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Solar Activity Prediction of Sunspot Numbers

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Predicted Solar Radio Flux

Mission Planning and Analysis Division

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Lyndon B. Johnson Space Center
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SHUTTLE PROGRAM

SOLAR ACTIVITY PREDICTION OF SUNSPOT NUMBERS

PREDICTED SOLAR RADIO FLUX

FM 15

By Gordon G. Johnson* and Samuel R. Newman,
Software Development Branch

Approved: Elric N. McHenry
Elric N. McHenry, Chief
Software Development

Approved: Ronald L. Berry
Ronald L. Berry, Chief
Mission Planning and Analysis Division

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*Senior Research Associate of the National Research Council at the NASA Lyndon B. Johnson Space Center and the University of Houston, Houston, Texas.

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1.0 INTRODUCTION

This document presents a solar activity prediction technique for monthly mean sunspot numbers over a period of approximately 10 years (from February 1979 to January 1989). This includes the predicted maximum epoch of solar cycle 21, approximately January 1980 and the predicted minimum epoch of solar cycle 22, approximately March 1987. Additionally, the solar radio flux 10.7-centimeter smooth values are included for the same time frame using the smooth 13-month (sunspot number/flux) empirical relationship from reference 1.

The incentive for predicting solar activity values is the requirement of solar flux (10.7-centimeter) data as input to upper atmosphere density models utilized in mission planning satellite orbital lifetime studies.

The mathematical analysis used for this technique is discussed in section 3.0.

2.0 SOLAR ACTIVITY HISTORY AND RELATIONSHIP

The solar activity sunspot numbers that have been observed and recorded since the year 1749 were obtained from reference 2 and are presented in figure 1. This solar record constitutes a total of 20 solar cycles plus part of the 21st cycle, which started in June 1976. These cycles seem to have an approximately 11-year cycle that can be identified with the 22-year magnetic period (fig. 2).

The Zurich listing of data (sunspot number) extends back to the year 1749; however, the reliable data start with solar cycle 8 or approximately 1834, as discussed in reference 3 and shown in figure 3. The first seven cycles are considered less reliable and were derived from historical data only after the Zurich sunspot number was defined. A description of sunspot number is given in appendix A.

Measurements of the 10.7-centimeter solar radio flux have been made since 1947. Because of this small data sample, a direct method of predicting the solar radio flux has not been developed (ref. 1). A description of solar radio flux is presented in appendix B.

3.0 MATHEMATICAL ANALYSIS

A mathematical technique employing in part the work of A. K. Paul (ref. 4) is given, which allows the reduction of monthly mean sunspot data to a residual curve that can be extropolated, thus allowing a short term prediction.

Suppose a, b, and h are numbers. Define, for all numbers t,

$$F(t) = a \cos 2\pi ht + b \sin 2\pi ht$$

and, as in references 4 and 5, for each number pair f, g; define, for all numbers t,

$$P(t;f,g) = (\exp(2\pi ift) - \exp(2\pi igt))/i\pi t$$

Observe that

$$P(t;f,g) = 2 \sin (\pi(f-g)t) \exp(2\pi i(f+g)t/2)/\pi t$$

and also that

$$\lim_{t \to 0} P(t;f,g) = 2(f - g)$$

Let $T(F;f,g)$ denote

$$\int_{-\infty}^{\infty} P(t;f,g)F(t)dt$$

In reference 4, it is shown that

$$T(F;f,g) = \begin{cases} 0 & \text{if} \quad h < g \\ (a + ib)/2 & \text{if} \quad g = h \\ a + ib & \text{if} \quad g < h < f \\ (a + ib)/2 & \text{if} \quad h = f \\ 0 & \text{if} \quad f < h \end{cases}$$

Let $c = (a^2 + b^2)^{1/2}$ and notice then that there is a unique number Θ such that

$$a = c \cos \Theta$$

and $$b = c \sin \Theta$$

from which it follows that

$$F(t) = c \cos (2\pi ht - \Theta) \quad \text{for all numbers } t$$

For each number s, define

$$\begin{aligned} F_s(t) &= F(t + s) \\ &= a \cos (2\pi h(t + s)) + b \sin (2\pi h(t+s)) \\ &= a' \cos 2\pi ht + b' \sin 2\pi ht \end{aligned}$$

and notice that $c = ((a')^2 + (b')^2)^{1/2}$. As before, there is a unique number Θ' such that

$$a' = c \cos \Theta'$$

and

$$b' = c \sin \Theta'$$

It then follows that

$$F_s (t) = c \cos (2\pi ht - \Theta')$$

Recall that $F_s(t) = F(t + s)$; therefore, $F_s(t) = c \cos(2\pi ht - (\Theta - 2\pi hs))$ and hence

$$\Theta' = \Theta - 2\pi hs$$

thus, $h = (\Theta - \Theta')/2\pi s$

As before

$$T(f_s;\ f,g) = \begin{cases} 0 & \text{if} \quad h < g \\ (a' + ib')/2 & \text{if} \quad g = h \\ a' + ib' & \text{if} \quad g < h < f \\ (a' + ib')/2 & \text{if} \quad h = f \\ 0 & \text{if} \quad f < h \end{cases}$$

An approximation to $T(F;\ f,\ g)$ is

$$\sum_{n=-\infty}^{\infty} (2/\pi Dn) \sin(\pi(f - g)nD) \exp(2\pi i(f+g)nD/2)\ F(nD)D$$

where D is a positive number. The term for $n = 0$ is $2(f - g)F(0)D$.

As only a finite amount of data are usually available, we further approximate $T(F;\ f,\ g)$ by

$$(2/\pi) \sum_{n=1}^{N} (1/n) \sin(\pi(f - g)nD)\exp(2\pi i(f + g)nD/2F(nD)$$

$$+ 2(f - g)\ F(0)D$$

$$+ (2/\pi) \sum_{n=-1}^{-N} (1/n) \sin(\pi(f - g)nD)\exp(2\pi i(f + g)nD/2)F(nD)$$

Let $\Delta = f - g$, $B = (f + g)/2$, $D = 1$

$F(n) = X_n$ and for each number s let $T_N(X,\ s,\ B,\ \Delta)$ be the complex number

$$(2/\pi) \sum_{n=1}^{N} (1/n) \sin(\pi\Delta n) \exp(2\pi iBn)\ X_{n+s}$$

$$+ 2(f - g)X_s$$

$$+ (2/\pi) \sum_{n=-1}^{-N} (1/n) \sin(\pi\Delta n)\exp(2\pi iBn)X_{n+s}$$

We now introduce several definitions that we shall need in order to predict sunspot numbers.

Definition 1. Suppose $\varepsilon > 0$ and f is function defined on $[a,b] = (x: a \leq x \leq b)$. The statement that f is ε-symmetric means

$$|f(x) - f(a + b - x)| < \varepsilon$$

for all x in $[a, b]$.

Definition 2. Suppose that R is a function defined on $[a, b]$. The statement that R^* is the symmetric extension of R to $[a, 2b - a]$ means

$$R^*(x) = \begin{cases} R(x) & \text{if } a \leq x \leq b \\ R(2b - x) & \text{if } b < x \leq 2b - a \end{cases}$$

Definition 3. Suppose $\varepsilon > 0$ and F is a function defined on $[u, v]$. The statement that F can be ε-reduced on $[u, v]$ means there is a finite sequence of periodic functions $\{g_i\}_{i=1}^{n}$ and a finite sequence of nonoverlapping intervals $\{I_i\}_{i=1}^{m}$ filling up $[u, v]$ such that the function R,

$$R = F - \sum_{i=1}^{n} g_i$$

is ε-symmetric on I_i for $i = 1,2....m$. Denote such a reduction by

$$\left\{\varepsilon, F, [u,v], \{g_i\}_{i=1}^{n}, \{I_i\}_{i=1}^{m}, R\right\}$$

Notice that there is no claim as to uniqueness of such a reduction.

Definition 4. Suppose $\varepsilon > 0$ and f is a function defined on $[a, b]$. The statement that f is short term ε-predictable means

a. There is an interval $[u, v]$ containing $[a, b]$ such that $b < v$, a function F defined on $[u, v]$ such that F can be ε-reduced on $[u, v]$ and $f(x) = F(x)$ for x in $[a, b]$.

b. There is an ε-reduction

$$\{\varepsilon,\ F,\ (u,\ v),\ \{g_i\}_{i=1}^{n}\ ,\ \{I_i\}_{i=1}^{m}\ ,\ R\}$$

and an integer j such that b is in the right-hand interior of I_j.

Theorem: If $\varepsilon > 0$ and f is short-term ε-predictable, then f can be extended to a function f^* such that $|F^*(X) - F(X)| < \varepsilon$ where F is the function stated in definition 4.

Proof: Suppose $\varepsilon > 0$, f is a function defined on (a,b) and F is a function defined on (u, v) such that F satisfies conditions of definition 4. Let

$$\{\varepsilon,\ F,\ (u,\ v),\ \{g_i\}_{i=1}^{n},\ \{I_i\}_{i=1}^{m},\ R\}$$

be a reduction of F satisfying definition 4. Then b is in the right-hand interior of I_j for some integer j. Let $I_j = (a_j, b_j)$ and c the center point of (a_j, b_j). Let $\overline{R}$ be the restriction of R to (a_j, c) and let $\overline{R}^*$ be symmetric extension of R^* to $(a_j, 2c - a_j)$ and notice that $2c - a_j = b_j$. Define

$$f^* = \overline{R}^* + \sum_{i=1}^{n} g_i \quad \text{on} \quad (a_j, b_j)$$

and $\quad f^* = f$

$$= R + \sum_{i=1}^{n} g_i \quad \text{on}\ (a,\ a_j)$$

Then $\quad |f^*(X) - F(X)| < \varepsilon \quad \text{on} \quad (a,\ b_j)$

Suppose that we have data $\{Y_r\}_{r=0}^{M}$ at equally spaced time intervals D (where $D=1$),

0 2N+1 M

Let $2N + 1$ be an integer less than M and for $-N \leq n \leq (M - (2N + 1))$

let $\quad X_n = Y_{N+n}$

$$\overline{\quad -N \qquad\qquad 0 \qquad\qquad N \qquad M - (N + 1) \quad}$$

Using the notions presented thus far, we have data from $(-N, M - (N + 1))$ and which to predict values for data indexed greater than $M - (2N + 1)$.

We shall assume that there is a function F defined on an interval (u, v) containing $(-N, M - (N + 1))$, which can be ε-reduced on (u, v) such that for some such reduction definition 4 is satisfied where the f in the definition is the known data function.

We shall generate the sequence of periodic functions $\{g_i\}_{i=1}^{n}$ and the sequence nonoverlapping intervals $\{I_i\}_{i=1}^{m}$ by applying the operator $T_N(X, s, B, \Delta)$ to the data X.

As we are seeking short-term prediction, we are seeking relatively high frequencies between 0.0012 and 0.0833, which correspond to one cycle per 5 years and one cycle per month.

Having the data X, we have three parameters, s, Δ and B that we can vary. The first, s, is a shift, Δ is a "bandwidth" and B is approximate "location."

Notice that $0 \leq s \leq M - (2N + 1)$. Let $s = 0$, select B and a Δ, then compute $T_N(X, \overline{0}, \overline{B}, \Delta)$, and iterate by reducing Δ. Then select new B and repeat. Continue this process.

Now select $0 < s < M - (2N + 1)$ and repeat above process.

Because $D = 1$ (1 month in case of sunspot data), we cannot determine frequencies that are higher that one cycle per month. Hence $0 < B \leq 0.0833$.

We continue in this manner, generating six columns of numbers the first being always 0,

$$s \qquad B \qquad \Delta \qquad a \qquad b \qquad c = (a^2 + b^2)^{1/2}$$

We now repeat for a value of $0 < s \leq M - (2N + 1)$. Hence, we have a subset of E^6, and we now search for two points within this set

$$V_1 = (0, B_1, \Delta_1, a_1, b_1, c_1)$$

0 as it first component and the second

$$V_1' = (s \neq 0, B', \Delta', a_1', b_1', c_1') \quad \text{where} \quad c_1' = c_1$$

and determine Θ_1 and Θ_1' from which a number $h_1 = (\Theta_1 - \Theta_1')/2\pi s$ is computed.

Let $R_1(n) = X_n - g_1(n)$ for $-N \leq n \leq N + (M - 2N + 1)$

where

$$g_1(n) = a_1 \cos (2\pi h_1 n) + b_1 \sin (2\pi h_1 n).$$

A check is now made to determine if X is ε-reduced on $\{u, v\}$ and that the reduction satisfies definition 4. If not, then we define

$$X_{1n} = R_1(n) \quad \text{for} \quad -N \leq n \leq N + (M - 2N -1)$$

Apply now

$$T(X_1, 0, B\Delta)$$

and $T(X_1, s, B, \Delta)$, and repeat above.

After each such determination of

$$g_j(n) = a_j \cos (2\pi h_j n) + b_j \sin (2\pi h_j n)$$

we set

$$R_j(n) = R_{j-1}(n) - g_j(n) \quad \text{for} \quad -N \leq n \leq N + (M - 2N - 1)$$

where

$$R_0(n) = Xn \quad \text{for} \quad -N \leq n \leq N + (M - 2N - 1)$$

At each step R_j is examined to determine if R_j is ε-reduced on $\{-N, N\}$.

Suppose now that we have succeeded in finding an ε-reduction such that definition 4 is satisfied. Then by a straightforward application of the theorem, we can extend, i.e., predict the future behavior.

4.0 APPLICATION OF METHOD OF SOLUTION

By applying the mathematical analysis of section 3.0 to the sunspot number data in figure 3, a model was obtained that is composed of 24 spectral lines, which were determined by numerous iterations.

a. Residual curves

The residual curves (R4, R8, R12, R16, and R20) obtained by subtracting out the frequency, amplitude, and phase (table I) demonstrate the changes in the residual curves (fig. 5.)

b. Final residual curve - R24

Figure 6(a) shows the final residual curve (R24) as a function of month/years with the last value at January 1979. Based on the technique from section 3.0., this curve was investigated for short-term symmetry as shown in figure 6(b). The shaded portions of the curve indicate symmetry from a minimum of 5.3 years to a maximum of 7 years.

In addition, based on previous tests cases, the final slope indicated the residual could be extrapolated for 7 or more years. And the data (residual values) are such at the end of the curve the slope has reversed its direction, therefore making it possible to extrapolate.

The 20th residual (R20) had the same criteria for extrapolation, except for the fact that only a 5- or 6-year prediction of sunspot number would have resulted. Therefore, the analysis was not terminated here. Instead, four more spectral lines were determined that yielded the final predicted curve R24. Figure 7 shows the last 49 years of the final R24 residual curve for enlargement purposes.

c. Determining the predicted residual

Because the summation of the 24 harmonics of the model with the final residual curve (R24) in figure 6 yields the original sunspot record, it was considered feasible that extrapolation of the R24 curve an additional 7 to 10 years results in a prediction of sunspot numbers for 7 to 10 years.

The method used in the extrapolation is explained as follows: first the midpoint of the curve was determined using the digital data of the residual (R24) before the last value terminated. By symmetric extension, the residual was extrapolated as shown in figure 8. Figure 9 presents the residual R24 from 1834 through January 1979 and its 10-year extrapolated residual through January 1989.

When the 24 harmonics are summed with the curve in figure 8, the original sunspot numbers (observed month mean values) and the predicted sunspot numbers are obtained as presented in figure 10.

NOTE: The curve shows the sunspot numbers through the maximum epoch of solar cycle 21 and minimum epoch of solar cycle 22. The minimum epoch is determined when the cycle passes through zero and reverses its algebraic sign.

5.0 COMPARISON OF PREDICTED SUNSPOT NUMBERS

A study was conducted to determine the accuracy of using this analysis. The same sunspot record was used but terminated at the data point (December 1972). The residual curve (R22) used for the final predictable residual is shown in figure 11 with the shaded areas for symmetry as in the case of the R24 residual. This allowed actual observed sunspot numbers to be compared with the predicted results (January 1973 to January 1979) for an accuracy check. The predicted sunspot numbers of this study is presented in figure 12. The data are shown in the customenary absolute value format (all positive) sunspot numbers (observed and predicted) as a function of month/years. The agreement between the two sets of data was deemed adequate for an accuracy check.

a. Sunspot number prediction (February 1979 to January 1989)

The 10-year sunspot number prediction from this analysis is presented in figure 13 and includes monthly observed data. This plot also shows the predicted maximum epoch of cycle 21 and minimum epoch of cycle 22.

b. Predicted smooth sunspot numbers (13-month smooth means)

For comparison purposes, 13-month smooth values were computed using the data in figure 13. The results are shown with the National Oceanic and Atmospheric Administration (NOAA) and Marshall Space Flight Center (MSFC) data in figure 14. The NOAA data obtained from reference 6 are in the form of predicted nominal and positive two-sigma (2σ) values. The MSFC predicted data (ref. 7) are shown in same format. In summation of this plot, the JSC predicted values agree comfortably when compared to the NOAA and MSFC (nominal and positive 2σ) values.

NOTE: The only way to truly evaluate these predictions is after several months of observed data have been recorded.

6.0 SUNSPOT NUMBER AND SOLAR RADIO FLUX EMPHERICAL RELATIONSHIP

One method of determining predicted solar flux values is to use the relationship between Ottawa 10.7-centimeter flux and Zurich smooth 13-month mean sunspot numbers as used by NOAA in reference 1. This relationship is presented in figure 15 and was used in this analysis.

NOTE: The 10.7-centimeter flux data are used by various organizations as input for upper atmosphere density models for orbital lifetime studies. Because the solar radio flux is also measured daily, it would be preferable to use it directly for predicting solar activity; however, this time series is too short (32 years) for a direct prediction technique like the sunspot number method.

7.0 COMPARISON OF PREDICTED SOLAR RADIO FLUX

The JSC predicted radio flux (fig. 16) was derived using the 13-month smooth sunspot number history in figure 14 and the flux/sunspot number curve in figure 15. This flux data are also included with the predicted NOAA and MSFC values, both nominal and plus 2 sigma (refs. 8 and 7, respectively). Solar flux predicted data from reference 8 are also included.

The difference between the predicted flux values shown in this plot is attributed to different sunspot number predictions. The predicted flux data other than the JSC values were obtained and plotted for this report in August 1979 and therefore do not represent any updated current predictions since that time. The JSC flux data are presented for a time period of approximately 10 years, starting in February 1979.

8.0 CONCLUSIONS

A method of predicting monthly sunspot numbers over a period of approximately 10 years is presented using the mathematical analysis technique from section 3.0, with an additional application step as discussed in section 4.0. The monthly mean sunspot numbers used in this study were obtained from reference 2 and are shown in figure 3. The final predicted sunspot numbers from February 1979 to January 1989 are presented in figure 13. Also included in this document is a comparison of the following solar activity data as follows:

a. Predicted and observed monthly mean sunspot numbers from January 1973 to January 1979

b. Predicted 13-month smooth sunspot numbers for JSC, MSFC, and NOAA

c. Predicted solar radio flux 10.7-centimeter data for JSC, MSFC, and NOAA

Prediction of solar activity sunspot numbers will yield prediction of the solar radio flux 10.7-centimeter values, which are used by various organizations as input for upper atmosphere density models for orbital lifetime studies.

9.0 REFERENCES

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TABLE I.- SUBTRACTION OF FREQUENCY, AMPLITUDE, AND PHASE FROM PREVIOUS RESIDUALS

R1 = SSN(I) - AMP1(COS(2π(FREQ1 x I - PHASE1)))
R2 = R1(I) - AMP2(COS(2π(FREQ2 x I - PHASE2)))
R3 = R2(I) - AMP3(COS(2π(FREQ3 x I - PHASE3)))
R3 = R3(I) - AMP4(COS(2π(FREQ4 x I - PHASE4)))
RN = R(N-1)(I) - AMPN(COS(2π(FREQN x I - PHASEN)))

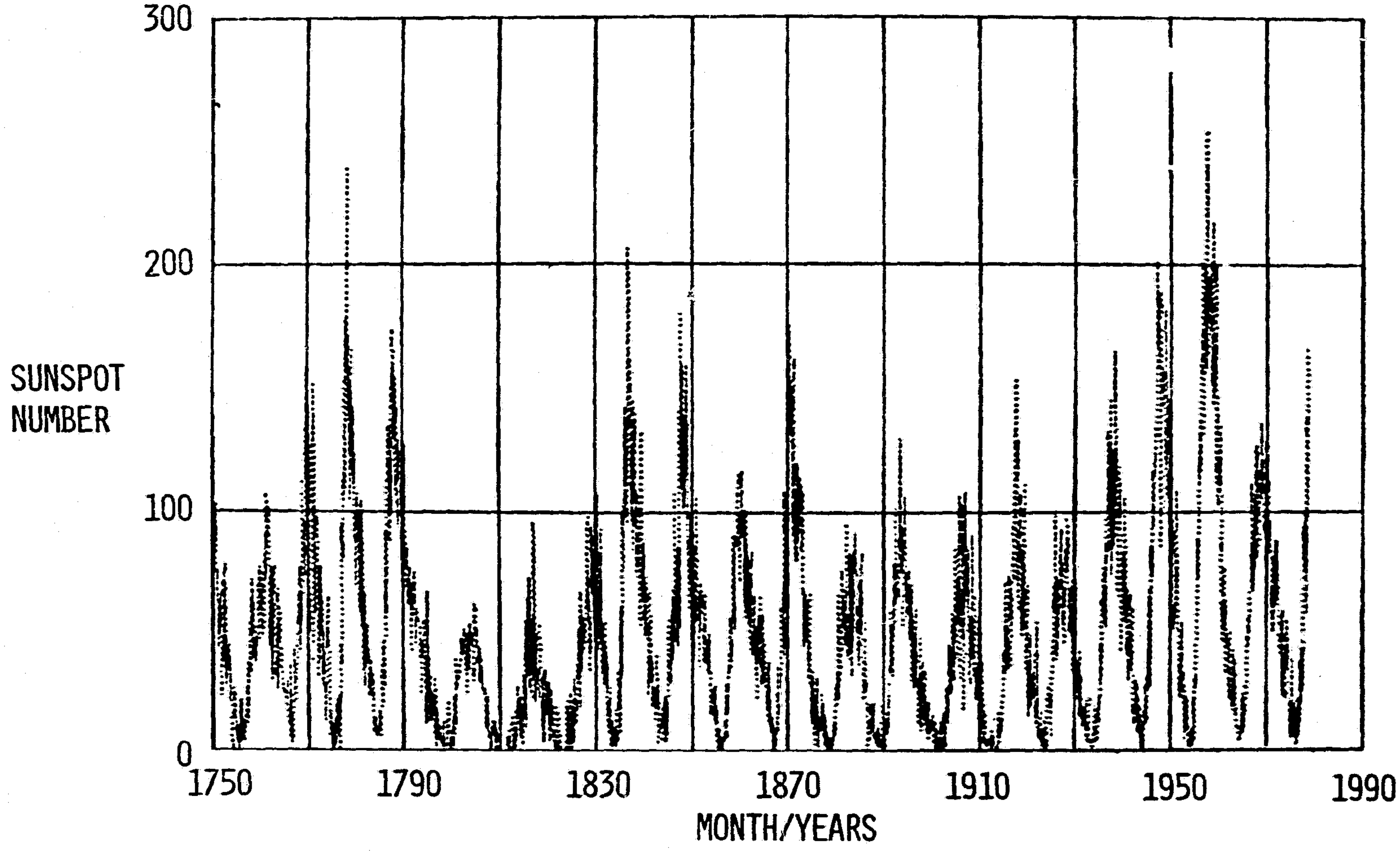


Figure 1.- Sunspot number record (1750-1979), monthly mean value.

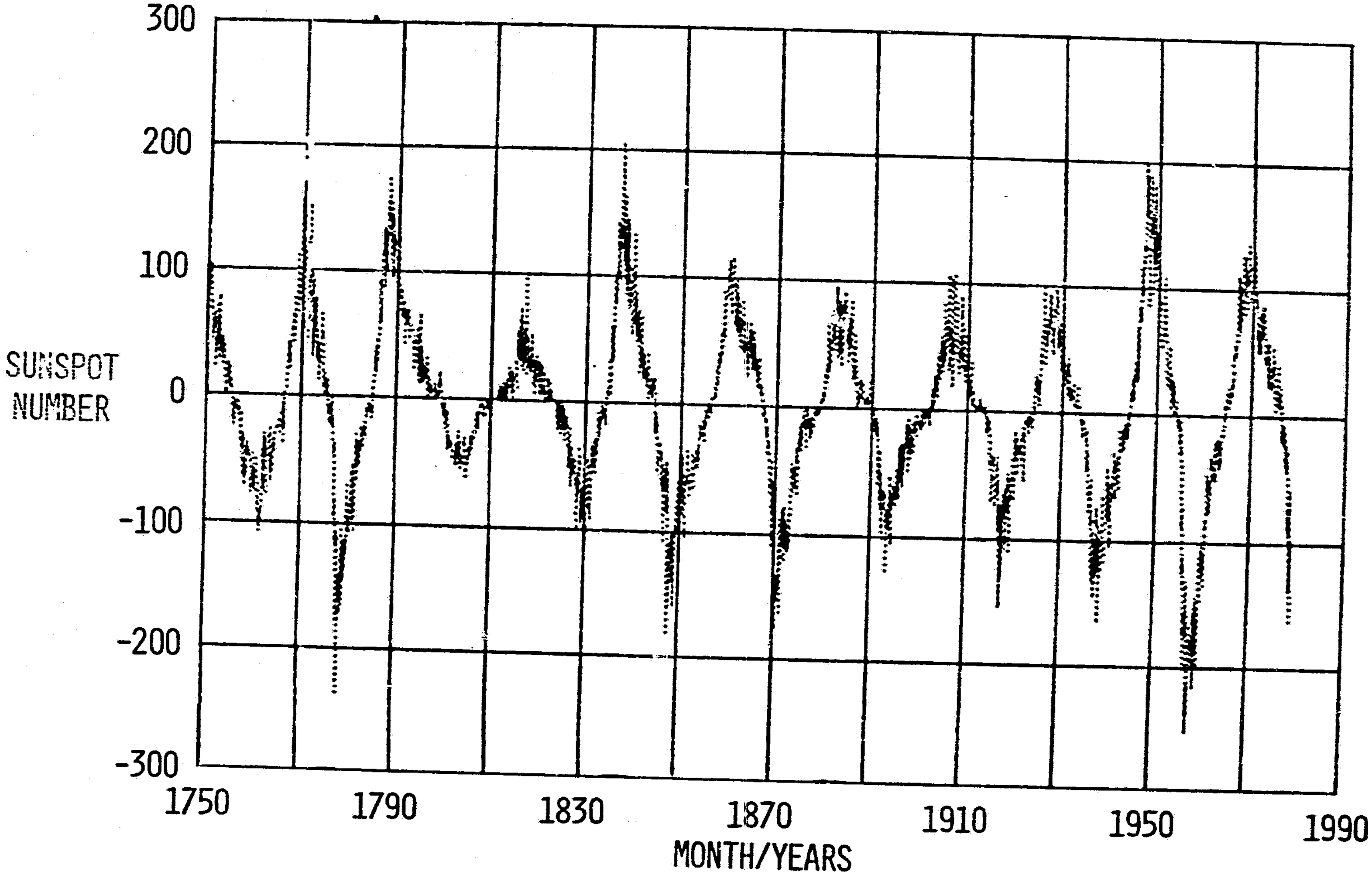


Figure 2.- Sunspot number solar cycles (1750-1979), monthly mean value.

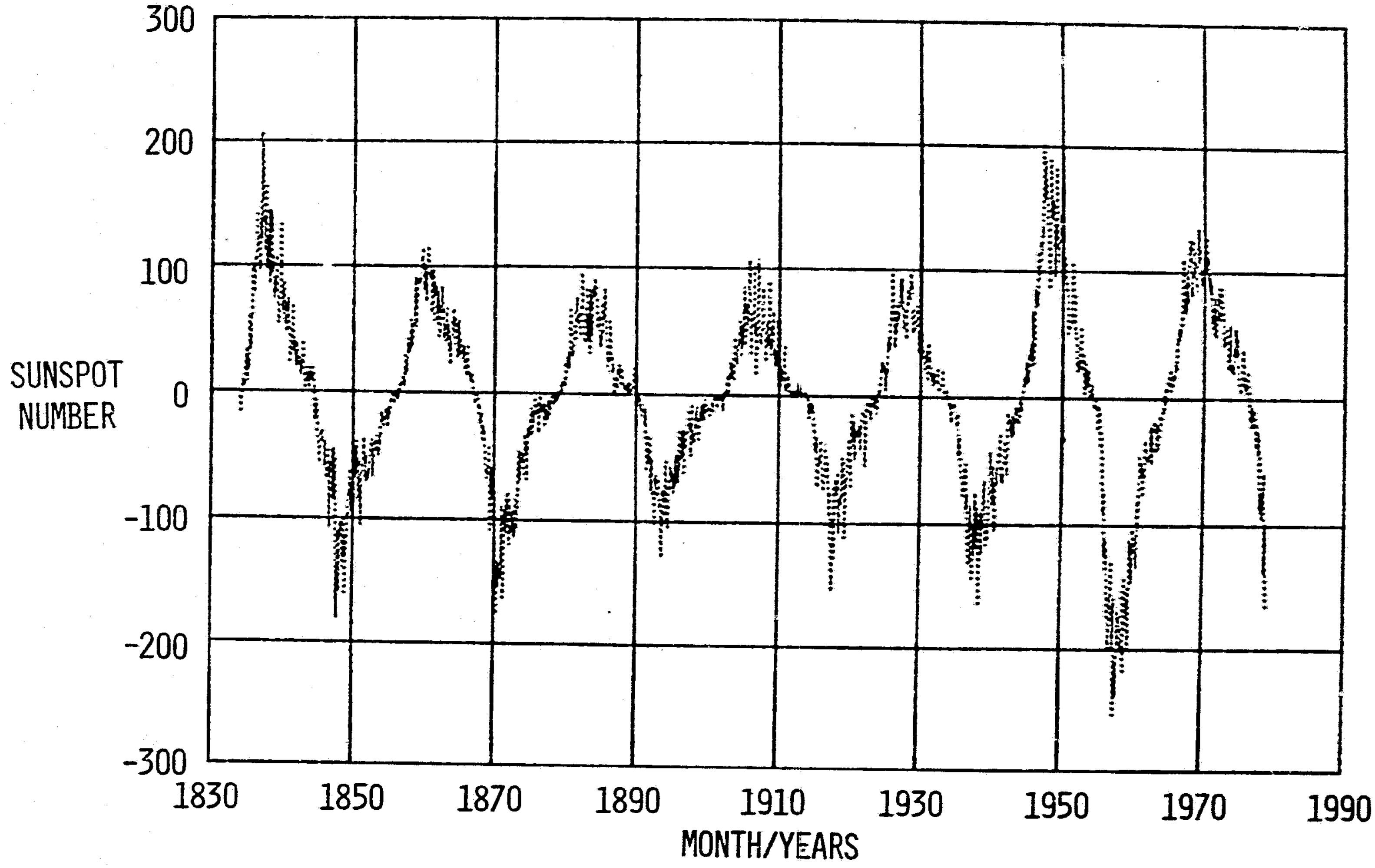
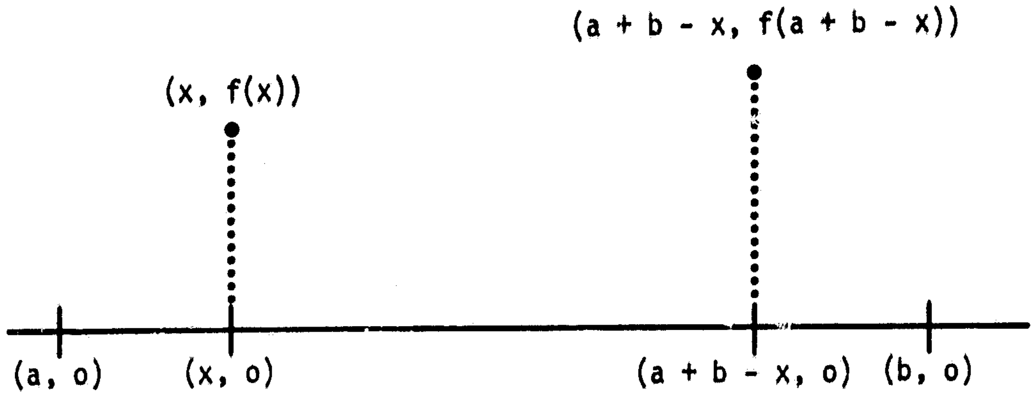
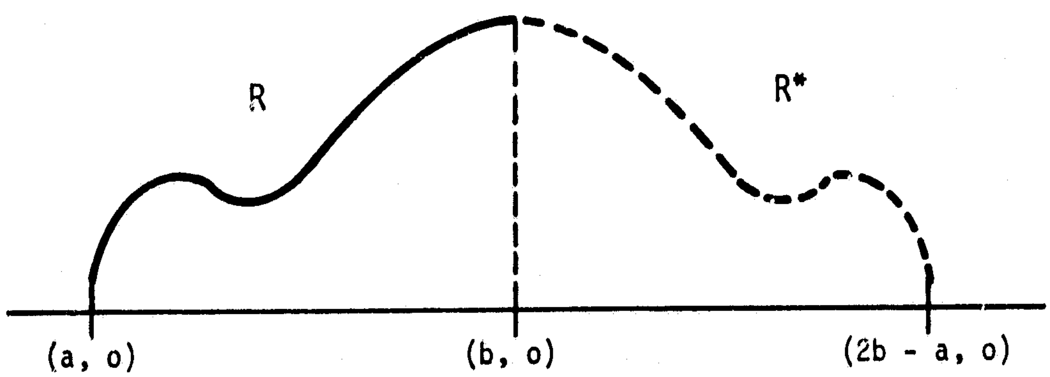


Figure 3.- Sunspot number solar cycles (1834-1979), monthly mean value.

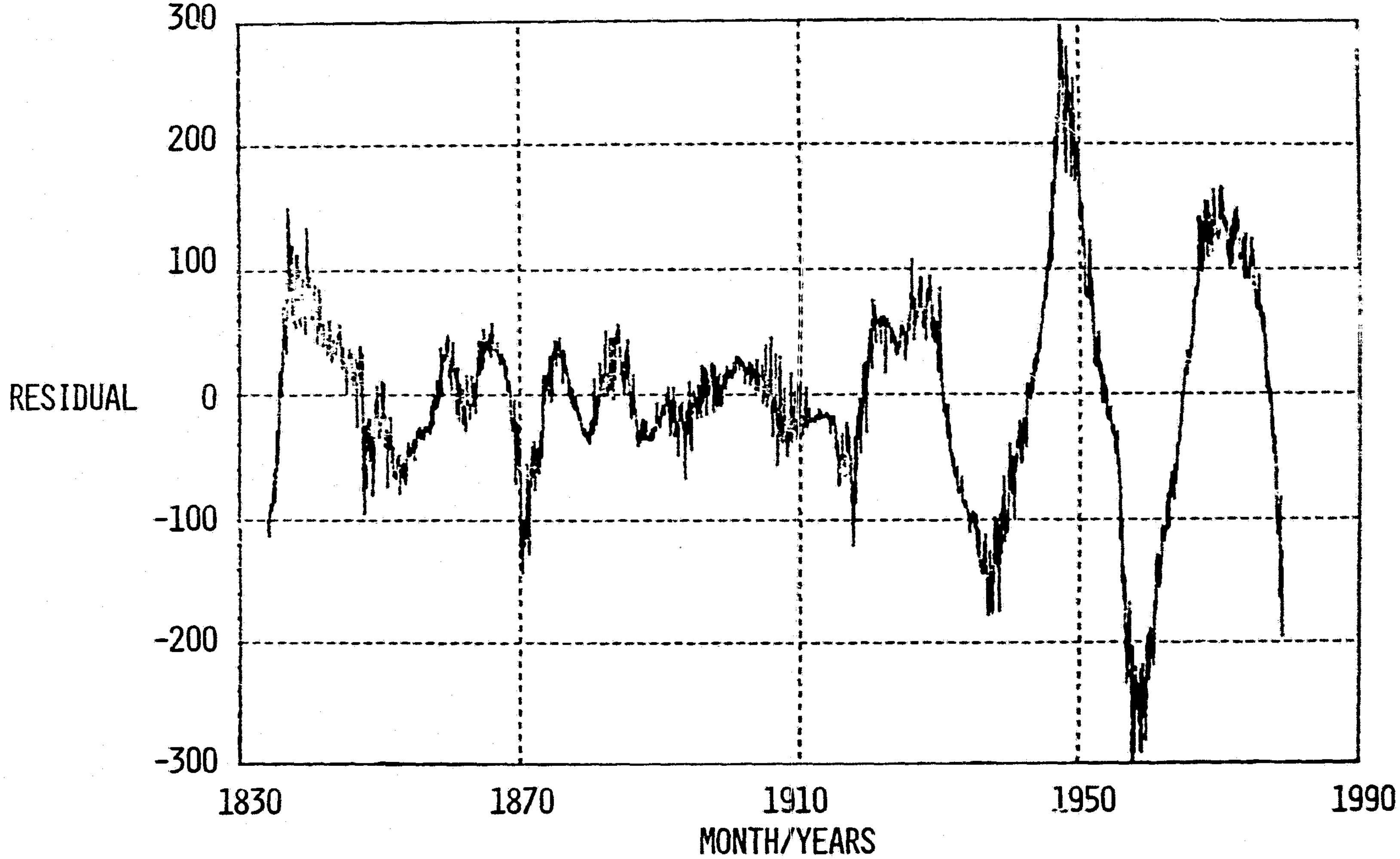


(a) Epsilon symmetry.



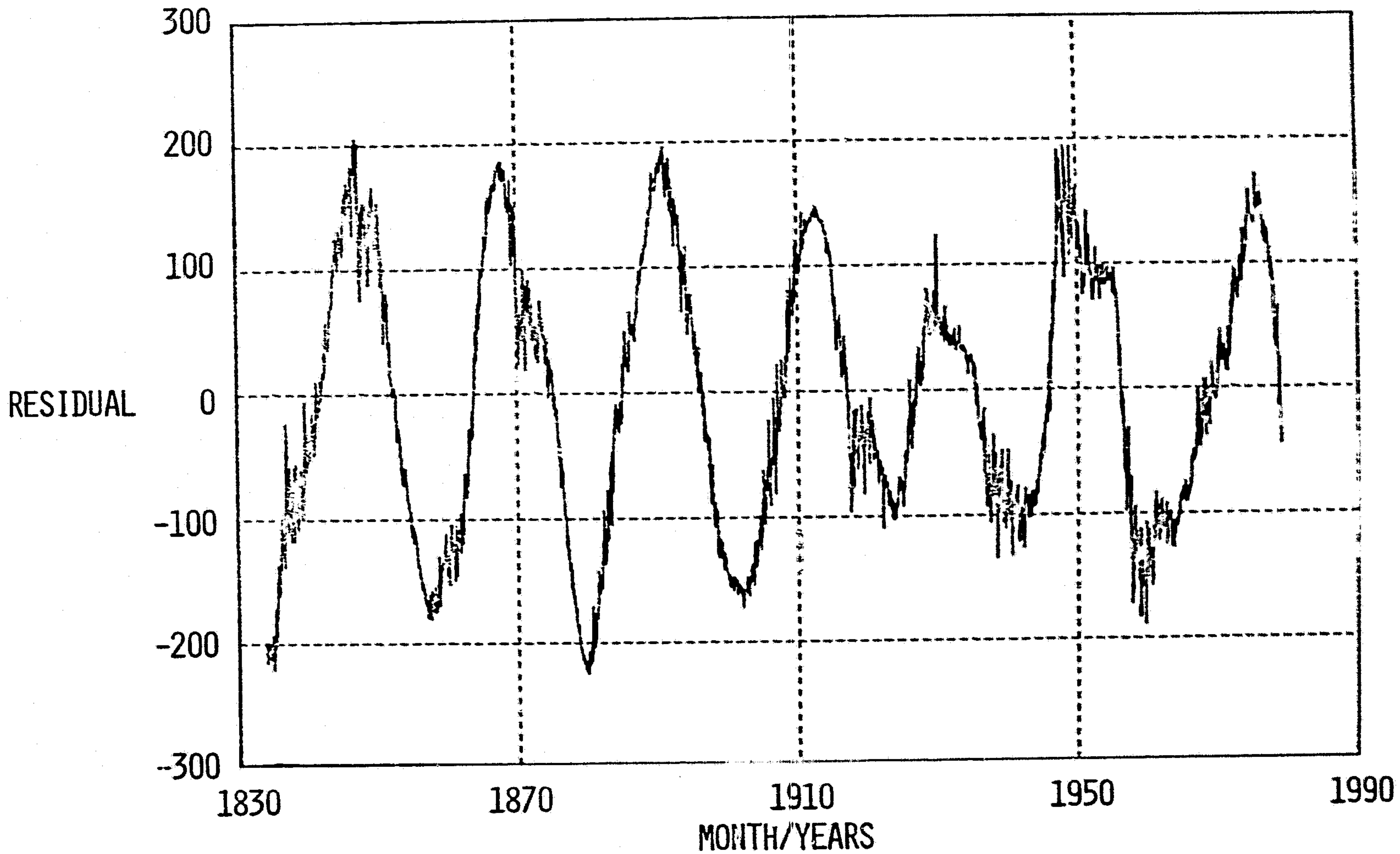
(b) Symmetric extension.

Figure 4.- Symmetry.



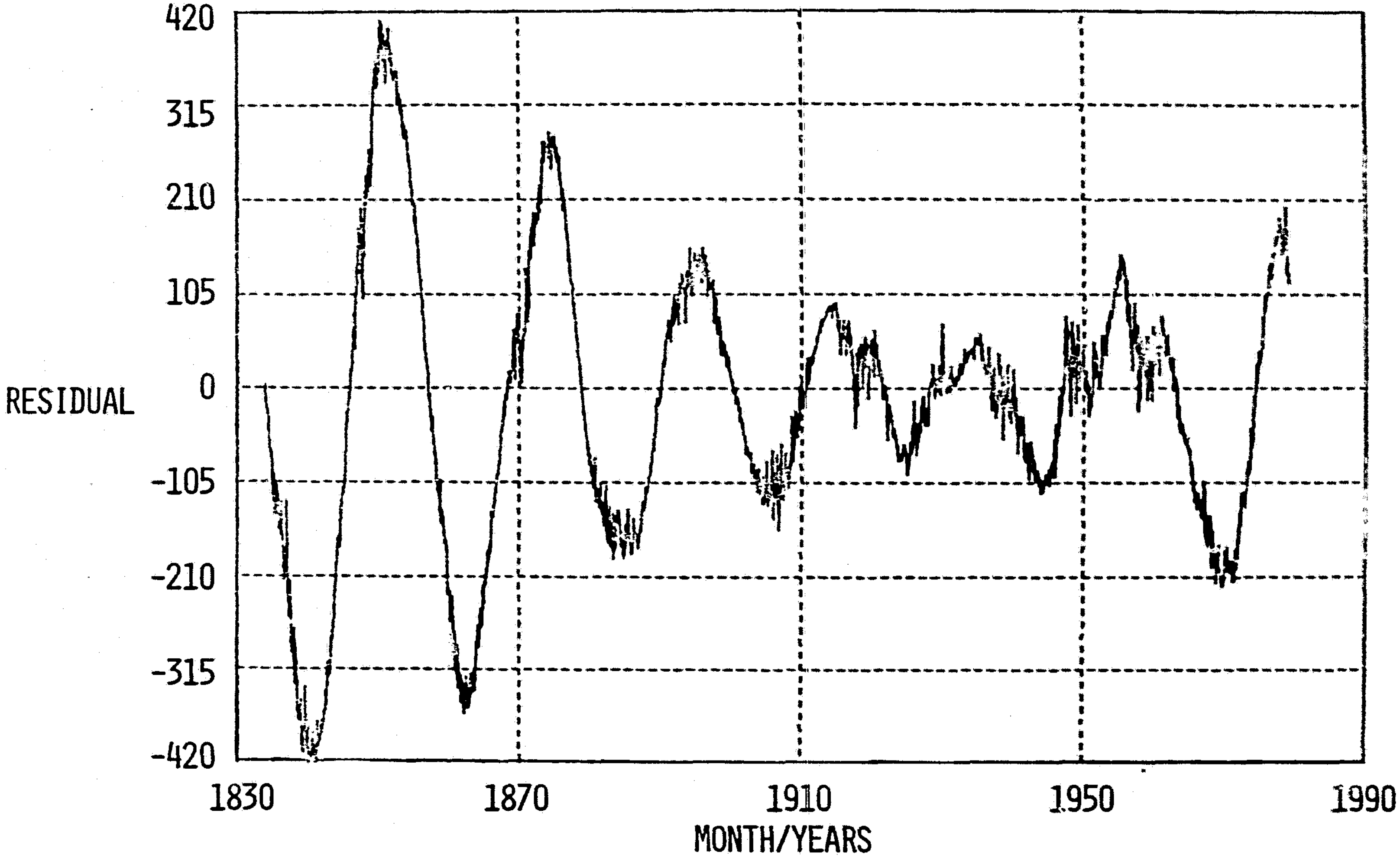
(a) Residual R4.

Figure 5.- Residuals as a function of time (1834-January 1979).



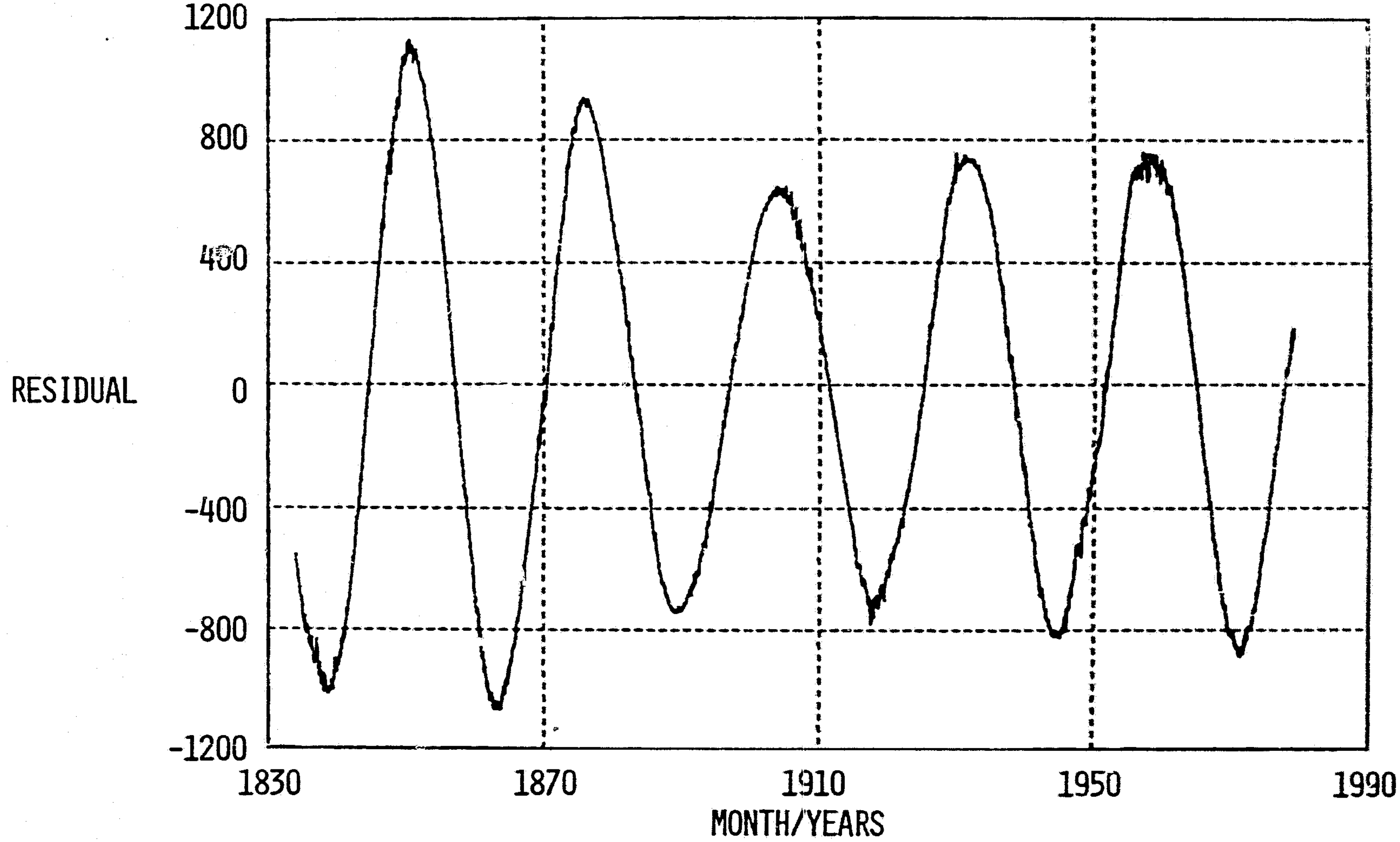
(b) Residual R8.

Figure 5.- Continued.



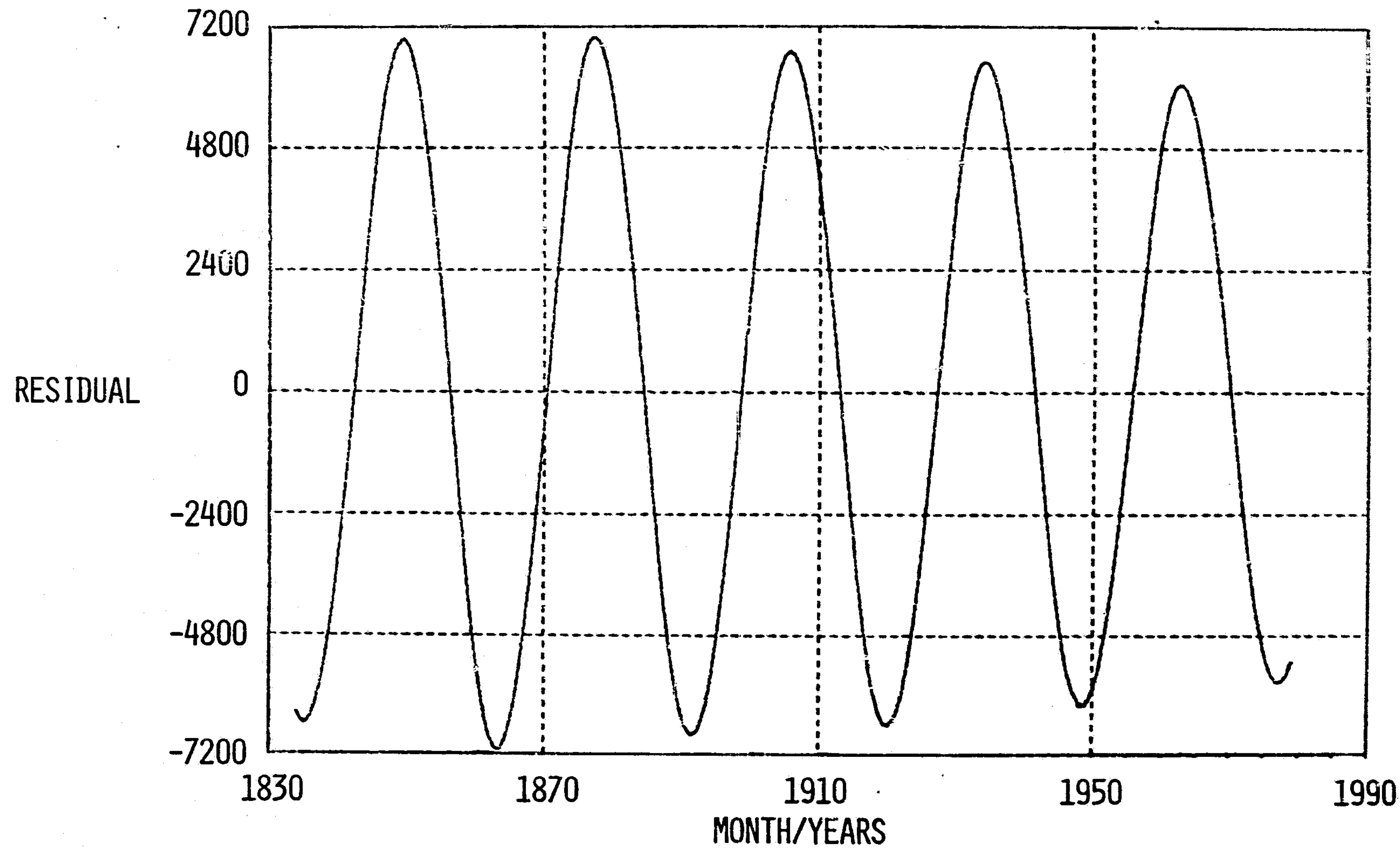
(c) Residual R12.

Figure 5.- Continued.



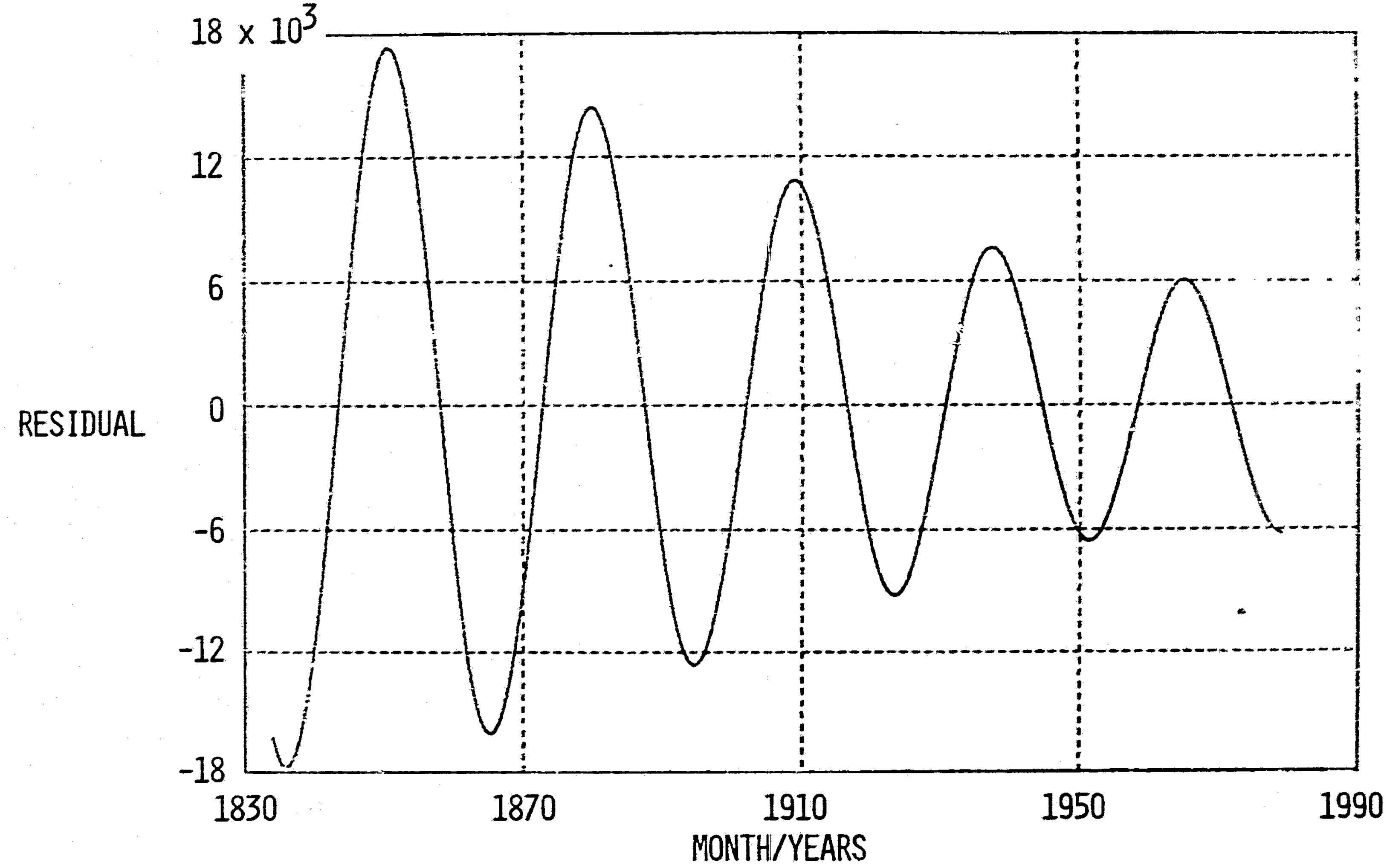
(d) Residual R16.

Figure 5.- Continued.



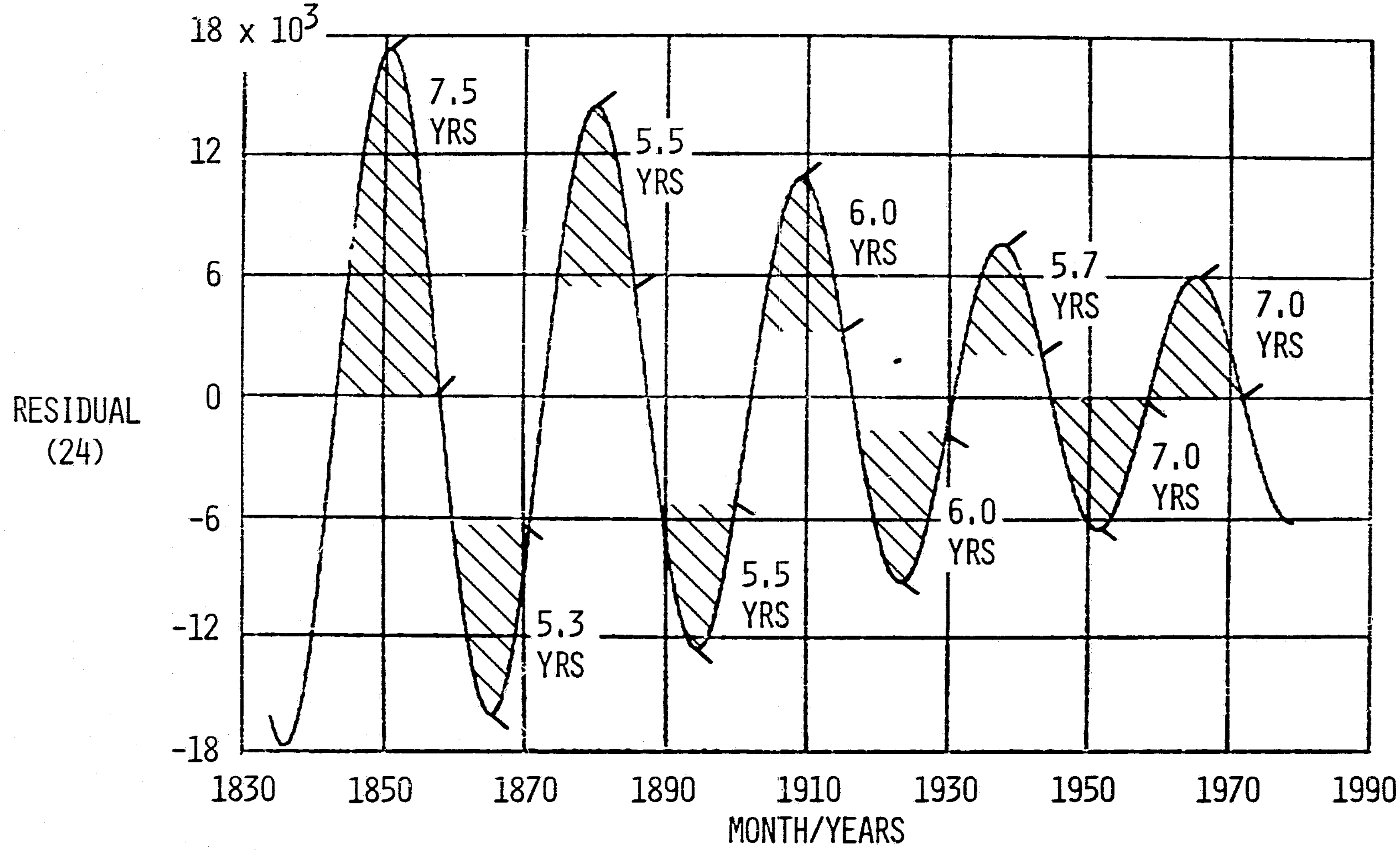
(e) Residual R20.

Figure 5.- Concluded.



(a) Final residual.

Figure 6.- Residual R24 (1834-January 1979).



(b) Final residual symmetry.

Figure 6.- Concluded.

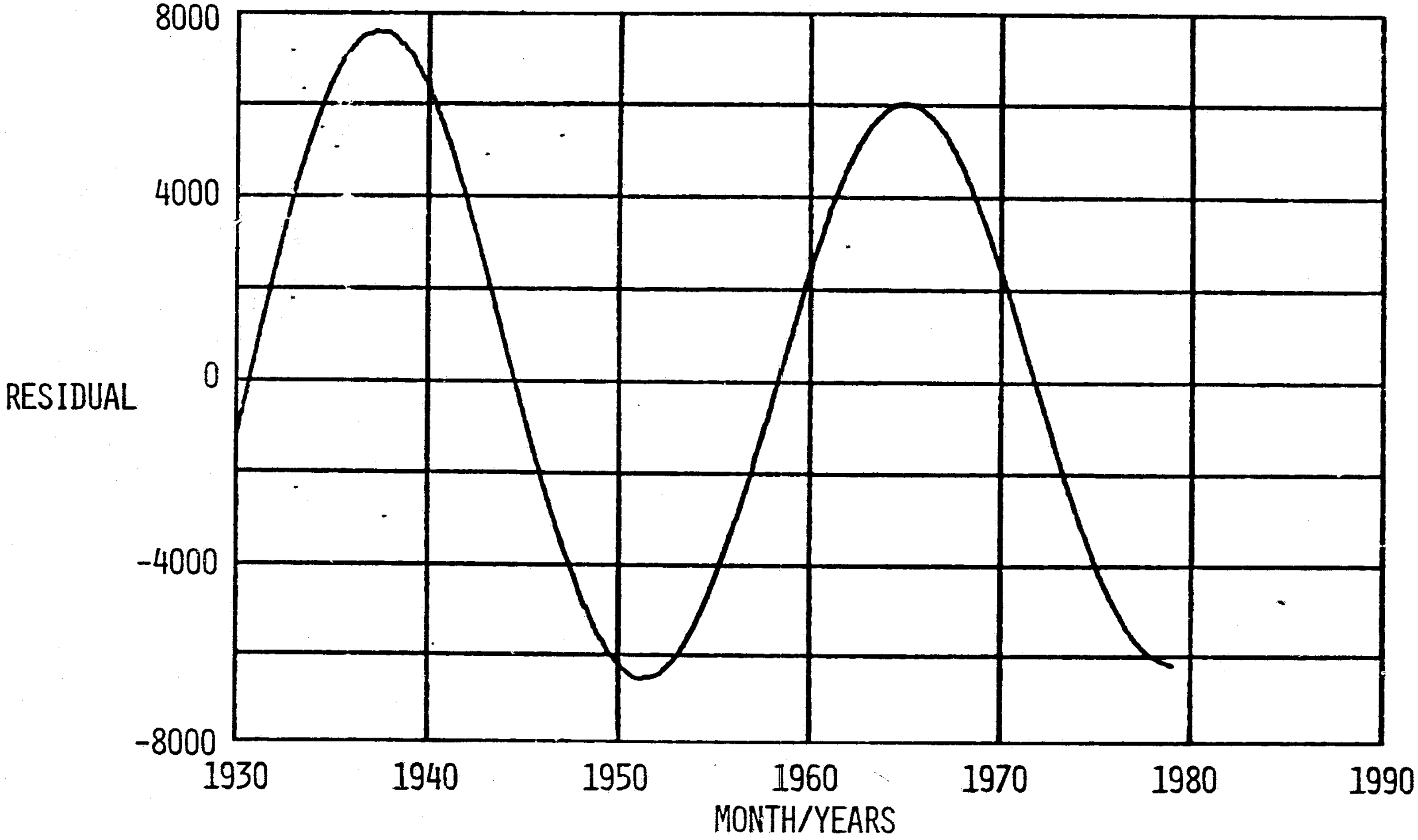


Figure 7.- Residual R24 (1930-January 1979).

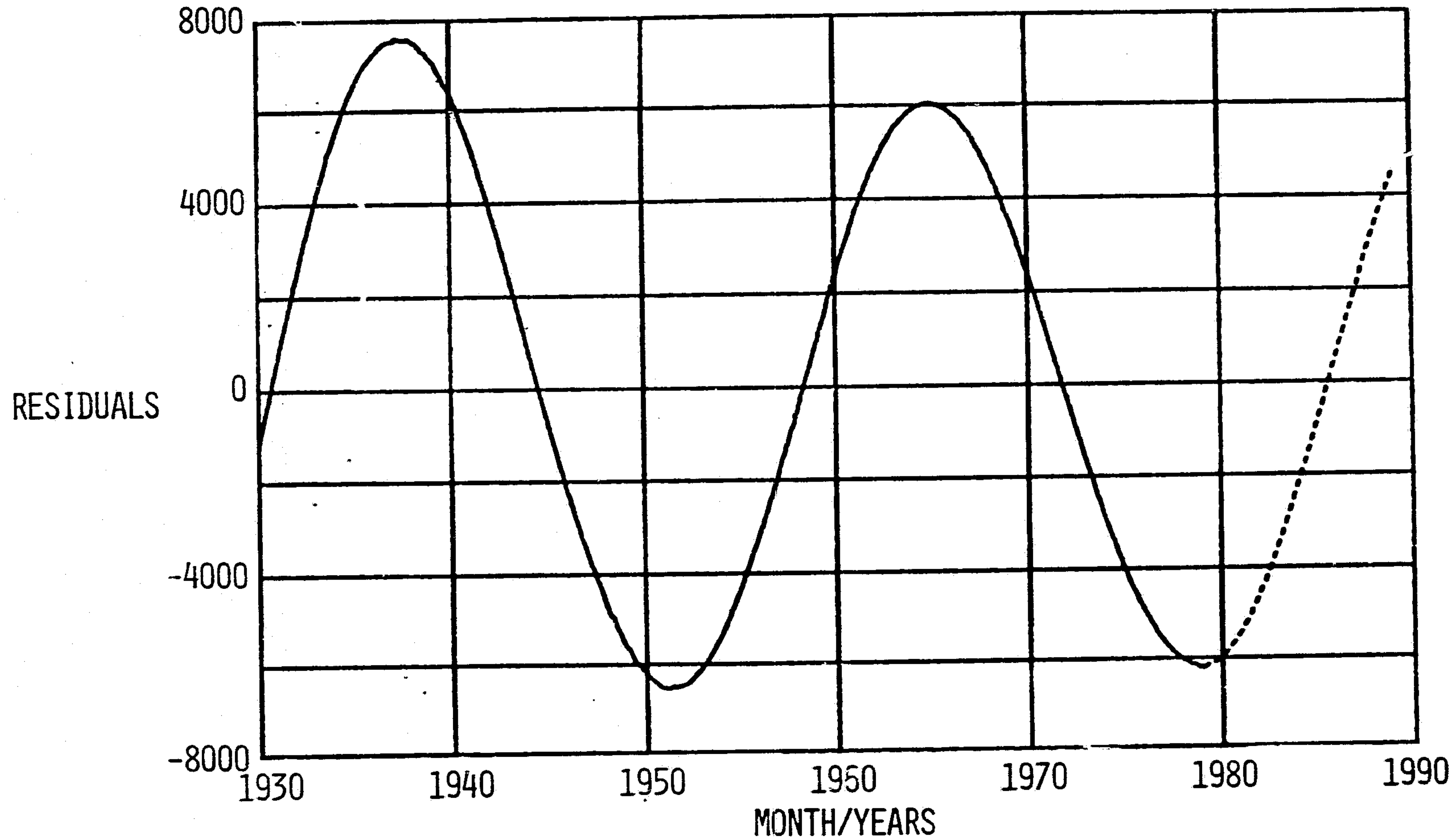


Figure 8.- Residual R24 and extrapolated residual (1930-January 1989).

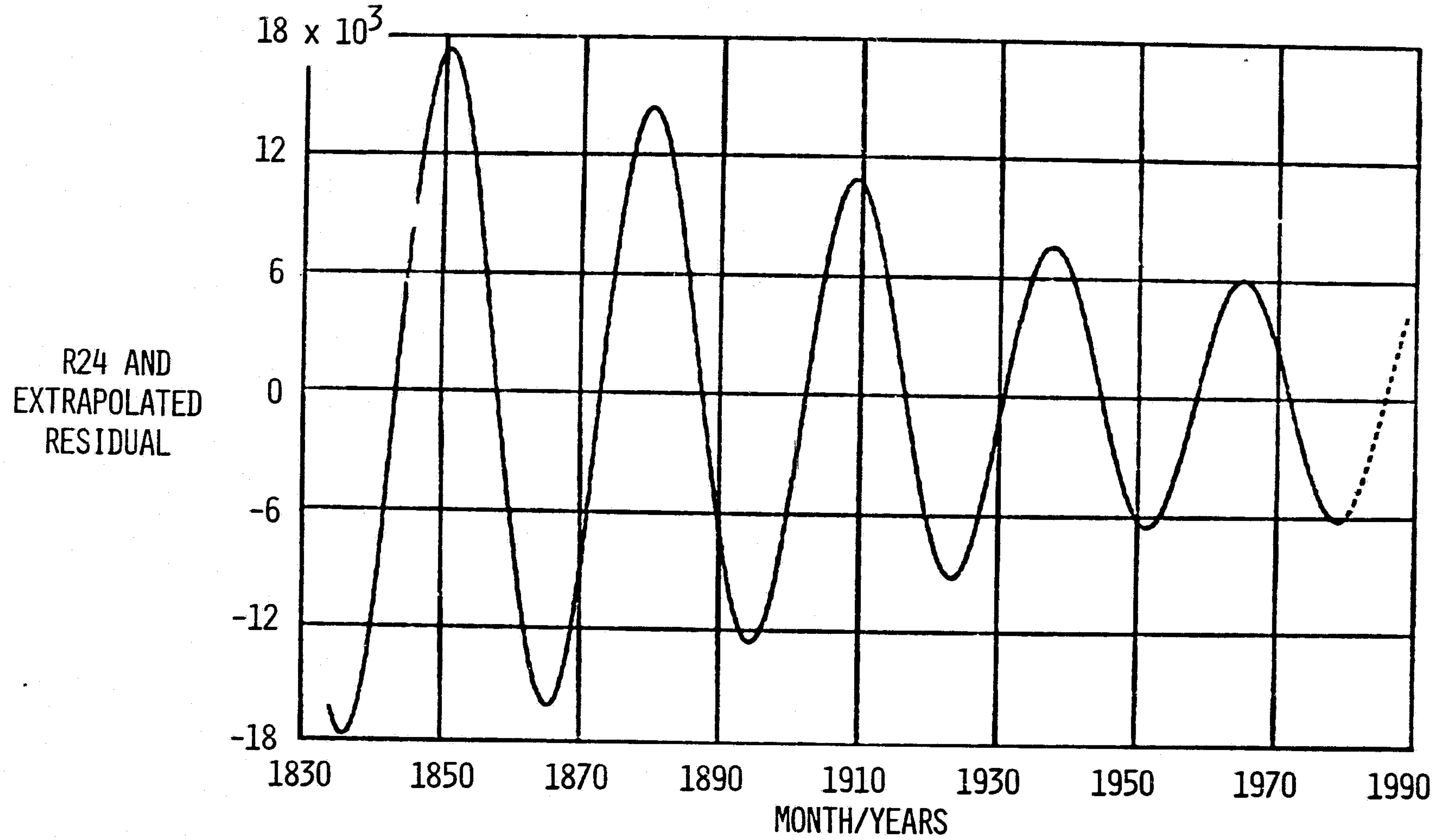


Figure 9.- Residual R24 and extrapolated residual (1834-January 1989).

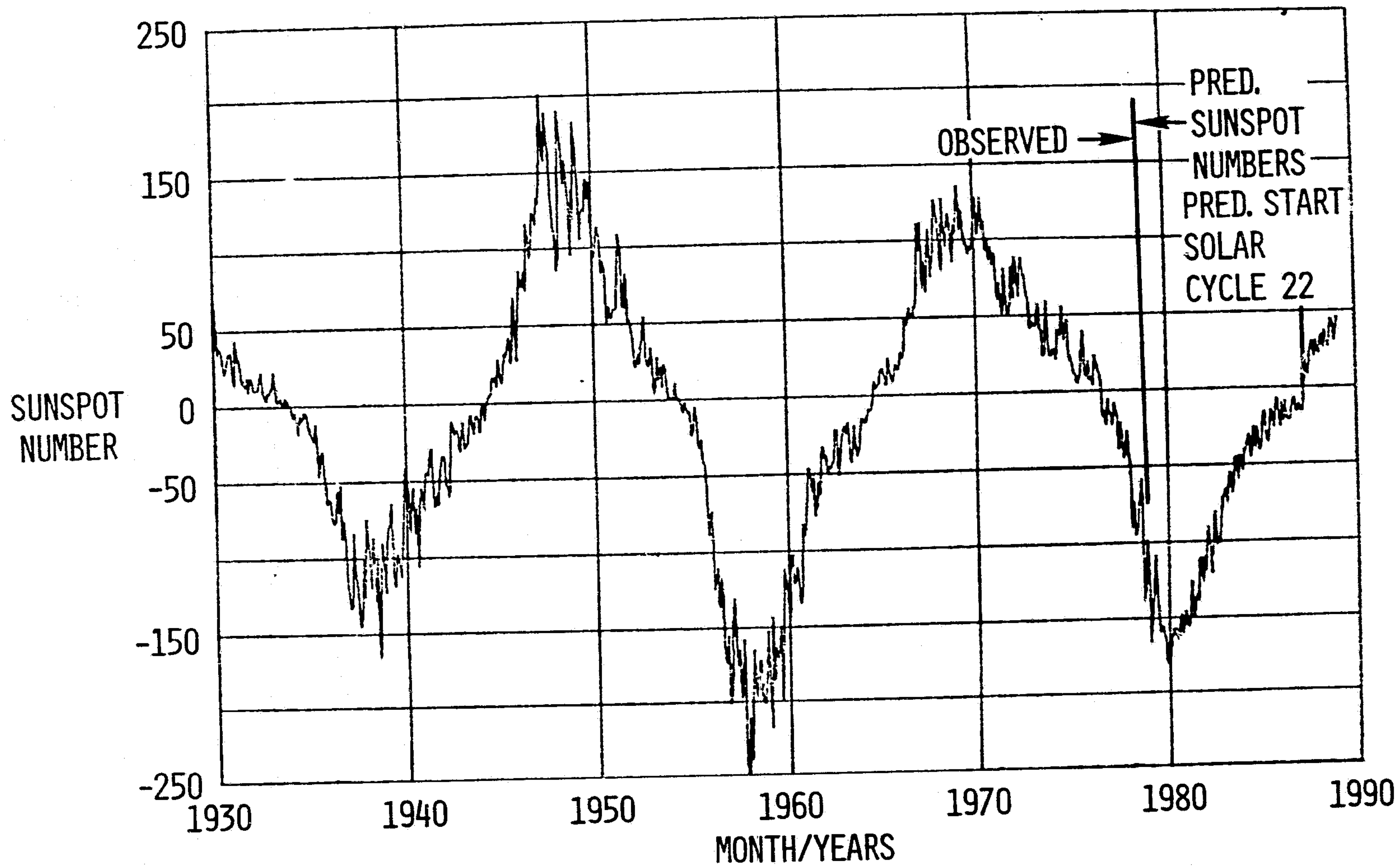


Figure 10.- Sunspot numbers observed and predicted (1930-January 1989).

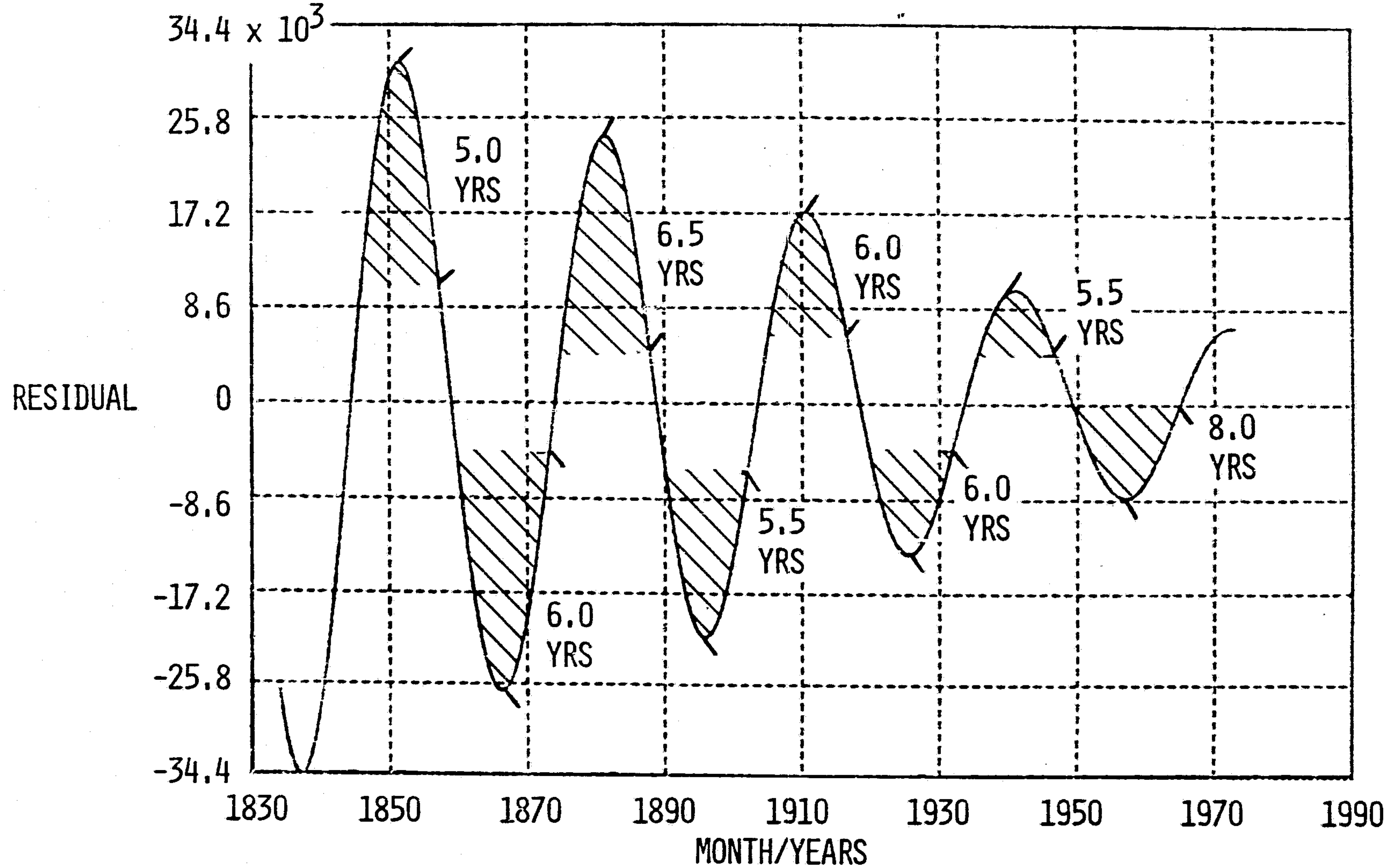


Figure 11.- Residual R22 symmetry.

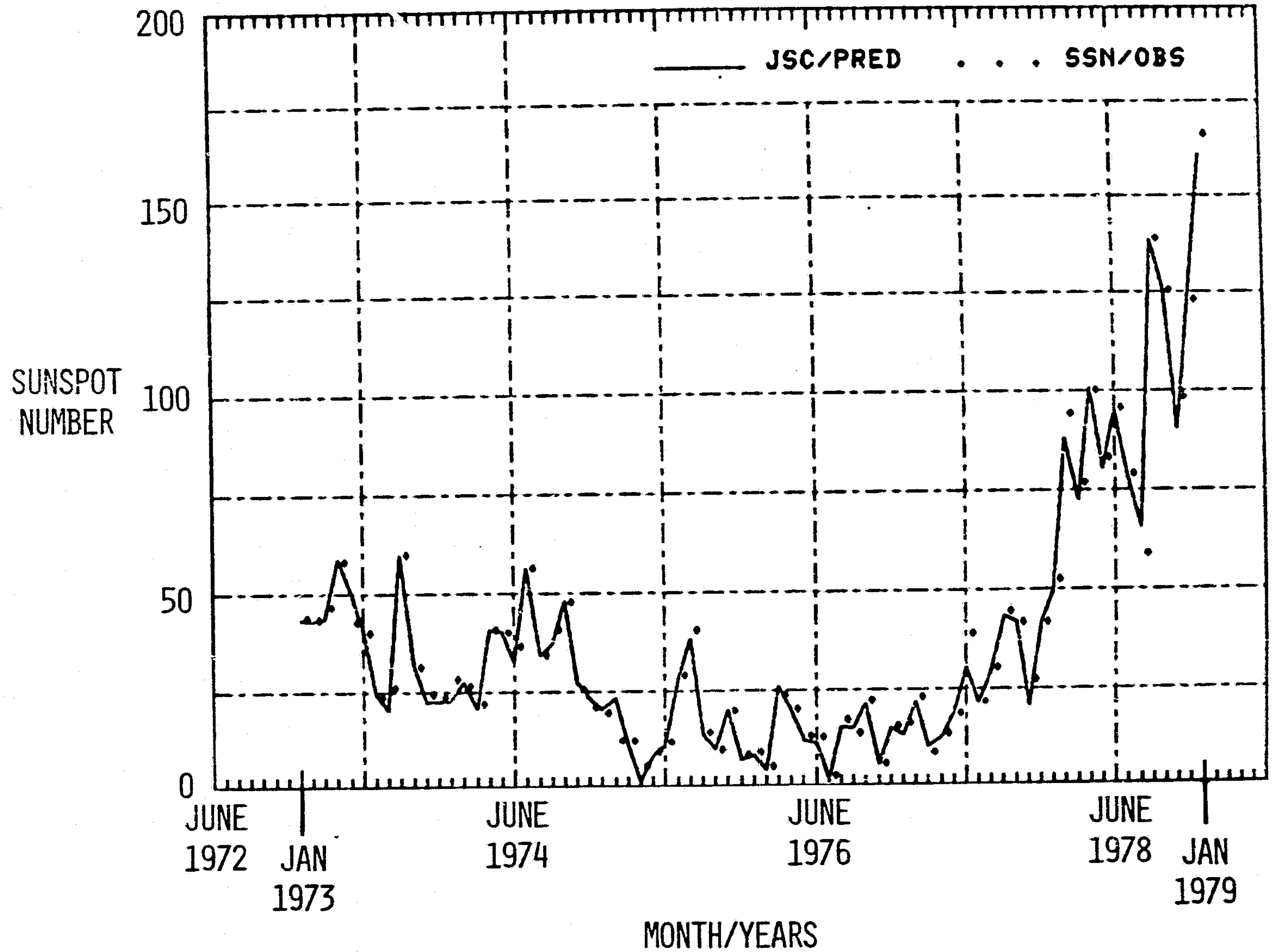


Figure 12.- Observed and predicted sunspot numbers (January 1972-January 1979).

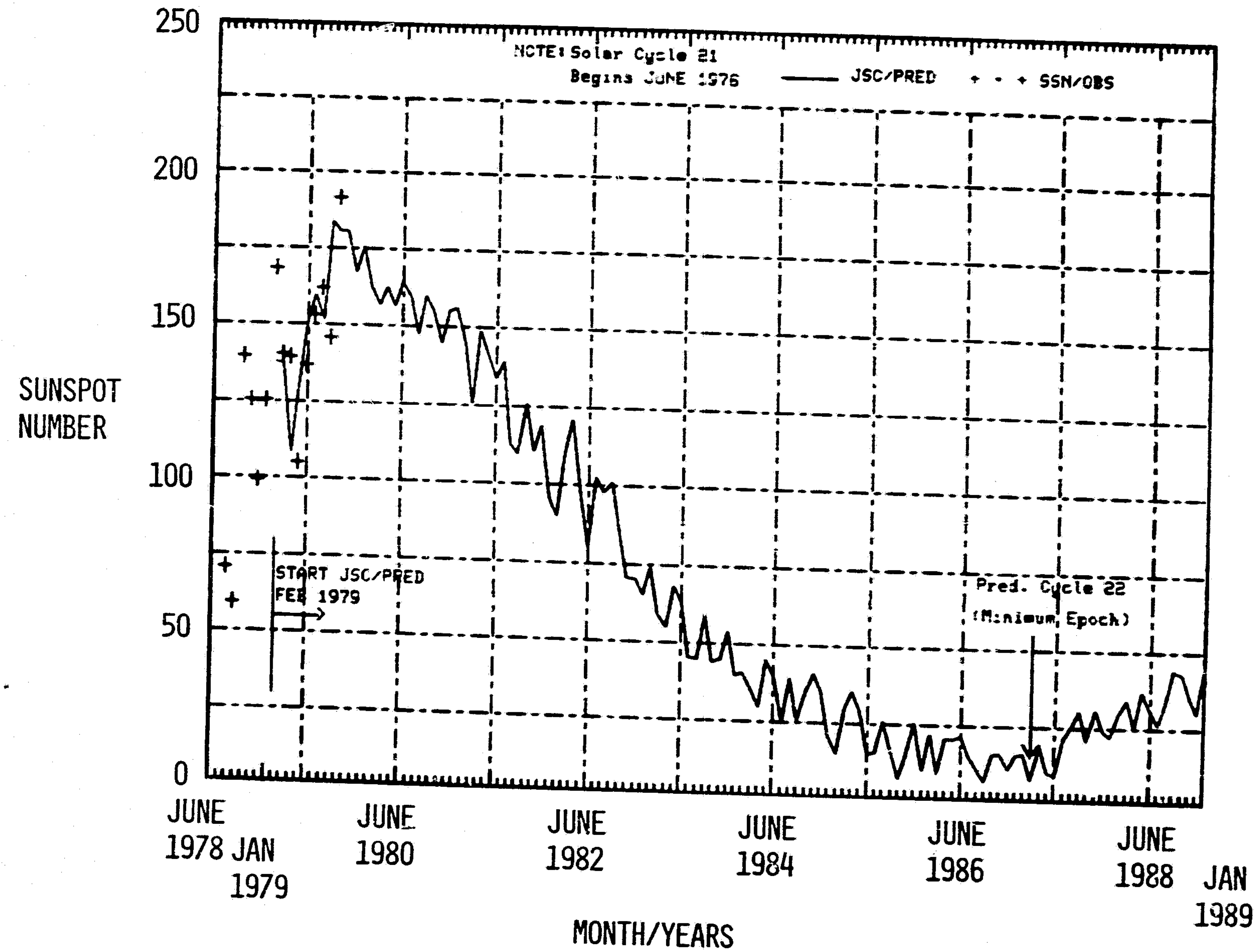


Figure 13.- Observed and predicted sunspot numbers (July 1978-January 1989).

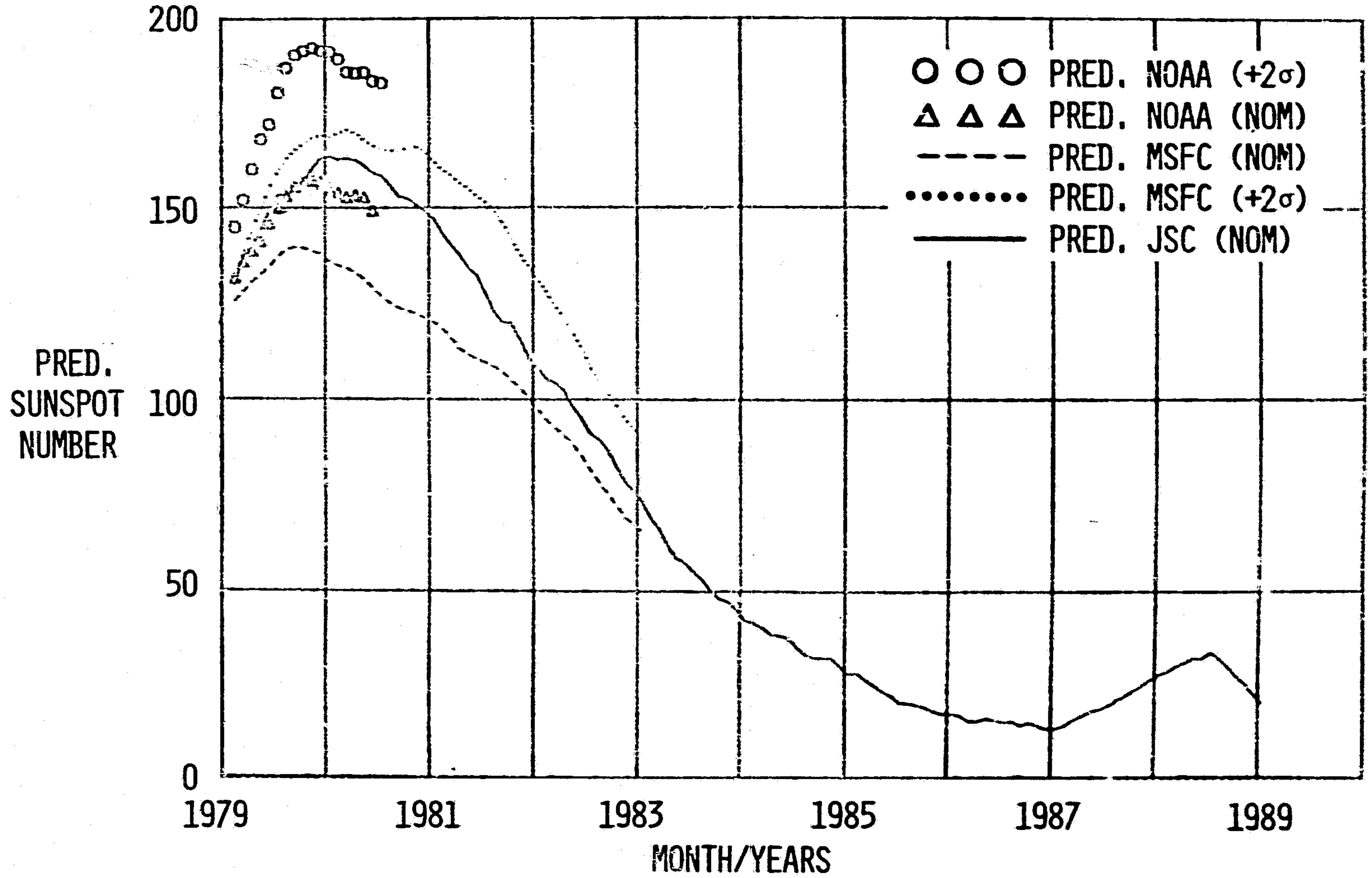


Figure 14.- Predicted smooth sunspot numbers (13-month smooth means) as a function of month/years.

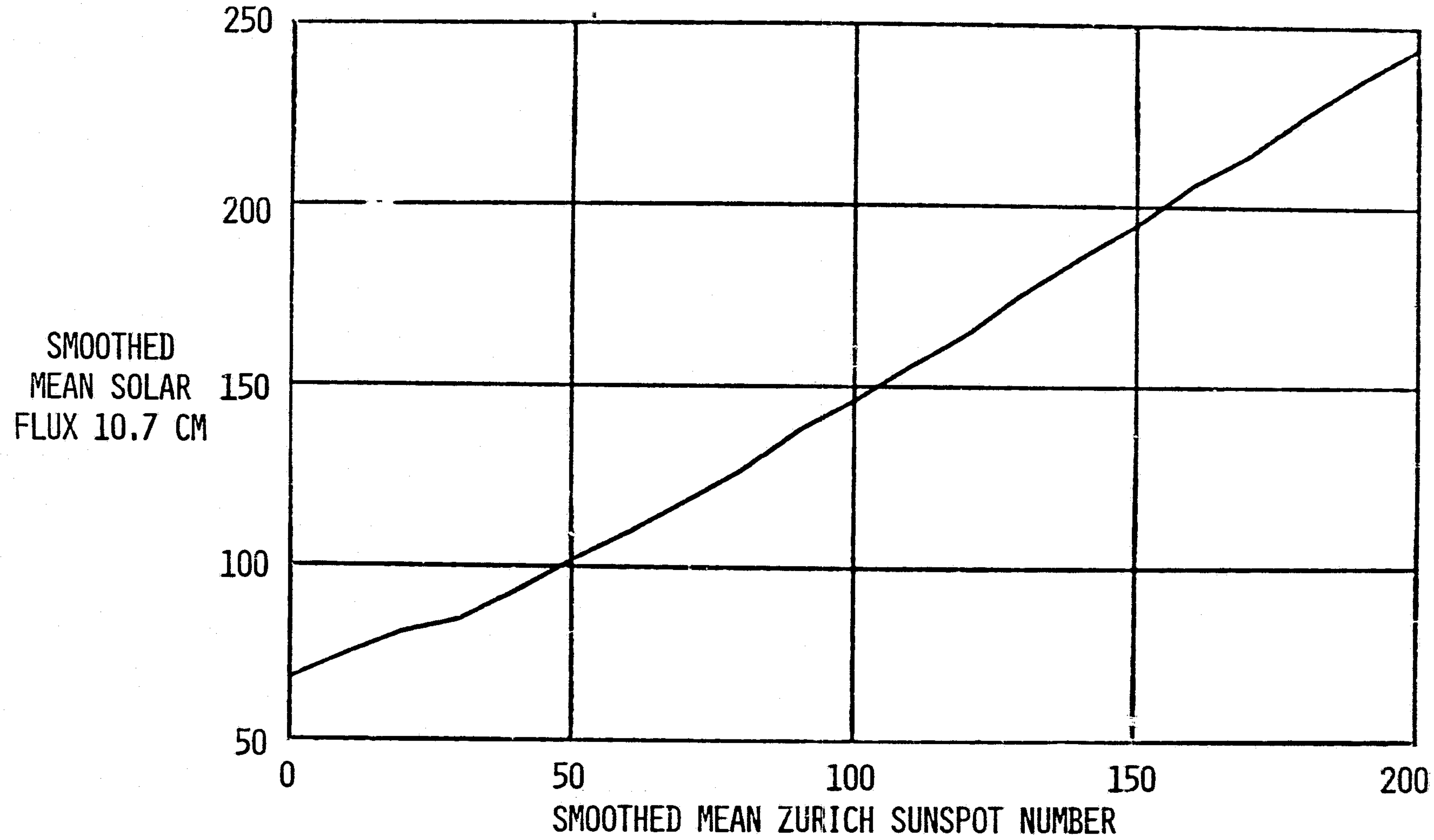


Figure 15.- Sunspot numbers/solar flux, 10.7-centimeter curve.

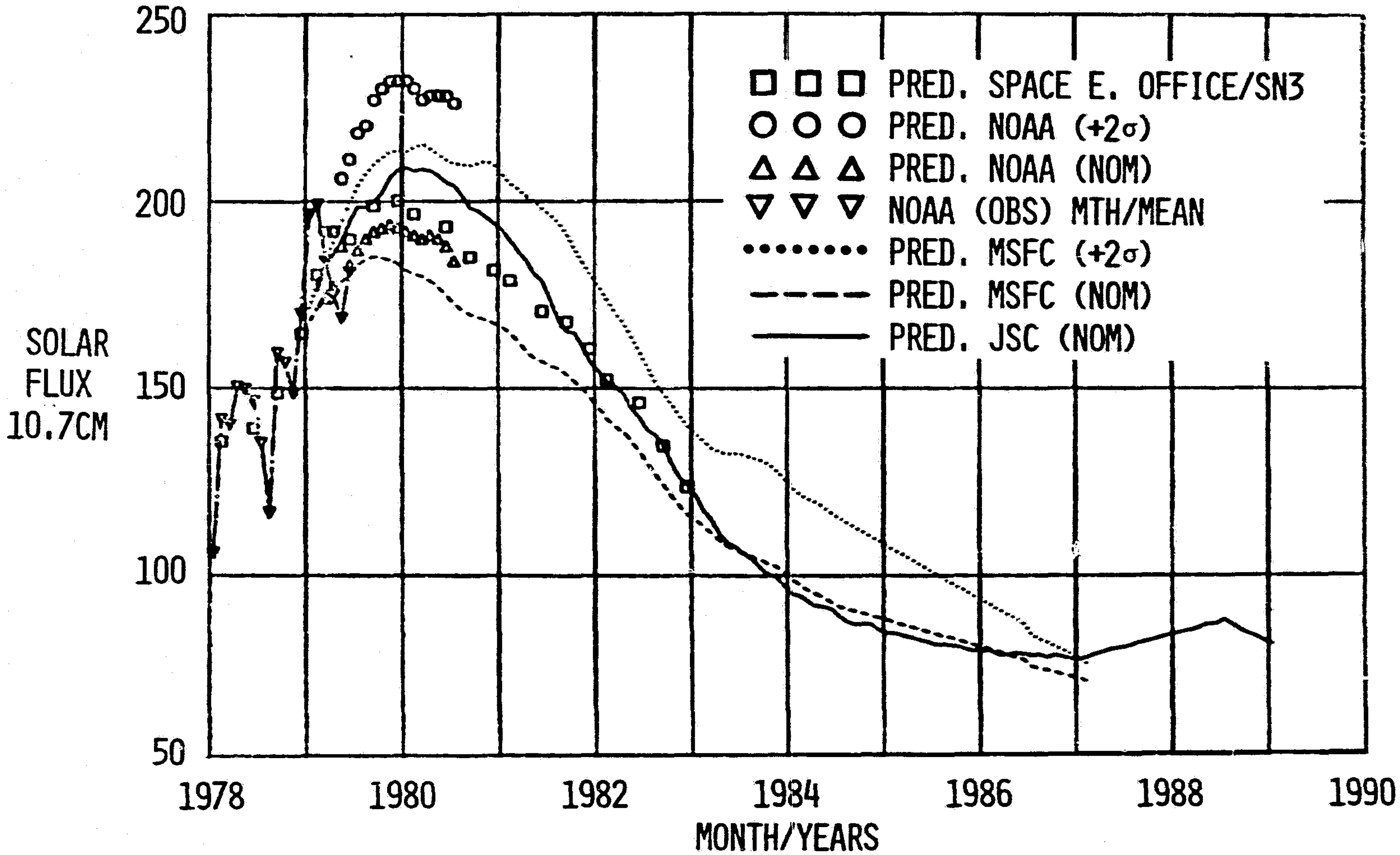


Figure 16.- Observed and predicted flux,10.7-centimeter values as a function of month/years.

APPENDIX A

DESCRIPTION OF SUNSPOT NUMBER

The Zurich relative sunspot number is an index of the activity of the entire visible disk of the Sun. It is determined each day without reference to preceding days. Each isolated cluster of sunspots is termed a sunspot group, and it may consist of one or a large number of distinct spots whose size can range from 10 or more square degrees of the solar surface down to the limit of 1/25 square degree.

The relative sunspot number is defined as:

$$R = K(10G + S)$$

Where G is the number of sunspot groups, S is the total number of distinct spots, and K is a scale factor.

APPENDIX B

DESCRIPTION OF SOLAR RADIO FLUX (10.7 cm)

Daily observations of the 2800 megahertz radio emissions that originate from the solar disk and from any active regions are made at the Algonquin Radio Observatory (ARO) of the National Research Council of Canada. These are a continuation of observations that commenced in Ottawa in 1947. Numerical values of flux are published by NOAA in the SGD documents in tables with units of $10^{-22} WM^{-2} H_z^{-1}$ and refer to a single calibration made near local noon at 1700 universal time. All flux data are then corrected to Sun-Earth distance of one astronautical unit.

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