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CONTROL-STRUCTURE INTERACTIONS IN A FREE BEAM

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Summary

A simple energy approach to study the problem of control-structure interactions in large space-structures is presented. For the illustrative case of a free-free beam, the vibrational energy imparted during operation of constant, step, and pulsed thrusters is found in a non-dimensional closed form. Then based on a parametric study, suggestions are made on the choice of parameters to minimize the control-structure interactions. The study of this simple system provides physical insight and understanding for more complex systems.

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Notation

E, E^*	energy and its non-dimensional form
$\overline{EI}$	flexure rigidity coefficient
F	force due to thruster
I	transverse moment of inertia
R, R_i	ratio of energy in flexure, flexure mode i
T	kinetic energy
U	strain energy
X	mode-shape
g_i	$\pi \tau_1 / \tau_i$
j	pulse number
k	roots of eq. (9) defining mode-shape
L	length
m	mass per unit length
n	total number of pulses
p	frequency
t	time
x	distance from one end of beam
y	transverse deflection
Σ	$\sum_{i=1}^{\infty}$
τ	time between pulses
τ_i	period of i^{th} mode
δ	pulse-width

Subscript

f	flexure mode
i	i^{th} mode
max	maximum
min	minimum
r	rigid mode

Dots indicate differentiation with respect to time and primes indicate differential with respect to x.

Introduction

The problem of control-structure interactions in large spacecraft has been of concern in recent years. A number of studies have been conducted on the subject by many authors. Generally one has to resort to extensive numerical simulation. Any a priori knowledge of the system helps in reducing the computational effort involved. This paper presents a simple energy approach which provides considerable insight into the problem. As an illustration a free-free uniform beam subjected to continuous, step and pulsed operation of a transverse thruster is chosen. It represents a rectangular solar power satellite or a slender rocket with large length to width/diameter ratio. To control the orientation (rigid-mode), transverse thrusters, usually placed near the ends, are operated. Their operation can impart significant undesirable vibrations.

Here the ratio of energy in flexure mode and rigid mode is found in a closed form. It is then used for an extensive parametric study. Non-dimensionalization makes the results applicable to a wide range of systems. Based on the analysis, suggestions are made for a strategy minimizing the control-structure interactions.

Analysis

Consider a free-free beam (Fig. 1a) subjected to a concentrated transverse load (thrust) F. Under the action of the force the beam will translate, rotate and vibrate. The first two are designated as rigid-mode, and the last is called flexure-mode of motion. Let us consider three types of loading - constant, step and pulsed.

A. Constant (continuous) Thrust

Ion-thrusters generate a loading of this nature (Fig. 1b). Under its action, energy in the rigid mode is given by

$$E_r = T_r = (I\,\omega^2 + m\ell\,v^2)/2 \tag{1}$$

where ω and v are rational and translational speed. If the thruster is at one end $(x_1 = \ell)$ we get

$$E_r = 2F^2t^2/m\ell \tag{2}$$

The kinetic and strain energy in the flexure mode are given by[1]

$$\begin{aligned} T_f &= \frac{1}{2}\int_0^\ell m\,\dot{y}^2\,dx \\ U_f &= \frac{1}{2}\int_0^\ell \overline{EI}\,(y'')^2\,dx \end{aligned} \tag{3}$$

Under concentrated loading the response of a beam can be written in terms of modal components as[1]

$$y = \sum \frac{X_i X_{i1}}{p_i \ell}\int_0^t q(t')\sin p_i(t-t')\,dt' \tag{4}$$

where X_{i1} is i^{th} mode calculated at the point of application of F, and $q = F/m$.
For a constant force, the response is

$$y = \frac{F}{m\ell} \sum \frac{X_i X_{i1}}{p_i^2} (1 - \cos p_i t) \tag{5}$$

For a free-free beam the normal modes can be taken as

$$X = \pm(\cos kx + \cosh kx) \pm c(\sin kx + \sinh kx) \tag{6}$$

where

$$c = \{(\sinh k\ell + \sin k\ell)/(\sinh k\ell - \sin k\ell)\}^{1/2} \tag{7}$$

$$p_i = \sqrt{EI/m}\, k_i^2 \tag{8}$$

$$\cos k\ell \cosh k\ell = 1 \tag{9}$$

Noting that

$$\int_0^\ell X_i^2 dx = \ell \quad ; \quad \int_0^\ell X_i X_j dx = 0 \quad i \neq j \tag{10}$$

and using (5) - (8), eq. (3) can be written as

$$T_f = \frac{F^2}{2m\ell} \sum X_{i1}^2 \frac{\sin^2 p_i t}{p_i^2}$$
$$U_f = \frac{F^2}{2m\ell} \sum X_{i1}^2 (1 - \cos p_i t)^2 / p_i^2 \tag{11}$$

The total energy in the flexure mode is therefore

$$E_f = T_f + U_f = \frac{F^2}{m\ell} \sum X_{i1}^2 \left(1 - \cos p_i t\right)/p_i^2 \tag{12}$$

For the force acting at one end $X_{i1}^2 = 4$, and

$$E_f = \frac{8F^2}{m\ell} \sum \left(\sin^2 p_i t/2\right)/p_i^2 \tag{13}$$

Eq. (2) and (13) can be written to yield non-dimensionalized components of energy as

$$E_r^* = E_r\, m\ell/2F^2 \tau_1^2 = \left(t/\tau_1\right)^2 \tag{14}$$

$$E_f^* = E_f\, m\ell/2F^2 \tau_1^2 = \sum \left(\tau_i/\pi\tau_1\right)^2 \sin^2(\pi t/\tau_i) \tag{15}$$

The energy-ratio is given by

$$R = E_f/E_r = \sum \left(\sin z_i / z_i\right)^2 = \sum R_i \tag{16}$$

where $z_i = \pi t/\tau_i$

Thus we obtain simple expressions for flexure-energy and energy-ratio. A high value of R corresponds to large control-structure interaction. Our aim is to minimize it. This will be discussed in the next section.

B. Step Thrust:

Generally, chemical thrusters are operated for a limited time like a step-function (Fig. 1c). During the application of force all the relations found above hold. After the termination of thrust at t_1, the energy will remain constant and will correspond to the value at $t=t_1$. Hence with t replaced by t_1, relations (12) - (16) are applicable. The response after t_1 is given by

$$y = \frac{F}{m\ell} \sum X_i X_{i1} \left\{ \cos p_i (t - t_1) - \cos p_i t \right\} / p_i^2 \tag{17}$$

C. Pulsed Thrust:

In many applications control is achieved using pulsed thrusters. In this case control-structure interactions can severe if the system parameters, namely thrust level F, pulse-width δ, period between pulses τ, and number of pulses n, are not chosen properly.

The response at the end of n uniform pulses can be expressed as

$$y = \frac{F}{m\ell} \sum \frac{X_i X_{i1}}{p_i^2} \sum_{j=1}^{n} \left\{ \cos p_i (t - t_{jf}) - \cos p_i (t - t_{jo}) \right\} \tag{18}$$

where t_{jf} and t_{jo} refer to the final and initial time of j^{th} pulse. They can be expressed as

$$t_{jf} = (j-1)\tau + \delta \quad , \quad t_{jo} = (j-1)\tau \tag{19}$$

Using (19), the j series in (18) can be summed[2] to yield

$$y = \frac{F}{m\ell} \sum \frac{X_i X_{i1}}{p_i^2} \frac{\sin p_i n\tau/2}{\sin p_i \tau/2} \left[\cos p_i \left(t - \delta - \frac{n-1}{2}\tau\right) - \cos p_i \left(t - \frac{n-1}{2}\tau\right) \right] \tag{20}$$

Using (20) in (3) with (6) and (10) yields the vibrational kinetic and strain energy at the end of n p ses as,

$$T_f = \frac{F^2}{2m\ell} \sum \left[\frac{X_{i1} \sin p_i n\tau/2}{p_i \sin p_i \tau/2} \left\{ \sin p_i \left(t - \tfrac{n-1}{2}\tau\right) - \sin p_i \left(t - \delta - \tfrac{n-1}{2}\tau\right) \right\} \right]^2$$

$$U_f = \frac{F^2}{2m\ell} \sum \left[\frac{X_{i1} \sin p_i n\tau/2}{p_i \sin p_i \tau/2} \left\{ \cos p_i \left(t - \delta - \tfrac{n-1}{2}\tau\right) - \cos p_i \left(t - \tfrac{n-1}{2}\tau\right) \right\} \right]^2 \qquad (21)$$

The total flexure-mode energy is

$$E_f = \frac{F^2}{m\ell} \sum \left(\frac{X_{i1} \sin p_i n\tau/2}{p_i \sin p_i \tau/2} \right)^2 \left(1 - \cos p_i \delta\right) \qquad (22)$$

With the thruster at one end, this can be written in non-dimensional form as

$$E_f^* = \frac{E_f m\ell}{2F^2 \tau_1^2} \sum \left\{ \frac{\tau_i \sin n\pi\tau/\tau_i \ \sin \pi\delta/\tau_i}{\pi \tau_1 \sin \pi \tau/\tau_i} \right\}^2 \qquad (23)$$

Corresponding component of energy in the rigid-mode is given by

$$E_r^* = \frac{E_r m\ell}{2F^2 \tau_1^2} = \left(n\delta/\tau_1\right)^2 \qquad (24)$$

From these, $\mathcal{R}$ can also be written. Note the similarity with (16) - (18). The limiting values of the energy introduced into the flexure-mode can be easily found from (23) as follows.

Since

$$\left(\frac{\sin n\pi\tau/\tau_i}{\sin \pi\tau/\tau_i} \right)_{max} = n \quad \text{at} \quad \tau/\tau_i = \text{integer}$$

$$\left(\quad \right)_{min} = 0 \quad \text{at} \quad n\tau/\tau_i = \text{integer, and } \tau/\tau_i \neq \text{integer} \qquad (25)$$

we have from (23),

$$E^{*}_{fi\,max} = \left(\frac{n}{g_i}\sin \pi\delta/\tau_i\right)^2 \quad ; \quad E^{*}_{fi\,min} = 0 \tag{26}$$

Also,

$$\begin{aligned}(E^{*}_{fi\,max})_{max} &= n^2/g_i^2 && \text{at } \delta/\tau_i = (N+1/2)\pi \\ &= E^{*}_{r} && \text{as } n \to \infty \\ (E_{fi\,max})_{min} &= 0 && \text{at } \delta/\tau_i = N\end{aligned} \tag{27}$$

where N is an integer. With a proper choice of δ, n, τ it is possible to minimize the total E_f, as discussed in the next section.

Parametric Study

In the case of continuous constant thrust, the only parameter under control is the thrust level F. Both E_r and E_f increase as square of F. The energy-ratio R varies with time. Initially for each mode, R_i is near unity. It goes down to zero when t equals the period of that mode (Fig. 2a). Fig. 2b compares the energy-ratio in the first 10 modes at three instants from the start. Generally the first three modes dominate. At some instants the higher modes may contain more energy than the lower ones. The overall energy ratio is very high in the beginning. Gradually, it reduces with the increase in time (Fig. 2c). Fig. 2d shows that initially only flexure modes are excited. Then the rigid-mode gains energy and keeps building up. Energy in the flexure-mode, however, keeps fluctuating between 0.01 and 0.11, which occur near $N\pi$ and $(N+1/2)\pi$, respectively (Fig. 2d).

These observations are applicable for the case of step-thrust also. Here, for a given amount of control (E_r constant), it is possible to trade between F and t_1 to minimize the energy introduced into the flexure-mode using Fig. 2d. Generally a choice of $t_1 \approx n\pi\tau_1$ will give minimum flexure-energy. After choosing t_1, F can be fixed. From eq. (13) and (15) one may also note that for a given t_1 and F more modes are excited for a more flexible system.

Use of pulsed thrust introduces 4 parameters, F, δ, τ, and n, any three of which can be chosen at will for a given amount of control. Fig. 3 and 4 generated using (23) show the effect of varying these parameters. Fig. 3(a) shows the maximum energy in a mode (Eq. 26) as a function of the pulse-width. As expected, for a pulse-width equal to an intepral multiple of the modal period, the energy in that mode is minimum. However, this may enhance energy in the other modes. So the choice of the pulse-width which minimizes the total flexure energy will require some effort and trade-offs. For a given thrust level and rigid-mode energy, Fig. 3(b) shows that the maximum energy in a mode increases as we increase the number of pulses (and reduce the pulse-width). They asymptotically reach the value corresponding to the rigid-mode. The energy in higher modes fluctuates between zero and a small value up to some n before increasing monotonically. Hence it seems good to keep n low. It is possible that energy in some modes may be more than in the lower modes. It is interesting to note that for the case considered only the first mode is excited at $n = 3$.

After fixing δ and n (and thereby, F for a given E_r) to yield the minimum of peak flexure-energy, it is possible to then reduce it further by a proper choice of the pulse-period τ. Generally it is desirable to keep τ as small as possible to complete the operation soon. However, one should avoid resonance with any mode, otherwise, the energy in flexure-mode can be very high. This is shown on Fig. 4, which presents variation of E_f^* as a function of τ for two cases ($n=4$, $\delta=0.25\,\tau_1$ and $n=10$, $\delta=0.1\,\tau_1$) with a given level of thrust and rigid-mode energy. With the increase in the number of pulses higher modes are also excited and the resonance peaks get higher and narrower. The choice of τ, therefore, becomes more critical. At $(\tau/\tau_1)_{min}$ the pulsed thrust case reduces to a step function. Fig. 4(c) shows the total energy in flexure-mode as a function of τ/τ_1. For $n=4$, $\tau/\tau_1=0.5$ results in minimum total energy. For $n=10$ minima occur at τ/τ_1 = .25, .5, .62, .8, etc. At higher n a slight change from the 'optimum τ' can result in a significant increase in E_f.

Based on these observations, a possible approach to minimize the control-structure interactions can be as follows:

- from attitude-control considerations, select the mode of control. This will also fix the energy per operation in the rigid-mode.
- thrust level is decided by the available hardware
- for the given E_r and F generate plots like Fig. 3 and select δ and n to give lowest $E^*_{f\,max}$.
- for the given E_r, F and n use eq. (23) to obtain a plot like Fig. 4(c). Then choose τ/τ_1 to yield the lowest E_f*.

Note that one may interchange the last two steps.

Alternatively, one may employ the variation of parameter approach to find the set of parameters corresponding to minimum E_f* (eq. 23). For this purpose the series may be truncated judiciously.

After establishing E_f, the maximum amplitude of vibration in a mode can be found by equating E_{fi} to the component of strain-energy in that mode.

Concluding Remarks

A simple approach to study the problem of control-structure interaction in large space-structure is presented. The energy considerations applied to the case of a free-free beam, subjected to continuous, step and pulsed thrust, yield much insight into the system behavior. This also leads to find a way to minimize the interactions. The approach can be used for many systems. It should reduce the computational effort involved in such analysis significantly.

References

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2. Dwight, H. B., Tables of Integrals and Other Mathematical Data, McMillan, 1961, p.92.

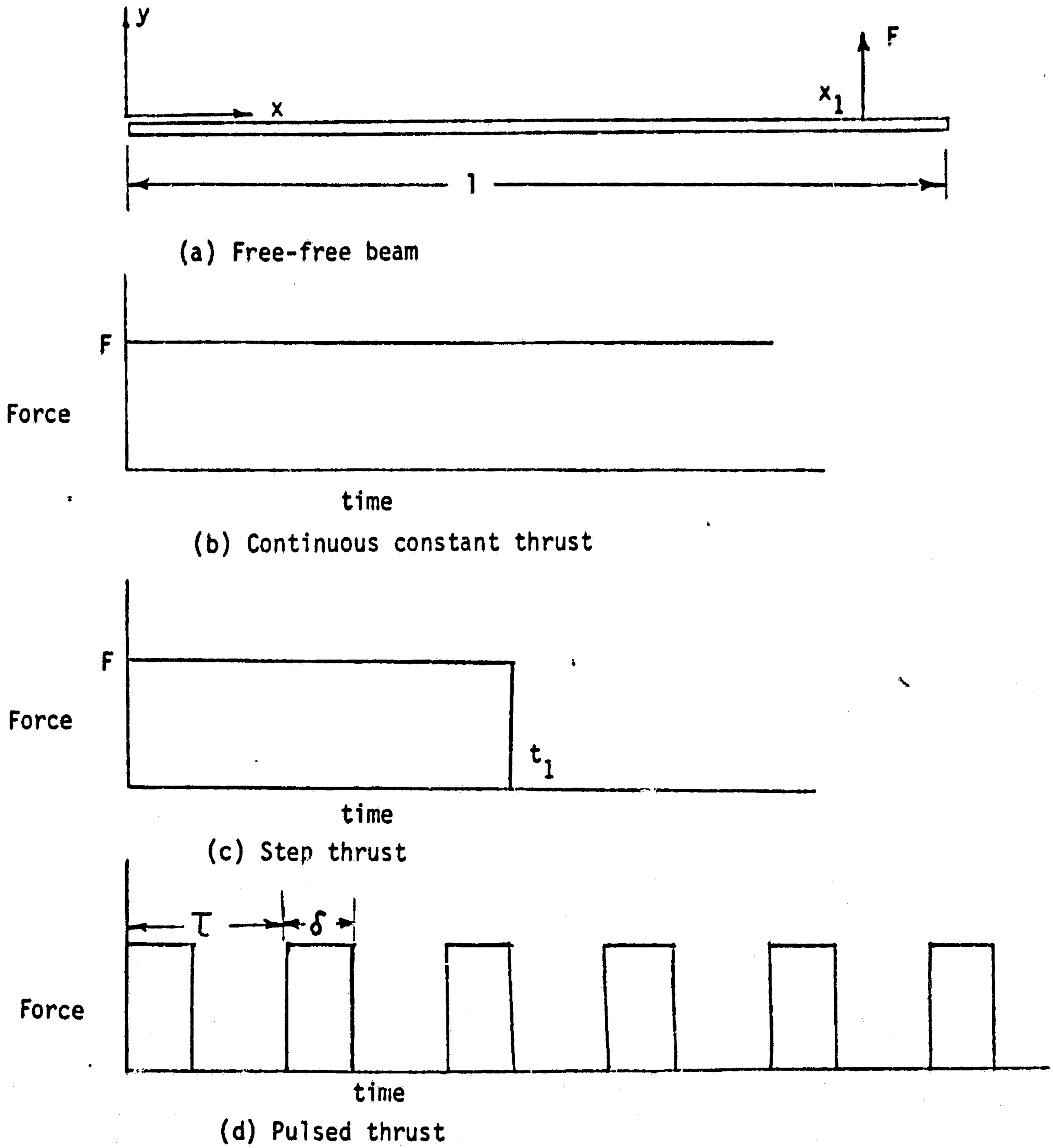

FIGURE 1: A FREE BEAM SUBJECTED TO CONCENTRATED LOAD

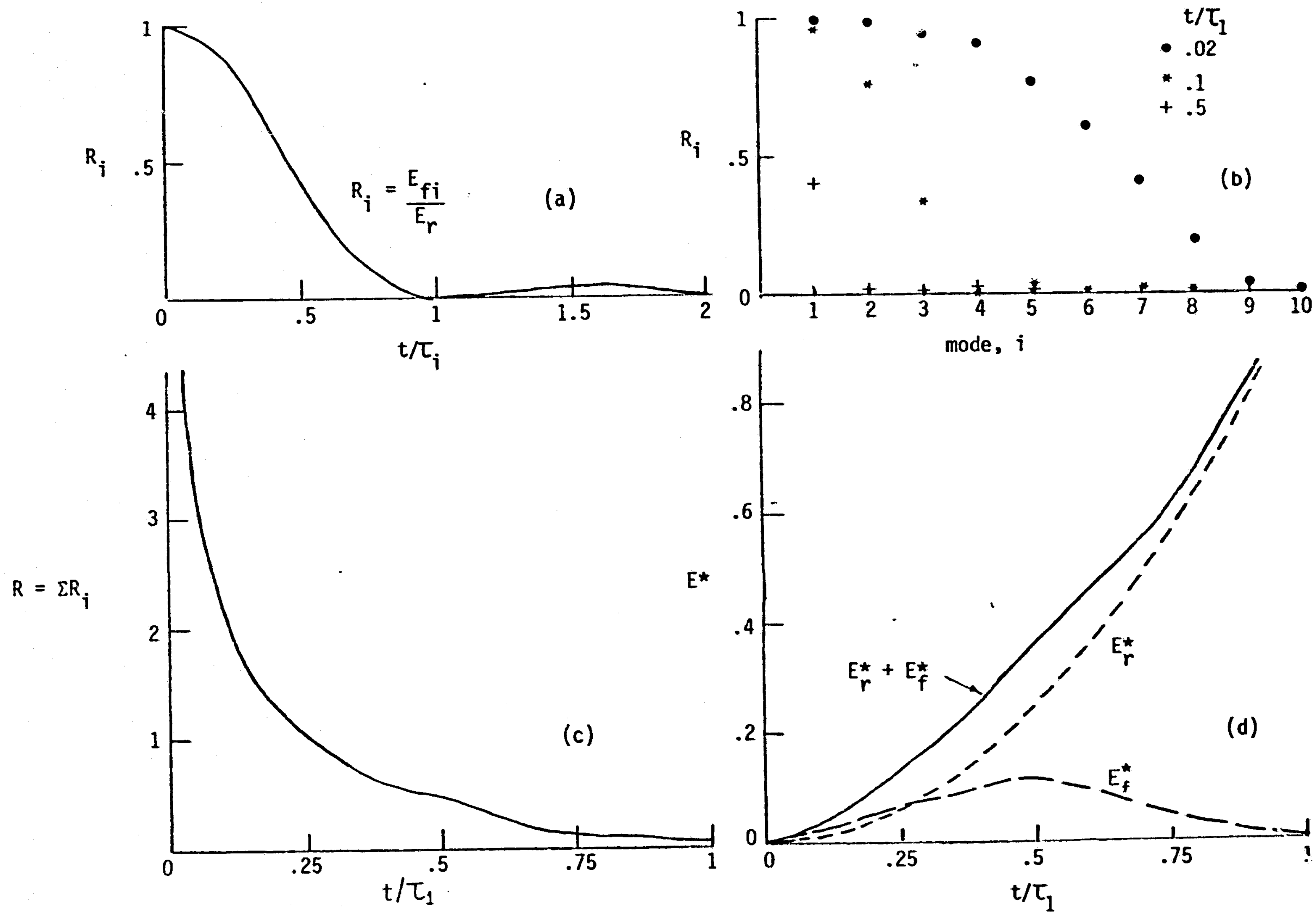

FIGURE 2: ENERGY IN A FREE-FREE BEAM SU..ECTED TO CANSTANT LOAD AT ONE END

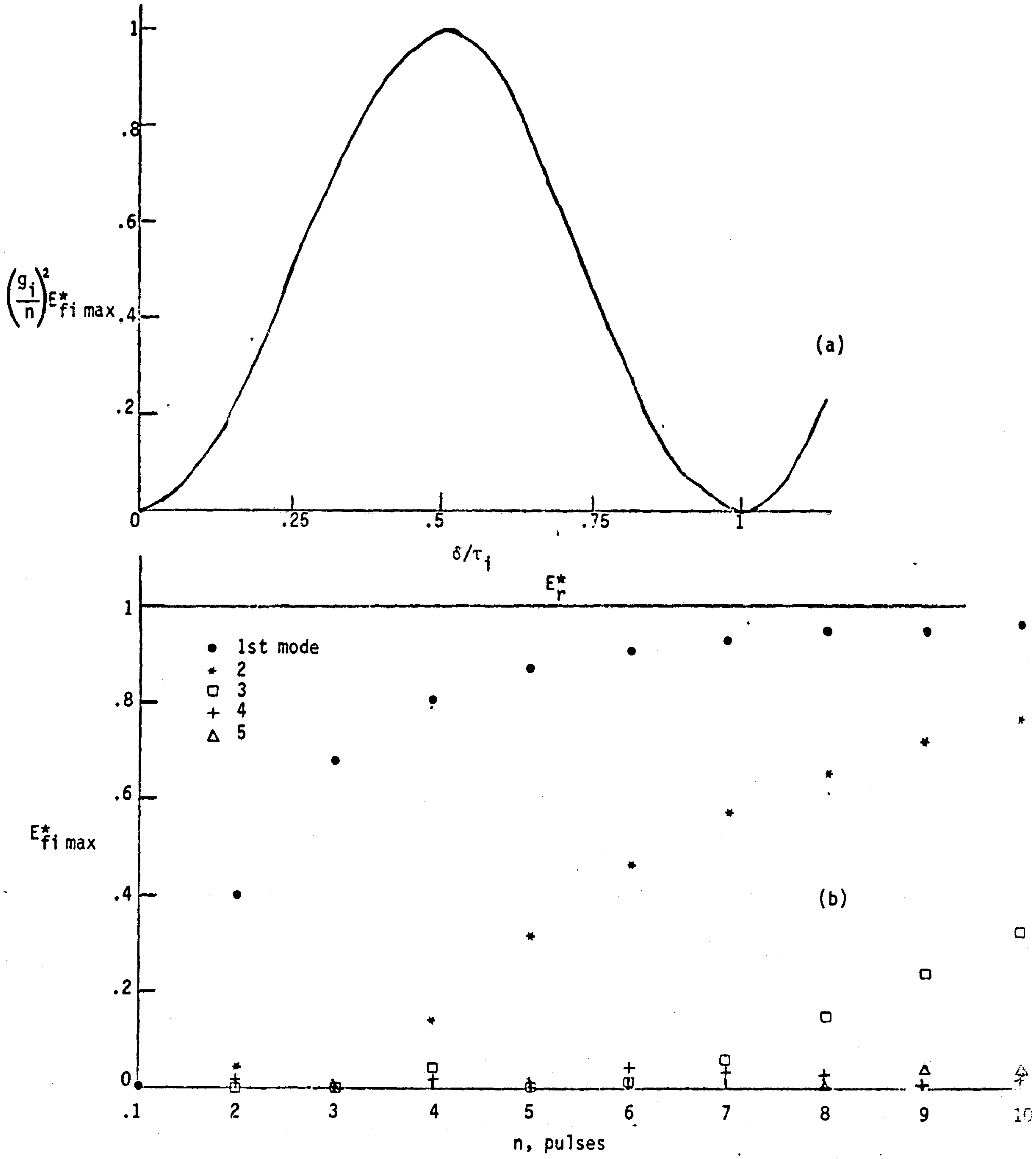

FIGURE 3: VARIATION OF PEAK ENERGY IN A MODE AS FUNCTION OF (a) PULSE-WIDTH

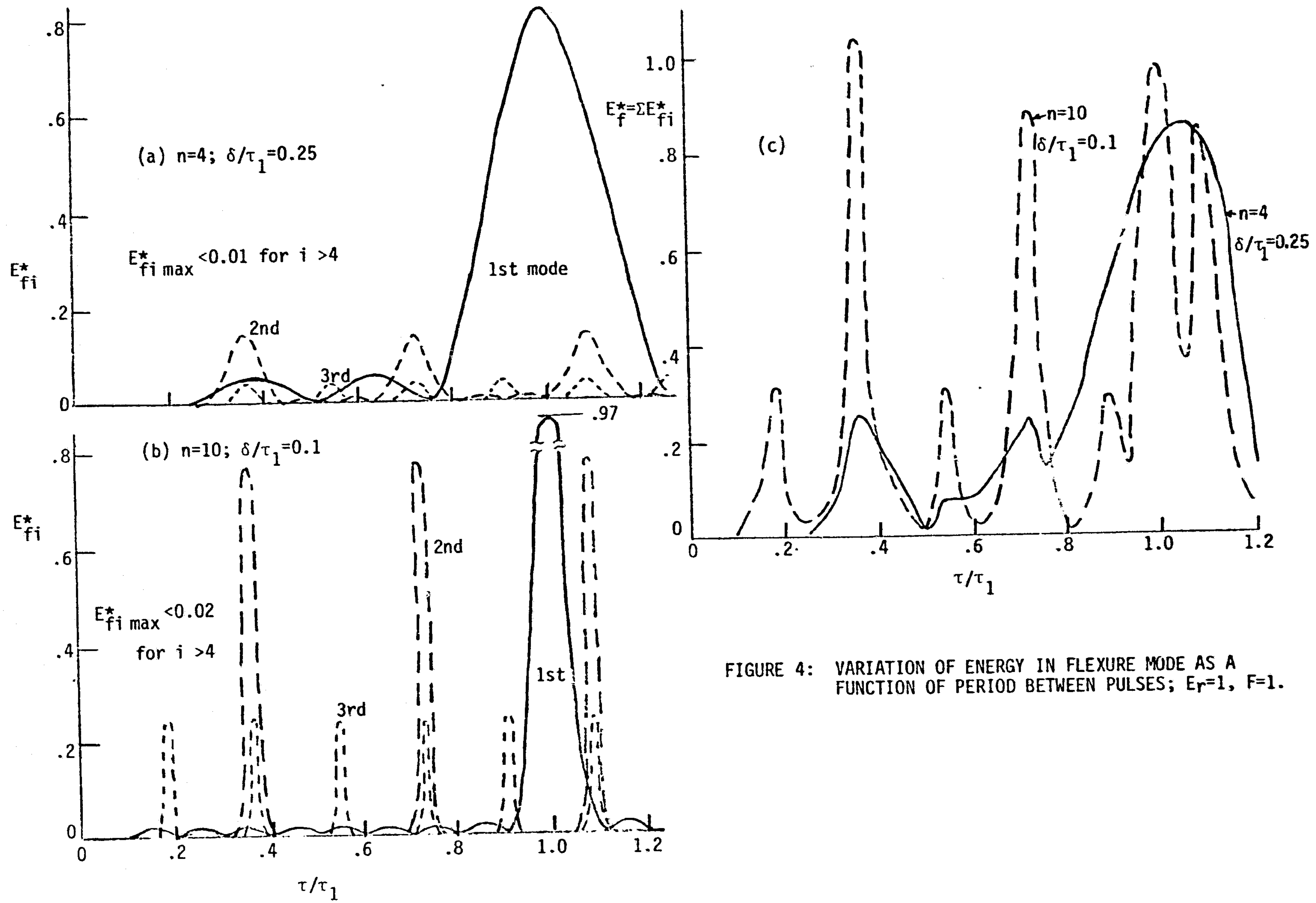

FIGURE 4: VARIATION OF ENERGY IN FLEXURE MODE AS A FUNCTION OF PERIOD BETWEEN PULSES; $E_r=1$, F=1.