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FULLY FLOODED ELASTOHYDRODYNAMIC LUBRICATED ELLIPTICAL CONTACTS

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FULLY FLOODED ELASTOHYDRODYNAMIC LUBRICATED ELLIPTICAL CONTACTS

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Elastohydrodynamic lubrication is a form of fluid-film lubrication where elastic deformation of bearing surfaces becomes significant. It is usually associated with highly stressed machine components of low conformity, such as gears and rolling-element bearings. This lubrication mechanism is also encountered with soft bearing materials, such as rubber seals and tires. The common factor in these applications is that local elastic deformation of the solids provides coherent fluid films and thus asperity interaction is prevented.

Historically, elastohydrodynamic lubrication may be viewed as one of the major developments in the field of tribology in the twentieth century. It not only revealed the existence of a previously unsuspected regime of lubrication in highly stressed and nonconformal machine elements, such as gears and rolling-element bearings, but it brought order to the understanding of the complete spectrum of lubrication ranging from boundary to hydrodynamic.

A chronology of the research on elastohydrodynamic lubrication (EHL) of elliptical contacts that the author has been involved with is given in table I. In this table the major research efforts and some of the results are given for the years 1974 to 1980. This particular lecture concentrates on fully flooded conjunctions. A fully flooded conjunction is one in which the film thickness is not significantly changed when the amount of lubricant is increased.

A brief description of the relevant equations used in the elastohydrodynamic lubrication analysis is given in this lecture. The most important practical aspect of the elastohydrodynamic lubrication theory is the determination of the minimum film thickness within the contact. The maintenance of a fluid film of adequate magnitude is an essential feature of the correct operation of lubricated machine elements. The results to be presented show the influence of contact geometry on minimum film thickness as expressed by the ellipticity parameter and the dimensionless speed, load, and materials parameters. These results are applied to materials of high elastic modulus, such as metal, and to materials of low elastic modulus, such as rubber. The solutions for materials of high elastic modulus are sometimes referred to as "hard EHL," and the solution for materials of low elastic modulus as "soft EHL." In addition to the film thickness equations that are developed, contour plots of pressure and film thickness are presented that show the essential features of elastohydrodynamically lubricated conjunctions. In particular the crescent-shaped region of minimum film thickness, with its side lobes in which the separation between the solids is a minimum, clearly emerges in the numerical solutions. These theoretical solutions for film thickness have all the essential features of previously reported experimental observations based on optical interferometry. Correlation between theory and experiments is also made.

RELEVANT EQUATIONS

The relevant equations used in elastohydrodynamic lubrication of elliptical contacts are as follows:

Lubrication equation (Reynolds equation)

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial y}\right) = 12u\frac{\partial}{\partial x}(\rho h) \tag{1}$$

where

$$u = \frac{(u_a + u_b)}{2}\text{, the mean surface velocity}$$

Viscosity variation

$$\eta = \eta_0 e^{\alpha p} \tag{2}$$

where

η_0 coefficient of absolute or dynamic viscosity at atmospheric pressure
α pressure-viscosity coefficient of fluid

Density variation (for mineral oils)

$$\rho = \rho_0\left(1 + \frac{0.6\,p}{1 + 1.7\,p}\right) \tag{3}$$

where

ρ_0 density at atmospheric conditions

Elasticity equation

$$w = \frac{2}{E'}\iint_A \frac{p(x,y)\,dA}{\sqrt{(x - x_1)^2 + (y - y_1)^2}} \tag{4}$$

where

$$E' = \frac{2}{\left(\dfrac{1 - \nu_a^2}{E_a} + \dfrac{1 - \nu_b^2}{E_b}\right)} \tag{5}$$

Film thickness equation

$$h = h_0 + S(x,y) + w(x,y) = h_0 + \frac{x^2}{2R_x} + \frac{y^2}{2R_y} + w(x,y) \tag{6}$$

The problem is to calculate the pressure distribution in the contact and at the same time allow for the effects that this pressure will have on the properties of the fluid and on the geometry of the elastic solids. The solution will also provide the shape of the lubricant film, particularly the minimum clearance between the solids. A detailed description of the elasticity model used is given in Dowson and Hamrock (1976), and the complete EHL theory is given in Hamrock and Dowson (1976).

DIMENSIONLESS GROUPING

The variables resulting from the isothermal elliptical contact theory developed in Hamrock and Dowson (1976) are

E'	effective elastic modulus, N/m^2
F	normal applied load, N
h	film thickness, m
R_x	effective radius in x (motion) direction, m
R_y	effective radius in y (transverse) direction, m
u	mean surface velocity in x direction, m/s
α	pressure-viscosity coefficient of fluid, m^2/N
η_0	atmospheric viscosity, N s/m^2

From the eight variables the following five dimensionless groupings were established:

Dimensionless film thickness

$$H = \frac{h}{R_x} \tag{7}$$

where

$$\frac{1}{R_x} = \frac{1}{r_{ax}} + \frac{1}{r_{bx}} \tag{8}$$

Ellipticity parameter

$$k = \frac{a}{b} = 1.03 \left(\frac{R_y}{R_x}\right)^{0.64} \tag{9}$$

where

$$\frac{1}{R_y} = \frac{1}{r_{ay}} + \frac{1}{r_{by}} \tag{10}$$

Equation (9) was obtained from Brewe and Hamrock (1977).

Dimensionless load parameter

$$W = \frac{F}{E'R_x^2} \tag{11}$$

where

F - normal applied load

Dimensionless speed parameter

$$U = \frac{\eta_0 u}{E'R_x} \tag{12}$$

Dimensionless materials parameter

$$G = \alpha E' \tag{13}$$

The dimensionless film thickness can thus be written as a function of the other four parameters

$$H = f(k,U,W,G) \tag{14}$$

The influence of the dimensionless parameters k, U, W, and G on minimum film thickness H_{min} is presented later for both hard and soft contacts.

The set of dimensionless groups $\{H,k,U,W,G\}$ is a useful collection of parameters for evaluating the results presented in this lecture. It is also comparable to the set of dimensionless parameters used in the initial elastohydrodynamic analysis of line contacts, and it has the merit that the physical significance of each term is readily apparent. However, a number of authors, for example, Moes (1965-66) and Theyse (1966), have noted that this set of dimensionless groups can be reduced by one parameter without any loss of generality. This approach is followed in the fourth lecture, where the film thicknesses to be expected in each of the four regimes of fluid-film lubrication are presented graphically.

HARD EHL RESULTS

By using the numerical procedures outlined in Hamrock and Dowson (1976) the influence of the ellipticity parameter and the dimensionless speed, load, and materials parameters on minimum film thickness has been investigated for hard EHL, fully flooded contacts (Hamrock and Dowson, 1977). The ellipticity parameter k was varied from 1 (a ball-on-plate configuration) to 8 (a configuration approaching a rectangular contact). The dimensionless speed U was varied over a range of nearly two orders of magnitude, and the dimensionless load parameter W over a range of one order of magnitude. Situations equivalent to using solid materials of bronze, steel, and silicon nitride and lubricants of paraffinic and naphthenic oils were considered in an investigation of the role of the dimensionless materials parameter G. The 34 cases used to generate the minimum-film-thickness formula are given in table II. In the table H_{min} corresponds to the minimum film thickness obtained from the EHL elliptical contact theory given in Hamrock and

Dowson (1976). The minimum-film-thickness formula obtained from a least-squares fit of the data was first given in Hamrock and Dowson (1977) and is given here as

$$\boxed{\tilde{H}_{min} = 3.63\ U^{0.68}G^{0.49}W^{-0.073}(1 - e^{-0.68k})} \tag{15}$$

In table II $\tilde{H}_{min}$ is the minimum film thickness obtained from equation (15). The percentage difference between H_{min} and $\tilde{H}_{min}$ is expressed by

$$D_1 = \left(\frac{\tilde{H}_{min} - H_{min}}{H_{min}}\right) 100 \tag{16}$$

In table II the values of D_1 are within ±5 percent.

It is interesting to compare the elliptical-contact, minimum-film-thickness formula (eq. (15)) with the corresponding equation generated by Dowson (1968) for nominal line or rectangular contacts

$$H_{min,r} = 2.65\ U^{0.70}G^{0.54}W_r^{-0.13} \tag{17}$$

where

$$W_r = \frac{F^*}{E'R_x} \tag{18}$$

$$F^* = \frac{F}{\ell}, \text{ load per unit length} \tag{19}$$

The powers of U, G, and W in equation (15) and U, G, and W_r in equation (17) are quite similar considering the different numerical procedures on which they are based. It is also worth noting that the power of W in equation (15) is extremely close to the value of -0.074 proposed by Archard and Cowking (1965-66) in their experimental studies of elliptical contacts.

In practical situations there is considerable interest in the central as well as the minimum film thickness in elastohydrodynamic contacts. This is particularly true when traction is considered since the surfaces in relative motion are separated by a film of almost constant thickness that is well represented by the central value over much of the Hertzian contact zone. The procedure used in obtaining the central film thickness was the same as that used in obtaining the minimum film thickness. The central-film-thickness formula for hard EHL contacts obtained from the numerical results is

$$\tilde{H}_c = 2.69\ U^{0.67}G^{0.53}W^{-0.067}(1 - 0.61\ e^{-0.73k}) \tag{20}$$

A representative contour plot of dimensionless pressure is shown in figure 1 for $k = 1.25$, $U = 0.168\times10^{-11}$, $W = 0.111\times10^{-6}$, and $G = 4522$. In this figure and in figure 2, the + symbol indicates the center of the Hertzian contact zone. The dimensionless representation of the X and Y coordinates causes the actual Hertzian contact ellipse to be a circle regardless of the value of the ellipticity parameter. The Hertzian contact circle is shown by asterisks. On the figure is a key showing the contour labels and each corresponding value of dimensionless pressure. The inlet region is to the left and the exit region is to the right.

The pressure gradient at the exit end of the conjunction is much larger than that in the inlet region. In figure 1 a pressure spike is visible at the exit of the contact.

Figure 2 shows contour plots of film thickness for $k = 1.25$, $U = 0.168 \times 10^{-11}$, $W = 0.111 \times 10^{-6}$, and $G = 4522$. In this figure two minimum-film-thickness regions occur in well-defined side lobes that follow, and are close to, the edge of the Hertzian contact ellipse. These results reproduce all the essential features of previously reported experimental observations based on optical interferometry (Cameron and Gohar, 1966).

The variation of pressure and film thickness in the direction of rolling quite close to the X axis near the midplane of the conjunction is shown in figure 3 for three values of U. The values of the dimensionless load, materials, and ellipticity parameters were held constant at $k = 6$, $W = 0.737 \times 10^{-6}$, and $G = 4522$. In figure 3(a) the dashed line corresponds to the Hertzian pressure distribution. This figure shows that the pressure at any location in the inlet region rises as the speed increases, a result that is also consistent with the elastohydrodynamic theory for line or rectangular contacts. Furthermore, as the speed decreases, the height of the pressure spike decreases and the hydrodynamic pressures gradually approach the semielliptical form of the Hertzian contact stresses. Note that the location of the pressure spike moves downstream toward the edge of the Hertzian contact ellipse as the speed decreases. For nominal line or rectangular contacts Dowson and Higginson (1966) showed results similar to those in figure 3(a).

The typical elastohydrodynamic film shape with an essentially parallel section in the central region is shown in figure 3(b). There is little sign of a reentrant region in this case, except perhaps at the lowest speed. Also, there is a considerable change in the film thickness as the dimensionless speed is changed, as indicated by equation (15). This illustrates most clearly the dominant effect of the dimensionless speed parameter U on the minimum film thickness in elastohydrodynamic contacts.

The variation of pressure and film thickness in the direction of motion along a line close to the midplane of the conjunction is shown in figure 4 for three values of dimensionless load parameter. The values of the dimensionless speed, materials, and ellipticity parameters were held fixed at $U = 0.168 \times 10^{-11}$, $G = 4522$, and $k = 6$. Note from figure 4(a) that the pressure at any location in the inlet region falls as the load increases. For the highest load ($W = 1.106 \times 10^{-6}$), figure 4(b), the film thickness rises between the central region and the outlet restriction. This reentrant effect is attributed to lubricant compressibility. Note also that at $W = 0.5228 \times 10^{-6}$ the film thickness is slightly smaller than at $W = 1.06 \times 10^{-6}$. This somewhat curious result is linked to the fact that the location of the minimum film thickness also changes drastically over this load range. At the lower load the minimum film thickness is located on the midplane of the conjunction downstream from the center of the contact; at the higher load it moves to the side lobes as described earlier.

COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL FILM THICKNESSES

The minimum- and central-film-thickness formulas developed in the previous section for fully flooded conjunctions are not only useful for design purposes, but they also provide a convenient means of assessing the influence of various parameters on the elastohydrodynamic film thickness. For the purpose of comparing theoretical film thicknesses with those found in actual elastohydrodynamic contacts, table III was obtained from Kunz and Winer (1977). The experimental apparatus consisted of a steel ball rolling and sliding on a sapphire plate; this generated a circular conjunction, or an ellipticity parameter of unity. Measurements were made by using the technique of optical interferometry. Table III shows the results of both calculations and measurements for three lubricants in pure sliding, each under two different loads and three speeds. The notation $\tilde{H}_{min}$ and $\tilde{H}_c$ is used to denote the dimensionless film thickness calculated from equations (15) and (20), respectively. The measured minimum and central film thicknesses obtained from Kunz and Winer (1977) are denoted by H_{min} and H_c. Figures 5 and 6 show comparisons between the calculated and measured film thicknesses for the two loads shown in table III.

For the smaller load ($W = 0.1238 \times 10^{-6}$) the results shown in figure 5 compare remarkably well if we bear in mind the difficulties associated with the experimental determination of such small quantities under arduous conditions and the error associated with the complex numerical evaluations of elastohydrodynamic conjunctions. The ratios between the central and minimum film thicknesses are similar for the calculations and the measurements, and the dependence of film thickness on speed thus appears to be well represented by equations (15) and (20).

For the larger load ($W = 0.9287 \times 10^{-6}$) the agreement shown in figure 6 is not quite so good, with the theoretical predictions of film thickness being consistently larger than the measured values. This discrepancy is sometimes attributed to viscous heating, as discussed by Greenwood and Kauzlarich (1973), or to non-Newtonian behavior, as discussed by Moore (1973). Viscous heating appears to enjoy the most support. Since the measurements were made in a condition of pure sliding, thermal effects might well be significant, particularly at the larger load. The value of viscosity used in the calculations reported in table III was that corresponding to the temperature of the lubricant bath. If thermal effects become important, the isothermal assumption used in deriving equations (15) and (20) is violated. But, although the value of the viscosity used in these expressions is necessarily somewhat arbitrary, there is evidence from previous studies of line or rectangular elastohydrodynamic conjunctions that the film thickness is determined by the effective viscosity in the inlet region. If this viscosity is known with reasonable accuracy, the predicted film thicknesses are quite reliable. In addition, at the more severe conditions imposed by the larger load the lubricant may no longer behave as a Newtonian liquid. This would violate the assumptions used in deriving equations (15) and (20).

The results of Kunz and Winer (1977) presented in table III and figures 5 and 6 suggest that at large loads there is a more rapid change in lubricant film thickness than would be predicted by isothermal elastohydrodynamic theory for elliptical conjunctions as presented in equations (15) and (20). This view is supported by the experimental results based on the X-ray technique reported by Parker and Kannel (1971) and the results of optical interferometry presented by Lee, et al. (1973). This observation, however, contradicts the results of Johnson and Roberts (1974), who used the spring dynamometer method described in the last section to estimate elastohydrodynamic film thickness. Johnson and Roberts found that, in spite of the approximate nature of their method, the results indicated strongly that elastohydrodynamic film thickness as predicted by equations (15) and (20) is maintained up to the highest contact pressures that are practical with ball bearing steel. Their results support the more precise measurements of Gentle and Cameron (1973) and extend the range of operating conditions to higher rolling speeds. The discrepancy between these investigations in regard to the influence of load on elastohydrodynamic film thickness has yet to be explained.

Another comparison between experimental findings and theoretical predictions can be based on the experimental results provided by Dalmaz and Godet (1978). They measured film thickness optically in a pure-sliding, circular-contact apparatus for different fire-resistant fluids. An example of the good correlation between the theoretical predictions based on the formulas developed in the previous section and these experimental results is shown in figure 7. The agreement between the experimentally determined variation of central film thickness with speed for mineral oil and water-glycol lubricants of similar viscosity and the theoretical predictions is most encouraging. The ellipticity parameter was unity and the applied load was 2.6 newtons for these experiments. Figure 7 also shows that for water-glycol the film thickness generated was barely one-third of that developed for mineral oil of similar viscosity. This drastic reduction in film thickness is attributed to the pressure-viscosity coefficient of water-glycol, which is only about one-fifth that of mineral oil.

Various theoretical predictions are compared with the experimental results of Archard and Kirk (1964) in figure 8. These results are presented in a form that indicates the influence of side leakage on film thickness in elliptical contacts by plotting a film reduction factor $H_{min}/H_{min,r}$ against the effective radius ratio R_x/R_y. The film reduction factor is defined as the ratio of the minimum film thickness achieved in an elliptical contact to that achieved in a rectangular contact formed between two cylinders in nominal line contact. The theoretical results of Archard and Cowking (1965-66), Cheng (1970), and Hamrock and Dowson (1977), along with the theoretical solution for rigid solids by Kapitza (1955), are presented in this figure. Archard and Cowking (1965-66) adopted an approach for elliptical contacts similar to that used by Grubin (1949) for a rectangular-contact condition. The Hertzian contact zone is assumed to form a parallel film region, and the generation of high pressure in the approach to the Hertzian zone is considered.

Cheng (1970) solved the coupled system of equations separately by first calculating the deformations from the Hertz equation. He then applied the Reynolds equation to this geometry. He did not consider a change in the lubricant density and assumed an exponential law for the viscosity change due to pressure. Hamrock and Dowson (1977) developed a procedure for the numerical solution of the complete isothermal elastohydrodynamic lubrication of elliptical contacts.

For $0.1 < R_x/R_y < 1$ the Archard and Cowking (1965-66) and Hamrock and Dowson (1977) predictions are in close agreement, and both overestimate the reduction in film thickness evident in the experimental results of Archard and Kirk (1964). For $1 < R_x/R_y < 10$ the Hamrock and Dowson (1977) predictions of film thickness exceed and gradually diverge from those of Archard and Cowking (1965-66). They are also in better agreement with the experimental results of Archard and Kirk (1964). Cheng's (1970) theoretical predictions are optimistic and differ significantly from the experimental data. Kapitza's (1955) rigid-contact theory is also optimistic about the reduction in film thickness compared with the experimental results, but it is in closer agreement with experiment than the Cheng (1970) results. These comparisons between theory and experiment offer reasonable grounds for confidence in the predictions of the complete three-dimensional elastohydrodynamic lubrication theory for elliptical contacts presented in the previous section.

SOFT EHL RESULTS

The earlier studies of elastohydrodynamic lubrication of conjunctions of elliptical form are applied to the particular and interesting situation exhibited by materials of low elastic modulus (soft EHL). The procedure used in obtaining the soft EHL results is given in Hamrock and Dowson (1978). The ellipticity parameter was varied from 1 (a ball-on-plate configuration) to 12 (a configuration approaching a nominal line or rectangular contact). The dimensionless speed and load parameters were varied by one order of magnitude. Seventeen different cases used to generate the minimum-film-thickness formula are given in table IV. In the table H_{min} corresponds to the minimum film thickness obtained from applying the EHL elliptical contact theory given in Hamrock and Dowson (1976) to the soft EHL contacts. The minimum-film-thickness formula obtained from a least-squares fit of the data was first given in Hamrock and Dowson (1978) and is given here as

$$\boxed{\tilde{H}_{min} = 7.43\ U^{0.65} W^{-0.21} (1 - 0.85\ e^{-0.31k})} \tag{21}$$

In table IV $\tilde{H}_{min}$ is the minimum film thickness obtained from equation (21). The percentage difference between H_{min} and $\tilde{H}_{min}$ is expressed by D_1, given in equation (16). The values of D_1 in table IV are within the range -8 to 3 percent.

It is interesting to compare the equation for materials of low elastic modulus (soft EHL, eq. (21)) with the corresponding equation for materials of high elastic modulus (hard EHL) given in equation (15). The powers of U in equations (21) and (15) are similar, but the power of W is much more significant for low-elastic-modulus materials. The expression showing the effect of the ellipticity parameter is of exponential form in both equations, but with different constants.

A major difference between equations (21) and (15) is the absence of a materials parameter G in the expression for low-elastic-modulus materials. There are two reasons for this. One is the negligible effect of pressure on the viscosity of the lubricating fluid, and the other is the way in which the role of elasticity is simply and automatically incorporated into the prediction of conjunction behavior through an increase in the size of the Hertzian contact zone corresponding to changes in load. As a check on the validity of this, case 9 of table IV was repeated with the material changed from nitrile to silicone rubber. The results of this change are recorded as case 17 in table IV.

The dimensionless minimum film thickness calculated from the full numerical solution to the elastohydrodynamic contact theory was 181.8×10^{-6}, and the dimensionless minimum film thickness predicted from equation (21) turned out to be 182.5×10^{-6}. This clearly indicates a lack of dependence of the minimum film thickness for low-elastic-modulus materials on the materials parameter.

There is interest in knowing the central film thickness, in addition to the minimum film thickness, in elastohydrodynamic contacts. The procedure used to obtain an expression for the central film thickness was the same as that used to obtain the minimum-film-thickness expression. The central-film-thickness formula for low-elastic-modulus materials (soft EHL) as obtained from Hamrock and Dowson (1978) is

$$\tilde{H}_c = 7.32(1 - 0.72\ e^{-0.28k})U^{0.64}W^{-0.22} \tag{22}$$

Comparing the central- and minimum-film-thickness equations, (22) and (21), reveals only slight differences. The ratio of minimum to central film thickness evident in the computed values given in Hamrock and Dowson (1978) ranged from 70 to 83 percent, the average being 77 percent.

Figure 9 shows the variation of the ratio $H_{min}/H_{min,r}$ where $H_{min,r}$ is the minimum film thickness for rectangular contacts, with the ellipticity parameter k for both high- and low-elastic-modulus materials. If it is assumed that the minimum film thickness obtained from the elastohydrodynamic analysis of elliptical contacts can only be obtained to an accuracy of 3 percent, we find that the ratio $H_{min}/H_{min,r}$ approaches the limiting value at $k = 5$ for high-elastic-modulus materials. For low-elastic-modulus materials the ratio approaches the limiting value more slowly, but it is reasonable to state that the rectangular-contact solution will give a very good prediction of the minimum film thickness for conjunctions in which k exceeds about 11.

The variation of pressure and film thickness in the direction of rolling along a line near the midplane of the conjunction is shown in figure 10 for two values of U. For all the solutions obtained at various speeds, the values of the dimensionless load, materials, and ellipticity parameters were held fixed. Figure 10 shows that the pressure at any point in the inlet region increases as the speed increases. The dominant effect of the dimensionless speed parameter on the minimum film thickness in elastohydrodynamic contacts for low-elastic-modulus materials (soft EHL) evident in equation (21) is reflected in figure 10. Similar results were found for high-elastic-modulus materials (hard EHL).

The variation of pressure and film thickness in the rolling direction along a line close to the midplane of the conjunction is shown in figure 11 for two values of the dimensionless load parameter. Once again the essential features of the minimum-film-thickness equation are reflected in this figure since a change in the dimensionless load parameter of one order of magnitude produces a considerable change in the pressure profile but not such a significant change in the film thickness. The small effect of load on minimum film thickness is a reasonable and important feature of elastohydrodynamic lubrication.

The pressure spikes found when dealing with materials of high elastic modulus (hard EHL) are not evident in these solutions for low-elastic-modulus materials (soft EHL). The absence of a pressure spike for low-elastic-modulus materials is due to the pressures generated for a given load in a contact between low-elastic-modulus materials being considerably lower than those generated in a contact between high-elastic-modulus materials.

CONCLUDING REMARKS

The emphasis of the present lecture was on fully flooded, elastohydrodynamic lubricated elliptical contacts. A fully flooded conjunction is one in which the film thickness is not significantly changed when the amount of lubricant is increased. A brief description of the relevant equations used in the elastohydrodynamic lubrication of elliptical contacts was given. The most important practical aspect of the elastohydrodynamic theory is the determination of the minimum film thickness within the contact. The maintenance of a fluid film of adequate magnitude is an essential feature of the correct operation of lubricated machine elements. The results presented show the influence of contact geometry on minimum film thickness as expressed by the ellipticity parameter and the dimensionless speed, load, and materials parameters. Film thickness equations are developed for materials of high elastic modulus, such as metal, and for materials of low elastic modulus, such as rubber. The solutions for high-elastic-modulus materials are sometimes referred to as "hard EHL," and the solutions for low-elastic-modulus materials as "soft EHL." In addition to the film thickness equations that were developed, plots of pressure and film thickness were presented. These theoretical solutions for film thickness have all the essential features of previously reported experimental observations based on optical interferometry. Correlation between theory and experiments was also made.

APPENDIX - SYMBOLS

a	semimajor axis of contact ellipse, m
b	semiminor axis of contact ellipse, m
D_1	$[(\tilde{H}_{min} - H_{min})/H_{min}]\ 100$
E	modulus of elasticity, N/m^2
E'	effective elastic modulus, $2\Big/\left[\dfrac{1-\nu_a^2}{E_a} + \dfrac{1-\nu_b^2}{E_b}\right]$, N/m^2
F	normal applied load, N
F^*	normal applied load per unit length, N/m
G	dimensionless materials parameter, $\alpha E'$
$\tilde{H}$	dimensionless film thickness, h/R_x
H	dimensionless film thickness obtained from least-squares fit of numerical data
h	film thickness, m
h_0	constant, m
k	ellipticity parameter
ℓ	unit length, m
P	dimensionless pressure, p/E'
p	pressure, N/m^2
R	effective radius, m
r	radius of curvature, m
S	geometric separation, m
U	dimensionless speed parameter, $\eta_0 u/E'R_x$
u	mean surface velocity in direction of motion, $(u_a + u_b)/2$, m/s
W	dimensionless load parameter, $F/E'R_x^2$
W_r	dimensionless load parameter for rectangular contact, $F^*/E'R_x$
x,y,X,Y	coordinate system
α	pressure-viscosity coefficient of lubricant, m^2/N
η	coefficient of absolute or dynamic viscosity, $N\ s/m^2$
η_0	coefficient of absolute or dynamic viscosity at atmospheric pressure, $N\ s/m^2$
ν	Poisson's ratio
ρ	lubricant density, $N\ s^2/m^4$
ρ_0	density at atmospheric pressure, $N\ s^2/m^4$

Subscripts:

a	solid a
b	solid b
c	central
min	minimum
r	for rectangular contact
x,y	coordinate system

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TABLE 1. - CHRONOLOGY OF RESEARCH ON ELASTOHYDRODYNAMIC LUBRICATION (EHL) OF ELLIPTICAL CONTACTS

Year	Major research effort	Some results
1974 -	(1) Review literature (2) Investigate various elasticity models	The elasticity model that assumes a uniform pressure over a rectangular area was chosen
1975	Develop EHL theory with the following assumptions: (1) Isothermal fluid (2) Newtonian fluid	For the first time a complete theoretical solution was obtained for elliptical contacts lubricated elastohydrodynamically. This involved successful coupling of the elasticity and Reynolds equations
1976	Apply EHL theory to <u>hard</u> contacts that have (1) Fully flooded or (2) Starved conjunctions	Contours describing the pressure and film thickness within the conjunction were developed Film thickness equations were formulated for the fully flooded and starved situations
1977	Apply EHL theory to <u>soft</u> contacts that have (1) Fully flooded or (2) Starved conjunctions	Same as above
1978	Investigate the four fluid-film lubrication regimes encountered in elliptical contacts: (1) Isoviscous rigid (2) Viscous rigid (3) Viscous elastic (hard EHL) (4) Isoviscous elastic (soft EHL)	(1) Film thickness equations were formulated for each regime in terms of dimensionless geometry, viscosity, and elasticity parameters (2) Maps of the four lubrication regimes were obtained The results enable both the mode of fluid-film lubrication and the magnitude of the film thickness to be ascertained for specified operating conditions
1979 & 1980	Incorporate the results from the previous 5 years of research into a book	EHL results are applied to the following applications: (1) Ball bearings (2) Wheel on a rail (3) Gears (4) Variable speed drive (5) Seals (6) Human joints (7) Roller bearings

TABLE II. - DATA SHOWING EFFECT OF ELLIPTICITY, LOAD, SPEED, AND MATERIALS PARAMETERS ON MINIMUM FILM THICKNESS FOR HARD EHL CONTACTS

Case	Ellipticity parameter, k	Dimensionless load parameter, W	Dimensionless speed parameter, U	Dimensionless materials parameter, G	Minimum film thickness		Difference between H_{min} and $\tilde{H}_{min}$, D_1, percent	Results
					Obtained from EHL elliptical contact theory, H_{min}	Obtained from least-squares fit, $\tilde{H}_{min}$		
1	1	0.1106×10^{-6}	0.1683×10^{-11}	4522	3.367×10^{-6}	3.514×10^{-6}	+4.37	Ellipticity
2	1.25				4.105	4.078	-.66	
3	1.5				4.565	4.554	-.24	
4	1.75				4.907	4.955	+.98	
5	2				5.255	5.294	+.74	
6	2.5				5.755	5.821	+1.15	
7	3				6.091	6.196	-1.72	
8	4				6.636	6.662	-.24	
9	6				6.969	7.001	+.46	
10	8				7.048	7.091	+.61	
11	6	0.2211	0.1683×10^{-11}	4522	6.492	6.656	+2.53	Load plus case 9
12		.3686			6.317	6.412	+1.50	
13		.5528			6.268	6.225	-.69	
14		.7371			6.156	6.095	-.99	
15		.9214			6.085	5.997	-1.45	
16		1.106			5.811	5.918	+1.84	
17		1.290			5.657	5.851	+3.43	
18	6	0.7371	0.08416	4522	3.926	3.805	-3.08	Speed plus case 14
19			.2525		8.372	8.032	-4.06	
20			.3367		9.995	9.769	-2.26	
21			.4208		11.61	11.37	-2.07	
22			.5892		14.39	14.29	-.69	
23			.8416		18.34	18.21	-.71	
24			1.263		24.47	24.00	-1.92	
25			1.683		29.75	29.18	-1.92	
26			2.104		34.58	33.96	-1.79	
27			2.525		39.73	38.44	-3.25	
28			2.946		43.47	42.69	-1.79	
29			3.367		47.32	46.76	-1.18	
30			4.208		54.57	54.41	-.29	
31			5.050		61.32	61.59	+.44	
32	6	0.7216	0.3296	2310	6.931	6.938	+0.10	Materials plus case 9
33	6	.7216	.9422	3491	17.19	17.59	+2.33	
34	6	.2456	.1122	6785	6.080	6.116	+.59	

TABLE III. - THEORETICAL AND EXPERIMENTAL FILM THICKNESS

[From Kunz and Winer (1977).]

Lubricant[a]	Dimensionless speed parameter, U	Central film thickness		Minimum film thickness	
		Theoretical, eq. (20), $\tilde{H}_c$	Experimental, H_c	Theoretical, eq. (15), $\tilde{H}_{min}$	Experimental, H_{min}
Dimensionless load parameter, W, 0.1238×10^{-6}					
A	0.1963×10^{-11}	6.84×10^{-6}	5.7×10^{-6}	3.87×10^{-6}	2.8×10^{-6}
	.3926	10.9	9.9	6.20	5.7
	.7866	17.3	16.0	9.3	11.0
B	.2637	11.0	15.0	6.58	8.4
	.5274	18.9	22.0	10.5	14.0
	1.057	30.2	34.0	17.0	24.0
C	.2268	8.04	8.2	4.53	5.0
	.4536	12.8	12.0	7.27	7.5
	.9089	20.6	18.0	11.6	13.0
Dimensionless load parameter, W, 0.9287×10^{-6}					
A	0.1963×10^{-11}	5.96×10^{-6}	4.3×10^{-6}	3.33×10^{-6}	2.8×10^{-6}
	.3926	9.50	7.1	5.35	4.3
	.7866	15.1	12.0	8.58	5.7
B	.2637	10.4	8.4	5.68	2.8
	.5274	16.6	12.0	9.11	5.6
	1.057	26.4	17.0	14.6	8.4
C	.2258	7.01	6.3	3.92	2.6
	.4536	11.2	9.4	6.27	3.8
	.9089	17.8	14.0	10.1	5.0

[a]Lubricant A is polyalkyl aromatic ($\alpha = 1.58\times10^{-8}$ $(N/m^2)^{-1}$; $\eta_0 = 0.0255$ N s/m^2; G = 4507); lubricant B is synthetic hydrocarbon ($\alpha = 3.11\times10^{-8}$ $(N/m^2)^{-1}$; $\eta_0 = 0.0343$ N s/m^2; G = 8874); lubricant C is modified polyphenyl ether ($\alpha = 1.79\times10^{-8}$ $(N/m^2)^{-1}$; $\eta_0 = 0.0295$ N s/m^2; G = 5107), where α is the pressure-viscosity constant, η_0 is the atmospheric viscosity, and G is the dimensionless materials parameter, $\alpha E'$.

TABLE IV. - DATA SHOWING EFFECT OF ELLIPTICITY, LOAD, SPEED, AND MATERIALS PARAMETERS ON MINIMUM FILM THICKNESS FOR SOFT EHL CONTACTS

Case	Ellipticity parameter, k	Dimensionless load parameter, W	Dimensionless speed parameter, U	Dimensionless materials parameter, G	Minimum film thickness		Difference between H_{min} and $\tilde{H}_{min}$, D_1, percent	Results
					Obtained from EHL elliptical contact theory, H_{min}	Obtained from least-squares fit, $\tilde{H}_{min}$		
1	1	0.4405×10^{-3}	0.1028×10^{-7}	0.4276	88.51×10^{-6}	91.08×10^{-6}	+2.90	Ellipticity
2	2				142.5	131.2	-7.93	
3	3				170.4	160.8	-5.63	
4	4				186.7	182.4	-2.30	
5	6				206.2	209.8	+1.75	
6	8				219.7	224.6	+2.23	
7	12				235.2	236.0	+.34	
8	6	0.4405×10^{-3}	0.05139	0.4276	131.8	133.7	+1.44	Speed plus case 5
9			.1542		268.1	273.1	+1.86	
10			.2570		381.6	380.7	-.24	
11			.05139		584.7	597.3	+2.15	
12	6	.2202	0.1028	0.4276	241.8	242.7	+0.37	Load plus case 5
13		.6607			190.7	192.7	+1.05	
14		1.101			170.5	173.1	+1.52	
15		1.542			160.4	161.3	+.56	
16		2.202			149.8	149.7	-.07	
17	6	0.1762	0.06169	1.069	181.8	182.5	+0.39	Material

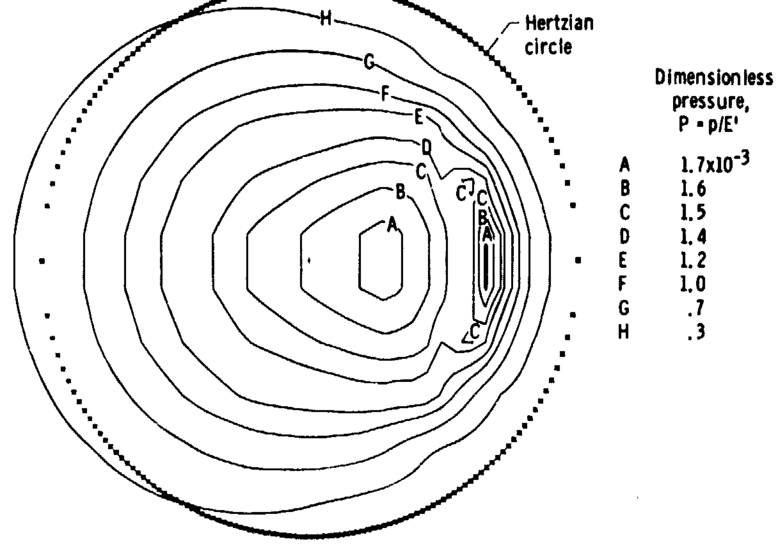


Figure 1. - Contour plot of dimensionless pressure. $k = 1.25$, $U = 0.168\times10^{-11}$, $W = 0.111\times10^{-6}$, and $G = 4522$.

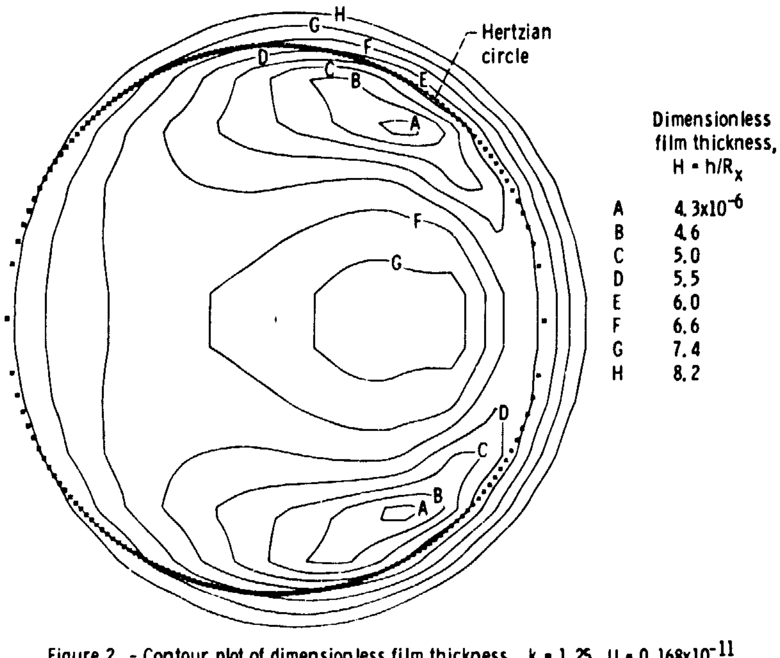


Figure 2. - Contour plot of dimensionless film thickness. $k = 1.25$, $U = 0.168\times10^{-11}$, $W = 0.111\times10^{-6}$, and $G = 4522$.

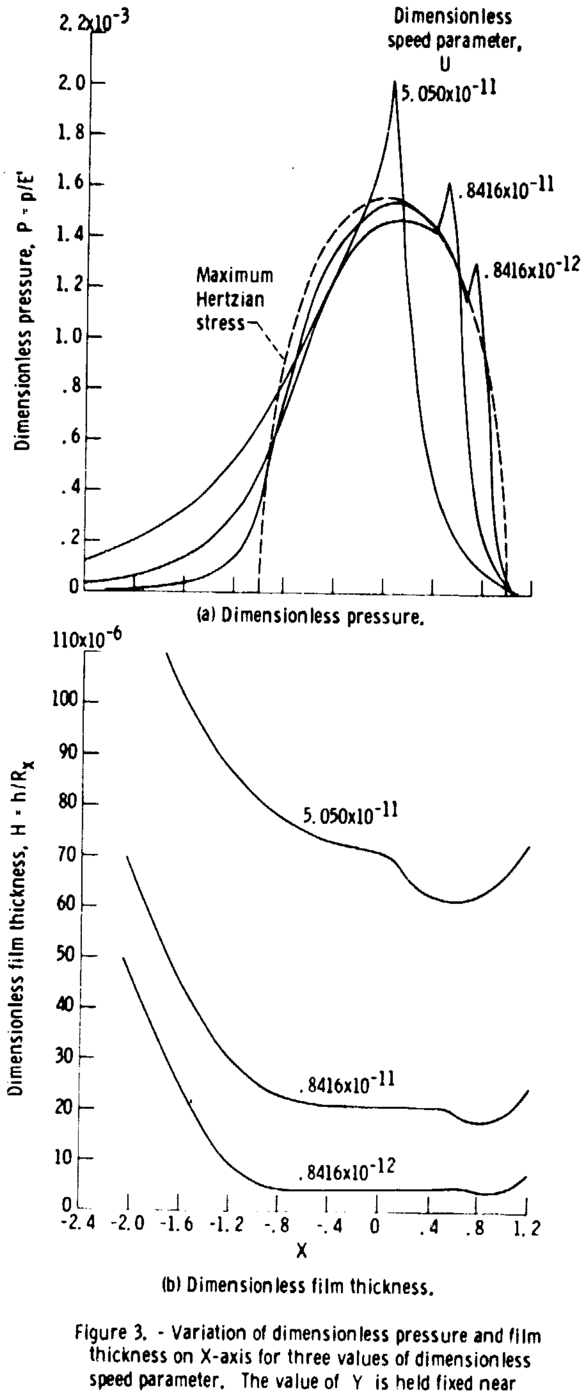


Figure 3. - Variation of dimensionless pressure and film thickness on X-axis for three values of dimensionless speed parameter. The value of Y is held fixed near axial center of contact.

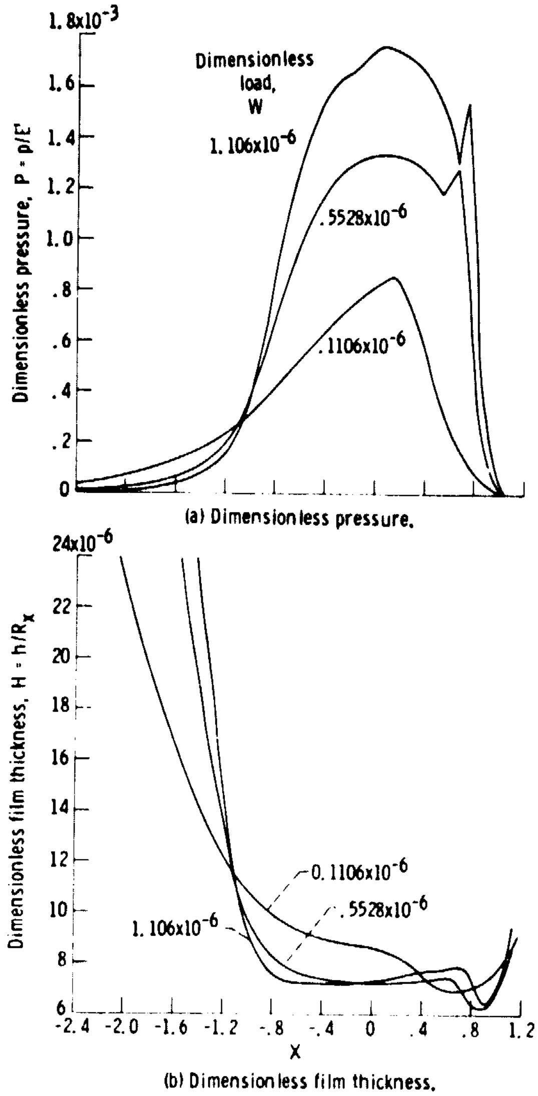


Figure 4. - Variation of dimensionless pressure and film thickness on X-axis for three values of dimensionless load parameter. The value of Y is held fixed near axial center of contact.

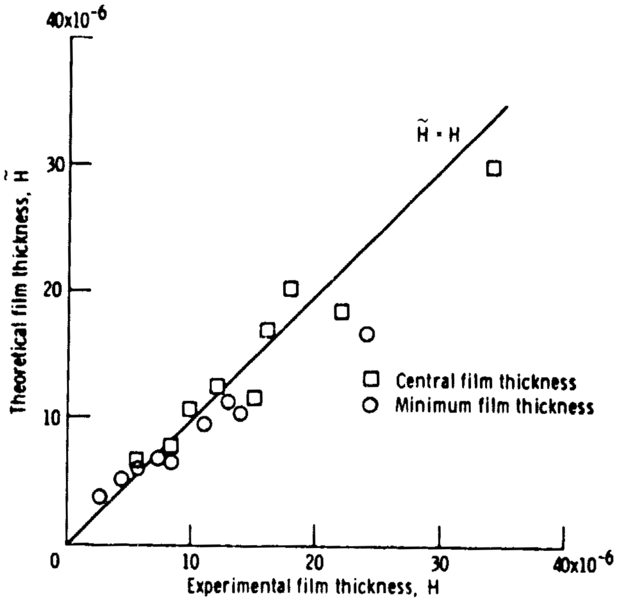


Figure 5. - Theoretical and experimental central and minimum film thicknesses for pure sliding. Dimensionless load parameter, W, 0.1238×10^{-6}. (From Kunz and Winer, 1977.)

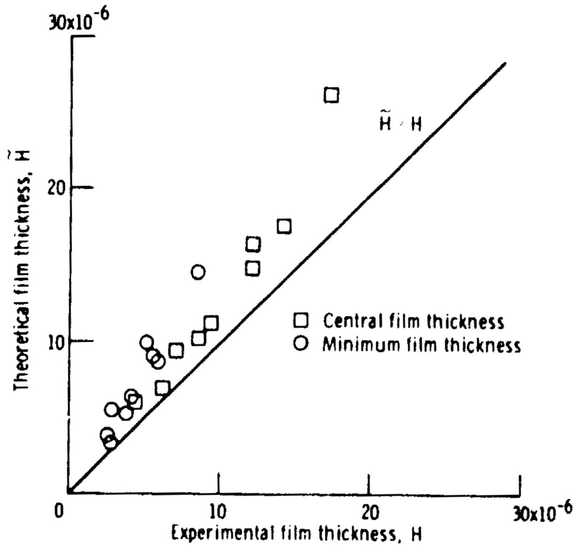
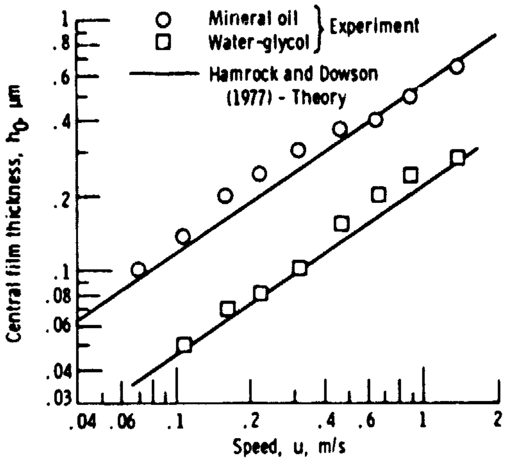


Figure 6. - Theoretical and experimental central and minimum film thicknesses for pure sliding. Dimensionless load parameter, W, 0.928×10^{-6}. (From Kunz and Winer, 1977.)



Figurr 7. - Effect of speed on central film thickness at constant load (2.6 N) for mineral oil and water-glycol lubricants of similar viscosity. (From Dalmaz and Godet, 1978.)

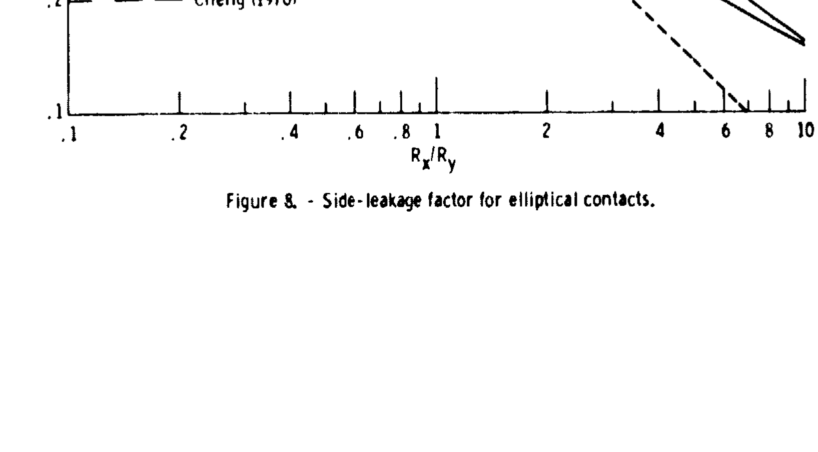


Figure 8. - Side-leakage factor for elliptical contacts.

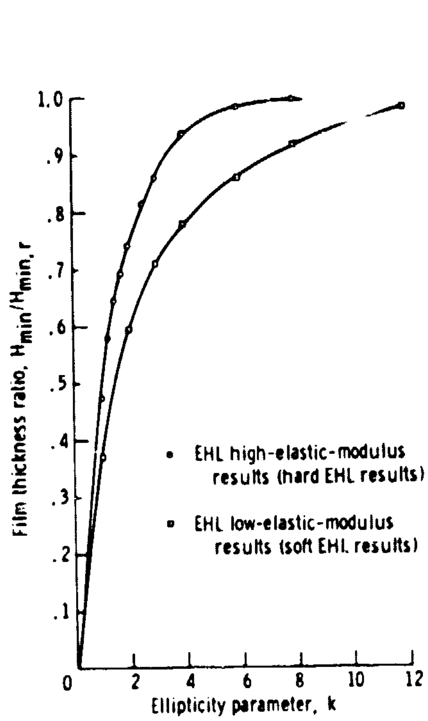


Figure 9. - Effect of ellipticity parameter on ratio of dimensionless minimum film thickness to dimensionless minimum film thickness for a line contact, for EHL high- and low-elastic-modulus analyses.

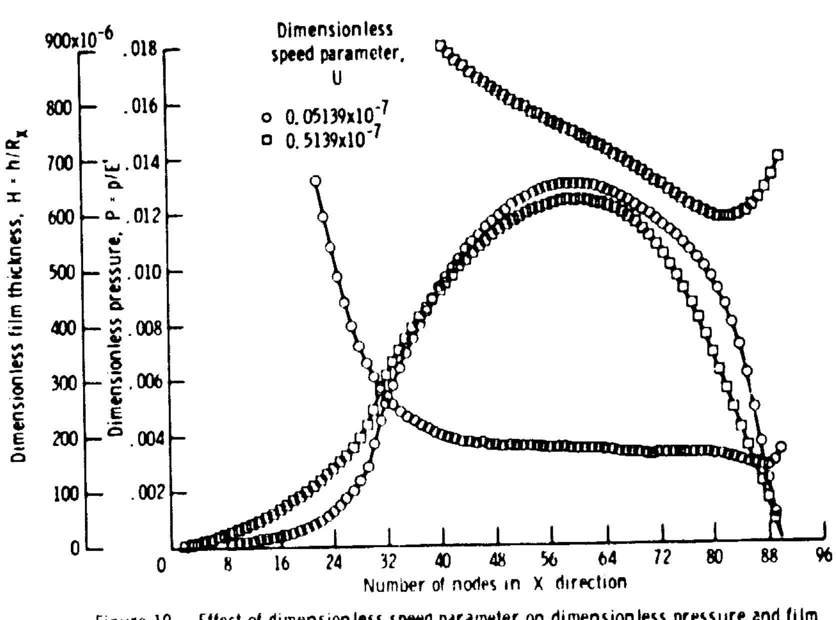


Figure 10. - Effect of dimensionless speed parameter on dimensionless pressure and film thickness on X-axis. The value of Y is held fixed near axial center of contact.

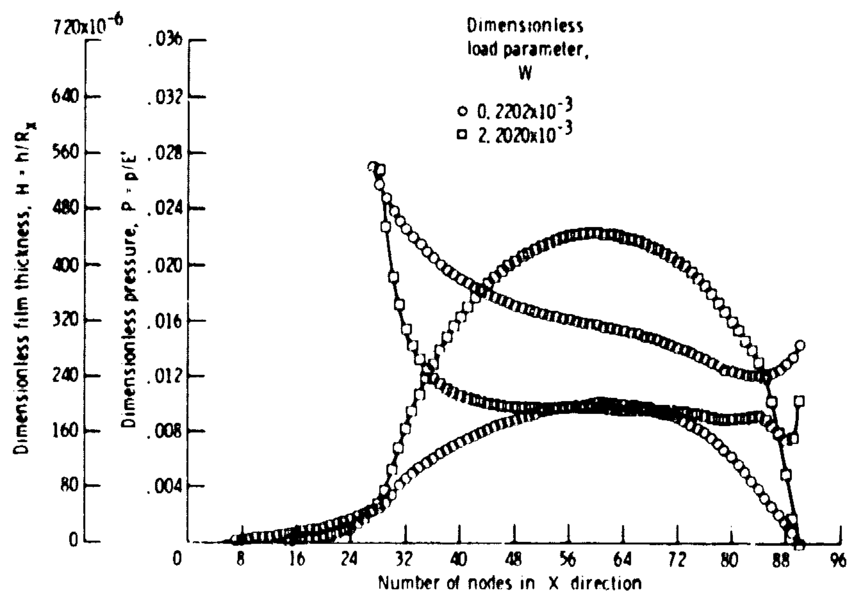


Figure 11. - Effect of dimensionless load parameter on dimensionless pressure and film thickness on X-axis. The value of Y is held fixed near axial center of contact.