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SUMMARY

Lateral deflections of beam columns on multiple, equally-spaced supports were calculated using the STAGS nonlinear structural analysis computer program. Three lateral loadings were considered: (1) uniform, (2) linear, and (3) uniform over only the center bay. The two types of boundary conditions considered at the end supports were clamped and simple support. Some of the results include the effect of an initial sinusoidal imperfection. Deflections in the center and end bays of the beam columns are presented as a function of applied axial compressive load.

These calculations were made to better understand the deflection pattern of structural panels on the space shuttle orbiter. Lateral deflections of the structural panels can become extremely important because the deflections can contribute to failure of the thermal protection system that is bonded to the structural panels.

INTRODUCTION

The thermal protection system on the space shuttle orbiter consists of ceramic tiles that are bonded to strain isolation pads (SIP) which, in turn, are bonded to aluminum structural panels on the surface of the shuttle orbiter. Lateral deflections of these structural panels can contribute to failure of the tiles at the tile-SIP interface. In many cases, these panels are stiffened panels in which the important deflections are those that occur between stiffeners. Panels are subjected to inplane and pressure loads, and have initial imperfections.

In an effort to get a better understanding of the interaction between pressure loadings, inplane loadings, initial imperfections, and resulting deflections of the aluminum structural panels, beam columns on multiple supports have been analyzed using a nonlinear structural analysis program. This report describes the analysis approach and discusses the results.

SYMBOLS

Values are given in both SI and U.S. Customary Units. The calculations were made in U.S. Customary Units.

E	Young's modulus
I	moment of inertia of beam of unit width, $\frac{t^3}{12}$
L	distance between supports
P	end load shown in figure 1
P_{cr}	buckling load of simply-supported beam of length L, $\frac{\pi^2 EI}{L^2}$
q	lateral load shown in figure 1
t	thickness of beam; thickness of skin between stiffeners
δ	peak-to-peak deflection illustrated in figure 4
δ_o	values of δ when $P = 0$

ANALYSIS APPROACH

Analysis Model

The analysis model, shown in figure 1, is a beam column on equally-spaced supports. The beam column model is chosen because many of the critical buckling

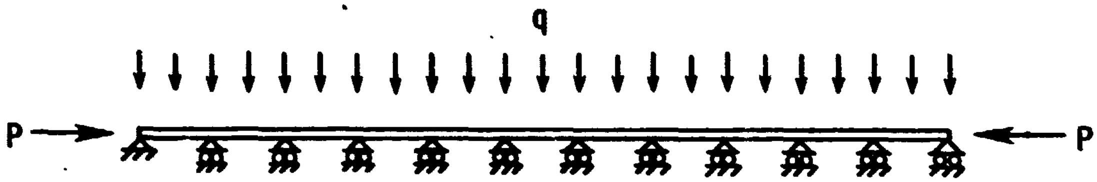


Figure 1 - Analysis model

conditions leading to large out-of-plane deflections involve high in-plane loads transverse to the stiffener direction. Since it is the deformation of the skin between stiffeners that is being examined, the stiffeners are represented by the supports. A section of a beam column between supports is termed a bay. Most calculations are made with eleven and thirteen bays. In some cases, check calculations were made with twenty-one bays. For all cases, the supports are at intervals of 5.44 cm (2.14 in.). The beam column has unit width, and has a thickness of .13 cm (.05 in.). Each bay is modeled with 20 finite elements. Young's modulus of the aluminum material is taken as 68.9 GPa (10×10^6 psi).

Analysis Procedure

The analyses were carried out with a nonlinear, finite element, structural analysis computer program denoted STAGS (ref. 1). The finite elements are based on von Karman-type plate theory and thus the program can account for moderately large rotations.

Loadings

The three types of lateral loadings shown in figure 2 were considered in the analyses. They are: (a) a uniform load, (b) a linearly varying load, and (c) a loading that is uniform over the single bay at the center and zero elsewhere. An axial compressive end load (fig. 1) is applied in all cases.

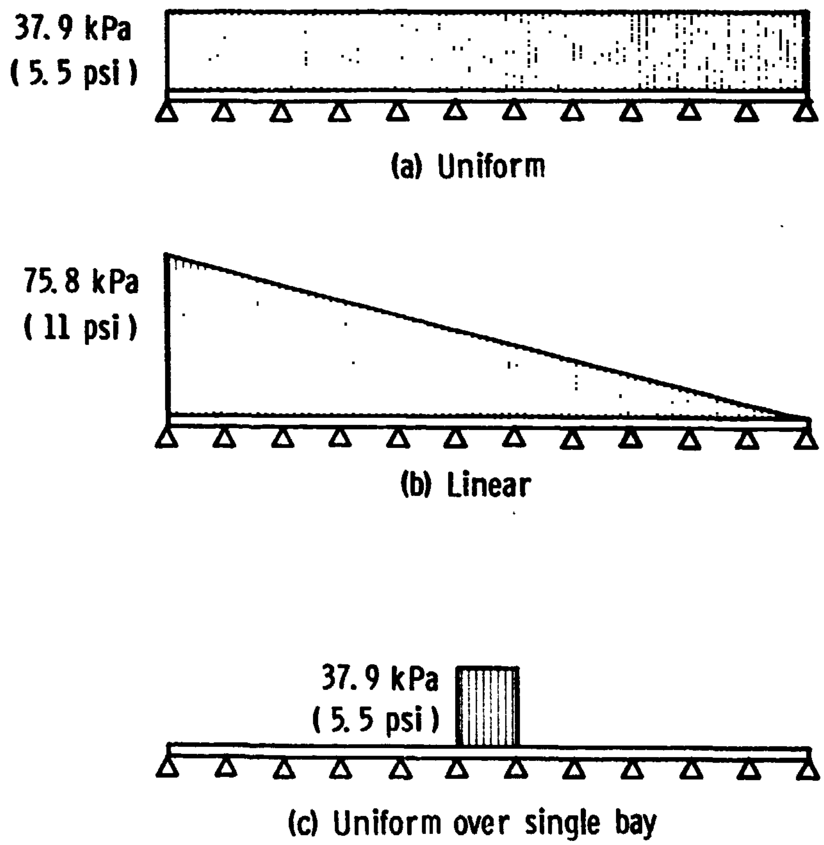


Figure 2. - Lateral loadings considered

Initial Imperfection

The one type of initial imperfection considered in the analyses is illustrated in figure 3. It is a sinusoidal imperfection with half-wavelength

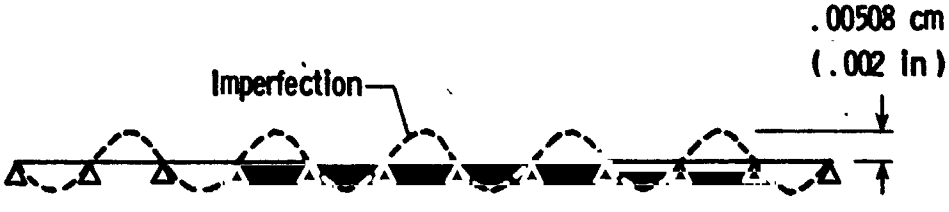


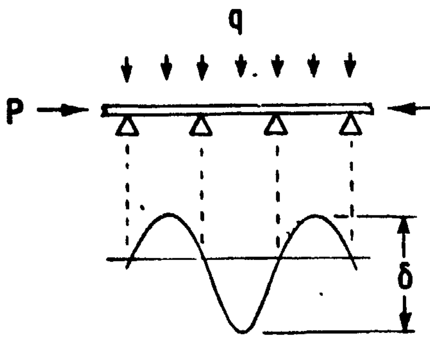
Figure 3. - Assumed initial imperfection

equal to the distance between supports and with amplitude equal to .00508 cm (.002 in.). This initial imperfection has the same shape as the buckle mode of a simply-supported beam on multiple, equally-spaced supports.

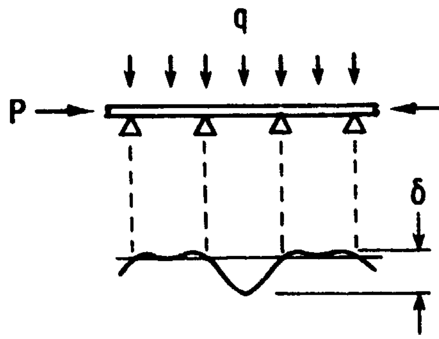
RESULTS

Definition of Deflection

The deformation that is examined in these examples is the peak-to-peak deflection δ shown in figure 4. In a well-developed deflection pattern, such as that shown in figure 4(a), the peak-to-peak deflections are measured at the center of adjacent bays. When the end load P is small compared with the lateral load q, the deflection pattern may be more like that illustrated in figure 4(b). In this case, the peak-to-peak deflections are not measured



(a) Well-developed deflection pattern



(b) Deflection pattern for loadings in which P is small compared with q

Figure 4. - Definition of deflection δ

at the center of adjacent bays. In cases involving an initial imperfection, the deflections shown in the figures include the initial imperfection.

Results for Beams with Clamped Ends, Deflections in Center Bays

In these results, which are given in figure 5, the beam is assumed to be clamped at the end supports. The peak-to-peak deflection δ is measured at the center bay relative to an adjacent bay. In the figure, δ is given as a function of P/P_{cr}, where P is the end load and P_{cr} is the value of P that causes buckling of a single bay of the beam assuming simple support boundary conditions. (See symbol list). All of these results were obtained with an eleven-bay model.

Initial imperfection. - In figure 5, the solid curve through the circular symbols is the only curve not calculated with STAGS. It is calculated with the magnification formula

$$\delta = \frac{\delta_o}{1 - P/P_{cr}} \tag{1}$$

in which δ_o is the initial value of δ, which in this case is caused solely by the initial imperfection. This curve is included for reference purposes.

Results obtained with STAGS for the case of an initial imperfection only — no lateral load — are given by the dashed curve through the square symbols. For small values of P/P_{cr}, these results are very nearly the same as those obtained with equation (1) — the solid curve with circular symbols. For that reason, the dashed curve does not begin until $P/P_{cr} \approx .6$. For larger values of P/P_{cr}, the results begin to differ. These differences are caused by dif-

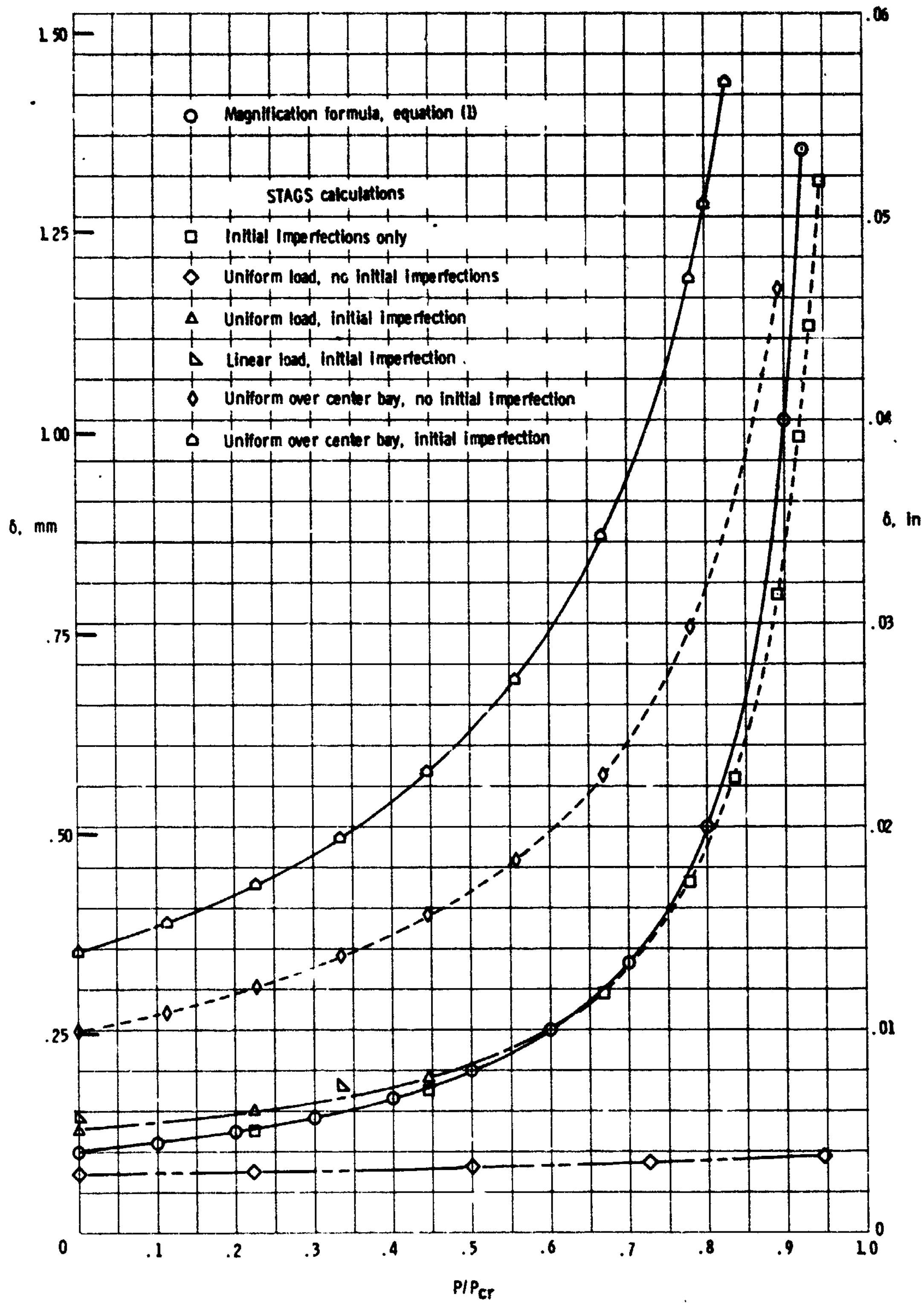


Figure 5. - Deflections in center bays for beam with clamped ends

ferences in boundary conditions. The dashed curve was calculated for a beam with clamped boundary conditions. The solid curve is essentially the same as would be obtained from STAGS with simple support boundary conditions.

Uniform load. - For the case of no initial imperfection, the results are given by the lowest curve in figure 5. These results are the same as one would obtain for a beam column (single bay) of length 5.44 cm (2.14 in.) with clamped boundary conditions. These deflections will not become large until $P = 4\ P_{cr}$ (the clamped buckling load for a bay). It is emphasized that the assumption of no initial imperfection is unrealistic.

For the case of an initial imperfection, plus a uniform load, the results for small values of P/P_{cr} are given by the centerline curve that passes through the triangular symbols. The curve ends at $P/P_{cr} \approx .6$. For values of $P/P_{cr} > .6$, the data falls on the dashed curve — the case of an initial imperfection and zero lateral load. These results could be expected since the deflections associated with the initial imperfection grow much faster than the deflections associated with the uniform load.

Linear load. - The linear load case has the same characteristics as the uniform load case. With no initial imperfection, the deflections grow very slowly and are essentially the same as the lowest curve. With an initial imperfection, the deflections are slightly larger than those of the uniform load case for small P/P_{cr} and are essentially equal to the uniform load case for large P/P_{cr}.

Uniform load over only center bay. - This loading produces a deflection pattern that is very different from either the uniform or linear load case. As can be seen by the dashed curve through the diamond symbols, deflections grow rapidly with P/P_{cr} when no initial imperfection is present. With an initial imperfection, the deflections grow even more rapidly.

Results for Beams with Simply-Supported Ends, Deflection in Center Bays

In these results, which are given in figures 6 and 7, the beam is assumed to be simply-supported at the end supports. The data in figures 6 and 7 are presented in the same way as in figure 5.

Initial imperfection. - The solid curve through the circular symbols in figure 6 is for an initial sinusoidal imperfection and no lateral load. It is the same as the corresponding curve in figure 5. The curve is calculated with the magnification formula, equation (1). The results are essentially the same as would be obtained from STAGS for this case.

Uniform load. - For the case of no initial imperfection, the results are given by the lowest curve in figure 6. Unlike the clamped case, in which the deflections do not grow rapidly as P/P_{cr} approaches 1.0, the deflections do grow rapidly as P/P_{cr} approaches 1.0 for the simply supported case.

For the case of an initial imperfection, plus a uniform load, the results are given in figure 6 by the centerline curve that passes through the triangular symbols. For all values of P/P_{cr}, these results are somewhat larger than the results obtained with only an initial imperfection.

The data for the two curves just discussed were obtained with a thirteen-bay model. These two cases were also examined with a twenty-one-bay model. In both models, the deflections associated with (1) the initial imperfection, (2) the lateral loading, and (3) the buckle mode were all in the same direction in the center bay.

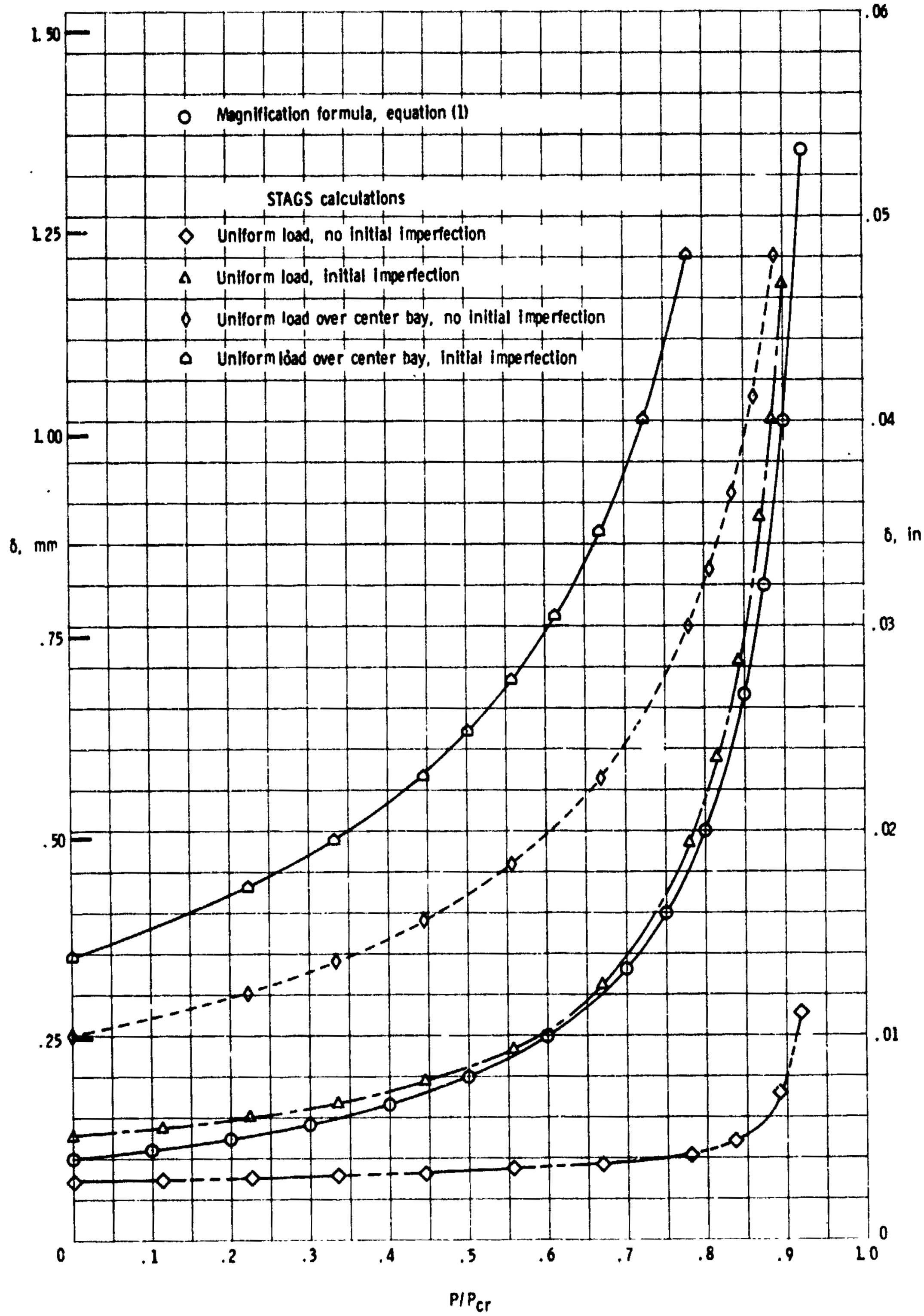


Figure 6. - Deflections in center bays for beam with simply-supported ends

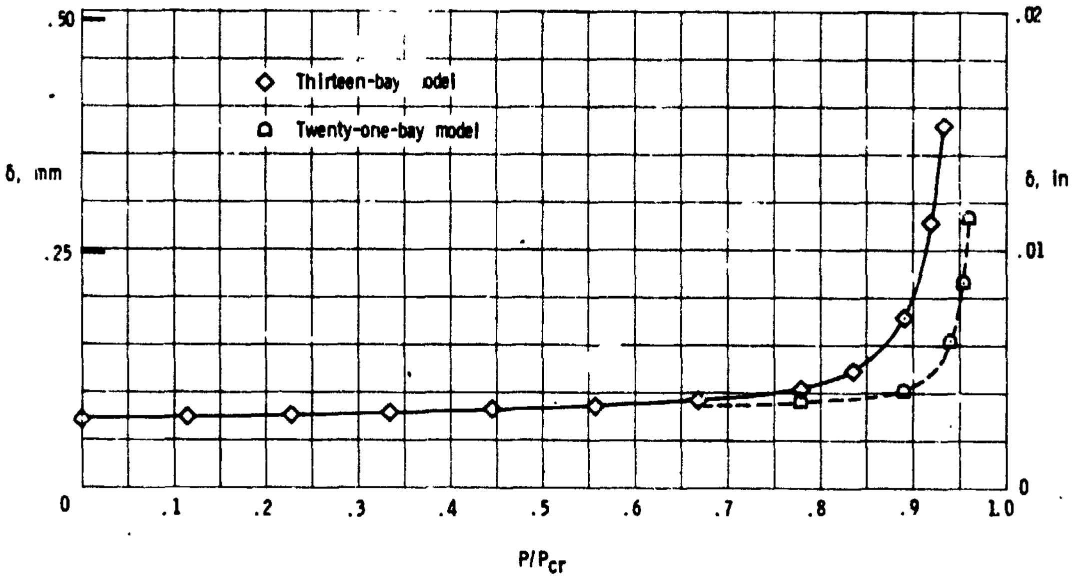


Figure 7.- Deflections in center bays for beam with simply supported ends, uniform load, no initial imperfection

With a uniform load and no initial imperfection, the deflections given by both the twenty-one-bay model and the thirteen-bay model are presented in figure 7. It can be concluded that the simple support boundary condition produces a deflection pattern that becomes very large as P/P_{cr} approaches 1.0. Also, the larger the number of bays, the nearer P/P_{cr} must be to 1.0 before the deflections at the center become large.

With a uniform load, plus an initial imperfection, the twenty-one-bay model gives the same results as the thirteen-bay model for small values of P/P_{cr}. For large values of P/P_{cr}, the twenty-one-bay model gives results that are smaller than those of the thirteen-bay model and slightly larger than the results obtained with the magnification formula considering only the initial imperfection. For example, at $P/P_{cr} = .877$, the STAGS results for the twenty-one-bay model are only 2.5 percent greater than results given by the magnification formula. It can be concluded that as the number of bays becomes large, it is the imperfection rather than the uniform pressure loading that produces large deflections in the center bays.

It should be pointed out that the deflections predicted by STAGS become too large as P approaches P_{cr}. An exact solution (refs. 2 and 3) does not give infinite deflections at $P = P_{cr}$. For the examples presented in this report, the STAGS results are probably acceptable to at least $P/P_{cr} = .9$, and the conclusions reached based on these results are valid.

Linear load. - The linear load case has the same characteristics and essentially the same values of the deflection as the uniform load case. Therefore, no results are shown for this case.

Uniform load over only center bay. - The results are presented in figure 6. Except for cases in which P/P_{cr} is very large, the results for simple support boundary conditions are the same as the results for clamped boundary conditions. With no imperfection, the deflections are large and grow as P/P_{cr} approaches 1.0. With an initial imperfection, the deflections are even larger. An eleven-bay model was used for these calculations.

Results for Beams with Simply-Supported Ends, Deflections in End Bays

The deflections in the end bays is presented in figure 8, which is organized in the same was as figure 5, 6, and 7.

Initial imperfection. - The solid curve through the circular symbols is for an initial sinusoidal imperfection and no lateral load. This same curve is presented in figures 5 and 6. The curve is calculated with the magnification formula, equation (1) and is essentially the same as would be obtained from STAGS for this case.

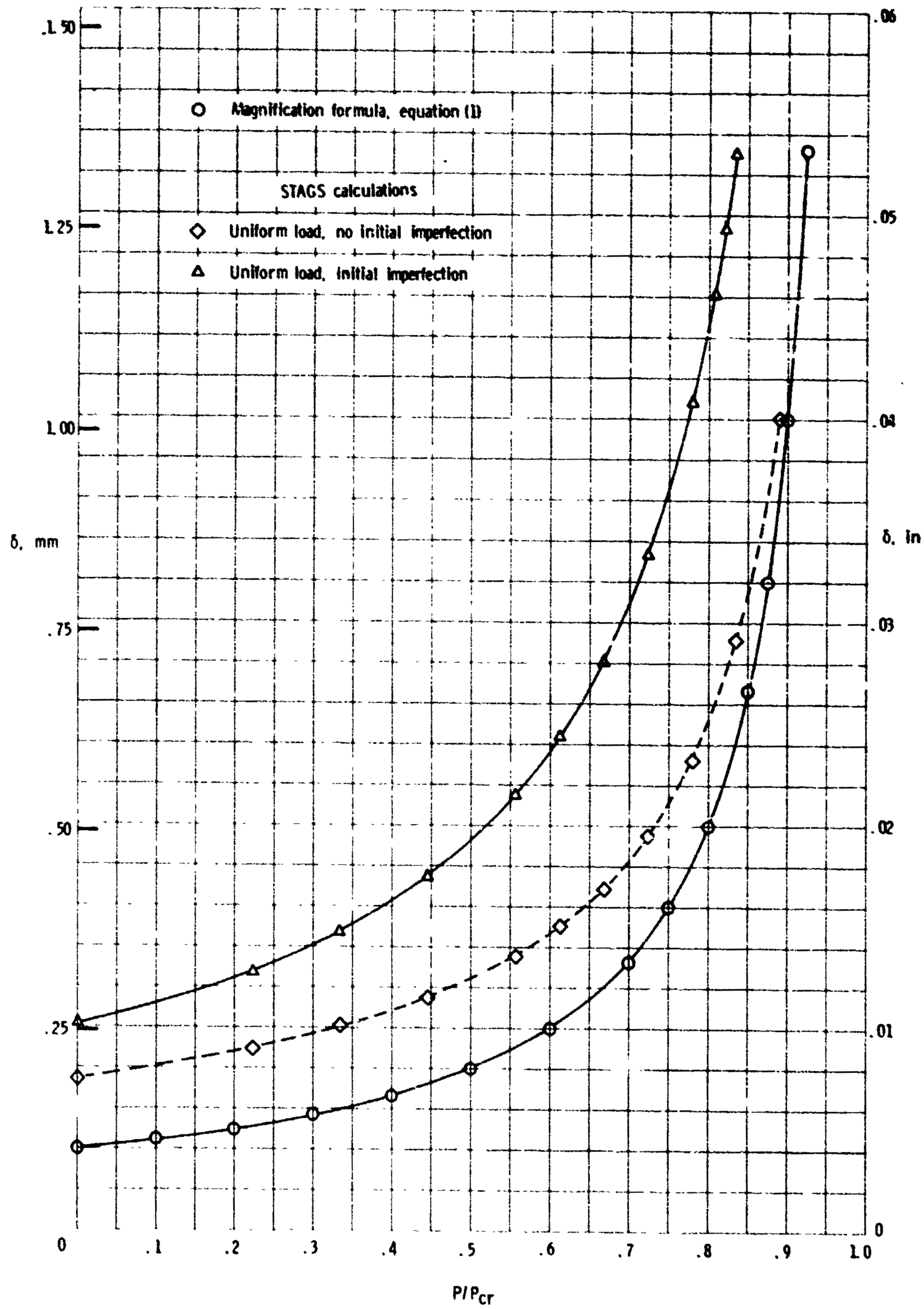


Figure 8. - Deflections in end bays for beam with simply-supported ends

Uniform load. - For the case of no initial imperfection, the results are given by the dashed curve through the diamond symbols. The deflections in the end bays are much larger than and grow much faster than the deflections in the center bays.

For the case of an initial imperfection, in addition to the uniform load, the results are given by the solid line that passes through the triangular symbols. Here, again, the deflections in the end bays are much larger than and grow much faster than the deflections in the center bays.

CONCLUDING REMARKS

Deflections of beam columns on multiple, equally-spaced supports were calculated for a variety of loadings and boundary conditions using the STAGS computer program. In all but two cases, the deflections became very large as the end load approached the buckling load of a simply-supported column with length equal to the distance between supports. Each of the two cases that were exceptions had clamped boundary conditions and no initial imperfection. In one of these cases, the lateral loading was uniform; in the other case, the lateral loading varied linearly along the length. In these two special cases, the deflections remained small for end loads exceeding the buckling load of a simply-supported column with length equal to the distance between supports. Since small initial imperfections always exist in practical structures, it is recommended that these two special cases be ignored for design purposes.

There are three other conclusions. (1) As the number of bays becomes large, the effect of boundary conditions on the deflections in the center bays diminishes. (2) For cases involving a uniform or linearly varying load, imperfections can have a much larger effect on deflections in the center bays than can lateral pressure. (3) For beams with clamped ends, the deflections in the center bays are representative of the deflections of the entire beam; however, for beams with ends that are not clamped, the end bays should be examined separately. In the case of simple-support boundary conditions, the deflections in the end bays are much larger than and grow much faster than the deflections in the center bays.

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