $m = -2$  (i.e. a wave travelling in the same direction as the rotation and symmetric with respect to the equator) he found that such a wave produced a discontinuous radial velocity curve, very sharp lines on the ascending branch of the radial velocity curve, and very diffuse lines around phase  $\phi = 0.75$ . This is characteristic of Beta Cephei stars such as BW Vulpeculae,  $\sigma$  Scorpii and 12 Lacertae. It appears, as first noted by Christy (1966) that a wave travelling in the same direction as rotation would explain such variations. Osaki thus proceeds to identify Struve's  $P_2$  oscillation (the one associated with variable line broadening,) as the  $P_2^2$  mode of oscillation.

The original calculation was made assuming an inclination of  $90^\circ$ . There appears to be little difference in the radial velocity curves for inclinations varying from  $45^\circ - 90^\circ$ , but there were large deviations for  $i = 30^\circ$ .

It should be noted that a wave travelling in the opposite direction to rotation ( $m = +2$ ) has the same characteristics as  $m = -2$ , but the sign of the radial velocity is reversed. Thus for a wave with  $m = +2$  the sharpest lines occur on the descending branch of the radial velocity curve and the

lines become diffuse on the ascending branch. But the observations contradict this, and so Osaki believes that this can probably be excluded.

Osaki believes that waves that are antisymmetric with respect to the equator ( $m = +1$ ) should not be excluded because there is simply no basis for doing so. In the case of  $m = -1$ , he finds that the shape of the velocity curves depends very strongly on the inclination. Nevertheless the actual shape of the line profile is very similar to that of the  $m = -2$  curve if viewed from near  $i = 45^\circ$ . In fact, the cases  $m = -2$  or  $m = -1$  cannot really be separated. Nevertheless, Osaki believes that  $m = -2$  should be identified as being the main oscillation mode. He feels that it is unlikely that the mode  $m = -1$  will be excited in preference to the symmetric mode  $m = -2$ . Furthermore, the profiles agree only when  $i \approx 45^\circ$ . It is possible that  $m = -1$  corresponds to one of the secondary oscillations.

The case where  $m = 0$  is the case of a mode which is axisymmetric, and so it does not couple with rotation. The period of variation in line broadening is half that of the radial velocity curve and this is clearly contradicted by the observations. This is the mode that Chandrasekhar and Lebovitz consider, and Osaki feels that this mode is clearly not as successful in matching the observations as the  $m = -2$  mode.

Osaki also considers the possibility of identifying the separate modes which give rise to the beat phenomenon. The Struve  $P_2$  oscillation associated with the variation in the line profiles generally has a shorter period and a larger velocity amplitude than the  $P_1$  oscillation. The exception is  $\beta$  Canis Majoris where the opposite applies. Osaki identifies  $P_2$  with  $m = -2$  (as do Ledoux and Christy). What about  $P_1$ ? We can have  $m = +2, +1, 0$ , and  $-1$ . We can exclude  $m = -1$  because a superposition does not give the beat phe-



nomenon where we have a long term amplitude modulation. Rather we get a long term modulation of the gamma velocity. Some Beta Cephei stars do exhibit such a variation ( $\sigma$  Scorpii). This variation has also been interpreted as being due to an orbital motion. Nevertheless, the alternate possibility should not be excluded. Also, a superposition of  $m = -2$  and 0 can bring about a beat in which the amplitude of the radial velocity curve is modulated. Osaki thus identifies the  $P_2$  wave as having  $m = -2$  (travelling in the same direction as rotation) and the  $P_1$  oscillation as a standing oscillation. He also considers  $m = -1$  and  $m = -2$  and finds that beats can occur. So this case is not totally excluded although the observed beat periods imply rotational velocities that are twice as large as those required for  $m = 0$ . Osaki speculates that  $m = -2$  is preferably excited compared with other modes and so the excitation mechanism may be closely related to the rotation.

Some of the line profiles computed by Osaki (for  $\ell = 2$ ,  $m = -2$ ) are shown in Figure 6. Similar computations have been performed by Stamford and Watson (1976) and by Kubiak (1978).

The line profiles predicted for the various nonradial modes with  $\ell = 2$  bear a strong resemblance to the type of profile encountered in the Beta Cephei stars that were originally noted for their large line-width variations. However, there are still some discrepancies. For instance, it is difficult to produce an outright splitting of the components with a nonradial oscillation alone. More importantly, all forms of nonradial pulsation (as well as radial pulsation) necessarily produce profile variations in which the direction of asymmetry is equally distributed between red and blue. In other words, if a certain asymmetric profile occurs at a given phase, its mirror image must

necessarily occur at another phase. Situations in which the asymmetry is always to the blue, as observed by Lesh and Karp (1977) for  $\nu$  Eri and  $\sigma$  Sco, cannot be explained by either a radial or nonradial oscillation alone.

For this reason, Lesh and Karp proposed that an accelerating velocity gradient in the stellar atmosphere (indicating the presence of a "stellar wind") is superimposed on the underlying radial or nonradial oscillation. This would mean that at least some Beta Cephei stars are losing mass to their surroundings. Lesh and Karp succeeded in producing consistently blueward asymmetries with a velocity field of the form  $v = -\alpha \log \tau + v_c$ , where  $\alpha$  (the acceleration term) = 40 km/sec and  $v_c$  (the radial pulsation amplitude) = 50 km/sec. Some of their profiles are shown in Figure 7. But the mass loss rate implied was of the order of  $10^{-5} M_{\odot}/\text{yr}$ , which is contradicted by the observations (these stars show no emission lines, P Cygni profiles, etc). Lesh and Karp suggested that the combination of a nonradial pulsation and a smaller velocity gradient might produce the desired asymmetry without requiring an unacceptably high mass loss rate.

Nonradial pulsation theory also can be used to predict the form of the radial velocity curve, which can then be compared with the observations. Thus far, such comparisons have not been very successful, partly because of the difficulty of measuring the radial velocity consistently when several components are present in the line profile, and partly because of the lack of a full hydrodynamic theory.

## VI. Conclusion

On the whole, then, the problem of mode identification in Beta Cephei stars cannot be said to have progressed to a state where it is a useful

indicator of the instability mechanism. Most simple, easily measured physical quantities are ambiguous in their interpretation - at best, they may serve to distinguish between radial and nonradial pulsation in general, or between fundamental modes and harmonics. The study of the line profiles holds out great promise for the identification of the actual pulsation mode, but more theoretical work and more observations are needed. The theoretical work should include a study of the effect of temperature and geometric change in the stellar surface due to the nonradial oscillation, and especially a study of the effect of the depth-dependent velocity field on the structure of the stellar atmosphere. On the observational side, high time and spectral resolution investigation of profile variations in additional Beta Cephei stars should be made, and the observations already made should be repeated to check for the reproducibility of the profiles at identical phases. The simultaneous observation of strong and weak lines in the same multiplets and of lines from neighboring ionization stages would help to disentangle the various effects in the atmosphere.

#### Bibliography

- Abhyankar, K. D. 1967, Ap.J. 141, 1056.
- Allison, A. M., Glaspey, J. W., and Fahlman, G. G. 1977, A.J., 82, 283.
- Chandrasekhar, S. and Lebovitz, N. R. 1962, Ap.J. 136, 1105.
- Christy, R. F. 1966, A. J. 72, 293.
- Clement, M. J. 1965, Ap.J. 141, 210.
- Clement, M. J. 1967, Ap.J. 150, 589.
- Dziembowski, W. 1977, Acta Astron. 27, 203.
- Frost, E. B. 1902, Ap. J. 15, 340.
- Frost, E. B. 1906, Ap. J. 24, 259.

- Goldberg, B. A., Walker, G. A. H., and Odgers, G. J. 1974, Astron. Astrophys. 32, 355.
- Goldberg, B. A., Walker, G. A. H., and Odgers, G. J. 1976, A.J. 81, 433.
- Karp, A. 1978, Ap.J. 222, 578.
- Jager, C. de 1953, BAN 12, 82,88,91.
- Kubiak, M. 1978, Acta Astron. 28, in press.
- Ledoux, P. 1951, Ap.J. 114, 373.
- Ledoux, P. 1958, Hd. d. Astrophys. 51, 579.
- Lesh, J. R. 1976, Ap.J. 208, 135.
- Lesh, J. R., and Aizenman, M. L. 1974, Astron. Astrophys. 34, 203.
- Lesh, J. R., and Aizenman, M. L. 1978, Ann. Rev. Astron. Astrophys. 16, 215.
- Lesh, J. R., and Karp, A. H. 1977, Veroeff. Remeis Sternw. Bamberg 11, No. 121, p. 625.
- Leung, K. C. 1968, Astrophys. Space Sci. 2, 302.
- Meyer, W. F. 1934, PASP 46, 202.
- Odgers, G. J. 1956, PDAO 10, 215.
- Schafgans, J. J., and Tinbergen, J. 1978, preprint.
- Stoeckly, R. 1965, Ap.J. 142, 208.
- Struve, O. 1955, PASP 67, 135.
- Stamford, P. A., and Watson, R. D. 1976, Proc. Ast. Soc. Australia 3, 75.
- Stamford, P. A., and Watson, R. D. 1977, MNRAS 180, 551.
- Watson, R. D. 1971, Ap.J. 170, 345.
- Wilson, O. C. 1935, Ap.J. 82, 233.
- Woltjer, L. 1935, MNRAS 95, 260.
- Woltjer, L. 1937, BAN 8, 193.
- Woltjer, L. 1943, BAN 9, 435, 441.
- Woltjer, L. 1946, BAN 10, 125, 130, 135.

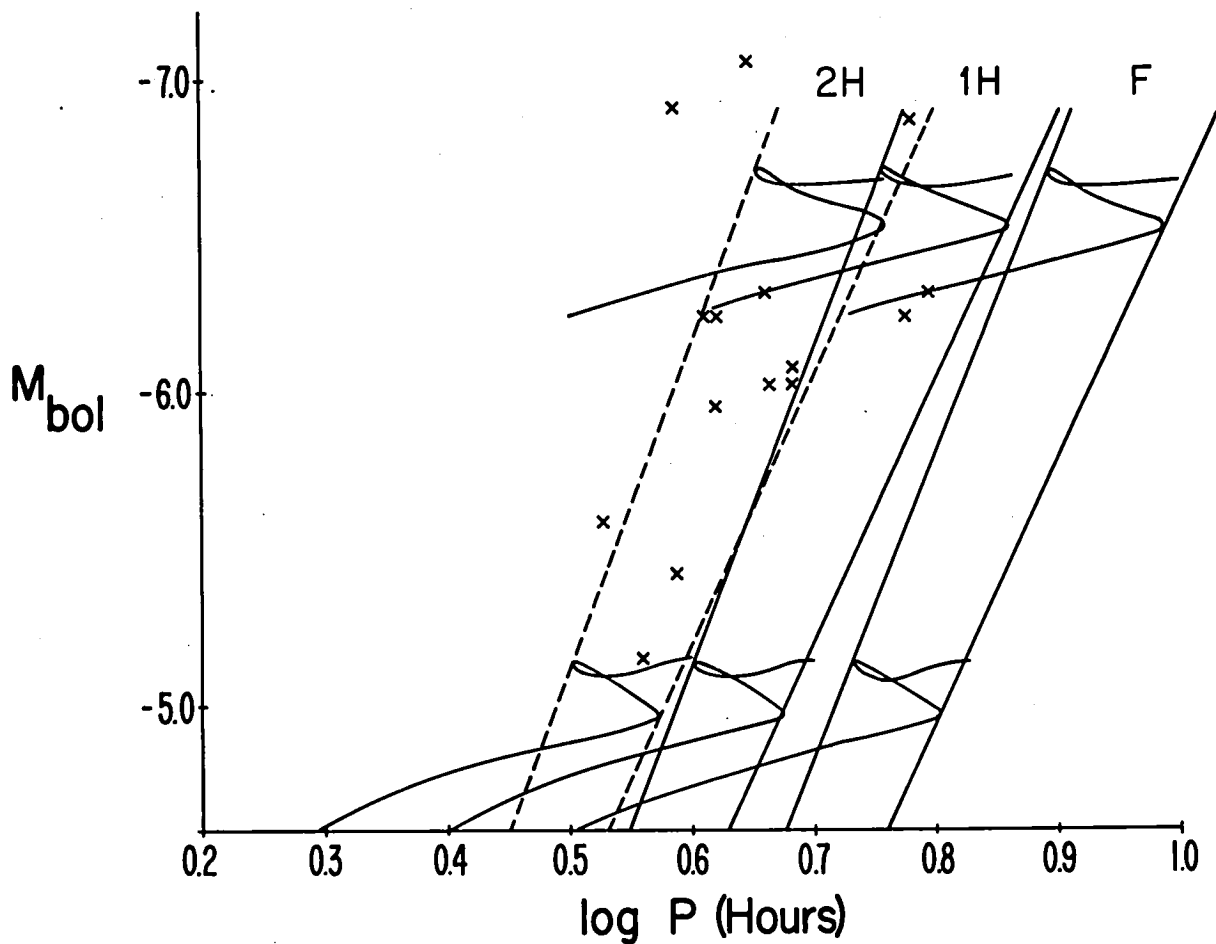


Figure 1. Period-luminosity diagram for observed Beta Cephei stars (crosses), and the "instability strips" calculated for the fundamental (F), first harmonic(1H), and second harmonic (2H) radial models (after Lesh and Aizerman 1974).

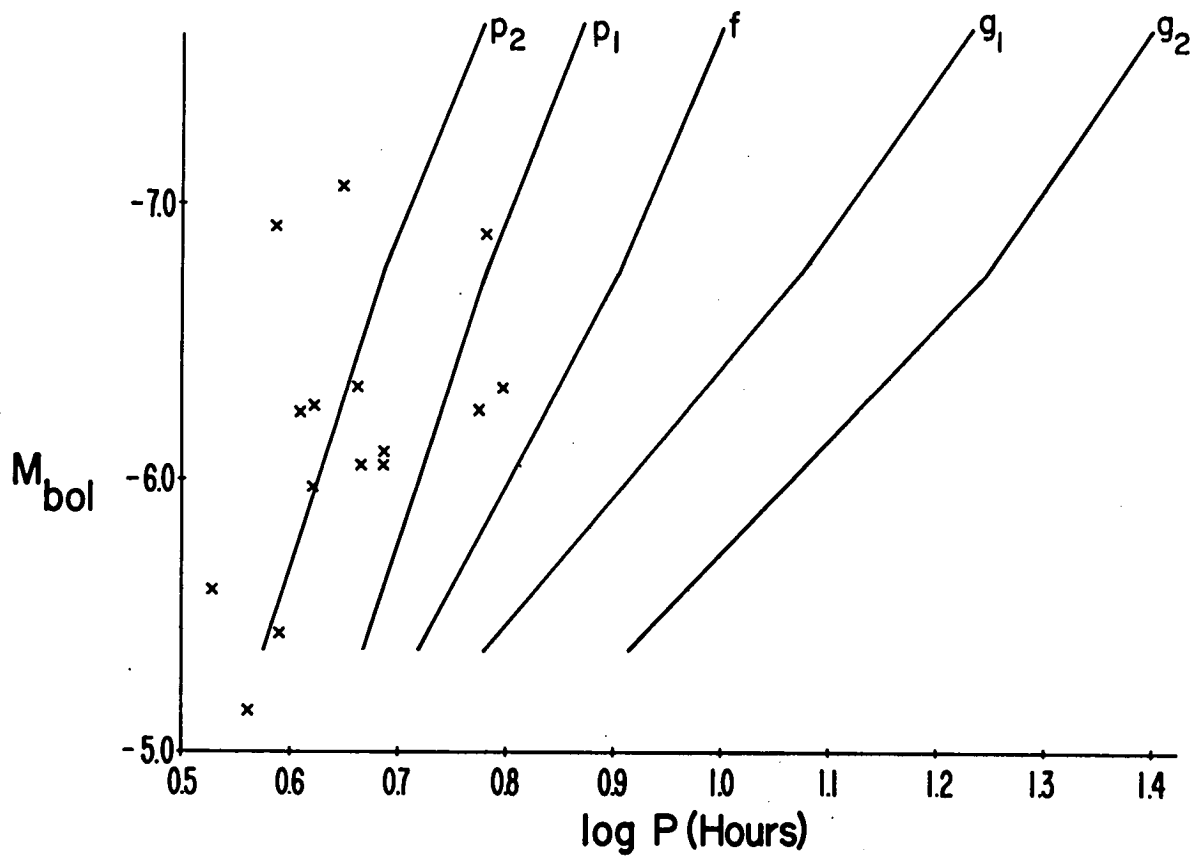


Figure 2. Period-luminosity diagram for observed Beta Cephei stars (crosses), and the right-hand edge of the predicted instability strips for nonradial p-, f-, and g-modes (after Lesh and Aizerman 1974).

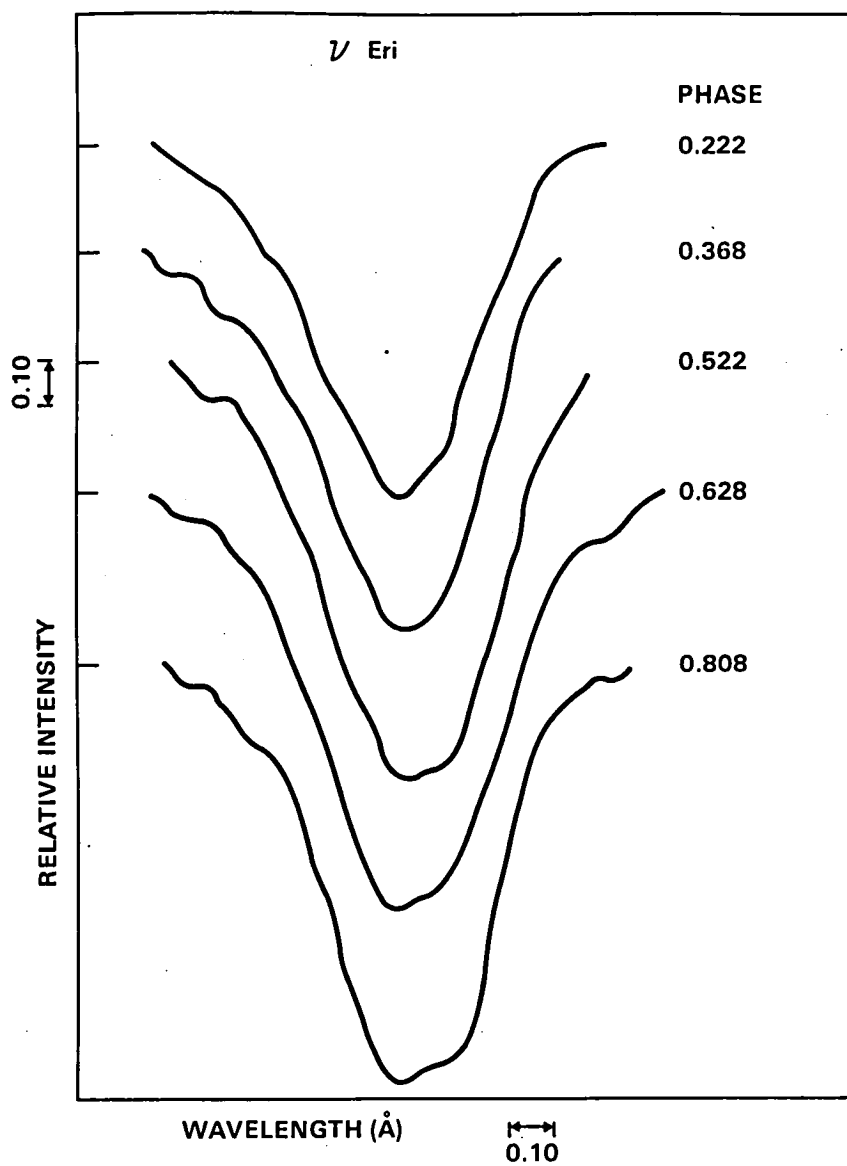


Figure 3. Profiles of the Si III 1294.5 $\text{\AA}$  line in  $\nu$  Eri, obtained with the Copernicus Ul scanner (after Lesh and Karp 1977).

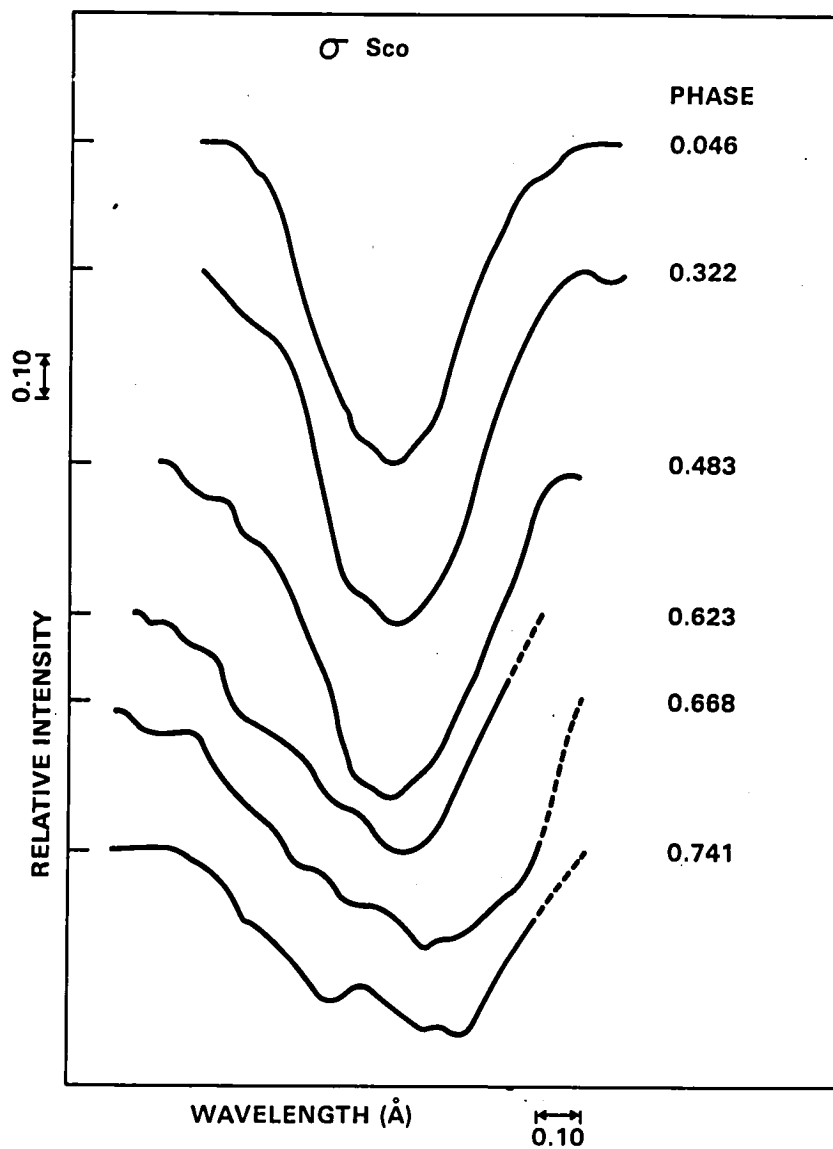


Figure 4. Profiles of the Si III 1294.5 Å line in  $\sigma$  Sco, obtained with the Copernicus U1 scanner (after Lesh and Karp 1977).



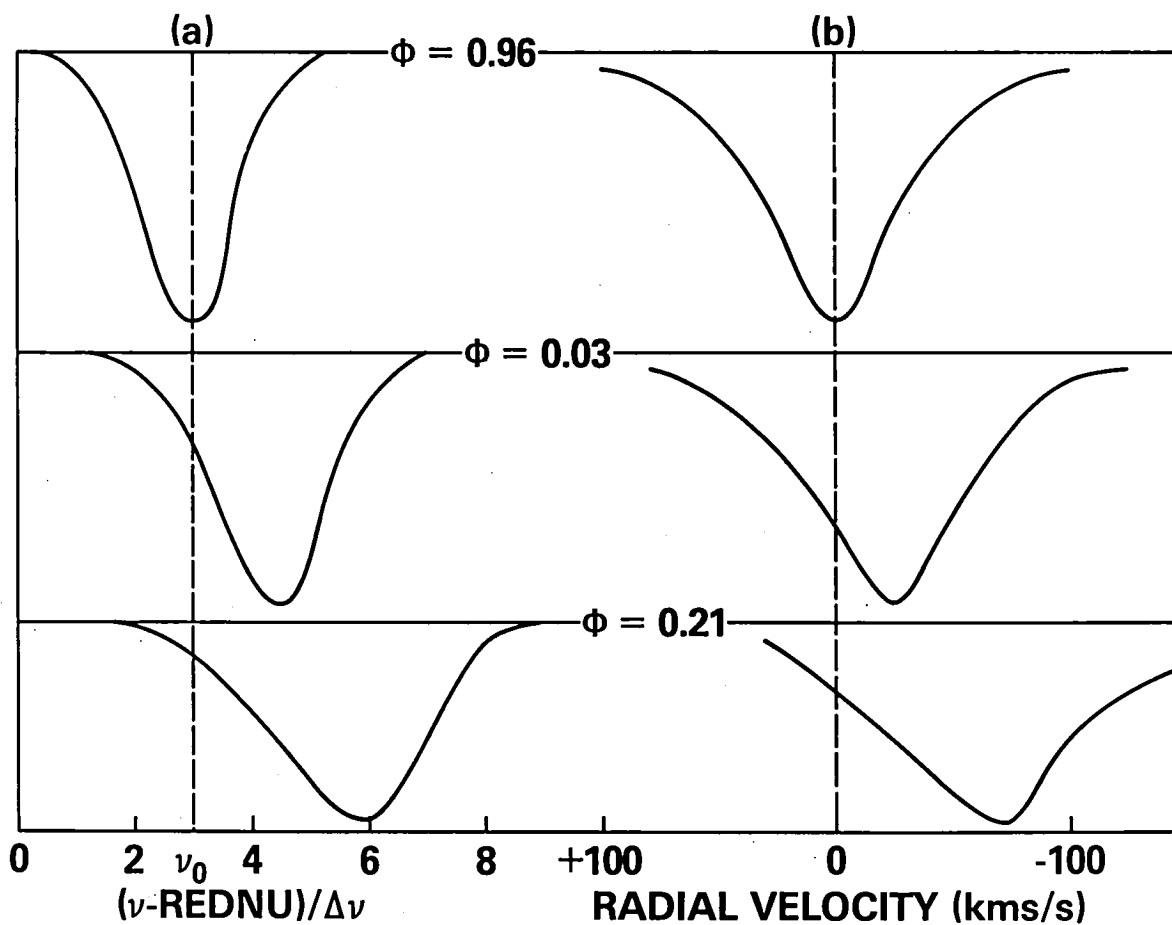


Figure 5. Synthetic line profiles for a star undergoing radial pulsation, with (a) differential expansion and (b) uniform expansion (after Goldberg, Walker and Odgers 1974).

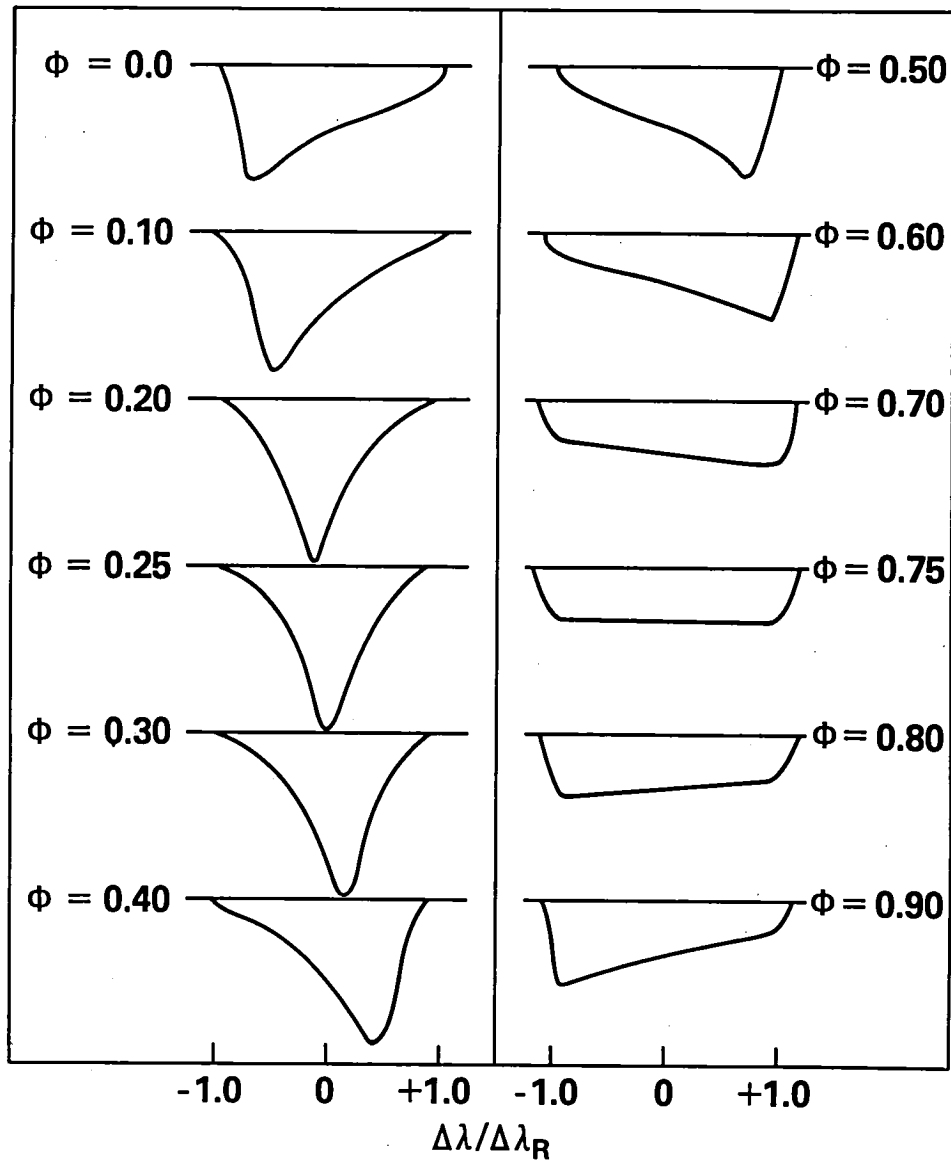


Figure 6. Synthetic line profiles for a rotating star undergoing nonradial pulsation in the  $\ell = 2$ ,  $m = -2$  mode (after Osaki 1971).

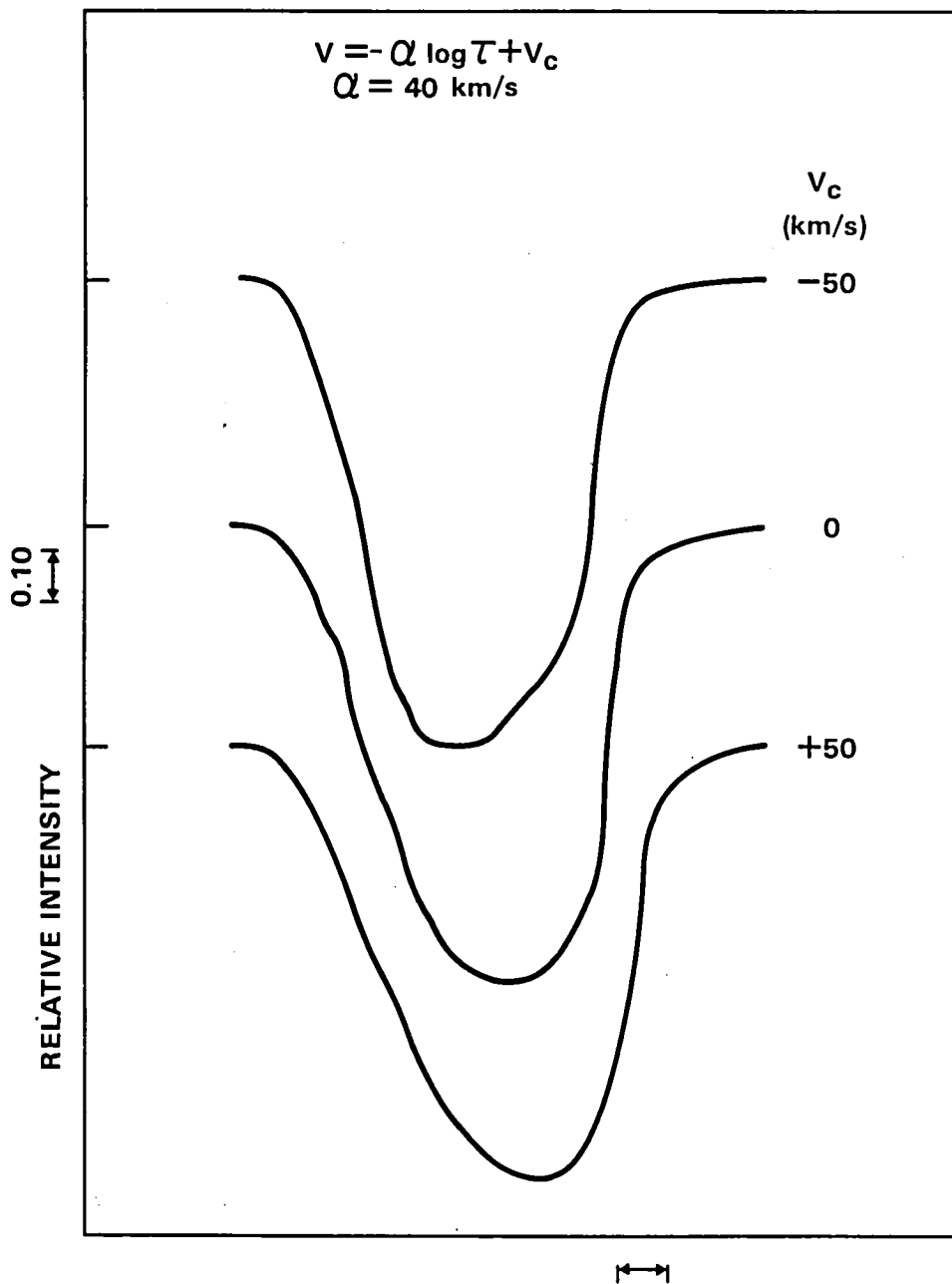


Figure 7. Synthetic line profiles for a star undergoing radial pulsation, with a superimposed stellar wind. The velocity field in the stellar atmosphere is of the form  $v = -\alpha \log \tau + v_c$  (after Lesh and Karp 1977).

## Discussion

M. Smith: I would like to second what you said about a shell in  $\sigma$  Sco. I would like to show you privately a little later some Reticon spectra of  $\sigma$  Sco that I observed which show emission P Cygni-type profiles at one particular phase in the star.

Lesh: That's very interesting. Of course, P Cygni profiles occur in stars where you have velocity gradients and mass loss, so perhaps we are finding evidence of the same phenomena.

Van Horn: To what extent is the symmetry in Osaki's theoretical profiles, which you showed, conditioned by an assumed symmetry of the underlying mode? In other words, could you get away from the comparable red and blue wings by going to a different order of spherical harmonics?

Lesh: I believe that Osaki considered only cases with  $\ell = 2$  in his paper, and he found this type of symmetry for all values of  $m$ . I don't know what would happen for  $\ell = 1$ , for example, or whether Osaki has considered this case. Do you know, Myron?

M. Smith: Yes, I have modeled various profile sequences from  $\ell = 1$  up to  $\ell = 4$ , and you find the same symmetry. It comes out of the spherical harmonic functions, which themselves have symmetrical properties. And this is true for intermediate inclinations as well.

Van Horn: Even the odd spherical harmonics?

M. Smith: Yes.

Shipman: Do all of the line profiles vary in the same way or are the variations different from one line to another?

Lesh: Normally, they all vary in the same way, with the occasional exception of hydrogen and helium. The Balmer lines tend to behave somewhat differently, and they are the only lines in which equivalent width variations are normally detected. But this is believed to be due to incipient emission at some phases, which causes the equivalent width apparently to decrease when, in fact, the line is simply being filled in. In stars in which the helium lines are very strong, the helium lines sometimes behave like the hydrogen lines, rather than like the metallic lines. But all of the metallic lines generally behave the same way.

A. Cox: What about abundance anomalies?

Lesh: As far as been observed, there are no abundance anomalies in these stars. People have done classical high-dispersion studies and have always found abundances in these stars that are comparable to those in other B stars. I myself have looked at the spectra of all these stars at classification dispersion and have never found any classification anomalies in them at all. So it would be nice to believe in the "helium fairy" [Laughter] who would increase the helium abundance in these stars quite a bit, because I am told that this is a possibility for creating an instability. But I would have to say, as an observer, that there is no evidence of very anomalous helium abundance or any other kind of abundance anomalies in these stars.

A. Cox: I am slightly worried that your lines may arise too high in the atmosphere. Have you tried to look at the variation in the weak lines which

come from very deep in the photosphere? In answer to another question, you said all lines behaved the same. Is that true of the weakest lines too?

Lesh: The lines that are observed in the visible are certainly weaker than the ones in the ultraviolet. The lines that we pick up in the ultraviolet tend to be very strong. Some people have tried to interpret the van Hoof effect as a means of measuring the velocity difference in different parts of the atmosphere, because there you are dealing with very gross effects -- hydrogen vs. helium vs. metals. However, Alan Karp has shown that the van Hoof effect is produced more by the ionization balance in the atmosphere than by the velocity field. In order to disentangle the velocity field from the ionization balance, you really have to observe a fairly large number of lines simultaneously. This hasn't been done yet, as far as I know, but it is something I am planning to do with IUE, if possible.

Hillendahl: I would just like to point out that the evidence for an outward velocity gradient increasing with radius doesn't necessarily mean that you have mass loss. This is an ordinary consequence of post-shock rarefaction waves. But you do get profiles like the ones that I showed in my paper. You do get outward flowing material, but it doesn't necessarily lead to mass loss. You have got to reach escape velocity for that.

Lesh: Yes, I agree with that, of course.

Hillendahl: Very elementary theory predicts that if you have pulsating stars with shock waves, you are likely to get a phenomenon called velocity doubling. That is, at various times you get apparent line cores that differ by a factor of two. I think it would be nice if people would indicate on their graphs a

Doppler shift corresponding to a factor of two, so you can see if this sort of thing is present. That will be one way of identifying a particular type of wave.

Lesh: Thank you. I will keep your suggestion in mind.

Baker: Have you tried to take the line shape change out of the radial velocity curve? What should we believe about the radial velocity curve, when there are these changes in line shape?

Lesh: I haven't done this myself. But it has been done by various people. If you are very careful about how you measure the radial velocity, curves which are apparently discontinuous -- like the one for BW Vul -- in fact, turn out to be continuous. The discontinuity can be fully explained by the change in the line shape or by the splitting into components. However, I suppose I should mention that BW Vul also shows a stillstand in its light curve which can't be explained away in the same way, and is probably due, in fact, to a shock wave in the atmosphere.