

DIFFERENTIAL CORRECTION CAPABILITY  
of the  
GTDS USING TDRSS DATA

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ABSTRACT

A differential correction (DC) capability was implemented in the Goddard Trajectory Determination System (GTDS) to process satellite tracking data acquired via the Tracking and Data Relay Satellite System (TDRSS). Configuration of the TDRSS will be reviewed, observation modeling will be presented, and major features of the capability will be discussed in this paper.

The following new types of TDRSS data can be processed by GTDS: 2-way relay range and Doppler measurements, hybrid relay range and Doppler measurements, one-way relay Doppler measurements, and differenced one-way relay Doppler measurements. These new types of data may be combined with conventional ground-based direct tracking data. By using Bayesian weighted-least-squares techniques, the new software allows the simultaneous determination of the trajectories of up to four different satellites--one user satellite and three relay satellites. In addition to satellite trajectories, the following parameters can be optionally solved for drag coefficient, reflectivity of a satellite for solar radiation pressure, transponder delay, station position, and biases. Signal travel time is corrected, and atmospheric refraction correction may be invoked optionally for the space-ground link. Finally, as an option, a statistical output report, which can be used for tracking system calibration and evaluation, will be generated.

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## 1.0 INTRODUCTION

Conventionally, satellite tracking data are obtained by direct observation of a satellite from ground tracking facilities on the surface of the Earth. The field of view, however, is limited by the local horizon. Thus, in order to have continuous tracking, it is necessary to have many ground tracking sites well distributed over the surface of the Earth. The installation, maintenance, and operation of these ground tracking facilities is very costly. One plausible solution to this cost problem is to use geosynchronous satellites to track other satellites. This scheme not only could eliminate all but one ground tracking facility, but could also provide nearly 100 percent continuous coverage of a user satellite (Reference 1).

Indeed, satellite-to-satellite tracking (SST) has been proved to be feasible after a number of years of successful experiments using Application Technology Satellite-6 (ATS-6, a geosynchronous satellite situated at 220 degrees East longitude and 0.4 degrees North latitude at an altitude of 35,800 kilometers) as a relay satellite in tracking GEOS, NIMBUS-6 and ISEE-3.

In December 1976, the National Aeronautics and Space Administration (NASA) contracted with Western Union for 10-year leased services of the Tracking and Data Relay Satellite System (TDRSS) to maintain its orbiting satellites. The system is scheduled to become operational in the 1980s (Reference 1).

This paper presents a brief description of current capabilities of GTDS for support of the TDRSS.

## 2.0 TDRS TRACKING SYSTEM

### 2.1 System Configuration of TDRSS

The system will consist of three geosynchronous satellites and one common ground tracking facility. Two of the satellites are operational satellites and

the other is an orbiting spare satellite. The spare satellite may be converted for use as an operational satellite or may be scheduled for service in conjunction with the two operational satellites.

Satellite TDRS East will be at 41 degrees West longitude, TDRS West at 171 degrees West longitude, and TDRS spare at 106 degrees West longitude. These satellites will have circular orbits around the equator at an altitude of 36000 kilometers. The antenna coverage of the TDRSS is shown in Figure 1 (from Reference 1). Above an altitude of 1200 kilometers, the coverage is 100 percent for user satellites within the TDRS antenna pointing limits. For single-access antennas, the pointing limits are  $\pm 22.5$  degrees east-west and  $\pm 31$  degrees north-south. For multiple-access antennas, the field of view is a 26 degree cone (Reference 1). Below 1200 kilometers, there is a shadow zone located between 50 degrees East longitude and 125 degrees East longitude. The maximum amount of coverage lost due to the Earth occultation is 20 percent for a user satellite as low as 200 kilometers.

The common ground tracking facility will be at White Sands, New Mexico, located at 106.5 degrees West longitude and 32.5 degrees North latitude. The tracking facility includes three 18-meter, steerable antennas operated at K-Band frequency. Each of these antennas is able to track any of the TDRSSs. The tracking equipment at the ground station is required to meet the following specifications (Reference 2):

- Systematic range light time error shall be less than  $\pm 20$  nanoseconds (corresponding to  $\pm 6$  meters).
- Maximum root-mean-square (rms) range light time noise shall be  $\pm 10$  nanoseconds (or  $\pm 3$  meters) for high data rate and  $\pm 20$  nanoseconds (or  $\pm 6$  meters) for low data rate.
- Maximum rms phase noise for Doppler measurement shall be  $\pm 0.1$  radians for high data rate and  $\pm 0.2$  radians for low data rate.

A sketch of the TDRSS ground tracking station at White Sands, reproduced from Reference 1, is shown in Figure 2.

Three TDRS antenna systems will be available for NASA use (Reference 1).

- TDRS to Tracking Station: a 2-meter antenna system operated at K-Band frequency (15 GHz)
- TDRS to Single Target: two 5-meter steerable single-access antenna systems operated at either K-Band or S-Band frequency (2 GHz); the steering range is  $\pm 22.5$  degrees in east/west direction, and  $\pm 31$  degrees in north/south direction; the target can be a user spacecraft or a ground transponder
- TDRS to Multiple Targets: a 30-element electronically steerable multiple-access antenna system operated at S-Band frequency; the field of view of the multiple-access antenna system is a cone of 26 degrees; a total of 20 targets can be tracked simultaneously

The TDRS spacecraft antenna configuration is shown in Figure 3, which is reproduced from Reference 1.

## 2.2 Tracking Configuration of TDRSS

Basically, there are three categories of tracking configuration in TDRSS currently supported by GTDS:

- Hybrid tracking configuration
- Two-way tracking configuration
- One-way tracking configuration

For descriptive purposes, the path of the tracking signal will be defined as a chain of nodes and legs. A NODE is either a station or a spacecraft which can transmit and/or receive a tracking signal. A LEG is the signal path between two nodes. The measurements related to these configurations are discussed separately in the following subsections.

### 2.2.1 Hybrid Relay Range and Doppler Measurements

Using the definitions for nodes and legs, the signal path of a hybrid relay range measurement is depicted schematically by Figure 4 (from Reference 1). The tracking signal originates and is transmitted from an antenna at White Sands

station (node 1) and is propagated through the forward-link TDRS (node 2). The signal then arrives at a target (node 3), is relayed to the return-link TDRS (node 4), and is finally received at an antenna at the White Sands station (node 5). The target being tracked by the TDRSS either can be an orbiting user-satellite or a ground transponder.

For a hybrid relay Doppler measurement, the signal path is similar to that of a range measurement, except that there is an extra node and an extra leg. A coherent Doppler signal is transmitted from the receiving antenna (node 6) and is mixed at the return-link TDRS (node 4) to maintain the phase coherency with the Doppler signal transmitted from the transmitting antenna (node 1). The mixed Doppler signal is finally received at the receiving antenna (node 5). Node 6 and node 5 physically are the same antenna but at different positions in the inertial coordinate system due to Earth rotation.

### 2.2.2 Two-Way Relay Range and Doppler Measurement

For a two-way relay range or Doppler measurement, the tracking signal also originates from a transmitting antenna, is propagated via a TDRS to a target, is retransmitted by the target back to the same TDRS, and is received by the same ground antenna. Figure 4 shows the two-way tracking configuration in which nodes 1, 5, and 6 are physically associated with the same antenna, and nodes 2 and 4 are associated with the same TDRS.

### 2.2.3 One-Way Relay Doppler Measurements

For a one-way relay Doppler measurement, the wide-beam tracking signal originates from the target (node 3), proceeds to the return-link TDRS (node 4), mixes with the coherent Doppler signal transmitted from the ground receiving antenna (node 6), and is finally received by the ground receiving antenna (node 5). Note that there are no one-way range measurements.

#### 2.2.4 Differenced One-Way Relay Doppler Measurements

A new type of measurement is feasible with the one-way tracking configuration. With a wide-beam antenna system, the one-way tracking signal generated by the user satellite may be received by all three TDRSs. By differencing two streams of one-way Doppler measurements, the oscillator frequency bias can be largely cancelled out. This is called differenced one-way relay Doppler measurement. With a multiple-access antenna system on TDRS, up to five user satellites can be tracked simultaneously with this type of measurement (Reference 1).

#### 2.3 Ground Transponder Tracking of TDRS

Theoretically, the target being tracked by TDRSS can either be in the sky (user satellite) or on the ground (ground transponder) for all configurations. The software design in GTDS does not impose any restrictions on a target in this regard. In practice, however, a ground transponder usually employs a highly directional antenna. Therefore, when a ground transponder is tracked with a TDRS, only a two-way tracking configuration is anticipated. This mode of tracking, using precisely surveyed ground locations of transponders, is primarily used for determining TDRS trajectories for calibration of TDRSS. With a multiple-access antenna system, the TDRS can track up to 10 ground transponders almost simultaneously because it has the capability to electronically steer the antenna beam from one transponder to another essentially instantaneously.

For hybrid and differenced one-way tracking configurations, the target must transmit with a wide-beam antenna so that more than one TDRS can pick up the signal to complete the configuration. Therefore, in practice the target is expected to be a user satellite instead of a ground transponder.

### 3.0 GTDS OBSERVATION MODELING

#### 3.1 Modeling of Range Observation

The TDRSS range observation is obtained by measuring the time delay for a reference time marker (pseudorandom code phase) to travel from the White Sands ground tracking station, to the TDRS, to the target, and then back to the same TDRS or a different TDRS and to the ground station. The measuring process only gives the fraction part of a pseudorandom (PN) code period. The ambiguity, i. e., the whole number of PN periods, must be resolved by the orbit determination process. The actual range measurement is halved by a data preprocessor before it is input to GTDS for modeling.

In GTDS, the time tag associated with a measurement is treated as the receive time of the tracking signal at the receiving station. Therefore, the backward signal trace method is used in determining the time the signal is transmitted from each node and the position of the node at the moment the signal is transmitted. During the course of signal tracing, signal delay time for propagation at the speed of light is iteratively corrected for each leg. After the actual transmit time is determined at node 1, one half of the distances (legs) between nodes are summed as the computed range observation. This computed range observation is compared with the observed ambiguous range to resolve the range ambiguity. Transponder delay, atmospheric refraction on ground-to-space legs, measurement bias, timing bias, or station geodetics bias can be invoked optionally during modeling. The formulation of the relay range measurement and the associated partial derivatives are given in Figures 5, 6, and 7. A more complete description of the relay range measurement is contained in Reference 3.

#### 3.2 Modeling of Doppler and Differenced Doppler Observations

Doppler measurements in TDRSS include hybrid, two-way, one-way, and differenced one-way. The raw data of the measurement consists of a nondestruct

Doppler count of a nominal bias frequency, 240 MHz, over a fixed time interval. The count is cumulative since the counter is not reset to zero between measurements.

A hybrid or a two-way Doppler measurement is performed by transmitting a signal at K-Band from the ground transmit station to a forward-link TDRS. The TDRS coherently translates the signal to the user spacecraft's tracking frequency in S- or K-Band and transmits it to the user spacecraft. The user coherently retransmits signal to the return-link TDRS at a ratio of either 240/221 for S-Band or 1600/1469 for K-Band. The TDRS then translates the signal to K-Band and transmits it to the ground receiving station (Reference 1).

The one-way Doppler measurement can be generated from either an autonomous spacecraft or a ground transponder. In the case of an autonomous spacecraft, the navigation might be performed over several days without commands from the ground. Any 10 of the 20 multiple-access service antennas of the TDRS may be simultaneously used for one-way Doppler measurements. Although the individual one-way Doppler measurements are dominated by oscillator frequency bias, a wide-beam antenna system on the autonomous spacecraft will allow the signal to be received by all three TDRSs with the same frequency bias being observed in each measurement. In differencing the measurements, this bias can be cancelled out. Thus, the tracking of a spacecraft can be as accurate as two-way measurements (Reference 1). The formulations of the relay Doppler and differenced Doppler measurements and their associated partial derivatives are given in Figures 5, 6, and 7. A more complete description of the Doppler measurements is contained in Reference 3.

#### 4.0 DC CAPABILITIES

##### 4.1 DC Solve for Parameters

Currently GTDS can solve for up to four satellite trajectories simultaneously, including one user satellite (target) and up to three TDRS relays in the TDRSS



observation processing mode of the Differential Correction (DC) Program. GTDS has the ability to solve for the following parameters simultaneously using any combination of the TDRSS measurement types in addition to the conventional ground-based direct tracking data of the TDRSs and the target:

- State vector of one user satellite
- State vectors of up to three TDRSs
- Drag on user satellite
- Reflectivity of the user satellite
- Reflectivity of the TDRSs being solved
- Measurement biases
- Time delay of ground transponder
- Time delay of satellite transponder
- Timing bias
- Geodetic location of tracking station and ground transponders
- Coefficients of geopotential harmonics

A Bayesian weighted-least-squares technique is employed by GTDS to process the observation data in the differential correction process. This is the same technique used in GTDS for all Differential Correction Program runs regardless of the type of tracking data being processed. The fundamentals of differential correction and the theory of estimation can be found in Reference 4.

#### 4.2 Integration Techniques for Equations of Motion

The equations of motion for all satellites will be numerically integrated using the 12th order Cowell integrator in GTDS. The Cowell sums and accelerations will be stored on GTDS ORBIT Files from which position and velocity components will be reconstructed during the processing of TDRS observation data. The relay ORBIT Files can optionally be created prior to a DC Program run and stored for use by all GTDS program users, alleviating the need to generate the reference orbits for the TDRS relays during each DC Program run.

### 4.3 TDRSS Observation Selection Capabilities

GTDS provides the user with a flexible observation selection capability to process both TDRS observation data and conventional direct ground tracking data in the same DC Program. The following criteria can be used in combination for data selection:

- Satellite ID: Data can be selected and processed according to the satellite identifier for the user satellite and any, or all, of the TDRS relay satellites included in a DC Program run
- Tracking Mode: Data to be processed can be conventional direct tracking, TDRS relay tracking, or a combination of both
- TDRS measurement identifiers including the following:
  - return-link TDRS identifier number
  - forward-link TDRS identifier number
  - ground transponder identifier (if a ground transponder is tracked)
  - equipment mode (selection based on whether the relay-to-user link is operating in the S- or K-Band)
- Tracker Type: Select data according to tracking station type (i.e., GRARR, C-Band, TDRSS, etc.)
- GTDS Measurement Type: Select data according to unique GTDS measurement number assigned to each supported measurement type
- Observation Time Span: Start and end times
- Data Rate

The data selection capabilities are made possible by the construction of an observation data working file created within the GTDS. This working file includes, for each observation, a self-contained data record consisting of the following information:

- Observation receive-time tag
- Satellite identifier number
- Transmit and receive station index number

- Actual measurement of GTDS measurement type
- Doppler count interval (if applicable)
- Data sampling information
- Observation validity flags
- Observation correction flags
- TDRSS observation information including:
  - forward-link TDRS identifier number
  - return-link TDRS identifier number
  - ground transponder identifier number (if applicable)
  - user-to-relay frequency
  - single access or multiple access antenna identifier

## 5.0 DC PROGRAM FLOW

The basic DC Program flow was maintained in GTDS for processing TDRSS observation data. (For a complete description of the DC flow see Reference 5). A major design change was made in the handling of up to four simultaneous satellite ephemerides. The normal mode of observation processing in GTDS is to integrate the equations of motion of a single satellite during the point-by-point processing of observation data in each DC iteration. The TDRSS processing mode creates up to four GTDS ORBIT Files (Reference 5) prior to the DC program execution or prior to each DC iteration. The state vector and transition matrix for each satellite involved in an observation is retrieved from the appropriate ORBIT File during the point-by-point observation data processing. The DC program flow remains the same as the previous GTDS flow after the retrieval of the satellite state vector and the transition matrix. Figure 8 shows the overall DC flow for processing TDRSS data in GTDS.

Upon completion of the DC program, as an option, a Statistical Output Report (SOR) can be generated. This report contains observation-dependent information, including weighted observation residuals, observation edit status,

standard deviations, associated orbit plane angles, and other pertinent information used for tracking system evaluation, validation, and calibration. An SOR can be generated for the input vector (first DC iteration) and/or the final vector (last DC iteration).

## 6.0 FUTURE TDRSS CAPABILITIES IN GTDS

In the future, the DC program will be able to use either the Brouwer or Brouwer-Lyddane orbit generators in GTDS to create satellite ephemerides for the user (target) satellite, thus removing the present restriction of the use of the Cowell orbit generator for all satellites. A logical extension will be the use of any GTDS orbit theory for integrating the equations of motion for the user satellite.

Observation processing for the TDRS RF Beam angles, spatial beam direction, and spacecraft orientation angles is currently being implemented in GTDS. These angular measurements will be used to make observation corrections due to the center of mass to antenna offset.

The interactive graphics capability of GTDS is being enhanced to provide operational satellite missions support with TDRSS configuration tracking data.

The use of GTDS ORBIT Files in the DC Program to process TDRSS observations allows for the creation of the relay ORBIT Files prior to a DC program run. These files, which contain precision satellite ephemerides for all TDRS relays will be concatenated over a specific time span (e.g., one month) and stored for retrieval by all GTDS program users. This alleviates the need to create satellite ephemerides in each DC program run, and it allows GTDS to treat the TDRS relays as if they were ground-based tracking stations with precisely known positions while solving for the trajectory of the user satellite. The SOR will be modified to process the statistics for the RF Beam angle measurements and for the associated orientation angle information.

## REFERENCES

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5. Computer Sciences Corporation, 3000-27900-02TM, Goddard Trajectory Determination System Design Manual, E. L. Zaveleta, et al., March 1975

## FIGURES

Figures 1 through 8, which were cited in the preceding text, are presented on the following pages.

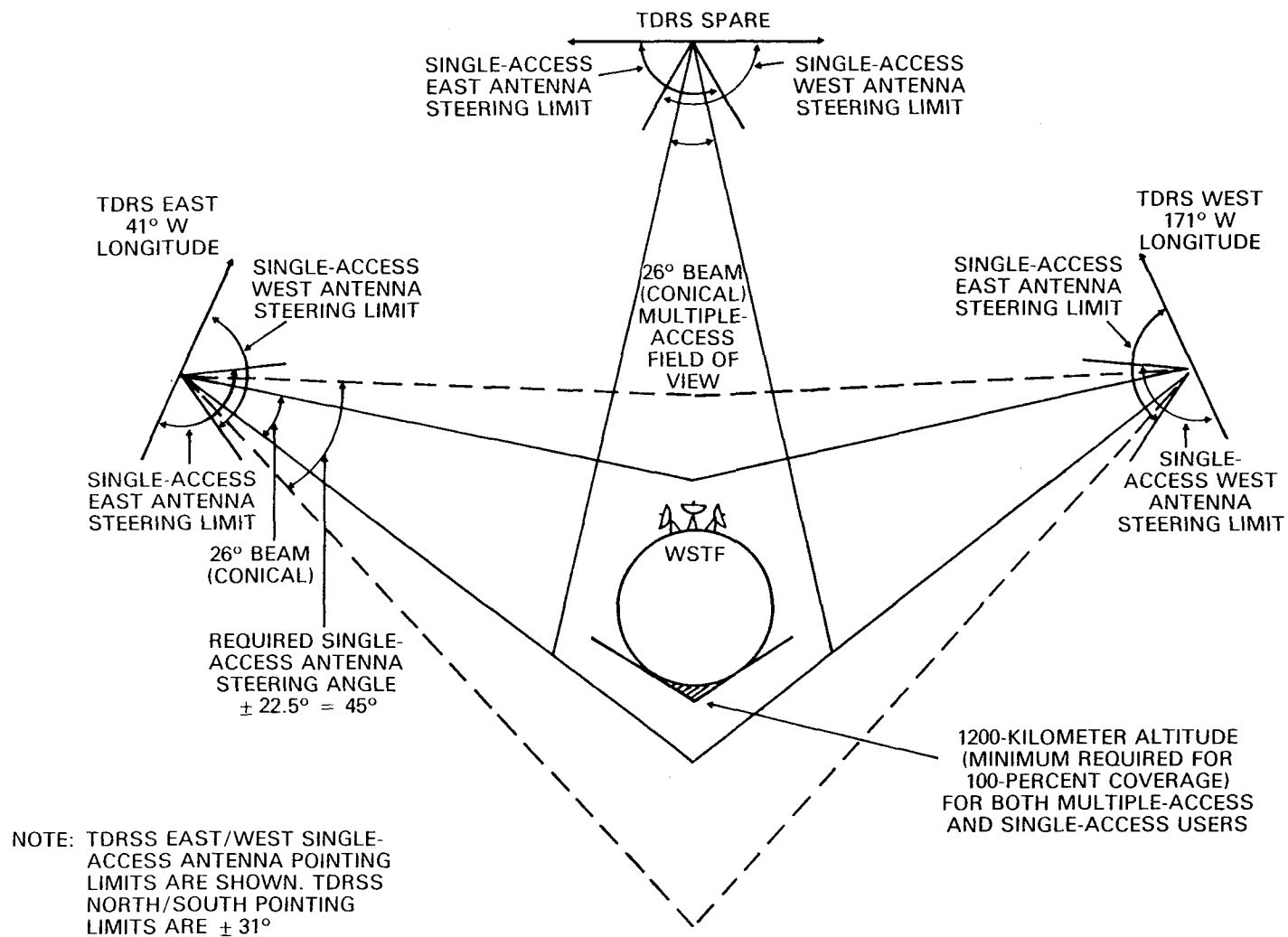


Figure 1. TDRSS Antenna Coverage

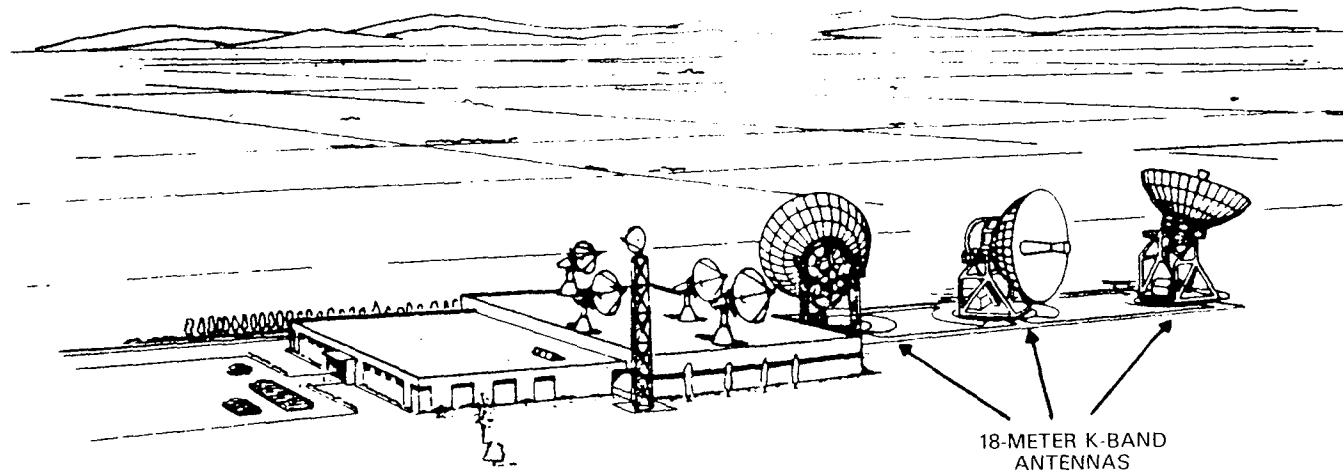


Figure 2. TDRSS Ground Station, White Sands

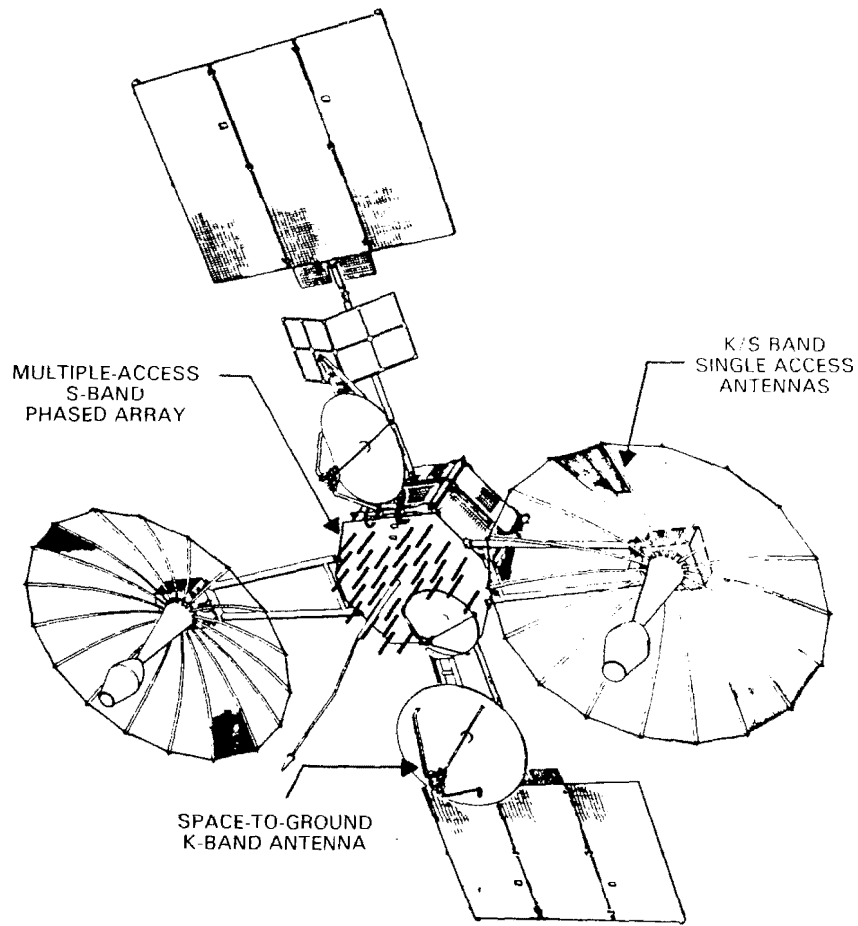


Figure 3. TDRS Spacecraft Configuration



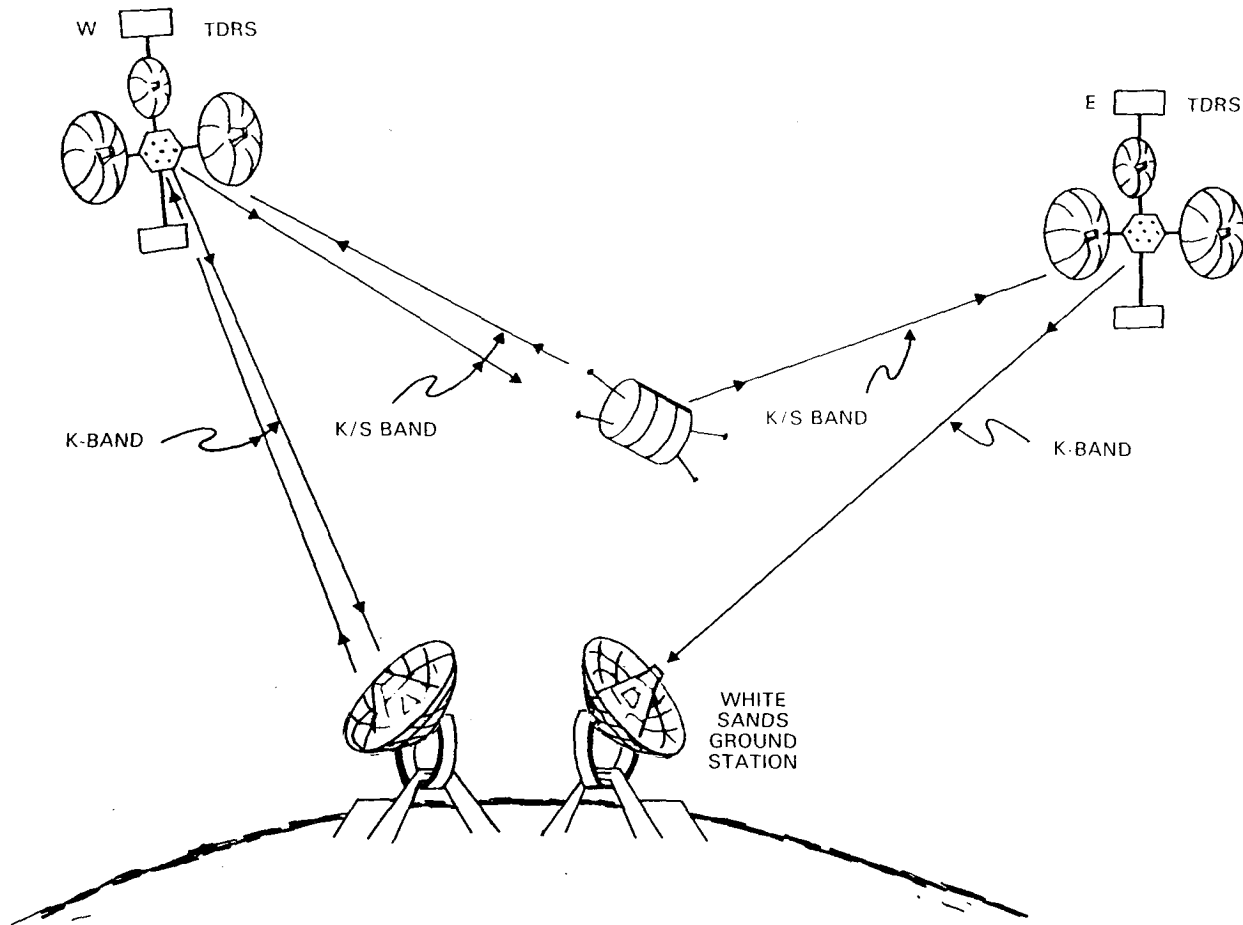


Figure 4. TDRSS Tracking Configuration

- **RELAY RANGE**

$$\rho(\mathbf{T}) = \frac{1}{2} \left[ \sum_{j=1}^4 \rho_j + c \sum_{k=2}^4 D_k \right]$$

- **RELAY DOPPLER**

$$\nu_{\mathbf{d}}(\mathbf{T}) = - \frac{1}{c\Delta T} \left[ \mathbf{A} \Delta \rho_{\mathbf{l}}(\mathbf{T}) + \mathbf{B} \Delta \rho_{\mathbf{s}}(\mathbf{T}) \right]$$

$$\text{WHERE } \Delta \rho(\mathbf{T}) = \sum \rho_j \Big|_{\mathbf{T}} - \sum \rho_j \Big|_{\mathbf{T} - \Delta \mathbf{T}}$$

- **DIFFERENCED ONE-WAY RELAY DOPPLER**

$$\Delta \nu_{\mathbf{d}}(\mathbf{T}) = \nu_{\mathbf{d}}(\mathbf{T}) \Big|_{\text{COMPARISON TDRS}} - \nu_{\mathbf{d}}(\mathbf{T}) \Big|_{\text{REFERENCE TDRS}}$$

Figure 5. TDRSS Measurements Modeling

- **RELAY RANGE**

$$\frac{\partial \rho(\mathbf{T})}{\partial \dot{\mathbf{X}}_j(t_0)} = \frac{\partial \rho(\mathbf{T})}{\partial \dot{\mathbf{X}}_j(t_j)} \Phi(t_j, t_0)$$

- **RELAY DOPPLER**

$$\frac{\partial \nu_{\mathbf{d}}(\mathbf{T})}{\partial \dot{\mathbf{X}}_j(t_0)} = - \frac{1}{c\Delta T} \left[ \mathbf{A} \frac{\partial \Delta \rho_{\mathbf{l}}(\mathbf{T})}{\partial \dot{\mathbf{X}}_j(t_0)} + \mathbf{B} \frac{\partial \Delta \rho_{\mathbf{s}}(\mathbf{T})}{\partial \dot{\mathbf{X}}_j(t_0)} \right]$$

- **DIFFERENCED ONE-WAY RELAY DOPPLER**

$$\frac{\partial \Delta \nu_{\mathbf{d}}(\mathbf{T})}{\partial \dot{\mathbf{X}}_j(t_0)} = \frac{\partial \nu_{\mathbf{d}}(\mathbf{T})}{\partial \dot{\mathbf{X}}_j(t_0)} \Big|_{\text{COMPARISON TDRS}} - \frac{\partial \nu_{\mathbf{d}}(\mathbf{T})}{\partial \dot{\mathbf{X}}_j(t_0)} \Big|_{\text{REFERENCE TDRS}}$$

Figure 6. Partial Derivatives of TDRSS Measurements with Respect to Solve-For Parameters

- $\rho(T)$  : RANGE AT OBSERVATION RECEIVE TIME T  
 $D_K$  : TRANSPONDER DELAY AT Kth NODE  
 $\nu_d(T)$  : DOPPLER FREQUENCY AT OBSERVATION  
 RECEIVE TIME T  
 $c$  : SPEED OF LIGHT  
 $\Delta T$  : DOPPLER COUNT INTERVAL  
 $\rho_l$  : "LONG PATH" RANGE  
 $\rho_s$  : "SHORT PATH" RANGE  
 $A$  : DOPPLER MULTIPLIER } HARDWARE RELATED  
 $B$  : DOPPLER MULTIPLIER } CONSTANTS  
 $\Phi(t_j, t_0)$  : TRANSITION MATRIX FROM TIME  $t_0$  TO  $t_j$

Figure 7. Definition of Symbols

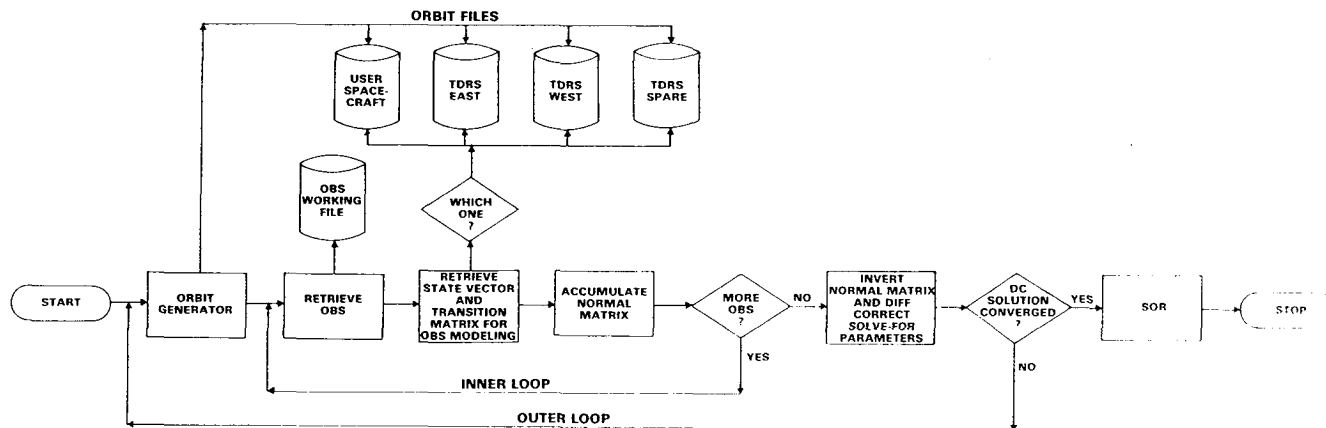


Figure 8. GTDS DC Flow for TDRSS Data



COVARIANCE ANALYSIS OF TDRS APPLICATIONS  
REQUIRING TDRS STATE PREDICTIONS

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ABSTRACT

This paper presents an initial look at the results of error analysis of TDRS applications requiring TDRS state prediction. Such a need might arise for a TDRS user requiring near-real-time ephemeris processing in the absence of available TDRS tracking data. Analysis thus far has considered several near-earth users in performing a standard covariance analysis of weighted least squares orbit determination. Results include plots of TDRS and user state errors as well as comparisons of varying parameter estimation scenarios.

