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FINAL REPORT

NASA GRANT: NSG-1414, Suppl. 2

THE DYNAMICS AND CONTROL OF LARGE
FLEXIBLE SPACE STRUCTURES-III

PART A: SHAPE AND ORIENTATION CONTROL OF
A PLATFORM IN ORBIT USING POINT
ACTUATORS

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THE DYNAMICS AND CONTROL OF LARGE
FLEXIBLE SPACE STRUCTURES-III

PART A: SHAPE AND ORIENTATION CONTROL OF
A PLATFORM IN ORBIT USING POINT
ACTUATORS

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ABSTRACT

The dynamics and attitude and shape control of a large thin flexible square platform in orbit are studied. Attitude and shape control is assumed to result from actuators placed perpendicular to the main surface and one edge and their effect on the rigid body and elastic modes is modelled to first order. The equations of motion are linearized about three different nominal orientations: (1) the platform following the local vertical with its major surface perpendicular to the orbital plane; (2) the platform following the local horizontal with its major surface normal to the local vertical; and (3) the platform following the local vertical with its major surface perpendicular to the orbit normal. The stability of the uncontrolled system is investigated analytically. Once controllability is established for a set of actuator locations, control law development is based on decoupling, pole placement, and linear optimal control theory. Frequencies and elastic modal shape functions are obtained using a finite element computer algorithm and two different approximate analytical methods and the results of the three methods compared.

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I. INTRODUCTION

The present grant represents a continuation of the effort attempted in the previous grant years (May 1977 - May 1979) and reported in Refs. 1 - 4*. Attitude control techniques for the pointing and stabilization of very large, inherently flexible spacecraft systems are being investigated in this research. First the attitude dynamics and control of a long, homogeneous flexible beam whose center of mass is assumed to follow a circular orbit have been treated ^{1,2}. In the initial phase, first-order effects of gravity-gradient were included, whereas external perturbations and related orbital station keeping maneuvers were ignored. Three mathematical models describing the system's rotations and deflections within the orbital plane have been developed--one model, which treats the beam as a number of discretized mass particles connected by massless links¹, and two continuum-type models.^{2,3} The natural (uncontrolled) dynamics of this system have been simulated. The concept of distributed modal control¹, which provides a means for controlling a particular system mode independently of all other modes, has been examined, along with other types of control laws including an application of optimal control theory and the use of decoupling techniques.³ The effect of varying the number of modes in our model as well as the number and location of control devices has been examined, analytically, where possible, and numerically for general cases.³

*For references cited in this report please see list of references after each chapter.

Towards the end of the second grant year the three dimensional model of a free-free plate in orbit was developed and a limited number of computer simulations of the uncontrolled dynamics in response to initial perturbations about a specific equilibrium orientation were performed.⁴ Frequency values associated with the basic structural modes of a square plate were obtained from energy considerations based on approximate expressions developed by Warburton.⁵ It was suggested at the final oral grant presentation that a comparison with results obtained using finite element methods and/or other analytical approaches should be examined to guarantee accuracy, particularly for higher order modes.

With this background and in accordance with our proposal to NASA dated January 25, 1979⁶, a plan of study was developed and has been extended to include the current grant year as outlined in Table I. The items indicated by a check mark have been completed by the end of the third grant year while those indicated by "IP" are currently in progress.

In this part of the 1979-80 final report (Part A) the control of an orbiting square shaped platform based on the continuum model of Ref. 2 with point actuators taken at selected locations on the platform surfaces is examined. A paper to be presented at the following conference forms the basis of Chapter II:

1980 AIAA/AAS Astrodynamics Conference, Danvers, Mass.,

Aug. 11-13, 1980 (only the contributions by A.S.S.R. Reddy,
P.M. Bainum, and R. Krishna are included here).

In Chapter III the results of two approximate analytical methods for predicting modal frequencies and modal shape functions are compared with the results obtained using a finite element computer algorithm using the homogeneous plate as an example.

TABLE I - STUDY PLAN 1977-1980

1. MODEL DEVELOPMENT

- ✓ A. Development of General Form of 3-Dimensional Equations for A Flexible Structure - Given the Modal Shape Functions
- ✓ B. Development of 3-Dimensional Equations of a Thin Homogenous Free-Free Beam
 - ✓ (1). The Case of No Longitudinal Vibrations: i.e. $\phi_x^{(n)} = 0$
 - ✓ (2). The Case of No Yaw: i.e. $\Psi = 0$
- C. Determination of Modal Shape Functions and Frequencies for Different Structural Models
 - ✓ (1). Circular Homogenous Membrane
 - ✓ (2). Rectangular Homogenous Membrane
 - ✓ (3). Rectangular Homogenous Plate (and Square Plate)
 - ✓ (4). Circular Homogenous Plate
 - ✓ (5). Shallow Spherical Shell Structure
- D. Implementation of One or More of the Structural Models for Digital Simulation
 - ✓ (1). Rectangular Homogenous Plate
 - ✓ (2). Thin-Homogenous Beam with Stabilizing Dumbbell (Local Horizontal Orientation)
 - (3). Square Plate with Stabilizing Dumbbell (Local Horizontal Orientation)
 - ✓ (4). Shallow Spherical Shell Structure with Stabilizing Dumbbell
 - ✓ (5). Circular Homogenous Plate with Stabilizing Dumbbell
- IP E. Provide Equations in a Form Suitable for Control Implementation
 - ✓ - Items completed
 - IP - Items in progress

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2. CONTROL CONCEPTS - LARGE FLEXIBLE SPACE STRUCTURES

A. Model Development

✓ (1). Concentrated on continuum model of large flexible beam in orbit (Santini and Howard University Formulation)

✓ (2). Modelled control devices as point actuators at specific locations along the beam

(3). Modelling of control devices as point or distributed actuators for other large flexible systems

✓ (a) Rectangular Homogenous Plate

(b) Circular Homogenous Plate

(c) Shallow Spherical Shell Structure

B. Control Concepts:

✓ (1). Modal Control - considered with discretized beam model during 1977-78

For independent control of all modes (N) retained in the model, the number of actuators (P) must be equal to N ($P=N$)

✓ (2). Establish relationship between P and N according to controllability requirements (applications of theorems developed by Balas) P can be less than N . (Applied to continuum beam model 1978-79).

✓ (3). Selection of control system gains - considers both position and rate feedback. (Applied to continuum beam model 1978-79)

✓ a. Develop criteria for complete decoupling of linearized controlled equations using the fundamental theorem of a system of N linear equations and P unknowns

For unique solution of gains, $P=N$ consistent with modal control; for non unique solution $P < N$

✓ b. Application of linear regulator problem to the original linearized and/or transformed equations

IP (4) Application of control concepts to more complex structures

IP C. Modelling of Sensors-the Problem of Observability

D. Treatment of Observation and Control Spillover

✓ Items completed

IP Items in progress

References are given separately for each chapter; symbols used in Chapter II are defined either in the text or in Appendix A of Chapter II, while symbols used in Chapter III are defined in the text where used.

Chapter IV describes general conclusions together with recommendations for future work.

Part B of this report, under separate cover, concentrates on the mathematical modelling and analysis of more complex structures such as beams and plates with connected gimballed dumbbells to provide gravitational stability about the local horizontal orientation, and also the analysis of the dynamics of a shallow shell-type structure in orbit.

I.1 References - Introduction

1. Bainum P.M. and Sellappan, R., "The Dynamics and Control of Large Flexible Space Structures," Final Report NASA Grant: NSG-1414, Part A: Discrete Model and Modal Control, Howard University, May 1978.
2. Bainum Peter M., Kumar, V.K., and James, Paul K., "The Dynamics and Control of Large Flexible Space Structures," Final Report, NASA Grant: NSG-1414. Part B: Development of Continuum Model and Computer Simulation, Howard University, May 1978.
3. Bainum, P.M. and Reddy, A.S.S.R., "The Dynamics and Control of Large Space Structures II," Final Report, NASA Grant NSG-1414, Suppl. I, Part A: Shape and Orientation Control Using Point Actuators, Howard University, June 1979.
4. Bainum, P.M., James P.K., Krishna, R., and Kumar, V.K., "The Dynamics and Control of Large Flexible Space Structures II" Final Report, NASA Grant NSG-1414, Suppl. I., Part B: Model Development and Computer Simulation, Howard University, June 1979.
5. Warburton, G.B., "The Vibration of Rectangular Plates," Proc. Institute of Mechanical Engineers, Vol. 168, No. 12, 1954, pp. 371-394.
6. Bainum, P.M., "Proposal for Research Grant on: The Dynamics and Control of Large Flexible Space Structures-III," Howard University, (Submitted to NASA), Jan. 25, 1979.

II. CONTROL OF A LARGE FLEXIBLE PLATE IN ORBIT

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Abstract

The dynamics and attitude and shape control of a large thin flexible platform in orbit are studied. Attitude and shape control is assumed to result from actuators placed perpendicular to the main surface and one edge and their effect on the rigid body and elastic modes is modelled to first order. The equations of motion are linearized about nominal orientations where the undeformed plate follows either the local vertical or local horizontal. The stability of the uncontrolled system is investigated analytically. Once controllability is established for a set of actuator locations, control law development is based on pole placement, decoupling, and linear optimal control theory.

1. Introduction

Large, flexible spacecraft systems have been proposed for future applications in widespread communications, electronic orbitally based mail systems, and as possible collectors of solar energy for transmittal to earth-based receiving stations.^{1,2} For such missions the size of the orbiting system may be several times larger than that of the earth-based receiving station(s), and both orientation and shape control of the orbiting system will be required.

In order to gain insight into the dynamics of such a large flexible system the equations of motion of a long, flexible free-free beam in orbit were developed³ using a slightly modified version of the general formulation of the dynamics of a general flexible orbiting body formulated by Santini.⁴ This specific example considered only the inplane rotations and deformations of the uncontrolled beam and demonstrated the possibility of instability for very small values of the ratio of the fundamental flexural frequency to the orbit angular velocity. Two related papers treated the modelling of point actuators located at specific points along the beam with the associated criteria for controllability⁵ and also the problem of selecting control law feedback gains based on decoupling techniques and application of the linear regulator problem.⁶ Also included were numerical results showing the effects of control spillover on the uncontrolled modes when the number of controllers is less than the number of modes in the modal, and the effects of inaccurate known age of

the control influence coefficients which lead to errors in the calculated feedback gains.⁶

In the present paper the two dimensional model considered in Refs. 3,5, and 6 is extended to three dimensions by developing the equations of motion for a large flexible rectangular plate (platform) in orbit. These equations include three rigid body equations plus the generic mode elastic equations.

2. Model Development

In the present paper three different nominal orientations of the platform in orbit are assumed about which attitude and shape control are to be achieved. These are:

- Case (i) the platform following the local vertical with its larger surface perpendicular to the plane of the orbit (Fig. 1a);
- Case (ii) the platform following the local horizontal with its larger surface area normal to the local vertical (Fig. 1b);
- Case (iii) the platform following the local vertical with its larger surface perpendicular to the orbit normal (Fig. 1c).

From the general formulation of Refs. 3 and 4, the equations of motion of the structure are obtained:

A. Rotational Equations of Motion:

$$\begin{aligned}\dot{\omega}_x &= \frac{I_y - I_z}{I_x} \omega_y \omega_z + \frac{G_x}{I_x} + \frac{T_x}{I_x} + \frac{C_x}{I_x} \\ \dot{\omega}_y &= \frac{I_z - I_x}{I_z} \omega_x \omega_z + \frac{G_y}{I_y} + \frac{T_y}{I_y} + \frac{C_y}{I_y} \\ \dot{\omega}_z &= \frac{I_x - I_y}{I_z} \omega_x \omega_y + \frac{G_z}{I_z} + \frac{T_z}{I_z} + \frac{C_z}{I_z}\end{aligned}\quad (1)$$

Using Euler angles to represent rigid body orientations relative to the local vertical (horizontal) system, the transformation from Euler angular rates to body rates is given by:

$$\begin{aligned}\omega_x &= \dot{\psi} + (\dot{\theta} + \omega_c) \sin\phi \\ \omega_y &= (\dot{\theta} + \omega_c) \cos\phi \cos\psi + \dot{\phi} \sin\psi \\ \omega_z &= \dot{\phi} \cos\psi - (\dot{\theta} + \omega_c) \sin\phi \cos\psi\end{aligned}\quad (2)$$

(Note: Symbols used are defined in Appendix A.)

G_{R_x} , G_{R_y} , G_{R_z} represent the gravity-gradient torques about the principal undeformed body axes and can be evaluated as:

$$\begin{aligned} G_{R_x} &= 3\omega_c^2(I_z - I_y)(-c\psi s\phi + s\psi s\phi)(s\psi s\phi + c\psi s\phi) \\ G_{R_y} &= 3\omega_c^2(I_z - I_x)(s\psi s\phi + c\psi s\phi)c\phi s\phi \\ G_{R_z} &= 3\omega_c^2(I_y - I_x)(-c\psi s\phi + s\psi s\phi)c\phi s\phi \end{aligned} \quad (3)$$

where $s(\cdot) = \sin(\cdot)$ and $c(\cdot) = \cos(\cdot)$.

B. Generic Mode Equations

The generic modal equations may be obtained for each of the three nominal orientations considered in terms of the modal amplitude (\tilde{A}_r):

For Case (i)

$$\ddot{\tilde{A}}_r + [\omega_r^2 - (\omega_x^2 + \omega_y^2) - M_{zz}] \tilde{A}_r = E_r / M_r \quad (4a)$$

where

$$M_{zz} = \omega_c^2 [3(s^2\psi s^2\phi c^2\phi + c^2\psi s^2\phi + 2s\psi s\phi c\phi s\phi) - 1]$$

For Case (ii)

$$\ddot{\tilde{A}}_r + [\omega_r^2 - (\omega_y^2 + \omega_z^2) - M_{xx}] \tilde{A}_r = E_r / M_r \quad (4b)$$

where

$$M_{xx} = \omega_c^2 [3c^2\phi c^2\phi - 1]$$

For Case (iii)

$$\ddot{\tilde{A}}_r + [\omega_r^2 - (\omega_x^2 + \omega_z^2) - M_{yy}] \tilde{A}_r = E_r / M_r \quad (4c)$$

where

$$M_{yy} = \omega_c^2 [3(c^2\psi s^2\phi c^2\phi + s^2\psi s^2\phi - 2c\psi s\phi s\phi) - 1]$$

C. Linearization

With the assumption of small amplitudes, the rotational equations of motion given by Eq. (1) become:

$$\begin{aligned} \ddot{\psi} &= \omega_c^2 \dot{\phi} \left[\frac{I_y - I_z}{I_x} - 1 \right] - \omega_c^2 \left(\frac{I_y - I_z}{I_x} \right) \psi + \frac{T_x}{I_x} + \frac{C_x}{I_x} \\ \ddot{\phi} &= \omega_c^2 \dot{\psi} \left[\frac{I_x - I_y}{I_z} + 1 \right] + 4\omega_c^2 \left(\frac{I_x - I_y}{I_z} \right) \phi + \frac{T_z}{I_z} + \frac{C_z}{I_z} \\ \ddot{\theta} &= 3\omega_c^2 \left(\frac{I_x - I_z}{I_y} \right) \theta + \frac{T_y}{I_y} + \frac{C_y}{I_y} \end{aligned} \quad (5)$$

For the present analysis, the platform is assumed to be square, thin and homogeneous, such that the following relationships among the principal moments of inertia are valid:

Case (i): $I_x = I_y$ and $I_z = 2I_x = 2I_y$

Case (ii): $I_y = I_z$ and $I_x = 2I_y = 2I_z$ (6)

Case (iii): $I_x = I_z$ and $I_y = 2I_x = 2I_z$

For small amplitude angles the generic mode equations become:

Case (i): $\ddot{\tilde{A}}_r + \omega_r^2 \tilde{A}_r = E_r / M_r$ (7)

Case (ii): $\ddot{\tilde{A}}_r + (\omega_r^2 - 3\omega_c^2) \tilde{A}_r = E_r / M_r$

Case (iii): $\ddot{\tilde{A}}_r + (\omega_r^2 - \omega_c^2) \tilde{A}_r = E_r / M_r$

D. Modelling of Point Actuators

For an actuator which can generate a force of the type

$$\bar{F} = f_x \hat{i} + f_y \hat{j} + f_z \hat{k} \quad (8)$$

and placed at a location (x, y, z) , the resultant control torque is given by

$$\bar{T} = \bar{F} \times \bar{r} \quad (9)$$

where $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ describes the position of the actuator on the surface (or edge) of the plate. Actuators can be placed perpendicular to the XY, YZ or ZX planes of the plate, so for an actuator whose force axis is perpendicular to the XY plane the torque is given by (since $f_x = f_y = 0$)

$$\bar{T} = yf_z \hat{i} - xf_z \hat{j} \quad (10)$$

For an actuator whose force axis is perpendicular to the YZ plane, the torque is given by (since $f_y = f_z = 0$)

$$\bar{T} = zf_x \hat{i} - yf_x \hat{k} \quad (11)$$

For an actuator perpendicular to the ZX plane, the torque is given by (since $f_z = f_x = 0$)

$$\bar{T} = -zf_y \hat{i} + xf_y \hat{k} \quad (12)$$

The generic force due to the r^{th} actuator on the r^{th} mode is given by

$$\begin{aligned} E_r &= \int \int W_r(x, y) \hat{k} \cdot \delta(x - x_i, y - y_i) f_i(t) \hat{k} dx dy \\ &= W_r(x_i, y_i) f_i(t) \end{aligned} \quad (13)$$

where $W_r(x, y)$ is the r^{th} modal (spatial) function of the deformed plate with vibrations assumed to occur along the Z direction, whose amplitudes are assumed to be much smaller than a characteristic plate length.

For n actuators placed on the XY plane of the plate with force axes normal to that deformed surface, the generic force on r^{th} mode is given by

$$E_r = \sum_{i=1}^n W_r(x_i, y_i) f_i \quad (14)$$

where x_i, y_i are the coordinates of the i^{th} actuator. An actuator placed normal to the X, Y plane won't produce a torque about the Z-axis; in order to obtain a direct torque about the Z-axis, actuators may have to be located on the other surfaces (edges) of the plate.

E. Modelling of Distributed Actuators

If the force is distributed along the surfaces of the plate, the force can be represented by

$$\begin{aligned} \bar{F} &= f_x(x, y, z, t) \hat{i} + f_y(x, y, z, t) \hat{j} \\ &\quad + f_z(x, y, z, t) \hat{k} \end{aligned} \quad (15)$$

where the force components are now both spatially and time dependent.

The torque due to such an actuator is given by

$$\bar{T} = \bar{R}\bar{x}\bar{f} \quad (16)$$

The total torque is given by

$$\bar{T} = \int (\bar{R}\bar{x}\bar{f}) dx dy dz \quad (17)$$

Using series expansions and separation of variables between spatially and time dependent functions, one can very accurately represent (e.g. for the x component),

$$f_x(x, y, z, t) = \sum_{k=1}^L f_{x_k}(x, y, z) g_{x_k}(t) \quad (18)$$

The integral for the torque is then given by

$$\begin{aligned} T = & \int \left[\left(\sum_{i=1}^N y f_{z_i}(x, y, z) g_{z_i}(t) - \sum_{i=1}^M y f_y(x, y, z) g_y(t) \right) \right. \\ & + \left(\sum_{k=1}^L x f_{z_k}(x, y, z) g_{z_k}(t) - \sum_{i=1}^N x f_{x_i}(x, y, z) g_{x_i}(t) \right) \dot{\psi} \\ & \left. + \left(\sum_{i=1}^N x f_y(x, y, z) g_y(t) - \sum_{k=1}^L x f_{x_k}(x, y, z) g_{x_k}(t) \right) \dot{\theta} \right] dx dy dz \quad (19) \end{aligned}$$

The resulting generic force is then obtained in the same manner as in Eq. (13) with the result,

$$E_r = \int [W_r(x, y)] \left[\sum_{i=1}^N f_{z_i}(x, y, z) g_{z_i}(t) \right] dx dy dz \quad (20)$$

3. Uncontrolled Motion-Numerical Example

The platform is assumed to have the following physical properties:

$$a = 100 \text{ m} \quad (\text{side of square plate})$$

$$M = 276800 \text{ kg}$$

$$\text{Minimum Moment of Inertia} = 2.354 \times 10^7 \text{ kg-m}^2$$

$$\text{Maximum Moment of Inertia} = 4.7088 \times 10^7 \text{ kg-m}^2$$

For an assumed orbital altitude of 250 n.mi. (circular)

$$\omega_c = 1.25 \times 10^{-3} \text{ rad/sec.}$$

The modal frequencies of the elastic modes have been obtained using a finite element computer algorithm.⁷ For the first three flexible modes:

$$\omega_1 = 2.0931947 \times 10^{-2} \text{ rad/sec}$$

$$\omega_2 = 3.0404741 \times 10^{-2} \text{ rad/sec}$$

$$\omega_3 = 3.9088122 \times 10^{-2} \text{ rad/sec}$$

The uncontrolled motion of the linear system through small amplitude deviations with respect to each of the three nominal orientations will now be considered.

$$\text{Case (i): } I_y = I_z, I_x = 2I_x = 2I_y$$

The rotational equations of motion and the generic modal equations are non-dimensionalized by the orbital period and the length variable ($\tau = \omega_c t$, $Z_r = A_r/a$, $\dot{\psi} = d\phi/dt$, etc.)

$$\dot{\psi}'' = [(I_y - I_z - I_x)/\omega_c I_x] \dot{\psi}' - [(I_y - I_z)/I_x] \dot{\psi} \quad (21)$$

$$\dot{\theta}'' = [(I_x - I_y + I_z)/\omega_c I_z] \dot{\psi}' + 4[(I_y - I_z)/I_z] \dot{\psi} \quad (22)$$

$$\dot{\theta}'' = 3\dot{\theta} \quad (23)$$

The generic mode equations become:

$$Z_r'' = -(\omega_r/\omega_c)^2 Z_r \quad (24)$$

The pitch and the generic mode equations are decoupled from roll and yaw. The pitch and generic modes exhibit simple harmonic motions. After substituting inertia values into the roll and yaw equations,

$$\dot{\psi}'' = -(2/\omega_c) \dot{\psi}' + \psi \quad (25)$$

$$\dot{\theta}'' = \dot{\psi}'/\omega_c \quad (26)$$

The characteristic equation for the system (25) and (26) is, $s^2(s^2 - 1 + 2/\omega_c^2) = 0$

It can be seen that the roll and yaw motion has a double pole at the origin and thus the uncontrolled roll/yaw motion is unstable. The analytical solution is obtained using Laplace transform techniques. A typical response for initial perturbations in both roll and yaw rate(s) is shown in Fig. 2.

$$\text{Case (ii): } I_y = I_z \text{ and } I_x = 2I_y$$

The rotational equations of motion are

$$\dot{\psi}'' = -(1/\omega_c) \dot{\psi}' \quad (27)$$

$$\dot{\theta}'' = (2/\omega_c) \dot{\psi}' + 4\dot{\psi} \quad (28)$$

$$\dot{\theta}'' = 3\dot{\theta} \quad (29)$$

The generic mode equations can be represented by,

$$Z_r'' = -[(\omega_r/\omega_c)^2 - 3] Z_r \quad (30)$$

From Eq. (29) the pitch amplitude increases exponentially in response to an initial displacement, whereas from Eq. (30), for $\omega_r/\omega_c > \sqrt{3}$ the generic modal amplitudes exhibit simple harmonic motion.

The characteristic equation for the combined roll/yaw motion is:

$$s^2(s^2 - 4 + 2/\omega_c^2) = 0 \quad (31)$$

The roll/yaw motion is characterized by a double pole at the origin and is thus unstable.

$$\text{Case (iii): } I_x = I_z \text{ and } I_y = 2I_x = 2I_z$$

The rotational equations of motion are

$$\dot{\psi}'' = -\dot{\psi}; \dot{\theta}'' = -4\dot{\psi}; \dot{\theta}'' = 0 \quad (32)$$

while the generic mode equations can be expressed by,

$$Z_r'' = -[(\omega_r/\omega_c)^2 - 1] Z_r \quad (33)$$

In this case, roll, yaw, pitch and the generic modes are decoupled from each other. The generic modes, roll and yaw exhibit simple harmonic motion, while the pitch amplitude increases linearly with time for a given initial pitch rate.

Case (i) Platform Following Local Vertical
With Major Surface Normal to the Orbit Plane.

The non zero elements of the A matrix are:

$$A_{1,1+6} = 1 \text{ for } i = 1, \dots, 6; A_{7,8} = 800;$$

$$A_{8,7} = -1600; A_{8,2} = 1; A_{9,3} = -3;$$

$$A_{10,4} = -277.414, A_{11,5} = -588.647;$$

$$A_{12,6} = -976.844$$

Location I (Lower Part of B matrix)

0.0	0.0	0.0	0.0	0.0	-0.22643
-0.4329	-0.4329	0.0	0.0	0.4329	0.0
0.4329	-0.4329	0.4329	-0.4329	0.4329	0.0
0.003126	-0.003126	0.0	0.0	-0.003126	0.0
0.0	0.0	-0.003084	-0.003084	0.0	0.0
-0.008786	-0.008786	-0.0113	-0.0113	-0.008786	0.0

Location II (Lower Part of B matrix)

0	0	0	0	0	-0.6796
-1.3592	-1.3592	1.3592	1.3592	0	0.0
1.3592	-1.3592	1.3592	-1.3592	1.3592	0.0
0.023	-0.023	-0.023	0.023	0.0	0.0
0.0	0.0	0.0	0.0	-0.023	0.0
0.023	0.023	0.023	0.023	0.023	0.0

Case (ii) Platform Along Local Horizontal

The elements of the A matrix that are different from Case (ii) are:

$$A_{7,1} = 4; A_{7,8} = 1600; A_{8,2} = 0; A_{8,7} = -800;$$

$$A_{9,3} = 3.$$

Location I (Lower Part of B matrix)

0.4392	0.4329	0.0	-2.0	-0.4329	0.0
0.0	0.0	0.0	0.0	0.0	0.22643
-0.4329	0.4329	-0.4329	0.4329	-0.4329	0.0
0.003126	-0.003126	0.0	0.0	-0.003126	0.0
0.0	0.0	-0.003084	-0.003084	0.0	0.0
-0.008786	-0.008786	-0.0113	-0.0113	-0.008786	0.0

Location II (Lower Part of B matrix)

1.3592	1.3592	-1.3592	-1.3592	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.6796
-1.3592	1.3592	-1.3592	1.3592	-1.3592	0.0
0.023	-0.023	-0.023	0.023	0.0	0.0
0	0.0	0.0	0.0	-0.023	0.0
0.023	0.023	0.023	0.023	0.023	0.0

Case (iii) Major Surface in Orbit Plane

The elements of the A matrix that are different from Case (ii) are:

$$A_{7,1} = -4; A_{7,8} = 0; A_{8,2} = -1; A_{8,7} = 0;$$

$$A_{9,3} = 0.$$

Location I (Lower Part of B matrix)

-0.4329	-0.4329	0.0	0.0	0.4329	0.0
0.4329	-0.4329	-0.4329	-0.4329	0.4329	0.0
0.0	0.0	0.0	0.0	0.0	-0.22643
0.003126	-0.003126	0.0	0.0	-0.003126	0.0
0.0	0.0	-0.003084	-0.003084	0.0	0.0
-0.008786	-0.008786	-0.0113	-0.0113	-0.008786	0.0

Location II (Lower Part of B matrix)

-1.3592	-1.3592	1.3592	1.3592	0.0	0.0
1.3592	-1.3592	1.3592	-1.3592	1.3592	0.0
0.0	0.0	0.0	0.0	0.0	-0.6796
0.023	-0.023	-0.023	0.023	0.023	0.0
0.0	0.0	0.0	0.0	-0.023	0.0
0.023	0.023	0.023	0.023	0.023	0.0

For all combinations considered above the gains are selected so as to produce 20% of critical damping in each of the rigid body modes and the first generic mode, and 10% of critical damping in the second and third generic modes. In order to provide a better transient response in the lower frequency fundamental elastic mode, the percentage of critical damping is selected to be twice that in the remaining flexible modes. The time response of the rigid body modes and the generic modal amplitudes for all combinations considered and for equal initial position displacements in all components of the state is illustrated in Fig. 4a.

As an example of the time history of the required control forces, Fig. 4b. shows such a time response for the exterior (II nd) location of the actuators with the platform nominally following the local vertical and the major surface area of the platform in the orbital plane. A complete summary of the maximum force amplitudes required for all combinations of actuator locations and platform orientations is given in Table I. In interpreting the results of Table I, it should be pointed out that, in the process of achieving both orientation and shape control, the maximum force(s) required of any actuator will vary with both the moment arm about the principal body axes and the value of the modal shape function at the particular actuator location for all modes contained within the mathematical modal.

(b) Stabilizing the System by Pole Clustering

The equations of motion of the platform when recast in state space format can be written as

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \quad (38), \quad \mathbf{x} \in \mathbb{R}^{(n+3)x1}$$

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Table I Maximum Force Amplitudes (Newtons) for Different Combinations of Cases with Actuator Locations

Force	Location I (interior)			Location II (exterior)			Indicates maximum level of control force required for each case.
	Case i	Case ii	Case iii	Case i	Case ii	Case iii	
f_1	-807.8	270.63	270.6	57.6	62.56	47.4	
f_2	270.0	136.2	-133.3	28.19	26.20	-17.2	
f_3	1141.0	387.3	-425.0	-65.89	-49.5	30.0	
f_4	527.9	195.6	163.7	17.10	-30.90	25.08	
f_5	489.09	-253.3	153.0	-44.6	-44.67	-44.67	
f_6	-264.58	-272.0	4.4	88.16	-88.06	1.47	

The control, $U = -KX$ is selected by using a digital computer algorithm such that $(A+RK)$ has the required identical negative real part in each of its eigenvalues. Although the number of actuators can be less than the number of modes (one half of the dimensionality of the state vector), a limitation of this algorithm is that the gains are selected such that all of the closed-loop poles lie on a line parallel to the imaginary axis. However this algorithm is useful when it is important that each mode in the system satisfy some minimum damping characteristics.

As an example of this technique we consider the system with four actuators and six modes where control about the first orientation (Case i) is desired. Three of the actuators are assumed to provide forces perpendicular to the major surface with the remaining actuator thrusting normal to an edge. The actuator coordinates in the body system (Fig. 1a) are: $f_1(-a/6, -a/6, 0)$; $f_2(a/6, -a/6, 0)$; $f_3(-a/6, 0, 0)$; and $f_4(a/2, a/6, 0)$ where $a = 100\text{m}$. It is assumed that the minimum damping requirement on the system has a time constant of (13.33 min or $(1/2\pi)$ dimensional orbital time). The control influence matrix is then calculated based on the assumed coordinates of the four actuators. The control $U = -KX$ can be calculated by the ORACLS pole clustering algorithm. Based on these gains time histories of the required control forces are then obtained.

The control influence matrix (lower part), closed loop poles, and maximum force amplitudes required are summarized as follows:

B matrix (Lower Part)

$$\begin{bmatrix} 0.0 & 0.0 & 0.0 & -0.22643 \\ -0.4529 & -0.4529 & 0.0 & 0.0 \\ 0.4529 & -0.4529 & 0.0 & 0.0 \\ 0.003126 & -0.003126 & 0.0 & 0.0 \\ 0.0 & 0.0 & -0.0030844 & 0.0 \\ -0.008786 & -0.008786 & -0.0115 & 0.0 \end{bmatrix}$$

Closed Loop Poles (Nondimensionalized)

The real part is -1.0 and the imaginary parts are ± 0.000485 , ± 0.993 , ± 16.82 , ± 24.26 , ± 31.33 and ± 1131.37 . The Maximum force amplitudes (Newtons) are calculated as

$$|f_1| = 78.5, |f_2| = 36.4, |f_3| = 169.5, \text{ and}$$

$$|f_4| = 35.3.$$

An interesting comparison can now be made between this result and that shown in Table I for case (i) and the first (I) location of the six actuators considered there. It can be seen that by using fewer actuators, appropriately placed, that better transient response characteristics can be obtained with smaller maximum force amplitudes. However a disadvantage of this method is that some of the controlled frequencies may be orders of magnitude greater than the highest frequency of the uncontrolled system (for this example compare 1131 with $\sqrt{76} = 31.24$). Depending on the nature of the expected disturbance forces this result could be very undesirable.

(c) Application of the Linear Regulator Theory

The control law, $U = -KX$, is selected such that the following performance index is minimized

$$J = \int_0^T (X^T Q X + U^T R U) dt \quad (39)$$

where Q and R are positive definite penalty matrices. The steady state solution of the matrix Riccati equation of dimension equal to the state has to be solved in order to evaluate the gain matrix, K .

A computer algorithm within the ORACLS⁸ software package is used to obtain the gain matrices K for different combinations of the Q and R penalty matrices. This algorithm utilizes the Newton Raphson method of solving the Riccati equation. In the examples considered here four actuators are assumed with the system represented by three rigid body and three flexible modes. The locations of the four actuators are taken to be the same as in Section (b), and control about the first nominal orientation (i) is considered.

The weighting matrix, Q , is selected based on the following considerations. For the example considered here it can be seen from Eq. (34), (35), and the B matrix that the uncontrolled system dynamics is either described by sets of uncoupled harmonic oscillators, or (in the case of roll/yaw motion) by a coupled two dimensional harmonic oscillator. The latter motion can be represented by

$$\begin{bmatrix} \omega_z' \\ \omega_x' \end{bmatrix} = \begin{bmatrix} 0 & a \\ -b & 0 \end{bmatrix} \begin{bmatrix} \omega_z \\ \omega_x \end{bmatrix} \quad (40)$$

where the system oscillates at the frequency $\Omega = \sqrt{ab}$. It is desired that the control remove a maximum "transverse" angular rate, $\omega_{T_{max}} = \max(\omega_z, \omega_x) = \sqrt{\omega_z^2 + \omega_x^2}$

so that a strategy for selecting the elements of Q could be^{9,10}

$$Q = \begin{bmatrix} f & 0 \\ 0 & f \end{bmatrix} \text{ where } f = \omega^2/\omega_c^2 \text{ at } \max \quad (41)$$

when the control penalty matrix is fixed. The remaining equations for any of the uncoupled oscillators can also be expressed by

$$\begin{bmatrix} z'_1 \\ z''_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -(\omega_1/\omega_c)^2 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z'_1 \end{bmatrix} \quad (42)$$

in the same format as Eq. (39), and thus the weights can be obtained in a similar manner.

The Q matrix for the case considered here (control about nominal orientation (1) with actuator locations as given in Section (b)) is obtained using the relations given by Eq. (41) and is a diagonal matrix, Q_0 , with the following elements:

$$\begin{aligned} Q_{1,1} &= 4.324 \times 10^9, \quad Q_{2,2} = 8.539 \times 10^9, \\ Q_{3,3} &= Q_{9,9} = 3.0 \times 10^4, \quad Q_{4,4} = Q_{10,10} = \\ 2.77414 &\times 10^6, \quad Q_{5,5} = Q_{11,11} = 5.88647 \times 10^6, \\ Q_{6,6} &= Q_{12,12} = 9.74844 \times 10^6, \quad Q_{7,7} = Q_{8,8} = \\ 2.222 &\times 10^3. \end{aligned}$$

The R_0 matrix is chosen as an identity matrix. A parametric study is done using various multiples of the Q_0 ($Q=cQ_0$) and R_0 matrices obtained above which are plotted against the negative real part of the least damped mode of the controlled system in Fig. 5. All the loci of the negative real part of the least damped mode approach unity and no significant improvement is observed by increasing the state penalty, $Q=cQ_0$, any further. Thus one wishes to operate on the horizontal line between the points (1) and (2). The maximum amplitude of the forces for $R = I$ and $R = 1000 I$ are calculated and plotted in Fig. 6. The closed loop poles of the controlled system at points (1) and (2) are virtually the same and are given as follows (nondimensionalized):

$$\begin{aligned} -1.0043, -1.8416.64, -2.16+124.20, -17.18+119.79, \\ -26.23, -36.22, -137.64 \text{ and } -38.66+11132.11 \end{aligned}$$

The maximum force amplitudes as shown in Fig. 6 are less than those corresponding to Case (1) - Location I of Table 1 for comparable transient responses, whereas these are high as compared to the forces obtained using the pole clustering technique (Section (b)). This is due to the large negative real parts of the other modes in the linear regulator case when compared to the pole clustering technique where all the poles have an equal negative real part (-1.0). Both the linear regulator and pole clustering technique have the draw back that the controlled frequencies can be quite high compared to the uncontrolled frequencies. On the otherhand, these techniques have the advantage that they can be applied to situations where the number of actuators is less than the number of modes in the mathematical model, in contrast to the decoupling technique of Section (a).

5. Conclusions

In this paper the dynamics, stability, and control of an orbiting homogeneous, flexible square platform are considered. Three different nominal orientations of the platform are examined. When the platform is nominally following the local vertical with its larger surface perpendicular to the orbital plane and also when the platform follows the local horizontal with its larger surface normal to the local vertical, it is seen that the uncontrolled roll/yaw motion is unstable. For the case where the platform follows the local vertical with its large surface perpendicular to the orbit normal, the uncontrolled pitch motion is found to be unstable.

Three different control techniques are considered for the selection of the control laws:

- The decoupling of the original state equations using state variable feedback eliminates the need of a transformation from the original coordinates to the modal coordinates and provides a method of specifying directly the amount of damping and frequency of the individual components of the state vector. However, with this technique the number of actuators must be equal to the number of coordinates (modes) in the model.
- The pole placement algorithm (ORACLS) guarantees the over-all required damping of the system and does not restrict the number of actuators to be equal to the number of modes in the model. However, it is seen that the closed-loop frequencies may be greatly increased when compared to the open-loop values which may cause problems with externally induced periodic excitations.
- The linear regulator theory can provide acceptable performance once the state and penalty matrices are properly selected, and the number of actuators can be less than the number of modes in the model. Computer capacity and accuracy limit the number of modes that can be considered. Here, too, an undesirable increase in the closed-loop frequencies may result in order to provide satisfactory responses with maximum allowable force amplitudes.

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M_r	r^{th} modal mass
T	Torque due to an actuator
I_x, I_y, I_z	Torque components
$w_r(x, y)$	r^{th} modal shape function
Z_r	Nondimensionalized r^{th} modal amplitude function
ω_c	Orbital frequency
$\omega_x, \omega_y, \omega_z$	Angular body rates
ϕ, ψ, θ	Roll, yaw, pitch, respectively
$\omega_1, \omega_2, \omega_3$	First three modal frequencies of the plate

Appendix A - Nomenclature

A_r	r^{th} modal amplitude function
B	Control influence matrix
C_x, C_y, C_z	Disturbance torques about the principal undeformed body axes
E_r	Generic force on r^{th} mode
f	Force due to an actuator
f_x, f_y, f_z	Force components due to an actuator
$G_{R_x}, G_{R_y}, G_{R_z}$	Gravity gradient torques about the principal undeformed body axes
I_x, I_y, I_z	Moments of inertia about the principal axes
K	Gain matrix
K_r, K_p	Rate and position feedback gain matrices

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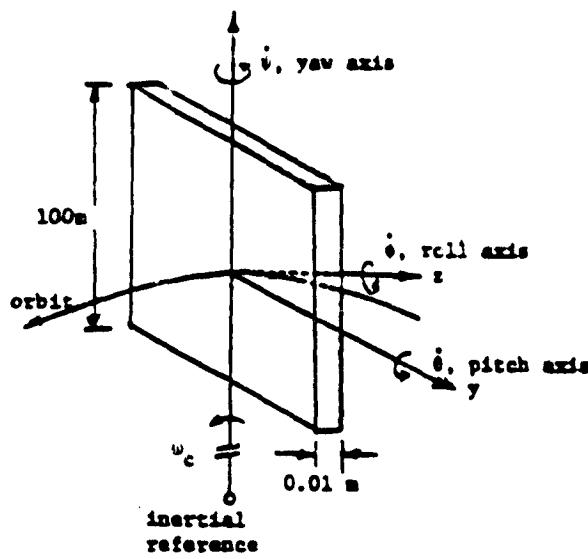


Fig. 1a. Platform following local vertical with major surface normal to the orbit plane - Case (i)

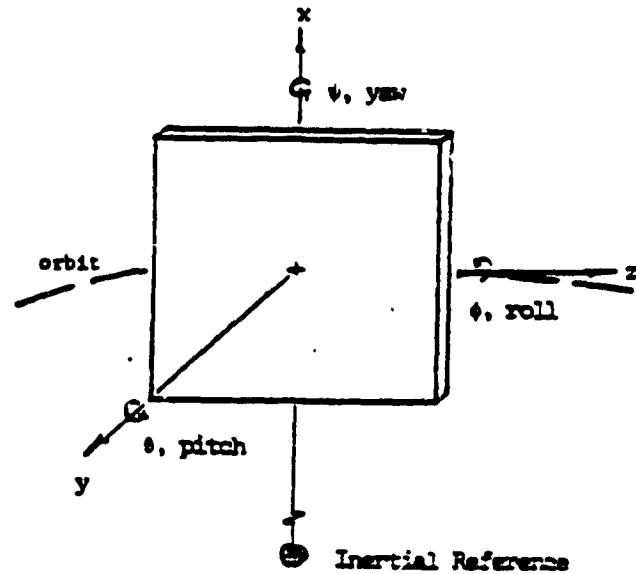


Fig. 1c. Platform following local vertical with major surface in the orbit plane - Case (iii)

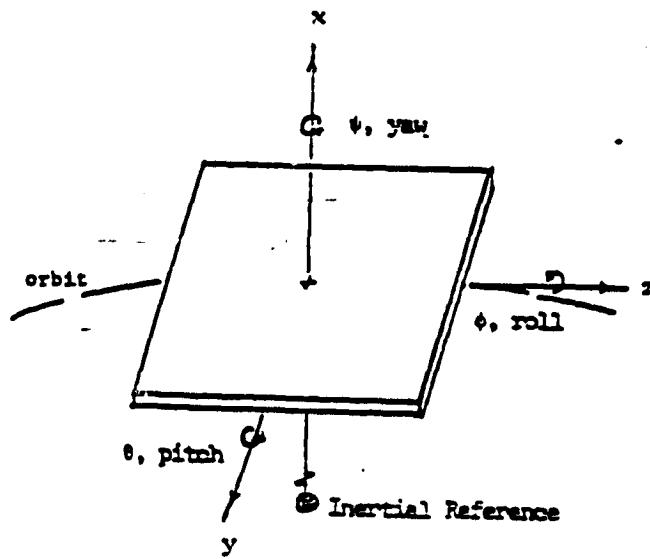


Fig. 1b. Platform along local horizontal - Case (ii)

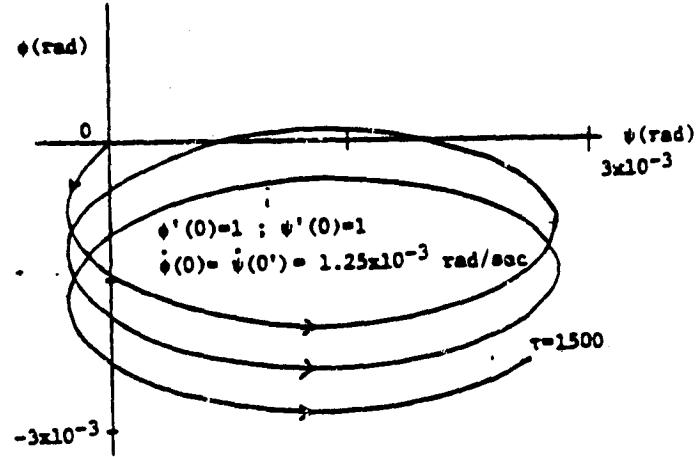


Fig. 2. Roll/yaw motion (uncontrolled) - Case (i)

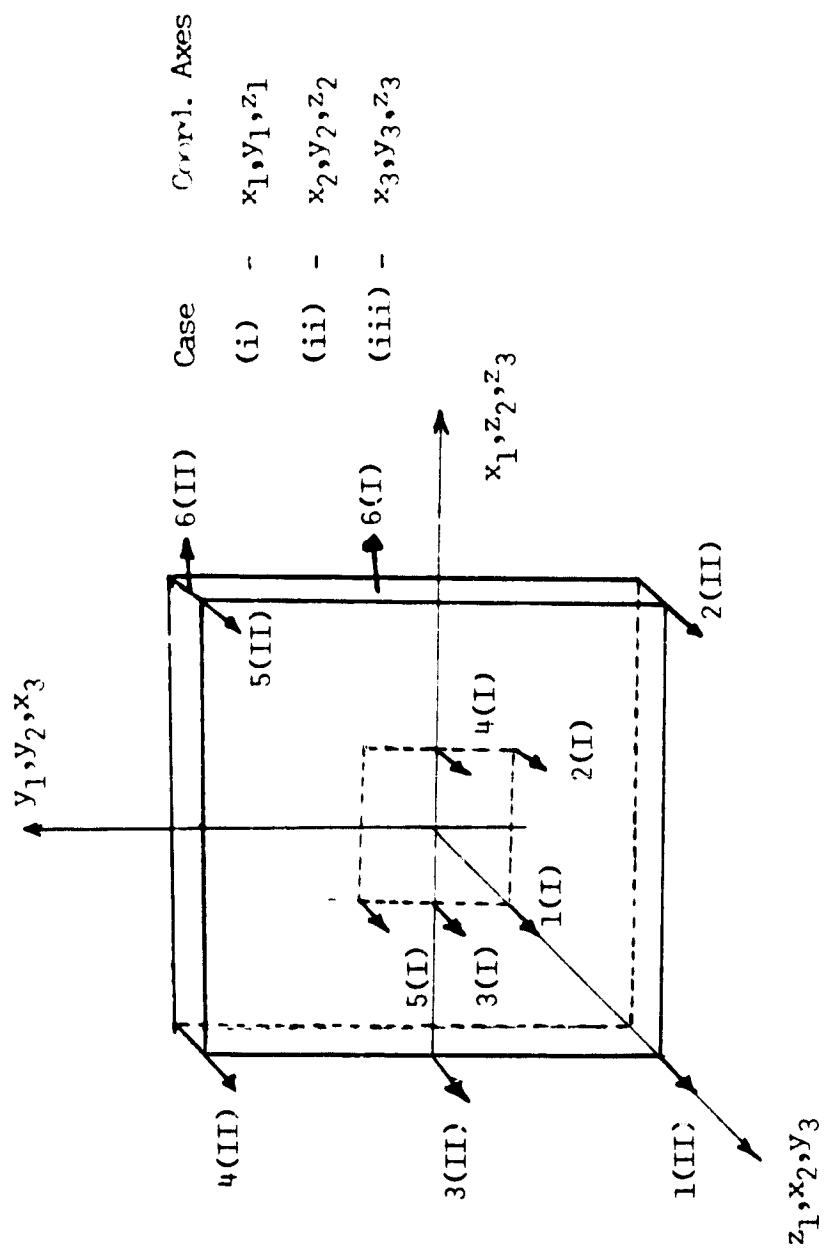


Fig 3. Location of two sets of actuators (I & II)

Non-dimensionalized rigid body modes
and three generic modes.

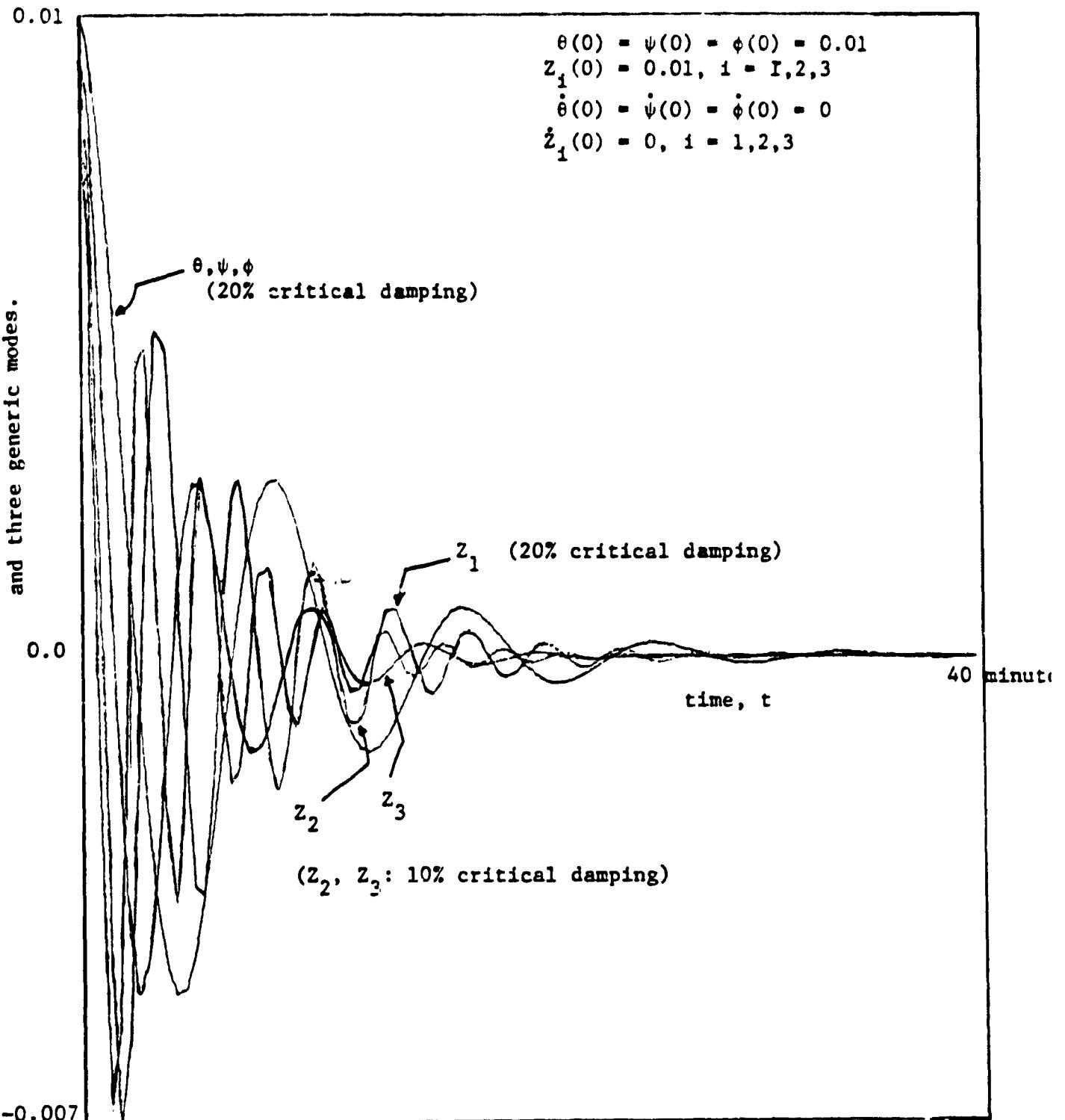


Fig 4a. Controlled state response for all combinations of orientations and actuator locations.

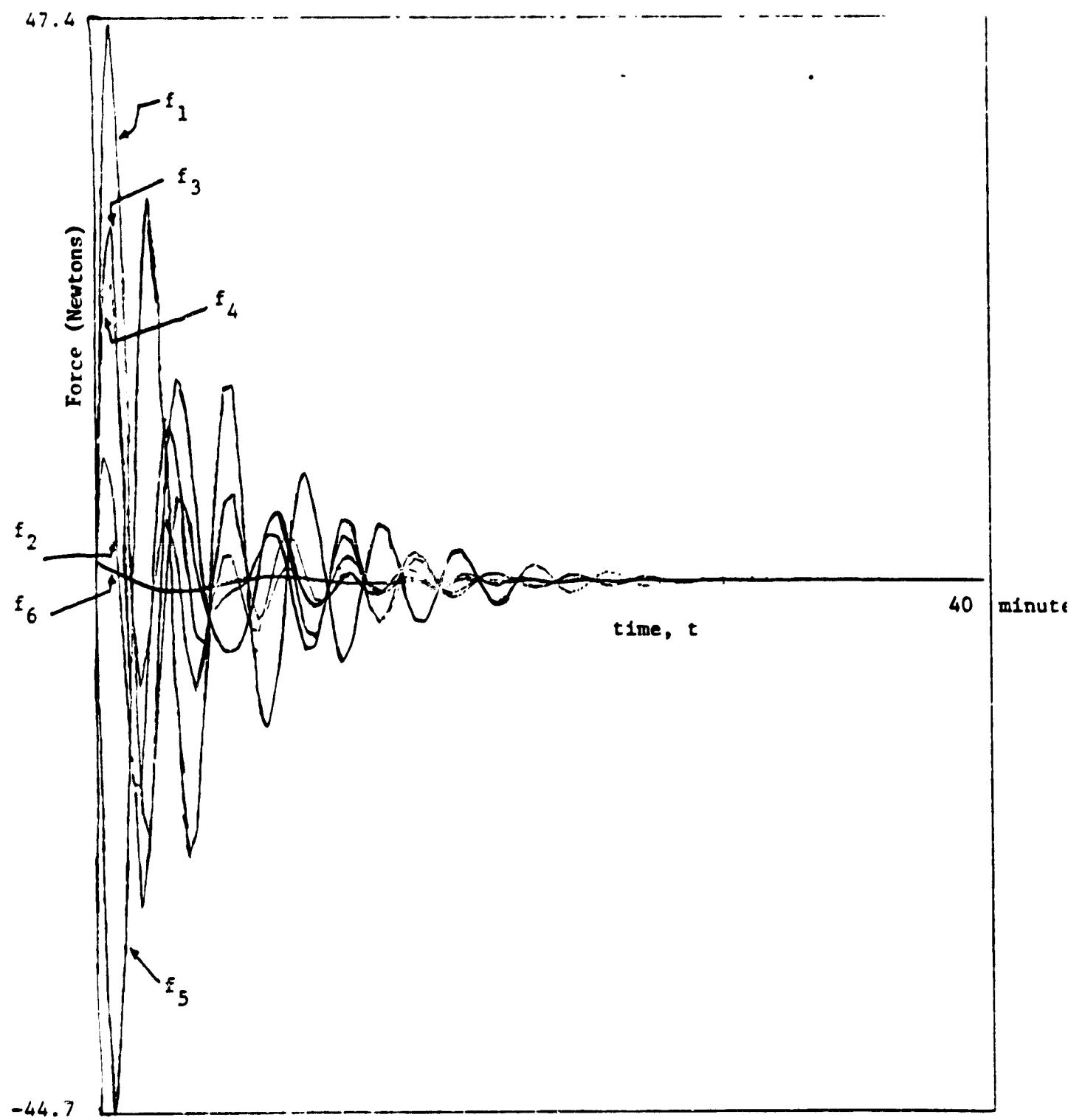


Fig 4b. Control force time history for Case (iii) - II

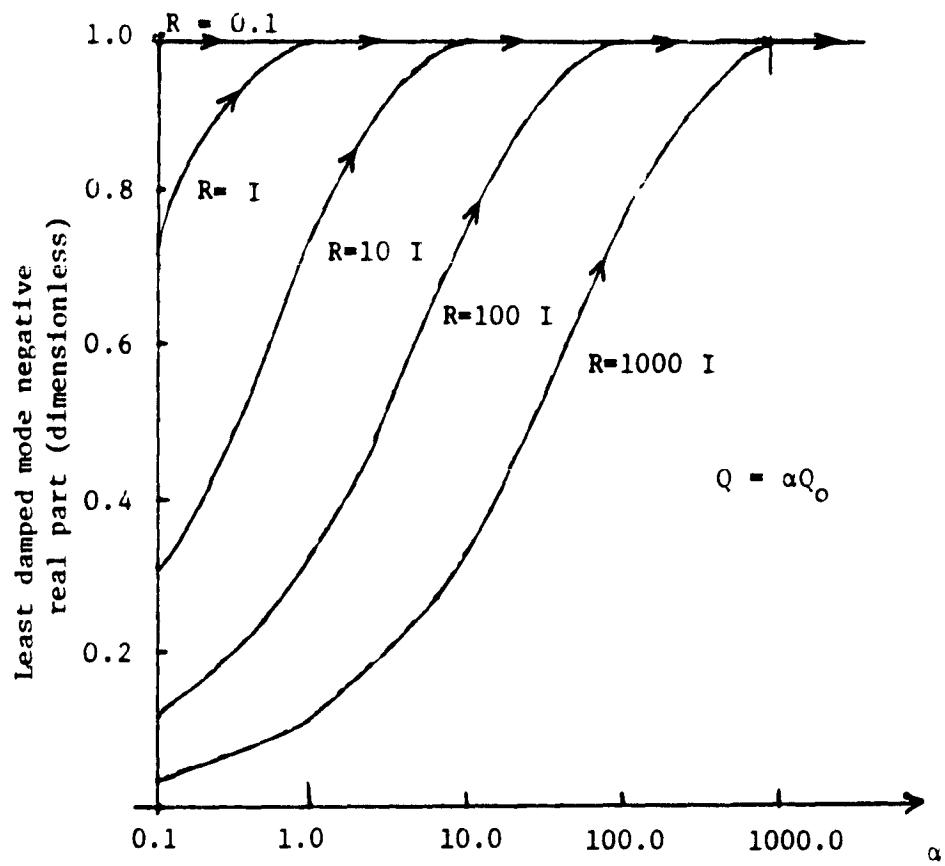


Fig. 5. Variation of least damped mode negative real part with α and R .

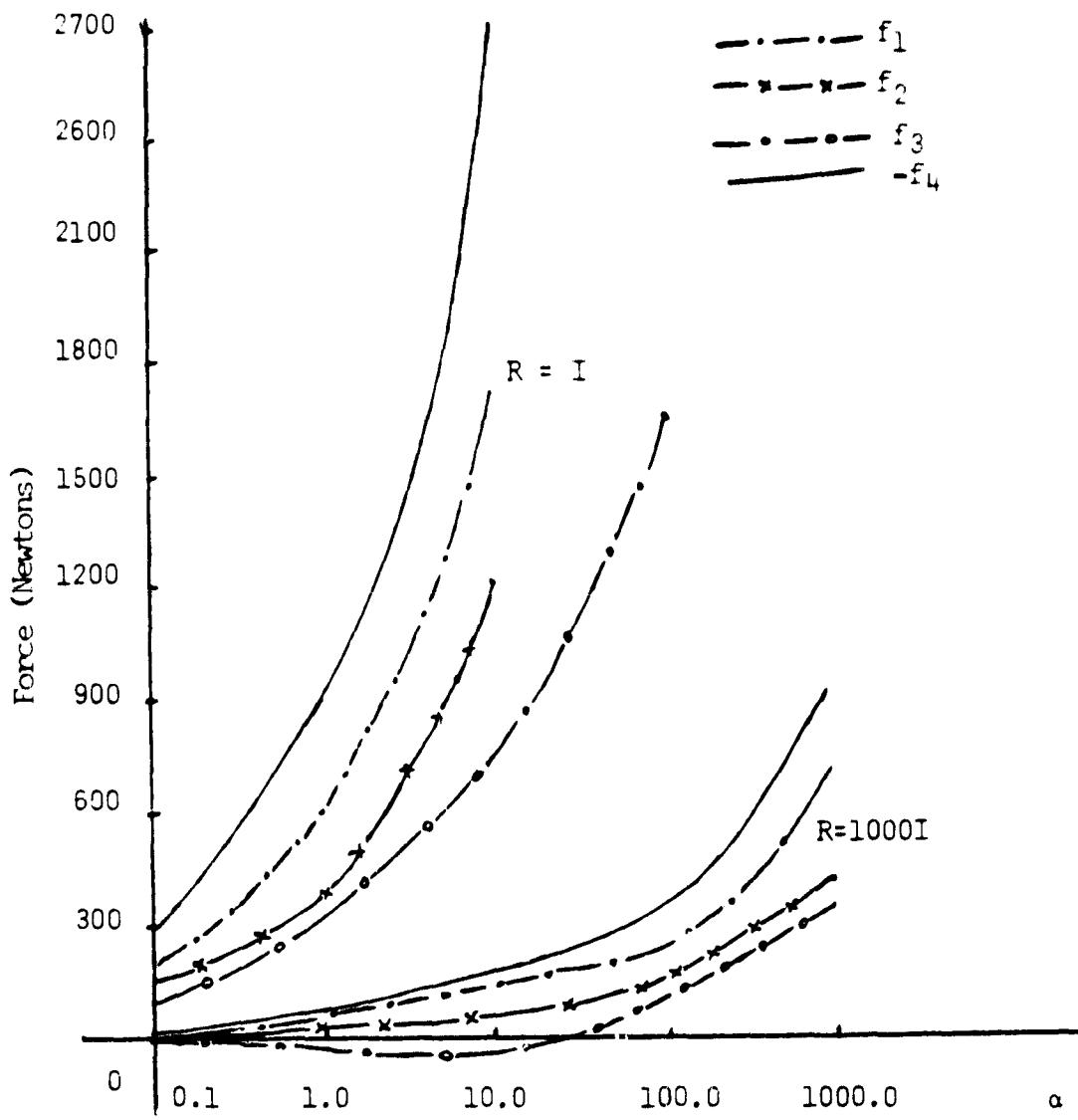


Fig 6. Maximum force amplitudes as a function of α and R for all actuators - (application of linear regulator theory).

III. FREQUENCIES AND MODE SHAPES FOR RECTANGULAR PLATES

The ability to determine accurately the frequencies and mode shapes is essential for the analysis and control of large structures in orbit. A thin rectangular plate, an important basic structure for several space applications, is considered for vibrational analysis. In the following sections the plate is assumed to be large, thin, and homogeneous, and all the edges are assumed to be free to vibrate. First, the approximate frequencies and mode shapes of a rectangular plate obtained by Warburton¹ is discussed. This analysis also includes the special case of a square plate. Next, the analytical results for a square plate using the method of Lemke² is considered. For a specific example of a square plate both analytical results are applied to determine the frequencies and mode shapes. An available finite element computer program³ is also used to obtain the frequencies and mode shapes of this plate. The results of both analytical methods and the computer routine are compared and discussed.

1. Formulation by Warburton

The approximate frequency formula is derived by applying the Raleigh method. The details of this method are given in the earlier contract report.⁴ The basic equation used was the plate vibrational equation in the cartesian co-ordinate system (x, y), with the length and width of the plate taken along the x and y directions, respectively, and is given as

$$\frac{\partial^4 W}{\partial x^4} + 2 \frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4} + \frac{12\rho(1-\sigma^2)}{Egh^2} \frac{\partial^2 W}{\partial t^2} = 0 \quad (\text{III-1})$$

where ρ , σ and E are the density, Poisson's ratio and Young's modulus of the plate material, respectively, h is the plate thickness, and g is the acceleration due to gravity. The displacement, w , at any point (x,y) at time t is given by

$$w = W \sin \omega t = A \theta(x) \phi(y) \sin \omega t \quad (\text{III-2})$$

$\theta(x)$ and $\phi(y)$ can be taken as the beam functions orthogonal to each other and can be used to approximate plate behavior. After taking the appropriate free-free beam functions for $\theta(x)$ and $\phi(y)$, the frequency expressions for a rectangular plate was derived as¹

$$\lambda^2 = \frac{\rho a^4 (2\pi f)^2 12(1-\sigma^2)}{\pi^4 E h^2 g} \quad (\text{III-3})$$

$$\text{and } \lambda^2 = G_x^{-4} + G_y^{-4} \frac{a^4}{b^4} + 2 \frac{a^2}{b^2} [\sigma H_x H_y + (1-\sigma) J_x J_y] \quad (\text{III-4})$$

where λ is a non-dimensional frequency factor, a and b are the length and width of the plate, and G_x , H_x , J_x , G_y , H_y , and J_y are functions associated with the number of nodal lines, m and n , parallel to x and y , respectively, for the beam functions $\theta(x)$ and $\phi(y)$, and are given in Table III-1. From Eq. (III-3), the frequency is obtained as

$$f = \frac{\lambda h \pi}{a^2} \left[\frac{E g}{48 \rho (1-\sigma^2)} \right]^{\frac{1}{2}} \quad (\text{cps}) \quad (\text{III-5})$$

Eq. (III-5) is valid for thin rectangular plates. However, for square plates, $(m,n) \pm (n,m)$ types of modes exist, and for these cases λ in Eq. (III-5) must be modified. These cases are discussed in detail in Ref. 1 and a few relevant results are given here.

Modes $(m, \sigma) + (\sigma, m)$, m is even

$$\lambda^2 = (m-\frac{1}{2})^4 \pm 2\sigma(m-\frac{1}{2})^2 \frac{8}{\pi^2}$$

Modes $(m, l) \pm (l, m)$, for $m = 3, 5, 7, \dots$ (III-6)

$$\begin{aligned} \lambda^2 &= (m-\frac{1}{2})^4 + 2(l-\sigma)(m-\frac{1}{2})^2 [1 + \frac{6}{(m-\frac{1}{2})\pi}] \frac{12}{\pi^2} \\ &\quad \pm 2\sigma(m-\frac{1}{2})^2 \frac{24}{\pi^2} [1 - \frac{2}{(m-\frac{1}{2})\pi}]^2 \pm 2(l-\sigma) \frac{192}{\pi^4} \end{aligned}$$

For any mode of vibration the nodal pattern is defined by m and n , the number of nodal lines in the x and y directions, respectively. The mode shapes are obtained by using the corresponding modal frequencies in the beam functions and then evaluating the product, $\theta(x)\phi(y)$, numerically.

2. Formulation by Lemke²

The frequencies and mode shapes were computed for a square plate using the Raleigh-Ritz method. The results are readily available only for six of the modes obtained by Warburton's method. Lemke uses displacement functions of the type,

$$W = \sum A_{m,n} \theta_m(x) \phi_n(y) \quad (\text{III-7})$$

where $\theta_m(x)$ and $\phi_n(y)$ are the free beam functions given as

$$\theta_m(x) = \frac{\cosh k_m \cos k_m x + \cos k_m \cosh k_m \bar{x}}{\sqrt{\cosh^2 k_m + \cos^2 k_m}} \quad (\text{m even}) \quad (\text{III-8})$$

$$= \frac{\sinh k_m \sin k_m x + \sin k_m \sinh k_m x}{\sqrt{\sinh^2 k_m - \sin^2 k_m}} \quad (\text{m odd})$$

$\phi_n(y)$ is obtained from Eq. (III-8) by replacing x by y and m by n .

The values, k_m , are the roots of the equations

$$\tan k_m + \tanh k_m = 0 \quad (m \text{ even})$$

$$\tan k_m - \tanh k_m = 0 \quad (m \text{ odd})$$

which result from the spatial boundary conditions. Further, it was shown by an energy principle that ^{2,4}

$$\omega^2 = \frac{U_{\max}}{\frac{\rho h}{2g} \int_0^a \int_0^b W^2 dx dy} \quad (\text{III-9})$$

where U_{\max} is the maximum potential energy due to bending. The coefficients, A_{mn} , in Eq. (III-7) are determined to make ω^2 in Eq. (III-9) a minimum. Lemke obtained the coefficients, A_{mn} , by taking six or more terms in the series (III-7) and using four different values of Poisson's ratio. Expressions for six mode shapes and frequencies along with the co-efficients, A_{mn} , are tabulated in Ref. 2. As an example the expression for the first mode is given here.

$$\begin{aligned} W(x,y) &= x_1 y_1 + 0.0325 (x_1 y_3 + x_3 y_1) - .005 x_3 y_3 \\ &\quad - .00257 (x_1 y_5 + x_5 y_1) + .00121 (x_3 y_5 + x_5 y_3) \\ &\quad - .000365 x_5 y_5 + \dots \end{aligned}$$

and $\omega = \frac{13.086}{a^2} \sqrt{\frac{E h^3}{12 \rho (1-\sigma^2)}} \text{ for } \sigma = .343$

3. Finite Element Computer Program

The computer program used is the Structural Design Language (STRUDL) which uses the finite element method to determine the mode shapes and the frequencies of vibration. The input to the computer routine is given by specifying the type of structure and supplying other physical properties and dimensions of the structure. For a rectangular plate, the finite elements can be specified as rectangular elements and the number of elements into which the plate should be divided depends upon the accuracy required. STRUDL gives deflections at each corner of the elements for all the modes from which the mode shapes can be determined. Further, a set of frequencies corresponding to the modes generated is obtained. In general, the accuracies of the frequencies and mode shapes will improve if the plate is modelled with a higher number of elements. However, computational errors due to truncation and round-off errors may predominate as the order of the elements increases beyond a limit. Further, the limitations of the computers will restrict the number of elements into which the plate can be divided to obtain more accurate results.

4. Discussion of Numerical Results

A square plate of sides 100 meters each and thickness 0.01 meters is considered to obtain the numerical results. The material of the plate is assumed to be aluminium with the following properties.

$$\text{density} = 2768.0 \text{ kg/m}^3$$

$$\text{Young's modulus} = 0.7441 \times 10^{10} \text{ kg/m}^2$$

$$\text{Poisson's ratio} = 0.33$$

Using Warburton's results, Eq. (III-4), Eq. (III-5), Table (III-1), and expressions for $\theta(x)$ and $\phi(y)$, frequencies and mode shapes are calculated for different combinations of the number of nodal lines, m and n , starting with combinations of $m=0$ and $n=1$, through $m=3$ and $n=3$. The first three combinations of nodal line numbers, $(0,0)$, $(1,0)$ and $(0,1)$, represent rigid body motion. The first fundamental flexural frequency is seen to be due to a combination of $m=1$ and $n=1$. The corresponding mode shape for the plate is obtained by multiplying the beam functions, $\theta(x)$ and $\phi(y)$, for (beam) mode numbers 1 and 1, respectively (Fig. 1). Since the plate is approximated by sets of orthogonal beams in the x and y directions, the nodal pattern is also obtained by plotting the nodal points of these beams for their first modes. The next two higher frequencies are obtained by combinations of $m=0$ and $n=2$, but the nodal patterns (Figs. 2,4) can not be visualized as before. This is because these frequencies are of a special type resulting from a combination of the $(2,0)$ and $(0,2)$ plate modes. It can be seen that when the mode corresponding to $(2,0)$ (Fig. 3(a)) is superimposed on the mode - $(0,2)$ (Fig. 3(b)) the mode shape depicted in Fig. 2 results. Similarly by superimposing the $(2,0)$ and $(0,2)$ modes the third mode shape (Fig. 4) is obtained. The two combinations of nodal patterns $m=1$ and $n=2$, give identical frequencies for the fourth and fifth mode and the corresponding shapes (Fig. 5) are as expected. The next two higher frequencies are also identical and result from combinations of the $(3,0)$ and $(0,3)$ modes. The eighth frequency is obtained from $m=2$ and $n=2$ and the mode shape obtained is shown in Fig. 7.

However, the ninth and tenth mode shapes obtained by the (3,1) and (1,3) combinations, are once again of a special type. The ninth mode shape is obtained by superimposing the (1,3) and (3,1) patterns (Fig. 8) and the tenth mode shape is obtained by superimposing (1,3) and (3,1) nodal patterns (Fig. 9). The next higher frequencies are obtained from combinations of the (3,2), (2,3) and (3,3) modes, respectively. The frequencies and nodal patterns obtained for all these modes are shown in Table 2.

Frequencies and mode shapes are also obtained by using the expressions for the six modes given by Lemke.² The first three frequencies and mode shapes obtained agree with the frequencies and mode shapes computed from Warburton's formulas (Table 1). However, the next three frequencies obtained by Lemke's method correspond to higher frequencies and mode shapes obtained by Warburton's method. Also the nodal patterns obtained by Lemke's method compare approximately with the nodal patterns obtained by Warburton's method although the frequencies do not correspond in all cases. The results obtained by Lemke do not show the four intermediate frequencies corresponding to the fourth, fifth, sixth and seventh modes obtained by Warburton's method. The frequencies and nodal pattern obtained by Lemke's method are shown in Table 2.

For implementation of the computer program STRUDL, first the plate is divided into four elements. The first six modes (in terms of increasing frequencies) as predicted by STRUDL are also apparent from Warburton's results. The plate is assumed to be divided into 9, 16, 36 and 64 elements, respectively. The results of STRUDL are tabulated in Table 2. It can be seen that STRUDL frequencies approach the frequencies obtained by Warburton's method as the number of plate elements is increased.

However, in the cases of 36 and 64 elements some of the frequencies show a tendency to oscillate about an average value. This probably is due to the computational round off errors which begin to dominate with the increasing computations associated with larger number of elements. Thus, the advantage of taking a large number of elements may not be fully realized due to numerical accuracy limitations. Computation with more elements requires more computation time and a larger computer memory. For the 64 elements case, it was not possible to obtain the mode shapes due to memory limitations. It was also observed that the convergence of the frequencies, with an increase in the number of elements, is faster than the convergence of the mode shapes. It can be seen from Table 2, that the numerical results of STRUDL using 36 elements correlate with the results of Warburton both in frequency and mode shapes.

Table 3 and Table 4 compare non-dimensionalized deflections at the nodes (corners of elements) obtained by the three methods for the second mode (Fig. 2). For locations where deflections exist and do not correspond to maximum amplitude (± 1.0) in all cases the results predicted by STRUDL lie in between the results obtained by the analytical methods of Lemke and Warburton.

The results of this comparative study give an indication of the types of modelling errors that would be expected in the estimation of the frequencies and mode shapes of the fundamental and lower order flexural modes of a large platform type structure in orbit. As an extension to this study the use of more powerful (and accurate) finite element computer algorithms, not currently available at Howard University, is recommended.

5. References

1. Warburton, G.B., "The Vibration of Rectangular Plates," Proc. Inst. Mech. Engrs., Vol. 168, No. 12, 1954, pp. 371-394.
2. Leissa, A.W., "Vibration of Plates", NASA SP-160, NASA, Washington, D.C., 1969, pp. 87-110.
3. ICES-STRUDL-II, Engineering Users Manual, Volume II, 1972.
4. Bainum, P.M. and James, P.K., "The Dynamics and Control of Large Flexible Space Structures - II. PART B: Model Development and Computer Simulation," June 1979.

TABLE III-1. Evaluation of Parameters in Frequency Expression (Warburton)

m	G _x	H _x	J _x	n	G _y	H _y	J _y
0	0	0	0	0	0	0	0
1	0	0	$\frac{12}{\pi^2}$	1	0	0	$\frac{12}{\pi^2}$
2	1.506	1.248	5.017	2	1.506	1.248	5.017
3 4 5 :	$m - \frac{1}{2}$	$(m - \frac{1}{2})^2 P$	$(m - \frac{1}{2})^2 Q$	$\frac{3}{4} \frac{5}{\dots}$	$(n - \frac{1}{2})$	$(n - \frac{1}{2})^2 P$	$(n - \frac{1}{2})^2 Q$

$$P = [1 - 2/(m - 0.5)\pi]$$

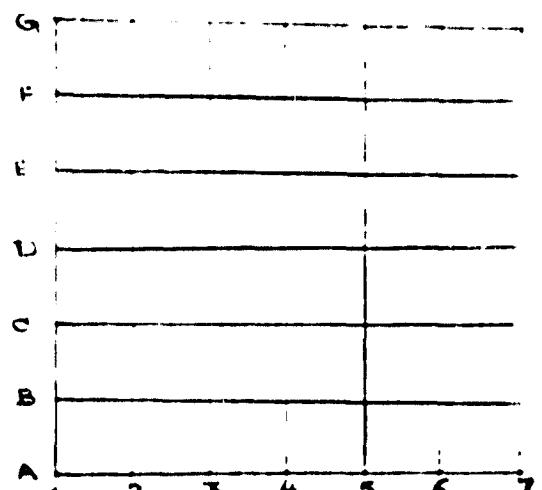
$$Q = [1 + 6/(m - 0.5)\pi]$$

STRUDL	NUMBER OF ELEMENTS	WARBURTON			LINE					
		4	9	16						
1	.003165	.003284	.003315	.00334	.003358	Nodal Pattern 	Freq. (cps) .003171	(m,n) (1,1)	Nodal Pattern 	Freq. (cps) .003281
2	.004291	.004664	.004776	.004839	.004356	Nodal Pattern 	Freq. (cps) .004846	(2,0)-(5,5)	Nodal Pattern 	Freq. (cps) .004931
3	.005549	.006133	.006221	.006221	.006207	Nodal Pattern 	Freq. (cps) .006175	(2,0)-(5,2)	Nodal Pattern 	Freq. (cps) .006175
4	.007619	.008453	.008632	.008690	.008677	Nodal Pattern 	Freq. (cps) .008600	(2,1)	Nodal Pattern 	Freq. (cps) .015744
5	.007619	.008453	.008632	.008690	.008692	Nodal Pattern 	Freq. (cps) .008690	(1,2)	Nodal Pattern 	Freq. (cps) .017161
6	.01323	.01512	.01600	.01596	.01579	Nodal Pattern 	Freq. (cps) .01542	(3,0)	Nodal Pattern 	Freq. (cps) .01944
7	.01512	.01600	.01596	.01579	.01579	Nodal Pattern 	Freq. (cps) .01542	(0,3)	Nodal Pattern 	Freq. (cps) .01542
8	.01584	.01618	.01615	.01607	.01607	Nodal Pattern 	Freq. (cps) .01656	(2,2)	Nodal Pattern 	Freq. (cps) .017161
9	.01630	.01727	.01750	.01746	.01746	Nodal Pattern 	Freq. (cps) .01715	(3,1)-(1,3)	Nodal Pattern 	Freq. (cps) .01715
10	.01843	.02022	.02031	.02005	.02005	Nodal Pattern 	Freq. (cps) .0208	(1,3)-(3,3)	Nodal Pattern 	Freq. (cps) .0208
11	.02465	.02693	.02693	.02700	.02700	Nodal Pattern 	Freq. (cps) .02726	(3,2)	Nodal Pattern 	Freq. (cps) .02726
12	.02465	.02673	.02643	.02700	.02700	Nodal Pattern 	Freq. (cps) .02726	(2,3)	Nodal Pattern 	Freq. (cps) .02726
13	.03362	.03029	.03162	.03104	.03104	Nodal Pattern 	Freq. (cps) .03341	(3,3)	Nodal Pattern 	Freq. (cps) .03341

TABLE-III-2. Frequencies and Nodal Patterns Obtained by the Three Methods.

Case 1: 4 Elements

LOCATION	STRUOL	LEMKE	WARBURTON
A1	0	0	0
A2	1.0	1.0	1.0
B1	-1.0	-1.0	-1.0
B2	0	0	0



grid used for 36 elements

Case 2: 9 Elements

LOCATION	STRUOL			LEMKE			WARBURTON			STRUOL	LEMKE			WARBURTON		
	1	2	3	1	2	3	1	2	3		1	2	3	1	2	3
A1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
A3	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
B1	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
B2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B3	0	0	0	0	0	0	0	0	0	0.2862	0.2673	0.3163	0	0	0	0
C1	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
C2	0	0	0	0	0	0	0	0	0	-0.2862	-0.2673	-0.3163	0	0	0	0
C3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE-III-3. Normalized Deflections at the Nodal Points Obtained by the Three Methods - Second Mode.

Case 4: 36 Elements

LOCATION	STRUOL	WARBU- RTON	LEMKE	LOCATION	STRUOL	WARBU- RTON	LEMKE
A ₁	0	0	0	C ₁	- .8590	- .524	- .9793
A ₂	.4874	.4733	.5346	C ₂	- .3595	- .3792	- .3447
A ₃	.8590	.8524	.9793	C ₃	0	0	0
A ₄	1.0	1.0	1.0	C ₄	.1334	.1475	.1207
B ₁	- .4873	- .4733	- .5346	D ₁	- 1.0	- 1.0	- 1.0
B ₂	0	0	0	D ₂	- .4938	- .5267	- .4654
B ₃	.3594	.3792	.3447	D ₃	- .1334	- .1475	- .1207
B ₄	.4933	.5267	.4654	D ₄	0	0	0

TABLE III-4. Normalized Deflections at the Nodal Points Obtained by the Three Methods - Second Mode.



Fig. 1 First Mode

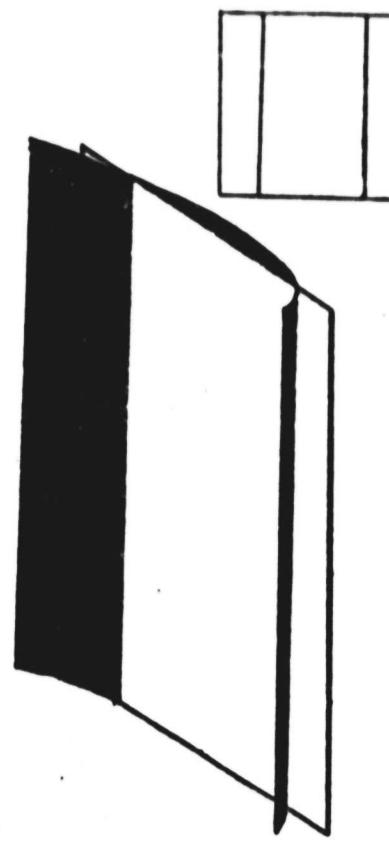


Fig. 3(a) (2,0) Mode

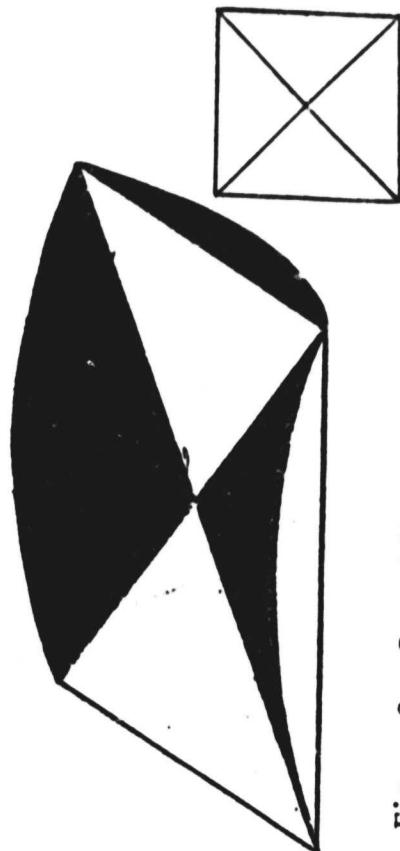
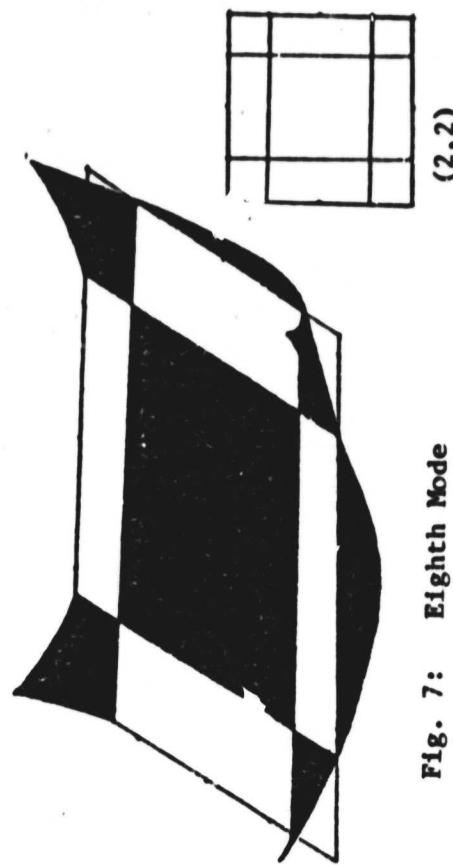
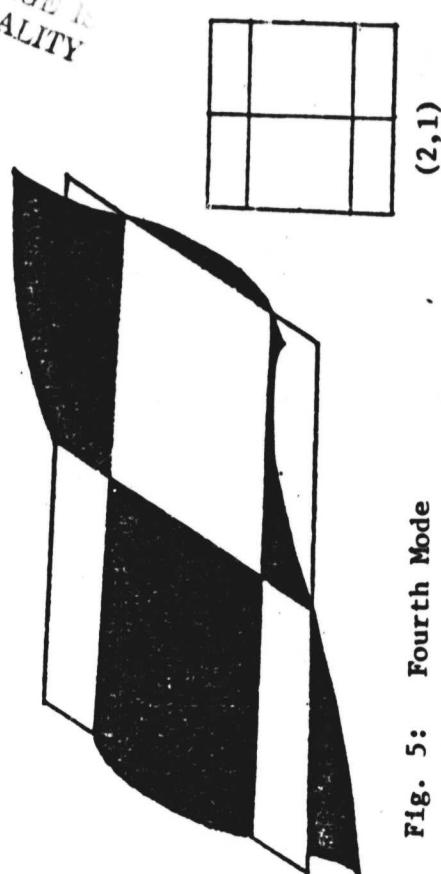
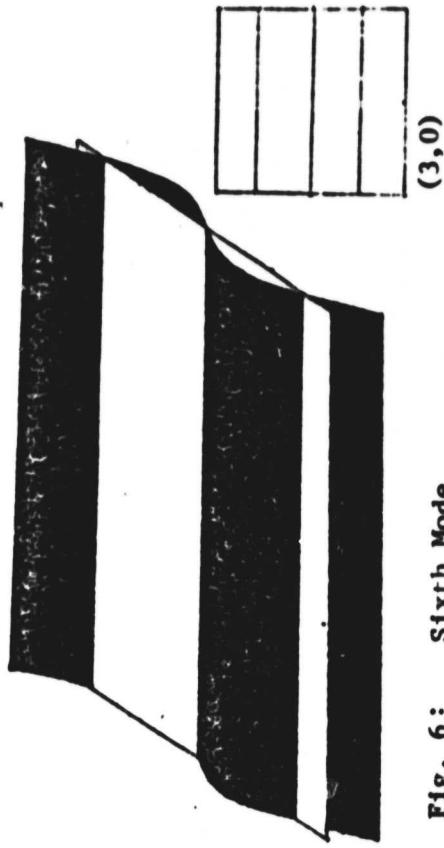
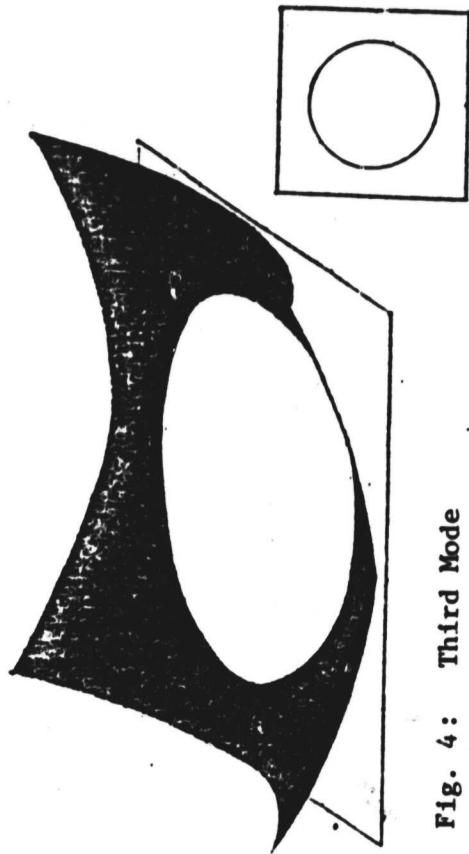


Fig. 2 Second Mode



Fig. 3(b) (0,2) Mode

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OF POOR QUALITY



Fig. 8: Ninth Mode
 $(1,3)-(3,1)$

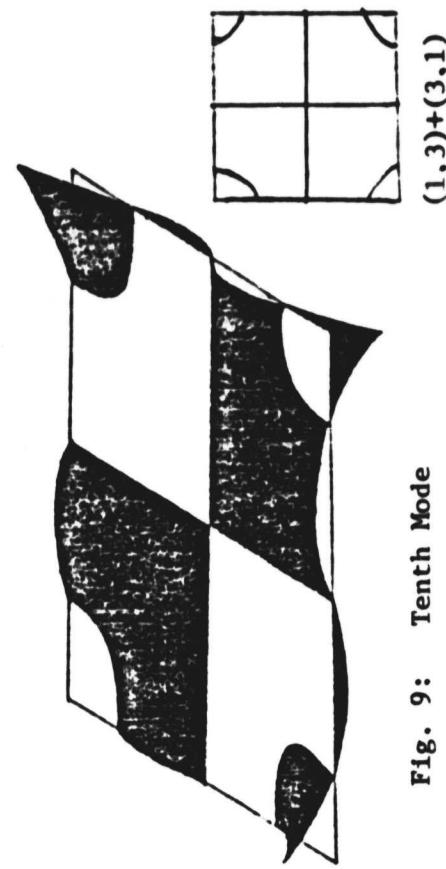


Fig. 9: Tenth Mode
 $(1,3)+(3,1)$

IV. GENERAL CONCLUSIONS AND RECOMMENDATIONS

A model is developed for predicting the dynamics of a large flexible free-free thin platform in orbit under the influence of control devices which are considered to be placed at specific locations on the major surface and one of the edges. Control about three different nominal orientations is considered. In the absence of control, for the case of a completely homogeneous platform instability in at least some of the modes is indicated for small amplitude motion about each of the three orientations. Once controllability is established, for a set of actuator locations, three different techniques are employed for the selection of actuator control laws:

- (1) the decoupling of the original state equations using state variable feedback;
- (2) a pole placement algorithm; and
- (3) an application of the linear regulator theory

It is seen that each of the three techniques have certain distinct advantages and also specific limitations), which are discussed in detail in Chapter II. For systems involving multi-degrees of freedom (such as in this application), the implementation of these techniques requires the extensive usage of computer algorithms.

As a logical extension to the present study which assumes perfect instantaneous knowledge of the state, the modelling of the sensor dynamics and related problem of observability should be considered, once specific information on the types of sensors required for monitoring the performance of large flexible systems is available.

The problems caused by both observation and control spillover could also be treated, perhaps by beginning with the simpler model of the control of a long, flexible beam in orbit and then extending this analysis to the three dimensional model of the platform.

A model of the uncontrolled dynamics of a large flexible shallow spherical shell (representative of an antenna dish or large radiometer) in orbit has been developed during the present grant year (see Part B this report). It is suggested that the effect of control devices be included in this model and that control laws could then be developed using different algorithms already in existence.

APPENDIX

Modifications to ORACLS Software Package

The ORACLS⁸ Software Package that was developed at Langley which operates on the Control Data Cyber Computer System was modified to suit the IBM 370/165 Computer System that is available at Howard University. The major modifications that were done are described below:

- (1) As the single precision accuracy on the CDC is approximately equal to the double precision on the IBM/370 System, the entire package was converted into double precision.
- (2) Some of the machine dependent constants were changed accordingly.
- (3) As the IBM System accepts only six letters for a subroutine/function name all the names that exceeded six letters were changed and the list of those subroutines is given below:

<u>Old Name</u>	<u>New Name</u>
(1) TESTSTA	TESTSA
(2) VARANCE	VARANC
(3) TRANSIT	TRNSIT
(4) DISCREG	DISREG
(5) CNTNREG	CNTREG
(6) RICTNWT	RICNWT
(7) ASYMREG	ASMREG
(8) ASYMFIL	ASMFIL
(9) EXPMDFL	EXPMDF
(10) IMPMDFL	IMPMDF

- (4) Some of the additional supporting subroutines/functions required were added and the names of these subroutines are given here:
 - (1) PNCH
 - (2) DIMAG
 - (3) DREAL
 - (4) BLOCK DATA

(5) None of the arguments of the subroutines were changed

The listing of the modified ORACLS package is given in the following pages. These routines have to be used in conjunction with Ref. (8). The numbers that appear in front of the FORTRAN statements are line numbers and have to be omitted.

1	SUBROUTINE RDTITL	RDT00010
2	IMPLICIT REAL*8 (A-H,O-Z)	RDT00020
3	COMMON/LINES/TITLE(10),TIL(3),NLP,LIN	RDT00030
4	COMMON/FORM/FMT1(2),FMT2(2),NEPR	RDT00040
5	COMMON/TOL/EPSAM,EPSBM,IACM	RDT00050
6	COMMON/CONV/SUMCV,RICTCV,SERCV,MAXSUM	RDT00060
7 C	NLP = NO. LINES/PAGE VARIES WITH THE INSTALLATION	RDT00070
8	READ(5,100,END=90,ERR=91) TITLE	RDT00080
9	100 FORMAT(10A8)	RDT00090
10	CALL LNCNT(100)	RDT00100
11	RETURN	RDT00110
12	90 CONTINUE	RDT00120
13	STOP 1	RDT00130
14	91 CONTINUE	RDT00140
15	STOP 2	RDT00150
	END	RDT00160

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```
SUBROUTINE LNCNT (N)          LNC00010
  IMPLICIT REAL*8 (A-H,O-Z)    LNC00020
2   COMMON/LINES/TITLE(10),TIL(3),NLP,LIN
  LIN=LIN+N                   LNC00030
5   IF (LIN.LE.NLP) GO TO 20    LNC00040
  WRITE(6,1010) TITLE,TIL      LNC00050
6   1010 FORMAT(1H1,10A8,3A8/)   LNC00060
  LIN=2+N                     LNC00070
9   IF (N.GT.NLP) LIN=2        LNC00080
  20 RETURN                    LNC00090
END                         LNC00100
                                LNC00110
```

```

1 SUBROUTINE READ(I,A,NA,B,NB,C,NC,D,ND,E,NE)          REA00010
2 IMPLICIT REAL*8 (A-H,O-Z)                           REA00020
3 DIMENSION A(1),B(1),C(1),D(1),E(1)                 REA00030
4 DIMENSION NA(2),NB(2),NC(2),ND(2),NE(2),NZ(2)       REA00040
5 READ(5,100) LAB,                                     NZ(1), NZ(2)      REA00050
6 CALL READ1(A, NA, NZ, LAB)                         REA00060
7 IF(I .EQ. 1) GO TO 999                           REA00070
8 READ(5,100) LAB,                                     NZ(1), NZ(2)      REA00080
9 CALL READ1(B, NB, NZ, LAB)                         REA00090
10 IF(I .EQ. 2) GO TO 999                          REA00100
11 READ(5,100) LAB,                                     NZ(1), NZ(2)      REA00110
12 CALL READ1(C, NC, NZ, LAB)                         REA00120
13 IF(I .EQ. 3) GO TO 999                          REA00130
14 READ(5,100) LAB,                                     NZ(1), NZ(2)      REA00140
15 CALL READ1(D, ND, NZ, LAB)                         REA00150
16 IF(I .EQ. 4) GO TO 999                          REA00160
17 READ(5,100) LAB,                                     NZ(1), NZ(2)      REA00170
18 CALL READ1(E, NE, NZ, LAB)                         REA00180
19 100 FORMAT(A4,4X,2I4)                            REA00190
20 999 RETURN                                         REA00200
21 END                                              REA00210

```

```

1      SUBROUTINE PRNT(A,NA,NAM,IOP)          PRN00010
2      IMPLICIT REAL*8 (A-H,O-Z)             PRN00020
3      DIMENSION A(1),NA(2)                  PRN00030
4      COMMON /FORM/FMT1(2),FMT2(2),NEPR    PRN00040
5      COMMON/LINES/TITLE(10),TIL(3),NLP,LIN  PRN00050
6      S C- NOTE NLP NO. LINES/PAGE VARIES WITH THE INSTALLATION.  PRN00060
7      DATA KZ,KW,KB /1H0,1H1,1H/           PRN00070
8      NAME = NAM                          PRN00080
9      II = IOP                           PRN00090
10     NR = NA(1)                         PRN00100
11     NC = NA(2)                         PRN00110
12     NLST = NR * NC                     PRN00120
13     IF( NLST .LT. 1 .OR. NR .LT. 1 ) GO TO 16  PRN00130
14     IF(NAME .EQ. 0) NAME = KB          PRN00140
15     C- SKIP HEADLINE IF REQUESTED.    PRN00150
16     GO TO (11,10,132,12), II          PRN00160
17     10 CALL LNCNT(100)                 PRN00170
18     11 CALL LNCNT(2)                  PRN00180
19     13 WRITE(6,177) KZ,NAME,NR,NC      PRN00190
20     177 FORMAT(A1,5X,A4,8H MATRIX,5X,I3,5H ROWS,5X,I3,8H COLUMNS)  PRN00200
21     GO TO 13                          PRN00210
22     12 CALL LNCNT(100)                 PRN00220
23     GO TO 13                          PRN00230
24     132 CALL LNCNT(2)                 PRN00240
25     WRITE (6,891)                      PRN00250
26     891 FORMAT (1H0)                   PRN00260
27     C- BELOW COMPUTE NR OF LINES/ ROW --DECIDE IF 1 EXTRA BLANK LINE  PRN00270
28     13 J=(NC-1)/NEPR+1                PRN00280
29     C- WHY ALWAYS ADD 1 LINE- BECAUSE IF MULTIPLE, USE 1 BLK LINE EXTRA.  PRN00290
30     NLPW$J
31     JST=1
32     C COMPUTE LAST ROW POSITION +1 BELOW          PRN00330
33     NLST = NLST -NR                      PRN00340
34     MN=NC
35     IF (NC.GT.NEPR) MN=NEPR            PRN00350
36     KLST=NR*(MN-1)                    PRN00360
37     91 CONTINUE
38     DO 912 J = JST, NR               PRN00370
39     CALL LNCNT(NLPW)                 PRN00380
40     KLST = KLST +1                  PRN00390
41     WRITE (6,FMT1) (A(N), N=J,KLST,NR)  PRN00400
42     IF (NC.LE.NEPR) GO TO 912        PRN00410
43     NLST = NLST +1                  PRN00420
44     KNR=KLST+NR                    PRN00430
45     WRITE (6,FMT2) (A(N), N=KNR,NLST,NR)  PRN00440
46     912 CONTINUE
47     RETURN
48     16 CALL LNCNT(1)                 PRN00450
49     WRITE (6,916) NAM,NA            PRN00460
50     916 FORMAT (' ERROR IN PRNT MATRIX ',A4,' HAS NA=',2I6)  PRN00470
51     RETURN
52     END.                            PRN00480

```

```
SUBROUTINE EQUATE(A,NA,R,NR) EQU00010
 2 IMPLICIT REAL*8 (A-H,O-Z) EQU00020
 3 DIMENSION A(1),B(1),NA(2),NB(2) EQU00030
 4 NH(1)=NA(1) EQU00040
 5 NR(2)=NA(2) EQU00050
 6 L=NA(1)*NA(2) EQU00060
 7 IF( NA(1).LT. 1 .OR. L.LT. 1 ) GO TO 999 EQU00070
 8 00 300 I=1,L EQU00080
 9 300 B(I)=A(I) EQU00090
10 1000 RETURN EQU00100
11 999 CALL LNCNT (1) EQU00110
12 WRITE (6,50) NA EQU00120
13 50 FORMAT (' DIMENSION ERROR IN EQUATE  NA*',2I6) EQU00130
14 RETURN EQU00140
15 END EQU00150
```

```

1      SUBROUTINE TRA00010
2      IMPLICIT REAL*8 (A=H,D-Z)
3      DIMENSION A(1),B(1),NA(2),NB(2)
4      NA(1)=NA(2)
5      NB(2)=NA(1)
6      NR=NA(1)
7      NC=NA(2)
8      L=NR+NC
9      IF( NR .LT. 1 .OR. L .LT. 1 ) GO TO 999
10     IR=0
11     DO 300 I=1,NR
12     IJ=I-NR
13     DO 300 J=1,NC
14     IJ=IJ+NR
15     IR=IR+1
16     300 B(IR)=A(IJ)
17     RETURN
18     999 CALL LNCNT(1)
19     WRITE (6,50) NA
20     50 FORMAT (' DIMENSION ERROR IN TRA00020
21     RETURN
22     END.          TRA00030
23                               TRA00040
24                               TRA00050
25                               TRA00060
26                               TRA00070
27                               TRA00080
28                               TRA00090
29                               TRA00100
30                               TRA00110
31                               TRA00120
32                               TRA00130
33                               TRA00140
34                               TRA00150
35                               TRA00160
36                               TRA00170
37                               TRA00180
38                               TRA00190
39                               TRA00200
40                               TRA00210
41                               TRA00220

```

SUBROUTINE SCALE (A, NA, M, NB, S)
IMPLICIT REAL*8 (A-M,C-Z)
DIMENSION A(1),B(1),NA(2),NB(2)
NA(1) = NA(1)
NB(2) = NA(2)
L = NA(1)*NA(2)
IF(NA(1) .LT. 1 .OR. L .LT. 1) GO TO 999
DO 300 I=1,L
300 B(I)=A(I)*S
1000 RETURN
999 CALL LNCNT(1)
WRITE (6,50) NA
50 FORMAT (' DIMENSION ERROR IN SCALE NA*',2I6)
RETURN
END

SCA00010
SCA00020
SCA00030
SCA00040
SCA00050
SCA00060
SCA00070
SCA00080
SCA00090
SCA00100
SCA00110
SCA00120
SCA00130
SCA00140
SCA00150

```
0      SUBROUTINE UNITY(A,NA)          UNI00010
1      IMPLICIT REAL=8 (A=H,0=Z)        UNI00020
2      DIMENSION A(1),NA(2)            UNI00030
3      IF(NA(1).NE.NA(2)) GO TO 999    UNI00040
4      L=NA(1)*NA(2)                 UNI00050
5      DO 100 IT=1,L                UNI00060
6      100 A(IT)=0.0                 UNI00070
7      J = - NA(1)                  UNI00080
8      NAX = NA(1)                  UNI00090
9      DO 300 I=1,NAX              UNI00100
10     J=NAX +J+1
11     300 A(J)=1.
12     GO TO 1000
13     999 CALL LNCNT (1)
14     NRITE(6, 50)(NA(I),I=1,2)
15     50 FORMAT (' DIMENSION ERROR IN UNITY  NA=',2I6)
16     1000 RETURN
17     END
```

```
0      SUBROUTINE NULL(A,NA)          NUL00010
1      IMPLICIT REAL*8 (A=H,0=Z)      NUL00020
2      DIMENSION A(1)                NUL00030
3      DIMENSION NA(2)              NUL00040
4      N=NA(1)*NA(2)                NUL00050
5      IF( NA(1) .LT. 1 .OR. N .LT. 1 ) GO TO 999
6      DO 10 I=1,N                 NUL00060
7      10 A(I) = 0.0                NUL00070
8      RETURN                         NUL00080
9      C
10     999 CONTINUE                  NUL00090
11     WRITE (6,50) NA               NUL00100
12     50 FORMAT(' DIMENSION ERROR IN NULL  NA =',2I6)
13     RETURN                         NUL00110
14     END                           NUL00120
                                         NUL00130
                                         NUL00140
                                         NUL00150
```

```
0      SUBROUTINE TRCE (A,NA,TR)          TRC00010
1      IMPLICIT REAL*8 (A-H,O-Z)          TRC00020
2      DIMENSION A(1)                   TRC00030
3      DIMENSION NA(2)                  TRC00040
4      IF (NA(1).NE.NA(2)) GO TO 600     TRC00050
5      N=NA(1)                         TRC00060
6      TR=0.0                           TRC00070
7      IF( N .LT. 1 ) GO TO 600         TRC00080
8      DO 10 I=1,N                     TRC00090
9      M3I=N*(I-1)                   TRC00100
10     10 TR=TR+A(M)                 TRC00110
11      RETURN                          TRC00120
12      600 CALL LNCNT(1)              TRC00130
13      WRITE (5,1600) NA              TRC00140
14      1600 FORMAT (' TRACE REQUIRES SQUARE MATRIX    NA=',2I6) TRC00150
15      RETURN                          TRC00160
16      END                            TRC00170
```

```
0 SUBROUTINE ADD (A,NA,B,NB,C,NC) ADD000010
1 IMPLICIT REAL*8 (A-H,O-Z) ADD000020
2 DIMENSION A(1),B(1),C(1),NA(2),NB(2),NC(2) ADD000030
3 IF( (NA(1) .NE. NB(1)) .OR. (NA(2) .NE. NB(2)) ) GO TO 999 ADD000040
4 NC(1)=NA(1) ADD000050
5 NC(2)=NA(2) ADD000060
6 L=NA(1)*NA(2) ADD000070
7 IF( NA(1) .LT. 1 .OR. L .LT. 1 ) GO TO 999 ADD000080
8 DO 300 I=1,L ADD000090
9 300 C(I)=A(I)+B(I) ADD000100
10 GO TO 1000 ADD000110
11 999 CALL LNCNT (1) ADD000120
12 WRITE(6,50) NA,NB ADD000130
13 50 FORMAT (' DIMENSION ERROR IN ADD      NA=',2I6,5X,'NB=',2I6) ADD000140
14 1000 RETURN ADD000150
15 END ADD000160
```

```

1      SUBROUTINE SUBT(A,NA,B,NB,C,NC)          SUB00010
2      IMPLICIT REAL*8 (A-H,O-Z)                SUB00020
3      DIMENSION A(1),B(1),C(1),NA(2),NB(2),NC(2)  SUB00030
4      IF((NA(1).NE.NB(1)),OR,(NA(2).NE.NB(2))) GO TO 999  SUB00040
5      NC(1)=NA(1)                            SUB00050
6      NC(2)=NA(2)                            SUB00060
7      L=NA(1)*NA(2)                          SUB00070
8      IF( NA(1) .LT. 1 .OR. L .LT. 1 ) GO TO 999  SUB00080
9      DO 300 I=1,L                         SUB00090
10     300 C(I)=A(I)-B(I)                  SUB00100
11     GO TO 1000                           SUB00110
12     999 CALL LNCNT (1)                  SUB00120
13     WRITE(6,50) NA,NB                  SUB00130
14     50 FORMAT (' DIMENSION ERROR IN SUBT      NA=',2I6,5X,'NB=',2I6)  SUB00140
15     1000 RETURN                         SUB00150
16     END                                SUB00160

```

```

0   SUBROUTINE MULT(A,NA,B,NB,C,NC)          MUL0001
1   IMPLICIT REAL*8 (A-H,O-Z)                MUL0002
2   DIMENSION A(1),B(1),C(1),NA(2),NB(2),NC(2) MUL0003
3   NC(1)=NA(1)                                MUL0004
4   NC(2)=NB(2)                                MUL0005
5   IF(NA(2).NE.NB(1)) GO TO 999              MUL0006
6   NAR=NA(1)                                  MUL0007
7   NAC=NA(2)                                  MUL0008
8   NBC=NB(2)                                  MUL0009
9   NAA=NAR*NAC                               MUL0010
0   NBB=NAR*NBC                               MUL0011
1   IF (NAR.LT.1.OR.NAA.LT.1.OR.NBB.LT.1) GO TO 999 MUL0012
2   IR=0                                      MUL0013
3   IK=NAC                                    MUL0014
4   DO 350 K=1,NBC                           MUL0015
5   IK=IK+NAC                                MUL0016
6   DO 350 J=1,NAR                           MUL0017
7   IR=IR+1                                   MUL0018
8   IJ=IK                                     MUL0019
9   JI=J-NAR                                 MUL0020
10  V1=0.0                                    MUL0021
11  DO 300 I=1,NAC                           MUL0022
12  JI=JI+NAR                                MUL0023
13  IB=IB+1                                   MUL0024
14  V3=A(JI)                                 MUL0025
15  V4=B(IB)                                 MUL0026
16  V2=V3*V4                                 MUL0027
17  V1=V1+V2                                 MUL0028
18  300 CONTINUE                             MUL0029
19  C(IR)=V1                                 MUL0030
20  350 CONTINUE                             MUL0031
21  GO TO 1000                               MUL0032
22  999 CALL LNCNT(1)                         MUL0033
23  WRITE(6,500) (NA(I),I=1,2),(NB(I),I=1,2) MUL0034
24  500 FORMAT (' DIMENSION ERROR IN MULT  NA=',2I6,'NB=',2I6) MUL0035
25  1000 RETURN                               MUL0036
26  END                                     MUL0037

```

```
0      SUBROUTINE MAXEL(A,NA,ELMAX)          MAX0001
1      IMPLICIT REAL*8 (A-H,O-Z)            MAX0002
2      DIMENSION A(1),NA(2)                  MAX0003
3 C
4      N = NA(1)*NA(2)                      MAX0004
5 C
6      ELMAX = DABS( A(1))                 MAX0005
7      DO 100 I = 2,N                       MAX0006
8      ELMAXI = DABS( A(I) )                MAX0007
9      IF( ELMAXI.GT.ELMAX) ELMAX=ELMAXI   MAX0008
10     100 CONTINUE                         MAX0009
11 C
12     RETURN                                MAX0010
13     END                                   MAX0011
```

```

0      SUBROUTINE NORMS(MAXROW,M,N,A,IOPT,RLNORM)          NOR0001
1      IMPLICIT REAL*8 (A-H,O-Z)                         NOR0002
2      DIMENSION A(1)                                     NOR0003
3 C
4 C  INITIALIZATION                                     NOR0004
5 C
6      RLNORM=0.                                         NOR0005
7      SUM=0.                                           NOR0006
8      I=MAXROW                                         NOR0007
9 C
10 C  TRANSFER TO APPROPRIATE LOOP TO COMPUTE THE DESIRED NORM NOR0008
11 C
12      IF(IOPT-2)5,20,30                                NOR0009
13 C
14 C  THIS LOOP COMPUTES THE ONE-NORM                  NOR0010
15 C
16      5 DO 15 K=1,N                                    NOR0011
17      I=I+MAXROW                                      NOR0012
18      DO 10 J=1,M                                    NOR0013
19      L=I+J                                         NOR0014
20      10 SUM=DABS(A(L))+SUM                         NOR0015
21      IF(SUM.GT.RLNORM)RLNORM=SUM                  NOR0016
22      15 SUM=0.                                       NOR0017
23      RETURN                                         NOR0018
24 C
25 C  THIS LOOP COMPUTES THE EUCLIDEAN NORM           NOR0019
26 C
27      20 DO 25 K=1,N                                    NOR0020
28      I=I+MAXROW                                      NOR0021
29      DO 25 J=1,M                                    NOR0022
30      L=I+J                                         NOR0023
31      SUM=A(L)                                       NOR0024
32      25 RLNORM=SUM*SUM+RLNORM                      NOR0025
33      RLNORM=DSQRT(RLNORM)                         NOR0026
34      RETURN                                         NOR0027
35 C
36 C  THIS LOOP COMPUTES THE INFINITY-NORM            NOR0028
37 C
38      30 DO 40 J=1,M                                    NOR0029
39      L=I+J                                         NOR0030
40      DO 35 K=1,N                                    NOR0031
41      L=L+MAXROW                                     NOR0032
42      35 SUM=DABS(A(L))+SUM                         NOR0033
43      IF(SUM.GT.RLNORM)RLNORM=SUM                  NOR0034
44      40 SUM=0.0                                     NOR0035
45      RETURN                                         NOR0036
46      END                                            NOR0037

```

```

0 SUBROUTINE JUXTC(A,NA,NB,C,NC) JUX0001
1 IMPLICIT REAL*8 (A=H,O=Z) JUX0002
2 DIMENSION A(1),B(1),C(1),NA(2),NB(2),NC(2) JUX0003
3 IF (NA(1).NE.NB(1)) GO TO 600 JUX0004
4 NC(1)=NA(1) JUX0005
5 NC(2)=NA(2)+NB(2) JUX0006
6 L=NA(1)*NA(2) JUX0007
7 NNC=NC(1)*NC(2) JUX0008
8 IF( NA(1) .LT. 1 .OR. L .LT. 1 ) GO TO 600 JUX0009
9 IF( NC(2) .LT. 1 ) GO TO 600 JUX0010
10 MS=NA(1)*NA(2) JUX0011
11 DO 10 I=1,MS JUX0012
12 10 C(I)=A(I) JUX0013
13 MBS=NA(1)*NB(2) JUX0014
14 DO 20 I=1,MBS JUX0015
15 J=MBS+I JUX0016
16 20 C(J)=B(I) JUX0017
17 RETURN JUX0018
18 600 CALL LNCNT(1) JUX0019
19 WRITE (6,1600) NA,NB JUX0020
20 1600 FORMAT (' DIMENSION ERROR IN JUXTC,  NA=',2I6,5X,'NB=',2I6) JUX0021
21 RETURN JUX0022
22 END JUX0023

```

```

0      SUBROUTINE JUXTR(A,NA,B,NB,C,NC)          JUX0001
1      IMPLICIT REAL*8 (A-H,O-Z)                  JUX0002
2      DIMENSION A(1),B(1),C(1),NA(2),NB(2),NC(2)  JUX0003
3      IF(NA(2).NE.NB(2))GO TO 600                JUX0004
4      NC(2)=NA(2)                                JUX0005
5      NC(1)=NA(1)+NB(1)                          JUX0006
6      L=NA(1)*NA(2)                            JUX0007
7      IF( NA(1) .LT. 1 .OR. L .LT. 1 ) GO TO 600  JUX0008
8      IF( NC(2) .LT. 1 ) GO TO 600              JUX0009
9      MCA=NA(2)                                JUX0010
10     MRA=NA(1)                                JUX0011
11     MRB=NB(1)                                JUX0012
12     MRC=NC(1)                                JUX0013
13     DO 10 I=1,MCA                           JUX0014
14     DO 10 J=1,MRA                           JUX0015
15     K=J+MRA*(I-1)                         JUX0016
16     L=J+MRC*(I-1)                         JUX0017
17     10 C(L)=A(K)                           JUX0018
18     DO 20 I=1,MCA                           JUX0019
19     DO 20 J=1,MRB                           JUX0020
20     K=J+MRB*(I-1)                         JUX0021
21     L=MRA+J+MRC*(I-1)                      JUX0022
22     20 C(L)=B(K)                           JUX0023
23     RETURN                                  JUX0024
24     600 CALL LNCNT(1)                      JUX0025
25     WRITE(6,1600) NA,NB                      JUX0026
26     1600 FORMAT(' DIMENSION ERROR IN JUXTR,  NA=',2I6,5X,'NB=',2I6) JUX0027
27     RETURN                                  JUX0028
28     END                                     JUX0029

```

```

0      SUBROUTINE FACTOR(Q,NQ,D,ND,IOP,IAC,DUMMY)          FAC0001
1      IMPLICIT REAL*8 (A-H,O-Z)                         FAC0002
2      DIMENSION Q(1),D(1),DUMMY(1)                      FAC0003
3      DIMENSION NQ(2),ND(2),NDUM(2)                     FAC0004
4      IOPT = 2                                         FAC0005
5      N = NQ(1)                                       FAC0006
6      M = N**2                                         FAC0007
7      N1 = M + 1                                      FAC0008
8      N2 = N1 + N                                     FAC0009
9 C
10     CALL EQUATE(Q,NQ,DUMMY,NQ)                      FAC0010
11     CALL SNVDEC(IOPT,N,N,N,N,DUMMY,NOS,B,IAC,ZTEST,DUMMY(N1),D,IRANK,AFAC0011
12     IPLUS,IERR)                                     FAC0012
13     IF( IERR .EQ. 0 ) GO TO 200                     FAC0013
14     CALL LNCNT(5)                                    FAC0014
15     IF( IERR .GT. 0 ) PRINT 100,IERR                FAC0015
16     IF( IERR .EQ. -1) PRINT 150,ZTEST,IRANK        FAC0016
17     100 FORMAT(//,' IN FACTOR , SNVDEC HAS FAILED TO CONVERGE TO THE ',I4,FAC0017
18     ' SINGULAR VALUE AFTER 30 ITERATIONS')          FAC0018
19     150 FORMAT(//,' IN FACTOR, THE MATRIX Q SUBMITTED TO SNVDEC IS CLOSE TFAC0019
20     TO A MATRIX OF LOWER RANK USING ZTEST = ',D16.8,/, ' IF THE ACCURACYFAC0020
21     2 IS REDUCED . THE RANK MAY ALSO BE REDUCED',/, ' CURRENT RANK = ',I4)FAC0021
22     NDUM(1)=N                                       FAC0022
23     NDUM(2)=1                                      FAC0023
24     IF(IERR .EQ. -1) CALL PRNT(DUMMY(N1),NDUM,4HSNVL,1) FAC0024
25     IF( IERR .GT. 0 ) RETURN                       FAC0025
26 C
27     200 CONTINUE
28     NDUM(1) = N                                     FAC0026
29 C
30     DO 250 J =1,N                                 FAC0027
31     M1 = (J-1)*N + 1                            FAC0028
32     M2 = J*N                                       FAC0029
33     DO 250 I =M1,M2                            FAC0030
34     K = N2+I-1                                  FAC0031
35     L = N1+J-1                                  FAC0032
36     IF( DUMMY(L) .EQ. 0.0) GO TO 300           FAC0033
37     DUMMY(K) = DSQRT(DUMMY(L))*DUMMY(I)       FAC0034
38     250 CONTINUE                                 FAC0035
39     NDUM(2)=N                                     FAC0036
40     GO TO 350                                   FAC0037
41 C
42     300 NDUM(2) = J - 1                         FAC0038
43     350 CONTINUE                                 FAC0039
44     IF( DUMMY(N2) .LT. 0.0 ) CALL SCALE(DUMMY(N2),NDUM,DUMMY(N2),NDUM,FAC0040
45     1-L,0)                                     FAC0041
46     CALL TRANP(DUMMY(N2),NDUM,D,ND)            FAC0042
47 C
48     IF( IOP .EQ. 0 ) RETURN                     FAC0043
49     CALL LNCNT(4)                                FAC0044
50     PRINT 400                                     FAC0045
51     400 FORMAT(//,' FACTOR Q AS (D TRANPOSE)XD ',/) FAC0046
52     CALL PRNT(Q,NQ,4H Q ,1)                      FAC0047
53     CALL PRNT(D,ND,4H D ,1)                      FAC0048
54     CALL MULT(DUMMY(N2),NDUM,D,ND,DUMMY,NQ)    FAC0049
55     CALL PRNT(DUMMY,NQ,4HDTXD,1)                 FAC0050
56 C
57     RETURN                                       FAC0051
58     END                                           FAC0052

```

```

0      SUBROUTINE EIGEN(MAX, N, A, ER, EI, ISV, ILV, V, WK, IERR)
1      IMPLICIT REAL*8 (A=H,0=Z)
2      DIMENSION A(MAX,N),ER(N),EI(N),V(MAX,1),WK(N,1)
3      INTEGER INT(20)
4      LOGICAL*1 SELECT(25)
5
6      C
7      C      PRELIMINARY REDUCTION
8
9      CALL BALANC (MAX,N,A,LOW,IGH,WK)
10     CALL ELMHES (MAX,N,LOW,IGH,A,INT(1))
11     IV = ISV + ILV
12     IF (IV .NE. 0) GO TO 10
13
14      C      COMPUTE ALL EIGENVALUES AND NO EIGENVECTORS
15
16     CALL HQR (MAX,N,LOW,IGH,A,ER,EI,IERR)
17     IF (IERR .NE. 0) GO TO 260
18     DO 5 I=1,N
19       WK(I,1) = ER(I)
20       WK(I,2) = EI(I)
21       WK(I,3) = ER(I)**2 + EI(I)**2
22     5 CONTINUE
23     IC = 0
24     GO TO 190
25
26      C      SAVE A MATRIX FOR INVERSE ITERATION AND INITIALIZE WK(I,4)
27      C      ARRAY WHICH WILL BE A LOGICAL ARRAY IN CALLED SUBROUTINES
28
29     DO 20 I=1,N
30     SELECT(I)=.FALSE.
31     JS = 1
32     IF (I .GE. 3) JS = I-1
33     DO 20 J=JS,N
34       WK(I,J+5) = A(I,J)
35   20 CONTINUE
36
37      C      COMPUTE ALL EIGENVALUES (UNORDERED)
38
39     CALL HQR (N,N,LOW,IGH,WK(1,6),ER,EI,IERR)
40     IF (IERR .NE. 0) GO TO 260
41     DO 30 I=1,N
42       WK(I,3) = ER(I)**2 + EI(I)**2
43   30 CONTINUE
44     IF (ILV .EQ. 0) GO TO 60
45
46      C      FIND LARGEST ILV EIGENVALUES AND FLAG THEM
47
48     DO 50 I=1,ILV
49       P = -1.0D0
50     DO 40 J=1,N
51       IF (WK(J,3) .LE. P) GO TO 40
52       K = J
53       P = WK(J,3)
54   40 CONTINUE
55     SELECT(K)=.TRUE.
56     WK(K,3) = -WK(K,3)
57
58   50 CONTINUE
59     IF (EI(K) .EQ. 0.) GO TO 60
60     IF (EI(K) .GT. 0.) GO TO 55
61     IF (SELECT(K-1)) GO TO 60
62     ILV = ILV+1
63     SELECT(K-1)=.TRUE.

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53      GO TO 60          EIG006-
54      55  CONTINUE      EIG006C
55      IF (.NOT.SELECT(K+1)) ILV = ILV+1 EIG006C
56      60  CONTINUE      EIG006C
57      IF (ISV .EQ. 0) GO TO 90 EIG006C
58 C    FIND SMALLEST ISV EIGENVALUES AND FLAG THEM EIG006C
59 C
60      DO 65 I=1,N EIG007C
61      WK(I,3) = DARS(WK(I,3)) EIG007C
62      65  CONTINUE      EIG007C
63      DO 80 J=1,ISV EIG007C
64      P = 1.074 EIG007C
65      DO 70 J=1,N EIG007C
66      IF (WK(J,3) .GE. P) GO TO 70 EIG007C
67      K = J EIG007C
68      P = WK(J,3) EIG007C
69      70  CONTINUE      EIG007C
70      SELECT(K)=.TRUE. EIG008C
71      WK(K,3) = 1.074 EIG008C
72      80  CONTINUE      EIG008C
73      IF (EI(K) .EQ. 0.) GO TO 90 EIG008C
74      IF (EI(K) .GT. 0.) GO TO 85 EIG008C
75      IF (SELECT(K-1)) GO TO 90 EIG008C
76      ISV = ISV+1 EIG008C
77      SELECT(K-1)=.TRUE. EIG008C
78      GO TO 90 EIG008C
79      85  CONTINUE      EIG008C
80      IF (.NOT.SELECT(K+1)) ISV = ISV+1 EIG008C
81      90  CONTINUE      EIG008C
82 C    FIND EIGENVECTORS FOR FLAGGED EIGENVALUES EIG009C
83 C
84      CALL INVIT (MAX,N,A,ER,EI,SELECT,N,M,V,IERR,WK(1,6),WK(1,3), EIG009C
85      1     WK(1,5)) EIG009C
86 C    BACK TRANSFORM EIGENVECTORS TO ORIGINAL MATRIX EIG009C
87 C
88      CALL ELMBAK (MAX,LOW,IGH,A,INT(1),M,V) EIG010C
89      CALL BALBAK (MAX,N,LOW,IGH,WK,M,V) EIG010C
90 C    SEPARATE FLAGGED EIGENVALUES FROM UNFLAGGED EIGENVALUES EIG010C
91 C
92      IV = ISV + ILV EIG010C
93      IF (IV .LE. N) GO TO 100 EIG010C
94      ILV = N-IV EIG010C
95      IV = N EIG010C
96      100 CONTINUE      EIG011C
97      IC = 0 EIG011C
98      JC = IV EIG011C
99      DO 150 I=1,N EIG011C
100     IF (SELECT(I)) GO TO 120 EIG011C
101     IF (EI(I) .GE. 0.) GO TO 110 EIG011C
102     IF (SELECT(I-1)) GO TO 120 EIG011C
103     110 CONTINUE      EIG011C
104     JC = JC+1 EIG011C
105     WK(JC,1) = ER(I) EIG012C
106     WK(JC,2) = EI(I) EIG012C
107     KC = JC EIG012C
108     GO TO 130 EIG012C
109     120 CONTINUE      EIG012C
110     IC = IC+1 EIG012C
111     WK(IC,1) = ER(I) EIG012C

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126      WK(IC,2) = EI(I)          EIG012
127      IC = IC                EIG012
128 130      CONTINUE           EIG012
129      WK(IC,3) = ER(I)**2 + EI(I)**2 EIG013
130 150      CONTINUE           EIG013
131 C
132 C      NORMALIZE VECTORS TO UNIT LENGTH AND STORE FOR REORDERING EIG013
133 C
134      J = 0                  EIG013
135 151      CONTINUE           EIG013
136      J = J+1                EIG013
137      IF (WK(J,2) .NE. 0.) GO TO 154 EIG013
138      SUM = 0.                EIG013
139      DO 152 I=1,N            EIG014
140          SUM = SUM + V(I,J)**2 EIG014
141 152      CONTINUE           EIG014
142      IF (SUM .EQ. 0.) GO TO 158 EIG014
143      SUM = DSQRT(SUM)        EIG014
144      DO 153 I=1,N            EIG014
145          WK(I,J+4) = V(I,J)/SUM EIG014
146 153      CONTINUE           EIG014
147      GO TO 158                EIG014
148 154      CONTINUE           EIG014
149      JP1 = J+1                EIG015
150      SUM = 0.                EIG015
151      DO 155 I=1,N            EIG015
152          SUM = SUM + V(I,J)**2 + V(I,JP1)**2 EIG015
153 155      CONTINUE           EIG015
154      IF (SUM .EQ. 0.) GO TO 157 EIG015
155      SUM = DSQRT(SUM)        EIG015
156      DO 156 I=1,N            EIG015
157          WK(I,J+4) = V(I,J)/SUM EIG015
158          WK(I,J+5) = V(I,JP1)/SUM EIG015
159 156      CONTINUE           EIG016
160 157      CONTINUE           EIG016
161      J = JP1                EIG016
162 158      CONTINUE           EIG016
163      IF (J .LT. IV) GO TO 151 EIG016
164      IC = 0                  EIG016
165      LC = 0                  EIG016
166      IF (ISV .EQ. 0) GO TO 190 EIG016
167 C
168 C      ORDER SMALLEST ISV EIGENVALUES AND EIGENVECTORS FOR OUTPUT EIG016
169 C
170      DO 190 I=1,ISV          EIG017
171          P = 1.074            EIG017
172          DO 160 J=1,IV        EIG017
173              IF (WK(J,3) .GE. P) GO TO 160 EIG017
174              K = J                EIG017
175              P = WK(J,3)          EIG017
176 160      CONTINUE           EIG017
177          IC = IC+1            EIG017
178          LC = LC+1            EIG017
179          ER(IC) = WK(K,1)      EIG018
180          EI(IC) = WK(K,2)      EIG018
181          DO 170 J=1,N          EIG018
182              V(J,LC) = WK(J,K+4) FIG018
183 170      CONTINUE           EIG018
184          WK(K,3) = 1.074      EIG018
185 180      CONTINUE           EIG018
186 190      CONTINUE           EIG018
187      IF (IV .EQ. N) GO TO 220 EIG018
188 C

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89 C ORDER UNFLAGGED EIGENVALUES FOR OUTPUT          EIG019
90 C
91    IV1 = IV+1                                     EIG019
92    IUF = N - IV                                  FIG019
93    DO 210 I=1,IUF                               EIG019
94    P = 1.074                                     EIG019
95    DO 200 J=IV1,N
96      IF (WK(J,3) .GE. P) GO TO 200             EIG019
97      K = J                                     EIG019
98      P = WK(J,3)                                EIG019
99 200 CONTINUE                                     EIG020
 00      IC = IC+1                                EIG020
201      ER(IC) = WK(K,1)                          EIG020
202      EI(IC) = WK(K,2)                          EIG020
203      WK(K,3) = 1.074                           EIG020
204 210 CONTINUE                                     EIG020
205 220 CONTINUE                                     EIG020
 06      IF (ILV .EQ. 0) GO TO 260                EIG020
 07 C
208 C ORDER LARGEST ILV EIGENVALUES AND EIGENVECTORS FOR OUTPUT EIG020
209 C
 10      DO 250 I=1,ILV                            EIG021
 11      P = 1.074                                EIG021
212      DO 230 J=1,IV
 13        IF (WK(J,3) .GE. P) GO TO 230           EIG021
 14        K = J
 15        P = WK(J,3)
 16 230 CONTINUE                                     EIG021
 17        IC = IC+1
 18        LC = LC+1
 19        ER(IC) = WK(K,1)                          EIG021
 20        EI(IC) = WK(K,2)                          EIG021
 21        DO 240 J=1,N
 22          V(J,LC) = WK(J,K+4)                   EIG022
 23 240 CONTINUE                                     EIG022
 24          WK(K,3) = 1.074                         EIG022
225 250 CONTINUE                                     EIG022
226 260 CONTINUE                                     EIG022
 27      RETURN                                     EIG022
 28      END                                       EIG022

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```

0      SUBROUTINE SYMPOS (MAXN,N,A,NRHS,B,IOPT,IFAC,DETERM,ISCALE,P,IERR)SYM000
1      IMPLICIT REAL*8 (A-H,O-Z)                                         SYM000
2      DIMENSION A(MAXN,N),B(MAXN,NRHS),P(N)                           SYM000
3 C
4      DATA R1,R2/1.00+75,1.00-75/                                         SYM000
5 C
6 C      TEST FOR A SCALAR MATRIX (IF COEFFICIENT MATRIX IS A           SYM000
7 C      SCALAR-- SOLVE AND COMPUTE DETERMINANT IF DESIRED)             SYM000
8 C      IERR = 0                                                       SYM000
9 C      NM1 = N-1                                                     SYM001
10     IF (NM1 .GT. 0) GO TO 20                                         SYM001
11 C
12     IF( A(1,1) ,LE, 0.0 ) IERR = 1                                     SYM001
13     ISCALE = 0                                                       SYM001
14     DETERM = A(1,1)                                                 SYM001
15     P(1) = 1.0/A(1,1)                                              SYM001
16     DO 10 J=1,NRHS                                                 SYM001
17     B(1,J) = B(1,J)/DETERM                                         SYM001
18   10 CONTINUE                                                       SYM001
19     RETURN                                                       SYM002
20 C
21 C      TEST TO DETERMINE IF CHOLESKY DECOMPOSITION OF COEFFICIENTS    SYM002
22 C      MATRIX IS DESIRED                                              SYM002
23   20 IF (IFAC .EQ. 1) GO TO 160                                     SYM002
24 C
25 C      INITIALIZE DETERMINANT EVALUATION PARAMETERS                   SYM002
26     DETERM=1.0                                                       SYM002
27     ISCALE=0                                                       SYM002
28 C
29 C      'LOOP' TO PERFORM CHOLESKY DECOMPOSITION ON THE COEF-          SYM003
30 C      FICIENT MATRIX A (I.E. MATRIX A WILL BE DECOMPOSED INTO       SYM003
31 C      THE PRODUCT OF A UNIT LOWER TRIANGULAR MATRIX (L), A           SYM003
32 C      DIAGONAL MATRIX (D), AND THE TRANSPOSE OF L (LTRANSPOSE).)    SYM003
33 C
34   30 DO 150 I=1,N                                                 SYM003
35     IM1 = I-1                                                       SYM003
36 C
37   30 DO 150 J=1,I                                                 SYM003
38     X = A(J,I)                                                    SYM003
39 C
40 C      DETERMINE IF ELEMENTS ARE ABOVE OR BELOW THE DIAGONAL        SYM004
41     IF (I .GT. J) GO TO 110                                         SYM004
42 C
43 C      USING THE DIAGONAL ELEMENTS OF MATRIX A, THIS SECTION          SYM004
44 C      COMPUTES DIAGONAL MATRIX AND DETERMINES IF MATRIX A IS         SYM004
45 C      SYMMETRIC POSITIVE DEFINITE                                SYM004
46     IF (IM1 .EQ. 0) GO TO 50                                         SYM004
47     DO 40 K=1,IM1                                                 SYM004
48     Y = A(I,K)                                                    SYM004
49     A(I,K) = Y*P(K)                                              SYM005
50     X = X - Y*A(I,K)                                            SYM005
51   40 CONTINUE                                                       SYM005
52 C
53 C      TEST IF COEFFICIENT MATRIX IS POSITIVE DEFINITE              SYM005
54   50 IF( X ,LE, 0.0 ) IERR = 1                                     SYM005
55 C
56 C      COMPUTE INVERSE OF DIAGONAL MATRIX D**=-1 = 1/P               SYM005
57     P(I) = 1.0 / X                                               SYM005
58 C
59 C      TEST TO SEE IF DETERMINANT IS TO BE EVALUATED                 SYM006
60   50 IF (IOPT .EQ. 0) GO TO 150                                     SYM006
61 C
62 C

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6 C SCALE THE DETERMINANT (COMPUTE THE DETERMINANT EVALUATION SYM0064
64 C PARAMETERS DETERM AND ISCALE) SYM0065
65 PIVOTI=X SYM0066
6 60 IF(DABS(DETERM),LT,R1) GO TO 70 SYM0067
6 DETERM = DETERM*R2 SYM0068
68 ISCALE = ISCALE+1 SYM0069
69 GO TO 60 SYM0070
7 70 IF(DABS(DETERM),GT,R2) GO TO 80 SYM0071
71 DETERM = DETERM*R1 SYM0072
72 ISCALE = ISCALE-1 SYM0073
73 GO TO 70 SYM0074
74 80 IF(DABS(PIVOTI),LT,R1) GO TO 90 SYM0075
75 PIVOTI = PIVOTI*R2 SYM0076
76 ISCALE = ISCALE+1 SYM0077
77 GO TO 80 SYM0078
78 90 IF(DABS(PIVOTI),GT,R2) GO TO 100 SYM0079
79 PIVOTI = PIVOTI*R1 SYM0080
80 ISCALE = ISCALE-1 SYM0081
81 GO TO 90 SYM0082
82 100 DETERM = DETERM*PIVOTI SYM0083
83 GO TO 150 SYM0084
84 C
85 C USING THE LOWER TRIANGULAR ELEMENTS OF MATRIX A, THIS SYM0085
86 C SECTION COMPUTES THE UNIT LOWER TRIANGULAR MATRIX SYM0086
87 110 JM1 = J-1 SYM0087
88 IF (JM1 .EQ. 0) GO TO 140 SYM0088
89 DO 120 K=1,JM1 SYM0089
90 X = X - A(I,K)*A(J,K) SYM0090
91 120 CONTINUE SYM0091
92 140 A(I,J) = X SYM0092
93 C
94 150 CONTINUE SYM0093
95 C
96 C SECTION TO APPLY BACK SUBSTITUTION TO SOLVE L*Y = B FOR SYM0094
97 C UNIT LOWER TRIANGULAR MATRIX AND CONSTANT COLUMN VECTOR B SYM0100
98 C
99 160 IF( IFAC ,EQ. 2 ) RETURN SYM0101
100 DO 180 I=2,N SYM0102
101 IM1=I-1 SYM0103
102 C
103 DO 180 J=1,NRHS SYM0104
104 X = B(I,J) SYM0105
105 C
106 DO 170 K=1,IM1 SYM0106
107 X = X - A(I,K)*R(K,J) SYM0107
108 170 CONTINUE SYM0108
109 C
110 B(I,J) = X SYM0109
111 180 CONTINUE SYM0110
112 C
113 C SECTION TO SOLVE (LYTRANSPOSE)*X = (D**-1)*Y FOR TRANSPPOSE SYM0116
114 C OF UNIT LOWER TRIANGULAR MATRIX AND INVERSE OF DIAGONAL SYM0117
115 C MATRIX SYM0118
116 C
117 Y = P(N) SYM0119
118 DO 190 J=1,NRHS SYM0120
119 B(N,J) = B(N,J)*Y SYM0121
120 190 CONTINUE SYM0122
121 C
122 200 I = NM1+1 SYM0123
123 Y = P(NM1) SYM0124

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1.26 C SYM012
127 DO 220 J=1,NRHS SYM012
128 X = A(NM1,J)*Y SYM012
1.29 C SYM013
130 DO 210 K=1,N SYM012
131 X = X - A(K,NM1)*B(K,J) SYM013
132 210 CONTINUE SYM013
133 C SYM013
134 B(NM1,J) = X SYM012
135 220 CONTINUE SYM013
1.36 C SYM012
.37 C SYM013
138 C TEST TO DETERMINE IF SOLUTIONS HAVE BEEN DETERMINED FOR SYM013
139 C ALL COLUMN VECTORS SYM014
140 NM1 = NM1-1 SYM014
141 IF (NM1 ,GT. 0) GO TO 200 SYM014
142 C SYM014
143 RETURN SYM014
144 END SYM014

```

0      SUBROUTINE GELM(NMAX,N,A,NRHS,B,IPIVOT,IFAC,WK,IERR)      GEL000
1      IMPLICIT REAL*8(A-H,O-Z)      GEL000
2      DIMENSION A(NMAX,1),B(NMAX,1),IPIVOT(1),WK(1)      GEL000
3      IERR=0      GEL000
4 C
5 C      TEST FOR L/U FACTORIZATION      GEL000
6 C
7      IF(IFAC.EQ.1)GO TO 10      GEL000
8      CALL DETFAC(NMAX,N,A,IPIVOT,IFAC,DETERM,ISCALE,WK,IERR)      GEL000
9      IF(IERR.GT.0)RETURN      GEL001
10     IF(IFAC.EQ.2) DETA=DETERM*(10.**((100+ISCALE)))      GEL001
11     10 NM1=N-1      GEL001
12 C
13 C      TEST FOR SCALAR A MATRIX      GEL001
14 C
15     IF(NM1.GT.0)GO TO 40      GEL001
16     IF(A(1,1).EQ.0.)GO TO 30      GEL001
17     DO 20 I=1,NRHS      GEL001
18     20 B(1,I)=B(1,I)/A(1,1)      GEL001
19     IF(IFAC.EQ.2) WK(1)=DETA      GEL001
20     RETURN      GEL001
21     30 IERR=1      GEL002
22     RETURN      GEL002
23 C
24     40 DO 100 M=1,NRHS      GEL002
25 C
26 C      PIVOT THE M-TH COLUMN OF B MATRIX      GEL002
27 C
28     DO 50 I=1,NM1      GEL002
29     KI=IPIVOT(I)      GEL002
30     P=B(KI,M)      GEL002
31     B(KI,M)=B(I,M)      GEL003
32     50 B(I,M)=P      GEL003
33 C
34 C      FORWARD SUBSTITUTION      GEL003
35 C
36     WK(1)=B(1,M)      GEL003
37 C
38     DO 70 I=2,N      GEL003
39     IM1=I-1      GEL003
40     P=0.0      GEL003
41     DO 60 K=1,IM1      GEL004
42     60 P=P+A(I,K)*WK(K)      GEL004
43     70 WK(I)=B(I,M)-P      GEL004
44 C
45 C      BACK SUBSTITUTION      GEL004
46 C
47     B(N,M)=WK(N)/A(N,N)      GEL004
48 C
49     DO 90 J=1,NM1      GEL004
50     I=N-J      GEL004
51     IP1=I+1      GEL005
52     P=WK(I)      GEL005
53     DO 80 K=IP1,N      GEL005
54     80 P=P-A(I,K)*B(K,M)      GEL005
55     90 B(I,M)=P/A(I,I)      GEL005
56 C
57     100 CONTINUE      GEL005
58     IF(IFAC.EQ.2) WK(1)=DETA      GEL005
59     RETURN      GEL005
60     END      GEL005

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0      SUBROUTINE SNVDEC(IOP,MD,ND,M,N,A,NOS,R,IAC,ZTEST,Q,V,IRANK,APLUS,SNV000
1      IERR)                                              SNV000
2      IMPLICIT REAL*8 (A-H,O-Z)                           SNV000
3      LOGICAL WITHU,WITHV                                SNV000
4      DIMENSION A(MD,N),V(ND,N),Q(N),E(150)              SNV000
5      DIMENSION B(MD,NOS),APLUS(ND,M)                   SNV000
6 C
7 C
8 C      TEST FOR SCALAR OR VECTOR A                      SNV000
9 C
10     IF( N .GE. 2 ) GO TO 3000                          SNV001
11 C
12     IERR = 0                                           SNV001
13     ZTEST = 10.**(-IAC)                               SNV001
14     SUM = 0.0                                         SNV001
15     DO 1000 I=1,M                                     SNV001
16     SUM = SUM + A(I,1)*A(I,1)                         SNV001
17   1000 CONTINUE                                       SNV001
18     SUM = DSQRT(SUM)                                 SNV001
19     IRANK = 0                                         SNV002
20     IF( SUM .GT. ZTEST ) IRANK = 1                  SNV002
21     Q(1) = SUM                                       SNV002
22 C
23     IF( IOP .EQ. 1 ) RETURN                          SNV002
24     V(1,1) = 1.0                                      SNV002
25     IF( IRANK .EQ. 0 ) GO TO 1200                  SNV002
26     DO 1100 I =1,M                                  SNV002
27     A(I,1) = A(I,1)/SUM                            SNV002
28   1100 CONTINUE                                       SNV002
29     GO TO 1300                                       SNV003
30   1200 CONTINUE                                       SNV003
31     A(1,1) = 1.0                                      SNV003
32   1300 CONTINUE                                       SNV003
33 C
34     IF( IOP .EQ. 2 ) RETURN                          SNV003
35     IF( IOP .EQ. 4 ) GO TO 1850                  SNV003
36     IF( IRANK .EQ. 0 ) GO TO 1600                  SNV003
37     DO 1500 J = 1,NOS                            SNV003
38     Z = 0                                           SNV003
39     DO 1400 I = 1,M                            SNV004
40     Z = Z + A(I,1)*B(I,J)/SUM                    SNV004
41   1400 CONTINUE                                       SNV004
42     B(1,J) = Z                                     SNV004
43   1500 CONTINUE                                       SNV004
44     GO TO 1800                                       SNV004
45   1600 CONTINUE                                       SNV004
46     DO 1700 J = 1,NOS                            SNV004
47     B(1,J) = 0.0                                    SNV004
48   1700 CONTINUE                                       SNV004
49   1800 CONTINUE                                       SNV005
50 C
51     IF( IOP .EQ. 3 ) RETURN                          SNV005
52   1850 CONTINUE                                       SNV005
53     IF( IRANK .EQ. 0 ) GO TO 2000                  SNV005
54     DO 1900 I = 1,M                            SNV005
55     APLUS(1,I) = A(I,1)/SUM                      SNV005
56   1900 CONTINUE                                       SNV005
57     RETURN                                         SNV005
58   2000 CONTINUE                                       SNV005
59     DO 2100 I=1,M                                SNV006
60     APLUS(1,I) = 0.0                            SNV006
61   2100 CONTINUE                                       SNV006
62     RETURN                                         SNV006

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63 C SNV006
64 C SNV006
65 3000 CONTINUE SNV006
66 C SNV006
67 C SNV006
68 C SNV006
69 TOL=1.00E-60 SNV007
70 SIZE=0.0 SNV007
71 NP1=N+1 SNV007
72 C SNV007
73 C COMPUTE THE E-NORM OF MATRIX A AS ZERO TEST FOR SINGULAR VALUES. SNV007
74 C SNV007
75 SUM=0.0 SNV007
76 DO 500 I=1,M SNV007
77 DO 500 J=1,N SNV007
78 500 SUM = SUM + A(I,J)**2 SNV007
79 ZTEST = DSQRT(SUM) SNV008
80 ZTEST = ZTEST*10.**(-IAC) SNV008
81 C SNV008
82 510 IF (IOP,NE,1) GO TO 515 SNV008
83 WITHU=.FALSE. SNV008
84 WITHV=.FALSE. SNV008
85 GO TO 520 SNV008
86 515 WITHU=.TRUE. SNV008
87 WITHV=.TRUE. SNV008
88 520 CONTINUE SNV008
89 G = 0.0 SNV009
90 X = 0.0 SNV009
91 DO 30 I = 1,N SNV009
92 C SNV009
93 C HOUSEHOLDER REDUCTION TO BIDIAGONAL FORM. SNV009
94 C SNV009
95 E(I) = G SNV009
96 S = 0.0 SNV009
97 L = I+1 SNV009
98 C SNV009
99 C ANNIHILATE THE I-TH COLUMN BELOW DIAGONAL. SNV010
100 C SNV010
101 DO 3 J = I,M SNV010
102 3 S = S + A(J,I)**2 SNV010
103 G = 0.0 SNV010
104 IF(S .LT. TOL) GO TO 10 SNV010
105 G = DSQRT(S) SNV010
106 F = A(I,I) SNV010
107 IF(F .GE. 0.0) G = -G SNV010
108 H = F*G -S SNV010
109 A(I,I) = F-G SNV011
110 IF(I .EQ. N) GO TO 10 SNV011
111 DO 9 J = L,N SNV011
112 S = 0.0 SNV011
113 DO 7 K = I,M SNV011
114 7 S = S + A(K,I)*A(K,J) SNV011
115 F = S/H SNV011
116 DO 8 K = I,M SNV011
117 8 A(K,J) = A(K,J) + F*A(K,I) SNV011
118 9 CONTINUE SNV011
119 10 Q(I) = G SNV012
120 IF(I .EQ. N) GO TO 20 SNV012
121 C SNV012
122 C ANNIHILATE THE I-TH ROW TO RIGHT OF SUPER-DIAG. SNV012
123 C SNV012
124 S = 0.0 SNV012
125 DO 11 J = L,N SNV012

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26 11 S = S + A(I,J)**2 SNV012
27 G = 0.0 SNV012
128 IF (S .LT. TOL) GO TO 20 SNV012
29 G = DSQRT(S) SNV013
30 F = A(I,I+1) SNV013
131 IF(F .GE. 0.0) G = -G SNV013
132 H = F*G -S SNV013
33 A(I,I+1) = F = G SNV013
34 DO 15 J = L,N SNV013
135 15 E(J) = A(I,J)/H SNV013
136 DO 19 J = L,M SNV013
37 S = 0.0 SNV013
138 DO 16 K = L,N SNV013
139 16 S = S + A(J,K) * A(I,K) SNV014
40 DO 17 K = L,N SNV014
41 17 A(J,K) = A(J,K) + S*E(K) SNV014
142 19 CONTINUE SNV014
43 20 Y = DARS(Q(I)) + DARS(E(I)) SNV014
44 IF(Y .GT. SIZE) SIZE = Y SNV014
145 30 CONTINUE SNV014
146 IF(.NOT. WITHV) GO TO 41 SNV014
47 C
148 C ACCUMULATION OF RIGHT TRANSFORMATIONS. SNV014
149 C
50 DO 40 II = 1,N SNV015
51 I = NP1 - II SNV015
152 IF(I .EQ. N) GO TO 39 SNV015
153 IF(G .EQ. 0.0) GO TO 37 SNV015
54 H = A(I,I+1)*G SNV015
55 DO 32 J = L,N SNV015
156 32 V(J,I) = A(I,J)/H SNV015
57 DO 36 J = L,N SNV015
58 S = 0.0 SNV015
159 DO 33 K = L,N SNV016
60 33 S = S + A(I,K)*V(K,J) SNV016
61 DO 34 K = L,N SNV016
62 34 V(K,J) = V(K,J) + S*V(K,I) SNV016
163 36 CONTINUE SNV016
64 37 DO 38 J = L,N SNV016
65 V(I,J) = 0.0 SNV016
166 38 V(J,I) = 0.0 SNV016
67 39 V(I,I) = 1.0 SNV016
68 G = E(I) SNV016
169 40 L = I SNV017
170 41 CONTINUE SNV017
71 IF(.NOT. WITHU) GO TO 53 SNV017
72 C
173 C ACCUMULATION OF LEFT TRANSFORMATIONS. SNV017
74 C
75 DO 52 II = 1,N SNV017
76 I = NP1 - II SNV017
77 L = I + 1 SNV017
78 G = Q(I) SNV017
79 IF(I .EQ. N) GO TO 43 SNV018
180 DO 42 J = L,N SNV018
81 42 A(I,J) = 0.0 SNV018
82 43 CONTINUE SNV018
183 IF(G .EQ. 0.0) GO TO 49 SNV018
184 IF(I .EQ. N) GO TO 47 SNV018
85 H = A(I,I)*G SNV018
186 DO 46 J = L,N SNV018
187 S = 0.0 SNV018
88 DO 44 K = L,M SNV018

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189   44 S = S + A(K,I)*A(K,J)          SNV019
190       F = S/H                         SNV019
191       DO 45 K = I,M                  SNV019
192   45 A(K,J) = A(K,J) + F*A(K,I)    SNV019
193       CONTINUE                      SNV019
194       DO 48 J = I,M                  SNV019
195   48 A(J,I) = A(J,I)/G            SNV019
196       GO TO 51                      SNV019
197   49 DO 50 J = I,M                  SNV019
198   50 A(J,I) = 0.0                  SNV020
199   51 A(I,I) = A(I,I) + 1.0        SNV020
200   52 CONTINUE                     SNV020
201   53 CONTINUE                     SNV020
202 C
203 C      DIAGONALIZATION OF BIDIAGONAL FORM.
204 C
205   DO 100 KK=1,N                  SNV020
206       K=NPI-KK                    SNV020
207       ITCNT=0                     SNV020
208       KP1=KK+1                   SNV020
209 C
210 C      TEST F SPLITTING.
211 C
212   59 CONTINUE                     SNV021
213   DO 60 LL=1,K                  SNV021
214       L=KP1-LL                  SNV021
215       IF((SIZE+DABS(E(L))).EQ.SIZE) GO TO 64 SNV021
216       LM1=LL-1                  SNV021
217       IF((SIZE+DABS(Q(LM1))).EQ.SIZE) GO TO 61 SNV021
218   60 CONTINUE                     SNV021
219 C
220 C      CANCELLATION OF E(L) IF L .GT. 1.
221 C
222   61 C=0.0                       SNV022
223       S=1.0                        SNV022
224       L1=L-1                      SNV022
225   DO 63 I=L,K                  SNV022
226       F=S*E(I)                  SNV022
227       E(I)=C*E(I)                SNV022
228       IF((SIZE+DABS(F)).EQ.SIZE) DO TO 64 SNV022
229       G=Q(I)                      SNV023
230       Q(I)=DSQRT(F*F+G*G)      SNV023
231       H=Q(I)                      SNV023
232       C=G/H                      SNV023
233       S=-F/H                    SNV023
234       IF(.NOT.WITHU) GO TO 63    SNV023
235   DO 62 J=1,M                  SNV023
236       Y=A(J,L1)                  SNV023
237       Z=A(J,I)                  SNV023
238       A(J,L1)=Y*C+Z*S          SNV023
239       A(J,I)=-Y*S+Z*C          SNV024
240   62 CONTINUE                     SNV024
241 C
242   63 CONTINUE                     SNV024
243 C
244 C      TEST F CONVERGENCE.
245 C
246   64 Z=Q(K)                      SNV024
247   IF(L.EQ.K) GO TO 75          SNV024
248   IF(ITCNT.LE.30) GO TO 65      SNV024
249   IFRR = KK                      SNV025
250   RETURN                         SNV025
251   65 ITCNT = ITCNT + 1          SNV025

```

```

152 C
153 C      SHIFT FROM LOWER 2X2.
154 C
155      X=Q(L)
156      Y=Q(K-1)
157      G=E(K-1)
158      H=E(K)
159      F=((Y-Z)*(Y+Z)+(G-H)*(G+H))/(2.0*M*V)
160      G=DSQRT(F*F+1.0)
161      IF(F.LT.0.0)  G=-G
162      F = ((X-Z)*(X+Z)+H*(Y/(F+G)-H))/X
163 C
164 C
165 C      NEXT QR TRANSFORMATION.
166 C
167      C=1.0
168      S=1.0
169      LP1=L+1
170      DO 73 I=LP1,K
171      G=E(I)
172      Y=Q(I)
173      H=S*G
174      G=C*G
175      Z=DSQRT(F*F+H*H)
176      E(I-1)=Z
177      C=F/Z
178      S=H/Z
179      F=X*C+G*S
180      G=-X*S+G*C
181      H=Y*S
182      Y=Y*C
183      IF(.NOT.WITHV)  GO TO 70
184      DO 68 J=1,N
185      X=V(J,I-1)
186      Z=V(J,I)
187      V(J,I-1)=X*C+Z*S
188      V(J,I)=-X*S+Z*C
189      68      CONTINUE
190 C
191      70      Z=DSQRT(F*F+H*H)
192      Q(I-1)=Z
193      C=F/Z
194      S=H/Z
195      F=C*G+S*Y
196      X=-S*G+C*Y
197      IF(.NOT.WITHV)  GO TO 73
198      DO 72 J=1,M
199      Y=A(J,I-1)
200      Z=A(J,I)
201      A(J,I-1)=Y*C+Z*S
202      A(J,I)=-Y*S+Z*C
203      72      CONTINUE
204 C
205 C
206      73      E(L) = 0.0
207      F(K)=F
208      Q(K)=X
209      GO TO 59
210 C
211 C      CONVERGENCE.
212 C
213      75      CONTINUE
214      IF(Z.GE.0.0)  GO TO 100

```

31 Q(K)=Z
 31 IF(.,NOT.WITHV) GO TO 100
 31 DO 76 J=1,N
 31 76 V(J,K)=V(J,K)
 31 100 CONTINUE
 32 C
 32 IERR=0
 32 DO 280 II=2,N
 32 I=II-1
 32 K=I
 32 P=Q(I)
 32 DO 250 J=II,N
 32 IF (Q(J),LE,P) GO TO 250
 32 K=J
 32 P=Q(J)
 32 250 CONTINUE
 33 C
 33 IF (K.EQ.I) GO TO 280
 33 Q(K) = Q(I)
 33 Q(I) = P
 33 C
 33 IF (IOP.EQ.1) GO TO 280
 33 C
 33 DO 260 J=1,N
 33 P= V(J,I)
 33 V(J,I)= V(J,K)
 33 V(J,K)= P
 33 260 CONTINUE
 33 C
 33 DO 270 J=1,M
 33 P = A(J,I)
 33 A(J,I)= A(J,K)
 33 A(J,K)= P
 33 270 CONTINUE
 34 C
 34 280 CONTINUE
 34 C
 34 J=N
 34 290 IF (Q(J).GT.ZTEST) GO TO 300
 34 Q(J)=0.0
 34 J=J-1
 34 GO TO 290
 34 300 IRANK =J
 34 TEMP = ZTEST/Q(J)
 34 IF (TEMP.GT..0625) IERR=-1
 34 C
 34 IF (IOP.LT. 3) RETURN
 34 IF(IOP.GT.3) GO TO 170
 34 DO 160 L=1,NOS
 34 DO 130 J=1,IRANK
 34 SUM=0.0
 34 DO 120 I=1,M
 34 120 SUM =SUM + A(I,J)*B(I,L)
 34 130 E(J)= SUM/Q(J)
 34 C
 34 DO 150 K=1,N
 34 SUM=0.0
 34 DO 140 I=1,IRANK
 34 140 SUM =SUM + V(K,I)*E(I)
 34 150 R(K,L)=SUM
 34 160 CONTINUE
 34 RETURN
 34 170 DO 200 J=1,M

SNV03160
 SNV03170
 SNV03180
 SNV03190
 SNV03200
 SNV03210
 SNV03220
 SNV03230
 SNV03240
 SNV03250
 SNV03260
 SNV03270
 SNV03280
 SNV03290
 SNV03300
 SNV03310
 SNV03320
 SNV03330
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 SNV03690
 SNV03700
 SNV03710
 SNV03720
 SNV03730
 SNV03740
 SNV03750
 SNV03760
 SNV03770
 SNV03780

37	-- 00 100 I=1,N	SNV03790
37	SUM=0.0	SNV03800
380	00 180 K=1,IRANK	SNV03810
381	180 SUM =SUM + V(I,K)*A(J,K)/Q(K)	SNV03820
38	190 APLUS(I,J)= SUM	SNV03830
383	200 CONTINUE	SNV03840
384	C	SNV03850
38	IF(IOP .EQ.4) RETURN	SNV03860
38	00 230 K=1,NOS	SNV03870
387	00 220 I=1,N	SNV03880
38	SUM=0.0	SNV03890
38	00 210 J=1,M	SNV03900
390	210 SIJM=SUM+ APLUS(I,J)*B(J,K)	SNV03910
391	220 E(I)=SUM	SNV03920
39	00 225 I=1,N	SNV03930
39	225 B(I,K)=E(I)	SNV03940
394	230 CONTINUE	SNV03950
3C	RETURN	SNV03960
3C	END	SNV03970

```

0      SUBROUTINE SUM(A,NA,B,NB,C,NC,IOP,SYM,DUMMY)          SUM00010
1      IMPLICIT REAL*8 (A-H,O-Z)                          SUM00020
2      DIMENSION A(1),B(1),C(1),DUMMY(1)                  SUM00030
3      DIMENSION NA(2),NB(2),NC(2)                      SUM00040
4      LOGICAL SYM                                     SUM00050
5      COMMON/CONV/SUMCV,RICTCV,SERCV,MAXSUM           SUM00060
6      IF( IOP .EQ. 0 ) GO TO 100                      SUM00070
7      PRINT 50                                         SUM00080
8      50 FORMAT(//,' LINEAR EQUATION SOLVER      X = AXC + B ')
9      CALL PRNT(A,NA,4H A ,1)                         SUM00100
10     IF( SYM ) GO TO 75                           SUM00110
11     CALL PRNT(C,NC,4H C ,1)                         SUM00120
12     GO TO 85                                         SUM00130
13     75 CONTINUE                                     SUM00140
14     PRINT 80                                         SUM00150
15     80 FORMAT(/, ' C = A TRANSPOSE ')             SUM00160
16     85 CONTINUE                                     SUM00170
17     CALL PRNT(B,NB,4H B ,1)                         SUM00180
18
19     C
20     100 CONTINUE                                    SUM00190
21     N1 = 1 + NA(1)*NC(1)                         SUM00200
22     I=1
23     200 CONTINUE                                    SUM00210
24     CALL MULT(A,NA,B,NB,DUMMY,NB)                 SUM00220
25     CALL MULT(DUMMY,NB,C,NC,DUMMY(N1),NB)       SUM00230
26     CALL MAXEL(B,NB,WNS)                         SUM00240
27     CALL MAXEL(DUMMY(N1),NB,WNDX)                SUM00250
28     IF(WNS .GE. 1.) GO TO 225                   SUM00260
29     IF( WNDX/WNS .LT. SUMCV ) GO TO 300         SUM00270
30     GO TO 235                                     SUM00280
31     225 IF( WNDX .LT. SUMCV ) GO TO 300         SUM00290
32     235 CONTINUE                                    SUM00300
33     CALL ADD(R,NB,DUMMY(N1),NB,B,NB)            SUM00310
34     CALL MULT(A,NA,A,NA,DUMMY,NA)                SUM00320
35     CALL EQUATE(DUMMY,NA,A,NA)                  SUM00330
36     IF( SYM ) GO TO 245                         SUM00340
37     CALL MULT(C,NC,C,NC,DUMMY,NC)                SUM00350
38     CALL EQUATE(DUMMY,NC,C,NC)                  SUM00360
39     GO TO 250                                     SUM00370
40     245 CONTINUE                                    SUM00380
41     CALL TRANP(A,NA,C,NC)                         SUM00390
42     250 CONTINUE                                    SUM00400
43     I=I+1                                         SUM00410
44     IF( I .LE. MAXSUM ) GO TO 200               SUM00420
45     CALL LNCNT(3)                                SUM00430
46     PRINT 275,MAXSUM                            SUM00440
47     275 FORMAT(//,' IN SUM, THE SEQUENCE OF PARTIAL SUMS HAS EXCEEDED STAGSUM00450
48     I E',I5,' WITHOUT CONVERGENCE')           SUM00460
49     300 CONTINUE                                    SUM00470
50     IF(IOP .EQ. 0) RETURN                         SUM00480
51     CALL PRNT(B,NB,4H X ,1)                      SUM00490
52     RETURN                                         SUM00500
53     END                                           SUM00510
54

```

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SUBROUTINE BILIN(A,NA,B,NB,C,NC,IOP,BETA,SYM,DUMMY)          BIL00010
IMPLICIT REAL*8 (A-H,O-Z)                                     BIL00020
DIMENSION A(1),B(1),C(1),DUMMY(1)                           BIL00030
DIMENSION NA(2),NB(2),NC(2),NDUM(2)                         BIL00040
DIMENSION IOP(2)                                              BIL00050
LOGICAL SYM                                                 BIL00060
IF( IOP(1) .EQ. 0 ) GO TO 300                               BIL00070
IF(SYM) GO TO 100                                           BIL00080
CALL LNCNT(3)                                              BIL00090
PRINT 50                                                 BIL00100
50 FORMAT(//,' LINEAR EQUATION SOLVER  AX + XB = C ')
CALL PRNT(A,NA,4H A ,1)
CALL PRNT(B,NB,4H B ,1)
GO TO 200
100 CONTINUE
CALL LNCNT(3)
PRINT 150
150 FORMAT(//,' LINEAR EQUATION SOLVER  ( B TRANSPOSE )X + XB = C ')
CALL TRANP(A,NA,DUMMY,NDUM)
CALL PRNT(DUMMY,NDUM,4H B ,1)
200 CONTINUE
CALL PRNT(C,NC,4H C ,1)
300 CONTINUE
C
IOPTT = 0
NENA(1)**2
MENA(1)**2
C
IF( IOP(2) .EQ. 0 ) GO TO 500
C
N1 = N + 1
CALL EQUATE(A,NA,DUMMY,NA)
N2 = N1 + NA(1)
N3 = N2 + NA(1)
ISV = 0
ILV = 0
NEVL = NA(1)
CALL EIGEN(NA(1),NA(1),DUMMY,DUMMY(N1),DUMMY(N2),ISV,ILV,V,DUMMY(NBIL00380
13),IERR)
IF (IERR .EQ. 0) GO TO 350
CALL LNCNT(3)
PRINT 325,IERR
325 FORMAT(//,' IN BILIN, THE ',I4,' EIGENVALUE OF A HAS NOT BEEN DETRIBIL00430
43 TERMINED AFTER 30 ITERATIONS')
IERR=1
CALL NORMS(NEVL,NEVL,NEVL,A,IERR,BETA)
BETA=2.*BETA
GO TO 385
350 CONTINUE
J= N1 + NEVL -1
K = N2 + NEVL -1
CO = DSQRT(DUMMY(N1)**2 + DUMMY(N2)**2)
CN = DSQRT(DUMMY(J)**2 + DUMMY(K)**2)
CD = DUMMY(J)-DUMMY(N1)
IF(CD .EQ. 0.0) GO TO 365
BETA = (DUMMY(N1)*CN-DUMMY(J)*CO)/CD
IF(BETA .LE. 0.0) GO TO 365
BETA = DSQRT(BETA)
GO TO 385
C
365 CONTINUE
BETA = 0.0

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61 DO 375 I = 1,NEVL BIL00640
62 J = N1 + I -1 BIL00650
63 K = N2 + I -1 BIL00660
64 IF(DUMMY(J) .GE. 0.0) GO TO 375 BIL00670
65 BETA = BETA + DSQRT(DUMMY(J)**2 + DUMMY(K)**2) BIL00680
66 375 CONTINUE BIL00690
67 BETA = BETA/NEVL BIL00700
68 C BIL00710
69 385 CONTINUE BIL00720
70 C BIL00730
71 IF( SYM ) GO TO 900 BIL00740
72 CALL EQUATE(B,NB,DUMMY,NB) BIL00750
73 N1=M+1 BIL00760
74 N2 = N1 +NB(1) BIL00770
75 N3 = N2 +NB(1) BIL00780
76 NEVL = NB(1) BIL00790
77 CALL EIGEN(NB(1),NB(1),DUMMY,DUMMY(N1),DUMMY(N2),ISV,ILV,V,DUMMY(NB) BIL00800
78 13),IERR) BIL00810
79 IF(IERR .EQ. 0) GO TO 450 BIL00820
80 CALL LNCNT(3) BIL00830
81 PRINT 400,IERR BIL00840
82 400 FORMAT(//,' IN BILIN, THE ',I4,' EIGENVALUE OF B HAS NOT BEEN FOUNDMIL00850
83 1D AFTER 30 ITERATIONS') BIL00860
84 IERR=1 BIL00870
85 CALL NORMS(NEVL,NEVL,NEVL,B,IERR,BETA1) BIL00880
86 BETA1=2.*BETA1 BIL00890
87 GO TO 485 BIL00900
88 450 CONTINUE BIL00910
89 J = N1 + NEVL -1 BIL00920
90 K = N2 + NEVL -1 BIL00930
91 CO = DSQRT(DUMMY(N1)**2 + DUMMY(N2)**2) BIL00940
92 CN = DSQRT(DUMMY(J)**2 + DUMMY(K)**2) BIL00950
93 CD = DUMMY(J)-DUMMY(N1) BIL00960
94 IF(CD .EQ. 0.0) GO TO 465 BIL00970
95 BETA1 = (DUMMY(N1)*CN - DUMMY(J)*CO)/CD BIL00980
96 IF(BETA1 .LE. 0.0) GO TO 465 BIL00990
97 BETA1 = DSQRT(BETA1) BIL01000
98 GO TO 485 BIL01010
99 C BIL01020
100 465 CONTINUE BIL01030
101 C BIL01040
102 104 BETA1 = 0.0 BIL01050
103 DO 475 I= 1,NEVL BIL01060
104 J = N1 + I -1 BIL01070
105 K = N2 + I -1 BIL01080
106 IF(DUMMY(J) .GE. 0.0) GO TO 475 BIL01090
107 BETA1 = BETA1 + DSQRT(DUMMY(J)**2 + DUMMY(K)**2) BIL01100
108 475 CONTINUE BIL01110
109 BETA1 = BETA1/NEVL BIL01120
110 C BIL01130
111 485 CONTINUE BIL01140
112 BETA = (BETA + BETA1)/2. BIL01150
113 C BIL01160
114 500 CONTINUE BIL01170
115 C BIL01180
116 117 C BIL01190
117 IF( IOP(1) .EQ. 0 ) GO TO 520 BIL01200
118 CALL LNCNT(4) BIL01210
119 PRINT 515,BETA BIL01220
120 515 FORMAT(//,' BETA = ',E16.8,/ BIL01230
121 520 CONTINUE BIL01240
122 C BIL01250
123 N1 = N+1 BIL01260

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126 CALL EQUATE(A,NA,DUMMY,NA) BIL01270
127 CALL EQUATE(A,NA,DUMMY(N1),NA) BIL01280
128 CALL SCALE(DUMMY,NA,DUMMY,NA,-1.) BIL01290
129 L = -NA(1) BIL01300
130 NAX = NA(1) BIL01310
131 DO 525 I=1,NAX BIL01320
132 L = L + NAX + 1 BIL01330
133 M1 = L + N BIL01340
134 DUMMY(L) = BETA - A(L) BIL01350
135 DUMMY(M1) = BETA + A(L) BIL01360
136 525 CONTINUE BIL01370
137 N2 = N1 + N BIL01380
138 CALL EQUATE(C,NC,DUMMY(N2),NDUM) BIL01390
139 NDUM(2) = NDUM(2) + NA(1) BIL01400
140 N3 = N2 + NC(1)*NC(2) BIL01410
141 GAM = -2.*BETA BIL01420
142 C BIL01430
143 IF( .NOT. SYM ) GO TO 600 BIL01440
144 C BIL01450
145 CALL UNITY(DUMMY(N3),NA) BIL01460
146 N4 = N3 + N BIL01470
147 NDUM(2) = NDUM(2) + NA(1) BIL01480
148 N5 = N4 + NA(1) BIL01490
149 IFAC = 0 BIL01500
150 CALL GELIM(NA(1),NA(1),DUMMY,NDUM(2),DUMMY(N1),DUMMY(N4),IFAC,DUMMB BIL01510
151 1Y(N5),IERR) BIL01520
152 IF( IERR .EQ. 1 ) PRINT 625 BIL01530
153 CALL EQUATE(DUMMY(N1),NA,DUMMY,NA) BIL01540
154 CALL EQUATE(DUMMY(N2),NC,C,NC) BIL01550
155 CALL TRANP(DUMMY,NA,DUMMY(N1),NA) BIL01560
156 CALL TRANP(DUMMY(N3),NA,DUMMY(N2),NA) BIL01570
157 CALL MULT(C,NC,DUMMY(N2),NA,DUMMY(N3),NA) BIL01580
158 CALL SCALE(DUMMY(N3),NC,C,NC,GAM) BIL01590
159 C BIL01600
160 C BIL01610
161 CALL SUM(DUMMY,NA,C,NC,DUMMY(N1),NA,IOPTT,SYM,DUMMY(N2)) BIL01620
162 GO TO 700 BIL01630
163 600 CONTINUE BIL01640
164 N4 = N3 + NA(1) BIL01650
165 IFAC = 0 BIL01660
166 CALL GELIM(NA(1),NA(1),DUMMY,NDUM(2),DUMMY(N1),DUMMY(N3),IFAC,DUMMB BIL01670
167 1Y(N4),IERR) BIL01680
168 IF(IERR .EQ. 1 ) PRINT 625 BIL01690
169 625 FORMAT(//,' IN BILIN, THE MATRIX (BETA)I - A IS SINGULAR, INCREASE BIL01700
170 1E BETA ') BIL01710
171 CALL EQUATE(DUMMY(N1),NA,DUMMY,NA) BIL01720
172 CALL EQUATE(DUMMY(N2),NC,C,NC) BIL01730
173 N2 = M + N BIL01740
174 CALL EQUATE(B,NB,DUMMY(N1),NB) BIL01750
175 CALL EQUATE(B,NB,DUMMY(N2),NB) BIL01760
176 CALL SCALE(DUMMY(N1),NB,DUMMY(N1),NB,-1.0) BIL01770
177 L=NB(1) BIL01780
178 NAX=NB(1) BIL01790
179 DO650I =1,NAX BIL01800
180 L=L + NAX + 1 BIL01810
181 L1 = L + N BIL01820
182 M1 = L + N2-1 BIL01830
183 DUMMY(L1) = BETA - B'L) BIL01840
184 DUMMY(M1) = BETA + B(L) BIL01850
185 650 CONTINUE BIL01860
186 C BIL01870
187 N3 = N2 + M BIL01880
188 CALL TRANP(DUMMY(N1),NB,DUMMY(N3),NB) BIL01890

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180 CALL EQUATE(DUMMY(N3),NB,DUMMY(N1),NR) BIL01900
181 CALL TRANP(DUMMY(N2),NB,DUMMY(N3),NB) BIL01910
182 CALL EQUATE(DUMMY(N3),NB,DUMMY(N2),NB) BIL01920
183 CALL TRANP(C,NC,DUMMY(N3),NDUM) BIL01930
184 NSDUM = NDUM(?) BIL01940
185 NDUM(2)= NDUM(2) + NB(2) BIL01950
186 IFAC = 0 BIL01960
187 N4=N3+NC(1)+NC(2) BIL01970
188 N5=N4+NB(1) BIL01980
189 CALL GELIM(NB(1),NB(1),DUMMY(N1),NDUM(2),DUMMY(N2),DUMMY(N4),IFAC,BIL01990
190 DUMMY(N5),IERR) BIL02000
191 IF(IFRR .EQ. 1 ) PRINT 675 BIL02010
192 675 FORMAT(//,'IN BILIN, THE MATRIX (BETA)I - B IS SINGULAR, INCREASE BIL02020
193 1 BETA ') BIL02030
194 CALL TRANP(DUMMY(N2),NB,DUMMY(N1),NB) BIL02040
195 NDUM(2)= NSDUM BIL02050
196 CALL TRANP(DUMMY(N3),NDUM,C,NC) BIL02060
197 CALL SCALE(C,NC,C,NC,GAM) BIL02070
198 N2 = N + M + 1 BIL02080
199 CALL SUM(DUMMY,NA,C,NC,DUMMY(N1),NB,IOPTT,SYM,DUMMY(N2)) BIL02090
200 C BIL02100
201 Z00 CONTINUE BIL02110
202 IF( IOP(1) .EQ. 0 ) RETURN BIL02120
203 CALL PRNT(C,NC,4H X ,1) BIL02130
204 RETURN BIL02140
205 END BIL02150

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1   SUBROUTINE BARSTW(A,NA,B,NB,C,NC,IOP,SYM,EPSA,EPSB,DUMMY)      BAR00010
2   IMPLICIT REAL*8 (A-H,O-Z)                                         BAR00020
3   DIMENSION A(1),B(1),C(1),DUMMY(1)                                 BAR00030
4   DIMENSION NA(2),NB(2),NC(2),NDUM1(2),NDUM2(2),NDUM3(2),NDUM4(2)  BAR00040
5   LOGICAL SYM                                                       BAR00050
6   IF ( IOP .EQ. 0 ) GO TO 250                                       BAR00060
7   IF(SYM) GO TO 100                                                 BAR00070
8   CALL LNCNT(3)                                                    BAR00080
9   PRINT 50                                                       BAR00090
10  50 FORMAT(//,' LINEAR EQUATION SOLVER    AX + XB = C ')        BAR00100
11  CALL PRNT(A,NA,4H A ,1)                                         BAR00110
12  CALL PRNT(B,NB,4H B ,1)                                         BAR00120
13  GO TO 200                                                       BAR00130
14  100 CONTINUE                                                 BAR00140
15  CALL LNCNT(3)                                                    BAR00150
16  PRINT 150                                                       BAR00160
17  150 FORMAT(//,' LINEAR EQUATION SOLVER ( B TRANSPOSE )X + XB = C')  BAR00170
18  CALL TRANP(A,NA,DUMMY,NDUM1)                                     BAR00180
19  CALL PRNT(DUMMY,NDUM1,4H B ,1)                                     BAR00190
20  200 CONTINUE                                                 BAR00200
21  CALL PRNT(C,NC,4H C ,1)                                         BAR00210
22  C
23  250 CONTINUE                                                 BAR00220
24  CALL EQUATE(A,NA,DUMMY,NDUM1)                                     BAR00230
25  N1=(NA(1)**2)+1                                              BAR00240
26  N2=N1+NA(1)-1                                              BAR00250
27  DO 300 I=N1,N2                                              BAR00260
28  DUMMY(I)=0.0                                                 BAR00270
29  300 CONTINUE                                                 BAR00280
30  C
31  NDUM1(2)=NDUM1(2)+1                                         BAR00290
32  NDUM2(1)=1                                                 BAR00300
33  NDUM2(2)=NDUM1(2)                                         BAR00310
34  N1=NDUM1(1)*NDUM1(2)+1                                     BAR00320
35  CALL NULL(DUMMY(N1),NDUM2)                                    BAR00330
36  LU=(NA(1)+1)**2 + 1                                         BAR00340
37  CALL JUXTR(DUMMY,NDUM1,DUMMY(N1),NDUM2,DUMMY(LU),NDUM3)       BAR00350
38  CALL EQUATE(DUMMY(LU),NDUM3,DUMMY,NDUM1)                     BAR00360
39  N=NA(1)+1                                                 BAR00370
40  C
41  IF(SYM) GO TO 500                                           BAR00380
42  C
43  CALL EQUATE(B,NB,DUMMY(LU),NDUM2)                           BAR00390
44  M1=LU+NB(1)**2                                         BAR00400
45  M2=M1+NB(1)-1                                         BAR00410
46  DO430 I=M1,M2                                         BAR00420
47  DUMMY(I)=0.0                                             BAR00430
48  400 CONTINUE                                                 BAR00440
49  C
50  NDUM2(2)=NDUM2(2)+1                                         BAR00450
51  NDUM3(1)=1                                                 BAR00460
52  NDUM3(2)=NDUM2(2)                                         BAR00470
53  M1=NDUM2(1)*NDUM2(2)+LU                                     BAR00480
54  CALL NULL(DUMMY(M1),NDUM3)                                    BAR00490
55  M2=LU+(NB(1)+1)**2                                         BAR00500
56  CALL JUXTR(DUMMY(LU),NDUM2,DUMMY(M1),NDUM3,DUMMY(M2),NDUM4)  BAR00510
57  CALL EQUATE(DUMMY(M2),NDUM4,DUMMY(LU),NDUM2)                BAR00520
58  M=NB(1)+ 1                                               BAR00530
59  LNB = LU                                                 BAR00540
60  LU = LU + (NB(1)+1)**2                                     BAR00550
61  LV = LU + NA(1)**2                                         BAR00560
62  CALL AXPXR(DUMMY,DUMMY(LU),N(1),N,NA(1),DUMMY(LNB),DUMMY(LV),NB(1)  BAR00570
63  1,NB(1),C,NC(1),EPSA,EPSB,NFAIL)                         BAR00580
64

```

67 C GO TO 600 BAR00640
68 C 500 CONTINUE BAR00650
69 C CALL TRANP(DUMMY,NDUM1,DUMMY(LU),NDUM2) BAR00660
70 C CALL EQUATE(DUMMY(LU),NDUM2,DUMMY,NDUM1) BAR00670
71 C CALL ATXPXA(DUMMY,DUMMY(LU),C,NA(1),N,NA(1),NC(1),EPSA,NFAIL) BAR00680
72 C 600 CONTINUE BAR00690
73 C IF(NFAIL .EQ. 0) GO TO 700 BAR00700
74 C CALL LNCNT(3) BAR00710
75 C PRINT 650 BAR00720
76 C 650 FORMAT(//,' IN BARSTW, EITHER THE SUBROUTINE AXPXB OR ATXPXA WAS
77 C IS UNABLE TO REDUCE A OR B TO SCHUR FORM ') BAR00730
78 C RETURN BAR00740
79 C 700 CONTINUE BAR00750
80 C IF(IOP .NE. 0) CALL PRNT(C,NC,4H X ,1) BAR00760
81 C RETURN BAR00770
82 C END BAR00780
83 C BAR00790
84 C BAR00800
85 C BAR00810
86 C BAR00820
87 C BAR00830

```

1 SURROUTINE TESTSA(A,NA,ALPHA,DISC,STABLE,IOP,DUMMY) TES00010
2 IMPLICIT REAL*8 (A-H,O-Z) TES00020
3 DIMENSION A(1),DUMMY(1) TES00030
4 DIMENSION NA(2),NDUM1(2),NDUM2(2) TES00040
5 LOGICAL DISC,STABLE TES00050
6 STABLE = .FALSE. TES00060
7 C TES00070
8 CALL EQUATE(A,NA,DUMMY,NA) TES00080
9 N1= NA(1)**2 + 1 TES00090
10 N2= N1+NA(1) TES00100
11 N3= N2+NA(1) TES00110
12 ISV = 0 TES00120
13 CALL EIGEN(NA(1),NA(1),DUMMY,DUMMY(N1),DUMMY(N2),ISV,ISV,V,DUMMY(NTES00130
14 13),IERR) TES00140
15 NEVL = NA(1) TES00150
16 IF( IERR .EQ. 0 ) GO TO 200 TES00160
17 CALL LNCNT(4) TES00170
18 PRINT 100,IERR TES00180
19 100 FORMAT(//,' IN TESTSA, THE ',I5,' EIGENVALUE OF A HAS NOT BEEN FO TES00190
20 UND AFTER 30 ITERATIONS',//) TES00200
21 RETURN TES00210
22 C TES00220
23 200 CONTINUE TES00230
24 NDUM1(1) = NEVL TES00240
25 NDUM1(2) = 1 TES00250
26 C TES00260
27 CALL JUXTC(DUMMY(N1),NDUM1,DUMMY(N2),NDUM1,DUMMY,NDUM2) TES00270
28 C TES00280
29 IF( DISC ) GO TO 400 TES00290
30 DO 300 I=1,NEVL TES00300
31 IF( DUMMY(I) .GE. ALPHA ) GO TO 600 TES00310
32 300 CONTINUE TES00320
33 GO TO 550 TES00330
34 400 CONTINUE TES00340
35 N = NDUM2(1)*NDUM2(2)+1 TES00350
36 DO 500 I =1,NEVL TES00360
37 K = I + NEVL TES00370
38 L=N +I -1 TES00380
39 DUMMY(L) = DSQRT((DUMMY(I)**2)+(DUMMY(K)**2)) TES00390
40 500 CONTINUE TES00400
41 C TES00410
42 IF( DUMMY(L) .GE. ALPHA ) GO TO 600 TES00420
43 STABLE =.TRUE. TES00430
44 600 CONTINUE TES00440
45 IF( IOP .EQ. 0 ) RETURN TES00450
46 CALL LNCNT(4) TES00460
47 PRINT 700 TES00470
48 700 FORMAT(//,' PROGRAM TO TEST THE RELATIVE STABILITY OF THE MATRIX ATES00490
49 1',//) TES00500
50 CALL PRNT(A,NA,4H A ,1) TES00510
51 CALL LNCNT(4) TES00520
52 PRINT 750 TES00530
53 750 FORMAT(//,' EIGENVALUES OF A ',/) TES00540
54 CALL PRNT(DUMMY,NDUM2,4HEVLA,1) TES00550
55 IF( .NOT. DISC ) GO TO 850 TES00560
56 CALL LNCNT(4) TES00570
57 PRINT 800 TES00580
58 800 FORMAT(//,' MODULI OF EIGENVALUES OF A ',/) TES00590
59 CALL PRNT(DUMMY(N),NDUM1,4HMODA,1) TES00600
60 C TES00610
61 850 CONTINUE TES00620
62 CALL LNCNT(4) TES00630

```

```
1      IF(STABLE) PRINT 900,ALPHA          TES00640
64     IF( .NOT. STABLE) PRINT 950,ALPHA    TES00650
15     900 FORMAT(//,' MATRIX A IS STABLE RELATIVE TO ',E16.8,/)
16     950 FQRMAT(//,' MATRIX A IS UNSTABLE RELATIVE TO ',E16.8,/)
67 C
68     RETURN
69     END
```

```

1 SUBROUTINE EXPSER(A,NA,EXPA,NEXPA,T,IOP,DUMMY)
2 IMPLICIT REAL*8 (A-H,O-Z)
3 DIMENSION A(1),EXPA(1),DUMMY(1)
4 DIMENSION NA(2),NEXPA(2)
5 COMMON/COV/SUMCV,PICTCV,SERCV,MAXSUM
6
7 C
8 N = NA(1)
9 L = (N**2) + 1
10 TT = T
11 NEXPA(1)=NA(1)
12 NEXPA(2)=NA(2)
13 C
14 CALL MAXEL(A,NA,ANAA)
15 ANAA = ANAA*TT
16 ANAA = DARS(ANAA)
17 IF( ANAA .GT. 1.E-15 ) GO TO 100
18 CALL UNITY(EXPA,NEXPA)
19 GO TO 800
20 C
21 100 CONTINUE
22 IOPT=2
23 CALL NORMS(N,N,N,A,IOPT,ZERO)
24 ZERO=ZERO/(2.*47)
25 CALL TRCE(A,NA,TR)
26 TR = TR/N
27 DO 200 I =1,N
28 M = I+N*(I-1)
29 A(M) = A(M) - TR
30 200 CONTINUE
31 C
32 IOPT = 1
33 CALL NORMS(N,N,N,A,IOPT,COL)
34 IOPT = 3
35 CALL NORMS(N,N,N,A,IOPT,ROW)
36 ANORM = ROW
37 IF( ANORM .GT. COL ) ANORM = COL
38 TMAX = 1./ANORM
39 K=0
40 300 CONTINUE
41 IF( TMAX = TT ) 325,350,350
42 325 CONTINUE
43 K=K+1
44 TT = T/(P**K)
45 IF( K = 1000 ) 300,700,700
46 350 CONTINUE
47 SC = TT
48 CALL SCALE(A,NA,A,NA,TT)
49 CALL UNITY(EXPA,NEXPA)
50 II = 2
51 CALL ADD(A,NA,EXPA,NEXPA,DUMMY,NA)
52 CALL EQUATE(A,NA,DUMMY(L),NA)
53 400 CONTINUE
54 CALL MULT(A,NA,DUMMY(L),NA,EXPA,NEXPA)
55 S = 1./II
56 CALL SCALE(EXPA,NEXPA,DUMMY(L),NA,S)
57 CALL ADD(DUMMY(L),NA,DUMMY,NA,EXPA,NEXPA)
58 CALL MAXEL(DUMMY,NA,TOT)
59 CALL MAXEL(DUMMY(L),NA,DELT)
60 IF( TOT .GT. 1.0 ) GO TO 500
61 IF( DELT/TOT .LT. SERCV ) GO TO 600
62 GO TO 550
63 500 CONTINUE
64 IF( DELT .LT. SERCV ) GO TO 600

```

EXP0001
EXP0002
EXP0003
EXP0004
EXP0005
EXP0006
EXP0007
EXP0008
EXP0009
EXP0010
EXP0011
EXP0012
EXP0013
EXP0014
EXP0015
EXP0016
EXP0017
EXP0018
EXP0019
EXP0020
EXP0021
EXP0022
EXP0023
EXP0024
EXP0025
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EXP0038
EXP0039
EXP0040
EXP0041
EXP0042
EXP0043
EXP0044
EXP0045
EXP0046
EXP0047
EXP0048
EXP0049
EXP0050
EXP0051
EXP0052
EXP0053
EXP0054
EXP0055
EXP0056
EXP0057
EXP0058
EXP0059
EXP0060
EXP0061
EXP0062
EXP0063

```

550 CONTINUE EXP00640
CALL EQUATE(EXPA,NEXPA,DUMMY,NA) EXP00650
65 II = II + 1 EXP00660
GO TO 400 EXP00670
C EXP00680
68 600 CONTINUE EXP00690
69 IF( K ) 625,675,650 EXP00700
70 625 CONTINUE EXP00710
71 CALL LNCNT(1) EXP00720
72 PRINT 635 EXP00730
73 635 FORMAT(' ERROR IN EXPSER, K IS NEGATIVE ') EXP00740
74 RETURN EXP00750
75 C EXP00760
76 650 CONTINUE EXP00770
77 DO 660 I = 1,K EXP00780
78 TT = 2*TT EXP00790
79 CALL EQUATE(EXPA,NEXPA,DUMMY,NA) EXP00800
80 CALL EQUATE(DUMMY,NA,DUMMY(L),NA) EXP00810
81 CALL MULT(DUMMY(L),NA,DUMMY,NA,EXPA,NEXPA) EXP00820
82 660 CONTINUE EXP00830
83 T = TT EXP00840
84 675 CONTINUE EXP00850
85 S = 1./SC EXP00860
86 CALL SCALE(A,NA,A,NA,S) EXP00870
87 DO 685 I = 1,N EXP00880
88 M = I + N*(I-1) EXP00890
89 A(M) = A(M) + TR EXP00900
90 IF( DABS(A(M)) .LE. ZERO ) A(M) = 0.0 EXP00910
91 685 CONTINUE EXP00920
92 C EXP00930
93 TR=TR*T EXP00940
94 S = DEXP(T) EXP00950
95 CALL SCALE(EXPA,NEXPA,EXPA,NEXPA,S) EXP00960
96 GO TO 800 EXP00970
97 C EXP00980
98 700 CONTINUE EXP00990
99 CALL LNCNT(1) EXP01000
100 PRINT 750 EXP01010
11 750 FORMAT(' ERROR IN EXPSER, K = 1000 ') EXP01020
12 RETURN EXP01030
103 C EXP01040
104 800 CONTINUE EXP01050
105 IF( IOP .EQ. 0 ) RETURN EXP01060
106 CALL LNCNT(4) EXP01070
107 PRINT 825 EXP01080
108 825 FORMAT(// ' COMPUTATION OF THE MATRIX EXPONENTIAL EXP(A T) BY THE EXP0109
109 SERIES METHOD ') EXP01100
110 CALL PRNT(A,NA,4H A ,1) EXP01110
111 CALL LNCNT(3) EXP01120
112 PPINT 850,T EXP01130
113 850 FORMAT(// ' T = ',016.8/) EXP01140
114 CALL PRNT(EXPA,NEXPA,4H EXPA,1) EXP01150
115 RETURN EXP01160
116 END EXP01170

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0_      SUBROUTINE EXPADE (MAX, N, A, EA, IDIG, WK, IERR)          EXP00010
1_      IMPLICIT REAL*8 (A-H,O-Z)                                EXP00020
2_      DIMENSION A(MAX,N),EA(MAX,N),WK(N,1),C(9)                EXP00030
3_      REAL*4 SDIGC,ALOG10                                         EXP00040
4_      IERR = 0                                                 EXP00050
5_      C      CALCULATE NORM OF A                               EXP00060
6_      ANORM = 0.                                              EXP00070
7_      DO 10 I=1,N                                           EXP00080
8_      S = 0.                                                 EXP00090
9_      DO 5 J=1,N                                           EXP00100
10_     S = S + DABS(A(I,J))                                 EXP00110
11_     5      CONTINUE                                     EXP00120
12_     IF (S .GT. ANORM) ANORM = S                         EXP00130
13_     10     CONTINUE                                     EXP00140
14_     C *****
15_     C      CALCULATE ACCURACY ESTIMATE                  EXP00150
16_     C *****
17_     DIGC = 24.*DFLOAT(N)                                EXP00160
18_     IF (ANORM .GT. 1.) DIGC = DIGC*ANORM                 EXP00170
19_     SDIGC=DIGC                                         EXP00180
20_     IDIG = 15 - IFIX(ALOG10(SDIGC) + .5)               EXP00190
21_     C      DETERMINE POWER OF TWO AND NORMALIZATION FACTOR EXP00200
22_     C *****
23_     M = 0                                                 EXP00210
24_     IF (ANORM .LE. 1.) GO TO 27                         EXP00220
25_     FACTOR = 2.                                         EXP00230
26_     DO 15 M=1,46                                         EXP00240
27_     IF (ANORM .LE. FACTOR) GO TO 20                     EXP00250
28_     FACTOR = FACTOR*2.                                   EXP00260
29_     15     CONTINUE                                     EXP00270
30_     GO TO 125                                         EXP00280
31_     20     CONTINUE                                     EXP00290
32_     C *****
33_     C      NORMALIZE MATRIX                           EXP00300
34_     C *****
35_     DO 25 I=1,N                                         EXP00310
36_     DO 25 J=1,N                                         EXP00320
37_     A(I,J) = A(I,J)/FACTOR                            EXP00330
38_     25     CONTINUE                                     EXP00340
39_     27     CONTINUE                                     EXP00350
40_     C *****
41_     C      SET COEFFICIENTS FOR (9,9) PADE TABLE ENTRY EXP00360
42_     C *****
43_     C(1) = .5                                         EXP00370
44_     C(2) = 1.1764705882352D-01                         EXP00380
45_     C(3) = 1.7156862745098D-02                         EXP00390
46_     C(4) = 1.7156862745098D-03                         EXP00400
47_     C(5) = 1.2254901960784D-04                         EXP00410
48_     C(6) = 6.2845651080945D-06                         EXP00420
49_     C(7) = 2.2444875386051D-07                         EXP00430
50_     C(8) = 5.1011080422845D-09                         EXP00440
51_     C(9) = 5.6678978247605D-11                         EXP00450
52_     C *****
53_     C      CALCULATE PADE NUMERATOR AND DENOMINATOR BY COLUMNS EXP00460
54_     C *****
55_     NP1 = N+1                                         EXP00470
56_     NP7 = N+7                                         EXP00480
57_     DO 95 J=1,N                                         EXP00490
58_     C *****
59_     C      COMPUTE JTH COLUMN OF FIRST NINE POWERS OF A EXP00500
60_     C *****
61_     DO 35 I=1,N                                         EXP00510
62_     S = 0.                                              EXP00520

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3      DO 30 L=1,N          EXP00640
4      S = S + A(I,L)*A(L,J)  EXP00650
5 30    CONTINUE            EXP00660
6 66    WK(I,NP1) = S      EXP00670
7 35    CONTINUE            EXP00680
8      DO 45 K=NP1,NP7      EXP00690
9      KP1 = K+1             EXP00700
0      DO 45 I=1,N           EXP00710
1      S = 0.                EXP00720
2 72    DO 40 L=1,N           EXP00730
3      S = S + A(I,L)*WK(L,K) EXP00740
4 40    CONTINUE            EXP00750
5      WK(I,KP1) = S        EXP00760
6 45    CONTINUE            EXP00770
7 C *****
8 C    COLLECT TERMS FOR JTH COLUMN OF NUMERATOR AND DENOMINATOR EXP00790
9 C *****
10     DO 85 I=1,N           EXP00800
11     S = 0.                EXP00810
12     U = 0.                EXP00820
13     DO 65 L=1,8           EXP00830
14     K = N+9-L              EXP00840
15     KN1 = K-N+1            EXP00850
16     P = C(KN1)*WK(I,K)    EXP00860
17     S = S + P              EXP00870
18     IEO = MOD(KN1,2)       EXP00880
19     IF (IEO,EQ.0) GO TO 55 EXP00890
20     U = U + P              EXP00900
21     GO TO 65               EXP00910
22 55    CONTINUE            EXP00920
23     U = U + P              EXP00930
24 65    CONTINUE            EXP00940
25     P = C(1)*A(I,J)       EXP00950
26     S = S + P              EXP00960
27     U = U - P              EXP00970
28     IF (I .NE. J) GO TO 80 EXP00980
29     S = S + 1.              EXP00990
30     U = U + 1.              EXP01000
31 80    CONTINUE            EXP01010
32     EA(I,J) = S            EXP01020
33     WK(I,J) = U            EXP01030
34 85    CONTINUE            EXP01040
35 95    CONTINUE            EXP01050
36 C *****
37 C    CALCULATE NORMALIZED EXP(A) BY WK * EXP(A) = EA            EXP01060
38 C *****
39     CALL GAUSEL (MAX,N,WK,N,EA,IERR)                         EXP01070
40     IF (IERR .NE. 0) GO TO 130                                EXP01080
41     IF (M .EQ. 0) GO TO 130                                EXP01090
42 C *****
43 C    TAKE OUT EFFECT OF NORMALIZATION ON EXP(A)                 EXP01100
44 C *****
45     DO 120 K=1,M          EXP01110
46     DO 110 I=1,N           EXP01120
47     DO 110 J=1,N           EXP01130
48     S = 0.                EXP01140
49     DO 105 L=1,N           EXP01150
50     S = S + EA(I,L)*EA(L,J)         EXP01160
51 105    CONTINUE            EXP01170
52     WK(I,J) = S            EXP01180
53 110    CONTINUE            EXP01190
54     DO 115 I=1,N           EXP01200
55     DO 115 J=1,N           EXP01210

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76	EAC(I,J) = WK(I,J)	EXP01270
77	115 CONTINUE	EXP01280
78	120 CONTINUE	EXP01290
29 C ****		EXP01300
0 C	UN-NORMALIZE A	EXP01310
-1 C ****		EXP01320
32	DO 122 I=1,N	EXP01330
33	DO 122 J=1,N	EXP01340
4	A(I,J) = A(I,J)*FACTOR	EXP01350
35	122 CONTINUE	EXP01360
36	GO TO 130.	EXP01370
7 C ****		EXP01380
38 C	NORM OF A IS EXCESSIVE	EXP01390
39 C ****		EXP01400
0	125 CONTINUE	EXP01410
1	IERR = 1	EXP01420
42 C ****		EXP01430
"3 C	EXIT ROUTINE	EXP01440
14 C ****		EXP01450
45	130 CONTINUE	EXP01460
46	RETURN	EXP01470
17	END	EXP01480

```

0      SUBROUTINE EXPINT(A,NA,B,NB,C,NC,T,IOP,DUMMY)          EXP0C
1      IMPLICIT REAL*8 (A-H,O-Z)                          EXP0C
2      DIMENSION A(1),B(1),C(1),DUMMY(1)                  EXP00
3      DIMENSION NA(2),NB(2),NC(2)                      EXP00
4      COMMON/CONV/SUMCV,RICTCV,SERCV,MAXSUM             EXP00
5      N = NA(1)                                         EXP00
6      L = (N**2)+1                                     EXP00
7      NC(1) = NA(1)                                    EXP00
8      NC(2) = NA(2)                                    EXP00
9      NB(1) = NA(1)                                    EXP00
10     NB(2) = NA(2)                                    EXP00
11     TT = T                                         EXP00
12 C
13     IOP = 1                                         EXP00
14     CALL NORMS(N,N,N,A,IOPP,COL)                   EXP00
15     IOP = 3                                         EXP00
16     CALL NORMS(N,N,N,A,IOPP,ROW)                   EXP00
17     ANAA = COL                                      EXP00
18     IF( ANAA .GT. ROW ) ANAA = ROW                EXP00
19     TMAX = 1./ANAA                                 EXP00
20     K = 0                                         EXP00
21     100 CONTINUE
22     IF( TMAX = TT ) 125,150,150                  EXP00
23     125 CONTINUE
24     K = K + 1                                     EXP00
25     TT = T/(2**K)                                EXP00
26     IF( K = 1000 ) 100,600,600                  EXP00
27 C
28     150 CONTINUE
29     SC = TT                                      EXP00
30     CALL SCALE(A,NA,A,NA,TT)                     EXP00
31     CALL UNITY(B,NB)                            EXP00
32     CALL SCALE(B,NB,DUMMY,NB,TT)                 EXP00
33     S = TT/2.                                     EXP00
34     CALL SCALE(A,NA,DUMMY(L),NA,S)              EXP00
35     II = 2                                       EXP00
36     CALL ADD(DUMMY,NA,DUMMY(L),NA,DUMMY(L),NA)   EXP00
37     CALL ADD(A,NA,B,NB,DUMMY,NA)                 EXP00
38     CALL EQUATE(A,NA,C,NC)                      EXP00
39     200 CONTINUE
40     CALL MULT(A,NA,C,NC,B,NB)                   EXP00
41     S = 1./II                                    EXP00
42     CALL SCALE(B,NB,C,NC,S)                     EXP00
43     CALL MAXEL(DUMMY,NA,TOT)                   EXP00
44     CALL MAXEL(C,NC,DELT)                      EXP00
45     IF( TOT .GT. 1.0 ) GO TO 300               EXP00
46     IF( DELT/TOT .LT. SERCV ) GO TO 400       EXP00
47     GO TO 350
48     300 CONTINUE
49     IF( DELT .LT. SERCV ) GO TO 400           EXP00
50     350 CONTINUE
51     S = TT/(II + 1)                           EXP00
52     CALL SCALE(C,NC,B,NB,S)                   EXP00
53     CALL ADD(B,NB,DUMMY(L),NB,DUMMY(L),NB)    EXP00
54     CALL ADD(C,NC,DUMMY,NC,DUMMY,NC)          EXP00
55     II = II + 1                               EXP00
56     GO TO 200
57 C
58     400 CONTINUE
59     CALL EQUATE(DUMMY,NB,B,NB)                 EXP00
60     IF( K ) 425,500,450                         EXP00
61     425 CONTINUE
62     CALL LNCNT(1)                                EXP00

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```

3      PRINT 435                                EXP00640
4      435 FORMAT(' ERROR IN EXPINT, K IS NEGATIVE')
5      RETURN
6 C
7      450 CONTINUE
8      DO 475 J = 1,K
9      TT = 2*TT
0      CALL EQUATE(B,NB,DUMMY,NB)
1      CALL MULT(DUMMY,NA,DUMMY(L),NA,C,NC)
2      CALL ADD(DUMMY(L),NC,C,NC,DUMMY(L),NC)
3      CALL MULT(DUMMY,NB,DUMMY,NB,B,NB)
4      475 CONTINUE
5      T = TT
6 C
7      500 CONTINUE
8      CALL EQUATE(DUMMY(L),NC,C,NC)
9      S = 1./SC
10     CALL SCALE(A,NA,A,NA,S)
11 C
82     IF( IOP .EQ. 0 ) RETURN
93     CALL LNCNT(5)
34     PRINT 550
85     550 FORMAT(//,' COMPUTATION OF THE MATRIX EXPONENTIAL EXP(A*T)',/,' AEXP0086
86     AND ITS INTEGRAL OVER (0,T) BY THE SERIES METHOD ',/,' EXP0087
37     CALL PRNT(A,NA,4H A, ,1)                  EXP0088
38     CALL LNCNT(3)                            EXP0089
89     PRINT 575, T                           EXP0090
70     575 FORMAT( /,' T = ',D16.8,/,' EXP0091
71     CALL PRNT(B,NB,4HEXPB,1)                 EXP0092
92     CALL PRNT(C,NC,4HINT ,1)                 EXP0093
93     RETURN
74 C
75     600 CONTINUE
96     CALL LNCNT(1)
77     PRINT 650
78     650 FORMAT( ' ERROR IN EXPINT, K > 1000 ')
99     RETURN
70 C
71     END

```

```

0   SUBROUTINE VARANC(A,NA,G,NG,Q,NQ,W,NW,IDENT,DISC,IOP,DUMMY)      VAR0001
1   IMPLICIT REAL*8 (A-H,O-Z)                                         VAR0002
2   DIMENSION A(1),G(1),Q(1),W(1),DUMMY(1)                           VAR0003
3   DIMENSION NA(2),NG(2),NQ(2),NW(2),NDUM1(2),IOP(3),IOPT(2)        VAR0004
4   LOGICAL IDENT,DISC,SYM                                         VAR0005
5   COMMON/TOL/EPSAM,EPSBM,IACM                                     VAR0006
6   IF( IOP(1) .EQ. 0 ) GO TO 100                                     VAR0007
7   CALL LNCNT(5)                                                 VAR0008
8   IF( DISC ) PRINT 25                                              VAR0009
9   IF( .NOT. DISC ) PRINT 35                                         VAR0010
10  25 FORMAT(//,' PROGRAM TO SOLVE FOR THE STEADY-STATE VARIANCE MATRIX'VAR0011
11  1,' FOR A LINEAR DISCRETE SYSTEM',//)                            VAR0012
12  35 FORMAT(//,' PROGRAM TO SOLVE FOR THE STEADY-STATE VARIANCE MATRIX'VAR0013
13  1,' FOR A LINEAR CONTINUOUS SYSTEM',//)                          VAR0014
14  CALL PRNT(A,NA,4H A ,1)                                         VAR0015
15  IF( .NOT. IDENT ) GO TO 55                                       VAR0016
16  CALL LNCNT(3)                                                 VAR0017
17  PRINT 45                                                 VAR0018
18  45 FORMAT(//,' G IS AN IDENTITY MATRIX',//)                      VAR0019
19  GO TO 65                                                 VAR0020
20  55 CONTINUE                                              VAR0021
21  CALL PRNT(G,NG,4H G ,1)                                         VAR0022
22  65 CONTINUE                                              VAR0023
23  IF( .NOT. IDENT ) GO TO 85                                       VAR0024
24  CALL LNCNT(3)                                                 VAR0025
25  PRINT 75                                                 VAR0026
26  75 FORMAT(//,' INTENSITY MATRIX FOR COVARIANCE OF PROCESS NOISE',//) VAR0027
27 C
28  85 CONTINUE                                              VAR0028
29  CALL PRNT(Q,NQ,4H Q ,1)                                         VAR0029
30 C
31  100 CONTINUE                                             VAR0030
32  IF( IDENT ) GO TO 200                                         VAR0031
33  CALL MULT(G,NG,Q,NQ,DUMMY,NG)                                VAR0032
34  N1 = NG(1)*NG(2) + 1                                         VAR0033
35  CALL TRANP(G,NG,DUMMY(N1),NDUM1)                             VAR0034
36  CALL MULT(DUMMY,NG,DUMMY(N1),NDUM1,Q,NQ)                   VAR0035
37 C
38  IF( IOP(1) .EQ. 0 ) GO TO 200                               VAR0036
39  CALL LNCNT(3)                                                 VAR0037
40  PRINT 75                                                 VAR0038
41  CALL PRNT(Q,NQ,4HGGGT,1)                                         VAR0039
42 C
43  200 CONTINUE                                              VAR0040
44  IF(.NOT. DISC) CALL SCALE(W,NW,W,NW,-1.0)                  VAR0041
45  IOPT(1) = IOP(2)                                         VAR0042
46  IOPT(2) = 1                                               VAR0043
47  SYM = .TRUE.                                              VAR0044
48  IF( DISC ) GO TO 300                                         VAR0045
49  IF( IOP(3) .EQ. 0 ) GO TO 250                               VAR0046
50  CALL BILIN(A,NA,A,NA,W,NW,IOP,BETA,SYM,DUMMY)             VAR0047
51  GO TO 400                                                 VAR0048
52 C
53  250 CONTINUE                                              VAR0049
54  CALL BARSTW(A,NA,A,NA,W,NW,IOPT,SYM,EPSA,EPSA,DUMMY)       VAR0050
55  GO TO 400                                                 VAR0051
56 C
57  300 CONTINUE                                              VAR0052
58  CALL EQUATE(A,NA,DUMMY,NA)                                 VAR0053
59  N = NA(1)**2                                         VAR0054
60  N1 = N + 1                                           VAR0055
61  CALL TRANP(A,NA,DUMMY(N1),NA)     A-50                 VAR0056
62  N2 = N1 + N                                         VAR0057

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63    CALL SUM(DUMMY,NA,W,NW,DUMMY(N1),NA,IOP,T,SYM,DUMMY(N2))      VAR006t
64 C
65 400 CONTINUE
66 IF( IOP(1) .EQ. 0 ) RETURN
67 CALL LNCNT(3)
68 PRINT 450
69 450 FORMAT( /, * VARIANCE MATRIX *, / )
70 CALL PRNT(W,NW,4H W ,1)
71 C
72 RETURN
73 END
```

```

0      SUBROUTINE CTROL(A,NA,R,NB,C,NC,IOP,IAC,IRANK,DUMMY)          CTR000
1      IMPLICIT REAL*8 (A-F,O-Z)                                     CTR000
2      DIMENSION A(1),B(1),C(1),DUMMY(1)                            CTR000
3      DIMENSION NA(2),NB(2),NC(2),NV(2),IOP(5)                      CTR000
4      N = NA(1)*NB(2)                                              CTR000
5      N1 = N+1                                                       CTR000
6      N2 = N1+N                                                       CTR000
7      K = NA(1)-1                                                 CTR000
8      J = 1                                                       CTR000
9      C
10     CALL EQUATE(B,NB,DUMMY(N2),NV)                                CTR001
11     CALL EQUATE(B,NB,DUMMY,NB)                                    CTR001
12 100  CONTINUE
13     CALL MULT(A,NA,DUMMY,NB,DUMMY(N1),NB)                         CTR001
14     CALL JUXT(C,DUMMY(N2),NV,DUMMY(N1),NB,C,NC)                  CTR001
15  C
16     IF( J .EQ. K ) GO TO 200                                     CTR001
17  C
18     CALL EQUATE(DUMMY(N1),NB,DUMMY,NB)                            CTR001
19     CALL EQUATE(C,NC,DUMMY(N2),NV)                                CTR002
20     J = J + 1                                                 CTR002
21     GO TO 100                                                 CTR002
22  C
23  200  CONTINUE
24  C
25     IF(IOP(1) .EQ. 0 ) GO TO 300                               CTR002
26     CALL PRNT(A,NA,4H A ,1)                                     CTR002
27     CALL PRNT(B,NB,4H B ,1)                                     CTR002
28     CALL LNCNT(4)                                              CTR002
29     PRINT 250                                                 CTR003
30  250  FORMAT(//,' THE MATRIX C IS THE CONTROLLABILITY MATRIX FOR THE A/CTR003
31  1B PAIR',/)
32     CALL PRNT(C,NC,4H C ,1)                                     CTR003
33  C
34  300  IF( IOP(2) .EQ. 0 ) RETURN                               CTR003
35     NOS = 0                                                       CTR003
36     IOPT = 2                                                       CTR003
37     K = NC(2)                                              CTR003
38     NC(2) = NB(2)*(NA(2)-NB(2)+1)                            CTR003
39     N = NC(1)*NC(2)                                         CTR004
40     CALL TRANP(C,NC,DUMMY,NV)                                 CTR004
41     NC(2) = K                                                 CTR004
42     N1 = N + 1                                              CTR004
43     N2 = N1 + NV(2)                                         CTR004
44     CALL SNVDEC(IOPT,NV(1),NV(2),NV(1),NV(2),DUMMY,NOS,B,IAC,ZTEST,DUMCTR004
45     !MY(N1),DUMMY(N2),IRANK,A,IERR)                           CTR004
46     IF( IERR .EQ. 0 ) GO TO 340                               CTR004
47     CALL LNCNT(5)                                              CTR004
48     IF( IERR .GT. 0 ) PRINT 310,IERR                          CTR004
49     IF( IERR .EQ. -1 ) PRINT 320,ZTEST,IRANK                 CTR005
50  310  FORMAT(//,' IN CTROL, SNVDEC HAS FAILED TO CONVERGE TO THE ',I4, 'CTR005
51     1 SINGULAR VALUE AFTER 30 ITERATIONS ')                  CTR005
52  320  FORMAT(//,' IN CTROL, THE MATRIX SUBMITTED TO SNVDEC USING ZTEST =CTR005
53     1',D16.8,' IS CLOSE TO A MATRIX WHICH IS OF LOWER RANK',/,,' IF THE CTR005
54     2ACCURACACY IS REDUCED THE RANK MAY ALSO BE REDUCED',/,,' CURRENT RACTR005
55     3NK =',I4)                                              CTR005
56     IF( IEPR .GT. 0 ) RETURN                                CTR005
57  C
58  340  CONTINUE
59     IF( IOP(3) .EQ. 0 ) GO TO 400                          CTR005
60     CALL LNCNT(6)                                              CTR006
61     PRINT 350,ZTEST,IRANK                                  CTR006
62  350  FORMAT(//,' BASED ON THE ZERO-TEST ',D16.8,' THE RANK OF THE CONTRCTR006

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63      10LLABILITY MATRIX IS ',I4,/, ' THE SINGULAR VALUES ARE ',/ ) CTR0
64      IOPT = 0 CTR0
65      NV(1)= NV(2) CTR0
66      NV(2)= 1 CTR0
67      CALL PRNT(DUMMY(N1),NV,IOPT,3) CTR0
68 C CTR0
69      400 IF( IOP(4) .EQ. 0 ) RETURN CTR0
70      N = NA(1)**2 CTR0
71      CALL EQUATE(DUMMY(N2),NA,DUMMY,NA) CTR0
72      N1 = N + 1 CTR0
73      N2 = N1 + N CTR0
74      CALL MULT(A,NA,DUMMY,NA,DUMMY(N1),NA) CTR0
75      CALL TRANP(DUMMY,NA,DUMMY(N2),NA) CTR0
76      CALL EQUATE(DUMMY(N2),NA,DUMMY,NA) CTR0
77      CALL MULT(DUMMY,NA,DUMMY(N1),NA,DUMMY(N2),NA) CTR0
78      CALL MULT(DUMM ,NA,B,NB,DUMMY(N1),NB) CTR0
79 C CTR0
80      IF( IOP(5) .EQ. 0 ) RETURN CTR0
81      CALL LNCNT(5) CTR0
82      PRINT 500 CTR0
83      500 FORMAT(//,' CONTROLLABILITY CANONICAL FORM ',/; ' (V TRANSPOSE) A CTR0
84      1 V') CTR0
85      CALL PRNT(DUMMY(N2),NA,IOPT,3) CTR0
86      CALL LNCNT(2) CTR0
87      PRINT 510 CTR0
88      510 FORMAT(//,' (V TRANSPOSE ) B ') CTR0
89      CALL PRNT(DUMMY(N1),NB,IOPT,3) CTR0
90      CALL LNCNT(2) CTR0
91      PRINT 520 CTR0
92      520 FORMAT(//,' V TRANSPOSE') CTR0
93      CALL PRNT(DUMMY,NA,IOPT,3) CTR0
94 C CTR0
95      RETURN CTR0
96      END CTR0

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0      SUBROUTINE TRANSIT(A,NA,B,NB,H,NH,G,NG,F,NF,V,NV,T,X,NX,DISC,STABL TRN001
1      1E,IOP,DUMMY) TRN001
2      IMPLICIT REAL*8 (A-H,O-Z) TRN001
3      DIMENSION A(1),B(1),H(1),G(1),F(1),V(1),X(1),DUMMY(1) TRN001
4      DIMENSION NA(2),NH(2),NG(2),NF(2),NV(2),NX(2),T(2),IOP(4) TRN001
5      DIMENSION NDUM1(2),NDUM2(2) TRN001
6      LOGICAL DISC,STABLE TRN001
7      N = NA(1)*NA(2) TRN001
8      N1 = N + 1 TRN001
9      N2 = N + N1 TRN001
10     N3 = N + N2 TRN001
11     N4 = N + N3 TRN001
12     N5 = N + N4 TRN001
13     N6 = N + N5 TRN001
14 C TRN001
15     CALL LNCNT(4) TRN001
16     IF(DISC) PRINT 100 TRN001
17     IF(.NOT. DISC) PRINT 120 TRN001
18     100 FORMAT(//,' COMPUTATION OF TRANSIENT RESPONSE FOR THE DIGITAL SYSTEM' TRN001
19     1EM ',') TRN002
20     120 FORMAT(//,' COMPUTATION OF TRANSIENT RESPONSE FOR THE CONTINUOUS TRN002
21     SYSTEM',) TRN002
22     CALL PRNT(A,NA,4H A ,1) TRN002
23     CALL PRNT(B,NB,4H B ,1) TRN002
24     IF( (IOP(1) .NE. 1) .AND. (IOP(1) .NE. 0) ) GO TO 180 TRN002
25     CALL LNCNT(3) TRN002
26     IF( IOP(1) .EQ. 0 ) PRINT 140 TRN002
27     IF( IOP(1) .EQ. 1 ) PRINT 160 TRN002
28     140 FORMAT(//,' H IS A NULL MATRIX ') TRN002
29     160 FORMAT(//,' H IS AN IDENTITY MATRIX ') TRN003
30     GO TO 200 TRN003
31     180 CONTINUE TRN003
32     CALL PRNT(H,NH,4H H ,1) TRN003
33     200 CONTINUE TRN003
34     IF( (IOP(2) .NE. 1) .AND. (IOP(2) .NE. 0) ) GO TO 260 TRN003
35     CALL LNCNT(3) TRN003
36     IF( IOP(2) .EQ. 0 ) PRINT 220 TRN003
37     IF( IOP(2) .EQ. 1 ) PRINT 240 TRN003
38     220 FORMAT(//,' G IS A NULL MATRIX') TRN003
39     240 FORMAT(//,' G IS AN IDENTITY MATRIX') TRN004
40     GO TO 280 TRN004
41     260 CONTINUE TRN004
42     CALL PRNT(G,NG,4H G ,1) TRN004
43     280 CONTINUE TRN004
44     CALL PRNT(F,NF,4H F ,1) TRN004
45     IF( (IOP(3) .NE. 0) .AND. (IOP(3) .NE. 1) ) GO TO 295 TRN004
46     CALL LNCNT(3) TRN004
47     IF(IOP(3).EQ.0) PRINT 285 TRN004
48     IF(IOP(3).EQ.1) PRINT 290 TRN004
49     285 FORMAT(//,' V IS A NULL MATRIX') TRN005
50     290 FORMAT(//,' V IS AN IDENTITY MATRIX') TRN005
51     GO TO 300 TRN005
52     295 CONTINUE TRN005
53     CALL PRNT(V,NV,4H V ,1) TRN005
54 C TRN005
55     300 CONTINUE TRN005
56     CALL EQUATE(A,NA,DUMMY(N6),NA) TRN005
57     CALL MULT(B,NB,F,NF,DUMMY,NA) TRN005
58     CALL SUBT(A,NA,DUMMY,NA,A,NA) TRN005
59 C TRN006
60     IF(DISC) GO TO 350 TRN006
61     NMAX = T(1)/T(2) A-54 TRN006
62     IOPT = 1 TRN006

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63      TT = T(2)                                TRN00
64      IF( IOP(3) .NE. 0 ) GO TO 315          TRN00
65      CALL EXPSER(A,NA,DUMMY,NA,TT,IOP,T,DUMMY(N1)) TRN00
66      GO TO 400                                TRN00
67      315 CONTINUE                            TRN00
68      CALL EXPINT(A,NA,DUMMY,NA,DUMMY(N1),NA,TT,IOP,T,DUMMY(N2)) TRN00
69      CALL MULT(DUMMY(N1),NA,B,NB,DUMMY(N2),NB)    TRN00
70      IF( IOP(3) .NE. 1 ) GO TO 325          TRN00
71      CALL EQUATE(DUMMY(N2),NB,DUMMY(N1),NX)     TRN00
72      GO TO 400                                TRN00
73      325 CONTINUE                            TRN00
74      CALL MULT(DUMMY(N2),NB,V,NV,DUMMY(N1),NX) TRN00
75      GO TO 400                                TRN00
76      350 CONTINUE                            TRN00
77      NMAX = IOP(4)                            TRN00
78      CALL EQUATE(A,NA,DUMMY,NA)               TRN00
79      IF( IOP(3) .EQ. 0 ) GO TO 400          TRN00
80      CALL MULT(B,NB,V,NV,DUMMY(N1),NX)     TRN00
81 C
82      400 CONTINUE                            TRN00
83      CALL LNCNT(4)                            TRN00
84      PRINT 420                               TRN00
85      420 FORMAT(//,' STRUCTURE OF PRINTING TO FOLLOW',//) TRN00
86      CALL LNCNT(6)                            TRN00
87      PRINT 440                               TRN00
88      440 FORMAT(' TIME OR STAGE ',//,' STATE - X TRANSPOSE - FROM DX = AX' TRN00
89      1 + BU',//,' OUTPUT - Y TRANSPOSE - FROM Y = HX + GU   IF DIFFERENT TRN00
90      2 FROM X',//,' CONTROL - U TRANSPOSE - FROM U = -FX + V',//) TRN00
91 C
92      K = 0                                  TRN00
93      L = 0                                  TRN00
94      CALL SCALE(F,NF,F,NF,-1.0)             TRN00
95 C
96      450 CONTINUE                            TRN00
97      IF( K .GT. NMAX ) GO TO 800          TRN00
98      CALL MULT(F,NF,X,NX,DUMMY(N2),NV)     TRN00
99      IF( IOP(3) .NE. 0 ) CALL ADD(DUMMY(N2),NV,V,NV,DUMMY(N2),NV) TRN010
100     CALL MULT(DUMMY,NA,X,NX,DUMMY(N3),NX) TRN010
01      IF( IOP(3) .EQ. 0 ) GO TO 475        TRN010
02      CALL ADD(DUMMY(N1),NX,DUMMY(N3),NX,DUMMY(N3),NX) TRN010
103     475 CONTINUE                            TRN010
04      IF( IOP(2) .EQ. 0 ) GO TO 525        TRN010
05      IF( IOP(2) .EQ. 1 ) GO TO 500        TRN010
06      CALL MULT(G,NG,DUMMY(N2),NV,DUMMY(N4),NDUM1) TRN010
107     GO TO 525                                TRN010
08      500 CONTINUE                            TRN010
09      CALL EQUATE(DUMMY(N2),NV,DUMMY(N4),NDUM1) TRN011
110     525 CONTINUE                            TRN011
11      IF( IOP(1) .EQ. 0 ) GO TO 575        TRN011
12      IF( IOP(1) .EQ. 1 ) GO TO 550        TRN011
113     CALL MULT(H,NH,X,NX,DUMMY(N5),NDUM1) TRN011
114     GO TO 575                                TRN011
15      550 CONTINUE                            TRN011
16      CALL EQUATE(X,NX,DUMMY(N5),NDUM1)     TRN011
117     575 CONTINUE                            TRN011
18      IF( IOP(2) .EQ. 0 ) GO TO 600        TRN011
19      IF( IOP(1) .EQ. 0 ) GO TO 700        TRN012
120     CALL ADD(DUMMY(N4),NDUM1,DUMMY(N5),NDUM1,DUMMY(N4),NDUM1) TRN012
121     GO TO 700                                TRN012
22      600 CONTINUE                            TRN012
123     IF( IOP(1) .NE. 0 ) CALL EQUATE(DUMMY(N5),NDUM1,DUMMY(N4),NDUM1) TRN012
124 C
25      700 CONTINUE                            TRN012

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126	CALL LNCNT(5)	TRN0
127	IF(.NOT. DISC) GO TO 720	TRN0
128	PRINT 710,K	TRN0
129	710 FORMAT(////,15)	TRN0
130	GO TO 740	TRN0
131	720 CONTINUE	TRN0
132	TIME=K*T(2)	TRN0
133	PRINT 730,TIME	TRN0
134	730 FORMAT(////,D16.7)	TRN0
135	740 CONTINUE	TRN0
136	CALL TRANP(X,NX,DUMMY(N5),NDUM2)	TRN0
137	CALL PRNT(DUMMY(N5),NDUM2,L,3)	TRN0
138	IF((IOP(2) .EQ. 0) .AND. ((IOP(1) .EQ. 0) .OR. (IOP(1) .EQ. 1))) GO TO 750	TRN0
139	1) CALL TRANP(DUMMY(N4),NDUM1,DUMMY(N5),NDUM2)	TRN0
140	CALL PRNT(DUMMY(N5),NDUM2,L,3)	TRN0
142	750 CONTINUE	TRN0
143	CALL TRANP(DUMMY(N2),NV,DUMMY(N5),NDUM2)	TRN0
144	CALL PRNT(DUMMY(N5),NDUM2,L,3)	TRN0
145 C	CALL EQUATE(DUMMY(N3),NX,X,NX)	TRN0
147	K = K + 1	TRN0
148	GO TO 450	TRN0
149 C		TRN0
150 C		TRN0
151	800 CONTINUE	TRN0
152	CALL SCALE(F,NF,F,NF,-1.0)	TRN0
153	IF(.NOT. STABLE .OR. IOP(3) .EQ. 0) GO TO 900	TRN0
154	IF(IOP(3) .EQ. 1) GO TO 820	TRN0
155	CALL MULT(B,NB,V,NV,DUMMY,NX)	TRN0
156	GO TO 840	TRN0
157	820 CONTINUE	TRN0
158	CALL EQUATE(B,NB,DUMMY,NX)	TRN0
159	840 CONTINUE	TRN0
160	IF(.NOT. DISC) GO TO 860	TRN0
161	CALL UNITY(DUMMY(N1),NA)	TRN0
162	CALL SUBT(DUMMY(N1),NA,A,NA,A,NA)	TRN0
163	860 CONTINUE	TRN0
164	IFAC = 0	TRN0
165	CALL GELIM(NA(1),NA(1),A,NX(2),DUMMY,DUMMY(N1),IFAC,DUMMY(N2),IERR)	TRN0
166	1)	TRN0
167	IF(IERR .EQ. 0) GO TO 880	TRN0
168	CALL LNCNT(3)	TRN0
169	IF(.NOT. DISC) PRINT 865	TRN0
170	IF(DISC) PRINT 870	TRN0
171	865 FORMAT(//," IN TRNSIT, THE MATRIX A-BF SUBMITTED TO GELIM IS SINGU")	TRN0
172	"LAR")	TRN0
173	870 FORMAT(//," IN TRNSIT, THE MATRIX I - (A-BF) SUBMITTED TO GELIM IT")	TRN0
174	"S SINGULAR")	TRN0
175	GO TO 900	TRN0
176	880 CONTINUE	TRN0
177	IF(.NOT. DISC) CALL SCALE(DUMMY,NX,DUMMY,NX,-1.0)	TRN0
78	CALL LNCNT(5)	TRN0
79	PRINT 890	TRN0
180	890 FORMAT(////," STEADY-STATE VALUE OF X TRANSPOSE")	TRN0
81	CALL TRANP(DUMMY,NX,DUMMY(N5),NDUM2)	TRN0
82	CALL PRNT(DUMMY(N5),NDUM2,L,3)	TRN0
183 C	900 CONTINUE	TRN0
184	CALL EQUATE(DUMMY(N6),NA,A,NA)	TRN0
185	RETURN	TRN0
186 C	END	TRN0
187		TRN0
88		TRN0

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0   SUBROUTINE SAMPL(A,NA,B,NB,Q,NQ,R,NR,W,NW,T,IOP,DUMMY)      SAM001
1   IMPLICIT REAL*8 (A-H,O-Z)                                     SAM001
2   DIMENSION A(1),B(1),Q(1),R(1),W(1),DUMMY(1)                 SAM001
3   DIMENSION NA(2),NB(2),NQ(2),NR(2),NW(2),IOP(2),NDUM(2)       SAM001
4   COMMON/CONV/SUMCV,RICTCV,SERCV,MAXSUM                         SAM001
5   IF( IOP(1) .EQ. 0 ) GO TO 100                                 SAM001
6 C
7   IF( IOP(2) .EQ. 0 ) GO TO 50                                  SAM001
8 C
9   CALL LNCNT(5)                                                 SAM001
10  PRINT 25                                                    SAM001
11  25 FORMAT(//,' COMPUTATION OF WEIGHTING MATRICES FOR THE OPTIMAL SAMP' SAM001
12  1LED-DATA REGULATOR PROBLEM',//)                                SAM001
13  CALL PRNT(A,NA,4H A ,1)                                         SAM001
14  CALL PRNT(B,NB,4H B ,1)                                         SAM001
15  CALL LNCNT(3)                                                 SAM001
16  PRINT 35                                                    SAM001
17  35 FORMAT(',- CONTINUOUS PERFORMANCE INDEX WEIGHTING MATRICES',/) SAM001
18  CALL PRNT(Q,NQ,4H Q ,1)                                         SAM001
19  CALL PRNT(R,NR,4H R ,1)                                         SAM002
20  CALL LNCNT(3)                                                 SAM002
21  PRINT 45,T                                                   SAM002
22  45 FORMAT(/,,' SAMPLE TIME = ',D16.8,/)                      SAM002
23 C
24  GO TO 100                                                 SAM002
25 C
26  50 CONTINUE                                              SAM002
27  CALL LNCNT(8)                                                 SAM002
28  PRINT 75                                                    SAM002
29  75 FORMAT(//,' COMPUTATION OF THE RECONSTRUCTIBILITY GRAMIAN',//,' FOR' SAM003
30  1 THE (A,H) SYSTEM OVER THE INTERVAL (0,T) ',,' THE MATRIX Q IS (SAM003
31  2 H TRANSPOSE ) X H',//)                                       SAM003
32  CALL PRNT(A,NA,4H A ,1)                                         SAM003
33  CALL PRNT(Q,NQ,4H Q ,1)                                         SAM003
34  CALL LNCNT(3)                                                 SAM003
35  PRINT 85,T                                                   SAM003
36  85 FORMAT(/,,' T = ',D16.8,/)                      SAM003
37 C
38  100 CONTINUE                                              SAM003
39 C
40  N = NA(1)                                                 SAM004
41  L = ( N**2)                                               SAM004
42  N1 = L + 1                                               SAM004
43  N2 = N1 + L                                             SAM004
44  TT = T                                                   SAM004
45 C
46  IOPT = 1                                                 SAM004
47  CALL NORMS(N,N,N,A,IOPT,ANORM)                            SAM004
48  IOPT = 3                                                 SAM004
49  CALL NORMS(N,N,N,A,IOPT,ROWA)                            SAM005
50  IF( ANORM .GT. ROWA ) ANORM = ROWA                         SAM005
51 C
52  IF( ANORM .LE. 1.E-15 ) GO TO 900                         SAM005
53 C
54  TMAX = 1.0/ANORM                                         SAM005
55  K = 0                                                    SAM005
56 C
57  125 CONTINUE                                              SAM005
58  IF( TMAX - TT ) 150,150,200                               SAM005
59  150 CONTINUE                                              SAM006
60  K = K + 1                                               SAM006
61  TT = T/( 2**K)                                           SAM006
62  IF( K = 1000 ) 125,800,800                               SAM006

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63 C SAMOC
 64 200 CONTINUE SAMOC
 65 C SAMOC
 66 I = 0 SAMOC
 67 SC = TT SAMOC
 68 CALL SCALE(A,NA,A,NA,TT) SAMOC
 69 CALL SCALE(Q,NQ,Q,NQ,TT) SAMOC
 70 CALL EQUATE(Q,NQ,DUMMY,NQ) SAMOC
 71 C SAMOC
 72 IF(IOP(2) .NE. 0) GO TO 500 SAMOC
 73 C SAMOC
 74 225 CONTINUE SAMOC
 75 II = I + 2 SAMOC
 76 I = I + 1 SAMOC
 77 F = 1.0/II SAMOC
 78 CALL SCALE(A,NA,DUMMY(N1),NA,F) SAMOC
 79 CALL MULT(DUMMY,NA,DUMMY(N1),NA,DUMMY(N2),NA) SAMOC
 80 CALL TRANP(DUMMY(N2),NA,DUMMY(N1),NA) SAMOC
 81 CALL ADD(DUMMY(N1),NA,DUMMY(N2),NA,DUMMY,NA) SAMOC
 82 C SAMOC
 83 CALL MAXEL(Q,NQ,TOT) SAMOC
 84 CALL MAXEL(DUMMY,NA,DELT) SAMOC
 85 IF(TOT .GT. 1.0) GO TO 250 SAMOC
 86 IF(DELT/TOT .LT. SERCV) GO TO 300 SAMOC
 87 GO TO 275 SAMOC
 88 250 CONTINUE SAMOC
 89 IF(DELT .LT. SERCV) GO TO 300 SAMOC
 90 275 CONTINUE SAMOC
 91 CALL ADD(Q,NQ,DUMMY,NA,Q,NQ) SAMOC
 92 GO TO 225 SAMOC
 93 C SAMOC
 94 300 CONTINUE SAMOC
 95 C SAMOC
 96 IF(K .EQ. 0) GO TO 400 SAMOC
 97 N3 = N2 + L SAMOC
 98 G = 1.0 SAMOC
 99 IOPT = 0 SAMOC
 100 CALL EXPSER(A,NA,DUMMY,NA,G,IOPT,DUMMY(N1)) SAMOC
 101 C SAMOC
 102 350 CONTINUE SAMOC
 103 IF(K .EQ. 0) GO TO 400 SAMOC
 104 K = K-1 SAMOC
 105 C SAMOC
 106 CALL TRANP(DUMMY,NA,DUMMY(N1),NA) SAMOC
 107 CALL MULT(Q,NQ,DUMMY,NA,DUMMY(N2),NA) SAMOC
 108 CALL MULT(DUMMY(N1),NA,DUMMY(N2),NA,DUMMY(N3),NA) SAMOC
 109 CALL ADD(Q,NQ,DUMMY(N3),NA,Q,NQ) SAMOC
 110 CALL MULT(DUMMY,NA,DUMMY,NA,DUMMY(N1),NA) SAMOC
 111 CALL EQUATE(DUMMY(N1),NA,DUMMY,NA) SAMOC
 112 C SAMOC
 113 GO TO 350 SAMOC
 114 C SAMOC
 115 400 CONTINUE SAMOC
 116 S = 1.0/SC SAMOC
 117 CALL SCALE(A,NA,A,NA,S) SAMOC
 118 C SAMOC
 119 IF(IOP(1) .EQ. 0) RETURN SAMOC
 120 CALL PRNT(Q,NQ,4HGRAM,1) SAMOC
 121 RETURN SAMOC
 122 C SAMOC
 123 500 CONTINUE SAMOC
 124 CALL SCALE(B,NB,B,NB,TT) SAMOC
 125 N3 = N2 + L SAMOC

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126      N4 = N3 + L          SAM012
127      N5 = N4 + L          SAM012
128      N6 = N5 + L          SAM012
129 C
130      525 CONTINUE        SAM013
131      II = I + 2          SAM013
132      I = I + 1          SAM013
133      F = 1.0/II          SAM013
134      CALL SCALE(A,NA,DUMMY(N1),NA,F)    SAM013
135      CALL TRANP(DUMMY(N1),NA,DUMMY(N2),NA)  SAM013
136      CALL MULT(DUMMY,NA,DUMMY(N1),NA,DUMMY(N3),NA)  SJM013
137      CALL TRANP(DUMMY(N3),NA,DUMMY(N1),NA)  SJM013
138      CALL MULT(DUMMY,NA,B,NB,DUMMY(N5),NW)    SAM013
139      CALL ADD(DUMMY(N1),NA,DUMMY(N3),NA,DUMMY,NA)  SAM014
140      CALL SCALE(DUMMY(N5),NW,DUMMY(N1),NW,F)    SAM014
141      IF( I .NE. 1 ) GO TO 550    SAM014
142      CALL EQUATE(DUMMY(N1),NW,W,NW)    SJM012
143      CALL EQUATE(DUMMY(N1),NW,DUMMY(N6),NW)    SAM014
144      CALL ADD(Q,NQ,DUMMY,NQ,Q,NQ)    SAM014
145      GO TO 525    SAM014
146 C
147      550 CONTINUE        SAM014
148      CALL MULT(DUMMY(N2),NA,DUMMY(N6),NW,DUMMY(N5),NW)  SAM014
149      CALL ADD(DUMMY(N5),NW,DUMMY(41),NW,DUMMY(N1),NW)  SAM015
150      CALL TRANP(B,NA,DUMMY(N2),NDUM)    SAM015
151      CALL SCALE(DUMMY(N2),NDUM,DUMMY(N2),NDUM,F)    SAM015
152      CALL MULT(DUMMY(N2),NDUM,DUMMY(N6),NW,DUMMY(N3),NR)  SAM015
153      CALL TRANP(DUMMY(N3),NR,DUMMY(N5),NR)    SAM015
154      CALL ADD(DUMMY(N3),NR,DUMMY(N5),NR,DUMMY(N3),NR)  SAM015
155      CALL EQUATE(DUMMY(N1),NW,DUMMY(N6),NW)    SAM015
156      IF( I .NE. 2 ) GO TO 575    SAM015
157      CALL ADD(Q,NQ,DUMMY,NQ,Q,NQ)    SAM015
158      CALL ADD(W,NW,DUMMY(N1),NW,W,NW)    SAM015
159      CALL EQUATE(DUMMY(N3),NR,DUMMY(N4),NR)    SAM016
160      GO TO 525    SJM016
161 C
162      575 CONTINUE        SAM016
163      CALL MAXEL(Q,NQ,TOT)    SAM016
164      CALL MAXEL(DUMMY,NQ,DELT)  SAM016
165      IF( TOT .GT. 1.0 ) GO TO 580    SAM016
166      IF( DELT/TOT .LT. SERCV ) GO TO 585    SAM016
167      GO TO 595    SAM016
168 C
169      580 CONTINUE        SAM017
170      IF( DELT .LT. SERCV ) GO TO 585    SAM017
171      GO TO 595    SAM017
172 C
173      585 CONTINUE        SAM017
174      CALL MAXEL(DUMMY(N4),NR,TOT)    SAM017
175      CALL MAXEL(DUMMY(N3),NR,DELT)  SAM017
176      IF( TOT .GT. 1.0 ) GO TO 590    SAM017
177      IF( DELT/TOT .LT. SERCV ) GO TO 600    SAM017
178      GO TO 595    SAM017
179 C
180      590 CONTINUE        SAM018
181      IF( DELT .LT. SERCV ) GO TO 600    SAM018
182 C
183      595 CONTINUE        SAM018
184      CALL ADD(Q,NQ,DUMMY,NQ,Q,NQ)  SAM018
185      CALL ADD(W,NW,DUMMY(N1),NW,W,NW)  SAM018
186      CALL ADD(DUMMY(N4),NR,DUMMY(N3),NR,DUMMY(N4),NR)  SAM018
187      GO TO 525    SAM018
188 C

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189 600 CONTINUE          SAM01
190 IF( K .EQ. 0 ) GO TO 700 SAM01
191 G = 1.0                  SAM01
192 IOPT = 0                 SAM01
193 CALL EXPINT(A,NA,DUMMY,NA,DUMMY(N1),NA,G,IOPT,DUMMY(N2)) SAM01
194 CALL MULT(DUMMY(N1),NA,B,NB,DUMMY(N2),NB) SAM01
195 CALL EQUATE(DUMMY(N2),NB,DUMMY(N1),NB) SAM01
196 C
197 650 CONTINUE          SAM01
198 IF( K .EQ. 0 ) GO TO 700 SAM01
199 K = K - 1                SAM02
200 CALL MULT(Q,NQ,DUMMY,NA,DUMMY(N2),NA) SAM02
201 CALL TRANP(DUMMY,NA,DUMMY(N3),NA) SAM02
202 CALL MULT(DUMMY(N3),NA,DUMMY(N2),NA,DUMMY(N5),NA) SAM02
203 CALL MULT(Q,NQ,DUMMY(N1),NB,DUMMY(N2),NB) SAM02
204 CALL ADD(Q,NQ,DUMMY(N5),NA,Q,NQ) SAM02
205 CALL MULT(DUMMY(N3),NA,DUMMY(N2),NB,DUMMY(N5),NB) SAM02
206 CALL MULT(DUMMY(N3),NA,W,NW,DUMMY(N6),NW) SAM02
207 CALL ADD(DUMMY(N5),NW,DUMMY(N6),NW,DUMMY(N5),NW) SAM02
208 CALL TPANP(DUMMY(N1),NB,DUMMY(N6),NDUM) SAM02
209 CALL MULT(DUMMY(N6),NDUM,W,NW,DUMMY(N3),NR) SAM02
210 CALL ADD(W,NW,DUMMY(N5),NW,W,NW) SAM02
211 CALL MULT(DUMMY(N6),NDUM,DUMMY(N2),NB,DUMMY(N5),NR) SAM02
212 CALL ADD(DUMMY(N5),NR,DUMMY(N3),NR,DUMMY(N5),NR) SAM02
213 CALL TRANP(DUMMY(N3),NR,DUMMY(N6),NR) SAM02
214 CALL ADD(DUMMY(N5),NR,DUMMY(N6),NR,DUMMY(N6),NR) SAM02
215 CALL SCALE(DUMMY(N4),NR,DUMMY(N4),NR,2.0) SAM02
216 CALL ADD(DUMMY(N6),NR,DUMMY(N4),NR,DUMMY(N4),NR) SAM02
217 CALL MULT(DUMMY,NA,DUMMY(N1),NB,DUMMY(N3),NB) SAM02
218 CALL ADD(DUMMY(N3),NB,DUMMY(N1),NB,DUMMY(N1),NB) SAM02
219 CALL MULT(DUMMY,NA,DUMMY,NA,DUMMY(N3),NA) SAM02
220 CALL EQUATE(DUMMY(N3),NA,DUMMY,NA) SAM02
221 GO TO 650                SAM02
222 C
223 700 CONTINUE          SAM02
224 CALL SCALE(R,NR,R,NR,T) SAM02
225 CALL ADD(R,NR,DUMMY(N4),NR,R,NR) SAM02
226 CALL SCALE(W,NW,W,NW,2.0) SAM02
227 S = 1.0/SC                SAM02
228 CALL SCALE(A,NA,A,NA,S) SAM02
229 CALL SCALE(B,NB,B,NB,S) SAM02
230 IF( IOP(1) .EQ. 0 ) RETURN SAM02
231 C
232 CALL LNCNT(3)           SAM02
233 PRINT 750                SAM02
234 750 FORMAT( /, ' DISCRETE PERFORMANCE INDEX WEIGHTING MATRICES' , /) SAM02
235 CALL PRNT(Q,NQ,4H Q ,1) SAM02
236 CALL PRNT(W,NW,4H W ,1) SAM02
237 CALL PRNT(R,NR,4H R ,1) SAM02
238 RETURN                   SAM02
239 C
240 800 CONTINUE          SAM02
241 CALL LNCFNT(1)           SAM02
242 PRINT 850                SAM02
243 850 FORMAT( ' ERROR IN SAMPL , K = 1000' ) SAM02
244 RETURN                   SAM02
245 C
246 900 CONTINUE          SAM02
247 CALL SCALE(Q,NQ,R,NQ,T) SAM02
248 IF( IOP(2) .NE. 0 ) GO TO 925 SAM02
249 IF( IOP(1) .NE. 0 ) CALL PRNT(Q,NQ,4HGRAM,1) SAM02
250 RETURN                   SAM02
51 C

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252	925 CONTINUE	SAM02
253	CALL MULT(Q,NQ,B,NB,W,NW)	SAM02
254	CALL SCALE(W,NW,W,NW,T)	SAM02
255	CALL TRANP(B,NB,DUMMY,NDUM)	SAM02
256	CALL MULT(DUMMY,NDUM,W,NW,DUMMY(N1),NP)	SAM02
257	TT = T/3.	SJM02
258	CALL SCALE(DUMMY(N1),NR,DUMMY,NR,TT)	SAM02
259	CALL SCALE(R,NR,R,NR,T)	SAM02
260	CALL ADD(R,NR,DUMMY,NR,R,NR)	SAM02
261	IF(IOP(1) .EQ. 0) RETURN	SAM02
262	CALL LNCNT(3)	SAM02
263	PRINT 750	SAM02
264	CALL PRNT(Q,NQ,4H Q ,1)	SAM02
265	CALL PRNT(W,NW,4H W ,1)	SAM02
266	CALL PRNT(R,NR,4H R ,1)	SAM02
267	RETURN	SAM02
268 C		SAM02
269	END	SAM02

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0   SUBROUTINE PREFIL(A,NA,B,NB,Q,NQ,W,NW,R,NR,F,NF,IOP,DUMMY)    PRE01
1   IMPLICIT REAL*8 (A-H,O-Z)                                         PRE01
2   DIMENSION A(1),B(1),Q(1),W(1),R(1),F(1),DUMMY(1)                PRE01
3   DIMENSION NA(2),NB(2),NQ(2),NW(2),NF(2),NR(2),IOP(3)             PRE01
4   IF( IOP(1) .EQ. 0 ) GO TO 100                                     PRE01
5   CALL LNCNT(5)                                                 PRE01
6   PRINT 25                                                       PRE01
7   25 FORMAT(//,' PROGRAM TO COMPUTE PREFILTER GAIN F TO ELEMINATE CROSSPRE01
8   IS-PRODUCT TERM ',/, ' IN QUADRATIC PERFORMANCE INDEX ',/)        PRE01
9   IF( IOP(3) .EQ. 0 ) GO TO 50                                     PRE01
10  CALL PRNT(A,NA,4H A ,1)                                           PRE01
11  CALL PRNT(B,NB,4H B ,1)                                           PRE01
12  50 CONTINUE                                                 PRE00
13  CALL PRNT(Q,NQ,4H Q ,1)                                           PRE00
14  CALL PRNT(W,NW,4H W ,1)                                           PRE00
15  CALL PRNT(R,NR,4H R ,1)                                           PRE00
16 C
17  100 CONTINUE                                                 PRE00
18  CALL TRANP(W,NW,F,NF)                                         PRE00
19  CALL SCALE(F,NF,F,NF,0.5)                                         PRE00
20  CALL EQUIATE(R,NR,DUMMY,NR)                                       PRE00
21  IOPT=0                                                       PRE00
22  IFAC=0                                                       PRE00
23  N1=NR(1)**2+1                                              PRE00
24  M = NR(1)                                                 PRE00
25  CALL SYMPDS(M,M,DUMMY,NF(2),F,IOPT,IFAC,DETERM,ISCALE,DUMMY(N1),IEPRE00
26  1RR)                                                       PRE00
27  IF( IERR .EQ. 0 ) GO TO 200                                     PRE00
28  CALL LNCNT(4)                                                 PRE00
29  PRINT 150                                                 PRE00
30  150 FORMAT(//,' IN PREFIL, THE MATRIX R IS NOT SYMMETRIC POSITIVE DEFIPRE00
31  INITE',/)                                                 PRE00
32  RETURN                                                 PRE00
33 C
34  200 CONTINUE                                                 PRE00
35  IF( IOP(2) .EQ. 0 ) GO TO 300                                     PRE00
36  CALL MULT(W,NW,F,NF,DUMMY,NQ)                                     PRE00
37  CALL SCALE(DUMMY,NQ,DUMMY,NQ,0.5)                                PRE00
38  CALL SUBT(Q,NQ,DUMMY,NQ,Q,NQ)                                    PRE00
39 C
40  300 CONTINUE                                                 PRE00
41  IF( IOP(3) .EQ. 0 ) GO TO 400                                     PRE00
42  CALL MULT(B,NB,F,NF,DUMMY,NA)                                     PRE00
43  CALL SUBT(A,NA,DUMMY,NA,A,NA)                                    PRE00
44 C
45  400 CONTINUE                                                 PRE00
46  IF( IOP(1) .EQ. 0 ) RETURN                                     PRE00
47  CALL PRNT(F,NF,4H F ,1)                                         PRE00
48  IF( IOP(2) .EQ. 0 ) GO TO 500                                     PRE00
49  CALL LNCNT(3)                                                 PRE00
50  PRINT 450                                                 PRE00
51  450 FORMAT(//, ' MATRIX Q = (W/2)F ',/)                         PRE00
52  CALL PRNT(Q,NQ,4HNEWQ,1)                                         PRE00
53 C
54  500 CONTINUE                                                 PRE00
55  IF( IOP(3) .EQ. 0 ) RETURN                                     PRE00
56  CALL PRNT(A,NA,4HNEWA,1)                                         PRE00
57  RETURN                                                 PRE00
58  END                                                       PRE00

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0   SUBROUTINE CSTAB(A,NA,B,NB,F,NF,IOP,SCLE,DUMMY)      CST01
1   IMPLICIT REAL*8 (A-H,O-Z)                          CST01
2   DIMENSION A(1),B(1),F(1),DUMMY()                  CST01
3   DIMENSION NA(2),NB(2),NF(2),IOP(3),NDUM(2)        CST01
4   DIMENSION IOPT(2)                                  CST01
5   LOGICAL SYM                                     CST01
6   COMMON/TOL/EPSA4,EPSBM,IACM                      CST01
7   N = NA(1)**2                                     CST01
8   N1=N+1                                         CST01
9 C
10  IF(IOP(2).EQ.0) GO TO 100                         CST01
11  CALL EQUATE(A,NA,DUMMY,NA)                        CST01
12  N2=N1+NA(1)                                      CST01
13  N3=N2+NA(1)                                      CST01
14  ISV=0                                           CST0
15  ILV=0                                           CST00
16  CALL EIGEN(NA(1),NA(1),DUMMY,DUMMY(N1),DUMMY(N2),ISV,ILV,V,DUMMY(NCST00
17  13),IERR)                                       CST00
18 C
19  M=NA(1)                                         CST00
20  IF(IERR.EQ.0) GO TO 50                           CST00
21  CALL LNCNT(3)                                    CST00
22  PRINT 25,IERR                                   CST00
23  25 FORMAT(//,' IN CSTAB, THE SUBROUTINE EIGEN FAILED TO DETERMINE THE CST00
24  1 ',I4,' EIGENVALUE FOR THE MATRIX A AFTER 30 ITERATIONS')    CST00
25  IERR=1                                         CST00
26  CALL NORMS(M,M,M,A,IERR,BETA)                  CST00
27  BETA=2.*BETA                                     CST00
28  GO TO 200                                       CST00
29  50 CONTINUE                                     CST00
30 C
31  BETA=0.0                                         CST00
32  DO 75 I=1,M                                     CST00
33  J=N1+I-1                                       CST00
34  RETA1=DARS(DUMMY(J))                           CST00
35  IF(BETA1.GT.BETA) BETA=BETA1                   CST00
36  75 CONTINUE                                     CST00
37  BETA=SCLE*(BETA+.001)                           CST00
38  GO TO 200                                       CST00
39 C
40  100 CONTINUE                                     CST004
41  BETA=SCLE                                     CST004
42  200 CONTINUE                                     CST004
43 C
44  CALL TRANP(B,NB,DUMMY,NDUM)                    CST004
45  CALL MULT(B,NB,DUMMY,NDUM,DUMMY(N1),NA)       CST004
46  CALL SCALE(C,DUMMY(N1),NA,DUMMY,NA,-2.0)     CST004
47  CALL SCALE(A,NA,DUMMY(N1),NA,-1.0)           CST004
48  J=-NA(1)                                       CST005
49  NAX=NA(1)                                       CST005
50  DO 225 I=1,NAX                                CST005
51  J=J+NAX+1                                     CST005
52  K=N1+J-1                                     CST005
53  DUMMY(K)=DUMMY(K)-BETA                     CST005
54  225 CONTINUE                                     CST005
55  N2=N1+N                                     CST005
56  SYM=.TRUE.                                     CST005
57  IOPT(1)=0                                     CST005
58 C
59  IF(IOP(3).NE.0) GO TO 300                     CST006
60  EPSA=EPSA4                                    CST006
61  CALL BARSTW(DUMMY(N1),NA,A,NA,DUMMY,NA,IOP,SYM,EPSA,EPSA,DUMMY(N2CST006
62  1))                                         CST006

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63. GO TO 350 CST01
64. 300 CONTINUE CST01
65. IOPT(2) = 1 CST01
66. CALL BILIN(DUMMY(N1),NA,A,NA,DUMMY,NA,IOPT,ASCL,SYM,DUMMY(N2)) CST01
67. 350 CONTINUE CST01
68 C CST01
69. CALL EQUATE(B,NB,DUMMY(N1),NB) CST01
70. IOPT(1) = 3 CST01
71. IAC = IACM CST01
72. N3 = N2 + NA(1) CST01
73. CALL SNVDEC(IOPT,NA(1),NA(1),NA(1),NA(1),DUMMY,NB(2),DUMMY(N1),IAC,CSTOC
74. ZTEST,DUMMY(N2),DUMMY(N3),IRANK,APLUS,IERR) CST01
75. IF(IERR .EQ. 0 ) GO TO 400 CST01
76. CALL LNCNT(5) CST01
77. IF(IERR .GT. 0 ) PRINT 360,IERR CST00
78. IF(IERR .EQ. -1) PRINT 370,ZTEST,IRANK CST00
79. 360 FORMAT(//,' IN CSTAB, SNVDEC HAS FAILED TO CONVERGE TO THE ',I4,' CST00
80. 1SINGULARVALUE AFTER 30 ITERATIONS',//) CST00
81. 370 FORMAT(//,' IN CSTAB, THE MATRIX SUBMITTED TO SNVDEC USING ZTEST =CST00
82. 1 ',D16.8,' IS CLOSE TO A MATRIX OF LOWER RANK ',//,' IF THE ACCURCST00
83. 2ACY IAC IS REDUCED THE RANK MAY ALSO BE REDUCED',//,' CURRENT RANK CST00
84. 3 =',I4) CST00
85. IF( IERR .GT. 0 ) RETURN CST00
86. NDUM(1) = NA(1) CST00
87. NDUM(2) =1 CST00
88. CALL PRNT(DUMMY(N2),NDUM,4HSGVL,1) CST00
89. 400 CONTINUE CST00
90 C CST00
91. CALL TRANP(DUMMY(N1),NB,F,NF) CST00
92. IF ( IOP(1) .EQ. 0 ) RETURN CST00
93. CALL LNCNT(4) CST00
94. PRINT 500 CST00
95. 500 FORMAT(//,' COMPUTATION OF F MATRIX SUCH THAT A=BF IS ASYMPTOTICALCST00
96. 1LY STABLE IN THE CONTINUOUS SENSE ',/) CST00
97. CALL PRNT(A,NA,4H A ,1) CST00
98. CALL LNCNT(4) CST00
99. PRINT 550,BETA CST01
100. 550 FORMAT(//,' BETA = ',D16.8,/) CST01
101. CALL PRNT(B,NB,4H B ,1) CST01
102. CALL PRNT(F,NF,4H F ,1) CST01
103. CALL MULT(B,NB,F,NF,DUMMY,NA) CST01
104. CALL SUBT(A,NA,DUMMY,NA,DUMMY,NA) CST01
105. CALL PRNT(DUMMY,NA,4HA-BF,1) CST01
106. N2 = N1+NA(1) CST01
107. N3 = N2+NA(1) CST01
108. ISV = 0 CST01
109. ILV = 0 CST01
110. CALL EIGEN(NA(1),NA(1),DUMMY,DUMMY(N1),DUMMY(N2),ISV,ILV,V,DUMMY(NCST01
111. 13),IERR) CST01
112. I = NA(1) CST01
113. IF( IERR .EQ. 0 ) GO TO 600 CST01
114. M = NA(1)-IERR CST01
115. CALL LNCNT(3) CST01
116. PRINT 25,IERR CST01
117. 600 CONTINUE CST01
118. CALL LNCNT(4) CST01
119. PRINT 650 CST01
120. 650 FORMAT(//,' EIGENVALUES OF A-BF',/) CST012
121. 675 FORMAT(10X,2D16.8) CST012
122. CALL LNCNT(M) CST012
123. DO 700 I=1,M CST012
124. I = N1+I-1 CST012
125. K = N2+I-1 CST012

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126 PRINT 675,DUMMY(J),DUMMY(K)
127 700 CONTINUE
128 C
129 RETURN
130 END

CS
CS
CS
CST
CST

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0      SUBROUTINE DSTAB(A,NA,B,NB,F,NF,SING,IOP,SCLE,DUMMY)      DST
1      IMPLICIT REAL*8 (A-H,O-Z)      DST
2      DIMENSION A(1),B(1),F(1),DUMMY(1)      DST
3      DIMENSION NA(2),NB(2),NF(2),NDUM(2),IOP(2),IOPT(3),NDUM1(2)  DST
4      LOGICAL SING,SYM      DST
5      COMMON/TOL/EPSAM,EPSBM,IACM      DST
6      N = NA(1)**2      DST
7      N1 = N + 1      DST
8      N2 = N1 + N      DST
9      IF( .NOT. SING ) GO TO 100      DST
10     IOPT(1)=IOP(1)      DST
11     IOPT(2) = 1      DST
12     IOPT(3) = 0      DST
13     CSCLE=1.05      DST
14     CALL CSTAB(A,NA,B,NB,F,NF,IOPT,CSCLE,DUMMY)      DST
15     CALL MULT(B,NB,F,NF,DUMMY,NA)      DST
16     CALL SUBT(A,NA,DUMMY,NA,DUMMY,NA)      DST
17     CALL EQUATE(DUMMY,NA,DUMMY(N1),NA)      DST
18     GO TO 200      DST
19 C
20   100 CONTINUE      DST
21     CALL EQUATE(A,NA,DUMMY,NA)      DST
22     CALL EQUATE(A,NA,DUMMY(N1),NA)      DST
23 C
24   200 CONTINUE      DST
25     IF( IOP(2) .EQ. 0 ) GO TO 300      DST
26     N3 = N2 + NA(1)      DST
27     N4 = N3 + NA(1)      DST
28     ISV = 0      DST
29     CALL EIGEN(NA(1),NA(1),DUMMY(N1),DUMMY(N2),DUMMY(N3),ISV,ISV,V,DUM DYSTO
30     MY(N4),IERR)      DST
31     CALL EQUATE(DUMMY,NA,DUMMY(N1),NA)      DST
32     M = NA(1)      DST
33     IF( IERR .EQ. 0 ) GO TO 250      DST
34     CALL LNCNT(3)      DST
35     PRINT 225, IERR      DST
36 225   FORMAT(//'IN DSTAB , THE PROGRAM EIGEN FAILED TO DETERMINE',
37           '15, 'EIGENVALUE FOR THE MATRIX A-BG AFTER 30 ITERATIONS ')
38     CALL PRNT(DUMMY,NA,4H A-BG,1)      DST
39     IF( SING ) CALL PRNT(F,NF,4H G ,1)      DST
40     RETURN      DST
41 C
42   250 CONTINUE      DST
43     ALPHA = 1.0      DST
44     DO 275 I =1,M      DST
45     I1 = N2 + I -1      DST
46     I2 = N3 + I -1      DST
47     ALPHA1 = DSQRT(DUMMY(I1)**2 + DUMMY(I2)**2)      DST
48     IF( ALPHA1 .LT. ALPHA .AND. ALPHA1 .NE. 0 ) ALPHA = ALPHA1      DST
49   275 CONTINUE      DST
50     ALPHA = SCLE*ALPHA      DST
51     GO TO 400      DST
52 C
53   300 CONTINUE      DST
54     ALPHA = SCLE      DST
55 C
56   400 CONTINUE      DST
57     J = -NA(1)      DST
58     NAX = NA(1)      DST
59     DO 425 I = 1,NAX      DST
60     J = J + NAX + 1      DST
61     K = N1 + J -1      DST
62     DUMMY(K) = DUMMY(J) - ALPHA      DST

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63 DUMMY(J) = DUMMY(J) + ALPHA          DST
64 425 CONTINUE                         DST
65 CALL EQUATE(B,NB,DUMMY(N2),NB)         DST
66 N3 = N2 + NA(1)*NB(2)                 DST
67 NRHS = NA(1)+NB(2)                   DST
68 N4 = N3 + NA(1)                      DST
69 IFAC = 0                             DST
70 CALL GELIM(NA(1),NA(1),DUMMY,NRHS,DUMMY(N1),DUMMY(N3),IFAC,DUMMY(NDST
71 14),IERR)                           DST
72 IF( IERR .EQ. 0 ) GO TO 500          DST
73 CALL LNCNT(3)                        DST
74 IF( .NOT. SING ) GO TO 445          DST
75 PRINT 435                           DST
76 435 FORMAT(//,' IN DSTAB, GELIM HAS FOUND THE MATRIX ( A-BG ) + ( ALPHA )DST
77 1I SINGULAR ')                     DST
78 CALL PRNT(A,NB,4H A ,1)              DST
79 CALL PRNT(F,NF,4H G ,1)              DST
80 GO TO 465                           DST
81 445 CONTINUE                         DST
82 CALL LNCNT(3)                        DST
83 PRINT 455                           DST
84 455 FORMAT(//,' IN DSTAB, GELIM HAS FOUND THE MATRIX A + ( ALPHA )I SINGDST
85 1ULAR ')                           DST
86 CALL PRNT(A,NA,4H A ,1)              DST
87 465 CONTINUE                         DST
88 CALL LNCNT(3)                        DST
89 PRINT 475,ALPHA                     DST
90 475 FORMAT(//,' ALPHA = ',D16.8)    DST
91 RETURN                               DST
92 C                                     DST
93 500 CONTINUE                         DST
94 CALL EQUATE(DUMMY(N1),NA,DUMMY,NA)   DST
95 CALL TRANP(DUMMY(N2),NB,DUMMY(N1),NDUM) DST
96 N3 = N2 + N                          DST
97 CALL MULT(DUMMY(N2),NB,DUMMY(N1),NDUM,DUMMY(N3),NA)   DST
98 CALL SCALE(DUMMY(N3),NA,DUMMY(N1),NA,4.0)             DST
99 SYM = .TRUE.                          DST
100 IOPT(1) = 0                          DST
101 EPSA=EPSAM                         DST
102 CALL BARSTW(DUMMY,NA,B,NB,DUMMY(N1),NA,IOPT,SYM,EPSA,EPSA,DUMMY(N2DST
103 1))                                DST
104 CALL EQUATE(DUMMY(N1),NA,DUMMY,NA)   DST
105 CALL TRANP(B,NB,DUMMY(N1),NDUM)      DST
106 CALL MULT(B,NB,DUMMY(N1),NDUM,DUMMY(N2),NA)           DST
107 CALL ADD(DUMMY,NA,DUMMY(N2),NA,DUMMY,NA)              DST
108 CALL EQUATE(A,NA,DUMMY(N1),NA)       DST
109 IF( .NOT. SING ) GO TO 600          DST
110 CALL MULT(B,NB,F,NF,DUMMY(N1),NA)   DST
111 CALL SUBT(A,NA,DUMMY(N1),NA,DUMMY(N1),NA)             DST
112 C                                     DST
113 600 CONTINUE                         DST
114 IOPT(1) = 3                          DST
115 M = NA(1)                           DST
116 IAC=IACM                            DST
117 CALL SNVDEC(IOPT,M,M,M,M,DUMMY,M,DUMMY(N1),IAC,ZTEST,DUMMY(N2),DUMMDST
118 1MY(N3),IRANK,APLUS,IERR)          DST
119 IF( IERR .EQ. 0 ) GO TO 700          DST
120 CALL LNCNT(5)                        DST
121 IF( IERR .GT. 0 ) PRINT 625,IERR    DST
122 IF( IEPP .EQ. -1) PRINT 650,ZTEST,IRANK  DST
123 625 FORMAT(//,' IN DSTAB, SNVDEC HAS FAILED TO CONVERGE TO THE ',I5,' DST
124 1SINGULAR VALUE AFTER 30 ITERATIONS')  DST
125 650 FORMAT(//,' IN DSTAB, THE MATRIX SUBMITTED TO SNVDEC, USING ZTEST DST

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126      I= ',D16.8,' , IS CLOSE TO A MATRIX OF LOWER RANK',/, ' IF THE ACCURST
127      2ACY IAC IS REDUCED THE RANK MAY ALSO BE REDUCED',/, ' CURRENT RANK DST
128      3 =',I4)
129      IF( IERR .GT. 0 ) RETURN
130      NDUM(1)= NA(1)
131      NDUM(2)= 1
132      CALL PRNT(DUMMY(N2),NDUM,4HSGVL,1)
133 C
134      700 CONTINUE
135      CALL TRANP(B,NB,DUMMY(N2),NDUM)
136      CALL MULT(DUMMY(N2),NDUM,DUMMY(N1),NA,DUMMY,NF)
137      IF( .NOT. SING ) GO TO 800
138      CALL ADD(F,NF,DUMMY,NF,F,NF)
139      GO TO 900
140 C
141      800 CONTINUE
142      CALL EQUATE(DUMMY,NF,F,NF)
143 C
144      900 CONTINUE
145      IF( IOP(1) .EQ. 0 ) RETURN
146      CALL LNCNT(4)
147      PRINT 1000
148      1000 FORMAT(//,' COMPUTATION OF F SUCH THAT A-BF IS ASYMPTOTICALLY STAB',DST0
149      1LE IN THE DISCRETE SENSE',/)
150      CALL PRNT(A,NA,4H A -L1)
151      CALL PRNT(B,NB,4H B -,1)
152      CALL LNCNT(4)
153      PRINT 1100,ALPHA
154      1100 FORMAT(//,' ALPHA = ',D16.8,/)
155      CALL PRNT(F,NF,4H F -,1)
156      CALL MULT(B,NB,F,NF,DUMMY,NA)
157      CALL SUBT(A,NA,DUMMY,NA,DUMMY,NA)
158      CALL PRNT(DUMMY,NA,4HA-BF,1)
159      CALL LNCNT(3)
160      PRINT 1200
161      1200 FORMAT(//,' EIGENVALUES OF A-BF')
162      NDUM(1) = NA(1)
163      NDUM(2) = 1
164      N2 = N1 + NA(1)
165      N3 = N2 + NA(1)
166      ISV = 0
167      CALL EIGEN(NA(1),NA(1),DUMMY,DUMMY(N1),DUMMY(N2),ISV,ISV,V,DUMMY(NDST01
168      13),IERR)
169      IF( IERR .EQ. 0 ) GO TO 1300
170      CALL LNCNT(3)
171      PRINT 1250
172      1250 FORMAT(//,' IN DSTAB, THE PROGRAM EIGEN FAILED TO DETERMINE THE ',DST01
173      115,' EIGENVALUE FOR THE A-BF MATRIX AFTER 30 ITERATIONS')
174      NDUM(1)=NA(1)-IERR
175 C
176      1300 CONTINUE
177      CALL JUXT(C(DUMMY(N1),NDUM,DUMMY(N2),NDUM,DUMMY,NDUM1)
178      CALL PRNT(DUMMY,NDUM1,4HEIGN,1)
179      CALL LNCNT(4)
180      PRINT 1400
181      1400 FORMAT(//,' MODULI OF EIGENVALUES OF A-BF',/)
182      M =NDUM(1)
183      DO 1500 I = 1,M
184      J = N1 + I - 1
185      K = N2 + I - 1
186      DUMMY(I)=DSQRT(DUMMY(J)**2 + DUMMY(K)**2)
187      1500 CONTINUE
188      CALL PRNT(DUMMY,NDUM,4HMOD ,1)

```

184 C

190

191

RETURN

END

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0   SUBROUTINE DISREG(A,NA,B,NB,H,NH,Q,NQ,R,NR,F,NF,P,NP,IOP,IDENT,OU DIS-
1   1MMY) DIS
2   IMPLICIT REAL*8 (A-H,O-Z) DIS
3   DIMENSION A(1),B(1),Q(1),R(1),F(1),P(1),DUMMY(1) CIS
4   DIMENSION NA(2),NB(2),NQ(2),NR(2),NF(2),NP(2) DIS
5   DIMENSION IOP(3) DIS
6   DIMENSION H(1),NH(2),NDUM(2) DIS
7   LOGICAL IDENT DIS
8   COMMON/TOL/EPSAM,EPSBM,IACM DIS
9   COMMON/CONV/SUMCV,RICTCV,SERCV,MAXSUM DIS
10  N = NA(1)**2 DIS
11  N1= N +1 DIS
12  N2= N1+N DIS
13  N3= N2+N DIS
14 C DIS
15  KSS = 0 DIS
16  I=IOP(3) DIS
17 C DIS
18  IF(IOP(1).EQ. 0) GO TO 85 DIS
19  CALL LNCNT(5) DIS
20  PRINT 25 DIS
21  25 FORMAT(//," PROGRAM TO SOLVE THE TIME-INVARIANT FINITE-DURATION OPDISO
22  ITIMAL",," DIGITAL REGULATOR PROBLEM WITH NOISE-FREE MEASUREMENTS"DISO
23  2,/) DIS
24  CALL PRNT(A,NA,4H A ,1) DIS
25  CALL PRNT(B,NB,4H B ,1) DIS
26  CALL PRNT(Q,NQ,4H Q ,1) DIS
27  IF(.NOT._IDENT ) GO TO 45 DIS
28  CALL LNCNT(3) DIS
29  PRINT 35 DIS
30  35 FORMAT(/," H IS AN IDENTITY MATRIX",/) DIS
31  GO TO 65 DIS
32  45 CONTINUE DIS
33  CALL PRNT(H,NH,4H H ,1) DIS
34  CALL MULT(Q,NQ,H,NH,DUMMY,NH) DIS
35  CALL TRANP(H,NH,DUMMY(N1),NF) DIS
36  CALL MULT(DUMMY(N1),NF,DUMMY,NH,Q,NQ) DIS
37  CALL LNCNT(3) DIS
38  PRINT 55 DIS
39  55 FORMAT(/," MATRIX ( H TRANSPOSE )QH",/) DIS
40  CALL PRNT(Q,NQ,4HHTQH,1) DIS
41  65 CONTINUE DIS
42  CALL PRNT(R,NR,4H R ,1) DIS
43  CALL LNCNT(4) DIS
44  PRINT 75 DIS
45  75 FORMAT(//," WEIGHTING ON TERMINAL VALUE OF STATE VECTOR",/) DIS
46  CALL PRNT(P,NP,4H P ,1) DIS
47 C DIS
48  85 CONTINUE DIS
49  IF((IOP(1).NE. 0) .OR. IDENT) GO TO 100 DIS
50  CALL MULT(Q,NQ,H,NH,DUMMY,NH) DIS
51  CALL TRANP(H,NH,DUMMY(N1),NF) DIS
52  CALL MULT(DUMMY(N1),NF,DUMMY,NH,Q,NQ) DIS
53 C DIS
54  100 CONTINUE DIS
55  I=I-1 DIS
56  CALL EQUATE(P,NP,DUMMY,NP) DIS
57  CALL MULT(P,NP,A,NA,DUMMY(N1),NA) DIS
58  CALL TRANP(B,NB,DUMMY(N2),NF) DIS
59  CALL MULT(DUMMY(N2),NF,DUMMY(N1),NA,F,NF) DIS
60  CALL MULT(P,NP,B,VA,DUMMY(N1),NB) DIS
61  CALL MULT(DUMMY(N2),NF,DUMMY(N1),NB,DUMMY(N3),NR) DIS
62  CALL ADD(R,NR,DUMMY(N3),NR,DUMMY(N1),NR) DIS

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63      IOPT = 3          DIS1
64      IAC=IACM          DIS1
65      MF = NR(1)        DIS1
66      CALL SNVDEC(IOPT,MF,MP,MP,MF,DUMMY(N1),NF(2),F,IAC,ZTEST,DUMMY(N2))DIS1
67      1,DIJMMY(N3),IRANK,APLUS,IERR)          DIS1
68      IF( IERR .EQ. 0 ) GO TO 300          DIS1
69      CALL LNCNT(5)          DIS1
70      IF(IERR .GT. 0) PRINT 200,IERR          DIS1
71      IF(IERR .EQ. -1) PRINT 250,ZTEST,IRANK          DIS1
72      200 FORMAT(//,' IN DISREG, SNVDEC HAS FAILED TO CONVERGE TO THE ',I4,DIS1
73      1'SINGULARVALUE AFTER 30 ITERATIONS',//)          DIS1
74      250 FORMAT(//,' IN DISREG, THE MATRIX SUBMITTED TO SNVDEC USING ZTEST DIS1
75      1=' ,D16.8,' IS CLOSE TO A MATRIX OF LOWER RANK',//,'IF THE ACCURACYDIS1
76      2 IAC IS REDUCED THE RANK MAY ALSO BE REDUCED',//,' CURRENT RANK = 'DIS1
77      3 ,I4)          DIS1
78      IF( IERR .GT. 0 ) RETURN          DIS1
79      NDOUM(1) = NA(1)          DIS1
80      NDOUM(2) = 1          DIS1
81      CALL PRNT(DUMMY(N2),NDOUM,4HSGVL,1)          DIS1
82 C
83      300 CONTINUE          DIS1
84      CALL MULT(R,NR,F,NF,DUMMY(N1),NF)          DIS1
85      CALL TRANP(F,NF,DUMMY(N2),NB)          DIS1
86      CALL MULT(DUMMY(N2),NB,DUMMY(N1),NF,P,NP)          DIS1
87      CALL ADD(Q,NQ,P,NP,P,NP)          DIS1
88      CALL MULT(B,NB,F,NF,DUMMY(N1),NA)          DIS1
89      CALL SUBT(A,NA,DUMMY(N1),NA,DUMMY(N1),NA)          DIS1
90      CALL MULT(DUMMY,NA,DUMMY(N1),NA,DUMMY(N2),NA)          DIS1
91      CALL TRANP(DUMMY(N1),NA,DUMMY(N3),NA)          DIS1
92      CALL MULT(DUMMY(N3),NA,DUMMY(N2),NA,DUMMY(N1),NA)          DIS1
93      CALL ADD(P,NP,DUMMY(N1),NA,P,NP)          DIS1
94 C
95      IF( IOP(2) .EQ. 0 ) GO TO 400          DIS01
96      CALL LNCNT(5)          DIS01
97      PRINT 350,I          DIS01
98      350 FORMAT(//,' STAGE ',I5,//)          DIS01
99      CALL PRNT(F,NF,4H F ,1)          DIS01
100     CALL PRNT(P,NP,4H P ,1)          DIS01
101 C
102     400 CONTINUE          DIS01
103     IF( I .EQ. 0 ) GO TO 600          DIS01
104     CALL MAXEL(DUMMY,NP,ANORM1)          DIS01
105     CALL SUBT(DUMMY,NP,P,NP,DUMMY(N2),NP)          DIS01
106     CALL MAXEL(DUMMY(N2),NP,ANORM2)          DIS01
107     IF( ANORM1 .NE. 0.0 ) GO TO 500          DIS01
108     GO TO 100          DIS01
109 C
110     500 CONTINUE          DIS01
111     IF(ANORM1 .GT. 1.0 ) GO TO 550          DIS01
112     IF( ANORM2/ANORM1 .LT. RICTCV ) KSS = 1          DIS01
113     GO TO 575          DIS01
114     550 CONTINUE          DIS01
115     IF( ANORM2 .LT. RICTCV ) KSS=1          DIS01
116     575 CONTINUE          DIS01
117     IF( KSS .EQ. 1) GO TO 600          DIS01
118     GO TO 100          DIS01
119 C
120     600 CONTINUE          DIS01
121     K = IOP(1) + IOP(2)          DIS01
122     IF( K .EQ. 0 ) RETURN          DIS01
123     IF( KSS .EQ. 0) GO TO 700          DIS01
124     CALL LNCNT(4)          DIS01
125     PRINT 650          DIS01

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126 650 FORMAT(//,' STEADY-STATE SOLUTION HAS BEEN REACHED IN DISREG',/) DIS01
127 C
128 700 CONTINUE DIS01
129 IF(IOP(2) .NE. 0) RETURN DIS01
130 IF(IOP(1) .EQ. 0) RETURN DIS01
131 CALL LNCNT(3) DIS01
132 I = IOP(3)-I DIS01
133 PRINT 800, I DIS01
134 800 FORMAT(//, ' F AND P AFTER ',I5, ' STEPS',/) DIS01
135 CALL PRNT(F,NF,4H F ,1) DIS01
136 CALL PRNT(P,NP,4H P ,1) DIS01
137 RETURN DIS01
138 END DIS01

```

0      SUBROUTINE CNTREG(A,NA,B,NB,H,NH,Q,NQ,R,NR,Z,W,LAMBDA,S,F,NF,P,NP CNT'
1      I,T,IOP,IDENT,DUMMY) CNT'
2      IMPLICIT REAL*8 (A-H,O-Z) CNT'
3 C
4      DIMENSION A(1),B(1),H(1),Q(1),R(1),Z(1),W(1),LAMBDA(1),S(1),F(1),PCNT' CNT'
5      I(1),T(1),DUMMY(1) CNT'
6      DIMENSION NA(2),NB(2),NH(2),NQ(2),NR(2),NF(2),NP(2),IOP(3),NDUM1(2CNT' CNT'
7      1),NDUM2(2) CNT'
8      LOGICAL IDENT CNT'
9      REAL*8 LAMBDA CNT'
10     COMMON/CONV/SUMCV,RICTCV,SERCV,MAXSUM CNT'
11 C
12     IF( IOP(1).EQ.-0 ) GO TO 65 CNT'
13     CALL LNCNT(5) CNT'
14     IF( IOP(3).EQ. 0 ) PRINT 25 CNT'
15     25 FORMAT(//'* PROGRAM TO SOLVE THE TIME-INVARIANT FINITE-DURATION CONCNTO CNT'
16     TINUOUS OPTIMAL*' REGULATOR PROBLEM WITH NOISE-FREE MEASUREMENTS*'CNTO CNT'
17     2) CNT'
18     IF( IOP(3).NE. 0 ) PRINT 30 CNT'
19     30 FORMAT(//'* PROGRAM TO SOLVE THE TIME-INVARIANT INFINITE-DURATION CNTO CNT'
20     IONTINUOUS OPTIMAL*' REGULATOR PROBLEM WITH NOISE-FREE MEASUREMENCTO CNT'
21     2TS') CNT'
22     CALL PRNT(A,NA,4H A ,1) CNT'
23     CALL PRNT(B,NB,4H B ,1) CNT'
24     CALL PRNT(Q,NQ,4H Q ,1) CNT'
25     IF( .NOT. IDENT ) GO TO 45 CNT'
26     CALL LNCNT(3) CNT'
27     PRINT 35 CNT'
28     35 FORMAT(/' H IS AN IDENTITY MATRIX') CNT'
29     GO TO 55 CNT'
30 C
31     45 CONTINUE CNT'
32     CALL PRNT(H,NH,4H H ,1) LNTOC CNT'
33     CALL MULT(Q,NQ,H,NH,DUMMY,NH) CNT'
34     N1= NH(1)*NH(2)+1 CNT'
35     CALL TRANP(H,NH,DUMMY(N1),NDUM1) CNT'
36     CALL MULT(DUMMY(N1),NDUM1,DUMMY,NH,Q,NQ) CNT'
37     CALL LNCNT(3) CNT'
38     PRINT 50 CNT'
39     50 FORMAT(//'* MATRIX (H TRANSPOSE)QH') CNT'
40     CALL PRNT(Q,NQ,0,3) CNT'
41     55 CONTINUE CNT'
42     CALL PRNT(R,NR,4H R ,1) CNT'
43 C
44     IF( IOP(3).NE. 0 ) GO TO 65 CNT'
45     CALL LNCNT(4) CNT'
46     PRINT 60 CNT'
47     60 FORMAT(//'* WEIGHTING ON TERMINAL VALUE OF STATE VECTOR') CNT'
48     CALL PRNT(P,NP,4H P ,1) CNT'
49 C
50     65 CONTINUE CNT'
51     CALL EQUATE(R,NR,DUMMY,NR) CNT'
52     N = NA(1)**2 CNT'
53     N1 = NR(1)*NB(2)+1 CNT'
54     CALL TRANP(B,NB,DUMMY(N1),NDUM1) CNT'
55     N2 = N1 + N CNT'
56     L = NR(1) CNT'
57     IOPT = 0 CNT'
58     IFAC = 0 CNT'
59     CALL SYMPDS(L,L,DUMMY,NB(1),DUMMY(N1),IOPT,IFAC,DET,ISCALE,DUMMY(NCNTOO CNT'
60     12),IERR) CNT'
61 C
62     IF( IERR.EQ. 0 ) GO TO 100 CNT'

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63 CALL LNCNT(4) CNT0
64 PRINT 75 CNT0
65 75 FORMAT(//'* IN CNTREG, THE SUBROUTINE SYMPOS HAS FOUND THE MATRIX CNT0
66 1 R NOT SYMMETRIC POSITIVE DEFINITE'*) CNT0
67 RETURN CNT0
68 C CNT0
69 100 CONTINUE CNT0
70 CALL EQUATE(DUMMY(N1),NDUM1,DUMMY,NDUM1) CNT0
71 CALL MULT(B,NB,DUMMY(N1),NDUM1,DUMMY(N2),NA) CNT0
72 CALL SCALE(DUMMY(N2),NA,DUMMY(N1),NA,-1.0) CNT0
73 N3 = N2 + N CNT0
74 IF( IDENT .OR. (IOP(1) .NE. 0) ) GO TO 200 CNT0
75 CALL MULT(Q,NQ,H,NH,DUMMY(N2),NH) CNT0
76 CALL TRANP(H,NH,DUMMY(N3),NDUM1) CNT0
77 CALL MULT(DUMMY(N3),NDUM1,DUMMY(N2),NH,Q,NQ) CNT0
78 C CNT0
79 200 CONTINUE CNT0
80 CALL SCALE(Q,NQ,Q,NQ,-1.0) CNT0
81 CALL JUXTR(A,NA,Q,NQ,Z,NDUM1) CNT0
82 CALL TRANP(A,NA,DUMMY(N2),NA) CNT0
83 CALL SCALE( DUMMY(N2),NA,DUMMY(N2),NA,-1.0) CNT0
84 L = 2*N + 1 CNT0
85 CALL JUXTR(DUMMY(N1),NA,DUMMY(N2),NA,Z(L),NDUM1) CNT0
86 CALL SCALE(Q,NQ,Q,NQ,-1.0) CNT0
87 NDUM2(1) = 2*NA(1) CNT0
88 NDUM2(2) = NDUM2(1) CNT0
89 IF( IOP(1) .NE. 0 ) CALL PRNT(Z,NDUM2,4H Z ,1) CNT0
90 CALL EQUATE(Z,NDUM2,DUMMY(N1),NDUM2) CNT0
91 M = 4*N CNT0
92 N2 = M + N1 CNT0
93 L = 2*NA(1) CNT0
94 N3 = N2 + L CNT0
95 N4 = N3 + L CNT0
96 ISV = L CNT0
97 ILV = 0 CNT0
98 CALL EIGEN(L,L,DUMMY(N1),DUMMY(N2),DUMMY(N3),ISV,ILV,W,DUMMY(N4),ICNT0) CNT0
99 1ERR) CNT0
100 IF( IERR .EQ. 0 ) GO TO 300 CNT0
101 CALL LNCNT(4) CNT0
102 IF( IERR .GT. 0 ) GO TO 250 CNT0
103 PRINT 225,IERR CNT0
104 225 FORMAT(//'* IN CNTREG, EIGEN FAILED TO COMPUTE THE ',I6,' EIGENVEC CNT0
105 1 TOR OF Z ') CNT0
106 RETURN CNT0
107 250 CONTINUE CNT0
108 PRINT 275,IERR CNT0
109 275 FORMAT(//'* IN CNTREG, THE ',I6,' EIGENVALUE OF Z HAS NOT BEEN FO CNT0
110 1UND AFTER 30 ITERATIONS IN EIGEN') CNT0
111 RETURN CNT0
112 C CNT0
113 300 CONTINUE CNT0
114 IF( IOP(1) .EQ. 0 ) GO TO 400 CNT0
115 CALL LNCNT(3) CNT0
116 PRINT 325 CNT0
117 325 FORMAT(//'* EIGENVALUES OF Z') CNT0
118 NDUM1(1) = L CNT0
119 NDUM1(2) = 2 CNT0
120 CALL PRNT(DUMMY(N2),NDUM1,0,3) CNT0
121 CALL LNCNT(3) CNT0
122 PRINT 350 CNT0
123 350 FORMAT(//'* CORRESPONDING EIGENVECTORS') CNT0
124 CALL PRNT(W,NDUM2,0,3) CNT0
25 C CNT0

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126 400 CONTINUE CNT
127 CALL EQUATE(W,NDUM2,DUMMY(N1),NDUM2) CNT
128 J1 = 1 CNT
129 J2 = 1 CNT
130 M = 2*N CNT
131 NDUM1(1) = L CNT
132 NDUM1(2) = 1 CNT
133 K4 = N4 CNT
134 C CNT
135 I=1 CNT
136 415 CONTINUE CNT
137 IF( I .GT. L ) GO TO 515 CNT
138 K1 = N2+I-1 CNT
139 K2 = N1+(I-1)*L CNT
140 K3 = N3+I-1 CNT
141 IF(DUMMY(K1) .GT. 0.0 ) GO TO 425 CNT
142 J = (J1-1)*L+M+1 CNT
143 J1 = J1+1 CNT
144 IF(DUMMY(K3) .NE. 0.0 ) J1=J1+1 CNT
145 GO TO 450 CNT
146 425 CONTINUE CNT
147 DUMMY(K4)=I CNT
148 K4 = K4+1 CNT
149 J = (J2-1)*L+1 CNT
150 J2 = J2+1 CNT
151 IF( DUMMY(K3) .NE. 0.0 ) J2 = J2 + 1 CNT
152 450 CONTINUE CNT
153 CALL EQUATE(DUMMY(K2),NDUM1,W(J),NDUM1) CNT
154 IF(DUMMY(K3) .EQ. 0.0 ) GO TO 500 CNT
155 I = I+1 CNT
156 K2 = K2+L CNT
157 J = J+L CNT
158 CALL EQUATE(DUMMY(K2),NDUM1,W(J),NDUM1) CNT
159 500 CONTINUE CNT
160 I=I+1 CNT
161 GO TO 415 CNT
162 515 CONTINUE CNT
163 C CNT
164 CALL NULL(LAMBDA,NA) CNT
165 K0 = -1 CNT
166 J = -NA(1) CNT
167 NAX = NA(1) CNT
168 I=1 CNT
169 520 CONTINUE CNT
170 IF( I .GT. NAX ) GO TO 530 CNT
171 J = NAX + J + 1 CNT
172 K0 = K0 + 1 CNT
173 K1 = N4 + K0 CNT
174 K2 = DUMMY(K1) CNT
175 K = N2+K2-1 CNT
176 LAMBDA(J) = DUMMY(K) CNT
177 K3 = N3+K2-1 CNT
178 IF( DUMMY(K3) .EQ. 0.0 ) GO TO 525 CNT
179 K4 = J+1 CNT
180 LAMBDA(K4) = -DUMMY(K3) CNT
181 K4 = K4+NAX CNT
182 LAMBDA(K4) = DUMMY(K) CNT
183 K4 = K4-1 CNT
184 LAMBDA(K4) = DUMMY(K3) CNT
185 K5 = M + (I-1)*L + 1 CNT
186 K6 = K5 + L CNT
187 CALL EQUATE(W(K5),NDUM1,DUMMY(N1),NDUM1) CNT
188 CALL EQUATE(W(K6),NDUM1,W(K5),NDUM1) CNT

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189     CALL EQUATE(NDUMMY(N1),NDUM1,W(K6),NDUM1)          CNT0
190     I = I+1                                              CNT0
191     J = NAX + J +1                                         CNT0
192     525 CONTINUE                                         CNT0
193     I=I+1                                              CNT0
194     GO TO 520                                           CNT0
195     530 CONTINUE                                         CNT0
196     C
197     IF( IOP(1) .EQ. 0 ) GO TO 700                         CNT0
198     CALL LNCNT(3)                                         CNT0
199     PRINT 535                                           CNT0
200     535 FORMAT(//'* REORDERED EIGENVECTORS')           CNT0
201     CALL PRNT(W,NDUM2,0,3)                               CNT0
202     CALL LNCNT(4)                                         CNT0
203     PRINT 545                                           CNT0
204     545 FORMAT(//'* LAMBDA MATRIX OF EIGENVALUES OF Z WITH POSITIVE REAL PA CNT0
205     IRTS*)                                              CNT0
206     CALL PRNT(LAMBDA,NA,0,3)                            CNT0
207     C
208     CALL MULT(Z,NDUM2,W,NDUM2,NDUMMY(N1),NDUM2)        CNT0
209     L = NDUM2(1)                                         CNT0
210     M = L**2                                              CNT0
211     N2 = N1+M                                           CNT0
212     CALL EQUATE(W,NDUM2,NDUMMY(N2),NDUM2)               CNT0
213     N3 = N2+M                                           CNT0
214     N4 = N3+L                                           CNT0
215     IFAC = 0                                            CNT0
216     CALL GELIM(L,L,NDUMMY(N2),L,NDUMMY(N1),NDUMMY(N3),IFAC,NDUMMY(N4),IER CNT0
217     1)
218     IF( IERR .EQ. 0 ) GO TO 600                         CNT0
219     CALL LNCNT(4)                                         CNT0
220     PRINT 550                                           CNT0
221     550 FORMAT(//'* IN CNTREG, GELIM HAS FOUND THE REORDERED MATRIX W TO 8 CNT0
222     1E SINGULAR *)                                     CNT0
223     600 CONTINUE                                         CNT0
224     CALL PRNT(NDUMMY(N1),NDUM2,4HWIZW,1)                CNT0
225     C
226     700 CONTINUE                                         CNT0
227     NDUM1(1) = 2*NA(1)                                 CNT0
228     NDUM1(2) = NA(1)                                  CNT0
229     N2 = 2*N + N1                                       CNT0
230     CALL TRANP(W,NDUM1,NDUMMY(N2),NDUM2)               CNT0
231     NW11 = N1                                         CNT0
232     NDUM1(1) = NA(1)                                  CNT0
233     CALL TRANP(NDUMMY(N2),NDUM1,NDUMMY(NW11),NDUM1)   CNT0
234     L = N2+N                                           CNT0
235     NW21 = NW11+N                                      CNT0
236     CALL TRANP(NDUMMY(L),NDUM1,NDUMMY(NW21),NDUM1)   CNT0
237     L = 2*N+1                                         CNT0
238     NDUM1(1)=2*NA(1)                                 CNT0
239     N3 = N2 + 2*N                                      CNT0
240     CALL TRANP(W(L),NDUM1,NDUMMY(N3),NDUM2)           CNT0
241     NDUM1(1) = NA(1)                                  CNT0
242     NW12 = NW21+N                                      CNT0
243     CALL TRANP(NDUMMY(N3),NDUM1,NDUMMY(NW12),NDUM1)   CNT0
244     L = N3 + N                                         CNT0
245     NW22 = NW12 + N                                     CNT0
246     CALL TRANP(NDUMMY(L),NDUM1,NDUMMY(NW22),NDUM1)   CNT0
247     C
248     IF( IOP(1) .EQ. 0 ) GO TO 800                     CNT0
249     CALL PRNT(NDUMMY(NW11),NA,4HW11 ,1)                CNT0
250     CALL PRNT(NDUMMY(NW21),NA,4HW21 ,1)                CNT0
251     CALL PRNT(NDUMMY(NW12),NA,4HW12 ,1)                CNT0

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    252      CALL PRNT(DUMMY(NW22),NA,4H W22 ,1)          CNT0
253 C
    254 900 CONTINUE
    255 IF( IOP(3) .NE. 0 ) GO TO 900
    256 N2 = N1+4*N
    257 CALL MULT(P,NP,DUMMY(NW12),NA,S,NA)
    258 CALL MULT(P,NP,DUMMY(NW11),NA,DUMMY(N2),NA)
    259 CALL SUBT(S,NA,DUMMY(NW22),NA,S,NA)
    260 CALL SUBT(DUMMY(NW21),NA,DUMMY(N2),NA,DUMMY(N2),NA)
    261 N3 = N2+N
    262 L = NA(1)
    263 IFAC = 0
    264 N4 = N3+NA(1)
    265 CALL GELIM(L,L,DUMMY(N2),L,S,DUMMY(N3),IFAC,DUMMY(N4),IERR)
    266 IF( IERR .EQ. 0 ) GO TO 850
    267 CALL LNCNT(4)
    268 PRINT 825
    269 825 FORMAT(//'* IN CNTREG, GELIM HAS FOUND THE MATRIX W21 - P1XW11 TO CNT0
270 1 BE SINGULAR'*)
    271 RETURN
272 C
    273 850 CONTINUE
    274 IF( IOP(1) .EQ. 0 ) GO TO 1000
    275 CALL PRNT(S,NA,4H S ,1)
    276 NDUM1(1) = NR(1)
    277 NDUM1(2) = NA(1)
    278 CALL LNCNT(3)
    279 PRINT 875
    280 875 FORMAT(//'* MATRIX (R INVERSE)X(B TRANSPOSE)*')
    281 CALL PRNT(DUMMY,NDUM1,0,3)
    282 GO TO 1000
283 C
    284 900 CONTINUE
    285 N2 = N1+4*N
    286 CALL TRANP(DUMMY(NW12),NA,DUMMY(N2),NA)
    287 CALL TRANP(DUMMY(NW22),NA,P,NP)
    288 N3 = N2+N
    289 IFAC = 0
    290 L = NA(1)
    291 N4 = N3 + NA(1)
    292 CALL GELIM(L,L,DUMMY(N2),L,P,DUMMY(N3),IFAC,DUMMY(N4),IERR)
    293 IF( IERR .EQ. 0 ) GO TO 950
    294 CALL LNCNT(4)
    295 PRINT 925
    296 925 FORMAT(//'* IN CNTREG, GELIM HAS FOUND THE MATRIX W12 TO BE SINGUL CNT02
297 1AR*)
    298 RETURN
299 950 CONTINUE
300 NDUM1(1) = NR(1)
301 NDUM1(2) = NA(1)
302 CALL MULT(DUMMY,NDUM1,P,NP,F,NF)
303 IF( IOP(1) .EQ. 0 ) RETURN
304 CALL PRNT(P,NP,4H P ,1)
305 CALL PRNT(F,NF,4H F ,1)
306 RETURN
307 C
308 1000 CONTINUE
309 NMAX = T(1)/T(2)
310 I = NMAX
311 CALL EQUATE(LAMBDA,NA,DUMMY(N2),NA)
312 TT = -T(2)
313 N4 = N3+N
314 N5 = N4+N

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315      N6 = N5+N  

316      N7 = N6+NA(1)  

317      KSS = 0  

318      NOUM1(1) = NR(1)  

319      NOUM1(2) = NA(1)  

320      CALL EXPSEI(DUMMY(N2),NA,DUMMY(N3),NA,TT,KSS,DUMMY(N4))  

321      CALL EQUATE(DUMMY(N3),NA,DUMMY(N2),NA)  

322      IF( IOP(1) .EQ. 0 ) GO TO 1075  

323      CALL LNCNT(3)  

324      PRINT 1050,T(2)  

325 1050 FORMAT(//' EXP(-LAMBDA X ^ ,D16.8,')')  

326      CALL PRNT(DUMMY(N2),NA,0,3)  

327 1075 CONTINUE  

328      IF( NMAX .LE. 0 ) RETURN  

329      CALL EQUATE(S,NA,DUMMY(N3),NA)  

330 1100 CONTINUE  

331      TIME = I*T(2)  

332      IF( I .NE. NMAX ) CALL EQUATE(DUMMY(N5),NA,P,NP)  

333      CALL MULT(DUMMY(N3),NA,DUMMY(N2),NA,DUMMY(N4),NA)  

334      CALL MULT(DUMMY(N2),NA,DUMMY(N4),NA,DUMMY(N3),NA)  

335      CALL MULT(DUMMY(NW1),NA,DUMMY(N3),NA,DUMMY(N4),NA)  

336      CALL ADD(DUMMY(NW12),NA,DUMMY(N4),NA,DUMMY(N4),NA)  

337      CALL TRNP(DUMMY(N4),NA,DUMMY(N5),NA)  

338      CALL EQUATE(DUMMY(N5),NA,DUMMY(N4),NA)  

339      CALL MULT(DUMMY(NW21),NA,DUMMY(N3),NA,DUMMY(N5),NA)  

340      CALL ADD(DUMMY(NW22),NA,DUMMY(N5),NA,DUMMY(N5),NA)  

341      CALL TRNP(DUMMY(N5),NA,DUMMY(N6),NA)  

342      CALL EQUATE(DUMMY(N6),NA,DUMMY(N5),NA)  

343      L = NA(1)  

344      IFAC = 0  

345      CALL GELIM(L,L,DUMMY(N4),L,DUMMY(N5),DUMMY(N6),IFAC,DUMMY(N7),IERR,CNT0  

346      1)  

347      IF( IERR .EQ. 0 ) GO TO 1200  

348      CALL LNCNT(3)  

349      PRINT 1150,TIME  

350 1150 FORMAT(//' IN CNTREG AT TIME ',D16.8,' P CANNOT BE COMPUTED DUE T CNT0  

351      10 MATRIX SINGULARITY IN GELIM')  

352      RETURN  

353 C  

354 1200 CONTINUE  

355      CALL MAXEL(P,NP,ANORM1)  

356      CALL SUBT(DUMMY(N5),NA,P,NP,DUMMY(N4),NA)  

357      CALL MAXEL(DLIMY(N4),NA,ANORM2)  

358      IF( ANORM1 .NE. 0.0 ) GO TO 1225  

359      GO TO 1300  

360 C  

361 1225 CONTINUE  

362      IF(ANORM1 .GT. 1.0 ) GO TO 1250  

363      IF( ANORM2/ANORM1 .LT. RICTCV ) KSS=1  

364      GO TO 1300  

365 1250 CONTINUE  

366      IF( ANORM2 .LT. RICTCV ) KSS=1  

367 C  

368 1300 CONTINUE  

369      CALL MULT(DUMMY,NOUM1,P,NP,F,NF)  

370      IF( IOP(2) .EQ. 0 ) GO TO 1400  

371      CALL LNCNT(5)  

372      PRINT 1350,TIME  

373 1350 FORMAT(///' TIME = ',D16.8/)  

374      CALL PRNT(P,NP,4H P ,1)  

375      IF( I .NE. NMAX ) CALL PRNT(F,NF,4H F ,1)  

376 C  

377 1400 CONTINUE

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378	IF(KSS .EQ. 1) GO TO 1500	CNT
379	I = I-1	CNT
380	IF(I .GE. 0) GO TO 1100	CNT
381	GO TO 1600	CNT
382	1500 CONTINUE	CNT
383	CALL LNCNT(4)	CNT
384	PRINT 1550	CNT
385	1550 FORMAT(//'* STEADY-STATE SOLUTION HAS BEEN REACHED IN CNTREG*')	CNT
386	C	CNT
387	1600 CONTINUE	CNT
388	IF(IOP(2) .NE. 0) RETURN	CNT
389	IF(IOP(1) .EQ. 0) RETURN	CNT
390	CALL LNCNT(5)	CNT
391	PRINT 1350,TIME	CNT
392	CALL PRNT(P,NP,4H P ,1)	CNT
393	CALL PRNT(F,NF,4H F ,1)	CNT
394	C	CNT
395	RETURN	LNT
396	END	CNT

```

0   SUBROUTINE RICNWT(A,NA,B,NB,H,NH,Q,NQ,R,NR,F,NF,P,NP,IOP,IDENT,DI RIC00
1   ISC,FNULL,DUMMY)
2   IMPLICIT REAL*8 (A-H,O-Z) RIC00
3   DIMENSION A(1),B(1),Q(1),R(1),F(1),P(1),DUMMY(1) RIC00
4   DIMENSION NA(2),NB(2),NQ(2),NR(2),NF(2),NP(2),IOP(3) RIC00
5   DIMENSION H(1),NH(2),IOPT(2) RIC00
6   LOGICAL IDENT,DISC,FNULL,SYM RIC00
7   COMMON/TOL/EPSAM,EPSSM,IACM RIC00
8   COMMON/CONV/SUMCV,RICTCV,SERCV,MAXSUM RIC00
9   I=1 RIC00
10  IOPT(1)=0 RIC00
11  SYM = .TRUE. RIC00
12 C RIC00
13  N = NA(1)**2 RIC00
14  N1 = N +1 RIC00
15  IF(.NOT. DISC) N1 = NA(1)*NR(1) + 1 RIC00
16  N2= N1+N RIC00
17  N3= N2+N RIC00
18  N4 = N3+N RIC00
19 C RIC00
20  IF( IOP(1) .EQ. 0 ) GO TO 210 RIC00
21  CALL LNCNT(4) RIC00
22  IF(.NOT. DISC)PRINT 100 RIC00
23  IF( DISC )PRINT 150 RIC00
24  100 FORMAT(//,' PROGRAM TO SOLVE CONTINUOUS STEADY-STATE RICCATI EQUATION BY THE NEWTON ALGORITHM',/) RIC00
25  100 FORMAT(//,' PROGRAM TO SOLVE DISCRETE STEADY-STATE RICCATI EQUATION BY THE NEWTON ALGORITHM',/) RIC00
26  150 FORMAT(//,' H IS AN IDENTITY MATRIX',/) RIC00
27  180 FORMAT(//,' H IS AN IDENTITY MATRIX',/) RIC00
28  CALL PRNT(A,NA,4H A ,1) RIC00
29  CALL PRNT(B,NB,4H B ,1) RIC00
30  CALL PRNT(Q,NQ,4H Q ,1) RIC00
31  IF( .NOT. IDENT )GO TO 185 RIC00
32  CALL LNCNT(3) RIC00
33  PRINT 180 RIC00
34  180 FORMAT(//,' H IS AN IDENTITY MATRIX',/) RIC00
35  GO TO 200 RIC00
36  185 CONTINUE RIC00
37  CALL PRNT(H,NH,4H H ,1) RIC00
38  CALL MULT(Q,NQ,H,NH,DUMMY,NH) RIC00
39  CALL TRANP(H,NH,DUMMY(N2),NP) RIC00
40  CALL MULT(DUMMY(N2),NP,DUMMY,NH,Q,NQ) RIC00
41  CALL LNCNT(3) RIC00
42  PRINT 195 RIC00
43  195 FORMAT(//,' MATRIX (H TRANSPOSE)QH ',/) RIC00
44  CALL PRNT(Q,NQ,4HHTQH,1) RIC00
45  200 CONTINUE RIC00
46  CALL PRNT(R,NR,4H R ,1) RIC00
47  IF( FNULL ) GO TO 210 RIC00
48  CALL LNCNT(3) RIC00
49  PRINT 205 RIC00
50  205 FORMAT(//,' INITIAL F MATRIX',/) RIC00
51  CALL PRNT(F,NF,4H F ,1) RIC00
52 C RIC00
53  210 CONTINUE RIC00
54  IF((IOP(1) .NE. 0) .OR. IDENT) GO TO 220 RIC00
55  CALL MULT(Q,NQ,H,NH,DUMMY,NH) RIC00
56  CALL TRANP(H,NH,DUMMY(N2),NP) RIC00
57  CALL MULT(DUMMY(N2),NP,DUMMY,NH,Q,NQ) RIC00
58  220 CONTINUE RIC00
59 C RIC00
60  IF (DISC) GO TO 900 RIC00
61 C RIC00
62  CALL TRANP(B,NB,P,NP) RIC00

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63     CALL EQUATE(R,NR,DUMMY,NR)          RIC
64     CALL SYMPOS(NR(1),NR(1),DUMMY,NP(2),P,IOPt,IOPt,DET,ISCALE,DUMMY(NR
65     1),IERR)                          RIC
66     IF(IERR .EQ. 0) GO TO 250          RIC
67     CALL LNCNT(3)                      RIC
68     PRINT 225                          RIC
69     225 FORMAT(/,' IN RICNWT, A MATRIX WHICH IS NOT SYMMETRIC POSITIVE DERIC
70     1FINITE HAS BEEN SUBMITTED TO SYMPOS',/) RIC
71     RETURN                             RIC
72 C
73     250 CONTINUE                      RIC
74     CALL EQUATE(P,NP,DUMMY,NF)          RIC
75     CALL MULT(B,NB,DUMMY,NF,DUMMY(N1),NA) RIC
76     CALL TRANP(DUMMY(N1),NA,DUMMY(N2),NA) RIC
77     CALL ADD(DUMMY(N1),NA,DUMMY(N2),NA,DUMMY(N1),NA) RIC
78     CALL SCALE(DUMMY(N1),NA,DUMMY(N1),NA,0.5)        RIC
79 C
80     IF(FNULL) GO TO 300               RIC
81 C
82     CALL MULT(B,NB,F,NF,DUMMY(N2),NA) RIC
83     CALL SUBT(A,NA,DUMMY(N2),NA,DUMMY(N2),NA) RIC
84     CALL TRANP(DUMMY(N2),NA,DUMMY(N3),NA) RIC
85     CALL EQUATE(DUMMY(N3),NA,DUMMY(N2),NA) RIC
86     CALL MULT(R,NR,F,NF,DUMMY(N3),NF)        RIC
87     CALL TRANP(F,NF,P,NP)                 RIC
88     CALL MULT(P,NP,DUMMY(N3),NF,DUMMY(N4),NA) RIC
89     CALL TRANP(DUMMY(N4),NA,DUMMY(N3),NA) RIC
90     CALL ADD(DUMMY(N4),NA,DUMMY(N3),NA,DUMMY(N3),NA) RIC
91     CALL SCALE(DUMMY(N3),NA,DUMMY(N3),NA,0.5)        RIC
92     CALL ADD(DUMMY(N3),NA,Q,NQ,P,NP)        RIC
93     CALL SCALE(P,NP,P,NP,-1.0)           RIC
94     GO TO 350                         RIC
95 C
96     300 CONTINUE                      RIC
97     CALL TRANP(A,NA,DUMMY(N2),NA)          RIC
98     CALL SCALE(Q,NQ,P,NP,-1.0)           RIC
99 C
100    350 CONTINUE                     RIC
101    IF(IOP(3) .NE. 0) GO TO 400         RIC
102    EPSA=EPSAM                         RIC
103    CALL BARSTW(DUMMY(N2),NA,B,NB,P,NP,IOPt,SYM,EPSA,EPSA,DUMMY(N3)) RIC
104    GO TO 450                         RIC
105 C
106    400 CONTINUE                     RIC
107    IOPt(2)=1                          RIC
108    CALL BILIN(DUMMY(N2),NA,B,NB,P,NP,IOPt,SCLE,SYM,DUMMY(N3)) RIC
109 C
110    450 CONTINUE                     RIC
111    CALL EQUATE(P,NP,DUMMY(N2),NP)        RIC
112    IF(IOP(2).EQ. 0) GO TO 550         RIC
113    CALL LNCNT(3)                      RIC
114    PRINT 500,I                         RIC
115    500 FORMAT(/,' ITERATION ',IS,/)      RIC
116    CALL PRNT(P,NP,4H P ,1)            RIC
117 C
118    550 CONTINUE                     RIC
119    CALL MULT(DUMMY(N1),NA,P,NP,DUMMY(N3),NA) RIC
120    CALL MULT(P,NP,DUMMY(N3),NA,DUMMY(N4),NA) RIC
121    CALL TRANP(DUMMY(N4),NA,P,NA)          RIC
122    CALL ADD(P,NP,DUMMY(N4),NA,P,NP)        RIC
123    CALL SCALE(P,NP,P,NP,0.5)           RIC
124    CALL ADD(Q,NQ,P,NP,P,NP)           RIC
125    CALL SCALE(P,NP,P,NP,-1.0)          RIC

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126 CALL SUBT(A,NA,DUMMY(N3),NA,DUMMY(N4),NA) RIC01
127 CALL TRANP(DUMMY(N4),NA,DUMMY(N3),NA) RIC01
128 C RIC01
129 IF(IOP(3).NE.0) GO TO 650 RIC01
130 CALL BARSTW(DUMMY(N3),NA,B,NB,P,NP,IOPT,SYM,EPSA,EPSA,DUMMY(N4)) RIC01
131 GO TO 675 RIC01
132 C RIC01
133 650 CONTINUE RIC01
134 CALL BILIN(DUMMY(N3),NA,B,NB,P,NP,IOPT,SCLE,SYM,DUMMY(N4)) RIC01
135 C RIC01
136 675 CONTINUE RIC01
137 I=I+1 RIC01
138 CALL MAXEL(DUMMY(N2),NA,ANORM1) RIC01
139 CALL SUBT(P,NP,DUMMY(N2),NA,DUMMY(N3),NA) RIC01
140 CALL MAXEL(DUMMY(N3),NA,ANORM2) RIC01
141 IF(ANORM1.GT.1.0) GO TO 700 RIC01
142 IF(ANORM2/ANORM1.LT.RICTCV) GO TO 800 RIC01
143 GO TO 750 RIC01
144 C RIC01
145 700 CONTINUE RIC01
146 IF(ANORM2.LT.RICTCV) GO TO 800 RIC01
147 C RIC01
148 750 CONTINUE RIC01
149 IF(I.LE.101) GO TO 450 RIC01
150 CALL LNCNT(3) RIC01
151 PRINT 775 RIC01
152 775 FORMAT(//,' THE SUBROUTINE RICNWT HAS EXCEEDED 100 ITERATIONS WITH RIC01
153 1UT CONVERGENCE',//) RIC01
154 IOP(1)=1 RIC01
155 C RIC01
156 800 CONTINUE RIC01
157 CALL MULT(DUMMY,NF,P,NP,F,NF) RIC01
158 GO TO 1300 RIC01
159 C RIC01
160 900 CONTINUE RIC01
161 IF(.NOT.FNULL) GO TO 950 RIC01
162 C RIC01
163 CALL EQUATE(Q,NQ,P,NP) RIC01
164 CALL EQUATE(A,NA,DUMMY(N1),NA) RIC01
165 CALL TRANP(A,NA,DUMMY(N2),NA) RIC01
166 GO TO 1000 RIC01
167 925 CONTINUE RIC01
168 C RIC01
169 I=I+1 RIC01
170 CALL EQUATE(P,NP,DUMMY,NP) RIC01
171 950 CONTINUE RIC01
172 C RIC01
173 CALL MULT(R,NR,F,NF,DUMMY(N1),NF) RIC01
174 CALL TRANP(F,NF,P,NP) RIC01
175 CALL MULT(P,NP,DUMMY(N1),NF,DUMMY(N2),NA) RIC01
176 CALL TRANP(DUMMY(N2),NA,DUMMY(N1),NA) RIC01
177 CALL ADD(DUMMY(N1),NA,DUMMY(N2),NA,DUMMY(N1),NA) RIC01
178 CALL SCALE(DUMMY(N1),NA,DUMMY(N2),NA,0.5) RIC01
179 CALL ADD(Q,NQ,DUMMY(N1),NA,P,NP) RIC01
180 CALL MULT(B,NB,F,NF,DUMMY(N1),NA) RIC01
181 CALL SUBT(A,NA,DUMMY(N1),NA,DUMMY(N1),NA) RIC01
182 CALL TRANP(DUMMY(N1),NA,DUMMY(N2),NA) RIC01
183 C RIC01
184 1000 CONTINUE RIC01
185 CALL SUM(DUMMY(N2),NA,P,NP,DUMMY(N1),NA,IOPT,SYM,DUMMY(N3)) RIC01
186 IF(IOP(2).EQ.0) GO TO 1100 RIC01
187 CALL LNCNT(3) RIC01
188 PRINT 500,I RIC01

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189 CALL PRNT(P,NP,4H P ,1) RICO
190 C RICO
191 1100 CONTINUE RICO
192 CALL MULT(P,NP,A,NA,DUMMY(N1),NA) RICO
193 CALL MULT(P,NP,B,NB,DUMMY(N2),NB) RICO
194 CALL TRANP(B,NB,DUMMY(N3),NF) RICO
195 CALL MULT(DUMMY(N3),NF,DUMMY(N1),NA,F,NF) RICO
196 CALL MULT(DUMMY(N3),NF,DUMMY(N2),NB,DUMMY(N1),NR) RICO
197 CALL TRANP(DUMMY(N1),NR,DUMMY(N2),NR) RICO
198 CALL ADD(DUMMY(N1),NR,DUMMY(N2),NR,DUMMY(N1),NR) RICO
199 CALL SCALE(DUMMY(N1),NR,DUMMY(N1),NR,0.5) RICO
200 CALL ADD(R,NR,DUMMY(N1),NR,DUMMY(N1),NR) RICO
201 CALL SYMPDS(NR(1),NR(1),DUMMY(N1),NA(1),F,IOP,T,IOP,DET,ISCALE,DUMRICO
202 1MY(N2),IERR) RICO
203 IF(IERR .EQ. 0) GO TO 1150 RICO
204 CALL LNCNT(3) RICO
205 PRINT 225 RICO
206 RETURN RICO
207 C RICO
208 1150 CONTINUE RICO
209 IF( I .EQ. 1) GO TO 925 RICO
210 CALL MAXEL(DUMMY,NA,ANORM1) RICO
211 CALL SUBT(P,NP,DUMMY,NA,DUMMY(N1),NA) RICO
212 CALL MAXEL(DUMMY(N1),NA,ANORM2) RICO
213 IF( ANORM1 .GT. 1.) GO TO 1200 RICO
214 IF( ANORM2/ANORM1 .LT. RICTCV ) GO TO 1300 RICO
215 GO TO 1250 RICO
216 1200 CONTINUE RICO
217 IF( ANORM2 .LT. RICTCV ) GO TO 1300 RICO
218 C RICO
219 1250 CONTINUE RICO
220 IF( I .LE. 101) GO TO 925 RICO
221 CALL LNCNT(3) RICO
222 PRINT 775 RICO
223 IOP(1) = 1 RICO
224 C RICO
225 1300 CONTINUE RICO
226 IF(IOP(1) .EQ. 0 ) RETURN RICO
227 CALL LNCNT(4) RICO
228 PRINT 1350,I RICO
229 1350 FORMAT(//,' FINAL VALUES OF P AND F AFTER',I5,' ITERATIONS TO CONVRICO
230 1ERGE',/) RICO
231 CALL PRNT(P,NP,4H P ,1) RICO
232 CALL PRNT(F,NF,4H F ,1) RICO
233 C RICO
234 RETURN RICO
235 END RICO

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0      SUBROUTINE ASMREG(A,NA,B,NB,M,NH,Q,NQ,R,NP,F,NF,P,NP,IDENT,DISC,N  ASM0
1      IENWT,STABLE,FNULL,ALPHA,IOP,DUMMY)  ASM0
2      IMPLICIT REAL*8 (A-M,O-Z)  ASM0
3      DIMENSION A(1),B(1),M(1),Q(1),R(1),F(1),P(1),DUMMY(1)  ASM0
4      DIMENSION NA(2),NB(2),NH(2),NQ(2),NR(2),NF(2),NP(2),IOP(5),IOP(3)ASM0
5      1,NDUM1(2),NDUM2(2),NDUM3(2)  ASM0
6      LOGICAL IDENT,DISC,NEWT,STABLE,FNULL,SING  ASM0
7      N = NA(1)**2  ASM0
8      N1= N+1  ASM0
9      IOPTT=0  ASM0
10     IF (.NOT. NEWT) GO TO 600  ASM0
11     IF( STABLE ) GO TO 500  ASM0
12     IF ( FNULL ) GO TO 100  ASM0
13     CALL MULT(B,NB,F,NF,DUMMY,NA)  ASM0
14     CALL SUBT(A,NA,DUMMY,NA,DUMMY,NA)  ASM0
15     CALL TESTSA(DUMMY,NA,ALPHA,DISC,STABLE,IOPTT,DUMMY(N1))  ASM0
16     GO TO 200  ASM0
17 100 CONTINUE  ASM0
18     CALL TESTSA(A,NA,ALPHA,DISC,STABLE,IOPTT,DUMMY)  ASM0
19 C
20 200 CONTINUE  ASM0
21     IF( STABLE ) GO TO 500  ASM0
22     IF( DISC ) GO TO 230  ASM0
23     J = -NA(1)  ASM0
24     NAX = NA(1)  ASM0
25     DO 210 I =1,NAX  ASM0
26     J = J + NAX +1  ASM0
27     A(J) = A(J)-ALPHA  ASM0
28 210 CONTINUE  ASM0
29     SCLE = 3.  ASM0
30     IOPT(1)=IOP(1)  ASM0
31     IOPT(2) = 1  ASM0
32     IOPT(3)=1  ASM0
33     CALL CSTAB(A,NA,B,NB,F,NF,IOPT,SCLE,DUMMY)  ASM0
34     J = -NA(1)  ASM0
35     DO 220 I=1,NAX  ASM0
36     J = J + NAX + 1  ASM0
37     A(J) = A(J) + ALPHA  ASM0
38 220 CONTINUE  ASM0
39 225 CONTINUE  ASM0
40     CALL MULT(B,NB,F,NF,DUMMY,NA)  ASM0
41     CALL SUBT(A,NA,DUMMY,NA,DUMMY,NA)  ASM0
42     CALL TESTSA(DUMMY,NA,ALPHA,DISC,STABLE,IOPTT,DUMMY(N1))  ASM0
43     GO TO 300  ASM0
44 C
45 230 CONTINUE  ASM0
46     J = 2*NA(1) + 1  ASM0
47     IF( .NOT. FNULL ) J = J + N  ASM0
48     SING = .FALSE.  ASM0
49     IF( DUMMY(J) .EQ. 0.0 ) SING = .TRUE.  ASM0
50     IOPT(1) = IOP(1)  ASM0
51     IOPT(2) = 1  ASM0
52     DSCL = 0.5  ASM0
53     ALPHAT = 1./ALPHA  ASM0
54     CALL SCALE(A,NA,A,NA,ALPHAT)  ASM0
55     CALL SCALE(B,NB,B,NB,ALPHAT)  ASM0
56     CALL DSTAB(A,NA,B,NB,F,NF,SING,IOPT,DSCL,DUMMY)  ASM0
57     CALL SCALE(A,NA,A,NA,ALPHA)  ASM0
58     CALL SCALE(B,NB,B,NB,ALPHA)  ASM0
59     GO TO 225  ASM0
60 C
61 300 CONTINUE  ASM0
62     IF( STABLE) GO TO 400  ASM0

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63     CALL LNCNT(5)          ASMO1
64     IF( DISC ) GO TO 330   ASMO1
65     PRINT 310,ALPHA       ASMO1
66     310 FORMAT(//' IN ASMREG, CSTAB HAS FAILED TO FIND A STABILIZING GAIN ASMO1
67     1 MATRIX (F) RELATIVE TO ',/, ' ALPHA = ',016.8/) ASMO1
68     RETURN                ASMO1
69     330 CONTINUE          ASMO1
70     PRINT 340,ALPHA       ASMO1
71     340 FORMAT(//' IN ASMREG, DSTAB HAS FAILED TO FIND A STABILIZING GAIN ASMO1
72     1 MATRIX (F) RELATIVE TO ',/, ' ALPHA = ',016.8/) ASMO1
73     RETURN                ASMO1
74 C
75     400 CONTINUE          ASMO1
76     FNULL = .FALSE.       ASMO1
77 C
78     500 CONTINUE          ASMO1
79     CALL RICNWT(A,NA,B,NB,H,NH,Q,NQ,R,NR,F,NF,P,NP,IOP,IDENT,DISC,FNU) ASMO1
80     1LL,DUMMY)           ASMO1
81     GO TO 750             ASMO1
82 C
83     600 CONTINUE          ASMO1
84     IF( DISC ) GO TO 700   ASMO1
85     NW = 4*N + 1          ASMO1
86     NLAM = NW + 4*N       ASMO1
87     NDUM = NLAM + N       ASMO1
88     IOP(3) = 1             ASMO1
89     CALL CNTREG(A,NA,B,NB,H,NH,Q,NQ,R,NR,DUMMY,DUMMY(NW),DUMMY(NLAM)) ASMO1
90     1S,F,NF,P,NP,IOP,IDENT,DUMMY(NDUM)) ASMO1
91     GO TO 750             ASMO1
92     700 CONTINUE          ASMO1
93     CALL DISREG(A,NA,B,NB,H,NH,Q,NQ,R,NR,F,NF,P,NP,IOP,IDENT,DUMMY) ASMO1
94 C
95     750 CONTINUE          ASMO1
96 C
97     IF( IOP(4) .EQ. 0 ) GO TO 1100 ASMO1
98 C
99     N2= N1 + N           ASMO1
100    N3= N2 + N           ASMO1
101 C
102    IF( DISC ) GO TO 800   ASMO1
103    CALL MULT(P,NP,B,NB,DUMMY,NB) ASMO1
104    CALL MULT(DUMMY,NB,F,NF,DUMMY(N1),NP) ASMO1
105    CALL TRANP(DUMMY(N1),NP,DUMMY,NP) ASMO1
106    CALL ADD(DUMMY,NP,DUMMY(N1),NP,DUMMY,NP) ASMO1
107    CALL SCALE(DUMMY,NP,DUMMY,NP,0.5) ASMO1
108    CALL SUBT(Q,NQ,DUMMY,NP,DUMMY,NP) ASMO1
109    CALL MULT(P,NP,A,NA,DUMMY(N1),NP) ASMO1
110    CALL ADD(DUMMY,NP,DUMMY(N1),NP,DUMMY,NP) ASMO1
111    CALL TRANP(DUMMY(N1),NP,DUMMY(N2),NP) ASMO1
112    CALL ADD(DUMMY,NP,DUMMY(N2),NP,DUMMY,NP) ASMO1
113    GO TO 900             ASMO1
114 C
115    800 CONTINUE          ASMO1
116    CALL MULT(R,NR,F,NF,DUMMY,NF) ASMO1
117    CALL TRANP(F,NF,DUMMY(N1),NB) ASMO1
118    CALL MULT(DUMMY(N1),NB,DUMMY,NF,DUMMY(N2),NA) ASMO1
119    CALL ADD(DUMMY(N2),NA,Q,NQ,DUMMY,NA) ASMO1
120    CALL MULT(B,NB,F,NF,DUMMY(N1),NA) ASMO1
121    CALL SUBT(A,NA,DUMMY(N1),NA,DUMMY(N1),NA) ASMO1
122    CALL MULT(P,NP,DUMMY(N1),NA,DUMMY(N2),NA) ASMO1
123    CALL TRANP(DUMMY(N1),NA,DUMMY(N3),NA) ASMO1
124    CALL MULT(DUMMY(N3),NA,DUMMY(N2),NA,DUMMY(N1),NA) ASMO1
125    CALL ADD(DUMMY,NA,DUMMY(N1),NA,DUMMY,NA) ASMO1

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26 CALL SUBT(P,NP,DUMMY,NA,DUMMY,NA) ASM01
27 C ASM01
128 900 CONTINUE ASM01
129 CALL LNCNT(4) ASM01
30 PRINT 1000 ASM01
131 1000 FORMAT(//' RESIDUAL ERROR IN RICCATI EQUATION ') ASM01
132 CALL PRNT(DUMMY,NP,4HEROR,1) ASM01
133 C ASM01
134 1100 CONTINUE ASM01
135 N2= N1+NA(1) ASM01
136 N3= N2+NA(1) ASM01
137 ISV = 0 ASM01
138 CALL EQUATE(P,NP,DUMMY,NP) ASM01
139 CALL EIGEN(NA(1),NA(1),DUMMY,DUMMY(N1),DUMMY(N2),ISV,ISV,V,DUMMY(NASM01
140 13),IERR) ASM01
141 NEVL = NA(1) ASM01
142 IF( IERR .EQ. 0 ) GO TO 1300 ASM01
143 NEVL=NA(1)-IERR ASM01
144 CALL LNCNT(4) ASM01
145 PRINT 1200,IERR ASM01
146 1200 FORMAT(//' IN ASMREG, THE ',IS,' EIGENVALUE OF P HAS NOT BEEN COASM01
147 MPUTED AFTER 30 ITERATIONS ') ASM01
148 C ASM01
149 1300 CONTINUE ASM01
150 NDUM1(1) = NEVL ASM01
151 NDUM1(2) = 1 ASM01
152 CALL EQUATE(DUMMY(N1),NDUM1,DUMMY,NDUM1) ASM01
153 N1 = NDUM1(1) +1 ASM01
154 CALL MULT(B,NB,F,NF,DUMMY(N1),NA) ASM01
155 CALL SUBT(A,NA,DUMMY(N1),NA,DUMMY(N1),NA) ASM01
156 N2 = N1+N ASM01
157 CALL EQUATE(DUMMY(N1),NA,DUMMY(N2),NA) ASM01
158 N3=N2+N ASM01
159 N4=N3+NA(1) ASM01
160 N5=N4+NA(1) ASM01
161 CALL EIGEN(NA(1),NA(1),DUMMY(N2),DUMMY(N3),DUMMY(N4),ISV,ISV,V,DUMASM01
162 1MY(N5),IERR) ASM01
163 NEVL = NA(1) ASM01
164 IF( IERR .EQ. 0 ) GO TO 1500 ASM01
165 NEVL=NA(1)-IERR ASM01
166 CALL LNCNT(4) ASM01
167 PRINT 1400,IERR ASM01
168 1400 FORMAT(//' IN ASMREG, THE ',IS,' EIGENVALUE OF A-BF HAS NOT BEEN CASM01
169 MPUTED AFTER 30 ITERATIONS ') ASM01
170 C ASM01
171 1500 CONTINUE ASM01
172 NDUM2(1) = NEVL ASM01
173 NDUM2(2) = 1 ASM01
174 CALL JUXTA(DUMMY(N3),NDUM2,DUMMY(N4),NDUM2,DUMMY(N2),NDUM3) ASM01
175 C ASM01
176 IF ( IOP(5) .EQ. 0 ) RETURN ASM01
177 C ASM01
178 CALL LNCNT(4) ASM01
179 PRINT 1600 ASM01
180 1600 FORMAT(//' EIGENVALUES OF P ') ASM01
181 CALL PRNT(DUMMY,NDUM1,4HEVLP,1) ASM01
182 CALL LNCNT(4) ASM01
183 PRINT 1700 ASM01
184 1700 FORMAT(//' CLOSED-LOOP RESPONSE MATRIX A-BF ') ASM01
185 CALL PRNT(DUMMY(N1),NA,4HA-BF,1) ASM01
186 CALL LNCNT(3) ASM01
187 PRINT 1800 ASM01
188 1800 FORMAT(//' EIGENVALUES OF A-BF') ASM01

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189 CALL PRNT('UMMY(N2),NOUM3,0,3)
190 C
191 RETURN
192 END

ASMO
ASMO
ASMO
ASMO

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0      SUBROUTINE ASMFILE(A,NA,G,NG,H,NH,Q,NQ,R,NR,F,NF,P,NP,IDENT,DISC,N AS
1      NEWT,STABLE,FNULL,ALPHA,IOP,DUMMY) AS
2      IMPLICIT REAL*8 (A-H,O-Z) AS
3      DIMENSION A(1),G(1),H(1),Q(1),R(1),F(1),P(1),DUMMY(1) AS
4      DIMENSION NA(2),NG(2),NH(2),NQ(2),NR(2),NF(2),NP(2),IOP(5),NDUM1(AS AS
5      12),IOP(1) AS
6      LOGICAL IDENT,DISC,NEWT,STABLE,FNULL AS
7      IF( IOP(1) .EQ. 0 ) GO TO 100 AS
8      CALL LNCNT(4) AS
9      IF(DISC) PRINT 15 AS
10     IF( .NOT. DISC ) PRINT 25 AS
11     15 FORMAT(//,' PROGRAM TO SOLVE THE DISCRETE INFINITE-DURATION OPTIMA AS
12     1L FILTER PROBLEM',//) AS
13     25 FORMAT(//,' PROGRAM TO SOLVE THE CONTINUOUS INFINITE-DURATION OPTI AS
14     1MAL FILTER PROBLEM',//) AS
15     CALL PRNT(A,NA,4H A ,1) AS
16     IF( .NOT. IDENT ) GO TO 35 AS
17     CALL LNCNT(3) AS
18     PRINT 30 AS
19     30 FORMAT(//,' G IS AN IDENTITY MATRIX',//) AS
20     GO TO 40 AS
21     35 CONTINUE AS
22     CALL PRNT(G,NG,4H G ,1) AS
23     40 CONTINUE AS
24     CALL PRNT(H,NH,4H H ,1) AS
25     CALL LNCNT(3) AS
26     PRINT 45 AS
27     45 FORMAT(//,' INTENSITY MATRIX FOR COVARIANCE OF MEASUREMENT NOISE',//) AS
28     CALL PRNT(R,NR,4H R ,1) AS
29 C
30     IF( .NOT. IDENT ) GO TO 65 AS
31     CALL LNCNT(3) AS
32     PRINT 55 AS
33     55 FORMAT(//,' INTENSITY MATRIX FOR COVARIANCE OF PROCESS NOISE',//) AS
34 C
35     65 CONTINUE AS
36     CALL PRNT(Q,NQ,4H Q ,1) AS
37 C
38     100 CONTINUE AS
39     IOP(1)=IOP(2) AS
40     IOP(2)=IOP(3) AS
41     IOP(3)=IOP(4) AS
42     IOP(4)=IOP(5) AS
43     IOP(5)=0 AS
44     K = 0 AS
45 C
46     200 CONTINUE AS
47     CALL TRANP(A,NA,DUMMY,NA) AS
48     CALL EQUATE(DUMMY,NA,A,NA) AS
49     CALL TRANP(H,NH,DUMMY,NDUM1) AS
50     CALL EQUATE(DUMMY,NDUM1,H,NH) AS
51     IF( IDENT ) GO TO 250 AS
52     CALL TRANP(G,NG,DUMMY,NDUM1) AS
53     CALL EQUATE(DUMMY,NDUM1,G,NG) AS
54     250 CONTINUE AS
55     IF ( K .EQ. 1 ) RETURN AS
56 C
57     K = K+1 AS
58     CALL ASMREG(A,NA,H,NH,G,NG,Q,NQ,R,NR,F,NF,P,NP,IDENT,DISC,NEWT,ST AS
59     TABLE,FNULL,ALPHA,IOP,DUMMY) AS
60 C
61     N1=(NA(1)**2)+3*NA(1)+1 AS
62     CALL TRANP(F,NF,DUMMY(N1),NDUM1) AS

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63 CALL EQUATE(DUMMY(N1),NDUM1,F,NF) ASMO
64 C ASMO
65 IF( IOP(1) .EQ. 0 ) GO TO 200 ASMO
66 C ASMO
67 IF(IDENT) GO TO 300 ASMO
68 CALL LNCNT(3) ASMO
69 PRINT 55 ASMO
70 CALL PRNT(Q,NQ,4HG0GT,1) ASMO
71 C ASMO
72 300 CONTINUE ASMO
73 CALL LNCNT(3) ASMO
74 PRINT 325 ASMO
75 325 FORMAT(/,' FILTER GAIN',/) ASMO
76 CALL PRNT(F,NF,4H F ,1) ASMO
77 CALL LNCNT(3) ASMO
78 PRINT 350 ASMO
79 350 FORMAT(/,' STEADY-STATE VARIANCE MATRIX OF RECONSTRUCTION ERROR',/) ASMO
80 CALL PRNT(P,NP,4H P ,1) ASMO
81 NDUM1(1)=NP(1) ASMO
82 NDUM1(2)=1 ASMO
83 CALL LNCNT(3) ASMO
84 PRINT 375 ASMO
85 375 FORMAT(/,' EIGENVALUES OF P ',/) ASMO
86 CALL PRNT(DUMMY,NDUM1,4HEVLP,1) ASMO
87 N1 = NP(1) + 1 ASMO
88 N = NA(1)**2 ASMO
89 N2 = N1 + N + 2*NA(1) ASMO
90 CALL TRAMP(DUMMY(N1),NA,DUMMY(N2),NA) ASMO
91 CALL PRNT(DUMMY(N2),NA,4HA=FH,1) ASMO
92 N2 = N1 + N ASMO
93 CALL LNCNT(3) ASMO
94 PRINT 385 ASMO
95 385 FORMAT(/,' EIGENVALUES OF A-FH MATRIX',/) ASMO
96 NDUM1(1) = NA(1) ASMO
97 NDUM1(2) = 2 ASMO
98 CALL PRNT(DUMMY(N2),NDUM1,0,3) ASMO
99 C ASMO
100 GO TO 200 ASMO
101 C ASMO
102 END ASMO

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0   SUBROUTINE EXPMDP (A,NA,B,NB,H,NH,AM,NAM,HM,NHM,Q,NQ,R,NR,F,NF,P, EXP0
1   INP,HIDENT,HMDENT,DISC,NEWT,STABLE,FNULL,ALPHA,IOP,DUMMY) EXP0
2   IMPLICIT REAL*8 (A-H,O-Z) EXP0
3   DIMENSION A(1),B(1),H(1),AM(1),HM(1),Q(1),R(1),F(1),P(1),DUMMY(1) EXP0
4   DIMENSION NA(2),NB(2),NH(2),NAM(2),NBM(2),NQ(2),NR(2),NF(2),NP(2),EXP0
5   IOP(1),IOP(5),NDUM1(2),NDUM2(2),NDUM3(2) EXP0
6   LOGICAL HIDENT,HMDENT,DISC,NEWT,STABLE,FNULL,SYM EXP0
7   COMMON/TOL/EPSAM,EPSBM,IACM EXP0
8   IF( IOP(1) .EQ. 0 ) GO TO 300 EXP0
9   CALL LNCNT(6) EXP0
10  IF( DISC ) PRINT 25 EXP0
11  IF( .NOT. DISC ) PRINT 50 EXP0
12  25 FORMAT(//,' PROGRAM TO SOLVE ASYMPTOTIC DISCRETE EXPLICIT MODEL-FOLEXP0
13  1LOWING PROBLEM',//,' PLANT DYNAMICS',/) EXP0
14  50 FORMAT(//,' PROGRAM TO SOLVE ASYMPTOTIC CONTINUOUS EXPLICIT MODEL-FEXP0
15  1OLLOWING PROBLEM',//,' PLANT DYNAMICS',/) EXP0
16  CALL PRNT(A,NA,4H A ,1) EXP0
17  CALL PRNT(B,NB,4H B ,1) EXP0
18  IF( HIDENT ) GO TO 75 EXP0
19  CALL PRNT(H,NH,4H H ,1) EXP0
20  GO TO 100 EXP0
21  75 CONTINUE EXP0
22  CALL LNCNT(3) EXP0
23  PRINT 85 EXP0
24  85 FORMAT(//,' H IS AN IDENTITY MATRIX',/) EXP0
25 C
26  100 CONTINUE EXP0
27  CALL LNCNJ(4) EXP0
28  PRINT 125 EXP0
29  125 FORMAT(//,' MODEL DYNAMICS',/) EXP0
30  CALL PRNT(AM,NAM,4H AM ,1) EXP0
31  IF( HMDENT ) GO TO 175 EXP0
32  CALL PRNT(HM,NHM,4H HM ,1) EXP0
33  GO TO 200 EXP0
34  175 CONTINUE EXP0
35  CALL LNCNT(3) EXP0
36  PRINT 185 EXP0
37  185 FORMAT(//,' HM IS AN IDENTITY MATRIX ',/) EXP0
38 C
39  200 CONTINUE EXP0
40  CALL LNCNT(4) EXP0
41  PRINT 225 EXP0
42  225 FORMAT(//,' WEIGHTING MATRICES ',/) EXP0
43  CALL PRNT(Q,NQ,4H Q ,1) EXP0
44  CALL PRNT(R,NR,4H R ,1) EXP0
45 C
46  300 CONTINUE EXP0
47  IF( IOP(2) .EQ. 0 ) GO TO 400 EXP0
48  NF(1) = NB(2) EXP0
49  NF(2) = NA(1) EXP0
50  NP(1) = NA(1) EXP0
51  NP(2) = NA(1) EXP0
52  IOP(1) = IOP(3) EXP0
53  IOP(2) = IOP(4) EXP0
54  IOP(3) = IUP(5) EXP0
55  IOP(4) = 0 EXP0
56  IOP(5) = 0 EXP0
57  N1 = NA(1)*NA(2) + 1 EXP0
58  CALL EQUATE(Q,NQ,DUMMY,NQ) EXP0
59  CALL ASMREG(A,NA,B,NB,H,NH,DUMMY,NQ,R,NR,F,NF,P,NP,HIDENT,DISC,NE EXP0
60  IWT,STABLE,FNULL,ALPHA,IOP,DUMMY(N1)) EXP0
61 C
62  400 CONTINUE EXP0

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63 IF( IOP(1) .EQ. 0 ) GO TO 600 EXP01
64 CALL LNCNT(4) EXP01
65 PRINT 425 EXP01
66 425 FORMAT(//,' CONTROL LAW U = F( COL.(X,XM) ), F = (F11,F12)',/) EXP01
67 CALL LNCNT(3) EXP01
68 PRINT 450 EXP01
69 450 FORMAT(//,' PART OF F MULTIPLYING X ',/) EXP01
70 CALL PRNT(F,NF,4H F11,1) EXP01
71 IF( .NOT. DISC .AND. IOP(2) .EQ. 0 ) GO TO 600 EXP01
72 CALL PRNT(P,NP,4H P11,1) EXP01
73 IF( IOP(2) .EQ. 0 ) GO TO 600 EXP01
74 CALL LNCNT(2) EXP01
75 PRINT 475 EXP01
76 475 FORMAT(//,' EIGENVALUES OF P11') EXP01
77 NDOUM1(1) = NA(1) EXP01
78 NDOUM1(2) = 1 EXP01
79 CALL PRNT(DUMMY(N1),NDOUM1,0,3) EXP01
80 N1 = N1 + NDOUM1(1) EXP01
81 NDOUM1(2) = NA(1) EXP01
82 CALL LNCNT(2) EXP01
83 PRINT 500 EXP01
84 500 FORMAT(//,' PLANT CLOSED-LOOP RESPONSE MATRIX A - BF11') EXP01
85 CALL PRNT(DUMMY(N1),NDOUM1,0,3) EXP01
86 CALL LNCNT(2) EXP01
87 PRINT 525 EXP01
88 525 FORMAT(//,' EIGENVALUES OF CLOSED-LOOP RESPONSE MATRIX') EXP01
89 N1 = N1 + NDOUM1(1)*NDOUM1(2) EXP01
90 NDOUM1(2) = 2 EXP01
91 CALL PRNT(DUMMY(N1),NDOUM1,0,3) EXP01
92 C EXP01
93 600 CONTINUE EXP01
94 NF(1)= NB(2) EXP01
95 NF(2)= NA(1) EXP01
96 CALL MULT(B,NB,F,NF,DUMMY,NA) EXP01
97 CALL SUBT(A,NA,DUMMY,NA,DUMMY,NA) EXP01
98 IF( IOP(1).EQ. 0 .OR. IOP(2) .NE. 0 ) GO TO 700 EXP01
99 CALL LNCNT(2) EXP01
100 PRINT 500 EXP01
101 CALL PRNT(DUMMY,NA,0,3) EXP01
102 C EXP01
103 700 CONTINUE EXP01
104 N1 = NA(1)**2 +1 EXP01
105 CALL TRANP(DUMMY,NA,DUMMY(N1),NA) EXP01
106 CALL EQUATE(DUMMY(N1),NA,DUMMY,NA) EXP01
107 NF(2) = NA(1) + NAM(1) EXP01
108 NP(2) = NF(2) EXP01
109 IF( .NOT. DISC .AND. IOP(2).EQ. 0 ) NP(2) = NAM(2) EXP01
110 IOPTT=0 EXP01
111 SYM = .FALSE. EXP01
112 CALL EQUATE( Q,NQ,DUMMY(N1),NDOUM2) EXP01
113 IF( HMDENT ) GO TO 725 EXP01
114 CALL MULT(Q,NQ,HM,NHM,DUMMY(N1),NDOUM2) EXP01
115 725 CONTINUE EXP01
116 IF( HIDENT ) GO TO 750 EXP01
117 N2 = N1 + NQ(1)*NHM(2) EXP01
118 CALL TRANP(H,NH,DUMMY(N2),NDOUM1) EXP01
119 N3 = N2 + NH(1)*NH(2) EXP01
120 CALL MULT(DUMMY(N2),NDOUM1,DUMMY(N1),NHM,DUMMY(N3),NDOUM2) EXP01
121 CALL EQUATE(DUMMY(N3),NDOUM2,DUMMY(N1),NDOUM2) EXP01
122 750 CONTINUE EXP01
123 N2 = NA(1)**2 + NA(1)*NHM(2) + 1 EXP01
124 N3 = NA(1)**2 + 1 EXP01
125 IF( .NOT. DISC .AND. IOP(2) .EQ. 0 ) N3 = 1 EXP01

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126 CALL EQUATE(DUMMY(N1),NDUM2,P(N3),NDUM2) EXP0
127 IF( DISC ) GO TO 800 EXP0
128 EPSA = EPSAM EXP0
129 CALL BARSTW(DUMMY,NA,AM,NAM,P(N3),NDUM2,IOPTT,SYM ,EPSA,EPSA,DUMMY EXP0
130 1(N2)) EXP0
131 GO TO 900 EXP0
132 C EXP0
133 800 CONTINUE EXP0
134 CALL SCALE(P(N3),NDUM2,P(N3),NDUM2,-1.0) EXP0
135 N4 = N2 +NAM(1)**2 EXP0
136 CALL EQUATE(AM,NAM,DUMMY(N2),NAM) EXP0
137 CALL SUM(DUMMY,NA,P(N3),NDUM2,DUMMY(N2),NAM,IOPTT,SYM,DUMMY(N4)) EXP0
138 C EXP0
139 900 CONTINUE EXP0
140 N2 = NB(2)*NA(1) + 1 EXP0
141 CALL TRANP(B,NB,DUMMY,NDUM1) EXP0
142 CALL MULT(DUMMY,NDUM1,P(N3),NDUM2,F(N2),NDUM3) EXP0
143 IF( .NOT. DISC ) GO TO 1000 EXP0
144 N1 = NB(1)*NB(2) + 1 EXP0
145 CALL MULT(DUMMY,NDUM1,P,NA,DUMMY(N1),NDUM2) EXP0
146 CALL MULT(DUMMY(N1),NDUM2,B,NB,DUMMY,NR) EXP0
147 CALL ADD(R,NR,DUMMY,NR,DUMMY,NR) EXP0
148 GO TO 1100 EXP0
149 C EXP0
150 1000 CONTINUE EXP0
151 CALL EQUATE(R,NR,DUMMY,NR) EXP0
152 C EXP0
153 1100 CONTINUE EXP0
154 N1 = NR(1)**2 + 1 EXP0
155 CALL SYMPDS(NR(1),NR(1),DUMMY,NHM(2),F(N2),IOPTT,IOPTT,DETERM,ISCA EXP0
156 1LE,DUMMY(N1),IERR) EXP0
157 IF( IERR .EQ. 0 ) GO TO 1200 EXP0
158 CALL LNCNT(3) EXP0
159 PRINT 1150 EXP0
160 1150 FORMAT(/,' IN EXPMDF, THE COEFFICIENT MATRIX FOR SYMPDS IS NOT SY EXP0
161 MMETRIC POSITIVE DEFINITE ',/) EXP0
162 RETURN EXP0
163 C EXP0
164 1200 CONTINUE EXP0
165 IF( .NOT. DISC ) GO TO 1300 EXP0
166 CALL MULT(F(N2),NDUM3,AM,NAM,DUMMY,NDUM1) EXP0
167 CALL EQUATE(DUMMY,NDUM1,F(N2),NDUM1) EXP0
168 1300 CONTINUE EXP0
169 IF( IOP(1) .EQ. 0 ) RETURN EXP0
170 CALL LNCNT(3) EXP0
171 PRINT 1325 EXP0
172 1325 FORMAT(/,' PART OF F MULTIPLYING XM ',/) EXP0
173 CALL PRNT(F(N2),NDUM3,4H F12,1) EXP0
174 NDUM1(1) = NA(1) EXP0
175 NDUM1(2) = NAM(1) EXP0
176 CALL PRNT(P(N3),NDUM1,4H P12,1) EXP0
177 RETURN EXP0
178 END EXP0

```

```

0      SUBROUTINE IMPMD( A,NA,B,NB,H,NH,AM,NAM,BM,NBM,Q,NQ,R,NR,F,NF,P,N )IMPO
1      IP,IDENT,DISC,NEWT,STABLE,FNULL,ALPHA,IOP,DUMMY)                         IMPO
2      IMPLICIT REAL*8 (A-H,O-Z)                                                 IMPO
3      DIMENSION A(1),B(1),H(1),AM(1),BM(1),Q(1),R(1),F(1),P(1),DUMMY(1)    IMPO
4      DIMENSION NA(2),NB(2),NH(2),NAM(2),NBM(2),NQ(2),NR(2),NF(2),NP(2),    IMPO
5      IOP(1),IOP(5),NDUM1(2)                                                 IMPO
6      LOGICAL IDENT,DISC,NEWT,STABLE,FNULL,HIDENT                           IMPO
7      IF( IOP(1) .EQ. 0 ) GO TO 200                                         IMPO
8      CALL LNCNT(6)                                                       IMPO
9      IF( DISC ) PRINT 25                                                 IMPO
10     IF( .NOT. DISC ) PRINT 50                                             IMPO
11     25 FORMAT(//,'PROGRAM TO SOLVE ASYMPTOTIC DISCRETE IMPLICIT MODEL-FOLLOWING PROBLEM',//,'PLANT DYNAMICS',//) IMPO
12     50 FORMAT(//,'PROGRAM TO SOLVE ASYMPTOTIC CONTINUOUS IMPLICIT MODEL-FOLLOWING PROBLEM',//,'PLANT DYNAMICS',//) IMPO
13     CALL PRNT(A,NA,4H A ,1)                                              IMPO
14     CALL PRNT(B,NB,4H B ,1)                                              IMPO
15     IF( IDENT ) GO TO 75                                                 IMPO
16     CALL PRNT(H,NH,4H H ,1)                                              IMPO
17     GO TO 100                                                       IMPO
18     75 CONTINUE                                                       IMPO
19     CALL LNCNT(3)                                                       IMPO
20     PRINT 55                                                       IMPO
21     85 FORMAT(//,'H IS AN IDENTITY MATRIX',//)                           IMPO
22     C
23     100 CONTINUE                                                       IMPO
24     CALL LNCNT(4)                                                       IMPO
25     PRINT 125                                                       IMPO
26     125 FORMAT(//,'MODEL DYNAMICS',//)                                     IMPO
27     CALL PRNT(AM,NAM,4H AM ,1)                                           RMP0
28     CALL PRNT(BM,NBM,4H BM ,1)                                           IMPO
29     CALL LNCNT(4)                                                       IMPO
30     PRINT 150                                                       IMPO
31     150 FORMAT(//,'WEIGHTING MATRICES',//)                                IMPO
32     CALL PRNT(Q,NQ,4H Q ,1)                                              IMPO
33     CALL PRNT(R,NR,4H R ,1)                                              IMPO
34     C
35     200 CONTINUE                                                       IMPO
36     N = NA(1)**2                                                       IMPO
37     N1 = N + 1                                                       IMPO
38     IF( .NOT. IDENT ) GO TO 300                                         IMPO
39     CALL SUBT(A,NA,AM,NAM,DUMMY,NA)                                       IMPO
40     CALL SUBT(B,NB,BM,NBM,DUMMY(N1),NB)                                     IMPO
41     GO TO 400                                                       IMPO
42     C
43     300 CONTINUE                                                       IMPO
44     CALL MULT(H,NH,A,NA,DUMMY,NH)                                         IMPO
45     CALL MULT(AM,NAM,H,NH,DUMMY(N1),NH)                                     IMPO
46     CALL SUBT(DUMMY,NH,DUMMY(N1),NH,DUMMY,NH)                            IMPO
47     CALL MULT(H,NH,B,NB,DUMMY(N1),NBM)                                     IMPO
48     CALL SUBT(DUMMY(N1),NBM,BM,NBM,DUMMY(N1),NBM)                          IMPO
49     C
50     400 CONTINUE                                                       IMPO
51     IF( IOP(1) .EQ. 0 ) GO TO 500                                         IMPO
52     CALL LNCNT(3)                                                       IMPO
53     PRINT 450                                                       IMPO
54     450 FORMAT(//,'MATRIX HA - AMH')                                     IMPO
55     CALL PRNT(DUMMY,NH,0,3)                                              IMPO
56     CALL LNCNT(3)                                                       IMPO
57     PRINT 475                                                       IMPO
58     475 FORMAT(//,'MATRIX HB - BM')                                     IMPO
59     CALL PRNT(DUMMY(N1),NBM,0,3)                                         IMPO
60     C
61     62 C

```

```

63 500 CONTINUE IMP
64 N2 = N1 + N IMP
65 N3 = N2 + N IMP
66 N4 = N3 + N IMP
67 CALL MULT(0,NQ,DUMMY,NH,DUMMY(N2),NH) IMP
68 CALL MULT(0,NQ,DUMMY(N1),NBM,DUMMY(N3),NBM) IMP
69 CALL TRANP(DUMMY,NH,DUMMY(N4),NDUM1) IMP
70 CALL MULT(DUMMY(N4),NDUM1,DUMMY(N2),NH,DUMMY,NA) IMP
71 CALL MULT(DUMMY(N4),NDUM1,DUMMY(N3),NBM,DUMMY(N2),NB) IMP
72 CALL TRANP(DUMMY(N1),NB,DUMMY(N4),NDUM1) IMP
73 CALL SCALE(DUMMY(N2),NB,DUMMY(N1),NB,2.0) IMP
74 CALL MULT(DUMMY(N4),NDUM1,DUMMY(N3),NR,DUMMY(N2),NR) IMP
75 CALL ADD(DUMMY(N2),NR,R,NDUM1,DUMMY(N2),NR) IMP
76 IF( IOP(1) .EQ. 0 ) GO TO 600 IMP
77 CALL LNCNT(3) IMP
78 PRINT 525 IMP
79 525 FORMAT(//,' MATRIX ( HA - AMH TRANSPOSE)Q( HA - AMH)' ) IMP
80 CALL PRNT(DUMMY,NA,0,3) IMP
81 CALL LNCNT(3) IMP
82 PRINT 550 IMP
83 550 FORMAT(//,' MATRIX 2( HA - AMH TRANSPOSE)Q( HB - BM)' ) IMP
84 CALL PRNT(DUMMY(N1),NB,0,3) IMP
85 CALL LNCNT(3) IMP
86 PRINT 575 IMP
87 575 FORMAT(//,' MATRIX ( HB - BM TRANSPOSE)Q( HB - BM ) + R' ) IMP
88 CALL PRNT(DUMMY(N2),NR,0,3) IMP
89 C IMP
90 600 CONTINUE IMP
91 IOPT(1)= 0 IMP
92 IOPT(2)= 1 IMP
93 IOPT(3)= 1 IMP
94 N5 = N4 + N IMP
95 CALL EQUATE(A,NA,DUMMY(N3),NA) IMP
96 CALL PREFIL(DUMMY(N3),NA,8,NB,DUMMY,NA,DUMMY(N1),NB,DUMMY(N2),NR,DIMPC
97 DUMMY(N4),NF,IOPT,DUMMY(N5)) IMP
98 IF( IOP(1) .EQ. 0 ) GO TO 700 IMP
99 CALL LNCNT(3) IMP
100 PRINT 625 IMP
101 625 FORMAT(//,' PREFILTER GAIN') IMP
102 CALL PRNT(DUMMY(N4),NF,0,3) IMP
103 CALL LNCNT(3) IMP
104 PRINT 650 IMP
105 650 FORMAT(//,' MATRIX A - B(PREFILTER)' ) IMP
106 CALL PRNT(DUMMY(N3),NA,0,3) IMP
107 CALL LNCNT(3) IMP
108 PRINT 675 IMP
109 675 FORMAT(//,' MODIFIED STATE VECTOR WEIGHTING MATRIX') IMP
110 CALL PRNT(DUMMY,NA,0,3) IMP
111 C IMP
112 700 CONTINUE IMP
113 CALL EQUATE(DUMMY(N4),NF,DUMMY(N1),NF) IMP
114 C IMP
115 IF( IOP(2) .EQ. -1000 ) RETURN IMP
116 C IMP
117 IOPT(1) = IOP(2) IMP
118 IOPT(2) = IOP(3) IMP
119 IOPT(3) = IOP(4) IMP
120 IOPT(4) = 0 IMP
121 IOPT(5) = 0 IMP
122 HIDENT = .TRUE. IMP
123 CALL ASMREG(DUMMY(N3),NA,B,NB,H,NH,DUMMY,NA,DUMMY(N2),NR,F,NF,P,N IMP
124 IP,HIDENT,DISC,NENT,STABLE,FNULL,ALPHA,IOPT,DUMMY(N4)) IMP
125 IF( IOP(1) .EQ. 0 ) GO TO 800 IMP

```

```

126 CALL LNCNT(3) IMPO
127 PRINT 725 IMPO
128 725 FORMAT(//, ' GAIN FROM ASMREG')
129 CALL PRNT(F,NF,0,3) IMPO
130 CALL LNCNT(3) IMPO
131 PRINT 750 IMPO
132 750 FORMAT(//, ' SOLUTION OF ASSOCIATED STEADY-STATE RICCATI EQUATION') IMPO
133 CALL PRNT(P,NP,0,3) IMPO
134 CALL LNCNT(3) IMPO
135 PRINT 775 IMPO
136 775 FORMAT(//, ' EIGENVALUES OF P')
137 NDUM1(1)= NA(1) IMPO
138 NDUM1(2)= 1 IMPO
139 CALL PRNT(DUMMY(N4),NDUM1,0,3) IMPO
140 C IMPO
141 800 CONTINUE IMPO
142 CALL ADD(F,NF,DUMMY(N1),NF,F,NF) IMPO
143 IF( IOP(1) .EQ. 0 ) RETURN IMPO
144 CALL LNCNT(4) IMPO
145 PRINT 825 IMPO
146 825 FORMAT(//, ' GAIN FOR MODEL-FOLLOWING CONTROL LAW, U = - F X , F = IMPO
147 L(PREFILTER) + (ASMREG) ', /) IMPO
148 CALL PRNT(F,NF,4H F ,1) IMPO
149 N6 = N4 + NA(1) IMPO
150 CALL PRNT(DUMMY(N6),NA,4H A-BF,1) IMPO
151 NDUM1(2) = 2 IMPO
152 N6 = N6 + N IMPO
153 CALL LNCNT(3) IMPO
154 PRINT 850 IMPO
155 850 FORMAT(//, ' EIGENVALUES OF A-BF')
156 CALL PRNT(DUMMY(N6),NDUM1,0,3) IMPO
157 C IMPO
158 RETURN IMPO
159 END IMPO

```

```
0      SUBROUTINE READ1 (A,NA,NZ,NAM)          REA
1      IMPLICIT REAL*8 (A-H,O-Z)              REA
2      DIMENSION A(1),NA(2),NZ(2)            REA
3      IF (NZ(1).EQ.0) GO TO 410           REA
4      NR=NZ(1)                            REA
5      NC=NZ(2)                            REA
6      NLST=NR*NC                          REA
7      IF( NLST .LT. 1 .OR. NR .LT. 1 ) GO TO 16 REA
8      DO 400 I = 1, NR                  REA
9      400 READ (5,101) (A(J), J = I,NLST,NR) REA
10     NA(1)=NR                           REA
11     NA(2)=NC                           REA
12     410 CALL PRNT (A,NA,NAM,1)          REA
13     101 FORMAT(8D10.2)                 REA
14     RETURN                             REA
15     16 CALL LNCNT(1)                  REA
16     WRITE (6,916) NAM, NR, NC          REA
17     916 FORMAT (' ERROR IN READ1 MATRIX ',A4,' HAS NA=',2I6) REA
18     RETURN                             REA
19     END                                REA
```

```

0      SUBROUTINE BALANC(NM,N,A,LOW,IGH,SCALE)          BAL00
1      IMPLICIT REAL*8 (A-H,O-Z)                      BAL00
2      INTEGER I,J,K,L,M,N,JJ,NM,IGH,LOW,IEXC        BAL00
3      DIMENSION A(NM,N),SCALE(N)                     BAL00
4 C      REAL C,F,G,R,S,B2,RADIX                      BAL00
5 C      REAL DABS                         BAL00
6 C      LOGICAL NOCONV                         BAL00
7 C
8 C
9 C      ***** RADIX IS A MACHINE DEPENDENT PARAMETER SPECIFYING    BAL00
10 C           THE BASE OF THE MACHINE FLOATING POINT REPRESENTATION.   BAL00
11 C
12 C
13 C      RADIX = 16.                         BAL00
14 C
15 C      B2 = RADIX * RADIX                  BAL00
16 C      K = 1                           BAL00
17 C      L = N                           BAL00
18 C      GO TO 100                      BAL00
19 C      ***** IN-LINE PROCEDURE FOR ROW AND             BAL00
20 C           COLUMN EXCHANGE *****                   BAL00
21 C      20 SCALE(M) = J                  BAL00
22 C      IF (J .EQ. M) GO TO 50            BJL00
23 C
24 C      DO 30 I = 1, L                 BAL00
25 C          F = A(I,J)                BAL00
26 C          A(I,J) = A(I,M)            BAL00
27 C          A(I,M) = F              BAL00
28 C      30 CONTINUE                  BAL00
29 C
30 C      DO 40 I = K, N                 BAL00
31 C          F = A(J,I)                BAL00
32 C          A(J,I) = A(M,I)            BAL00
33 C          A(M,I) = F              BAL00
34 C      40 CONTINUE                  BAL00
35 C
36 C      50 GO TO (80,130), IEXC        BAL00
37 C      ***** SEARCH FOR ROWS ISOLATING AN EIGENVALUE    BAL00
38 C           AND PUSH THEM DOWN *****                  BAL00
39 C      80 IF (L .EQ. 1) GO TO 280        BAL00
40 C      L = L - 1                  BAL00
41 C      ***** FOR J=L STEP -1 UNTIL 1 DO *****       BAL00
42 C      100 DO 120 JJ = 1, L            BAL00
43 C          J = L + 1 - JJ            BAL00
44 C
45 C      DO 110 I = 1, L               BAL00
46 C          IF (I .EQ. J) GO TO 110        BAL00
47 C          IF (A(J,I) .NE. 0.000) GO TO 120        BAL00
48 C      110 CONTINUE                  BAL00
49 C
50 C      M = L                         BAL00
51 C      IEXC = 1                      BAL00
52 C      GO TO 20                      BAL00
53 C      120 CONTINUE                  BAL00
54 C
55 C      GO TO 140                    BAL00
56 C      ***** SEARCH FOR COLUMNS ISOLATING AN EIGENVALUE    BAL00
57 C           AND PUSH THEM LEFT *****                  BAL00
58 C      130 K = K + 1                BAL00
59 C
60 C      140 DO 170 J = K, L            BJL00
61 C
62 C      DO 150 I = K, L              BAL00

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```

63 IF (I .EQ. J) GO TO 150                                BAL00
64 IF (A(I,J) .NE. 0.000) GO TO 170                         BJL00
65 150 CONTINUE                                         BAL00
66 C
67 M = K                                              BAL00
68 IEYC = 2                                            BAL00
69 GO TO 20                                           BAL00
70 170 CONTINUE                                         BAL00
71 C      ***** NOW BALANCE THE SUBMATRIX IN ROWS K TO L ***** BAL00
72 DO 180 I = K, L                                     BAL00
73 180 SCALE(I) = 1.000                               BAL00
74 C      ***** ITERATIVE LOOP FOR NORM REDUCTION ***** . BJL00
75 190 NOCONV = .FALSE.                                BAL00
76 C
77 DO 270 I = K, L                                     BAL00
78 C = 0.000                                         BAL00
79 R = 0.000                                         BAL00
80 C
81 DO 200 J = K, L                                     BAL00
82 IF (J .EQ. I) GO TO 200                           BAL00
83 C = C + DABS(A(J,I))                            BAL00
84 R = R + DABS(A(I,J))                            BAL00
85 200 CONTINUE                                         BAL00
86 C      ***** GUARD AGAINST ZERO C OR R DUE TO UNDERFLOW ***** BAL00
87 IF (C .EQ. 0.000 .OR. R .EQ. 0.000) GO TO 270       BAL00
88 G = R / RADIX                                      BAL00
89 F = 1.000                                         BAL00
90 S = C + R                                         BAL00
91 210 IF (C .GE. G) GO TO 220                      BAL00
92 F = F * RADIX                                     BAL00
93 C = C * B2                                         BAL00
94 GO TO 210                                         BAL00
95 220 G = R * RADIX                                 BAL00
96 230 IF (C .LT. G) GO TO 240                      BAL00
97 F = F / RADIX                                     BAL00
98 C = C / B2                                         BAL00
99 GO TO 230                                         BAL01
100 C      ***** NOW BALANCE *****                   BAL01
101 240 IF ((C + R) / F .GE. 0.95 * S) GO TO 270     BAL01
102 G = 1.000 / F                                     BAL01
103 SCALE(I) = SCALE(I) * F                          BAL01
104 NOCONV = .TRUE.                                    BAL01
105 C
106 DO 250 J = K, N                                   BAL01
107 250 A(I,J) = A(I,J) * G                         BAL01
108 C
109 DO 260 J = 1, L                                   BAL01
110 260 A(J,I) = A(J,I) * F                         BAL01
111 C
112 270 CONTINUE                                         BAL01
113 C
114 IF (NOCONV) GO TO 190                           BAL01
115 C
116 280 LOW = K                                       BAL01
117 IGH = L                                           BAL01
118 RETURN                                         BAL01
119 C      ***** LAST CARD OF BALANC *****          BAL01
120 END                                             BAL01

```

```

0      SUBROUTINE ELMHES(NM,N,LOW,IGH,A,INT)
1      IMPLICIT REAL*8 (A-H,O-Z)
2      INTEGER I,J,M,N,LA,NM,IGH,KP1,LOW,MM1,MP1
3      DIMENSION A(NM,N)
4 C      REAL X,Y
5 C      REAL DABS
6      INTEGER INT(IGH)
7 C
8      LA = IGH - 1
9      KP1 = LOW + 1
10     IF (LA .LT. KP1) GO TO 200
11 C
12    DO_180 M = KP1, LA
13      MM1 = M - 1
14      X = 0.000
15      I = M
16 C
17      DO_100 J = M, IGH
18      IF (DABS(A(J,MM1)) .LE. DABS(X)) GO TO 100
19      X = A(J,MM1)
20      I = J
21 100   CONTINUE
22 C
23      INT(M) = I
24      IF (I .EQ. M) GO TO 130
25 C      ***** INTERCHANGE ROWS AND COLUMNS OF A *****
26      DO_110 J = MM1, N
27      Y = A(I,J)
28      A(I,J) = A(M,J)
29      A(M,J) = Y
30 110   CONTINUE
31 C
32      DO_120 J = 1, IGH
33      Y = A(J,I)
34      A(J,I) = A(J,M)
35      A(J,M) = Y
36 120   CONTINUE
37 C      ***** END INTERCHANGE *****
38 130   IF (X .EQ. 0.000) GO TO 180
39      MP1 = M + 1
40 C
41      DO_160 I = MP1, IGH
42      Y = A(I,MM1)
43      IF (Y .EQ. 0.000) GO TO 160
44      Y = Y / X
45      A(I,MM1) = Y
46 C
47      DO_140 J = M, N
48 140   A(I,J) = A(I,J) - Y * A(M,J)
49 C
50      DO_150 J = 1, IGH
51 150   A(J,M) = A(J,M) + Y * A(J,I)
52 C
53 160   CONTINUE
54 C
55 180 CONTINUE
56 C
57 200 RETURN
58 C      ***** LAST CARD OF ELMHES *****
59      END

```

```

0      SUBROUTINE QR(NM,N,LOW,IGH,H,WR,WI,IFRR)          HQRO
1      IMPLICIT REAL*8 (A-H,O-Z)                      HQR00
2          REAL*8 NORM,MACHEP                         HQR0
3          INTEGER I,J,K,L,M,N,EN,LL,MM,NA,NM,IGH,ITS,LOW,MP2,ENM2,IERR   HQR0
4          DIMENSION H(NM,N),WR(N),WI(N)                HQR00
5 C          REAL P,Q,R,S,T,W,X,Y,ZZ,NORM,MACHEP        HQR00
6          REAL*8 DSQRT,DABS,DSIGN                     HQR0
7 C          INTEGER MINO                           HQR0
8          LOGICAL NOTLAS                         HQR00
9 C
10 C
11 C          ***** MACHEP IS A MACHINE DEPENDENT PARAMETER SPECIFYING    HQR00
12 C          THE RELATIVE PRECISION OF FLOATING POINT ARITHMETIC.       HQR00
13 C
14 C
15 C          MACHEP = 16.**(-13)                                HQR00
16 C
17 C          IERR = 0                                     HQR0
18 C          NORM = 0.000                                HQR00
19 C          K = 1                                     HQR0
20 C          ***** STORE ROOTS ISOLATED BY BALANC        HQR0
21 C          AND COMPUTE MATRIX NORM *****             HQR00
22 DO 50 I = 1, N                                 HQR00
23 C
24 DO 40 J = K, N                                 HQR0
25 40     NORM = NORM + DABS(H(I,J))            HQR00
26 C
27     K = I                                     HQR0
28     IF (I .GE. LOW .AND. I .LE. IGH) GO TO 50   HQR00
29     WR(I) = H(I,I)                            HQR0
30     WI(I) = 0.000.                            HQR0
31 50 CONTINUE                                HQR00
32 C
33     EN = IGH                                  HQR0
34     T = 0.000                                HQR0
35 C          ***** SEARCH FOR NEXT EIGENVALUES *****    HQR00
36 60 IF (EN .LT. LOW) GO TO 1001               HQR0
37     ITS = 0                                 HQR0
38     NA = EN - 1                            HQR00
39     ENM2 = NA - 1                          HQR00
40 C          ***** LOOK FOR SINGLE SMALL SUB-DIAGONAL ELEMENT      HQR0
41 C          FOR L=EN STEP -1 UNTIL LOW DO -- *****           HQR00
42 70 DO 80 LL = LOW, EN                         HQR00
43     L = EN + LOW - LL                        HQR0
44     IF (L .EQ. LOW) GO TO 100               HQR0
45     S = DABS(H(L-1,L-1)) + DABS(H(L,L))    HQR00
46     IF (S .EQ. 0.000) S = NORM              HQR0
47     IF (DABS(H(L,L-1)) .LE. MACHEP * S) GO TO 100   HQR0
48 80 CONTINUE                                HQR00
49 C          ***** FORM SHIFT *****                  HQR00
50 100 X = H(EN,EN)                            HQR0
51     IF (L .EQ. EN) GO TO 270               HQR0
52     Y = H(NA,NA)                            HQR00
53     W = H(EN,NA) * H(NA,EN)                HQR0
54     IF (L .EQ. NA) GO TO 280               HQR0
55     IF (ITS .EQ. 30) GO TO 1000            HQR00
56     IF (ITS .NE. 10 .AND. ITS .NE. 20) GO TO 130   HQR00
57 C          ***** FORM EXCEPTIONAL SHIFT *****      HQR0
58     T = T + X                               HQR0
59 C
60 DO 120 I = LOW, EN                         HQR0
61 120 H(I,I) = H(I,I) - X                  HQR0
62 C

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63      S = DABS(H(EN,NA)) + DABS(H(NA,ENM2))      HQR0
64      X = 0.75 * S      HQR00
65      Y = X      HQR0A
66      W = -0.4375 * S * S      HQR0
67      130 IT5 = IT5 + 1      HQR00
68 C      ***** LOOK FOR TWO CONSECUTIVE SMALL      HQR00
69 C      SUB-DIAGONAL ELEMENTS.      HQR0
70 C      FOR M<EN-2 STEP -1 UNTIL L DO -- *****      HQR0
71      DO 140 MM = L, ENM2      HQR0C
72      M = ENM2 + L - MM      HQR0
73      ZZ = H(M,M)      HQR0
74      R = X - ZZ      HQR00
75      S = Y - ZZ      HQR00
76      P = (R * S - W) / H(M+1,M) + H(M,M+1)      HQR0
77      Q = H(M+1,M+1) - ZZ - R - S      HQR00
78      R = H(M+2,M+1)      HQR00
79      S = DABS(P) + DABS(Q) + DABS(R)      HQR0
80      P = P / S      HQR0
81      Q = Q / S      HQR00
82      R = R / S      HQR0
83      IF (M .EQ. L) GO TO 150      HQR0
84      IF (DABS(H(M,M-1)) * (DABS(Q) + DABS(R)) .LE. MACHEP * DABS(P))      HQR00
85      X * (DABS(H(M-1,M-1)) + DABS(ZZ) + DABS(H(M+1,M+1)))) GO TO 150      HQR00
86      140 CONTINUE      HQR0
87 C      *****      HQR0J
88      150 MP2 = M + 2      HQR00
89 C      *****      HQR0
90      DO 160 I = MP2, EN      HQR0
91      H(I,I-2) = 0.000      HQR00
92      IF (I .EQ. MP2) GO TO 160      HQR0C
93      H(I,I-3) = 0.000      HQR0
94      160 CONTINUE      HQR0J
95 C      ***** DOUBLE QR STEP INVOLVING ROWS L TO EN AND      HQR00C
96 C      COLUMNS M TO EN *****      HQR0
97      DO 260 K = M, NA      HQR0
98      NOTLAS = K .NE. NA      HQR00C
99      IF (K .EQ. M) GO TO 170      HQR0
100     P = H(K,K-1)      HQR0
101     Q = H(K+1,K-1)      HQR01
102     R = 0.000      HQR0
103     IF (NOTLAS) R = H(K+2,K-1)      HQR0
104     X = DABS(P) + DABS(Q) + DABS(R)      HQR0
105     IF (X .EQ. 0.000) GO TO 260      HQR01
106     P = P / X      HQR0
107     Q = Q / X      HQR0
108     R = R / X      HQR0
109     170 S = DSIGN(DSQRT(P*P+Q*Q+R*R),P)      QR0
110     IF (K .EQ. M) GO TO 180      HQR0
111     H(K,K-1) = -S * X      HQR01
112     GO TO 190      HQR0
113     180 IF (L .NE. M) H(K,K-1) = -H(K,K-1)      QR0
114     190 P = P / S      HQR0
115     X = P / S      HQR01
116     Y = Q / S      HQR0
117     ZZ = R / S      HQR0
118     Q = Q / P      HQR01
119     R = R / P      HQR0
120 C      ***** ROW MODIFICATION *****      HQR0
121     DO 210 J = K, EN      HQR0
122     P = H(K,J) + Q * H(K+1,J)      HQR01
123     IF (.NOT. NOTLAS) GO TO 200      HQR0
124     P = P + R * H(K+2,J)      HQR0
125     H(K+2,J) = H(K+2,J) - P * ZZ      HQR01

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126 200 H(K+1,J) = H(K+1,J) - P * Y
127 H(K,J) = H(K,J) - P * X
128 210 CONTINUE
129 C J = MIN0(EN,K+3)
130 ***** COLUMN MODIFICATION *****
131 C DO 230 I = L, J
132 P = X + H(I,K) + Y + H(I,K+1)
133 IF (.NOT. NOTLAS) GO TO 220
134 P = P + ZZ + H(I,K+2)
135 H(I,K+2) = H(I,K+2) - P * R
136 H(I,K+1) = H(I,K+1) - P * Q
137 220 H(I,K) = H(I,K) - P
138 230 CONTINUE
139 C
140 C
141 260 CONTINUE
142 C
143 GO TO 70
144 C ***** ONE ROOT FOUND *****
145 270 WR(EN) = X + T
146 WI(EN) = 0.000
147 EN = NA
148 GO TO 60
149 C ***** TWO ROOTS FOUND *****
150 280 P = (Y - X) / 2.000
151 Q = P * P + W
152 ZZ = DSQRT(DABS(Q))
153 X = X + T
154 IF (Q .LT. 0.000) GO TO 320
155 C ***** REAL PAIR *****
156 ZZ = P + DSIGN(ZZ,P)
157 WR(NA) = X + ZZ
158 WR(EN) = WR(NA)
159 IF (ZZ .NE. 0.000) WR(EN) = X - W / ZZ
160 WI(NA) = 0.000
161 WI(EN) = 0.000
162 GO TO 330
163 C ***** COMPLEX PAIR *****
164 320 WR(NA) = X + P
165 WR(EN) = X + P
166 WI(NA) = ZZ
167 WI(EN) = -ZZ
168 330 EN = ENM2
169 GO TO 60
170 C ***** SET ERROR -- NO CONVERGENCE TO AN
171 C EIGENVALUE AFTER 30 ITERATIONS *****
172 1000 IERR = EN
173 1001 RETURN
174 C ***** LAST CARD OF HQR *****
175 END

```

```

0      SUBROUTINE INVIT(NM,N,A,WR,WI,SELECT,MM,M,Z,IERR,RM1,RV1,RV2)    INVO
1      IMPLICIT REAL*8 (A-H,O-Z)    INVO
2      REAL*8 NORM,NORMV,ILAMBD,MACHEP    INVO
3      INTEGER I,J,K,L,M,N,S,II,IP,MM,MP,NM,NS,N1,UK,IP1,ITS,KM1,IERR    RNVO
4      DIMENSION A(NM,N),WR(4),WI(N),Z(NM,MM),RM1(N,N),RV1(N),RV2(N)    INVO
5 C      REAL T,W,X,Y,EPS3,NORM,NORMV,GROWTO,ILAMBD,MACHEP,RLAMBD,UKROOT    INVO
6      REAL*8 DSQRT,CDABS,DAB8,DFLOAT    INVO
7      INTEGER IABS    RNV0
8      LOGICAL*1 SELECT(N)    INVO
9      COMPLEX*16 Z3,DCMPLX    INVO
10     REAL*8 DREAL,DIMAG    INVO
11 C
12     MACHEP = 16.**(-13)    INVO
13 C
14     IERR = 0    INVO
15     UK = 0    RNV0
16     S = 1    INVO
17 C      ***** IP = 0, REAL EIGENVALUE    INVO
18 C      1, FIRST OF CONJUGATE COMPLEX PAIR    INVO
19 C      -1, SECOND OF CONJUGATE COMPLEX PAIR *****    INVO
20     IP = 0    INVO
21     N1 = N - 1    RNV0
22 C
23     DO 980 K = 1, N    INVO
24     IF (WI(K) .EQ. 0.000 .OR. MP .LT. 0) GO TO 100    INVO
25     IP = 1    INVO
26     IF (SELECT(K) .AND. SELECT(K+1)) SELECT(K+1) = .FALSE.    INVO
27     100    IF (.NOT. SELECT(K)) GO TO 960    INVO
28     IF (WI(K) .NE. 0.000) S = S + 1    INVO
29     IF (S .GT. MM) GO TO 1000    INVO
30     IF (UK .GE. K) GO TO 200    INVO
31 C      ***** CHECK FOR POSSIBLE SPLITTING *****    INVO
32     DO 120 UK = K, N    INVO
33     IF (UK .EQ. N) GO TO 140    INVO
34     IF (A(UK+1,UK) .EQ. 0.000) GO TO 140    INVO
35     120    CONTINUE    INVO
36 C      ***** COMPUTE INFINITY NORM OF LEADING UK BY UK    INVO
37 C      (HESSENBERG) MATRIX *****    INVO
38     140    NORM = 0.000    INVO
39     MP = 1    INVO
40 C
41     DO 180 I = 1, UK    INVO
42     X = 0.000    INVO
43 C
44     DO 160 J = MP, UK    INVO
45     X = X + DAB8(A(I,J))    INVO
46 C
47     IF (X .GT. NORM) NORM = X    INVO
48     MP = I    INVO
49     180    CONTINUE    INVO
50 C      ***** EPS3 REPLACES ZERO PIVOT IN DECOMPOSITION    INVO
51 C      AND CLOSE ROOTS ARE MODIFIED BY EPS3*****    &RNVO
52     IF (NORM .EQ. 0.000) NORM = 1.000    INVO
53     EPS3 = MACHEP * NORM    INVO
54 C      ***** GROWTO IS THE CRITERION FOR THE GROWTH *****    INVO
55     UKROOT = DSQRT(DFLOAT(UK))    INVO
56     GROWTO = 1.00-1 / UKROOT    INVO
57     200    RLAMBD = WR(K)    INVO
58     ILAMBD = WI(K)    INVO
59     IF (K .EQ. 1) GO TO 280    INVO
60     KM1 = K - 1    INVO
61     GO TO 240    INVO
62 C      ***** PEPTURB EIGENVALUE IF IT IS CLOSE    INVO

```

63 C TO ANY PREVIOUS EIGENVALUE *****
 64 220 RЛАМБО = RЛАМБО + EP93
 65 C ***** FOR I=K-1 STEP -1 UNTIL 1 DO -- *****
 66 240 DO 260 II = I, KMI
 67 I = K - II
 68 IF (SELECT(I) .AND. DABS(WR(I)-RЛАМБО) .LT. EPS3 .AND.
 69 X DABS(WI(I)-ILAMBD) .LT. EPS3) GO TO 220
 70 260 CONTINUE
 71 C
 72 WR(K) = RЛАМБО
 73 C ***** PERTURB CONJUGATE EIGENVALUE TO MATCH *****
 74 IP1 = K + IP
 75 WR(IP1) = RЛАМБО
 76 C ***** FORM UPPER HESSENBERG A=RЛАМБО*I (TRANSPOSED)
 77 C AND INITIAL REAL VECTOR *****
 78 280 MP = 1
 79 C
 80 DO 320 I = 1, UK
 81 C
 82 DO 300 J = MP, UK
 83 300 RM1(J,I) = A(I,J)
 84 C
 85 RM1(I,I) = RM1(I,I) - RЛАМБО
 86 MP = I
 87 RV1(I) = EPS3 ??
 88 320 CONTINUE
 89 C
 90 ITS = 0
 91 IF (ILAMBD .NE. 0.000) GO TO 520
 92 C ***** REAL EIGENVALUE.
 93 C TRIANGULAR DECOMPOSITION WITH INTERCHANGES,
 94 C REPLACING ZERO PIVOTS BY EPS3 *****
 95 IF (UK .EQ. 1) GO TO 420
 96 C
 97 DO 400 I = 2, UK
 98 MP = I - 1
 99 IF (DABS(RM1(MP,I)) .LE. DABS(RM1(MP,MP))) GO TO 360
 100 C
 101 DO 340 J = MP, UK
 102 Y = RM1(J,I)
 103 RM1(J,I) = RM1(J,MP)
 104 RM1(J,MP) = Y
 105 340 CONTINUE
 106 C
 107 360 IF (RM1(MP,MP) .EQ. 0.000) RM1(MP,MP) = EPS3
 108 X = RM1(MP,I) / RM1(MP,MP)
 109 IF (X .EQ. 0.000) GO TO 400
 110 C
 111 DO 380 J = I, UK
 112 380 RM1(J,I) = RM1(J,I) - X * RM1(J,MP)
 113 C
 114 400 CONTINUE
 115 C
 116 420 IF (RM1(UK,UK) .EQ. 0.000) RM1(UK,UK) = EPS3
 117 C ***** BACK SUBSTITUTION FOR REAL VECTOR
 118 C FOR I=UK STEP -1 UNTIL 1 DO -- *****
 119 440 DO 500 II = 1, UK
 120 I = UK + I - II
 121 Y = RV1(I)
 122 IF (I .EQ. UK) GO TO 480
 123 IP1 = I + 1
 124 C
 125 DO 460 J = IP1, UK

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126 460 Y = Y - RM1(J,I) * RV1(J) INV0
 127 C INV0
 128 480 RV1(I) = Y / RM1(I,I) INV0
 129 500 CONTINUE INV0
 130 C INV0
 131 GO TO 740 INV0
 132 C ***** COMPLEX EIGENVALUE. INV0
 133 C TRIANGULAR DECOMPOSITION WITH INTERCHANGES, RNV0
 134 C REPLACING ZERO PIVOTS BY EPS3. STORE IMAGINARY INV0
 135 C PARTS IN UPPER TRIANGLE STARTING AT (1,3) ***** INV0
 136 520 NS = N - S RNV0
 137 Z(1,S-1) = -ILAMBD INV0
 138 Z(1,S) = 0.000 INV0
 139 IF (N .EQ. 2) GO TO 550 INV0
 140 RM1(1,3) = -ILAMBD INV0
 141 Z(1,S-1) = 0.000 INV0
 142 IF (N .EQ. 3) GO TO 550 INV0
 143 C INV0
 144 DO 540 I = 4, N INV0
 145 540 RM1(I,I) = 0.000 INV0
 146 C INV0
 147 550 DO 640 I = 2, UK INV0
 148 MP = I - 1 INV0
 149 W = RM1(MP,I) INV0
 150 IF (I .LT. N) T = RM1(MP,I+1) INV0
 151 IF (I .EQ. N) T = Z(MP,S-1) INV0
 152 X = RM1(MP,MP) * RM1(MP,MP) + T * T INV0
 153 IF (W * W .LE. X) GO TO 580 INV0
 154 X = RM1(MP,MP) / W INV0
 155 Y = T / W INV0
 156 RM1(MP,MP) = W INV0
 157 IF (I .LT. N) RM1(MP,I+1) = 0.000 INV0
 158 IF (I .EQ. N) Z(MP,S-1) = 0.000 INV0
 159 C INV0
 160 DO 560 J = I, UK INV0
 161 W = RM1(J,I) INV0
 162 RM1(J,I) = RM1(J,MP) - X * W INV0
 163 RM1(J,MP) = W INV0
 164 IF (J .LT. N) GO TO 550 INV0
 165 L = J - NS INV0
 166 Z(I,L) = Z(MP,L) - Y * W INV0
 167 Z(MP,L) = 0.000 INV0
 168 GO TO 560 INV0
 169 555 RM1(I,J+2) = RM1(MP,J+2) - Y * W INV0
 170 RM1(MP,J+2) = 0.000 INV0
 171 560 CONTINUE INV0
 172 C INV0
 173 RM1(I,I) = RM1(I,I) - Y * ILAMBD INV0
 174 IF (I .LT. N) GO TO 570 INV0
 175 L = I - NS INV0
 176 Z(MP,L) = -ILAMBD INV0
 177 Z(I,L) = Z(I,L) + X * ILAMBD INV0
 178 GO TO 640 INV0
 179 570 RM1(MP,I+2) = -ILAMBD RNV0
 180 RM1(I,I+2) = RM1(I,I+2) + X * ILAMBD INV0
 181 GO TO 640 INV0
 182 580 IF (X .NE. 0.000) GO TO 600 INV0
 183 RM1(MP,MP) = EPS3 INV0
 184 IF (I .LT. N) RM1(MP,I+1) = 0.000 INV0
 185 IF (I .EQ. N) Z(MP,S-1) = 0.000 INV0
 186 T = 0.000 INV0
 187 Y = EPS3 * EPS3 INV0
 188 600 W = W / X INV0

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189      X = HM1(MP,MP) * W           INV01
190      Y = -T * W                  INV01
191 C
192      DO 620 J = I, UK
193          IF (J .LT. N1) GO TO 610   INV01
194          L = J - NS               INV01
195          T = Z(MP,L)             INV01
196          Z(I,L) = -X * T - Y * RM1(J,MP)   INV01
197          GO TO 615               INV01
198 610      T = RM1(MP,J+2)         INV01
199          RM1(I,J+2) = -X * T - Y * RM1(J,MP)   INV02
200 615      RM1(J,I) = RM1(J,I) - X * RM1(J,MP) + Y * T   INV02
201 620      CONTINUE              INV02
202 C
203          IF (I .LT. N1) GO TO 630   INV01
204          L = I - NS               INV02
205          Z(I,L) = Z(I,L) - ILAMBD   INV02
206          GO TO 640               INV02
207 630      RM1(I,I+2) = RM1(I,I+2) - ILAMBD   INV02
208 640      CONTINUE              INV02
209 C
210          IF (UK .LT. N1) GO TO 650   INV02
211          L = UK - NS               INV02
212          T = Z(UK,L)             INV02
213          GO TO 655               INV02
214 650      T = RM1(UK,UK+2)         INV02
215 655      IF (RM1(UK,UK) .EQ. 0.0D0 .AND. T .EQ. 0.0D0) RM1(UK,UK) = EPS3INV02
216 C ***** BACK SUBSTITUTION FOR COMPLEX VECTOR   INV02
217 C          FOR I=UK STEP -1 UNTIL 1 00 == *****      INV02
218 660      DO 720 II = 1, UK           INV02
219          I = UK + 1 - II            INV02
220          X = RV1(I)               INV02
221          Y = 0.0D0                RNV02
222          IF (I .EQ. UK) GO TO 700   INV02
223          IP1 = I + 1               INV02
224 C
225      DO 680 J = IP1, UK
226          IF (J .LT. N1) GO TO 670   INV02
227          L = J - NS               INV02
228          T = Z(I,L)             INV02
229          GO TO 675               INV02
230 670      T = RM1(I,J+2)         INV02
231 675      X = X - RM1(J,I) * RV1(J) + T * RV2(J)   INV02
232          Y = Y - RM1(J,I) * RV2(J) - T * RV1(J)   RNV02
233 680      CONTINUE              INV02
234 C
235 700      IF (I .LT. N1) GO TO 710   INV02
236          L = I - NS               INV02
237          T = Z(I,L)             INV02
238          GO TO 715               INV02
239 710      T = RM1(I,I+2)         INV02
240 715      Z3 = DCMLPX(X,Y) / DCMLPX(RM1(I,I),T)   INV02
241          RV1(I) = DREAL(Z3)        INV02
242          RV2(I) = DIMAG(Z3)        INV02
243 720      CONTINUE              INV02
244 C ***** ACCEPTANCE TEST FOR REAL OR COMPLEX   INV02
245 C          EIGENVECTOR AND NORMALIZATION *****   INV02
246 740      ITS = ITS + 1           INV02
247      NORM = 0.0D0                INV02
248      NORMV = 0.0D0               INV02
249 C
250      DO 780 I = 1, UK
251          IF (ILAMBD .EQ. 0.0D0) X = DABS(RV1(I))   INV02

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252 IF (ILAMBD .NE. 0.000) X = CDABS(DCMPLX(RV1(I),RV2(I))) INVO
253 IF (NORMV .GE. X) GO TO 760 INVO
254 NORMV = X INVO
255 J = I INVO
256 760 NORM = NORM + X INVO
257 780 CONTINUE INVO
258 C INVO
259 IF (NORM .LT. GROWTH) GO TO 840 INVO
260 C ***** ACCEPT VECTOR *****
261 X = RV1(J) INVO
262 IF (ILAMBD .EQ. 0.000) X = 1.000 / X INVO
263 IF (ILAMBD .NE. 0.000) Y = RV2(J) RNV02
264 C INVO
265 DO 820 I = 1, UK INVO
266 IF (ILAMBD .NE. 0.000) GO TO 800 INVO
267 Z(I,S) = RV1(I) * X INVO
268 GO TO 820 INVO
269 800 Z3 = DCMPLX(RV1(I),RV2(I)) / DCMPLX(X,Y) INVO
270 Z(I,S-1) = DREAL(Z3) RNV02
271 Z(I,S) = DIMAG(Z3) INVO
272 820 CONTINUE INVO
273 C INVO
274 IF (UK .EQ. N) GO TO 940 INVO
275 J = UK + 1 INVO
276 GO TO 900 INVO
277 C ***** IN-LINE PROCEDURE FOR CHOOSING INVO
278 C A NEW STARTING VECTOR *****
279 840 IF (ITS .GE. UK) GO TO 880 INVO
280 X = UKROOT INVO
281 Y = EPS3 / (X + 1.000) RNV02
282 RV1(1) = EPS3 INVO
283 C INVO
284 DO 860 I = 2, UK INVO
285 860 RV1(I) = Y INVO
286 C INVO
287 J = UK - ITS + 1 INVO
288 RV1(J) = RV1(J) - EPS3 * X INVO
289 IF (ILAMBD .EQ. 0.000) GO TO 440 INVO
290 GO TO 660 INVO
291 C ***** SET ERROR -- UNACCEPTED EIGENVECTOR *****
292 880 J = 1 INVO
293 IERR = -K INVO
294 C ***** SET REMAINING VECTOR COMPONENTS TO ZERO *****
295 900 DO 920 I = J, N INVO
296 Z(I,S) = 0.000 INVO
297 IF (ILAMBD .NE. 0.000) Z(I,S-1) = 0.000 INVO
298 920 CONTINUE INVO
299 C INVO
300 940 S = S + 1 INVO
301 960 IF (IP .EQ. (-1)) IP = 0 ORIGINAL PAGE IS
302 IF (IP .EQ. 1) IP = -1 OF POOR QUALITY
303 980 CONTINUE INVO
304 C INVO
305 GO TO 1001 INVO
306 C ***** SET ERROR -- UNDERESTIMATE OF EIGENVECTOR INVO
307 C SPACE REQUIRED *****
308 1000 IF (IERR .NE. 0) IERR = IERR - N INVO
309 IF (IERR .EQ. 0) IERR = -(2 * N + 1) INVO
310 1001 M = S - 1 - IABS(IP) INVO
311 RETURN INVO
312 C ***** LAST CARD OF INVIT *****
313 END INVO

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0      SUBROUTINE ELMBAK(NM,LOW,IGH,A,INT,M,Z)          ELM00
1      IMPLICIT REAL*8 (A-H,O-Z)                      ELM000
2      INTEGER I,J,M,LA,MM,MP,NM,IGH,KP1,LOW,MP1       ELM00F
3      DIMENSION A(NM,IGH),Z(NM,M)                   ELM00
4 C      REAL X                                         ELM000
5      INTEGER INT(IGH)                                ELMJ00
6 C
7 C
8      IF (M .EQ. 0) GO TO 200                         ELM00
9      LA = IGH - 1                                     ELM00
10     KP1 = LOW + 1                                    ELM00
11     IF (LA .LT. KP1) GO TO 200                     ELM001
12 C     ***** FOR MP=IGH-1 STEP -1 UNTIL LOW+1 DO -- *****
13     DO 140 MM = KP1, LA                            ELM001
14     MP = LOW + IGH - MM                           ELM001
15     MP1 = MP + 1                                    ELM001
16 C
17     DO 110 I = MP1, IGH                           ELM00
18     X = A(I,MP-1)                                 ELM001
19     IF (X .EQ. 0.0) GO TO 110                     ELM002
20 C
21     DO 100 J = 1, M                             ELM002
22     100    Z(I,J) = Z(I,J) + X * Z(MP,J)        ELM002
23 C
24     110    CONTINUE                                ?
25 C
26     I = INT(MP)                                 ELM002
27     IF (I .EQ. MP) GO TO 140                     ELM00
28 C
29     DO 130 J = 1, M                           ELM002
30     X = Z(I,J)                               ELM00
31     Z(I,J) = Z(MP,J)                           ELM00
32     Z(MP,J) = X                               ELM003
33     130    CONTINUE                                ELM00
34 C
35     140 CONTINUE                                ELM003
36 C
37     200 RETURN                                  ELM00
38 C     ***** LAST CARD OF ELMBAK *****           ELM00S
39     END                                         ELM004

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0      SUBROUTINE BALBAK(NM,N,LOW,IGH,SCALE,M,Z)          BALOU
1      IMPLICIT REAL*8 (A-H,O-Z)                      BALOO
2      INTEGER I,J,K,M,N,II,NM,IGH,LOW                RALO
3      DIMENSION SCALE(N),Z(NM,M)                   RALO
4 C      REAL S                                     BALOO
5      IF (M .EQ. 0) GO TO 200                      BALOO
6      IF (IGH .EQ. LOW) GO TO 120                  BALO
7 C
8      DO 110 I = LOW, IGH                         BALOO
9      S = SCALE(I)                                BALO
10 C     ***** LEFT HAND EIGENVECTORS ARE BACK TRANSFORMED    BALO
11 C     IF THE FOREGOING STATEMENT IS REPLACED BY        BALOO
12 C     S=1.0/SCALE(I), *****                         BALO
13      DO 100 J = 1, M                           BALO
14      100   Z(I,J) = Z(I,J) * S                 BALOO
15 C
16      110 CONTINUE                               BALO
17 C     ***** FOR I=LOW-1 STEP -1 UNTIL 1,             BALOO
18 C     IGH+1 STEP 1 UNTIL N DO -- *****           BALOO
19      120 DO 140 II = 1, N                      BJLO
20      I = II                                     BALO
21      IF (I .GE. LOW .AND. I .LE. IGH) GO TO 140    BALOO
22      IF (I .LT. LOW) I = LOW - II               BALO
23      K = SCALE(I)                                BALO
24      IF (K ,EQ. I) GO TO 140                  BALOO
25 C
26      DO 130 J = 1, M                           RALO
27      S = Z(I,J)                                BALO
28      Z(I,J) = Z(K,J)                            BALOO
29      . Z(K,J) = S                             BALO
30      130 CONTINUE                               BALO
31 C
32      140 CONTINUE                               BALO
33 C
34      200 RETURN                                 BALO
35 C     ***** LAST CARD OF BALBAK *****            BALOO
36      END                                     BALO

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0.      SUBROUTINE DETFAC(NMAX,N,A,IPIVOT,IDET,DETERM,ISCALE,WK,IERR)    DETC
1.      IMPLICIT REAL*8 (A-H,O-Z)    DETO
2.      DIMENSION A(NMAX,1),IPIVOT(1),WK(1)    DETO
3 C.      ISCALE=0    DETC
4 C.      NM1=N-1    DETO
5 C.      IERR=0    DETC
6 C.      DETERMINANT CALCULATION TEST    DETO
7 C.      IF(IDET.EQ.1)GO TO 230    DETO
8 C.      TEST FOR A SCALAR MATRIX    DETO
9 C.      IF(NM1.GT.0)GO TO 20    DETO
10 C.      DETERM=A(1,1)    DETC
11 C.      RETURN    DETC
12 C.      COMPUTE SCALING FACTORS    DETO
13 C.      CON;INUE    DETO
14 C.      DO 60 I=1,N    DETO
15 C.      P=0.0    DETO
16 C.      DO 30 J=1,N    DETO
17 C.      Q=DMAX1(P,DABS(A(I,J)))    DETO
18 C.      IF(Q.GT.P)P=Q    DETO
19 C.      CONTINUE    DETO
20 C.      IF(P).GT.0.40,60    DETO
21 C.      DETERM=0.0    DETO
22 C.      IERR=1    DETO
23 C.      RETURN    DETO
24 C.      WK(I)=P    DETO
25 C.      DO 210 M=1,NM1    DETO
26 C.      PIVOTAL LOGIC SETUP    DETO
27 C.      P=0.0    DETO
28 C.      DO 110 I=M,N    DETO
29 C.      Q=DABS(A(I,M))/WK(I)    DETO
30 C.      IF(Q-P).GT.110,110,100    DETO
31 C.      P=Q    DETO
32 C.      IP=I    DETO
33 C.      110 CONTINUE    DETO
34 C.      IPIVOT(M)=IP    DETO
35 C.      IF(P.EQ.0.)GO TO 40    DETO
36 C.      IF(M.EQ.IP)GO TO 155    DETO
37 C.      PIVOT THE M-TH ROW OF THE A MATRIX    DETO
38 C.      DO 150 I=1,N    DETO
39 C.      P=A(IP,I)    DETO
40 C.      A(IP,I)=A(M,I)    DETO
41 C.      150 A(M,I)=P    DETO
42 C.      P=WK(IP)    DETO
43 C.      WK(IP)=WK(M)    DETO
44 C.      WK(M)=P    DETO
45 C.      155 MP1=M+1    DETO

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```

63 C      L/U FACTORIZATION LOGIC
64 C
65 C
66      P=A(M,M)
67      DO 180 I=MP1,N
68      A(I,M)=A(I,M)/P
69      Q=A(I,M)
70      DO 180 K=MP1,N
71      180 A(I,K)=A(I,K)-Q*A(M,K)
72 C
73      210 CONTINUE
74 C
75      IPIVOT(N)=N
76      IF (A(N,N) .EQ. 0.0) GO TO 40
77 C
78 C      CALCULATION OF THE DETERMINANT OF A
79 C
80      IF(IDET.EQ.0)RETURN
81 C
82      230 SIGN=1.0
83      DETERM=1.0
84 C
85 C      ADJUST SIGN OF DETERMINANT DUE TO PIVOTAL STRATEGY
86 C
87      DO 250 I=1,NM1
88      IF(I-IPIVOT(I))240,250,240
89      240 SIGN=-SIGN
90      250 CONTINUE
91 C
92      DO 340 I=1,N
93      P=A(I,I)
94 C
95      260 CONTINUE
96      IF(R1.GT.DABS(P))GO TO 280
97      P=P*R2
98      ISCALE=ISCALE+1
99      GO TO 260
100 C
101     280 CONTINUE
102     IF(R2.LT.DABS(P))GO TO 290
103     P=P*R1
104     ISCALE=ISCALE-1
105     GO TO 280
106 C
107     290 DETERM=DETERM*P
108 C
109     300 CONTINUE
110     IF(R1.GT.DABS(DETERM))GO TO 320
111     DETERM=DETERM*R2
112     ISCALE=ISCALE+1
113     GO TO 300
114 C
115     320 CONTINUE
116     IF(R2.LT.DABS(DETERM))GO TO 340
117     DETERM=DETERM*R1
118     ISCALE=ISCALE-1
119     GO TO 320
120 C
121     340 CONTINUE
122 C
123     DETERM=DETERM*SIGN
124 C
125     RETURN

```

126

END

DET

QUALITY

```

0      SUBROUTINE AXPYB(A,U,M,NA,NU,B,V,N,NB,NV,C,NC,EPSA,
1 1EPSB,FAIL)
2      IMPLICIT REAL*8 (A-H,O-Z)
3      DIMENSION
4      1A(NA,1),U(NU,1),B(NB,1),V(NV,1),C(NC,1)
5      INTEGER
6 C
7      1 FAIL
8      M1 = M+1
9      MM1 = M-1
10     N1 = N+1
11     NM1 = N-1
12 C IF REQUIRED, REDUCE A TO UPPER REAL SCHUR FORM.
13 C
14      IF(EPSA .LT. 0.) GO TO 35
15      DO 10 I=1,M
16          DO 10 J=I,M
17              TEMP = A(I,J)
18              A(I,J) = A(J,I)
19              A(J,I) = TEMP
20      10 CONTINUE
21      CALL HSHLDR(A,M,NA)
22      CALL BCKMLT(A,U,M,NA,NU)
23      IF(MM1 .EQ. 0) GO TO 25
24      DO 20 I=1,MM1
25          A(I+1,I) = A(I,M1)
26      20 CONTINUE
27      CALL SCHUR(A,U,M,NA,NU,EPSA,FAIL)
28      IF(FAIL .NE. 0) RETURN
29      25 DO 30 I=1,M
30          DO 30 J=I,M
31              TEMP = A(I,J)
32              A(I,J) = A(J,I)
33              A(J,I) = TEMP
34      30 CONTINUE
35 C
36 C IF REQUIRED, REDUCE B TO UPPER REAL SCHUR FORM.
37 C
38      35 IF(EPSB .LT. 0.) GO TO 45
39      CALL HSHLDR(B,N,NB)
40      CALL BCKMLT(B,V,N,NB,NV)
41      IF(NM1 .EQ. 0) GO TO 45
42      DO 40 I=1,NM1
43          B(I+1,I) = B(I,N1)
44      40 CONTINUE
45      CALL SCHUR(B,V,N,NB,NV,EPSB,FAIL)
46      FAIL = -FAIL
47      IF(FAIL .NE. 0) RETURN
48 C
49 C TRANSFORM C.
50 C
51      45 DO 60 J=1,N
52          DO 50 I=1,M
53              A(I,M1) = 0.
54              DO 50 K=1,M
55                  A(I,M1) = A(I,M1) + U(K,I)*C(K,J)
56      50 CONTINUE
57          DO 60 I=1,M
58              C(I,J) = A(I,M1)
59      60 CONTINUE
60          DO 80 I=1,M
61              DO 70 J=1,N
62                  B(N1,J) = 0.

```

```

63      DO 70 K=1,N          AXP0
64      B(N1,J) = B(N1,J) + C(I,K)*V(K,J)
65 70 CONTINUE               AXP0
66      DO 80 J=1,N           AXP0
67      C(I,J) = B(N1,J)       AXP0
68 80 CONTINUE               AXP0
69 C
70 C SOLVE THE TRANSFORMED SYSTEM.   AXP0
71 C
72 CALL SHRSLV(A,B,C,M,N,NA,NB,NC) AXP0
73 C
74 C TRANSFORM C BACK TO THE SOLUTION. AXP0
75 C
76      DO 100 J=1,N          AXP0
77      DO 90 I=1,M           AXP0
78      A(I,M1) = 0.           AXP0
79      DO 90 K=1,M           AXP0
80      A(I,M1) = A(I,M1) + U(I,K)*C(K,J) AXP0
81 90 CONTINUE               AXP0
82      DO 100 I=1,M          AXP0
83      C(I,J) = A(I,M1)       AXP0
84 100 CONTINUE              AXP0
85      DO 120 I=1,M          AXP0
86      DO 110 J=1,N           AXP0
87      B(N1,J) = 0.           AXP0
88      DO 110 K=1,N           AXP0
89      B(N1,J) = B(N1,J) + C(I,K)*V(J,K) AXP0
90 110 CONTINUE              AXP0
91      DO 120 J=1,N           AXP0
92      C(I,J) = B(N1,J)       AXP0
93 120 CONTINUE              AXP0
94      RETURN                AXP0
95      END                   AXP0

```

```

0      SUBROUTINE SHRSLV(A,B,C,M,N,NA,NB,NC)          SHR00
1      IMPLICIT REAL*8 (A-H,O-Z)                      SHR00
2      DIMENSION
3      1A(NA,1),B(NB,1),C(NC,1)                      SHR00
4      INTEGER
5      1 DK,DL
6      COMMON/SLVBLK/T(5,5),P(5),NSYS               SHR00
7      L = 1                                         SHR00
8      10 LM1 = L-1                                  SHR00
9      DL = 1                                         SHR00
10     IF(L .EQ. N) GO TO 15                         SHR00
11     IF(B(L+1,L) .NE. 0.) DL = 2                  SHR00
12     15 LL = L+DL-1                                SHR00
13     IF(L .EQ. 1) GO TO 30                         SHR00
14     DO 20 J=L,LL                                  SHR00
15     DO 20 I=1,M                                  SHR00
16     DO 20 IB=1,LM1                               SHR00
17     C(I,J) = C(I,J) - C(I,IB)*B(IB,J)           SHR00
18     20 CONTINUE                                 SHR00
19     30 K = 1                                     SHR00
20     40 KM1 = K-1                                SHR00
21     DK = 1                                         SHR00
22     IF(K .EQ. M) GO TO 45                         SHR00
23     IF(A(K,K+1) .NE. 0.) DK = 2                  SHR00
24     45 KK = K+DK-1                                SHR00
25     IF(K .EQ. 1) GO TO 60                         SHR00
26     DO 50 I=K,KK                                  SHR00
27     DO 50 J=L,LL                                  SHR00
28     DO 50 JA=1,KM1                               SHR00
29     C(I,J) = C(I,J) - A(I,JA)*C(JA,J)           SHR00
30     50 CONTINUE                                 SHR00
31     60 IF(DL .EQ. 2) GO TO 80                   SHR00
32     IF(DK .EQ. 2) GO TO 70                         SHR00
33     T(1,1) = A(K,K) + B(L,L)                     SHR00
34     IF(T(1,1) .EQ. 0.) STOP                      SHR00
35     C(K,L) = C(K,L)/T(1,1)                       SHR00
36     GO TO 100                                    SHR00
37     70 T(1,1) = A(K,K) + B(L,L)                   SHR00
38     T(1,2) = A(K,KK)                            SHR00
39     T(2,1) = A(KK,K)                           SHR00
40     T(2,2) = A(KK,KK) + B(L,L)                   SHR00
41     P(1) = C(K,L)                             SHR00
42     P(2) = C(KK,L)                            SHR00
43     NSYS = 2                                    SHR00
44     CALL SYSSLV                                SHR00
45     C(K,L) = P(1)                             SHR00
46     C(KK,L) = P(2)                            SHR00
47     GO TO 100                                    SHR00
48     80 IF(DK .EQ. 2) GO TO 90                   SHR00
49     T(1,1) = A(K,K) + B(L,L)                   SHR00
50     T(1,2) = B(LL,L)                           SHR00
51     T(2,1) = B(L,LL)                           SHR00
52     T(2,2) = A(K,K) + B(LL,LL)                 SHR00
53     P(1) = C(K,L)                             SHR00
54     P(2) = C(K,LL)                            SHR00
55     NSYS = 2                                    SHR00
56     CALL SYSSLV                                SHR00
57     C(K,L) = P(1)                             SHR00
58     C(K,LL) = P(2)                            SHR00
59     GO TO 100                                    SHR00
60     90 T(1,1) = A(K,K) + B(L,L)                 SHR00
61     T(1,2) = A(K,KK)                           SHR00
62     T(1,3) = B(LL,L)                           SHR00

```

63	T(1,4) = 0.	SHROC
64	T(2,1) = A(KK,K)	SHROC
65	T(2,2) = A(KK,KK) + B(L,L)	SHROC
66	T(2,3) = 0.	SHROC
67	T(2,4) = T(1,3)	SHROC
68	T(3,1) = B(L,LL)	SHROC
69	T(3,2) = 0.	SHROC
70	T(3,3) = A(K,K) + B(LL,LL)	SHROC
71	T(3,4) = T(1,2)	SHROC
72	T(4,1) = 0.	SHROO
73	T(4,2) = T(3,1)	SHROC
74	T(4,3) = T(2,1)	SHROC
75	T(4,4) = A(KK,KK) + B(LL,LL)	SHROC
76	P(1) = C(K,L)	SHROO
77	P(2) = C(KK,L)	SHROO
78	P(3) = C(K,LL)	SHROO
79	P(4) = C(KK,LL)	SHROO
80	NSYS = 4	SHROO
81	CALL SYSSLV	SHROO
82	C(K,L) = P(1)	SHROC
83	C(KK,L) = P(2)	SHROO
84	C(K,LL) = P(3)	SHROO
85	C(KK,LL) = P(4)	SHROO
86	100 K = K + DK	SHROO
87	IF(K .LE. M) GO TO 40	SHROO
88	L = L + DL	SHROO
89	IF(L .LE. N) GO TO 10	SHROO
90	RETURN	SHROO
91	END	SHROO

```

0      SUBROUTINE ATXPXA(A,U,C,N,NA,NU,NC,EPS,FAIL)          ATX0
1      IMPLICIT REAL*8 (A-H,O-Z)                          ATX0C
2      DIMENSION                                         ATX0C
3      1A(NA,1),U(NU,1),C(NC,1)                         ATX0C
4      INTEGER                                           ATX0C
5      I FAIL                                         ATX0C
6      N1 = N+1                                       ATX0C
7      NM1 = N-1                                       ATX0C
8      C
9      C IF REQUIRED, REDUCE A TO LOWER REAL SCHUR FORM.   ATX0C
10     C
11     IF(EPS .LT. 0.) GO TO 15                           ATX0C
12     CALL HSHLDR(A,N,NA)                                ATX0C
13     CALL BCKMLT(A,U,N,NA,NU)                            ATX0C
14     DO 10 I=1,NM1                                     ATX0C
15     A(I+1,I) = A(I,N1)                               ATX0C
16     10 CONTINUE                                     ATX0C
17     CALL SCHUR(A,U,N,NA,NU,EPS,FAIL)                 ATX0C
18     IE(FAIL .NE. 0) RETURN                           ATX0C
19     C
20     C TRANSFORM C.                                 ATX0C
21     C
22     15 DO 20 I=1,N                                  ATX0C
23     C(I,I)=C(I,I)/2.                                ATX0C
24     20 CONTINUE                                     ATX0C
25     DO 40 I=1,N                                   ATX0C
26     DO 30 J=1,N                                   ATX0C
27     A(N1,J) = 0.                                    ATX0C
28     DO 30 K=I,N                                   ATX0C
29     A(N1,J) = A(N1,J) + C(I,K)*U(K,J)             ATX0C
30     30 CONTINUE                                     ATX0C
31     DO 40 J=1,N                                   ATX0C
32     C(I,J) = A(N1,J)                             ATX0C
33     40 CONTINUE                                     ATX0C
34     DO 60 J=1,N                                   ATX0C
35     DO 50 I=1,N                                   ATX0C
36     A(I,N1) = 0.                                    ATX0C
37     DO 50 K=1,N                                   ATX0C
38     A(I,N1) = A(I,N1) + U(K,I)*C(K,J)             ATX0C
39     50 CONTINUE                                     ATX0C
40     DO 60 I=1,N                                   ATX0C
41     C(I,J) = A(I,N1)                             ATX0C
42     60 CONTINUE                                     ATX0C
43     DO 70 I=1,N                                   ATX0C
44     DO 70 J=I,N                                   ATX0C
45     C(I,J) = C(I,J) + C(J,I)                     ATX0C
46     C(J,I) = C(I,J)                             ATX0C
47     70 CONTINUE                                     ATX0C
48     C
49     C SOLVE THE TRANSFORMED SYSTEM.                ATX0C
50     C
51     CALL SYMSLV(A,C,N,NA,NC)                      ATX0C
52     C
53     C TRANSFORM C BACK TO THE SOLUTION.           ATX0C
54     C
55     DO 80 I=1,N                                   ATX0C
56     C(I,I) = C(I,I)/2.                            ATX0C
57     80 CONTINUE                                     ATX0C
58     DO 100 I=1,N                                  ATX0C
59     DO 90 J=1,N                                   ATX0C
60     A(N1,J) = 0.                                    ATX0C
61     DO 90 K=I,N                                   ATX0C
62     A(N1,J) = A(N1,J) + C(I,K)*U(J,K)             ATX0C

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63	90	CONTINUE	ATX0C
64		DO 100 J=1,N	ATX0C
65		C(I,J) = A(N1,J)	ATX0C
66	100	CONTINUE	ATX0C
67		DO 120 J=1,N	ATX0C
68		DO 110 I=1,N	ATX0C
69		A(I,N1) = 0.	ATX0C
70		DO 110 K=1,N	ATX0C
71		A(I,N1) = A(I,N1) + U(I,K)*C(K,J)	ATX0C
72	110	CONTINUE	ATX0C
73		DO 120 I=1,N	ATX0C
74		C(I,J) = A(I,N1)	ATX0C
75	120	CONTINUE	ATX0C
76		DO 130 I=1,N	ATX0C
77		DO 130 J=I,N	ATX0C
78		C(I,J) = C(I,J) + C(J,I)	ATX0C
79		C(J,I) = C(I,J)	ATX0C
80	130	CONTINUE	ATX0C
81		RETURN	ATX0C
82		END	ATX0C

```

0      SUBROUTINE SYMSLV(A,C,N,NA,NC)          SYM0
1      IMPLICIT REAL*8 (A-H,O-Z)              SYM0
2      DIMENSION                         SYM0
3      IA(NA,1),C(NC,1)                   SYM0
4      INTEGER                           SYM0
5      DK,DL                           SYM0
6      COMMON/SLVBLK/T(5,5),P(5),NSYS    SYM0
7      L = 1                            SYM0
8      10     DL = 1                      SYM0
9      IF(L .EQ. N) GO TO 20             SYM0
10     IF(A(L+1,L) .NE. 0.) DL = 2       SYM0
11     20     LL = L+DL-1                SYM0
12     K = L                            SYM0
13     30     KM1 = K-1                 SYM0
14     DK = 1                            SYM0
15     IF(K .EQ. N) GO TO 35             SYM0
16     IF(A(K+1,K) .NE. 0.) DK = 2       SYM0
17     35     KK = K+DK-1                SYM0
18     IF(K .EQ. L) GO TO 45             SYM0
19     DO 40 I=K,KK                  SYM0
20         DO 40 J=L,LL                SYM0
21         DO 40 IA=L,KM1               SYM0
22         C(I,J) = C(I,J) - A(IA,I)*C(IA,J)   SYM0
23     40     CONTINUE                  SYM0
24     45     IF(DL .EQ. 2) GO TO 60           SYM0
25     IF(DK .EQ. 2) GO TO 50             SYM0
26     T(1,1) = A(K,K) + A(L,L)          SYM0
27     IF(T(1,1) .EQ. 0.) STOP            SYM0
28     C(K,L) = C(K,L)/T(1,1)          SYM0
29     GO TO 90                          SYM0
30     50     T(1,1) = A(K,K) + A(L,L)          SYM0
31     T(1,2) = A(KK,K)                 SYM0
32     T(2,1) = A(K,KK)                 SYM0
33     T(2,2) = A(KK,KK) + A(L,L)          SYM0
34     P(1) = C(K,L)                   SYM0
35     P(2) = C(KK,L)                  SYM0
36     NSYS = 2                        SYM0
37     CALL SYSSLV                     SYM0
38     C(K,L) = P(1)                   SYM0
39     C(KK,L) = P(2)                  SYM0
40     GO TO 90                          SYM0
41     60     IF(DK .EQ. 2) GO TO 70           SYM0
42     T(1,1) = A(K,K) + A(L,L)          SYM0
43     T(1,2) = A(LL,L)                 SYM0
44     T(2,1) = A(L,LL)                 SYM0
45     T(2,2) = A(K,K) + A(LL,LL)        SYM0
46     P(1) = C(K,L)                   SYM0
47     P(2) = C(K,LL)                  SYM0
48     NSYS = 2                        SYM0
49     CALL SYSSLV                     SYM0
50     C(K,L) = P(1)                   SYM0
51     C(K,LL) = P(2)                  SYM0
52     GO TO 90                          SYM0
53     70     IF(K .NE. L) GO TO 80           SYM0
54     T(1,1) = A(L,L)                 SYM0
55     T(1,2) = A(LL,L)                SYM0
56     T(1,3) = 0.                      SYM0
57     T(2,1) = A(L,LL)                SYM0
58     T(2,2) = A(L,L) + A(LL,LL)        SYM0
59     T(2,3) = T(1,2)                  SYM0
60     T(3,1) = 0.                      SYM0
61     T(3,2) = T(2,1)                  SYM0
62     T(3,3) = A(LL,LL)                SYM0

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63      P(1) = C(L,L)/2.          SYMO
64      P(2) = C(LL,L)           SYMO
65      P(3) = C(LL,LL)/2.        SYMO
66      NSYS = 3                 SYMO
67      CALL SYSSLV              SYMO
68      C(L,L) = P(1)            SYMO
69      C(LL,L) = P(2)           SYMO
70      C(L,LL) = P(2)           SYMO
71      C(LL,LL) = P(3)          SYMO
72      GO TO 90                SYMO
73      80      T(1,1) = A(K,K) + A(L,L)   SYMO
74      T(1,2) = A(KK,K)          SYMO
75      T(1,3) = A(LL,L)          SYMO
76      T(1,4) = 0.               SYMO
77      T(2,1) = A(K,KK)          SYMO
78      T(2,2) = A(KK,KK) + A(L,L)   SYMO
79      T(2,3) = 0.               SYMO
80      T(2,4) = T(1,3)          SYMO
81      T(3,1) = A(L,LL)          SYMO
82      T(3,2) = 0.               SYMO
83      T(3,3) = A(K,K) + A(LL,LL)  SYMO
84      T(3,4) = T(1,2)          SYMO
85      T(4,1) = 0.               SYMO
86      T(4,2) = T(3,1)          SYMO
87      T(4,3) = T(2,1)          SYMO
88      T(4,4) = A(KK,KK) + A(LL,LL)  SYMO
89      P(1) = C(K,L)            SYMO
90      P(2) = C(KK,L)           SYMO
91      P(3) = C(K,LL)           SYMO
92      P(4) = C(KK,LL)          SYMO
93      NSYS = 4                 SYMO
94      CALL SYSSLV              SYMO
95      C(K,L) = P(1)            SYMO
96      C(KK,L) = P(2)           SYMO
97      C(K,LL) = P(3)           SYMO
98      C(KK,LL) = P(4)          SYMO
99      90      K = K + DK       SYMO1
100     IF(K .LE. N) GO TO 30    SYMO1
101     LDL = L + DL           SYMO1
102     IF(LDL .GT. N) RETURN   SYMO1
103     DO 120 J=LDL,N          SYMO1
104     DO 100 I=L,LL            SYMO1
105     C(I,J) = C(J,I)          SYMO1
106     100      CONTINUE        SYMO1
107     DO 120 I=J,N             SYMO1
108     DO 110 K=L,LL            SYMO1
109     C(I,J) = C(I,J) - C(I,K)*A(K,J) - A(K,I)*C(K,J)  SYMO1
110     110      CONTINUE        SYMO1
111     C(J,I) = C(I,J)          SYMO1
112     120      CONTINUE        SYMO1
113     L = LDL                SYMO1
114     GO TO 10                SYMO1
115     END                     SYMO1

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```

0      SUBROUTINE HSHLDL(A,N,NA)          HSH0
1      IMPLICIT REAL*8 (A-H,O-Z)          HSH0
2      DIMENSION A(NA,1)                  HSH0
3      REAL*8 MAX                      HSH0
4 C
5      NM2 = N-2                      HSH0
6      N1 = N+1                      HSH0
7      IF(N .EQ. 1) RETURN          HSH0
8      IF(N .GT. 2) GO TO 5          HSH0
9      A(1,N1) = A(2,1)          HSH0
10     RETURN                         HSH0
11     5 DO 80 L=1,NM2            HSH0
12         L1 = L+1                  HSH0
13         MAX = 0.                  HSH0
14         DO 10 I=L1,N            HSH0
15             MAX = DMAX1(MAX,DABS(A(I,L))) HSH0
16     10 CONTINUE                  HSH0
17         IF(MAX .NE. 0.) GO TO 20 HSH0
18         A(L,N1) = 0.            HSH0
19         A(N1,L) = 0.            HSH0
20         GO TO 80                HSH0
21     20 SUM = 0.                  HSH0
22         DO 30 I=L1,N            HSH0
23             A(I,L) = A(I,L)/MAX HSH0
24             SUM = SUM + A(I,L)**2    HSH0
25     30 CONTINUE                  HSH0
26         S = DSIGN(DSQRT(SUM),A(L1,L)) HSH0
27         A(L,N1) = -MAX*S        HSH0
28         A(L1,L) = S + A(L1,L)    HSH0
29         A(N1,L) = S*A(L1,L)    HSH0
30         DO 50 J=L1,N            HSH0
31             SUM = 0.            HSH0
32             DO 40 I=L1,N            HSH0
33                 SUM = SUM + A(I,L)*A(I,J) HSH0
34     40 CONTINUE                  HSH0
35         P = SUM/A(N1,L)        HSH0
36         DO 50 I=L1,N            HSH0
37             A(I,J) = A(I,J) - A(I,L)*P HSH0
38     50 CONTINUE                  HSH0
39         DO 70 I=1,N            HSH0
40             SUM = 0.            HSH0
41             DO 60 J=L1,N            HSH0
42                 SUM = SUM + A(I,J)*A(J,L) HSH0
43     60 CONTINUE                  HSH0
44         P = SUM/A(N1,L)        HSH0
45         DO 70 J=L1,N            HSH0
46             A(I,J) = A(I,J) - P*A(J,L) HSH0
47     70 CONTINUE                  HSH0
48     80 CONTINUE                  HSH0
49         A(N-1,N1) = A(N,N-1)    HSH0
50         RETURN                   HSH0
51     END                         HSH0

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0   SUBROUTINE BCKMLT(A,U,N,NA,NU)          BCKO
1   IMPLICIT REAL*8 (A-H,O-Z)              BCKO
2   DIMENSION                         BCKO
3   1A(NA,1),U(NU,1)                   BCKO
4 C
5   N1 = N+1                           BCKO
6   NM1 = N-1                          BCKO
7   NM2 = N-2                          BCKO
8   U(N,N) = 1.                        BCKO
9   IF(NM1 .EQ. 0) RETURN             BCKO
10  U(NM1,N) = 0.                      BCKO
11  U(N,NM1) = 0.                      BCKO
12  U(NM1,NM1) = 1.                    BCKO
13  IF(NM2 .EQ. 0) RETURN             BCKO
14  DO 40 LL=1,NM2                  BCKO
15  L = NM2-LL+1                     BCKO
16  L1 = L+1                          BCKO
17  IF(A(N1,L) .EQ. 0.) GO TO 25    BCKO
18  DO 20 J=L1,N                     BCKO
19  SUM = 0.                           BCKO
20  DO 10 I=L1,N                   BCKO
21  SUM = SUM + A(I,L)*U(I,J)       BCKO
22  10 CONTINUE                      BCKO
23  P = SUM/A(N1,L)                 BCKO
24  DO 20 I=L1,N                   BCKO
25  U(I,J) = U(I,J) - A(I,L)*P     BCKO
26  20 CONTINUE                      BCKO
27  25 DO 30 I=L1,N               BCKO
28  U(I,L) = 0.                      BCKO
29  U(L,I) = 0.                      BCKO
30  30 CONTINUE                      BCKO
31  U(L,L) = 1.                      BCKO
32  40 CONTINUE                      BCKO
33  RETURN                           BCKO
34  END                               BCKO

```

```

0      SUBROUTINE SCHUR(H,U,NN,NH,NU,EPS,FAIL)          SC
1      IMPLICIT REAL*8 (A-H,O-Z)                      SC
2      DIMENSION                                     SC
3      H(NH,1),U(NU,1)                            SC
4      INTEGER                                     SC
5      I FAIL                                     SC
6      LOGICAL                                     SC
7      ILAST                                     SC
8      N = NN                                     SC
9      HN = 0.                                     SC
10     DO 20 I=1,N                                SC
11     JL = MAX0(1,I-1)                          SC
12     RSUM = 0.                                  SC
13     DO 10 J=JL,N                                SC
14     RSUM = RSUM + DABS(H(I,J))                SC
15     10 CONTINUE                               SC
16     HN = DMAX1(HN,RSUM)                         SC
17     20 CONTINUE                               SC
18     TEST = EPS*HN                             SC
19     IF(HN .EQ. 0.) GO TO 230                  SC
20     30 IF(N .LE. 1) GO TO 230                  SC
21     ITS = 0.                                 SC
22     NA = N-1                                 SC
23     NM2 = N-2                                 SC
24     40 DO 50 LL=2,N                           SC
25     L = N-LL+2                               SC
26     IF(DABS(H(L,L-1)) .LE. TEST) GO TO 60    SC
27     50 CONTINUE                               SC
28     L = 1                                    SC
29     GO TO 70                                 SC
30     60 H(L,L-1) = 0.                          SC
31     70 IF(L .LT. NA) GO TO 72                SC
32     N = L-1                                 SC
33     GO TO 30                                 SC
34     72 X = H(N,N)/HN                         SC
35     Y = H(NA,NA)/HN                         SC
36     R = (H(N,NA)/HN)*(H(NA,N)/HN)           SC
37     IF(ITS .LT. 30) GO TO 75                SC
38     FAIL = N                                 SC
39     RETURN                                   SC
40     75 IF(ITS.EQ.10 .OR. ITS.EQ.20) GO TO 80   SC
41     S = X + Y                               SC
42     Y = X*Y - R                            SC
43     GO TO 90                                 SC
44     80 Y = (DABS(H(N,NA)) + DABS(H(NA,NM2)))/HN SC
45     S = 1.5*Y                               SC
46     Y = Y**2                                 SC
47     90 ITS = ITS + 1                         SC
48     DO 100 MM=L,NM2                         SC
49     M = NM2-MM+L                           SC
50     X = H(M,M)/HN                         SC
51     R = H(M+1,M)/HN                         SC
52     Z = H(M+1,M+1)/HN                       SC
53     P = X*(X-S) + Y + R*(H(M,M+1)/HN)       SC
54     Q = R*(X+Z-S)                           SC
55     R = R*(H(M+2,M+1)/HN)                   SC
56     W = DABS(P) + DABS(Q) + DABS(R)         SC
57     P = P/W                                 SC
58     Q = Q/W                                 SC
59     R = R/W                                 SC
60     IF(M .EQ. L) GO TO 110                  SC
61     IF(DABS(H(M,M-1))*(DABS(Q)+DABS(R)) .LE. DABS(P)*TEST) SC
62     GO TO 110                               SC

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63 100 CONTINUE          SCHOC
64 110 M2 = M+2          SCHOC
65 M3 = M+3              SCHOC
66 DO 120 I=M2,N         SCHOC
67 H(I,I-2) = 0.          SCHOC
68 120 CONTINUE          SCHOC
69 IF(M3 .GT. N) GO TO 140 SCHOC
70 DO 130 I=M3,N         SCHOC
71 H(I,I-3) = 0.          SCHOC
72 130 CONTINUE          SCHOC
73 140 DO 220 K=M,NA     SCHOC
74 LAST = K.EQ.NA        SCHOC
75 IF(K .EQ. M) GO TO 150 SCHOC
76 P = H(K,K-1)          SCHOC
77 Q = H(K+1,K-1)        SCHOC
78 R = 0.                 SCHOC
79 IF(.NOT.LAST) R = H(K+2,K-1) SCHOC
80 X = DABS(P) + DABS(Q) + DABS(R) SCHOC
81 IF(X .EQ. 0.) GO TO 220 SCHOC
82 P = P/X               SCHOC
83 Q = Q/X               SCHOC
84 R = R/X               SCHOC
85 150 S = DSQRT(P**2 + Q**2 + R**2) SCHOC
86 IF(P .LT. 0.) S = -S SCHOC
87 IF(K .NE. M) H(K,K-1) = -S*X SCHOC
88 IF(K.EQ.M .AND. L.NE.M) H(K,K-1) = -H(K,K-1) SCHOC
89 P = P + S              SCHOC
90 X = P/S               SCHOC
91 Y = Q/S               SCHOC
92 Z = R/S               SCHOC
93 Q = Q/P               SCHOC
94 R = R/P               SCHOC
95 DO 170 J=K,NN          SCHOC
96 P = H(K,J) + Q*H(K+1,J) SCHOC
97 IF(LAST) GO TO 160      SCHOC
98 P = P + R*H(K+2,J)      SCHOC
99 H(K+2,J) = H(K+2,J) - P*Z SCHOC
100 160 H(K+1,J) = H(K+1,J) - P*Y SCHOC
101 H(K,J) = H(K,J) - P*X SCHOC
102 170 CONTINUE          SCHOC
103 J = MIN0(K+3,N)        SCHOC
104 DO 190 I=1,J          SCHOC
105 P = X*H(I,K) + Y*H(I,K+1) SCHOC
106 IF(LAST) GO TO 180      SCHOC
107 P = P + Z*H(I,K+2)      SCHOC
108 H(I,K+2) = H(I,K+2) - P*R SCHOC
109 180 H(I,K+1) = H(I,K+1) - P*Q SCHOC
110 H(I,K) = H(I,K) - P     SCHOC
111 190 CONTINUE          SCHOC
112 DO 210 I=1,NN          SCHOC
113 P = X*U(I,K) + Y*U(I,K+1) SCHOC
114 IF(LAST) GO TO 200      SCHOC
115 P = P + Z*U(I,K+2)      SCHOC
116 U(I,K+2) = U(I,K+2) - P*R SCHOC
117 200 U(I,K+1) = U(I,K+1) - P*Q SCHOC
118 U(I,K) = U(I,K) - P     SCHOC
119 210 CONTINUE          SCHOC
120 220 CONTINUE          SCHOC
121 GO TO 40               SCHOC
122 230 FAIL = 0           SCHOC
123 RETURN                SCHOC
124 END                   SCHOC

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```

0      SUBROUTINE SYSSLV          SYS0
1      IMPLICIT REAL*8 (A-H,O-Z)   SYS0
2 C
3      COMMON/SLVBLK/A(5,5),B(5),N   SYS0
4      REAL*8 MAX                 SYS0
5      NM1 = N - 1                SYS0
6      N1 = N+1                   SYS0
7 C
8 C COMPUTE THE LU FACTORIZATION OF A.          SYS0
9      DO 80 K=1,N                SYS0
10     KM1 = K-1                  SYS0
11     IF(K.EQ.1) GO TO 20        SYS0
12     DO 10 I=K,N                SYS0
13     DO 10 J=1,KM1              SYS0
14     A(I,K) = A(I,K) - A(I,J)*A(J,K)    SYS0
15     10 CONTINUE                SYS0
16     20 IF(K.EQ.N) GO TO 100      SYS0
17     KP1 = K+1                  SYS0
18     MAX = DABS(A(K,K))        SYS0
19     INTR = K                   SYS0
20     DO 30 I=KP1,N              SYS0
21     AA = DAHS(A(I,K))         SYS0
22     IF(AA .LE. MAX) GO TO 30      SYS0
23     MAX = AA                   SYS0
24     INTR = I                   SYS0
25     30 CONTINUE                SYS0
26     IF(MAX .EQ. 0.) STOP       SYS0
27     A(N1,K) = INTR            SYS0
28     IF(INTR .EQ. K) GO TO 50      SYS0
29     DO 40 J=1,N                SYS0
30     TEMP = A(K,J)             SYS0
31     A(K,J) = A(INTR,J)         SYS0
32     A(INTR,J) = TEMP          SYS0
33     40 CONTINUE                SYS0
34     50 DO 80 J=KP1,N           SYS0
35     IF(K.EQ.1) GO TO 70        SYS0
36     DO 60 I=1,KM1              SYS0
37     A(K,J) = A(K,J) - A(K,I)*A(I,J)    SYS0
38     60 CONTINUE                SYS0
39     70 A(K,J) = A(K,J)/A(K,K)    SYS0
40     80 CONTINUE                SYS0
41 C
42 C INTERCHANGE THE COMPONENTS OF B.          SYS0
43 C
44     100 DO 110 J=1,NM1          SYS0
45     INTR = A(N1,J)             SYS0
46     IF(INTR .EQ. J) GO TO 110      SYS0
47     TEMP = B(J)                SYS0
48     B(J) = BINTR)               SYS0
49     BINTR) = TEMP              SYS0
50     110 CONTINUE                SYS0
51 C
52 C SOLVE LX = B.                  SYS0
53 C
54     200 B(1) = B(1)/A(1,1)      SYS0
55     DO 220 I=2,N                SYS0
56     IM1 = I-1                  SYS0
57     DO 210 J=1,IM1              SYS0
58     B(I) = B(I) - A(I,J)*B(J)    SYS0
59     210 CONTINUE                SYS0
60     B(I) = B(I)/A(I,I)          SYS0
61     220 CONTINUE                SYS0
62 C

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63 C SOLVE UX = B. SY80
64 C SY80
65 300 DO 310 II=1,NM1 SY80
66 I = NM1-II+1 SY80
67 II = I+1 SY80
68 DO 310 J=II,N SY80
69 B(I) = B(I) - A(I,J)*B(J) SY80
70 310 CONTINUE SY80
71 RETURN SY80
72 END SY80

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0      SUBROUTINE GAUSEL (MAX, N, A, NR, B, IERP)          GAU
1      IMPLICIT REAL*8 (A-H,O-Z)                         GAU
2 C      FUNCTION                                         - COMPUTES SOLUTION TO A SET OF SIMULTANEOUS   GAU
3 C                                         LINEAR EQUATIONS (DOES NOT GIVE PIVOT OR   GAU
4 C                                         DETERMINANT DATA)                                GAU
5 C      USAGE                                           - CALL GAUSEL (MAX,N,A,NR,B,IERR)                  GAU
6 C      PARAMETERS MAX                               - MAXIMUM ROW DIMENSION OF B                      GAU
7 C                                         N                               - ORDER OF A                                     GAU
8 C                                         A(N,N)                         - INPUT MATRIX OF COEFFICIENTS (DESTROYED)       GAU
9 C                                         NR                            - NUMBER OF COLUMNS IN B                         GAU
10 C                                         B(MAX,NR)                    - MATRIX OF CONSTANTS (REPLACED BY SOLUTIONS)    GAU
11 C                                         IERR                           - INTEGER ERROR CODE                           GAU
12 C                                         = 0     NORMAL RETURN                         GAU
13 C                                         = 2     INPUT MATRIX IS SINGULAR                 GAU
14 C      REQUIRED ROUTINES                         - NONE                                         GAU
15 C
16 C      SOURCE                                         NASA, LRC, ANALYSIS AND COMPUTATION DIVISION SUBPROGRAM  GAU
17 C                                         LIBRARY                                         GAU
18 C
19 C *****                                         GAUO
20      DIMENSION A(N,N),B(MAX,NR)                   GAUO
21      NM1 = N-1                                     GAUO
22      IF (NM1 .EQ. 0) GO TO 140                  GAUO
23 C *****                                         GAUO
24 C      FIND LARGEST REMAINING ELEMENT IN I-TH COLUMN FOR PIVOT  GAUO
25 C *****
26      DO 100 I=1,NM1                           GAUO
27      BIG = 0.                                    GAUO
28      DO 20 K=I,N                               GAUO
29      TERM = DABS(A(K,I))                     GAUO
30      IF (TERM - BIG) 20,20,10                GAUO
31      10      BIG = TERM                      -
32      L = K                                     GAUO
33      20      CONTINUE                         GAUO
34      IF (BIG) 40,30,40                         GAUO
35      30      IERR = 2                        GAUO
36      RETURN                                     GAUO
37      40      IF (I-L) 50,80,50                GAUOC
38 C *****                                         GAUOC
39 C      PIVOT ROWS OF A AND B                  GAUOC
40 C *****
41      50      CONTINUE                         GAUOC
42      DO 60 J=1,N                               GAUOC
43      TEMP = A(I,J)                           GAUOC
44      A(I,J) = A(L,J)                         GAUOC
45      A(L,J) = TEMP                           GAUOC
46      60      CONTINUE                         GAUOC
47      DO 70 J=1,NR                           GAUOC
48      TEMP = B(I,J)                           GAUOC
49      B(I,J) = B(L,J)                         GAUOC
50      B(L,J) = TEMP                           GAUOC
51      70      CONTINUE                         GAUOC
52      80      CONTINUE                         GAUOC
53 C *****
54 C      STORE PIVOT AND PERFORM COLUMN OPERATIONS ON A AND B  GAUOC
55 C *****
56      IP1 = I+1                                GAUOC
57      DO 100 II=IP1,N                          GAUOC
58      A(II,I) = A(II,I)/A(I,I)                GAUOC
59      X3 = A(II,I)                           GAUOC
60      DO 90 K=IP1,N                          GAUOC
61      A(II,K) = A(II,K) - X3*A(I,K)           GAUOC
62      90      CONTINUE                         GAUOC

```

63	DO 100 K=1,NR	GAUO
64	B(II,K) = B(II,K) - X3*B(I,K)	GAUO
65	100 CONTINUE	GAUO
66	C ****	GAUO
67	C PERFORM BACK SUBSTITUTION	GATJO
68	C ****	GAUO
69	DO 110 IC=1,NR	GAUO
70	B(N,IC) = B(N,IC)/A(N,N)	GAUO
71	110 CONTINUE	GAUO
72	DO 130 KK=1,NM1	GAUO
73	I = N-KK	GAUO
74	IP1 = I+1	GAUO
75	DO 130 J=1,NR	GAUO
76	SUM = B(I,J)	GAUO
77	DO 120 K=IP1,N	GAUO
78	SUM = SUM - A(I,K)*B(K,J)	GAUO
79	120 CONTINUE	GAUO
80	B(I,J) = SUM/A(I,I)	GAUOC
81	130 CONTINUE	GAUOC
82	RETURN	GAUOC
83	140 CONTINUE	GAUOC
84	IF (A(1,1) .EQ. 0.) GO TO 300	GAUOC
85	DO 150 J=1,NR	GAUOC
86	B(1,J) = B(1,J)/A(1,1)	GAUOC
87	150 CONTINUE	GAUOC
88	RETURN	GAUOC
89	300 IERR = 2	GAUOC
90	RETURN	GAUOC
91	END	GAUOC

```
0      SUBROUTINE PNCH (A,NA,NAM,IOP)
1 C IOP(1)=0, SKIP TITLE; IOP(2)=N, SKIP LINES; IOP(3)=1, TAB 25 SPACES.
2      IMPLICIT REAL*8 (A-H,O-Z)
3      DIMENSION A(1),IOP(4),NA(2)
4      NR=NA(1)
5      NC=NA(2)
6      NMAX=NR+NC
7      NSKIP=IOP(2)
8      IF (IOP(2).EQ.0) GO TO 205
9      DO 200 I=1,NSKIP
10     200 WRITE(7,150)
11     150 FORMAT(2X)
12     205 CONTINUE
13     IF (IOP(1).EQ.0) GO TO 210
14     WRITE(7,151) NAM, NR, NC
15     151 FQRMAT(A4,/,215)
16     210 CONTINUE
17     DO 250 I=1,NR
18     IF (IOP(3).EQ.0) WRITE(7,152) (A(J),J=I,NMAX,NR)
19     IF (IOP(3).NE.0) WRITE(7,153) (A(J),J=I,NMAX,NR)
20     250 CONTINUE
21     152 FORMAT(6(1PD13.5))
22     153 FORMAT(25X,6(1PD13.5))
23     RETURN
24     END
```

0	FUNCTION DIMAG(Z)	DIMC
1	REAL*8 A(2),DIMAG	DIMC
2	COMPLEX*16 Z,B	DIMC
3	EQUIVALENCE (A,B)	DIMC
4	B=Z	DIMC
5	DIMAG=A(2)	DIMC
6	RETURN	DIMO
7	END	DIMO

```
0      FUNCTION DREAL(Z)
1      REAL*8 A(2),DREAL
2      COMPLEX*16 Z,B
3      EQUIVALENCE (A,B)
4      B=Z
5      DREAL=A(1)
6      RETURN
7      END
```

```
DREAL
DREAL
DREAL
DREAL
DREAL
DREAL
DREAL
DREAL
DREAL
```

0	BLOCK DATA	MOD000
1	IMPLICIT REAL*8 (A-H,0-2)	MOD000
2	COMMON/LINES/TITLE(10),TIL(3),NLP,LIN	MOD000
3	COMMON/FOPM/FMT1(2),FMT2(2),NEPR	MOD000
4	COMMON/TOL/EPSAM,EPSBM,IACM	MOD000
5	COMMON/CONV/SUMCV,RICTCV,SERCV,MAXSUM	MOD000
6	DATA LIN,NLP/1,58/	MOD000
7	DATA NEPR,FMT1/7,8H(1P7D16.,8H7) /	MOD000
8	DATA TIL/RM ORA,8HCLS PRO,8HGRAM /	MOD000
9	DATA FMT2/8H(3X,1P7D,9H10.7) /	MOD000
10	DATA EPSAM/1.E-10/	MOD000
11	DATA EPSBM/1.E-10/	MOD000
12	DATA IACM/12/	MOD000
13	DATA SUMCV/1.E-8/	MOD000
14	DATA RICTCV/1.E-8/	MOD000
15	DATA SERCV/1.E-8/	MOD000
16	DATA MAXSUM/50/	MOD000
17	END	MOD000