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*The Electric Potential Of Particles in Interstellar
Space Released From A Nuclear Waste Payload*



for

**NASA/George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama 35812**

Grant No. NSG-8068

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THE ELECTRIC POTENTIAL OF PARTICLES IN INTERSTELLAR
SPACE RELEASED FROM A NUCLEAR WASTE PAYLOAD

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FOREWORD

This technical report is the final documentation of work performed by Alabama A. & M. University for NASA/Marshall Space Flight Center under Grant No. NSG-8068. The period of performance covers July, 1979 through February, 1980. The results presented in this report are intended to assist NASA in its feasibility study of space disposal of nuclear waste materials.

TABLE OF CONTENTS

	Page
1. INTRODUCTION AND SUMMARY	1
1.1 Objectives and Scope	1
1.2 Summary of Results	1
2. CHARGING RATES	3
2.1 First Order Processes	3
2.1.1 Sticking of Solar Wind Electrons	3
2.1.2 Sticking of Solar Wind Ions	4
2.1.3 Ejection of Electrons by the Photoelectric Effect	4
2.1.4 Escape of Alpha Particles	5
2.1.5 Escape of Beta Particles	8
2.2 Higher Order Processes	9
2.2.1 Secondary Electron Emission by Alpha and Beta Radiation	10
2.2.2 Secondary Electron Emission by Gamma Radiation	11
3. EQUILIBRIUM POTENTIALS ON GRAINS	14
3.1 General Equation	14
3.2 Dependence Upon Distance from Sun	14
3.3 Dependence Upon Size of Grain	16
3.4 Dependence Upon Time	16
3.5 Maximum Potential Without Electrostatic Explosion	20
4. RELATIVE SIGNIFICANCE OF LORENTZ AND GRAVITATIONAL FORCES	22
5. NUMERICAL RESULTS WITH PW-4b REFERENCE WASTE	25
5.1 Significant Beta and Alpha Emitters	25
5.2 Equilibrium Potentials on the Grains	27
APPENDIX I — RANGE OF ALPHA AND BETA PARTICLES	28
APPENDIX II — FRACTION OF ALPHA AND BETA PARTICLES TO ESCAPE FROM A FINITE SIZE GRAIN	30
APPENDIX III — ENERGY SPECTRUM OF BETA PARTICLES	34
APPENDIX IV — EQUILIBRIUM POTENTIAL ON GRAINS	37
REFERENCES	41

THE ELECTRIC POTENTIAL OF PARTICLES IN INTERSTELLAR SPACE RELEASED FROM A NUCLEAR WASTE PAYLOAD

1. INTRODUCTION AND SUMMARY

1.1 Objectives and Scope

NASA is conducting an initial assessment regarding the desirability of disposing of certain high-level nuclear wastes in space as an augmentation of the currently planned national nuclear waste management program. Several space disposal concepts have been considered, including solar system escape, injection into the Sun, placement on the Moon, and injection into a stable solar orbit [1]. The currently favored concept is injection into a circular 0.85 astronomical unit (A.U.) solar orbit, that is, about halfway between the Earth and Venus. Orbital calculations indicate that at least for a million years, and probably more, this orbit is stable with respect to the Earth and Venus and would not intersect the orbits of either one [2].

The effect of the nuclear waste payload disintegration into small particles either by accidental, intentional, or natural means and the resulting long-term Earth reentry risk of the particles, however, require further analysis. Besides gravitational forces due to the Sun and nearby planets, these particles will be subjected to several perturbing forces. The most important of these are (1) the Poynting-Robertson drag; (2) the bombardment drag arising from the impact of the solar wind; (3) the Coulombic drag originating in the momentum transfer from passing solar wind components; and (4) the Lorentz force arising from the interaction with the interplanetary magnetic field.

If the grains were electrically neutral, only the first two perturbing interactions would have to be taken into account. If they are charged, however, the last two must also be considered. The relative significance of these depends upon the actual electric potential carried by them. It is desirable, then, to determine as accurately as possible the potential that would exist on such grains. This problem is addressed in this report.

1.2 Summary of Results

It is found that there are many mechanisms for charging a grain in the interplanetary medium. The most important processes are (1) capture of solar wind electrons; (2) capture of solar wind protons; (3) ejection of electrons through the photoelectric effect due to the solar

radiation; (4) escape of beta particles from beta emitters in the grain; and (5) escape of alpha particles from alpha emitters in the grain.

If the grains were non-radioactive (N.R.), only the first three processes would be present. For such grains, the potentials on them would have the following characteristics: (1) small (≤ 12 V); (2) independent of particle size and time; (3) independent of distance from Sun for distances greater than 1 A.U., and (4) for distances less than 1 A.U., the potentials decrease with distance. Appendix IV contains tables for the potentials on grains as a function of the distance from the Sun.

For radioactive grains, the last two processes would have to be taken into account also. In this case, the potentials on them have these characteristics: (1) dependent upon particle size, distance from the Sun, and time; (2) for spherical grains with radius less than 1 micron and located at a distance of 1 A.U. or less from the Sun, the potentials are the same as they are for the N.R. grains; (3) for grains with radii less than or equal to 0.05 micron, the potentials are the same as for the N.R. grains; (4) for any given distance from the Sun, the potential of a grain increases as the radius of the grain increase up to some maximum potential (this maximum is equal to the smaller of the potential which causes an electrostatic explosion and the equilibrium potential for the particular grain); (5) the characteristic time interval over which the equilibrium potentials change appreciably is 10 years; and (6) for waste which is over 1000 years old, the potentials on grains with a radius less than or equal to 10 microns are the same as for N.R. grains as far as this solar system is concerned. Numerical results are presented in Section 5 where the waste mix is assumed to be PW-4b [3].

2. CHARGING RATES

2.1 First Order Processes

In considering the rates at which charge will accumulate on a grain in interplanetary space, various mechanisms of charging must be looked at. The primary or first order processes are defined to be those which can change the equilibrium potential on a grain by at least 1 V. These are (1) the sticking of solar wind electrons; (2) the sticking of solar wind ions; (3) the ejection of electrons by the photoelectric effect; (4) the escape of alpha particles from alpha emitters; and (5) the escape of beta particles from beta emitters. Each of these will now be looked at individually, and the equation which determines the rate of charging for each will be given.

2.1.1 Sticking of Solar Wind Electrons

The solar wind consists of electrons and ions moving outward from the Sun as an aggregate at average speeds which increase as the distance from the Sun increases, eventually leveling off at some asymptotic value (usually 1 A.U.). The temperature of the solar wind is such that the average thermal speed of the electrons is very much greater than the mean speed of the wind itself. This means that for the electrons a Maxwellian distribution of speeds can be assumed. With this assumption, the rate at which units of positive charge will accumulate on a grain of radius a is given by [5]

$$\left(\frac{dN}{dt}\right)_e = -\xi_e \left(\frac{8\pi kT}{m_e}\right)^{\frac{1}{2}} n_e a^2 \times \begin{cases} 1 + \gamma & ; V > 0 \\ e^\gamma & ; V < 0 \end{cases} \quad (2.1)$$

where

ξ_e = fraction of incident electrons which remain on the grain

k = Boltzman's constant

T = temperature of the solar wind

m_e = mass of an electron

n_e = number of electrons/cm³

γ = eV/kT

e = magnitude of the charge on an electron

V = potential on the grain.

The sticking probability, f_e , for electrons of energies up to about 15 eV incident upon metallic surfaces lie in the range from 0.5 to 1.0 [5]. The number density, n_e , is about $5/\text{cm}^3$ at 1 A.U. and varies inversely with both the speed of the solar wind and the square of the solar distance [4]. Hence,

$$n_e = n_o \frac{V_{WE}}{V_W} \frac{r_o^2}{r^2} \quad (2.2)$$

where $n_o = 5/\text{cm}^3$, V_{WE} = speed of solar wind at $r_o = 1$ A.U., and V_W = speed of solar wind at a distance r .

2.1.2 Sticking of Solar Wind Ions

The ions in the solar wind are about 90 percent protons. Hence, it will be assumed that the solar wind consists of only electrons and protons and that it is electrically neutral overall. The thermal speeds of the protons are much smaller than the average speed of the solar wind aggregate. It follows, then, that the velocity of the protons is the same as that of the wind. In this case, the rate of gain of positive charge on a grain of radius a and potential V is given by [6]

$$\left(\frac{dN}{dt}\right)_p = \begin{cases} \epsilon_p n_p V_W \pi a^2 \left[1 - \frac{2eV}{n_p V_W^2}\right] & \frac{1}{2} m_p V_W^2 > eV \\ 0 & \frac{1}{2} m_p V_W^2 \leq eV \end{cases} \quad (2.3)$$

where ϵ_p = fraction of incident protons which remains on the grain and n_p = number of protons/ cm^3 .

2.1.3 Ejection of Electrons by the Photoelectric Effect

When radiation from the Sun is incident upon grains, electrons are ejected from them through the well known photoelectric effect.

The rate at which electrons are ejected depends upon the chemical composition of the grains as well as the potential that they carry. The theoretical rate for the rate of gain of positive units of charge for a grain of radius a , cutoff wavelength λ_1 , and located a distance r from the sun is given by [6]

$$\left(\frac{dN}{dt}\right)_{P.E.} = \frac{\pi a^2 r_0^2}{r^2} \int_0^{\lambda^*} \phi_0(\lambda) Y(\lambda) [1 - C(\lambda)] d\lambda ; \quad (2.4)$$

where

$\phi_0(\lambda) d\lambda$ = flux of solar quanta ($\text{cm}^{-2} \text{sec}^{-1}$) with wavelength between λ and $\lambda + d\lambda$ at an arbitrary distance r_0 from the Sun;

$Y(\lambda)$ = the photoelectric efficiency = $\frac{\text{number of ejected electrons}}{\text{number of incident photons}}$;

$$hc/\lambda^* = \begin{cases} hc/\lambda_1 + eV & V > 0 \\ hc/\lambda_1 & V \leq 0 \end{cases}$$

$C(\lambda)$ = the function which takes into account the fact that the ejected electrons will leave the surface with a certain energy distribution:

$$= \begin{cases} \frac{e^2 V^2 \lambda_1^2}{h^2 c^2 (\lambda_1/\lambda - 1)} & V > 0 \\ 0 & V \leq 0 \end{cases} \quad (2.5)$$

2.1.4 Escape of Alpha Particles

In this section, the rate of gain of positive units of charge due to the escape of alpha particles from the alpha emitters in the grain will be calculated. If there are n different such emitters, this rate is given by

$$\left(\frac{dN}{dt}\right)_\alpha = -2 \sum_{i=1}^n \frac{dN_{i,\alpha}}{dt} F_{\text{Esc}}(\alpha,i) , \quad (2.6)$$

where

$$\begin{aligned} \frac{dN_{i,\alpha}}{dt} &= \text{number of disintegrations per second of nucleus } i \\ &= N_{0,i,\alpha} \lambda_{i,\alpha} e^{-\lambda_{i,\alpha} t} \end{aligned} \quad (2.7)$$

$N_{0,i,\alpha}$ = total number of nucleus i present at $t = 0$

$\lambda_{i,\alpha}$ = disintegration time constant for α -decay of nucleus i

$F_{\text{Esc}}(\alpha, i)$ = fraction of the emitted alpha particles from nucleus i which escapes from the grain.

The function $F_{\text{Esc}}(\alpha, i)$ depends upon the potential and radius of the grain as well as the range of alpha particles in the grain material. This function is determined in Appendix II to be the following:

a) $V \geq 0$

$$F_{\text{Esc}}(\alpha, i) = \begin{cases} F_i(0, E_{i,\alpha}) & R_{i,\alpha} < 2a \\ 1 & R_{i,\alpha} \geq 2a \end{cases} \quad (2.8)$$

b) $V < 0; R_{i,\alpha} < 2a$

$$F_{\text{Esc}}(\alpha, i) = \begin{cases} F_i(-2eV, E_{i,\alpha}) & E_{i,\alpha} > -2eV \\ 0 & E_{i,\alpha} \leq -2eV \end{cases} \quad (2.9)$$

c) $V < 0; R_{i,\alpha} \geq 2a$

$$F_{\text{Esc}}(\alpha, i) = \begin{cases} F_i(-2eV, E_{i,\alpha}) & E_{i,\alpha} < \frac{-2eV}{(1 - 2a/R_{i,\alpha})} \\ 1 & E_{i,\alpha} \geq \frac{-2eV}{(1 - 2a/R_{i,\alpha})} \\ 0 & E_{i,\alpha} \leq -2eV \end{cases} \quad (2.10)$$

In the above equations, $R_{i,\alpha}$ is the average range of the alpha particles emitted from nucleus i (see Appendix I) and $E_{i,\alpha}$ is the kinetic energy of these emitted particles. The function F is given by

$$F_i(E_1, E_2) = 4(1 - E_1/E_2)^3 \frac{(R_{i,\alpha})^3}{(2a)^3} [1 - 3/4 (1 - E_1/E_2) (R_{i,\alpha}/2a)] . \quad (2.11)$$

Putting equations (2.6) and (2.7) together the rate of increase of charge due to alpha emission becomes

$$\left(\frac{dN}{dt}\right)_\alpha = -2 \sum_i N_{o,i,\alpha} \lambda_{i,\alpha} F_{Esc}(\alpha,i) e^{-\lambda_{i,\alpha} t} . \quad (2.12)$$

The number of alpha-emitting nuclei at $t = 0$, $N_{o,i,\alpha}$, can be written in terms of the mass of nucleus i present at $t = 0$, $M_{i,\alpha}$; the atomic mass of element i , $A_{i,\alpha}$; and Avogadro's number, N_A :

$$N_{o,i,\alpha} = \frac{M_{i,\alpha}}{A_{i,\alpha}} N_A . \quad (2.13)$$

If the grain has mass M_g , and a fraction $P_{i,\alpha}$ of this mass is element i , then

$$M_{i,\alpha} = P_{i,\alpha} M_g . \quad (2.14)$$

If the grain is assumed to be spherical with radius a and has a uniform mass density d , then $M_{i,\alpha}$ can be written, setting $M_g = 4/3 \pi a^3 d$, as

$$M_{i,\alpha} = \frac{4}{3} \pi a^3 d P_{i,\alpha} .$$

Assuming that this grain was at one time part of a larger quantity with total mass M_T , with a known distribution of individual nuclei masses, $m_{i,\alpha}$, then

$$M_{i,\alpha} = \frac{4}{3} \pi a^3 d \frac{m_{i,\alpha}}{M_T} \quad (2.15)$$

Equation (2.12) becomes

$$\left(\frac{dN}{dt}\right)_\alpha = \frac{4\pi}{3} \frac{a^3 d}{M_T} N_A \sum_{i=1}^n \frac{m_{i,\alpha} \lambda_{i,\alpha}}{A_{i,\alpha}} F_{Esc}(\alpha,i) e^{-\lambda_{i,\alpha} t} \quad (2.16)$$

which is the desired equation for the rate of increase of positive charge on a grain due to the emission of alpha particles.

2.1.5 Escape of Beta Particles

The escape of beta particles from the grain is handled analogously to the alpha escape process. However, beta emission is complicated by the fact that the kinetic energies of beta particles from any particular nucleus form a continuum rather than being discrete as in the alpha emitting case. The general equation that determines the rate of increase of position units of charge due to beta emission can, nevertheless, be written as

$$\left(\frac{dN}{dt}\right)_\beta = \frac{4\pi}{3} \frac{a^3 d}{M_T} N_A \sum_{i=1}^{n'} \frac{m_{i,\beta} \lambda_{i,\beta}}{A_{i,\beta}} F_{Esc}(\beta,i) e^{-\lambda_{i,\beta} t} \quad (2.17)$$

where n' is the number of different beta emitters, and the other symbols represent either the same or the corresponding quantities for beta emitters as they do in the alpha emitting case.

The complexity arising from the fact that the kinetic energy spectrum is a continuum is embedded in the function $F_{Esc}(\beta,i)$ which gives the fraction of the beta particles from nucleus i that escapes from the grain. This function is determined in Appendix II to be given by the following (for a grain of radius a and potential V):

a) $V < 0$

$$F_{\text{Esc}}(\beta, i) = F_i'(0, E_{i, \beta}) \quad (2.18)$$

b) $V \geq 0$

$$F_{\text{Esc}}(\beta, i) = \begin{cases} 0 & E_{i, \beta} \leq eV \\ F_i'(eV, E_{i, \beta}) & E_{i, \beta} > eV \end{cases}, \quad (2.19)$$

where

$E_{i, \beta}$ = the maximum kinetic energy of the beta spectrum for nucleus i ;

$$F_i'(E_1, E_2) = \frac{\int_{E_1}^{E_2} F_i(E_1, E) N_i(E) dE}{\int_0^{E_{i, \beta}} N_i(E) dE}; \quad (2.20)$$

$N_i(E) dE$ = number of beta particles emitted from nucleus i with kinetic energy between E and $E + dE$ (see Appendix III);

$$F_i(E_1, E) = \begin{cases} 4 \left(1 - \frac{E_1}{E}\right)^3 \left(\frac{R}{2a}\right)^3 \left[1 - \frac{3}{4} \left(1 - \frac{E_1}{E}\right) \left(\frac{R}{2a}\right)\right] & R < 2a \\ 1 & R > 2a \end{cases} \quad (2.21)$$

$R = R(E)$ = the average range of a beta particle with energy E in the grain's material (see Appendix I).

2.2 Higher Order Processes

There are various higher order processes which tend to charge grains in interstellar space. The most dominant of these result from the emission of secondary electrons as charged particles or gamma rays pass

through them. The relative significance of the emission of such electrons by incident solar wind electrons and protons has already been investigated by Wyatt [6]. He found that their rate of emission is about two orders of magnitude less than that of electron ejection by solar quanta.

Spitzer [7] discussed the emission of secondary electrons by cosmic rays and concluded that this rate is negligible compared to the rate of sticking of solar wind electrons. Two other mechanisms for secondary electron emission must be investigated. These are (1) the ejection caused by emitted alpha and beta particles and (2) the ejection of electrons by emitted gamma rays.

2.2.1 Secondary Electron Emission by Alpha and Beta Radiation

When an alpha particle of speed V collides with an electron at rest, it is a straightforward calculation to show that the speed of the electron after the collision is given by

$$u = \frac{2 M_{\alpha}}{M_{\alpha} + M_e} V \cos \theta \quad (2.22)$$

where θ is the scattering angle of the electron. Since $M_e/M_{\alpha} \approx 0$, this becomes

$$u = 2 V \cos \theta \quad (2.23)$$

Hence, the maximum velocity of the electrons is given by

$$u_{\max} = 2 V \quad (2.24)$$

This means that the maximum kinetic energy of the scattered electrons is

$$(K.E.)_{\max} = \frac{4 M_e}{M_{\alpha}} E_{\alpha} = \frac{E_{\alpha}}{1836} \quad (2.25)$$

Using $E_{\alpha} = 6 \text{ MeV}$ for the alpha particle energy yields a value of 3.3 keV for the maximum electron energy. According to Appendix I, the range of these electrons is only 0.027 micron. Hence, even the maximum energy electrons will be stopped before escaping from all but the smallest of grains.

Furthermore, the number of electrons scattered by an angle θ is proportional to $\tan^2\theta$ [8] which means that most of the electrons are emitted around 90° with nearly zero energy. In fact, most of the electrons will be scattered with energies between zero and a few volts. It is concluded from this that secondary electron emission by alpha particles can be neglected.

For incident beta particles, the scattered electrons will have a speed given by $U = V \cos \theta$. Because beta particles, in general, are much faster than alpha particles, the number of secondary electrons emitted by the beta is much smaller. Again, the energy of the majority of these electrons is only a few volts [13]. Therefore, they will either not traverse to the surface of the grains or not have enough energy to escape from the Coulombic attraction of the positively charged grains.

2.2.2 Secondary Electron Emission by Gamma Radiation

Gamma particles that are emitted by the various isotopes in the grain can interact with the electrons by the photoelectric and the Compton scattering processes. These gammas are emitted in conjunction with an alpha or a beta particle. Hence, if the number of electrons that are ejected by each gamma is not too large, then the isotopes which are most significant in producing secondary electrons are the same as those which are significant for the emission of beta and alpha particles. These isotopes are given in the tables in Section 5.

In determining the relative significance of the secondary electrons, it is sufficient to determine the fraction of emitted gamma quanta which produces such electrons. This is true because if the gammas originate with beta emitters, this fraction is identical with the ratio of the number of secondary electrons to the number of primary electrons. Also, if the gammas are emitted by alpha emitters this fraction is the ratio of secondary electrons to primary alphas which, if small, would mean an even smaller secondary to primary electron ratio since the number of alphas emitted is significantly smaller than the number of betas.

For Compton scattering, if N represents the number of electrons scattered per unit time, and N_0 represents the number of incident quanta per unit time per unit area then the scattering cross section per electron is given by

$$\sigma_S = N/N_0 \quad (2.26)$$

This means that in a length ℓ the number of electrons scattered by the Compton effect relative to the number of emitted gammas is given by

$$f_S = \sigma_S d_e \ell \quad , \quad (2.27)$$

where d_e is the number of electrons per unit volume. The length ℓ can be taken as the radius of the grain.

An upper limit to the number density of electrons can be obtained by assuming that the entire mass of the grain comes from an equal number of protons, neutrons, and electrons. If d is the mass density of the grain, one obtains for d_e the following:

$$d_e = \frac{d}{2 M_n} \quad , \quad (2.28)$$

where M_n is the mass of a nucleon (1.67×10^{-24} gm). Taking d as 6.7 gm/cm^3 and the radius of the grain as 1 micron, the fraction of scattered electrons becomes

$$f_S = (2 \times 10^{20}) \sigma_S \quad . \quad (2.29)$$

The Compton scattering cross section is a function of the gamma energy but is not dependent on the atomic number of the scattering medium [9]. An upper limit for σ_S is about $5 \times 10^{-25} \text{ cm}^2/\text{electron}$. This gives an upper limit for f_S as 10^{-4} which means that Compton scattered electrons are negligible relative to the number of primary electrons.

For the photoelectric effect, the absorption cross section σ_γ per atom depends critically upon the energy of the gamma as well as the atomic number of the absorbing material [9]. In fact σ_γ increases as the atomic number increases and as the energy decreases. Similar to equation (2.27), the number of electrons ejected by the photoelectric effect is given by

$$f_S = \sigma_\gamma d_A \ell \quad , \quad (2.30)$$

where d_A is the number of atoms per unit volume.

To obtain an upper limit for f_S , one can assume that the grain is a uniform distribution of a large Z element such as lead. Then the number density of atoms is given by

$$d_A = \frac{d}{M_{pb}} \quad , \quad (2.31)$$

where M_{pb} is the mass of a lead atom (3.44×10^{-22} gm). Again, taking d as 6.7 gm/cm^3 and ℓ as 1.0 micron, one obtains

$$f_S = (2 \times 10^{18}) \sigma_\gamma \quad . \quad (2.32)$$

The range of σ_γ in lead for the energies of interest lies between 10^{-22} and $10^{-20} \text{ cm}^2/\text{atom}$. This gives 10^{-2} as an upper limit value for f_S .

Hence, as previously discussed since this number is small, the relative significance of these secondary electrons is negligible.

3. EQUILIBRIUM POTENTIALS ON GRAINS

3.1 General Equation

The processes which add or subtract positive charge from particles in the interplanetary medium have been considered in Section 2. Concentrating on the first order processes, the ones that add positive charge are the sticking of solar wind protons, the ejection of electrons by the photoelectric effect, and the escape of beta particles from the beta emitters that may be present. The rate at which each of these processes takes place is a function of the potential on the grain. They all decrease as the potential becomes more and more positive, and they either increase or remain constant as the potential becomes more negative.

The processes which add negative charge to the grains are the sticking of solar wind electrons and the escape of alpha particles from the alpha emitters. The rates of these two decrease as the potential becomes more negative and either increases or remains constant as the potential becomes more positive.

Consequently, there exists an equilibrium potential at which the rate of gain of positive charge and the rate of gain of negative charge balance each other. At this potential, the net rate of gain of positive units of charge is equal to zero:

$$\left(\frac{dN}{dt}\right)_{\beta} + \left(\frac{dN}{dt}\right)_{\alpha} + \left(\frac{dN}{dt}\right)_{e} + \left(\frac{dN}{dt}\right)_{p} + \left(\frac{dN}{dt}\right)_{P.E.} = 0 \quad (3.1)$$

3.2 Dependence Upon Distance from Sun

The rates of emission of alpha and beta particles do not depend upon the distance from the Sun, but the other processes do. From equations (2.1) and (2.2), the rate that electrons will stick to the grain is inversely proportional to the square of the solar distance plus any dependence upon this distance of the solar wind speed. As was mentioned in Section 2.1, the solar wind is assumed to be dependent upon the solar distance up to some maximum distance r_0 after which it is constant. For distances less than 1 A.U., data [4] indicate that the average speed of the solar wind is approximately proportional to the square root of the distance. Hence, the solar distance (r) dependence of the electron capture rate is given by

$$\left(\frac{dN}{dt}\right)_{e|r} = \left(\frac{r_0}{r}\right)^{2+n} \left(\frac{dN}{dt}\right)_{e r_0} \quad (3.2)$$

where

$$n = \begin{cases} 0.5 & ; \quad r \leq r_0 \\ 0 & ; \quad r > r_0 \end{cases} .$$

From equations (2.2) and (2.3), taking the proton number density the same as the electron's, it is seen that the rate of capture of solar wind protons is inversely proportional to the square of the solar distance. Also, as indicated in equation (2.4), the rate of photo-ejection of electrons is inversely proportional to the square of this distance.

Hence,

$$\left(\frac{dN}{dt}\right)_p \Big|_r = \left(\frac{r_0}{r}\right)^2 \left(\frac{dN}{dt}\right)_p \Big|_{r_0} \quad (3.3)$$

and

$$\left(\frac{dN}{dt}\right)_{P.E.} \Big|_r = \left(\frac{r_0}{r}\right)^2 \left(\frac{dN}{dt}\right)_{P.E.} \Big|_{r_0} \quad (3.4)$$

Substituting equations (3.2), (3.3), and (3.4) into equation (3.1) and multiplying by $(r/r_0)^2$, one obtains the following equation for the equilibrium potential:

$$\left(\frac{r}{r_0}\right)^2 \left[\left(\frac{dN}{dt}\right)_p + \left(\frac{dN}{dt}\right)_r \right] + \left(\frac{r_0}{r}\right)^n \left(\frac{dN}{dt}\right)_e \Big|_{r_0} + \left(\frac{dN}{dt}\right)_p \Big|_{r_0} + \left(\frac{dN}{dt}\right)_{P.E.} \Big|_{r_0} = 0 \quad (3.5)$$

This equation shows that the radioactivity of the grains becomes more and more important as the solar distance increases. At very large distances, the last three terms become negligible, and the equilibrium potential becomes independent of the solar distance. For small distances ($r < r_0$), the first two terms which are due to the radioactivity of the grains become less and less important. However, even if the radioactivity were negligible, the equilibrium potential would still depend upon r for $r < r_0$ because of the factor $(r_0/r)^n$.

3.3 Dependence Upon Size of Grain

Looking at equations (2.1), (2.3), and (2.4), it is seen that the electron sticking rate, the proton sticking rate, and the photo-ejection rate all are proportional to the square of the radius a of the grain. That is,

$$\left(\frac{dN}{dt}\right)_q \Big|_a = \left(\frac{a}{a_0}\right)^2 \left(\frac{dN}{dt}\right)_q \Big|_{a_0} \quad (3.6)$$

where $q = e, P, \text{ or } P.E.$ Putting this into equation (3.1) and multiplying by $(a_0/a)^2$, the following equation for the equilibrium potential is obtained:

$$\begin{aligned} \left(\frac{a_0}{a}\right)^2 \left[\left(\frac{dN}{dt}\right)_\alpha (a) + \left(\frac{dN}{dt}\right)_\beta (a) \right] + \left(\frac{dN}{dt}\right)_e \Big|_{a_0} + \left(\frac{dN}{dt}\right)_P \Big|_{a_0} \\ + \left(\frac{dN}{dt}\right)_{P.E.} \Big|_{a_0} = 0 \quad (3.7) \end{aligned}$$

The dependence of the radioactive escape rates upon the radius of the grain is complex as was shown in sections 2.1.4 and 2.1.5. For grain radii that are very much less than the average range of the emitted particles, however, these rates are essentially proportional to the cube of the radius. In the present case of interest (see Section 5), for $a < 1$ micron the radioactive escape rates become proportional to a^3 . This means the smaller the grain, the less important the radioactive rates are. Conversely, the larger the grains are, the more important the radioactivity becomes relative to the other processes.

3.4 Dependence Upon Time

For a given grain size at a given distance from the Sun, the charging rates due to the sticking of electrons and protons and to the ejection of electrons by the photoelectric effect are independent of time. The rates of emission of radioactive particles, however, are dependent upon time. This will give the equation for the equilibrium potential a non-factorable time dependence and, hence, the equilibrium potential is a function of time.

According to equations (2.16) and (2.17), the time dependence of the rates of charging due to the emission of beta or alpha particles is given by

$$\frac{dN}{dt} = \sum_i c_i e^{-\lambda_i t} = \sum_i D_i(t) \quad (3.8)$$

This shows that each emitter i contributes to the charging rate with a time dependent term $D_i(t)$ which decreases exponentially with a time constant.

$$\frac{1}{\lambda_i} = \frac{\text{half-life}}{\text{Ln}(2)} \quad (3.9)$$

Hence, at times near $t = 0$, the shorter half-life nuclei could contribute appreciably, but as time goes on, only nuclei with relative long half lives would contribute.

Equation (3.8), however, only takes into account the fact that parent isotopes are being depleted because they are decaying away. It does not include the fact that daughter nuclei are building up due to this decay of the parent. If the daughter nucleus is radioactive, it must also be included in the terms in equation (3.8). The significance of this can be really pronounced if the parent is short-lived and the daughter is long-lived. To take this into account quantitatively, one must look at the differential equation which governs the number of daughter nuclei present at an arbitrary time.

Let N_p and N_d be the number of parent nuclei and daughter nuclei present at time t , respectively. Also, let N_{p0} and N_{d0} be these same numbers at the initial time $t = 0$. If the parent decays with a disintegration constant λ_p and the daughter decays with the constant λ_d , then the equations that give N_p and N_d are the following:

$$N_p = N_{p0} e^{-\lambda_p t} \quad (3.10)$$

$$\frac{dN_d}{dt} = -\lambda_d N_d + \lambda_p N_p \quad (3.11)$$

Solving equation (3.11) for N_d with the initial condition that $N_d = N_{do}$ at $t = 0$ yields

$$N_d = \frac{\lambda_p}{\lambda_d - \lambda_p} N_{po} \left(e^{-\lambda_p t} - e^{-\lambda_d t} \right) + N_{do} e^{-\lambda_d t} \quad (3.12)$$

The rate of decay of the daughter nucleus is the first term in equation (3.11), or

$$\left(\frac{dN_d}{dt} \right)_{\text{decay}} = -\lambda_d N_d \quad (3.13)$$

Using equation (3.12) for N_d in equation (3.13) gives the following equation:

$$\left(\frac{dN_d}{dt} \right)_{\text{decay}} = -\lambda_d N_{do} e^{-\lambda_d t} \left[1 + \left(\frac{\lambda_p}{\lambda_d - \lambda_p} \right) \frac{N_{po}}{N_{do}} \right] - \left(\frac{\lambda_d}{\lambda_d - \lambda_p} \right) \lambda_p N_{po} e^{-\lambda_p t} \quad (3.14)$$

This equation is of the form

$$\left(\frac{dN_d}{dt} \right)_{\text{decay}} = A \left(\frac{dN_d}{dt} \right)_{\text{decay}}^0 + B \left(\frac{dN_p}{dt} \right)_{\text{decay}}^0 \quad (3.15)$$

where $(dN/dt)_{\text{decay}}^0$ is the decay rate when buildup is not taken into account. Hence, the net effect of the buildup of the daughter nucleus is that the constant D_i in equation (3.8) for this nucleus gets multiplied by the factor A, and the constant D_i for the parent nucleus is multiplied by B. That is,

$$D_i \text{ (parent)} \rightarrow B D_i \text{ (parent)} \quad (3.16)$$

$$D_i \text{ (daughter)} \rightarrow A D_i \text{ (daughter)} \quad (3.17)$$

where

$$A = 1 + \frac{\lambda_d}{\lambda_d - \lambda_p} \frac{N_{po}}{N_{do}} \quad (3.18)$$

and

$$B = \frac{\lambda_d}{\lambda_d - \lambda_p} \quad (3.19)$$

Note that the substitutions (3.16) and (3.17) could possibly include the following cases: (1) the daughter nucleus may not have been present before the buildup began, and (2) the parent nucleus may be a different type of emitter than the daughter, but it will still be included in equation (3.8) as if it were the same.

According to equation (2.13) the number of nuclei present at $t = 0$ can be written in terms of the initial mass M_o , the atomic mass A , and Avogadro's number N_A as

$$N_o = \frac{M_o}{A} N_A \quad (3.20)$$

Hence, the constant A can be written as

$$A = 1 + \frac{\lambda_p}{\lambda_d - \lambda_p} \frac{M_{po} A_d}{M_{do} A_p} \quad (3.21)$$

In summary, the equilibrium potential is time dependent because of the radioactivity of the grains. The rates of charging due to this radioactivity is given by equation (3.8). The constant C_i in this equation is obtained for alpha and beta emitters by comparing it with equations (2.16) and (2.17), respectively. When buildup of any particular daughter nucleus is taken into account, the constants C_i for the daughter and its parent must be multiplied by the factors A and B , respectively. Sometimes, this may require that either the parent or the daughter be added to the equation. In the case of the parent, it may have to be added even if it is not the same type of emitter as the daughter. It should also be stated that, in general, chains longer than two nuclei would have

to be considered. These longer chains, however, were ruled out for the specific application of this work by checking the effects of other members of the chain numerically.

3.5 Maximum Potential Without Electrostatic Explosion

For a given grain size and composition, there exists a maximum electrical potential which it can withstand before it breaks up. The magnitude of this potential depends upon the tensile strength of the grain.

To calculate the maximum potential, it will be assumed that at the critical point the total charge Q is divided into two equal parts of $Q/2$ each. Also the two parts are assumed to be separated by one radius length a . The Coulombic force of repulsion between the two parts is then given by (CGS units)

$$F_c = \frac{Q^2}{4a^2} \quad . \quad (3.22)$$

The force that is tending to hold the two parts together is the tensile strength T times the cross sectional area:

$$F_t = T \pi a^2 \quad . \quad (3.23)$$

At the point of breakup, these two forces are equal:

$$F_c = F_t \quad (3.24)$$

or

$$\frac{Q^2}{4a^2} = T \pi a^2 \quad . \quad (3.25)$$

This says that the maximum potential then is given by

$$V_{\max} = \frac{Q}{a} = 2a (\pi T)^{\frac{1}{2}} \quad . \quad (3.26)$$

The tensile strengths for most metals lie in the range 10^8 dyne/cm² to 10^{10} dyne/cm² [10]. Hence, if a typical value of 2×10^9 dyne/cm² is taken, then

$$V_{\max} = (1.585 \times 10^5) a \quad . \quad (3.27)$$

If a is expressed in microns and V_{\max} is in V, then

$$V_{\max} = (4655)a \text{ V} \quad . \quad (3.28)$$

Since the maximum potential depends upon the square root of the tensile strength, even if it changed by a factor of 4 or 5, the maximum given in equation (3.28) would still be larger than the potential expected on grains (see Section 5).

4. RELATIVE SIGNIFICANCE OF LORENTZ AND GRAVITATIONAL FORCES

It is of interest to determine the relative significance of the Lorentz and gravitational forces on a grain in interplanetary space as a function of the grain's size and voltage. For a grain of radius a , density d , and charge Q , the magnitudes of the solar gravitational and Lorentz forces are given by (CGS units)

$$F_G = \frac{G M_S}{r^2} \left(\frac{4}{3} \pi a^3 d \right) \quad (4.1)$$

and

$$F_L = \frac{QvB}{c} \quad (4.2)$$

respectively. Here, r is the distance from the Sun, M_S is the Sun's mass, v is the solar wind's speed, B is the component of the interplanetary magnetic field which is perpendicular to the direction of the solar wind velocity (radial), and c is the speed of light (3×10^{10} cm/sec).

Both B and v depend upon the solar distance r , but B can be written in terms of v as [11]

$$B = \frac{B_0}{vr} \quad (4.3)$$

where B_0 is a constant. There are field fluctuations as a function of time, but for comparison purposes typical values can be used. Hence, at the orbit of the Earth, a typical value for the magnetic field is 3×10^{-5} gauss and the speed of the solar wind is 450 km/sec. This gives

$$B = \left(3 \times 10^{-5} \frac{v_0}{v} \frac{r_0}{r} \right) \text{ gauss} \quad (4.4)$$

and

$$F_L = \frac{3 \times 10^{-5}}{c} Q v_0 \frac{r_0}{r} \quad (4.5)$$

Putting in $v_0 = 450 \text{ km/sec} = 450 \times 10^5 \text{ cm/sec}$, this becomes

$$F_L = 1350 \frac{Q}{c} \frac{r_0}{r} \quad (4.6)$$

where $r_0 = 1 \text{ A.U.}$,

Taking the ratio of equation (4.6) to equation (4.1), one obtains

$$\frac{F_L}{F_G} = (1.81 \times 10^{-8}) \frac{Q}{a} \frac{r}{a^2 d} \quad (4.7)$$

where r is measured in A.U., and a and d are in cm and gm/cm^3 , respectively. Since Q/a is the voltage V on the grain, equation (4.7) can be written as

$$\frac{F_L}{F_G} = \frac{(0.006) V r}{a^2 d} \quad (4.8)$$

where V is measured in V; r in A.U.; a in microns; and d in gm/cm^3 . Taking the reference value of 6.7 gm/cm^3 as d , this ratio becomes [3]

$$\frac{F_L}{F_G} = (8.96 \times 10^{-4}) \frac{V r}{a^2} \quad (4.9)$$

The Lorentz force can be said to be significant when F_L/F_G is greater than or equal to 0.1. Using this as a criterion, one can find the maximum potential that a grain of d given radius and solar distance can have without perturbing the gravitational force significantly. This maximum is given for various radii and solar distances in Table 1. The equation used, in general, is

$$V_{\text{max}} = 111.61 \frac{a^2}{r} \quad (4.10)$$

TABLE 1. MAXIMUM POTENTIAL THAT GRAINS CAN HAVE WITHOUT
 PERTURBING THE SOLAR GRAVITATIONAL FORCE SIGNIFICANTLY
 (r = solar distance; a = radius of grain)

r (A.U.)	a (microns)	V _{max} (V)
0.1	0.01	0.11
	0.10	11.16
	1.00	1116.10
0.5	0.01	0.02
	0.10	2.23
	1.00	223.22
1.0	0.01	0.01
	0.10	1.12
	1.00	111.61
5.0	0.01	0.002
	0.10	0.22
	1.00	22.32
	10.00	2232.20
10.0	0.01	0.001
	0.10	0.11
	1.00	11.16
	10.00	1116.10

5. NUMERICAL RESULTS WITH PW-4b REFERENCE WASTE

5.1 Significant Beta and Alpha Emitters

As was discussed in Section 3, the equilibrium potentials on the grains is found by solving equation (3.1). The radioactive decay rates in this equation, however, are time dependent as described in equation (3.8). In the PW-4b waste mix, there are many radioactive isotopes, but only a few of them contribute significantly to the decay rates at any given time.

Neglecting the buildup of daughter nuclei, for the moment, the isotopes which contribute significantly are shown in Tables 2 through 7.

TABLE 2. BETA EMITTERS FOR $0 < t < 200$ YEARS

Isotope	E_{\max} (MeV)
Sr ⁹⁰	0.546
Y ⁹⁰	2.270
Cs ¹³⁴	0.658
Cs ¹³⁷	0.514
Pm ¹⁴⁷	0.225
Eu ¹⁵⁴	0.580

TABLE 3. BETA EMITTERS FOR $200 < t < 500$ YEARS

Isotope	E_{\max} (MeV)
Sr ⁹⁰	0.546
Cs ¹³⁷	0.514
Sm ¹⁵¹	0.076

TABLE 4. BETA EMITTERS FOR $500 < t < 700$ YEARS

Isotope	E_{\max} (MeV)
Sr ⁹⁰	0.546
Cs ¹³⁷	0.514
Sm ¹⁵¹	0.076
Zr ⁹³	0.063
Tc ⁹⁹	0.292
Cs ¹³⁵	0.210

TABLE 5. BETA EMITTERS FOR $700 < t < 7 \times 10^6$ YEARS

Isotope	E_{\max} (MeV)
Sm ¹⁵¹	0.076
Zr ⁹³	0.063
Tc ⁹⁹	0.292
Cs ¹³⁵	0.210
Pd ¹⁰⁷	0.035

TABLE 6. ALPHA EMITTERS FOR $0 < t < 1000$ YEARS

Isotope	E (MeV)
Am ²⁴¹	5.49
Cm ²⁴⁴	5.81

TABLE 7. ALPHA EMITTERS FOR $t > 1000$ YEARS

Isotope	E (MeV)
Am ²⁴¹	5.49
Np ²³⁷	4.52
Pu ²³⁹	5.16
Pu ²⁴⁰	5.17
Am ²⁴³	5.28

Taking into account the buildup of daughter nuclei, it is found that the decay schemes for which either the factor A is significantly different from unity or the factor B is significantly different from zero [see equation (3.15)] are the following:



For the reaction in equation (5.1), $A = -0.25$ and $B = 1.0$. This means that for the times when Y^{90} is important, its contribution is multiplied by -0.25 , and when Sr^{90} is important, another term will be added to the beta emitters which has the same properties as Sr^{90} , but the energy of the emitted particles is that of Y^{90} .

For the reaction in equation (5.2), $A = 1.167$ and $B = 0$. Hence, at times when Np^{237} is important, its scaling factor $D_i(t)$ in equation (3.8) should be multiplied by 1.167 .

For the reaction in equation (5.3), $A = -0.86$ and $B = 1.0$. Therefore, when Pa^{233} is important, its scaling factor is to be multiplied by -0.86 . Also, Np^{237} must be added to the beta emitters although it is an alpha emitter. When it is added, it will retain the same scaling factor, but the energy spectrum of the emitted particles is to be taken as the same as that of Pa^{233} . It does not become significant, however, until after 500 years.

5.2 Equilibrium Potentials on the Grains

Solving equation (3.1) numerically, the equilibrium potentials on grains as a function of time, grain size, and solar distance were determined. These results are tabulated in Appendix IV. By comparing the potentials found there with the maximum potential in Table 1 or equation (4.10), one can determine for which size grains one has to be concerned about the charge on them for a given solar distance and time. For example, assume one wants to determine for what times and distances that the charge on a 5 micron radius grain become important. Using equation (4.10), it is found that the maximum potential that the grain can have without becoming significant is 558.05 V at $r = 5$ A.U. and 279.03 V at $r = 10$ A.U. From Appendix IV, it is seen that at $r = 5$ A.U., the potential on this size grain is less than the needed maximum for all times, and at 10 A.U. the potential is significant for all times less than 20 years.

It can also be seen in this way that for solar distances less than or equal to 2 A.U. and for all times, the maximum size grain with which one has to be concerned has a radius of 0.5 micron.

APPENDIX I

RANGE OF ALPHA AND BETA PARTICLES

A. Beta Particles

The range-energy relation for electrons is not strongly dependent upon the atomic number of the stopping material [12]. This means that the ranges for all metallic type materials are essentially the same, and hence one can use the practical range of aluminum as found experimentally [13]:

$$R_{\beta} = \begin{cases} \frac{0.412}{d} E^n \text{ cm} & E \leq 2.5 \text{ MeV} \\ \frac{(.530 E - .106) \text{ cm}}{d} & 2.5 \text{ MeV} \leq E \leq 20 \text{ MeV} \end{cases} \quad (I.1)$$

where, E is the kinetic energy of the beta particle, d is the density in gm/cm^3 of the stopping material, and $n = 1.265 - 0.0954 \ln E$.

B. Alpha Particles

The ranges of heavy particles such as alpha particles do depend upon the atomic number of the stopping material. The stopping material that is of interest in this work is heterogeneous; hence, an average value for the atomic number must be obtained. The particular manner in which this averaging is carried out is really not very important since it is found that the contribution due to the escape of alpha particles is actually a second order effect (see Section 5).

If one simply calculates the average atomic number of the PW-4b mix [3] as it exists in oxide form, a value around that of lead is obtained. Hence, for this work the ranges of the alpha particles will be those for lead.

In general, the range of alpha particles in a material is related to the range of protons in that material by [12]

$$R_{\alpha}(E) = R_p \left(\frac{E}{4} \right) \quad (I.2)$$

The energies E of the alpha particles that are of interest all lie in the range from 4 to 8 MeV. Hence, the corresponding proton energies are

in the range from 1 to 2 MeV. The range of protons in lead at the energies 1 and 2 MeV are 7.90 mg/cm^2 and 2.505 mg/cm^2 , respectively [12]. It is reasonable to assume that the average range of protons between 1 and 2 MeV as a function of the energy is a straight line. This gives

$$R_p(E') = (17.15E' - 9.25) \text{ mg/cm}^2; 1 \text{ MeV} < E' < 2 \text{ MeV}. \quad (\text{I.3})$$

Equation (I.2) then becomes

$$R_\alpha(E) = (4.288 E - 9.25) \text{ mg/cm}^2; 4 \text{ MeV} < E < 8 \text{ MeV} .$$

In terms of cm, the range of alpha particles can be written using the density d of the material as

$$R_\alpha(E) = \frac{(4.288 \times 10^{-3} E - 9.25 \times 10^{-3}) \text{ cm}}{d} .$$

APPENDIX II

FRACTION OF ALPHA AND BETA PARTICLES TO
ESCAPE FROM A FINITE SIZE GRAIN

Consider the situation where alpha or beta particles are emitted uniformly throughout a spherical volume of radius a . Each particle is assumed to have initial energy E_0 and average range $R = R(E_0)$. Then if a particle travels a distance X ($\leq 2a$) to reach the surface, its energy at the surface is given by

$$E = E_0 (1 - X/R) \quad . \quad (II.1)$$

The fraction of particles $f(X) dX$ which travel a distance in the range $(X, X+dX)$ to reach the surface is given by the fraction of the total volume that contains points with distances to the surface in this range. It is a straightforward calculation to show that this volume is given by

$$V(X, X+dX) dX = 2\pi X^2 (1 - X/2a) dX \quad (II.2)$$

and, hence

$$f(X) dX = \frac{3}{2} \frac{X^2}{a^3} (1 - X/2a) dX \quad . \quad (II.3)$$

The fraction of particles $f'(E) dE$ with energy in the range $(E, E+dE)$ is the same as the fraction that travels a distance in the range $(X, X + dX)$. That is,

$$f'(E) dE = f(X) dX \quad . \quad (II.4)$$

Using equation (II.1) to obtain

$$X = R(1 - E/E_0) \quad (II.5)$$

and

$$\frac{dX}{dE} = -R/E_0, \quad (II.6)$$

equation (II.4) gives

$$f'(E) dE = 12 \frac{(E_0 - E)^2}{E_0^3} \left(\frac{R}{2a}\right)^3 \left(1 - (1 - E/E_0) \frac{R}{2a}\right) dE \quad (II.7)$$

for all E such that

$$(1 - E/E_0) R/(2a) \leq 1$$

or

$$E \geq E_0 (1 - 2a/R) \quad (II.8)$$

Condition (II.8) takes into account the fact that some particles may lose all their energy before they reach the surface.

If there is some minimum energy E_{\min} that the particles need to have at the surface to escape, then the fraction of the emitted particles with initial energy E_0 to escape is given by

$$F(E_{\min}, E_0) = \int_{E_{\min}}^{E_0} f'(E) dE \quad (II.9)$$

Using equation (II.7) for $f(E)$, this fraction is easily found to be

$$F(E_{\min}, E_0) = \begin{cases} 4 (1 - E_{\min}/E_0)^3 \left(\frac{R}{2a}\right)^3 \left[1 - \frac{3}{4} (1 - E_{\min}/E_0) \left(\frac{R}{2a}\right)\right] & E_{\min} \geq E_0 \\ 0 & E_{\min} \geq E_0 \\ 1 & E_{\min} \leq E_0 (1 - 2a/R) \end{cases} \quad (II.10)$$

The fact that $F(E_{\min}, E_0) = 1$ when $E_{\min} \leq E_0 (1 - 2a/R)$ comes from condition (II.8) which indicates that all the particles reach the surface with a certain minimum energy. If that minimum is negative, then all the particles do not reach the surface.

Since the initial energies of beta particles emitted from a nucleus form a continuum with number density $N(E_0)$ (see Appendix III), whereas the emitted alpha particles are essentially monoenergetic with an energy E_0 , then it is necessary to look at alpha particles and beta particles separately. Also whether particles that reach the surface will escape or not depends upon the sign and the magnitude of the electric potential V on the surface. Hence, it is necessary to examine four different cases:

- 1) Alpha Particles; $V \geq 0$.

In this case, the minimum energy needed for an alpha particle is zero. Hence the fraction to escape is given by

$$F_{\text{Esc}} = F(0, E_0) \quad . \quad (\text{II.11})$$

- 2) Alpha Particles; $V < 0$.

When the potential is negative, the alpha particles that reach the surface need a minimum energy of $2e|V|$ to overcome the Coulombic attraction. The fraction to escape is then

$$F_{\text{Esc}} = F(2e|V|, E_0) \quad . \quad (\text{II.12})$$

- 3) Beta Particles; $V \leq 0$.

In general, beta particles are emitted with an energy distribution $N(E_0)$ where the energies range from zero up to a maximum E_{\max} . In general, then, the fraction of beta particles to escape is given by

$$F_{\text{Esc}}(E_{\min}, E_{\max}) = \frac{\int_0^{E_{\max}} F(E_{\min}, E_0) N(E_0) dE_0}{\int_0^{E_{\max}} N(E_0) dE_0} \quad . \quad (\text{II.13})$$

For $V \leq 0$, the minimum energy needed is zero and, therefore, the fraction of beta particles to escape is given by

$$F_{\text{Esc}} = F_{\text{Esc}}(0, E_{\text{max}}) \quad (\text{II.14})$$

4) Beta Particles; $V > 0$.

In this case, the beta particles need a minimum energy of eV to overcome the Coulombic attraction. Hence, the fraction to escape in this case is

$$F_{\text{Esc}} = F_{\text{Esc}}(eV, E_{\text{max}}) \quad (\text{II.15})$$

APPENDIX III
ENERGY SPECTRUM OF BETA PARTICLES

A. Momentum Spectrum

The momentum spectrum of beta particles emitted by a nucleus with atomic number $Z+1$ and mass number A is given by [14]

$$N'(P) dP = C(Z,A) F(Z,A,E) P^2 (E_{MAX} - E)^2 dP, \quad (III.1)$$

where:

$N'(P) dP$ = number of particles emitted with momentum in the range $(P, P+dP)$

$C(Z,A)$ = a constant for each nucleus (depends upon the length of time the nucleus has been emitting particles)

E_{MAX} = maximum kinetic energy of the beta particles

$F(Z,A,E)$ = the Coulomb factor which to 1 percent accuracy is given by

$$F(Z,A,E) = \frac{4\pi (1+S)}{(2S)!} (2 P \Psi / mc)^{2S-2} (S^2 + \eta^2)^{S-\frac{1}{2}} \\ \times \text{Exp} (2\phi\eta - 2S + S/(6(S^2 + \eta^2))) \quad (III.2)$$

where:

$$S = (1 - (Ze^2/\hbar C)^2)^{\frac{1}{2}} \quad (III.3)$$

$$\Psi = R/\hbar C; R = \text{nuclear radius} \quad (III.4)$$

$$\eta = \frac{Ze^2}{\hbar v}; v = \text{beta particle's speed} \quad (III.5)$$

$$\phi = \text{Tan}^{-1} (S/\eta) \quad (III.6)$$

B. Transformation to an Energy Spectrum

To transfer equation (III.1) to an energy spectrum, one uses the fact that the number of particles with energy in the range $(E, E+dE)$ is equal to the number of particles with momentum in the range $(P, P+dP)$. That is,

$$N(E) dE = N'(P) dP \quad . \quad (\text{III.7})$$

Hence,

$$N(E) = N'(P) \frac{dP}{dE} = N'(P) (dE/dP)^{-1} \quad . \quad (\text{III.8})$$

The relativistic kinematical equations which are of interest here are the following:

$$E = mc^2 (\gamma - 1) \quad (\text{III.9})$$

$$P = m \gamma v \quad (\text{III.10})$$

$$\gamma = (1 - v^2/c^2)^{-\frac{1}{2}} \quad . \quad (\text{III.11})$$

These can be used to give

$$N(E) = C(Z,A) m^2 c F(Z,A,E) (1 + E/mc^2) [(1 + E/mc^2)^2 - 1]^{\frac{1}{2}} (E_{\text{MAX}} - E)^2 \quad . \quad (\text{III.12})$$

If one defines

$$B = (1 + E/mc^2) \quad (\text{III.13})$$

and

$$\alpha = e^2/\hbar c \quad (\text{III.14})$$

then the quantities which define the Coulomb factor F can be written as

$$\eta = \frac{Z \alpha B}{(B^2 - 1)^{\frac{1}{2}}} \quad (\text{III. 15})$$

$$S = (1 - Z^2 \alpha^2)^{\frac{1}{2}} \quad (\text{III. 16})$$

$$\phi = \text{Tan}^{-1} (S/\eta) \quad (\text{III. 17})$$

Also, if a new overall constant is defined by

$$C'(Z, A) = \frac{4\pi (1 + S)}{(2S!)^2} (2\psi)^{2S-2} e^{-2S} m^4 c^5 C(Z, A) \quad (\text{III. 18})$$

then the energy spectrum is given by

$$\begin{aligned} N(E) dE = & C'(Z, A) B(B^2 - 1)^{S-\frac{1}{2}} (S^2 + \eta^2)^{S-\frac{1}{2}} [(E_{\text{max}} - E_0)/mc^2] \\ & \times \text{Exp} (2\phi\eta + S/6(S^2 + \eta^2)) dE \quad (\text{III. 19}) \end{aligned}$$

This is the desired equation for the energy spectrum.

APPENDIX IV
EQUILIBRIUM POTENTIAL ON GRAINS

I. Notation

- a = radius of grain (microns)
r = distance from the Sun (A.U.)
t = age of reprocessed waste (yr)
 V_{eq} = Equilibrium Potential (V)

II. Reference Values Used

- 1) Waste form -- as given by Battelle Laboratories Progress Report (September 14, 1979) pg. 3.
- 2) Waste Mix -- Commercial PW-4b.
- 3) Waste reprocessed after 10 years from reactor.

III. Equilibrium Potential

A) Submicron Particles ($a \leq 1.0$)

- 1) $r \leq 1.0$

Here, the radioactivity of the particles is not significant enough to affect the equilibrium potentials. This means that the voltages are time independent and are the same as they are for non-radioactive particles:

r	V_{eq}
1.00	12.0
0.75	10.5
0.50	9.5
0.25	8.0
0.10	7.0

- 2) $r > 1.0$

The relative importance of the radioactivity of the particles becomes more significant as r increases (also as a increases).

(i) $a = 1.0$

<u>r</u>	<u>t</u>	<u>V_{eq}</u>
2.0	0	14
	10	14
	100	12
	N.R.	12
5.0	0	32
	10	35
	100	12
	N.R.	12
10.0	0	175
	10	95
	100	17
	1000	12
	N.R.	12
50.0	0	5,000
	10	2,700
	100	300
	1000	12
	N.R.	12

(ii) $a = 0.1$

<u>r</u>	<u>t</u>	<u>V_{eq}</u>
2.0	0	12
	N.R.	12
5.0	0	13
	10	12
	N.R.	12
10.0	0	23
	10	16
	100	12
	N.R.	12
50.0	0	450
	10	250
	100	25
	1000	12
	N.R.	12

(iii) $a < 0.05$

For particles of this size, the radioactivity is not significant for any reasonable distance from the Sun. ($r \leq 100$)

B) Particles larger than 1 micron ($a > 1.0$)

1) $r \leq 1.0$, $a = 10.0$

<u>r</u>	<u>t</u>	<u>V_{eq}</u>
1.00	0	19.0
	10	15.5
	100	12.0
	N.R.	12.0
0.75	0	13.0
	10	11.0
	100	10.5
	N.R.	10.5
0.50	0	10.5
	10	9.5
	N.R.	9.5
0.25	0	8.0
	N.R.	8.0
0.10	0	7.0
	N.R.	7.0

2) $r > 1.0$

(i) $a = 5.0$

<u>r</u>	<u>t</u>	<u>V_{eq}</u>
2.0	0	116
	10	67
	100	15.5
	1000	12.0
	N.R.	12.0
5.0	0	190
	10	112
	100	19.0
	1000	12.0
	N.R.	12.0

<u>r</u>	<u>t</u>	<u>V_{eq}</u>
10.0	0	810
	10	500
	100	45
	1000	12.0
	N.R.	12.0
50.0	0	20,500
	10	12,800
	100	1,450
	1000	8.5*
	N.R.	12.0
(ii) a = 10.0		
<u>r</u>	<u>t</u>	<u>V_{eq}</u>
2.0	0	48.0
	10	30.0
	100	13.0
	N.R.	12.0
5.0	0	350
	10	230
	100	25
	1000	12.0
	N.R.	12.0
10.0	0	1500
	10	950
	100	100
	1000	12.0
	N.R.	12.0
50.0	0	37,000
	10	25,000
	100	2,700
	1000	7.0*
	N.R.	12.0

* At t = 1000 yrs the alpha emission is greater than the beta emission whereas for 0, 10, and 100 yrs, it is not.

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