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SYMBOLS

a_e	error in estimate of a_{zw}, m/s^2
a_{xb}, a_{yb}, a_{zb}	accelerometer measurements in F_B, m/s^2
a_{xw}, a_{yw}, a_{zw}	aerodynamic reaction components in F_W (including thrust), m/s^2
b	vector in F_B
C_{L_α}, C_{N_α}	derivatives of lift and normal-force coefficients with respect to α
F_B, F_E, F_V, F_W	reference frames: body fixed, earth fixed, vehicle-carried vertical, air trajectory (wind)
g	specific force of gravity, m/s^2
h	altitude (-z), m
L_{BV}, L_{WV}, L_{WB}	transformation matrices for rotations $(\psi_b, \theta_b, \phi_b)$, $(\psi_w, \theta_w, \phi_w)$, $(0, -\alpha, \beta)$
$L_x(\cdot)$, $L_y(\cdot)$, $L_z(\cdot)$	matrices for single-axis rotations
l_{ij}	ijth element of $L_{VB} = L_{BV}^T$
m	mass, kg
p_b, q_b, r_b	body-axis angular velocities, r/s
p_w, q_w, r_w	wind-axis angular velocities, r/s
Q	dynamic pressure, n/m^2
S	reference area, m^2
V	true airspeed, m/s
v	vector in F_W
w_x, w_y, w_z	wind velocities in F_E, m/s
x, y, z	positions in F_E: north, east, down, m
α, α_o	angle of attack, rad
β	sideslip angle, rad

ϕ_b, θ_b, ψ_b	body axis Euler angles, rad
ϕ_e	error in estimate of ϕ_w, rad
ϕ_w, θ_w, ψ_w	wind-axis Euler angles, rad
ρ	density of atmosphere, kg/m^3
O_B, O_E, O_V, O_W	origins of reference frames F_B, F_E, F_V, F_W

EQUATIONS FOR DETERMINING AIRCRAFT MOTIONS FROM ACCIDENT DATA

Ralph E. Bach, Jr., and Rodney C. Wingrove

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SUMMARY

This paper describes procedures for determining a comprehensive accident scenario from a limited data set. The analysis techniques accept and process data from either of two sources: an Air Traffic Control (ATC) radar-tracking system or a foil flight-data recorder. Local meteorological information at the time of the accident and aircraft performance data are also utilized.

Equations for the desired aircraft motions (roll, pitch, yaw) and forces (lift, excess thrust) are given in terms of elements of the measurement set and certain of their time derivatives. The principal assumption made in the development presented here is that aircraft side force and sideslip angle are negligible.

An estimation procedure is outlined for use with each data source. For the foil case, a discussion of exploiting measurement redundancy is given. Since either formulation requires estimates of measurement time derivatives, an algorithm for least-squares smoothing is provided

INTRODUCTION

For several years Ames Research Center has been assisting the National Transportation Safety Board (NTSB) and the military services in their investigations of aircraft accidents. During this period, methods have been developed to determine aircraft motions from the limited data available following an accident. It is the purpose of this report to derive the equations used in the analysis of such data and to summarize their application. A companion paper (ref. 1) presents an experimental assessment of the accuracy of the methods and results from several accident analyses. To the authors' knowledge, these are the first expositions of accident analysis techniques based on the full aircraft kinematic model that have appeared in the open literature.

The data sources considered here are the ground-based radar tracking system and the onboard foil flight recorder. A radar tracking system provides time histories of aircraft position, including altitude (when the vehicle is equipped with an altitude transponder). Radar data are recorded by an Air Traffic Control (ATC) station in the vicinity of the accident site (ref. 2). A foil record contains time histories of indicated airspeed, magnetic heading, barometric altitude, and normal acceleration. Flight recorders of the metal-foil type are carried by many airliners and some military aircraft. They are of rugged construction, designed to withstand the rigors of severe accidents (refs 3 and 4).

By 1974, NTSB scientists had developed digital computer programs for analysis of both ATC radar data and foil records. These programs corrected the data for calibration errors and nonstandard atmospheric conditions and provided time-history estimates of such parameters as groundspeed and groundtrack and flightpath angles (ref. 5). The NASA contribution has been to show how either data source, in combination with meteorological and aircraft data, can be utilized to estimate wind-axis specific forces (excess thrust, lift) and body-axis Euler angles (roll, pitch, yaw). Thrust and lift forces are important aircraft performance parameters (ref. 6), which, along with the Euler angle time histories, are valuable aids to the investigator in visualizing an accident trajectory.

This report is organized as follows: 1) wind-axis force and angle relations are derived for a vehicle governed by basic rigid-body dynamics, referred to a flat, nonrotating earth; 2) then, formulas for expressing body-axis Euler angles in terms of wind-axis angles and the aerodynamic angles are presented; 3) the specialized applications of the equations to the analysis of either radar or foil data are then summarized; pertinent axis systems and transformations are defined in Appendix A; and finally, since both analyses depend on estimates of first and second derivatives of certain measurement time histories, an algorithm for least-squares smoothing is presented in Appendix B.

FORCE AND ATTITUDE EQUATIONS

The key steps in processing accident data from ATC stations or foil recorders are indicated in figure 1. Smoothing the data provides estimates of first and second time derivatives from which wind-axis forces and angles can be determined. The forces, airspeed, and specific information about the airplane (lift characteristics) permit estimating the angle of attack, which can then be used with the wind-axis angles to determine time histories of the body-axis Euler angles. The assumption of negligible side force ($a_{y_W} = 0$) and sideslip angle ($\beta = 0$) is necessary to obtain the desired solutions. There may be, of course, accident situations in which this assumption is not valid. In the following paragraphs, we derive the equations for force and attitude estimation. The vehicle dynamics introduced here are discussed in reference 7.

Wind-Axis Forces and Angles

It is convenient first to develop expressions for wind-axis angles and forces in terms of quantities derived from vehicle position derivatives and local wind estimates. The rationale for this approach should become evident as the development proceeds. Orientation of the wind-axis frame F_W, relative to the vehicle-carried vertical frame F_V, is defined by Euler angles (ψ_W, θ_W, ϕ_W). As described in Appendix A, both F_W and F_V have their origins at the vehicle mass center and move with it. Frame F_W has axis Ox_W along the velocity vector and axis Oz_W in the vehicle plane of symmetry. The axes of

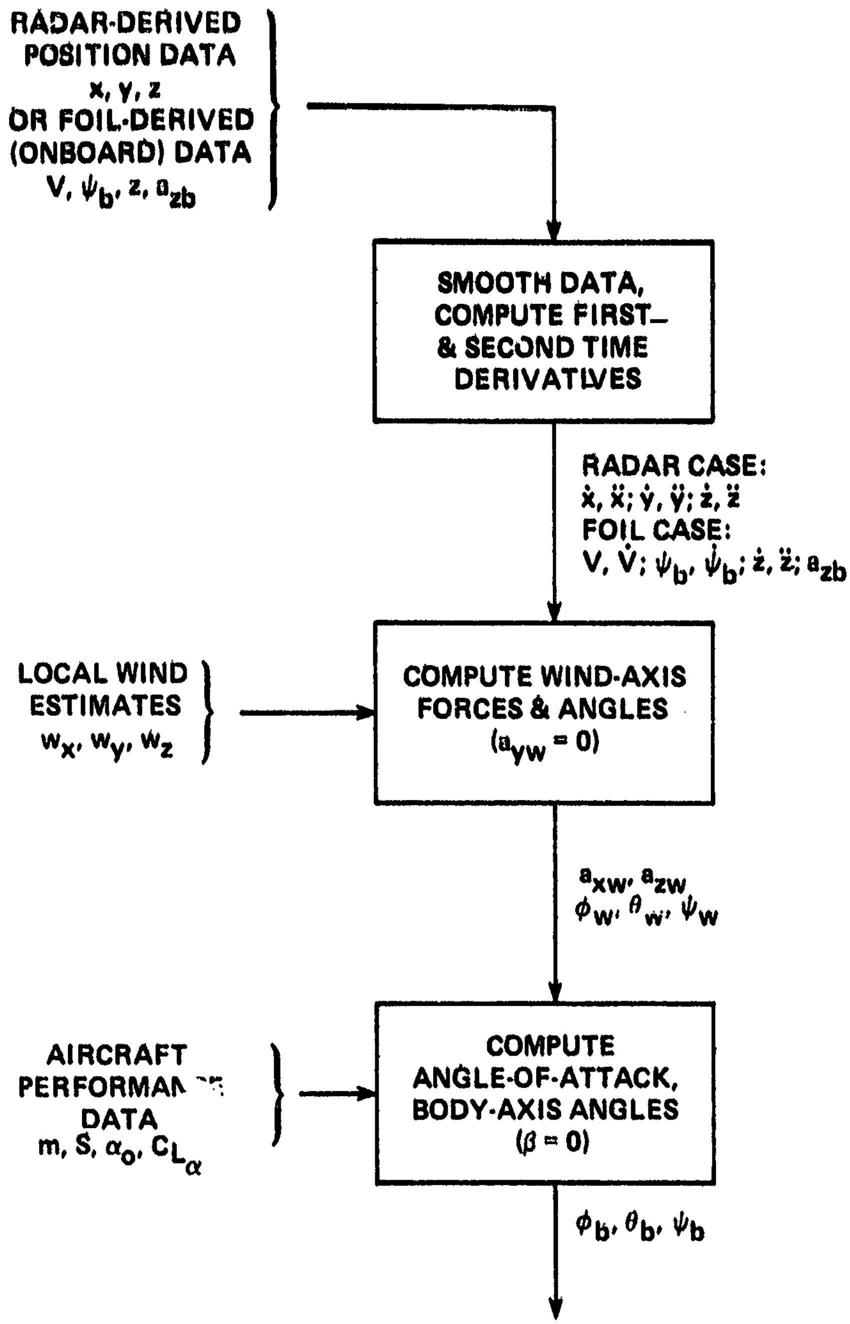

Figure 1.– Flow chart for processing accident data to obtain wind-axis forces and body-axis Euler angles.

F_V are parallel to the stationary earth-surface frame F_E. In this development we assume that the earth's curvature and rotation relative to an inertial frame can be neglected. Definitions of the Euler angles and a review of important axis transformations can also be found in Appendix A.

Expressions for wind-axis angles ψ_W and θ_W are easily derived from the relationship between vehicle velocity in F_E and F_W, given by

$$\begin{bmatrix} \dot{x} - w_x \\ \dot{y} - w_y \\ \dot{z} - w_z \end{bmatrix} = L_{VW} \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} V \cos\theta_W \cos\psi_W \\ V \cos\theta_W \sin\psi_W \\ -V \sin\theta_W \end{bmatrix} \tag{1}$$

where $(\dot{x}, \dot{y}, \dot{z})$ are time derivatives of vehicle position in F_E, (w_x, w_y, w_z) are winds in F_E, and V is true airspeed. It should be noted that, along a space trajectory, winds would be ideally characterized as functions of time and position, i.e., $w_x = w_x(t,x,y,z)$; $w_y = w_y(t,x,y,z)$; $w_z = w_z(t,x,y,z)$. When the second equation of (1) is divided by the first, it is seen that

$$\tan\psi_W = (\dot{y} - w_y)/(\dot{x} - w_x) \tag{2}$$

while the third equation yields

$$\sin\theta_W = -(\dot{z} - w_z)/V \tag{3}$$

Note that the airspeed can be written as

$$V = \left[(\dot{x} - w_x)^2 + (\dot{y} - w_y)^2 + (\dot{z} - w_z)^2\right]^{1/2} \tag{4}$$

which is obtained by premultiplying the vector in equation (1) by its transpose.

The relationship for vehicle aerodynamic reaction in terms of the acceleration of its center of mass and its gravitational attraction is used to complete the wind-axis Euler angle set and determine the forces. Hence,

$$\begin{bmatrix} a_{xw} \\ a_{yw} \\ a_{zw} \end{bmatrix} = a_{cw} - L_{WV} \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \tag{5}$$

Aerodynamic reaction as defined in reference 7 is

$$\begin{bmatrix} a_{xw} \\ a_{yw} \\ a_{zw} \end{bmatrix} = \begin{bmatrix} T_{xw} - D \\ T_{yw} - C \\ T_{zw} - L \end{bmatrix} /m \tag{6}$$

where (T_{xw}, T_{yw}, T_{zw}) are thrust terms; D is drag; C is side force; L is lift; and m is vehicle mass. For an aircraft in cruising flight, only T_{xw}, D, and L are significant forces. Often, a_{xw} is referred to as "excess thrust." The acceleration of the vehicle center of mass can be written

$$a_{cw} = \begin{bmatrix} \dot{V} \\ r_w V \\ -q_w V \end{bmatrix} + L_{WV} \begin{bmatrix} \dot{w}_x \\ \dot{w}_y \\ \dot{w}_z \end{bmatrix} \tag{7}$$

where $\dot{V}$ and $(\dot{w}_x, \dot{w}_y, \dot{w}_z)$ are time derivatives of airspeed and winds. The angular velocities q_w and r_w are given by

$$\left.\begin{aligned} q_w &= -\dot{\theta}_w \sin\phi_w + \dot{\psi}_w \cos\theta_w \cos\phi_w \\ r_w &= \dot{\theta}_w \cos\phi_w + \dot{\psi}_w \cos\theta_w \sin\phi_w \end{aligned}\right\} \tag{8}$$

Note that expressions for the derivatives $\dot{\psi}_w, \dot{\theta}_w, \dot{V}$ can be obtained by differentiating equations (2) through (4), which yields

$$\dot{\psi}_w = \left[(\ddot{y} - \dot{w}_y)\cos\psi_w - (\ddot{x} - \dot{w}_x)\sin\psi_w\right]/V \cos\theta_w \tag{9}$$

$$\dot{\theta}_w = -(\ddot{z} - \dot{w}_z + \dot{V} \sin\theta_w)/V \cos\theta_w \tag{10}$$

$$\dot{V} = \left[(\ddot{x} - \dot{w}_x)\cos\psi_w + (\ddot{y} - \dot{w}_y)\sin\psi_w\right]\cos\theta_w - (\ddot{z} - \dot{w}_z)\sin\theta_w \tag{11}$$

where $(\ddot{x}, \ddot{y}, \ddot{z})$ are second time derivatives of vehicle position in F_E.

We now multiply both sides of equation (5) by $L_x(-\phi_w)$ and use equations (7), (8), and (10) to obtain

$$a_{xw} = \dot{V} + (g - \dot{w}_z)\sin\theta_w + (\dot{w}_x \cos\psi_w + \dot{w}_y \sin\psi_w)\cos\theta_w \tag{12}$$

and

$$\left.\begin{aligned} -a_{zw} \sin\phi_w + a_{yw} \cos\phi_w &= C_1 \\ a_{yw} \sin\phi_w + a_{zw} \cos\phi_w &= C_2 \end{aligned}\right\} \tag{13}$$

where

$$\left.\begin{aligned} C_1 &= \dot{\psi}_w V \cos\theta_w + (\dot{w}_y \cos\psi_w - \dot{w}_x \sin\psi_w) \\ C_2 &= (\ddot{z} - g + a_{xw} \sin\theta_w)/\cos\theta_w \end{aligned}\right\} \tag{14}$$

Next, we solve equation (13) for $\sin\phi_w$, $\cos\phi_w$ as

$$\left.\begin{aligned} \sin\phi_w &= (C_2 a_{yw} - C_1 a_{zw})/(a_{yw}^2 + a_{zw}^2) \\ \cos\phi_w &= (C_1 a_{yw} + C_2 a_{zw})/(a_{yw}^2 + a_{zw}^2) \end{aligned}\right\} \tag{15}$$

from which we obtain the expression

$$\tan\phi_w = \frac{(a_{yw}/a_{zw}) + (C_1/-C_2)}{1 - (a_{yw}/a_{zw})(C_1/-C_2)} \tag{16}$$

It should be noted that side reaction a_{yw} is usually small compared with a_{zw}. In fact, in order to estimate ϕ_w and a_{zw} by using the formulation of equations (13) and (14), it is necessary to assume that the ratio (a_{yw}/a_{zw}) is negligible. In that case

$$\tan\phi_w = (C_1/-C_2) \tag{17}$$

Finally, solution of equation (13) for a_{zw} yields

$$a_{zw} = C_2 \cos\phi_w - C_1 \sin\phi_w \tag{18}$$

An analysis of the errors in ϕ_w and a_{zw} incurred by neglecting the ratio (a_{yw}/a_{zw}) shows that

$$\left.\begin{aligned} \tan\phi_e &= (a_{yw}/a_{zw}) \\ (a_e/a_{zw}) &= 1 - \sec\phi_e \end{aligned}\right\} \tag{19}$$

where ϕ_e is the error in the estimate of ϕ_w and a_e is the error in the estimate of a_{zw}.

Body-Axis Euler Angles

We have shown that wind-axis Euler angles $(\psi_w, \theta_w, \phi_w)$ and specific forces (a_{xw}, a_{yw}, a_{zw}) may be determined from vehicle position derivatives and wind information. It is, however, the attitude of the body frame F_B relative to F_V as described by the Euler angles $(\psi_b, \theta_b, \phi_b)$ that is of primary interest in the analysis of vehicle motion. Recall that the frames F_B and F_W are displaced by angle of attack α and sideslip angle β (see Appendix A). In this development, we assume that sideslip is negligible, which is consistent with the statement $a_{yw} \approx 0$. In the next section, we obtain an estimate of α by using a_{xw}, a_{zw}, V and performance data for a particular aircraft. Hence, given $(\psi_w, \theta_w, \phi_w)$ and (α, β), it should be possible to determine expressions for the body-axis angles $(\psi_b, \theta_b, \phi_b)$. To conclude this section, therefore, we shall review the transformation between frames F_B and F_W and derive the expressions relating body and wind-axis Euler angles.

The body-axis system F_B is carried into the wind-axis system F_W by the rotation sequence $(0, -\alpha, \beta)$. The transformation matrix is given by

$$L_{WB} = L_z(\beta)L_y(-\alpha) \tag{20}$$

Now, note that a vector b in F_B may be transformed into F_V in either of two equivalent ways:

$$v = L_{VB}\, b \quad \text{or} \quad v = L_{VW}L_{WB}\, b \tag{21}$$

Here v is a vector with coordinates in F_V, and L_{VB} represents the transformation

$$L_{VB} = L_z(-\psi_b)L_y(-\theta_b)L_x(-\phi_b) \tag{22}$$

Hence we can express

$$L_{VB} = L_{VW}L_{WB} \tag{23}$$

so that given $(\phi_w, \theta_w, \psi_w)$ and (α, β) we should be able to solve for $(\phi_b, \theta_b, \psi_b)$. In order to do this, we form the matrix product indicated by equation (22):

$$L_{VB} = \begin{bmatrix} \cos\theta_b \cos\psi_b & \sin\phi_b \sin\theta_b \cos\psi_b - \cos\phi_b \sin\psi_b & \cos\phi_b \sin\theta_b \cos\psi_b + \sin\phi_b \sin\psi_b \\ \cos\theta_b \sin\psi_b & \sin\phi_b \sin\theta_b \sin\psi_b + \cos\phi_b \cos\psi_b & \cos\phi_b \sin\theta_b \sin\psi_b - \sin\phi_b \cos\psi_b \\ -\sin\theta_b & \sin\phi_b \cos\theta_b & \cos\phi_b \cos\theta_b \end{bmatrix} \tag{24}$$

from which it follows that

$$\tan\phi_b = \ell_{32}/\ell_{33} \tag{25}$$

$$\sin\theta_b = -\ell_{31} \tag{26}$$

$$\tan\psi_b = \ell_{21}/\ell_{11} \tag{27}$$

where the $\{\ell_{ij}\}$ are elements of L_{VB}. (An equivalent representation can be found in ref. 8.) It is convenient to use the trigonometric identity for $\tan(\psi_b - \psi_w)$ with equation (27) to obtain the expression

$$\tan(\psi_b - \psi_w) = (\ell_{21} \cos\psi_w - \ell_{11} \sin\psi_w)/(\ell_{11} \cos\psi_w + \ell_{21} \sin\psi_w) \tag{28}$$

Finally, we evaluate the matrix product of equation (23) as shown in table I and substitute the required elements of $\{\ell_{ij}\}$ into equations (25), (26), and (28) to obtain the desired body-angle formulas

$$\tan\phi_b = \frac{\cos\beta \sin\phi_w \cos\theta_w - \sin\beta \sin\theta_w}{(\cos\alpha \cos\phi_w - \sin\alpha \sin\beta \sin\phi_w)\cos\theta_w - \sin\alpha \cos\beta \sin\theta_w} \tag{29}$$

$$\sin\theta_b = \cos\alpha \cos\beta \sin\theta_w + (\sin\alpha \cos\phi_w + \cos\alpha \sin\beta \sin\phi_w)\cos\theta_w \tag{30}$$

$$\tan(\psi_b - \psi_w) = \frac{\sin\alpha \sin\phi_w - \cos\alpha \sin\beta \cos\phi_w}{\cos\alpha \cos\beta \cos\theta_w - (\sin\alpha \cos\phi_w + \cos\alpha \sin\beta \sin\phi_w)\sin\theta_w} \tag{31}$$

TABLE I.— ELEMENTS OF MATRIX L_{VB}

$$L_{VB} = L_{VW}L_{WB} = L_z(-\psi_w)L_y(-\theta_w)L_x(-\phi_w)L_z(\beta)L_y(-\alpha) = \begin{bmatrix} \ell_{11} & \ell_{12} & \ell_{13} \\ \ell_{21} & \ell_{22} & \ell_{23} \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix}$$

$$\ell_{11} = \cos\alpha \cos\beta \cos\theta_w \cos\psi_w - \cos\alpha \sin\beta (\sin\phi_w \sin\theta_w \cos\psi_w - \cos\phi_w \sin\psi_w) - \sin\alpha (\cos\phi_w \sin\theta_w \cos\psi_w + \sin\phi_w \sin\psi_w)$$

$$\ell_{21} = \cos\alpha \cos\beta \cos\theta_w \sin\psi_w - \cos\alpha \sin\beta (\sin\phi_w \sin\theta_w \sin\psi_w + \cos\phi_w \cos\psi_w) - \sin\alpha (\cos\phi_w \sin\theta_w \sin\psi_w - \sin\phi_w \cos\psi_w)$$

$$\ell_{31} = \cos\alpha \cos\beta \sin\theta_w - \cos\alpha \sin\beta \sin\phi_w \cos\theta_w - \sin\alpha \cos\phi_w \cos\theta_w$$

$$\ell_{12} = \sin\beta \cos\theta_w \cos\psi_w + \cos\beta (\sin\phi_w \sin\theta_w \cos\psi_w - \cos\phi_w \sin\psi_w)$$

$$\ell_{22} = \sin\beta \cos\theta_w \sin\psi_w + \cos\beta (\sin\phi_w \sin\theta_w \sin\psi_w + \cos\phi_w \cos\psi_w)$$

$$\ell_{32} = - \sin\beta \sin\theta_w + \cos\beta \sin\phi_w \cos\theta_w$$

$$\ell_{13} = \sin\alpha \cos\beta \cos\theta_w \cos\psi_w - \sin\alpha \sin\beta (\sin\phi_w \sin\theta_w \cos\psi_w - \cos\phi_w \sin\psi_w) + \cos\alpha (\cos\phi_w \sin\theta_w \cos\psi_w + \sin\phi_w \sin\psi_w)$$

$$\ell_{23} = \sin\alpha \cos\beta \cos\theta_w \sin\psi_w - \sin\alpha \sin\beta (\sin\phi_w \sin\theta_w \sin\psi_w + \cos\phi_w \cos\psi_w) + \cos\alpha (\cos\phi_w \sin\theta_w \sin\psi_w - \sin\phi_w \cos\psi_w)$$

$$\ell_{33} = -\sin\alpha \cos\beta \sin\theta_w - \sin\alpha \sin\beta \sin\phi_w \cos\theta_w + \cos\alpha \cos\phi_w \cos\theta_w$$

MOTIONS DERIVED FROM ACCIDENT DATA

In this section, procedures for determining aircraft motions and forces from radar or foil data are summarized; the procedures are based on the equations developed in the previous section. We assume that certain types of data anomalies, such as dropouts, wild points, etc., have been removed, and that corrections for temperature, pressure, etc., have been made to air data in so far as possible. Meteorological data, of course, will be imprecise — wind measurements, if available, may have been recorded many miles from the crash site and at a different time from that of the accident. Usually, data sufficient to determine only w_x and w_y (as functions of altitude) are obtained. Hence, it must often be assumed that $w_z = 0$. These limitations notwithstanding, we can then proceed to compute wind-axis Euler angles and forces along the trajectory, to estimate an angle-of-attack-time history, and to determine the body-axis Euler angles. The necessary assumption for a solution with either data set is that vehicle side force a_{yw} and sideslip angle β are negligible.

Analysis of Radar Data

We assume that time histories of vehicle position (x, y, z) and winds (w_x, w_y, w_z) are available. First, "smooth" the data (see Appendix B) to obtain estimates of vehicle velocity ($\dot{x}$, $\dot{y}$, $\dot{z}$) and acceleration ($\ddot{x}$, $\ddot{y}$, $\ddot{z}$). We then use equations (2) through (4) to determine wind-axis angles ψ_w and θ_w and airspeed V:

$$\left.\begin{aligned} \psi_w &= \tan^{-1}\left[(\dot{y} - w_y)/(\dot{x} - w_x)\right] \\ \theta_w &= \sin^{-1}\left[-(\dot{z} - w_z)/V\right] \\ V &= \left[(\dot{x} - w_x)^2 + (\dot{y} - w_y)^2 + (\dot{z} - w_z)^2\right]^{1/2} \end{aligned}\right\} \tag{32}$$

Next, we determine excess thrust by using equation (11) in equation (12), which yields

$$a_{xw} = (\ddot{x} \cos\psi_w + \ddot{y} \sin\psi_w)\cos\theta_w - (\ddot{z} - g)\sin\theta_w \tag{33}$$

and then we solve for lift and wind-axis roll angle with

$$\left.\begin{aligned} a_{zw} &= C_2 \cos\phi_w - C_1 \sin\phi_w \\ \phi_w &= \tan^{-1}(C_1/-C_2) \end{aligned}\right\} \tag{34}$$

where equation (9) has been used in equation (14) to give

$$\left.\begin{aligned} C_1 &= \ddot{y} \cos\psi_w - \ddot{x} \sin\psi_w \\ C_2 &= (\ddot{z} - g + a_{xw} \sin\theta_w)/\cos\theta_w \end{aligned}\right\} \tag{35}$$

Notice that in equations (33) and (35) there is no dependence on wind-derivative ($\dot{w}_x$, $\dot{w}_y$, $\dot{w}_z$) information.

Finally, in order to obtain the body-axis angles (ψ_b, θ_b, ϕ_b), an estimate of angle of attack is needed. In the linear region, that estimate can be approximated by

$$\left.\begin{aligned} \alpha &= \alpha_o - m\,a_{zw}/QS\,C_{L_\alpha} \\ Q &= (1/2)\rho V^2 \end{aligned}\right\} \qquad (36)$$

where m is mass, S is wing area, ρ is air density, and C_{L_α} is the derivative of the lift coefficient with respect to α. Values of α_o and C_{L_α} depend primarily on aircraft configuration and Mach number and are tabulated for a given aircraft. For high angles of attack, a flat-plate relationship yields a good approximation (refs. 9 and 10):

$$\alpha = \tan^{-1}(a_{xw}/a_{zw}) \qquad (37)$$

Now the body angles can be determined by using equations (29) through (31), with $\beta = 0$:

$$\left.\begin{aligned} \psi_b &= \psi_w + \tan^{-1}\left[\sin\alpha\ \sin\phi_w/(\cos\alpha\ \cos\theta_w - \sin\alpha\ \cos\phi_w\ \sin\theta_w)\right] \\ \theta_b &= \sin^{-1}(\cos\alpha\ \sin\theta_w + \sin\alpha\ \cos\theta_w\ \cos\phi_w) \\ \phi_b &= \tan^{-1}\left[\cos\theta_w\ \sin\phi_w/(\cos\alpha\ \cos\theta_w\ \cos\phi_w - \sin\alpha\ \sin\theta_w)\right] \end{aligned}\right\} \qquad (38)$$

Analysis of Foil Data

The data set derived from a foil record consists of true airspeed V, heading angle ψ_b, altitude $-z$, and normal specific force $-a_{zb}$. These quantities, along with time derivatives $\dot{V}$, $\dot{\psi}_b$, $\dot{z}$, and $\ddot{z}$, wind information, and aircraft performance data, can be utilized to provide an accident scenario. First, we use equation (3) to determine the wind-axis pitch angle θ_w:

$$\theta_w = \sin^{-1}\left[-(\dot{z} - w_z)/V\right] \qquad (39)$$

Next, the forces a_{xw} and a_{zw} and the roll angle ϕ_w are estimated by using equations (12) through (14):

$$a_{xw} = \dot{V} + (g - \dot{w}_z)\sin\theta_w + (\dot{w}_x\ \cos\psi_w + \dot{w}_y\ \sin\psi_w)\cos\theta_w \qquad (40)$$

$$\left.\begin{aligned} a_{zw} &= C_2\ \cos\phi_w - C_1\ \sin\phi_w \\ \phi_w &= \tan^{-1}(C_1/-C_2) \end{aligned}\right\} \qquad (41)$$

where

$$C_1 = \dot{\psi}_w V \cos\theta_w + \dot{w}_y \cos\psi_w - \dot{w}_x \sin\psi_w$$
$$C_2 = (g - \beta + a_{xw} \sin\theta_w)/\cos\theta_w \qquad (42)$$

Notice that the expressions for a_{xw}, a_{zw}, and ϕ_w depend on wind-derivative quantities ($\dot{w}_x$, $\dot{w}_y$, $\dot{w}_z$), estimates of which are generally unreliable (or unavailable). Since w_z is usually not measured, and w_x, w_y are recorded as functions only of altitude, the best estimates of the wind derivatives are

$$\dot{w}_x = \frac{dw_x}{dz}\dot{z}; \qquad \dot{w}_y = \frac{dw_y}{dz}\dot{z}; \qquad \dot{w}_z = 0 \qquad (43)$$

Notice also that the factor $\dot{\psi}_w$ in equation (42) must be approximated by the time derivative of ψ_b, which is included in the data set. (If wind-derivative effects in equations (40) and (42) are to be included, let $\psi_w \doteq \psi_b$ also.). Finally, after estimating angle of attack as in the radar case, we solve for the unknown Euler angles:

$$\theta_b = \sin^{-1}(\cos\alpha \sin\theta_w + \sin\alpha \cos\theta_w \cos\phi_w)$$
$$\phi_b = \tan^{-1}\left[\cos\theta_w \sin\phi_w/(\cos\alpha \cos\theta_w \cos\phi_w - \sin\alpha \sin\theta_w)\right] \qquad (44)$$
$$\psi_w = \psi_b - \tan^{-1}\left[\sin\alpha \sin\phi_w/(\cos\alpha \cos\theta_w - \sin\alpha \sin\theta_w \cos\phi_w)\right]$$

The foil record apparently contains redundancy, since the measurement a_{zb} has not appeared in any of the equations. The redundancy may be utilized to provide an independent estimate of angle of attack. In the linear region that estimate can be approximated by

$$\alpha = \alpha_o - m\, a_{zb}/QSC_{N_\alpha} \qquad (45)$$

where m, Q, and S have been previously defined and C_{N_α} is the derivative of the normal force coefficient. Again, α_o and C_{N_α} would be found as tabulated functions of configuration and Mach number for a given aircraft. Outside of the linear region, α may be obtained from a flat-plate relation equivalent to equation (37), which is

$$\alpha = \sin^{-1}(a_{xw}/a_{zb}) \qquad (46)$$

Using the estimate of α so obtained, we can determine the wind-axis reaction term a_{zw} independently of ϕ_w. That expression is given by

$$a_{zw} = a_{zb}/\cos\alpha - a_{xw} \tan\alpha \qquad (47)$$

One way that the redundancy implied by equation (47) can be utilized is to provide a check on the assumption that the ratio (a_{yw}/a_{zw}) is negligibly small. If each of the relations in equation (15) is squared and added, the result is

$$a_{yw}^2 + a_{zw}^2 = C_1^2 + C_2^2 \qquad (48)$$

from which the magnitude

$$|a_{yw}/a_{zw}| = \left[(C_1^2 + C_2^2)/a_{zw}^2 - 1\right]^{1/2} \tag{49}$$

is obtained. The redundancy can also be used to compensate for anomalies in the data. Onboard instruments are subject to unusual operating conditions during an accident, and portions of the recovered foil record may contain significant errors. If the altimeter is suspect, then the roll angle ϕ_w can be computed from

$$\phi_w = \sin^{-1}(-C_1/a_{zw}) \tag{50}$$

When data from the directional gyro are considered unreliable, the magnitude of ϕ_w can be determined from the expression

$$|\phi_w| = \tan^{-1}\left[(a_{zw}/C_2)^2 - 1\right]^{1/2} \tag{51}$$

The alternate solutions for roll angle given by equations (50) and (51) could, of course, be used to provide a data-consistency check of the four foil measurements. In this regard, one further observation should be noted: Manipulation of equation (41) yields

$$C_2 = a_{zw}\cos\phi_w \tag{52}$$

from which an expression for vertical acceleration

$$\ddot{z} = a_{zw}\cos\phi_w\cos\theta_w - a_{xw}\sin\theta_w + g \tag{53}$$

follows. Hence, equation (53) may be integrated twice, and with appropriate choice of constants the result should match the altitude record.

CONCLUDING REMARKS

This report presents the equations necessary to construct a comprehensive scenario of aircraft motions by using data from an Air Traffic Control radar system or a foil flight-data recorder. Vehicle performance data and local meteorological information are also utilized.

Expressions for angles and forces are first obtained in a wind-axis frame, in terms of the measured variables and certain of their time derivatives. Estimation of angle of attack then permits determination of the desired body axis Euler angles. The derivations require an assumption of negligible side force and sideslip; error estimates have been included.

The procedures described herein have been implemented to assist the National Transportation Safety Board and the military services in their investigations of several aircraft accidents.

APPENDIX A

AXIS SYSTEMS AND TRANSFORMATIONS

In the text, the four reference frames F_E, F_V, F_W, F_B are utilized. They are defined as follows:

(a) The earth-surface frame F_E, with axes $O_E x_E y_E z_E$, has $O_E z_E$ directed vertically down. $O_E x_E y_E$ is a local horizontal plane, with $O_E x_E$ pointing north and $O_E y_E$ pointing east.

(b) The vehicle-carried vertical frame F_V, with axes $O_V x_V y_V z_V$, has its origin fixed at the mass center of the vehicle. For the applications considered herein the curvature of the earth is neglected, and the axes of F_V are taken parallel to those of F_E.

(c) The wind-axis frame F_W, with axes $O_W x_W y_W z_W$, has its origin fixed at the mass center and has its axis $O_W x_W$ directed along the velocity vector of the vehicle relative to the atmosphere. Axis $O_W z_W$ lies in the vehicle plane of symmetry.

(d) The body-axis frame F_B, with axes $O_B x_B y_B z_B$, has its origin fixed at the mass center and has its axis $O_B x_B$ parallel to the aerodynamic reference direction (zero-lift line). Axis $O_B z_B$ lies in the vehicle plane of symmetry.

In flight dynamics, the Euler angles describe the orientation of the vehicle-carried vertical frame F_V with respect to either the wind-axis system F_W or the body-axis system F_B. The angles (ψ_w, θ_w, ϕ_w) rotate F_V into coincidence with F_W; the angles (ψ_b, θ_b, ϕ_b) rotate F_V into coincidence with F_B. The following steps describe the sequence of rotations illustrated in figure 2.

(a) A rotation is made about Oz_V, carrying the axes to $Ox_2y_2z_2$. This is the yaw angle ψ.

(b) A rotation is made about Oy_2, carrying the axes to $Ox_3y_3z_3$. This is the pitch angle θ.

(c) A rotation is made about Ox_3, carrying the axes to their final position $Oxyz$. This is the roll angle ϕ.

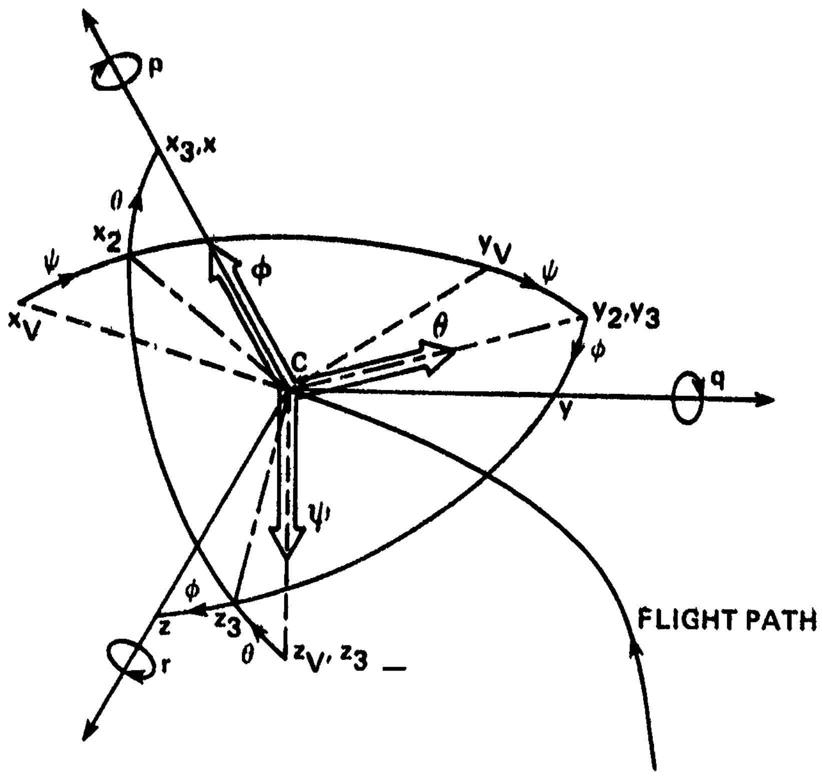

Figure 2.-- Euler angles.

The matrices required to transform a vector from F_V into either F_W or F_B correspond to the sequence of rotations (ψ, θ, ϕ) and are given by

$$L_{WV} = L_x(\phi_w)L_y(\theta_w)L_z(\psi_w) \tag{A1}$$

for the wind axes, and

$$L_{BV} = L_x(\phi_b)L_y(\theta_b)L_z(\psi_b) \tag{A2}$$

for the body axes. The transformations associated with a single rotation about each of the coordinate axes are

$$L_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \tag{A3}$$

$$L_y(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \tag{A4}$$

$$L_z(\psi) = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (A5)$$

It should be noted that the products $L_x(\phi)L_x(-\phi)$, $L_y(\theta)L_y(-\theta)$, and $L_z(\psi)L_z(-\psi)$ each reduce to the unit matrix. Thus, the transformations of equations (A1) and (A2) are orthogonal, that is:

$$\left.\begin{aligned} L_{VW} &= L_z(-\psi_w)L_y(-\theta_w)L_x(-\phi_w) = L_{WV}^T \\ L_{VB} &= L_z(-\psi_b)L_y(-\theta_b)L_x(-\phi_b) = L_{BV}^T \end{aligned}\right\} \quad (A6)$$

where the superscript T denotes the matrix transpose.

Finally, it should be noted that frames F_W and F_B are displaced by angle of attack α and sideslip angle β. Specifically, the body-axis system F_B is carried into the wind-axis system F_W by the rotation sequence $(0, -\alpha, \beta)$. The transformation matrix is given by

$$L_{WB} = L_z(\beta)L_y(-\alpha) \quad (A7)$$

APPENDIX B

A SMOOTHING ALGORITHM

SMOOTH is a routine that "smooths" a data record and provides estimates of its first and second derivatives with respect to the independent variable. The algorithm, adapted from reference 11, passes a "least-squares moving arc" through the data. The arc is a second-degree polynomial spanning an interval of NS (odd) equally spaced data points such that

$$\left.\begin{aligned} x_i &= a_0 + a_1(i - p) + a_2(i - p)^2 \\ p &= j - (NS - 1)/2 \end{aligned}\right\} \qquad \text{(B1)}$$

where (a_0, a_1, a_2) are chosen to minimize

$$J = \sum_{i=j-NS+1}^{i=j} (z_i - x_i)^2 \qquad \text{(B2)}$$

The data record consists of the samples z_j, j = 1, ..., NPTS. Values of the polynomial and its first two derivatives are computed at the central point p of each interval as the arc is passed through the data. Estimates near the beginning and end of the data record are obtained by extrapolation of the first and last polynomials.

A FORTRAN IV listing of the program is given in figure 3. The subroutine call statement is CALL SMOOTH (Z, X, Y, W, NS, H, NPTS), where the parameters are

Z	input vector of length NPTS containing data record to be smoothed
X	output vector of length NPTS containing smoothed data record
Y	output vector of length NPTS containing first derivative of X
W	output vector of length NPTS containing second derivative of X
NS	number of points desired in smoothing interval; must be odd and $3 \le NS \le NPTS$
H	sample interval, sec
NPTS	number of points in data record

```
      SUBROUTINE SMOOTH(Z,X,Y,W,NS,H,NPTS)
      DIMENSION Z(NPTS),X(NPTS),Y(NPTS),W(NPTS)
C
C     REF: WHITE SANDS HB OF DR METHODS, PG 160
C
      IF(NS.GT.NPTS) NS=NPTS
      C3=2./(H**2)
C
C     COEFFICIENT CALCULATION
C
      N=NS
      ENSQ=N**2
      D1=N*(ENSQ-4.)
      D2=ENSQ-1.
      C11=.75*(3.*ENSQ-7.)/D1
      C13=-15./D1
      C22=12./(N*D2)
      C33=-12.*C13/D2
C
C     INITIALIZE SUM CALCULATION
C
      AP=Z(1)
      BP=0.
      CP=0.
C
C     COMPUTE SUMS RECURSIVELY FOR N=3,5,---,NS
C
      DO 100 N=3,NS,2
      NM1=(N-1)/2
      NP1=(N+1)/2
      NM1SQ=NM1**2
      NP1SQ=NP1**2
      NXT1=N-1
      NXT2=N
      AP=AP+Z(NXT1)+Z(NXT2)
      BP=BP-AP+NM1*Z(NXT1)+NP1*Z(NXT2)
      CP=CP-2.*BP-AP+(NM1SQ)*Z(NXT1)+(NP1SQ)*Z(NXT2)
100   CONTINUE
      NSTP=NPTS-NM1
C
C     SMOOTHED VALUES FOR I=1,2,--,NP1
C
      A0=C11*AP+C13*CP
      A1=C22*BP
```

Figure 3.— Listing of smoothing routine.

```
      A2=C13*AP+C33*CP
      DO 150 I=1,NP1
      L=I-NP1
      X(I)=A0+L*(A1+L*A2)
      Y(I)=(A1+2.*L*A2)/H
      W(I)=C3*A2
150   CONTINUE
      IF(NS.EQ.NPTS) GO TO 250
C
C     INTERIOR SMOOTHING (I.GT.NP1; I.LE.NSTP)
C
      M=NP1+1
      DO 200 I=M,NSTP
      LST=I-NP1
      NXT=I+NM1
      AP=AP-Z(LST)+Z(NXT)
      BP=BP-AP+NM1*Z(LST)+NP1*Z(NXT)
      CP=CP-2.*BP-AP-NM1SQ*Z(LST)+NP1SQ*Z(NXT)
      A0=C11*AP+C13*CP
      A1=C22*BP
      A2=C13*AP+C33*CP
      X(I)=A0
      Y(I)=A1/H
      W(I)=C3*A2
200   CONTINUE
250   CONTINUE
C
C     SMOOTHED VALUES FOR I=NSTP+1,...,NPTS
C
      M=NSTP+1
      DO 300 I=M,NPTS
      L=I-NSTP
      X(I)=A0+L*(A1+L*A2)
      Y(I)=(A1+2.*L*A2)/H
      W(I)=C3*A2
300   CONTINUE
      RETURN
      END
```

Figure 3.— Concluded.

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