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A STATISTICAL TEST PROCEDURE FOR DETECTING MULTIPLE OUTLIERS IN A DATA SET

Job Order 73-715

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For

EARTH OBSERVATIONS DIVISION

SPACE AND LIFE SCIENCES DIRECTORATE

National Aeronautics and Space Administration

LYNDON B. JOHNSON SPACE CENTER

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1. INTRODUCTION

Data contamination is a fairly common problem. When analyzing data, it is sometimes desirable to examine whether or not observations have come from the same distribution and to detect the potential outliers in the data. So far, any solution to this problem has been limited to the detection of a specified number of outliers. The number of outliers is generally unknown and cannot be specified in advance. The major drawbacks of testing for a fixed number of outliers are discussed in reference 1.

The sample size is an important influence on the number of observations likely to be outliers. It is reasonable to think of outliers as a minority, hence not more than 50 percent of a set of observations can be outliers. It has been suggested (ref. 1) that a certain percentage of the data should be considered for potential outliers; however, the percentage should be variable rather than fixed. For example, it is reasonable to consider 3 potential outliers in a data set of 10 observations, but it is unrealistic to expect 30 outliers out of a data set of 100 observations. In the latter case, the outlier detection problem becomes one of discrimination between two or more classes of data.

Several test statistics have been suggested for the significance test for detecting outliers (refs. 1 through 4). Among all the suggested tests, the extreme studentized deviate (ESD) test procedure is most favored, as it is shown to have more power against certain alternatives for the number of outliers and their distributions, and this procedure is computationally simple. The problem of obtaining the joint distribution of the ESD test statistics for the multiple outliers detection is still intractable; however, certain percentage points in the case of testing for the existence of specifically one or two outliers have been obtained for the test statistics using the Monte Carlo technique (ref. 1).

In the present Monte Carlo analysis, Rosner's table (ref. 1) is extended to include percentage points for the 5-percent significance level for testing as many as 19 outliers in a data set for which the primary distribution is normal. Also given are certain empirically developed relationships which can be used to easily obtain the percentage points of the ESD test statistics for various combinations of sample size and the hypothetical number of outliers.

2. THE TEST PROCEDURE

The test procedure based on ESD statistics is described in this section. Following Rosner (ref. 1), who has given a general formulation of many outlier test procedures, let $X_1, X_2, \cdots, X_n$ be a data set with k possible outliers. Consider the sequence of subsets $A_0, A_1, \cdots, A_k$, where $A_0 = \left\{X_1, X_2, \cdots, X_n\right\}$ and $A_{i+1} = A_i - \left\{X^{(i)}\right\}$, $i = 1, 2, \cdots, k-1$ where $X^{(i)}$ is defined by

$$\left| X^{(i)} - \overline{X}(A_i) \right| = \max_{X_j \varepsilon A_i} \left| X_j - \overline{X}(A_i) \right|$$

$$\overline{X}(A_i) = \frac{1}{n - i} \sum_{X_j \varepsilon A_i} X_j$$

Thus, $A_n \subset A_{n-1} \cdots \subset A_1 \subset A_0$ and A_i is obtained by deleting from A_{i-1} the data point farthest away from the mean of A_{i-1}. The test statistic $t(A_{i+1})$ applied to assess the significance of the most outlying observation in A_i is defined by

$$t(A_{i+1}) = \frac{\max_{X_j \varepsilon A_i} \left| X_j - \overline{X}(A_i) \right|}{S(A_i)} \qquad (1)$$

where

$$S^2(A_i) = \frac{1}{n - i - 1} \sum_{X_j \varepsilon A_i} \left[X_j - \overline{X}(A_i)\right]^2 \quad ; \quad i = 0, 1, 2, \cdots, k - 1$$

Considering H_0, the hypothesis of no outlier present in the data, the significance test procedures is to reject the hypothesis and declare that the data contains some outliers if

$$t(A_i) > \lambda_i, \text{ for some } i \ \varepsilon \{1, 2, \cdots, k\} \qquad (2)$$

where $\lambda_1, \lambda_2, \cdots, \lambda_k$ are determined by

$$\text{Prob}\left\{\bigcup_{i=1}^{k}\left[t(A_i) > \lambda_i\right] \middle| H_0\right\} = \alpha \tag{3}$$

with α as the desired significance level. Clearly, the determination of λ_i requires knowing the joint distribution of $t(A_1), t(A_2), \cdots, t(A_k)$ under H_0. Moreover, the solution of equation (3) forms a hyperplane and thus there is no unique set of λ_i satisfying equation (3). To achieve a unique solution, another requirement besides equation (3) is introduced by considering critical regions to have a fixed significance level β in each dimension. As such, the procedure is to find β and $\lambda_{ik}(\beta)$, $i = 1, 2, \cdots, k$ so that, given H_0,

$$\text{Prob}\left\{t(A_i) > \lambda_{ik}(\beta)\right\} = \beta \tag{4}$$

and

$$\text{Prob}\left\{\bigcup_{i=1}^{k}\left[t(A_i) > \lambda_{ik}(\beta)\right]\right\} = \alpha \tag{5}$$

If $t(A_i) < \lambda_{ik}(\beta)$ simultaneously for all i, no outliers in the data are declared. Otherwise, if $\bigcup_{i=1}^{k}\left[t(A_i) > \lambda_{ik}(\beta)\right]$ holds and

$$m = \max_{i=1,2,\cdots,k}\left\{i: \ t(A_i) > \lambda_{ik}(\beta)\right\}$$

then $X^{(0)}, X^{(1)}, \cdots, X^{(m-1)}$ are declared as outliers, i.e., the data points excluded to form A_m are to be declared as outliers.

It may be pointed out that though β in equation (4) is chosen independent of i, it can be considered different for different outliers. Unless there is a specific reason, it is appropriate to attach equal significance to different outliers and to have the same β for all i in equation (4).

3. DETERMINATION OF CRITICAL VALUES FOR $t(A_i)$

The joint distribution of $t(A_i)$, $i = 1, 2, \cdots, k$, is needed to obtain the critical values of $\lambda_{ik}(\beta)$ for equation (5). No exact derivation of the distribution is possible; instead, the critical values for a 5-percent significance level are evaluated by the Monte Carlo procedure. For each sample size n = 3(1)7, 10(5) 30(10) 100, 1000 samples of ordered normally distributed observations were generated. Considering detection of k outliers, where $k = 1, 2, \cdots, \left[\frac{n}{2}\right] \leq 19$, the empirical distribution of $t(A_i)$ was obtained with intervals of probability of size 0.001, enabling us to find β and $\lambda_{ik}(\beta)$ such that

$$\text{Prob}\left[t(A_i) > \lambda_{ik}(\beta)\right] = \beta \quad ; \quad \text{for } i = 1, 2, \cdots, k$$

and

$$\text{Prob}\left\{\bigcup_{i=1}^{k} \left[t(A_i) > \lambda_{ik}(\beta)\right]\right\} = 0.05$$

The critical values of $\lambda_{ik}(\beta)$ are presented in table I. For k = 2, these values are also given in reference 1. A comparison between the values given here and corresponding values given in table 7, reference 1, indicates a difference at the second decimal place. Since a larger number of samples are used in computations by Rosner (ref. 1), the critical values given in his paper for the case of k = 2 should be regarded as being more accurate.

In order to obtain the critical values for the 5-percent significance level for any combination (n, k), where $n \leq 100$ and $k \leq 19$, a model was empirically developed for $\lambda_{ik}(\beta)$ as a function of n and k. A least-square fit led to the following equation for approximating the critical values.

$$\lambda_{ikn} = A_{ik} + B_{ik} \log\left(\frac{n}{2} - k\right) \quad ; \quad i = 1, 2, \cdots, k \tag{6}$$

where values for A_{ik} and B_{ik} are given in tables II and III, respectively. Although equation (6) can be used to obtain approximate critical values, the relative difference of an approximated value from the actual could be as high as 5 percent. When the outliers are outstanding, it is safe to use equation (6) to approximate the critical values of the significance test. On the other hand, more accurate critical values may need to be computed and used to detect outliers that are not so outstanding. Considering that errors in the distributional assumption could invalidate the test even if its exact critical values are known, equation (6) can be regarded as a practical solution to the problem of determining the critical values of a significance test at the 5-percent level for detecting as many as 19 outliers in a data set.

TABLE I.— CRITICAL VALUES AT 5-PERCENT SIGNIFICANCE LEVEL

n	k	Critical values $\lambda_{jk}(\varepsilon)$																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
3	1	1.15																		
4	1	1.48																		
5	1	1.71																		
	2	1.73	1.49																	
6	1	1.89																		
	2	1.93	1.74																	
7	1	2.00																		
	2	2.08	1.93																	
	3	2.10	1.95	1.75																
10	1	2.27																		
	2	2.33	2.16																	
	3	2.37	2.20	2.13																
	4	2.40	2.22	2.17	2.08															
15	1	2.58																		
	2	2.69	2.40																	
	3	2.76	2.42	2.33																
	4	2.78	2.50	2.35	2.34															
	5	2.81	2.52	2.40	2.37	2.27														
	6	2.89	2.54	2.44	2.43	2.30	2.25													
	7	2.90	2.57	2.47	2.45	2.32	2.26	2.25												
20	1	2.67																		
	2	2.77	2.54																	
	3	2.86	2.58	2.50																
	4	2.91	2.61	2.51	2.39															
	5	2.92	2.62	2.53	2.40	2.38														
	6	2.93	2.63	2.54	2.43	2.39	2.36													
	7	3.01	2.64	2.54	2.46	2.40	2.37	2.37												
	8	3.03	2.65	2.54	2.48	2.42	2.38	2.38	2.44											
	9	3.04	2.65	2.55	2.49	2.45	2.38	2.38	2.44	2.36										

TABLE I.— Continued.

n	k	Critical values $\lambda_{ik}(\beta)$																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
25	1	2.83																		
	2	2.98	2.58																	
	3	3.10	2.68	2.50																
	4	3.14	2.71	2.52	2.43															
	5	3.16	2.73	2.53	2.45	2.49														
	6	3.18	2.76	2.54	2.45	2.51	2.37													
	7	3.19	2.81	2.57	2.46	2.53	2.39	2.34												
	8	3.22	2.82	2.60	2.47	2.55	2.42	2.34	2.35											
	9	3.33	2.82	2.62	2.50	2.56	2.43	2.37	2.35	2.37										
	10	3.34	2.84	2.66	2.52	2.57	2.43	2.38	2.38	2.38	2.39									
	11	3.35	2.85	2.68	2.54	2.59	2.43	2.39	2.41	2.40	2.39	2.55								
	12	3.37	2.85	2.69	2.57	2.59	2.45	2.40	2.41	2.40	2.39	2.57	2.35							
30	1	2.88																		
	2	3.01	2.65																	
	3	3.08	2.81	2.51																
	4	3.13	2.85	2.54	2.51															
	5	3.18	2.89	2.55	2.54	2.43														
	6	3.21	2.94	2.56	2.58	2.44	2.39													
	7	3.24	2.99	2.58	2.60	2.45	2.41	2.36												
	8	3.24	3.00	2.58	2.62	2.46	2.42	2.39	2.38											
	9	3.27	3.00	2.61	2.63	2.48	2.42	2.43	2.38	2.41										
	10	3.30	3.01	2.63	2.65	2.48	2.42	2.43	2.41	2.43	2.39									
	11	3.32	3.01	2.63	2.67	2.49	2.43	2.43	2.41	2.43	2.39	2.45								
	12	3.35	3.01	2.63	2.69	2.50	2.43	2.43	2.41	2.43	2.39	2.46	2.37							
	13	3.37	3.02	2.64	2.71	2.50	2.43	2.44	2.41	2.44	2.40	2.48	2.37	2.38						
	14	3.38	3.02	2.65	2.72	2.50	2.43	2.45	2.41	2.45	2.41	2.50	2.38	2.38	2.42					

TABLE I.— Continued.

n	k	Critical values $\lambda_{ik}(\beta)$																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
40	1	3.06																		
	2	3.15	2.76																	
	3	3.20	2.82	2.60																
	4	3.26	2.88	2.62	2.51															
	5	3.39	2.88	2.66	2.53	2.47														
	6	3.43	2.89	2.69	2.55	2.49	2.36													
	7	3.44	2.91	2.72	2.56	2.49	2.38	2.41												
	8	3.45	2.94	2.73	2.58	2.50	2.39	2.42	2.43											
	9	3.46	2.95	2.73	2.59	2.51	2.40	2.42	2.43	2.34										
	10	3.46	2.95	2.75	2.59	2.52	2.41	2.44	2.44	2.35	2.30									
	11	3.47	2.95	2.77	2.59	2.52	2.42	2.47	2.44	2.36	2.31	2.26								
	12	3.48	2.96	2.78	2.59	2.53	2.42	2.47	2.46	2.36	2.32	2.27	2.30							
	13	3.48	2.98	2.79	2.60	2.54	2.43	2.47	2.47	2.36	2.33	2.28	2.31	2.27						
	14	3.49	2.98	2.80	2.60	2.56	2.47	2.48	2.47	2.36	2.34	2.28	2.32	2.27	2.37					
	15	3.49	2.99	2.81	2.60	2.57	2.48	2.48	2.47	2.37	2.34	2.28	2.33	2.27	2.38	2.31				
	16	3.50	3.00	2.81	2.60	2.59	2.51	2.49	2.48	2.37	2.35	2.28	2.34	2.28	2.39	2.31	2.36			
	17	3.50	3.01	2.82	2.61	2.60	2.53	2.49	2.48	2.37	2.35	2.29	2.36	2.28	2.41	2.32	2.36	2.34		
	18	3.50	3.01	2.82	2.61	2.60	2.54	2.49	2.48	2.37	2.35	2.30	2.37	2.29	2.42	2.32	2.36	2.34	2.28	
	19	3.51	3.02	2.83	2.62	2.61	2.56	2.49	2.48	2.38	2.35	2.30	2.38	2.29	2.43	2.33	2.37	2.35	2.29	2.35
50	1	3.11																		
	2	3.29	2.87																	
	3	3.35	2.91	2.69																
	4	3.39	2.93	2.72	2.59															
	5	3.47	2.98	2.74	2.60	2.51														
	6	3.49	3.00	2.75	2.61	2.53	2.44													
	7	3.51	3.01	2.76	2.61	2.54	2.45	2.46												
	8	3.54	3.02	2.78	2.62	2.55	2.46	2.46	2.41											
	9	3.54	3.02	2.79	2.65	2.56	2.47	2.46	2.42	2.45										
	10	3.55	3.05	2.81	2.65	2.56	2.47	2.46	2.42	2.46	2.35									
	11	3.56	3.06	2.85	2.66	2.57	2.48	2.47	2.43	2.47	2.35	2.33								
	12	3.58	3.07	2.88	2.67	2.58	2.50	2.47	2.44	2.47	2.35	2.34	2.32							
	13	3.59	3.08	2.92	2.68	2.60	2.51	2.47	2.44	2.47	2.35	2.35	2.32	2.28						
	14	3.61	3.13	2.92	2.70	2.61	2.51	2.47	2.45	2.49	2.39	2.36	2.32	2.29	2.35					
	15	3.64	3.17	2.92	2.73	2.61	2.52	2.48	2.46	2.50	2.42	2.38	2.33	2.30	2.36	2.33				
	16	3.66	3.18	2.93	2.74	2.61	2.52	2.49	2.48	2.51	2.43	2.38	2.34	2.31	2.36	2.35	2.29			
	17	3.67	3.19	2.93	2.75	2.61	2.52	2.50	2.49	2.51	2.43	2.38	2.34	2.31	2.36	2.35	2.29	2.31		
	18	3.68	3.20	2.93	2.75	2.62	2.53	2.50	2.50	2.52	2.43	2.39	2.34	2.32	2.36	2.36	2.30	2.31	2.34	
	19	3.68	3.21	2.94	2.75	2.62	2.55	2.50	2.51	2.52	2.44	2.40	2.34	2.34	2.36	2.36	2.31	2.31	2.34	2.28

TABLE I.— Continued.

n	k	Critical values $\lambda_{ik}(\beta)$																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
60	1	3.22																		
	2	3.37	2.94																	
	3	3.41	3.00	2.75																
	4	3.58	3.02	2.77	2.60															
	5	3.61	3.03	2.82	2.60	2.56														
	6	3.62	3.06	2.83	2.62	2.56	2.47													
	7	3.63	3.07	2.84	2.64	2.57	2.47	2.42												
	8	3.65	3.08	2.85	2.66	2.58	2.48	2.44	2.37											
	9	3.66	3.09	2.86	2.66	2.59	2.48	2.45	2.38	2.37										
	10	3.69	3.10	2.87	2.67	2.60	2.50	2.45	2.40	2.38	2.35									
	11	3.69	3.10	2.88	2.67	2.61	2.51	2.46	2.40	2.38	2.36	2.33								
	12	3.71	3.10	2.89	2.67	2.62	2.51	2.47	2.40	2.39	2.37	2.34	2.29							
	13	3.74	3.10	2.90	2.67	2.62	2.51	2.47	2.41	2.39	2.37	2.34	2.30	2.33						
	14	3.77	3.10	2.90	2.68	2.62	2.52	2.48	2.41	2.39	2.37	2.35	2.30	2.34	2.34					
	15	3.78	3.10	2.90	2.68	2.62	2.52	2.49	2.42	2.39	2.37	2.35	2.31	2.34	2.34	2.24				
	16	3.80	3.10	2.93	2.68	2.63	2.56	2.50	2.42	2.39	2.38	2.35	2.32	2.35	2.34	2.25	2.24			
	17	3.82	3.10	2.95	2.68	2.64	2.60	2.51	2.42	2.39	2.38	2.35	2.33	2.36	2.35	2.26	2.24	2.28		
	18	3.84	3.10	2.96	2.68	2.64	2.62	2.51	2.42	2.39	2.38	2.35	2.33	2.37	2.35	2.26	2.25	2.28	2.26	
	19	3.84	3.11	2.96	2.68	2.65	2.63	2.52	2.42	2.40	2.39	2.36	2.33	2.37	2.35	2.27	2.25	2.28	2.26	2.26
70	1	3.22																		
	2	3.40	2.92																	
	3	3.47	2.98	2.75																
	4	3.51	3.00	2.76	2.67															
	5	3.53	3.02	2.79	2.68	2.58														
	6	3.53	3.03	2.83	2.70	2.59	2.52													
	7	3.57	3.04	2.83	2.71	2.60	2.53	2.45												
	8	3.58	3.05	2.84	2.71	2.60	2.54	2.46	2.44											
	9	3.60	3.06	2.85	2.71	2.60	2.55	2.46	2.45	2.45										
	10	3.66	3.07	3.86	2.72	2.60	2.55	2.46	2.45	2.45	2.38									
	11	3.71	3.07	2.86	2.72	2.60	2.55	2.47	2.45	2.45	2.38	2.39								
	12	3.73	3.07	2.87	2.72	2.61	2.56	2.47	2.45	2.46	2.39	2.39	2.33							
	13	3.73	3.08	2.87	2.73	2.61	2.56	2.47	2.45	2.46	2.40	2.39	2.34	2.30						
	14	3.74	3.09	2.88	2.73	2.61	2.57	2.48	2.45	2.47	2.41	2.39	2.34	2.31	2.27					
	15	3.76	3.10	2.88	2.73	2.62	2.58	2.48	2.46	2.47	2.43	2.40	2.34	2.31	2.29	2.29				
	16	3.78	3.11	2.88	2.74	2.62	2.59	2.48	2.48	2.47	2.44	2.40	2.34	2.31	2.30	2.29	2.27			
	17	3.79	3.11	2.88	2.74	2.63	2.59	2.48	2.48	2.47	2.45	2.40	2.34	2.31	2.30	2.29	2.27	2.27		
	18	3.80	3.12	2.89	2.75	2.63	2.59	2.50	2.50	2.47	2.46	2.40	2.35	2.33	2.31	2.30	2.27	2.28	2.31	
	19	3.81	3.12	2.89	2.75	2.64	2.59	2.51	2.51	2.47	2.47	2.40	2.35	2.33	2.31	2.30	2.28	2.28	2.31	2.26

TABLE I.— Continued.

n	k	Critical values $\lambda_{ik}(\varepsilon)$																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
80	1	3.27																		
	2	3.41	2.92																	
	3	3.53	2.97	2.80																
	4	3.55	2.98	2.82	2.68															
	5	3.59	3.03	2.85	2.70	2.58														
	6	3.62	3.07	2.88	2.71	2.58	2.54													
	7	3.68	3.09	2.89	2.71	2.59	2.57	2.48												
	8	3.70	3.10	2.90	2.72	2.59	2.57	2.48	2.41											
	9	3.73	3.11	2.94	2.73	2.59	2.57	2.49	2.41	2.40										
	10	3.78	3.13	2.95	2.77	2.63	2.57	2.49	2.42	2.41	2.40									
	11	3.80	3.13	2.96	2.78	2.63	2.58	2.50	2.42	2.41	2.41	2.37								
	12	3.81	3.14	2.96	2.78	2.65	2.59	2.50	2.42	2.41	2.41	2.37	2.36							
	13	3.82	3.14	2.96	2.78	2.64	2.60	2.50	2.42	2.42	2.42	2.38	2.36	2.31						
	14	3.82	3.16	2.96	2.79	2.65	2.60	2.51	2.42	2.42	2.43	2.39	2.37	2.32	2.32					
	15	3.82	3.17	2.96	2.79	2.65	2.60	2.51	2.43	2.42	2.44	2.40	2.38	2.32	2.32	2.31				
	16	3.82	3.18	2.97	2.79	2.65	2.61	2.52	2.43	2.43	2.44	2.40	2.38	2.32	2.32	2.31	2.28			
	17	3.82	3.18	2.97	2.79	2.65	2.61	2.52	2.43	2.43	2.44	2.40	2.38	2.32	2.32	2.31	2.28	2.28		
	18	3.84	3.18	2.97	2.79	2.66	2.62	2.52	2.44	2.43	2.45	2.41	2.39	2.33	2.32	2.32	2.28	2.28	2.30	
	19	3.87	3.18	2.98	2.79	2.67	2.64	2.53	2.45	2.44	2.45	2.42	2.40	2.33	2.33	2.32	2.29	2.28	2.31	2.26
90	1	3.29																		
	2	3.42	2.97																	
	3	3.51	3.03	2.85																
	4	3.55	3.04	2.87	2.77															
	5	3.59	3.06	2.89	2.80	2.62														
	6	3.62	3.09	2.89	2.81	2.64	2.54													
	7	3.67	3.10	2.91	2.82	2.66	2.56	2.54												
	8	3.70	3.13	2.92	2.85	2.66	2.58	2.55	2.47											
	9	3.73	3.14	2.92	2.86	2.67	2.59	2.56	2.48	2.45										
	10	3.75	3.16	2.92	2.86	2.67	2.59	2.57	2.49	2.45	2.41									
	11	3.77	3.17	2.92	2.86	2.67	2.60	2.57	2.49	2.46	2.42	2.37								
	12	3.78	3.18	2.92	2.87	2.68	2.60	2.58	2.49	2.46	2.44	2.38	2.39							
	13	3.78	3.18	2.92	2.87	2.68	2.60	2.58	2.49	2.45	2.44	2.38	2.39	2.36						
	14	3.79	3.19	2.93	2.87	2.68	2.61	2.58	2.50	2.46	2.45	2.39	2.39	2.37	2.35					
	15	3.79	3.19	2.93	2.87	2.68	2.61	2.59	2.50	2.46	2.45	2.39	2.39	2.37	2.35	2.37				
	16	3.79	3.19	2.93	2.87	2.68	2.61	2.59	2.50	2.46	2.45	2.39	2.39	2.37	2.35	2.37	2.37			
	17	3.80	3.21	2.93	2.88	2.68	2.62	2.59	2.50	2.46	2.45	2.39	2.39	2.37	2.36	2.38	2.37	2.33		
	18	3.80	3.22	2.93	2.88	2.68	2.62	2.60	2.51	2.46	2.45	2.40	2.39	2.37	2.37	2.38	2.37	2.33	2.27	
	19	3.81	3.22	2.93	2.88	2.68	2.62	2.60	2.51	2.46	2.45	2.40	2.39	2.37	2.37	2.38	2.37	2.33	2.27	2.31

TABLE I.— Concluded.

n	k	Critical values $\lambda_{ik}(\beta)$																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
100	1	3.38																		
	2	3.50	3.08																	
	3	3.59	3.13	2.86																
	4	3.66	3.16	2.89	2.75															
	5	3.69	3.18	2.90	2.76	2.67														
	6	3.77	3.27	2.94	2.78	2.69	2.58													
	7	3.78	3.30	2.96	2.78	2.70	2.59	2.57												
	8	3.79	3.31	2.96	2.79	2.70	2.59	2.58	2.51											
	9	3.80	3.34	2.96	2.80	2.70	2.59	2.58	2.52	2.47										
	10	3.81	3.36	2.97	2.80	2.70	2.59	2.58	2.52	2.47	2.44									
	11	3.87	3.36	2.98	2.82	2.71	2.60	2.58	2.53	2.47	2.45	2.44								
	12	3.82	3.36	2.99	2.82	2.71	2.60	2.58	2.54	2.48	2.46	2.45	2.37							
	13	3.82	3.37	2.99	2.82	2.72	2.60	2.59	2.54	2.48	2.46	2.45	2.37	2.38						
	14	3.82	3.37	3.00	2.83	2.72	2.61	2.60	2.54	2.50	2.46	2.45	2.37	2.38	2.38					
	15	3.82	3.37	3.01	2.83	2.73	2.62	2.60	2.55	2.51	2.46	2.45	2.38	2.39	2.39	2.32				
	16	3.82	3.38	3.01	2.83	2.73	2.62	2.60	2.56	2.52	2.46	2.45	2.39	2.39	2.39	2.32	2.30			
	17	3.82	3.38	3.01	2.83	2.73	2.63	2.61	2.57	2.52	2.46	2.46	2.39	2.39	2.39	2.32	2.30	2.23		
	18	3.82	3.38	3.01	2.83	2.73	2.63	2.61	2.57	2.53	2.46	2.46	2.40	2.39	2.39	2.32	2.30	2.23	2.24	
	19	3.83	3.40	3.02	2.84	2.73	2.63	2.61	2.58	2.53	2.46	2.46	2.40	2.39	2.39	2.32	2.30	2.23	2.24	2.25

TABLE II.— INTERCEPT VALUES (A_{ik})

	k = 1	k = 2	k = 3	k = 4	k = 5	k = 6	k = 7	k = 8	k = 9	k = 10	k = 11	k = 12	k = 13	k = 14	k = 15	k = 16	k = 17	k = 18	k = 19
i = 1	1.55	1.96	2.25	2.37	2.49	2.68	2.91	2.84	3.02	3.07	3.18	3.34	3.21	3.34	3.25	3.33	3.37	3.45	3.52
i = 2		1.78	2.08	2.22	2.32	2.42	2.60	2.56	2.65	2.71	2.78	2.88	2.88	2.95	2.78	2.83	2.89	2.95	3.01
i = 3			1.95	2.14	2.20	2.30	2.45	2.39	2.48	2.48	2.57	2.66	2.58	2.66	2.70	2.74	2.77	2.80	2.84
i = 4				2.08	2.18	2.30	2.41	2.35	2.44	2.42	2.49	2.58	2.58	2.64	2.40	2.45	2.50	2.55	2.60
i = 5					2.18	2.25	2.34	2.36	2.43	2.44	2.49	2.54	2.43	2.48	2.45	2.50	2.53	2.56	2.59
i = 6						2.20	2.28	2.29	2.34	2.32	2.36	2.42	2.35	2.40	2.35	2.42	2.47	2.50	2.54
i = 7							2.27	2.28	2.34	2.32	2.36	2.40	2.39	2.42	2.36	2.39	2.41	2.44	2.46
i = 8								2.36	2.38	2.35	2.38	2.41	2.40	2.41	2.41	2.42	2.43	2.45	2.46
i = 9									2.34	2.36	2.38	2.40	2.40	2.42	2.32	2.34	2.36	2.37	2.39
i = 10										2.34	2.35	2.37	2.33	2.37	2.25	2.28	2.31	2.33	2.36
i = 11											2.49	2.40	2.40	2.42	2.18	2.20	2.23	2.27	2.30
i = 12												2.34	2.33	2.35	2.25	2.28	2.31	2.34	2.36
i = 13													2.32	2.33	2.19	2.22	2.25	2.27	2.30
i = 14														2.41	2.38	2.40	2.41	2.43	2.41
i = 15															2.27	2.29	2.31	2.32	2.33
i = 16																2.36	2.34	2.34	2.34
i = 17																	2.36	2.36	2.35
i = 18																		2.31	2.31
i = 19																			2.38

TABLE III.— SLOPE VALUES (B_{ik})

	k = 1	k = 2	k = 3	k = 4	k = 5	k = 6	k = 7	k = 8	k = 9	k = 10	k = 11	k = 12	k = 13	k = 14	k = 15	k = 16	k = 17	k = 18	k = 19
i = 1	0.49	0.41	0.35	0.33	0.32	0.27	0.21	0.24	0.20	0.19	0.17	0.12	0.17	0.13	0.17	0.15	0.13	0.12	0.10
i = 2		0.34	0.26	0.23	0.21	0.20	0.15	0.17	0.15	0.13	0.11	0.09	0.09	0.07	0.14	0.12	0.11	0.09	0.07
i = 3			0.24	0.19	0.18	0.16	0.12	0.15	0.12	0.13	0.10	0.08	0.11	0.09	0.08	0.07	0.06	0.05	0.04
i = 4				0.17	0.15	0.12	0.08	0.11	0.09	0.10	0.08	0.05	0.06	0.04	0.12	0.11	0.10	0.08	0.07
i = 5					0.12	0.10	0.08	0.08	0.06	0.06	0.04	0.03	0.07	0.05	0.07	0.05	0.04	0.04	0.03
i = 6						0.09	0.07	0.07	0.06	0.07	0.06	0.04	0.07	0.05	0.08	0.06	0.05	0.04	0.03
i = 7							0.06	0.06	0.05	0.06	0.05	0.04	0.04	0.03	0.06	0.05	0.04	0.04	0.03
i = 8								0.02	0.02	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01
i = 9									0.03	0.02	0.02	0.01	0.01	0.01	0.05	0.04	0.04	0.03	0.03
i = 10										0.01	0.01	0.01	0.02	0.02	0.06	0.05	0.04	0.04	0.03
i = 11											-0.04	-0.04	-0.01	-0.01	0.07	0.07	0.06	0.05	0.04
i = 12												0.00	0.01	0.00	0.04	0.03	0.02	0.01	0.01
i = 13													0.00	-0.00	0.05	0.04	0.03	0.03	0.02
i = 14														-0.02	-0.01	-0.02	-0.02	-0.02	-0.02
i = 15															0.02	0.01	0.00	0.00	-0.00
i = 16																-0.01	-0.02	-0.02	-0.01
i = 17																	-0.03	-0.03	-0.02
i = 18																		-0.01	-0.01
i = 19																			-0.02

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