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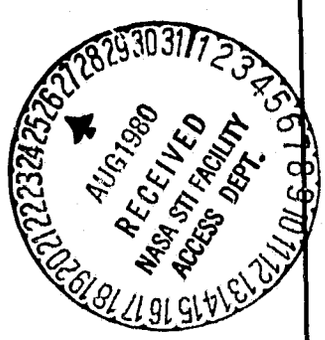

NASA
National Aeronautics and Space Administration

Lyndon B. Johnson Space Center
Houston, Texas

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SHUTTLE PROGRAM

A COMPUTER PROGRAM FOR DETERMINING TRUNCATION ERROR COEFFICIENTS FOR RUNGE-KUTTA METHODS

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TABLES

1.0 INTRODUCTION

The solution of the initial value problem for ordinary differential equations (ODE's)

$$\frac{dy}{dt} = f(t,y), \quad y(t_o) = y_o \tag{1}$$

may be treated by several different numerical methods. Runge-Kutta (RK) algorithms are a type of method well suited to solving equation (1) for many classes of functions, f, because of their simplicity and their accuracy. The RK algorithm is derived using a direct comparison with a truncated Taylor series, giving the accuracy of the Taylor series without the difficulty of determining complicated partial derivatives. The comparison between the Taylor series expansion of the solution vector and the solution determined by the RK algorithm results in a number of expressions referred to as truncation error coefficients, $T_{i,j}$. Associated with each term of order i in the Taylor series (or with each power of h, the integration stepsize) are λ_i truncation error coefficients. For an RK algorithm to be of order p, the $T_{i,j}$ coefficients must be identically zero for $i = 1,\ldots,p$; $j = 1,\ldots,\lambda_i$. These vanishing truncation error coefficients are referred to as equations of condition. The nonvanishing error coefficients, however, are of equally great importance since they indicate how closely the RK solution approximates a Taylor series solution of higher order. The equations of condition determine the validity of an RK algorithm; the nonvanishing error coefficients explain the differences between particular RK algorithms of the same order. While a user may apply an RK algorithm, never considering the truncation error coefficients, awareness of the effect of these terms is important both in the selection of a specific algorithm and in the analysis of difficulties encountered during the solution of a particular ODE.

D. G. Bettis,[a] has developed an algorithm for generating the truncation error coefficients for RK methods designed to treat systems of both first- and second-order ODE's directly. The recursive nature of this algorithm lends itself readily to computer programing, generating high order error coefficients with little added difficulty. Such an algorithm, implemented in a numerical code, is an essential tool for anyone developing coefficients for RK algorithms and is of interest to the user of RK methods in analyzing the effectiveness of specific RK algorithms. A Fortran subroutine, RKEQN, written to accompany reference 1, generated the truncation error coefficients through order 10 but required a great amount of storage location, particularly when a double precision version of the program was needed. The basic structure of this original program has been reformulated to reduce storage requirements significantly and to accommodate variable dimensioning. This new Fortran program, SUBROUTINE RKEQ, determines truncation error coefficients for RK algorithms in the sequence presented in reference 1 for orders 1 through 10 and extends the order of coefficients

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[a]From a private communication with D. G. Bettis, 1978.

through 12 with the 11th- and 12th-order terms determined following the patterns used to establish the lower order coefficients. Both subroutines (RKEQN and RKEQ) are also written to treat RK m-fold methods (refs. 2 and 3) which utilize m known derivatives of f to increase the order of the algorithm. Setting $m = 0$ gives the classical RK algorithm.

2.0 THE GENERATION OF TRUNCATION ERROR COEFFICIENTS

The solution of equation (1) at $t_1 = t_o + h$, using the RK algorithm, is written

$$y_1 = y_o + \sum_{k=o}^{\rho} C_k f_k \tag{2}$$

where

$$f_o = f(t_o, y_o)$$

$$f_k = f(t_o + \alpha_k h,\ y_o + h \sum_{\lambda=o}^{k-1} \beta_{k,\lambda} f_\lambda)$$

where $\rho + 1$, the number of evaluations of f computed, is referred to as the number of stages. The truncation error coefficients, $T_{i,j}$, determined by comparing the Taylor series expansion of equation (2) with the Taylor series expansion of the solution about t_o, are nonlinear combinations of the C, α, and β coefficients. For the classical RK algorithm, the jth error coefficient of order i assumes the form

$$T_{i,j} = \left\{F_{i,j} - 1/(pA_{i,j})\right\} /B_{i,j} \tag{3}$$

$j = 1,\ldots,\lambda_i$ where $p = i$. For the m-fold algorithm, $p = m + i$ and corresponds to the order of the term. (For $m = 0$, the m-fold algorithm is identical to the classical RK formula.) The $A_{i,j}$ and $B_{i,j}$ terms are constants (or functions of m for m-fold methods) and may be determined by recursive relations. (One should note that while references to m-fold RK algorithms may appear to complicate matters, the inclusion of these methods in RKEQ (or RKEQN) involves the insertion of only a few additional lines of coding. Once these additions are made, the classical RK error coefficients and the m-fold error coefficients are determined identically.) The complicated expression to generate in equation (3) is the $F_{i,j}$ term, which is a combination of the C, α, and β coefficients

$$F_{i,j} = \sum_{k=k_o}^{\rho} C_k S_{i,j,k} \tag{4}$$

with $S_{i,j,k}$ being a combination of α and β coefficients and where k_o depends upon the number of summations embedded in $S_{i,j,k}$.

The algorithm developed by Bettis and used for generating these $S_{i,j,k}$, $A_{i,j}$, and $B_{i,j}$ terms is documented in reference 1, where the $S_{i,j,k}$ terms are written in an abbreviated notation, e.g., $S_{8,13} = \alpha^2\beta\alpha\beta\beta\alpha$, with the subscripting and embedded summations being suppressed. The rules for writing the entire $S_{i,j,k}$ terms are also described thoroughly in reference 1. For the sake of interpreting the program, however, a few features need to be known about generating the terms in abbreviated notation. Denoting the number of truncation error coefficients of order μ, by λ_μ, and suppressing the k subscript, the first $2\lambda_{\mu-1}$ $S_{i,j,k}$ expressions are generated from the $S_{i-1,j,k}$ terms. The remaining $\lambda_\mu - 2\lambda_{\mu-1}$ expressions, referred to as composite sums, are formulated as products of lower order S terms. The $A_{i,j}$ and $B_{i,j}$ constants are also generated from simple relationships involving previous A and B terms. In generating the first λ_{i-1} terms of order i, the $S_{i-1,j}$ expressions are premultiplied by an α. (Adjacent α's represent actual multiplication.) Thus, $S_{9,13} = \alpha S_8$, $S_{13} = \alpha^3\beta\alpha\beta\beta\alpha$. The next λ_{i-1} terms, $S_{i,j}$, $j = \lambda_{i-1} + 1,\ldots,2\lambda_{i-1}$ are generated by premultiplying the $S_{i-1,j-\lambda_{i-1}}$ expressions by a β, e.g., $S_{9,128} = \beta S_{8,13} = \beta\alpha^2\beta\alpha\beta\beta\alpha$. (Adjacent β's do not represent multiplication of the β coefficients since a summation sign precedes each β when the entire S term is written, e.g.,

$$S_{6,4,k} = \alpha^2\beta\beta\alpha = \alpha_k{}^2 \sum_{\ell=2}^{k-1} \beta_{k,\ell} \sum_{m=1}^{\ell-1} \beta_{\ell,m}\alpha_m \qquad (5)$$

($S_{6,4,k} \neq \alpha^2\beta^2\alpha$.)) The recursion relations, then, for the first λ_{i-1} terms of order i, ($i > 2$), are

$$S_{i,j} = \alpha S_{i-1,j}$$

$$A_{i,j} = A_{i-1,j}$$

$$B_{i,j} = \mu B_{i-1,j}$$

for $j = 1,2,\ldots,\lambda_{i-1}$, where μ is the power of the leading α in the $S_{i-1,j}$ term, and

$$S_{i,j} = \beta\, S_{i-1,j-\lambda_{i-1}}$$

$$A_{i,j} = (m+i-1)\, A_{i-1,j-\lambda_{i-1}}$$

$$B_{i,j} = B_{i-1,j-\lambda_{i-1}}$$

for $j = \lambda_{i-1} + 1,\ldots,2\lambda_{i-1}$. ($S_{1,1} = 1$, and $S_{2,1} = \alpha$.) The first λ_{i-1} S expressions are referred to as alpha terms in subroutine RKEQ, while the next λ_{i-1} expressions are called beta terms. The remaining terms, composite sums, are generated by considering the weight factors of the S terms. The $S_{\mu,j}$ expressions have a weight factor of $\mu-1$ (i.e., the number of α and β coefficients included in the S term). The composite sums of order i are all products of $S_{\mu,j}$ terms having initial β coefficients, whose weight factors add up to i-1. Subroutine RKEQ determines these composite sums in a separate block of the subprogram, calling subroutine CROSS to perform the multiplication of the S, A, and B terms. The $A_{i,j}$ and $B_{i,j}$ terms of a composite sum are the products of the A and B constants whose corresponding S terms form the composite sum. (When an S term is raised to the power k, an additional k! multiplies the B constant.)

3.0 DESCRIPTION OF THE FORTRAN PROGRAM

Subroutine RKEQ determines the truncation error coefficients (TEC) for a given set of RK coefficients and returns TERROR,

$$\text{TERROR} = \left\{ \sum_{j=1}^{\lambda_i} T_{i,j}^2 \right\}^{1/2}$$

for a specified order i. Since RK algorithms with embedded pairs of solutions, e.g. RK-Fehlberg formulas, are often studied, RKEQ is written to treat two algorithms simultaneously, which use identical α and β coefficients but different C_k and $\hat{C}_k$ coefficients. (TERROR is formed using the $\hat{C}_k$ coefficients.) The Greek letters α_i and β_i are replaced by A(I) and B(I) and the $A_{i,j}$ and $B_{i,j}$ constants are denoted AA(I,J) and BB(I,J), respectively.

The input parameters for RKEQ are:

(1) The RK coefficients

(a) A(K) α_k, the alpha coefficients

(b) CO, C(K) C_o, C_k, the C_k coefficients for the first solution

(c) CHO, CH(K) $\hat{C}_o$, $\hat{C}_k$, the $\hat{C}_k$ coefficients for the second solution used to form TERROR

(d) BO(K), B(K,L) $\beta_{k,o}$, $\beta_{k,\ell}$, the beta coefficients

where K = 1,2,...,R, L = 1,2,..., K - 1, R an integer with R + 1 being the number of stages of the algorithm, and

(2) The integers controlling orders and options

(a) R = the index for dimensioning the RK coefficients

(b) IORDER = the maximum order to be treated

(c) ITERR = the order of TEC used to form TERROR

(d) IOPT = the options for operating the program. For IOPT = 1, RKEQ computes and prints all TEC(I,J) for I = 1,...,IORDER. For IOPT = 2, RKEQ computes and prints TEC(IORDER, J) only. For IOPT = 3, RKEQ computes but does not print TEC(ITERR, J). (For all options TERROR is computed, which may require internal adjustments to the order.)

(e) MFOLD = an integer giving the number of known derivatives of f. For the classical RK algorithm, MFOLD = 0.

(f) LS = a dimensioning index for the work arrays, S, AA and BB.

The output parameter for RKEQ is TERROR, the Euclidean norm of the TEC of order ITERR. Depending upon the option used, RKEQ may print values of TEC, but these are not returned to the main program.

Parameters S, AA, and BB are used internally by RKEQ to compute the TEC terms. To take advantage of variable dimensioning, these parameters are given in the calling sequence with dimensions S(LS,R), AA(LS), BB(LS). The RK coefficients should be dimensioned A(R), C(R), CH(R), B(R, R), BO(R), R an integer, where R + 1 is the number of stages for the algorithm.

The calling sequence for RKEQ is

SUBROUTINE RKEQ(A, C, CO, CH, CHO, B, BO, R, IORDER, ITERR, IOPT, MFOLD, TERROR, S, LS, AA, BB).

which, if a printing option is used, will give the TEC from the C solution in the first column and the TEC from the CH solution in the second column. Integers IORDER, ITERR, and IOPT are reset within the subroutine, and any adjustments made to protect against exceeding dimension or option limits are made to the new variables, so that the user may enter constant values in the calling sequence of the driving program.

A sample calling sequence for a six-stage, fifth-order algorithm is

CALL RKEQ(A, C, CO, CH, CHO, B, BO, 5, 7, 6, 1, 0, TERROR, S, 48, AA, BB)

which computes and prints all TEC through order 7 for the classical RK formulas, using the 6th-order terms of the CH solution to form TERROR. Using the calling sequence

CALL RKEQ(A, CH, CHO, C, CO, B, BO, 5, 7, 6, 1, 0, TERROR, S, 48, AA, BB)

generates similar information except that TERROR is formed by the C solution

(and the $\hat{C}$ TEC terms are now printed in the first column.)

The minimum value of LS for a given ORDER, I, is found in table I. A listing of subroutines RKEQ and CROSS may be found in the appendix.

TABLE I.- DIMENSIONING PARAMETER, LS, FOR ORDERS 1 THROUGH 12

ORDER	1	2	3	4	5	6	7	8	9	10	11	12
LS_{MIN}	1	1	2	4	9	20	48	115	286	719	1842	4766

4.0 CONCLUDING REMARKS

A program, which evaluates the truncation error coefficients, is an essential tool in the development of Runge-Kutta algorithms and in the comparison of existing RK algorithms. By structuring the routine in the given form, a substantial savings in storage occurs in generating these truncation error coefficients using the recursive formulation presented by D. G. Bettis (ref. 1). The extension to orders higher than 12 is relatively simple but not of great practical use at the present time.

REFERENCES

1. Bettis, D. G.; and Horn, M. K.: Computation of Truncation Error Terms for Runge-Kutta Methods. TICOM Report 77-14, December 1977.

2. Fehlberg, E.: New High-Order Runge-Kutta Formulas with Step-size Control for Systems of First- and Second-Order Differential Equations. ZAMM, Vol. 44, 1964.

3. Fehlberg, E.: New High-Order Runge-Kutta Formulas with an Arbitrarily Small Truncation Error. ZAMM, Vol. 6, 1966.

APPENDIX A

SUBROUTINE RKEQ

```
      SUBROUTINE RKEQ(A,C,CO,CH,CHO,B,BO,R,
     1  IORDER,ITERR,IOPT,MFOLD,TERROR,S,LS,AA,BB)                    00000200
      INTEGER R,ORDER,OPTION                                        00000300
      DOUBLE PRECISION A(R),C(R),CH(R),B(R,R),BO(R)                 00000400
      DOUBLE PRECISION CO CHO                                       00000500
      DOUBLE PRECISION S(LS,R),AA(LS),BB(LS)                        00000600
      DOUBLE PRECISION SUM1,SUM3,X1,ZERO,UNITY,TWO,                 00000700
     1  MP(12),MM(12),FACT(12)                                      00000800
      DOUBLE PRECISION P,PP1,PM1,PM2,ZAPP,TERROR                    00000900
      DIMENSION M(12),LIMIT(12),INDEX(11)                           00001000
      LOGICAL EVEN                                                  00001100
C                                                                   00001200
      DATA LIMIT/1,1,2,4,9,20,48,115,286,719,1842,4766/             00001300
      DATA INDEX/1,1,2,5,11,28,67,171,433,1123,2924/                00001400
      DATA ZERO,UNITY,TWO,ZAPP/0.0D0,1.0D0,2.0D0,1.0D-14/           00001500
      DATA MAXORD/12/                                               00001600
C                                                                   00001700
C*********************************                                  00001800
C                                                                   00001900
      DATA KPRINT/0/                                                00002000
C                                                                   00002100
C*********************************                                  00002200
C                                                                   00002300
C                                                                   00002400
C                                                                   00002500
C     SUBROUTINE RKEQ IS A FORTRAN SUBROUTINE WRITTEN BY            00002600
C     M.K. HORN WHICH IMPLEMENTS THE ALGORITHM DEVELOPED            00002700
C     BY D.G. BETTIS TO GENERATE TRUNCATION ERROR COEFFI-           00002800
C     IENTS FOR RUNGE-KUTTA ALGORITHMS.                             00002900
C                                                                   00003000
C     REFERENCE: BETTIS,D.G. AND M.K. HORN, 'COMPUTATION            00003100
C     OF TRUNCATION ERROR TERMS FOR RUNGE-KUTTA METHODS,'           00003200
C     TICOM REPORT 77-14, DECEMBER, 1978.                           00003300
C                                                                   00003400
C     SUBROUTINE RKEQ DETERMINES THE TRUNCATION ERROR               00003500
C     COEFFICIENTS (TEC) FOR A RUNGE-KUTTA ALGORITHM HAVING         00003600
C     AN EMBEDDED PAIR OF SOLUTIONS                                 00003700
C                                                                   00003800
C                    R                                              00003900
C                                                                   00004000
C     Y  = Y  + H SUM C  F                                          00004100
C      1    0          K  K                                        00004200
C                  K=0                                              00004300
C                                                                   00004400
C                                                                   00004500
C                    R                                              00004600
C                                                                   00004700
C     YH  = Y  + H SUM CH  F                                        00004800
C       1    0           K  K                                       00004900
```

```
C                  K=0                                                  00005000
C                                                                       00005100
C                                                                       00005200
C                                                                       00005300
C     WHERE      F  = F(T ,Y )                                          00005400
C                 0       0  0                                        00005500
C                                                                       00005600
C                                                K-1                    00005700
C                                                                       00005800
C                F  = F(T  + A H, Y  + H * SUM B     F  )               00005900
C                 K       0    K    0            K,L  L               00006000
C                                                L=0                    00006100
C                                                                       00006200
C     FOR MFOLD = 0, RKEQ FORMS THE TEC FOR THE CLASSICAL RK            00006300
C     FORMULAS OF ORDER = IORDER.  FOR MFOLD = M, RKEQ FORMS            00006400
C     THE TEC FOR MFOLD-RK FORMULAS OF ORDER = M + IORDER.              00006500
C                                                                       00006600
C                                                                       00006700
C                                                                       00006800
C -------                                                               00006900
C                                                                       00007000
C     OPTION .EQ. 1   COMPUTES AND PRINTS ALL TRUNCATION                00007100
C                     ERROR COEFFICIENTS THROUGH ORDER =                00007200
C                     IORDER                                            00007300
C     OPTION .EQ. 2   COMPUTES AND PRINTS ONLY T.E.C. OF                00007400
C                     ORDER = IORDER                                    00007500
C     OPTION .EQ. 3   COMPUTES T.E.C. AS IN OPTION = 2 BUT              00007600
C                     DOES NOT PRINT                                    00007700
C                                                                       00007800
C -------                                                               00007900
C                                                                       00008000
C                                                                       00008100
C     ADJUSTS INPUT PARAMETERS IF THESE ARE NOT WITHIN                  00008200
C     ALLOWABLE RANGE                                                   00008300
C                                                                       00008400
C                                                                       00008500
      ORDER = IORDER                                                    00008600
      LIMITS = ITERR                                                    00008700
      IF (ORDER .LT. LIMITS)  ORDER = LIMITS                            00008800
      IF (LS. LT. LIMIT(MAXORD))  GO TO 1                               00008900
      IF (ORDER .LE. MAXORD)  GO TO 1                                   00009000
      ORDER = MAXORD                                                    00009100
      PRINT 507,MAXORD                                                  00009200
  507 FORMAT(52H ORDER REQUESTED IS BEYOND CAPABILITY OF THE PROGRAM    00009300
     1 ,/,24H ORDER LOWERED--ORDER =  ,I2)                              00009400
      IF (ITERR .LE. ORDER)   GO TO 1                                   00009500
      LIMITS = ORDER                                                    00009600
      PRINT 526,LIMITS                                                  00009700
  526 FORMAT(45H ITERR REQUESTED IS LARGER THAN MAXIMUM ORDER            00009800
     1  ,/,35H ITERR HAS BEEN REDUCED TO ORDER =  ,I2  )                00009900
```

```
C                                                                       00010000
.
    1 CONTINUE                                                          00010100
C                                                                       00010200
      TERROR = ZERO                                                     00010300
      OPTION = IOPT                                                     00010400
      IF (IOPT .LE. 0 .OR. IOPT .GT. 3)  OPTION = 1                     00010500
      IF (LS .GE. LIMIT(ORDER))  GO TO 3                                00010600
    2 CONTINUE                                                          00010700
      ORDER = ORDER - 1                                                 00010800
      IF (LS .LT. LIMIT(ORDER))  GO TO 2                                00010900
      PRINT 505,ORDER,IORDER,LIMIT(IORDER)                              00011000
  505 FORMAT(50H THE ORDER SPECIFIED HAS BEEN REDUCED TO ORDER =        00011100
     1  ,I3,/,32H BECAUSE OF INSUFFICIENT STORAGE  ,/,                  00011200
     2  14H FOR ORDER =  ,I3,20H  LS MUST BE .GE.   ,I5,                00011300
     3  /,56H TERROR = SQRT(SUM(T.E.C.(I,J)*T.E.C.(I,J))) IS COMPUTED   00011400
     4  ,/,14H FOR I = ORDER )                                          00011500
C                                                                       00011600
      IF (LIMITS .LE. ORDER)  GO TO 3                                   00011700
      LIMITS = ORDER                                                    00011800
      PRINT 527,LIMITS                                                  00011900
  527 FORMAT(45H ITERR REQUESTED IS LARGER THAN THE PROVIDED            00012000
     1,/,50H DIMENSIONING.  ITERR HAS BEEN REDUCED TO ITERR = ,         00012100
     2  I2)                                                             00012200
    3 CONTINUE                                                          00012300
C                                                                       00012400
C                                                                       00012500
C                                                                       00012600
      IF (ORDER .GT. MAXORD)   ORDER = MAXORD                           00012700
C                                                                       00012800
C                                                                       00012900
C -------                                                               00013000
C                                                                       00013100
C     MP(J) = (MFOLD+J)          D.P.                                   00013200
C     M(J) = J                   INTEGER                                00013300
C     FACT(J) = J                D.P.                                   00013400
C     MM(J) = (MFOLD+J)          D.P.                                   00013500
C                                                                       00013600
C     INDEX--COUNTS THE NUMBER OF S(J,K) WITH A GIVEN A**K              00013700
C            AS THE FIRST TERM IN THE EXPRESSION, E.G.,                 00013800
C            FOR J=7, THERE ARE 1 A**6, 1 A**4, 2 A**3,                 00013900
C            4 A**2, AND 9 A**1 @S                                      00014000
C                                                                       00014100
C -------                                                               00014200
C                                                                       00014300
      X1 = A(1) - B0(1)                                                 00014400
      JJ = 1                                                            00014500
  550 FORMAT(15H ERROR IN BETA(,I2,10H)   SUM =  ,D15.7)                00014600
      IF (DABS(X1) .GE. ZAPP)  PRINT 550,JJ,X1                          00014700
      DO 5 J = 2,R                                                      00014800
      X1 = A(J) - B0(J)                                                 00014900
```

```
      JLOW = J-1                                                        00015000
.
      DO 4 K = 1,JLOW                                                   00015100
    4 X1 = X1 - B(J,K)                                                  00015200
      IF (DABS(X1) .GE. ZAPP)  PRINT 550,J,X1                           00015300
    5 CONTINUE                                                          00015400
C                                                                       00015500
      IF (KPRINT .EQ. 1) READ PRINT 499                                 00015600
C                                                                       00015700
C                                                                       00015800
C     SETS VALUES OF FACTORIALS USED IN THE PROGRAM.                    00015900
C     ADJUSTMENTS IN THE PROGRAM TO ACCOMODATE M-FOLD                   00016000
C     RUNGE-KUTTA ALGORITHMS OCCUR HERE.  IF MFOLD = 0,                 00016100
C     THE CLASSICAL RUNGE-KUTTA T.E.C. OCCUR                            00016200
C                                                                       00016300
C                                                                       00016400
      IF (MFOLD .GT. 0)  GO TO 6                                        00016500
      EVEN = .TRUE.                                                     00016600
      GO TO 8                                                           00016700
    6 CONTINUE                                                          00016800
      N1 = MFOLD / 2                                                    00016900
      N2 = 2 * N1                                                       00017000
      EVEN = .FALSE.                                                    00017100
      IF (N2 .EQ. MFOLD)  EVEN = .TRUE.                                 00017200
    8 CONTINUE                                                          00017300
      MP(1) = DFLOAT(MFOLD)+UNITY                                       00017400
      M(1) = 1                                                          00017500
      X1 = UNITY                                                        00017600
      FACT(1) = UNITY                                                   00017700
      DO 10 I = 2,ORDER                                                 00017800
      X1 = X1 + UNITY                                                   00017900
      FACT(I) = FACT(I-1)*X1                                            00018000
      MP(I) = MP(I-1) + UNITY                                           00018100
   10 M(I) = M(I-1) + 1                                                 00018200
      M2 = 1                                                            00018300
      M1 = 0                                                            00018400
   11 CONTINUE                                                          00018500
      M1 = M1 + 1                                                       00018600
      M2 = M2*M1                                                        00018700
      IF (M1 .LT. MFOLD+1)  GO TO 11                                    00018800
      MM(1) = DFLOAT(M2)                                                00018900
      DO 12 I = 2,ORDER                                                 00019000
   12 MM(I) = MP(I)* MM(I-1)                                            00019100
C                                                                       00019200
C                                                                       00019300
      IF (KPRINT .EQ. 1) PRINT 499                                      00019400
C                                                                       00019500
C     SETS INITIAL VALUES OF S(I,J) EQUAL TO ZERO                       00019600
C     AND SETS AA(J) AND BB(J) EQUAL TO UNITY                           00019700
C                                                                       00019800
      DO 14 J = 1,LS                                                    00019900
```

```
      AA(J) = UNITY                                                    00020000
C
      BB(J) = UNITY                                                    00020100
      DO 14 K = 1,R                                                    00020200
   14 S(J,K) = ZERO                                                    00020300
C                                                                      00020400
      IF (OPTION .GT. 1 .AND. LIMITS .NE. 1)  GO TO 21                 00020500
C                                                                      00020600
C     EVALUATES T.E.C. OF ORDER 1                                      00020700
C                                                                      00020800
      SUM1 = UNITY/DFLOAT(MFOLD + 1) -  C0                             00020900
      SUM3 = UNITY/DFLOAT(MFOLD + 1) - CH0                             00021000
      DO 20 I = 1,R                                                    00021100
      SUM1 = SUM1 - C(I)                                               00021200
      SUM3 = SUM3 - CH(I)                                              00021300
   20 CONTINUE                                                         00021400
C                                                                      00021500
      JJ1 = 1                                                          00021600
      IF (LIMITS .EQ. 1)  TERROR = SUM3*SUM3                           00021700
      IF (OPTION .EQ. 3)  GO TO 21                                     00021800
      PRINT 500,JJ1                                                    00021900
      PRINT 501,JJ1,JJ1,SUM1,SUM3                                      00022000
  500 FORMAT(36H TRUNCATION ERROR TERMS   X-ORDER = ,I2,2X             00022100
     1  ,//)                                                          00022200
  501 FORMAT(2(2X,I4),2(D15.7))                                        00022300
C                                                                      00022400
   21 CONTINUE                                                         00022500
C                                                                      00022600
C     SETS S(1,J) TERMS                                                00022700
C                                                                      00022800
      KOUNT = 1                                                        00022900
C                                                                      00023000
      DO 26 I = 1,R                                                    00023100
      IF (MFOLD .GT. 0)  GO TO 22                                      00023200
      S(1,I) = A(I)                                                    00023300
      GO TO 26                                                         00023400
   22 CONTINUE                                                         00023500
      IF (EVEN) GO TO 24                                               00023600
      S(1,I) = DABS(A(I))**MP(1)                                       00023700
      GO TO 26                                                         00023800
   24 CONTINUE                                                         00023900
      X1 = DABS(A(I))**MP(1)                                           00024000
      S(1,I) = DSIGN(X1,A(I))                                          00024100
   26 CONTINUE                                                         00024200
C                                                                      00024300
C     EVALUATES T.E.C. FOR ORDER = KOUNT = 2,3                         00024400
C                                                                      00024500
C                                                                      00024600
      AA(1) = UNITY                                                    00024700
      BB(1) = MM(1)                                                    00024800
   28 CONTINUE                                                         00024900
```

```
      P = TWO + DFLOAT(MFOLD)                                          00025000
*
      PP1 = P+UNITY                                                    00025100
   30 CONTINUE                                                         00025200
      IF (LIMITS .EQ. KOUNT)  JJ1 = KOUNT - 1                         00025300
      IF (OPTION .GT. 1 .AND. LIMITS .NE. KOUNT)  GO TO 34             00025400
      JJ2 = 0                                                          00025500
      JJ1 = KOUNT + 1                                                  00025600
      PRINT 500,JJ1                                                    00025700
      DO 33 K = 1,KOUNT                                                00025800
      JJ2 = JJ2 + 1                                                    00025900
      SUM1 = UNITY/(AA(K)*P)                                           00026000
      SUM3 = SUM1                                                      00026100
      DO 32 I = 1,R                                                    00026200
      SUM1 = SUM1 -   C(I)*S(K,I)                                      00026300
      SUM3 = SUM3 -  CH(I)*S(K,I)                                      00026400
   32 CONTINUE                                                         00026500
      IF (LIMITS .EQ. KOUNT)  TERROR = TERROR + SUM3*SUM3              00026600
      IF (OPTION .EQ. 3)  GO TO 33                                     00026700
      PRINT 501,JJ1,JJ2,SUM1,SUM3                                      00026800
   33 CONTINUE                                                         00026900
C                                                                      00027000
   34 CONTINUE                                                         00027100
C                                                                      00027200
      IF (KOUNT .EQ. 2)  GO TO 38                                      00027300
C                                                                      00027400
C     SETS S(1,J), S(2,J) FOR THIRD ORDER T.E.C.                       00027500
C                                                                      00027600
      KOUNT = 2                                                        00027700
      P = P + UNITY                                                    00027800
      PP1 = PP1 + UNITY                                                00027900
      DO 35 I = 2,R                                                    00028000
      S(2,I) = ZERO                                                    00028100
      IM1 = I-1                                                        00028200
      DO 35 J = 1,IM1                                                  00028300
   35 S(2,I) = S(2,I) + B(I,J)*S(1,J)                                  00028400
      DO 36 I = 1,R                                                    00028500
   36 S(1,I) = S(1,I)*A(I)                                             00028600
      AA(2) = TWO                                                      00028700
      BB(1) = MM(2)                                                    00028800
      BB(2) = MM(1)                                                    00028900
      GO TO 30                                                         00029000
   38 CONTINUE                                                         00029100
      IF (ORDER .LE. 3)  GO TO 182                                     00029200
      LIM1 = 3                                                         00029300
C                                                                      00029400
C     EVALUATES T.E.C. FOR ORDERS GREATER THAN THREE                   00029500
C                                                                      00029600
      DO 180 J = 4,ORDER                                               00029700
      P = P + UNITY                                                    00029800
      PP1 = P + UNITY                                                  00029900
```

```
      PM1 = P - UNITY                                                   00030000

      PM2 = P - TWO                                                     00030100
      IF (KPRINT .EQ. 1) READ 497,II                                    00030200
      LIM1 = LIM1 + 1                                                   00030300
      LIMA = LIMIT(J-1)                                                 00030400
      LIMB = LIMA                                                       00030500
C                                                                       00030600
C     COMPUTES S(1,I)--ALL OTHER S(J,K) TERMS INVOLVING A               00030700
C                      LEADING ALPHA ARE ALREADY DETERMINED             00030800
C                      EXCEPT FOR THE POWER OF ALPHA WHICH              00030900
C                      IS DETERMINED BY IN INDEX AND J                  00031000
C                                                                       00031100
C     AA(1) = AA(1)                                                     00031200
      BB(1)  = BB(1) * MP(LIM1-1)                                       00031300
      DO 42 I = 1,R                                                     00031400
   42 S(1,I) = S(1,I)*A(I)                                              00031500
C                                                                       00031600
C     LIM1 = INTEGER P                                                  00031700
C                                                                       00031800
C                                                                       00031900
C                                                                       00032000
C     BETA TERMS                                                        00032100
C                                                                       00032200
      MARKB = LIMA+1                                                    00032300
      DO 61 I = 2,R                                                     00032400
      S(MARKB,I) = ZERO                                                 00032500
      IM1 = I-1                                                         00032600
      DO 61 K = 1,IM1                                                   00032700
   61 S(MARKB,I) = S(MARKB,I)+B(I,K)*A(K)**(MFOLD+LIM1-2)               00032800
      AA(MARKB)=PM1                                                     00032900
      BB(MARKB) = MM(LIM1-2)                                            00033000
      IND2 = 1                                                          00033100
      IND1 = MARKB                                                      00033200
      LL = LIM1 - 4                                                     00033300
      IF (LIM1 .EQ. 4)  GO TO 66                                        00033400
      DO 65 K = 1,LL                                                    00033500
      LL1 = INDEX(K+1)                                                  00033600
      IPOW = LL-K+1                                                     00033700
      DO 65 KK = 1,LL1                                                  00033800
      IND1 = IND1 + 1                                                   00033900
      IND2 = IND2 + 1                                                   00034000
      DO 64 I = 2,R                                                     00034100
      S(IND1,I) = ZERO                                                  00034200
      IM1 = I-1                                                         00034300
      DO 64 L = 1,IM1                                                   00034400
   64 S(IND1,I) = S(IND1,I)+B(I,L)*S(IND2,L)*A(L)**M(IPOW)              00034500
      BB(IND1) = BB(IND2)*FACT(IPOW)                                    00034600
   65 AA(IND1) = AA(IND2)*PM1                                           00034700
   66 CONTINUE                                                          00034800
      LL = LIMB - IND2                                                  00034900
```

```
      DO 68 K = 1,LL                                                    00035000
      IND1 = IND1 + 1                                                   00035100
      IND2 = IND2 + 1                                                   00035200
      DO 67 I = 2,R                                                     00035300
      S(IND1,I) = ZERO                                                  00035400
      IM1 = I-1                                                         00035500
      DO 67 L = 1,IM1                                                   00035600
   67 S(IND1,I) = S(IND1,I)+B(I,L)*S(IND2,L)                            00035700
      BB(IND1) = BB(IND2)                                               00035800
   68 AA(IND1) = AA(IND2)*PM1                                           00035900
  883 CONTINUE                                                          00036000
      JM3 = J - 3                                                       00036100
      GO TO (150,100,105,110,115,120,125,130,135),JM3                   00036200
                                                                        00036300
C                                                                       00036400
C     CROSS PRODUCT TERMS                                               00036500
C                                                                       00036600
  100 CONTINUE                                                          00036700
C                                                                       00036800
C     5 TH ORDER TERMS                                                  00036900
C                                                                       00037000
      IND = 2*LIMIT(4)+1                                                00037100
      CALL CROSS(LS,R,S,AA,BB,IND,2,2,0,0)                              00037200
      GO TO 150                                                         00037300
C                                                                       00037400
C      6 TH ORDER TERMS                                                 00037500
  105 CONTINUE                                                          00037600
      IND = 2*LIMIT(5)+1                                                00037700
      CALL CROSS(LS,R,S,AA,BB,IND,2,1,3,4)                              00037800
      GO TO 150                                                         00037900
                                                                        00038000
C                                                                       00038100
C      7 TH ORDER TERMS                                                 00038200
C                                                                       00038300
  110 CONTINUE                                                          00038400
      IND = 2*LIMIT(6)+1                                                00038500
      CALL CROSS(LS,R,S,AA,BB,IND,2,1,5,8)                              00038600
      CALL CROSS(LS,R,S,AA,BB,IND,2,3,0,0)                              00038700
      CALL CROSS(LS,R,S,AA,BB,IND,3,2,0,0)                              00038800
      CALL CROSS(LS,R,S,AA,BB,IND,4,2,0,0)                              00038900
      CALL CROSS(LS,R,S,AA,BB,IND,3,1,4,4)                              00039000
      GO TO 150                                                         00039100
C                                                                       00039200
C      8 TH ORDER TERMS                                                 00039300
C                                                                       00039400
  115 CONTINUE                                                          00039500
      IND = 2*LIMIT(7)+1                                                00039600
      CALL CROSS(LS,R,S,AA,BB,IND,2,1,10,18)                            00039700
      CALL CROSS(LS,R,S,AA,BB,IND,3,1,5,8)                              00039800
      CALL CROSS(LS,R,S,AA,BB,IND,4,1,5,8)                              00039900
```

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      CALL CROSS(LS,R,S,AA,BB,IND,2,2,3,4)                          00040000
*
      GO TO 150                                                     00040100
C                                                                   00040200
C     9 TH ORDER TERMS                                              00040300
  120 CONTINUE                                                      00040400
      IND = 2*LIMIT(8)+1                                            00040500
      CALL CROSS(LS,R,S,AA,BB,IND,2,1,21,40)                        00040600
      CALL CROSS(LS,R,S,AA,BB,IND,3,1,10,18)                        00040700
      CALL CROSS(LS,R,S,AA,BB,IND,4,1,10,18)                        00040800
      DO 121 K = 5,8                                                00040900
  121 CALL CROSS(LS,R,S,AA,BB,IND,K,1,K,8)                          00041000
      CALL CROSS(LS,R,S,AA,BB,IND,9,1,5,8)                          00041100
      CALL CROSS(LS,R,S,AA,BB,IND,2,4,0,0)                          00041200
      CALL CROSS(LS,R,S,AA,BB,IND,2,1,46,48)                        00041300
      GO TO 150                                                     00041400
C                                                                   00041500
C    10 TH ORDER TERMS                                              00041600
C                                                                   00041700
  125 CONTINUE                                                      00041800
      IND = 2*LIMIT(9) + 1                                          00041900
C                                                                   00042000
      CALL CROSS(LS,R,S,AA,BB,IND,2,1,49,96)                        00042100
      CALL CROSS(LS,R,S,AA,BB,IND,3,1,21,40)                        00042200
      CALL CROSS(LS,R,S,AA,BB,IND,4,1,21,40)                        00042300
      DO 126 K = 5,9                                                00042400
  126 CALL CROSS(LS,R,S,AA,BB,IND,K,1,10,18)                        00042500
      CALL CROSS(LS,R,S,AA,BB,IND,2,1,106,113)                      00042600
      CALL CROSS(LS,R,S,AA,BB,IND,2,3,3,4)                          00042700
      CALL CROSS(LS,R,S,AA,BB,IND,3,3,0,0)                          00042800
      CALL CROSS(LS,R,S,AA,BB,IND,4,3,0,0)                          00042900
      CALL CROSS(LS,R,S,AA,BB,IND,4,2,3,3)                          00043000
      CALL CROSS(LS,R,S,AA,BB,IND,3,2,4,4)                          00043100
C                                                                   00043200
      GO TO 150                                                     00043300
C                                                                   00043400
  130 CONTINUE                                                      00043500
C                                                                   00043600
C     11TH ORDER TERMS                                              00043700
C                                                                   00043800
      IND = 2*LIMIT(10) + 1                                         00043900
C                                                                   00044000
      CALL CROSS(LS,R,S,AA,BB,IND,2,1,116,230)                      00044100
      CALL CROSS(LS,R,S,AA,BB,IND,3,1,49,96)                        00044200
      CALL CROSS(LS,R,S,AA,BB,IND,4,1,49,96)                        00044300
      DO 131  K = 5,8                                               00044400
  131 CALL CROSS(LS,R,S,AA,BB,IND,K,1,21,40)                        00044500
      DO 132  K = 10,18                                             00044600
  132 CALL CROSS(LS,R,S,AA,BB,IND,K,1,K,18)                         00044700
      CALL CROSS(LS,R,S,AA,BB,IND,2,2,21,40)                        00044800
      CALL CROSS(LS,R,S,AA,BB,IND,19,1,10,18)                       00044900
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      CALL CROSS(LS,R,S,AA,BB,IND,20,1,10,18)                          00045000
.
      CALL CROSS(LS,R,S,AA,BB,IND,2,1,269,278)                         00045100
      DO 133 K = 46,48                                                 00045200
  133 CALL CROSS(LS,R,S,AA,BB,IND,K,1,5,8)                             00045300
      CALL CROSS(LS,R,S,AA,BB,IND,2,3,5,8)                             00045400
      CALL CROSS(LS,R,S,AA,BB,IND,2,2,46,48)                           00045500
      CALL CROSS(LS,R,S,AA,BB,IND,2,5,0,0)                             00045600
C                                                                      00045700
      GO TO 150                                                        00045800
C                                                                      00045900
C     12TH ORDER TERMS                                                 00046000
C                                                                      00046100
  135 CONTINUE                                                         00046200
C                                                                      00046300
      IND = 2*LIMIT(11) + 1                                            00046400
C                                                                      00046500
C                                                                      00046600
      CALL CROSS(LS,R,S,AA,BB,IND,2,1,287,572)                         00046700
      CALL CROSS(LS,R,S,AA,BB,IND,3,1,116,230)                         00046800
      CALL CROSS(LS,R,S,AA,BB,IND,4,1,116,230)                         00046900
      DO 141 K = 5,8                                                   00047000
  141 CALL CROSS(LS,R,S,AA,BB,IND,K,1,49,96)                           00047100
      DO 142 K = 10,18                                                 00047200
  142 CALL CROSS(LS,R,S,AA,BB,IND,K,1,21,40)                           00047300
      CALL CROSS(LS,R,S,AA,BB,IND,2,2,49,96)                           00047400
      CALL CROSS(LS,R,S,AA,BB,IND,19,1,21,40)                          00047500
      CALL CROSS(LS,R,S,AA,BB,IND,20,1,21,40)                          00047600
      DO 143 K = 41,48                                                 00047700
  143 CALL CROSS(LS,R,S,AA,BB,IND,K,1,10,18)                           00047800
      CALL CROSS(LS,R,S,AA,BB,IND,114,1,5,8)                           00047900
      CALL CROSS(LS,R,S,AA,BB,IND,115,1,5,8)                           00048000
      DO 144 K = 269,278                                               00048100
  144 CALL CROSS(LS,R,S,AA,BB,IND,K,1,3,4)                             00048200
      CALL CROSS(LS,R,S,AA,BB,IND,3,3,2,2)                             00048300
      CALL CROSS(LS,R,S,AA,BB,IND,4,3,2,2)                             00048400
      CALL CROSS(LS,R,S,AA,BB,IND,3,2,20,20)                           00048500
      CALL CROSS(LS,R,S,AA,BB,IND,4,2,19,19)                           00048600
      CALL CROSS(LS,R,S,AA,BB,IND,2,4,3,4)                             00048700
  150 CONTINUE                                                         00048800
C                                                                      00048900
C     TEMPORARY INSERT TO CHECK VALUES OF AA AND BB COEFF              00049000
C                                                                      00049100
C     LL = LIMIT(LIM1)                                                 00049200
C     IFAKE = 0                                                        00049300
C     DO 153 K = 1,LL                                                  00049400
C     IFAKE = IFAKE + 1                                                00049500
C     IF (IFAKE .LT. 40)  GO TO 153                                    00049600
C     IF (KPRINT .EQ. 1) READ 497,II                                   00049700
C     IFAKE = 0                                                        00049800
C 153 PRINT 506,K,AA(K),K,BB(K)                                        00049900
```

```
C 506 FORMAT( 4H AA(,I4, 4H) = ,D15.7,2X,4H BB(,                        00050000

C     1   I4,4H) = ,D15.7)                                              00050100
      IF (KPRINT .EQ. 1) PRINT 499                                      00050200
C                                                                       00050300
C                                                                       00050400
C                                                                       00050500
      IF (LIMITS .EQ. J)  JJ1 = J - 1                                   00050600
      IF (OPTION .GT. 1 .AND. LIMITS .NE. J)  GO TO 180                 00050700
C                                                                       00050800
      JJ2 =  1                                                          00050900
      JJ1 = J                                                           00051000
C                                                                       00051100
C     EVALUATES FIRST T.E.C. OF ORDER J                                 00051200
C                                                                       00051300
      SUM1 = UNITY/(AA(1)*P)                                            00051400
      SUM3 = SUM1                                                       00051500
      DO 154 I = 1,R                                                    00051600
      SUM1 = SUM1 -   C(I)*S(1,I)                                       00051700
      SUM3 = SUM3 -  CH(I)*S(1,I)                                       00051800
  154 CONTINUE                                                          00051900
      SUM1 = SUM1 / BB(1)                                               00052000
      SUM3 = SUM3 / BB(1)                                               00052100
      IF (LIMITS .EQ. J)  TERROR = TERROR + SUM3*SUM3                   00052200
      IF (OPTION .EQ. 3)  GO TO 155                                     00052300
      PRINT 501,JJ1,JJ2,SUM1,SUM3                                       00052400
  155 CONTINUE                                                          00052500
C                                                                       00052600
C                                                                       00052700
C     EVALUATES T.E.C. FOR S(I,J) TERMS WITH ALPHA AS                   00052800
C     LEADING COEFFICIENT  (I .NE. 2)                                   00052900
C                                                                       00053000
      IFAKE = 1                                                         00053100
      K = 1                                                             00053200
      KNT = 2                                                           00053300
      IPOW = LIM1 - 3                                                   00053400
      LIMD = INDEX(KNT)                                                 00053500
  156 CONTINUE                                                          00053600
      DO 159 KK = 1,LIMD                                                00053700
      K = K + 1                                                         00053800
      SUM1 = UNITY/(AA(K)*P)                                            00053900
      SUM3 = SUM1                                                       00054000
      DO 157 I = 1,R                                                    00054100
      SUM1 = SUM1 -   C(I)*A(I)**IPOW*S(K,I)                            00054200
      SUM3 = SUM3 -  CH(I)*A(I)**IPOW*S(K,I)                            00054300
  157 CONTINUE                                                          00054400
      SUM1 = SUM1 / (BB(K)*FACT(IPOW))                                  00054500
      SUM3 = SUM3 / (BB(K)*FACT(IPOW))                                  00054600
      IF (LIMITS .EQ. J)  TERROR = TERROR + SUM3*SUM3                   00054700
      IF (OPTION .EQ. 3)  GO TO 159                                     00054800
      JJ2 = JJ2 + 1                                                     00054900
```

```
      IFAKE = IFAKE + 1                                                 00055000

      IF (IFAKE .LT. 40)  GO TO 158                                     00055100
      IFAKE = 0                                                         00055200
      IF (KPRINT .EQ. 1) PRINT 499                                      00055300
  158 CONTINUE                                                          00055400
  497 FORMAT(I3)                                                        00055500
      PRINT 501,JJ1,JJ2,SUM1,SUM3                                       00055600
  159 CONTINUE                                                          00055700
C                                                                       00055800
      KNT = KNT + 1                                                     00055900
      LIMD = INDEX(KNT)                                                 00056000
      IPOW = IPOW - 1                                                   00056100
      IF (IPOW .GE. 1)  GO TO 156                                       00056200
C                                                                       00056300
      LIMD =  LIMIT(LIM1) - LIMA                                        00056400
C                                                                       00056500
C     EVALUATES T.E.C. FOR S(I,J) TERMS WITH BETA AS                    00056600
C     LEADING COEFFICIENT                                               00056700
C                                                                       00056800
      DO 162 KK = 1,LIMD                                                00056900
      K = K + 1                                                         00057000
      SUM1 = UNITY/(AA(K)*P)                                            00057100
      SUM3 = SUM1                                                       00057200
      DO 160 I = 1,R                                                    00057300
      SUM1 = SUM1 -   C(I)*S(K,I)                                       00057400
      SUM3 = SUM3 -  CH(I)*S(K,I)                                       00057500
  160 CONTINUE                                                          00057600
      SUM1 = SUM1 / BB(K)                                               00057700
      SUM3 = SUM3 / BB(K)                                               00057800
      IF (LIMITS .EQ. J)  TERROR = TERROR + SUM3*SUM3                   00057900
      IF (OPTION .EQ. 3)  GO TO 162                                     00058000
      IFAKE = IFAKE + 1                                                 00058100
      IF (IFAKE .LT. 40)  GO TO 161                                     00058200
      IFAKE = 0                                                         00058300
      IF (KPRINT .EQ. 1) PRINT 499                                      00058400
  161 CONTINUE                                                          00058500
      JJ2 = JJ2 + 1                                                     00058600
C     PRINT 556,K,LIM1,AA(K),P,PP1                                      00058700
  556 FORMAT(2(2X,I4),3(2X,D15.7))                                      00058800
      PRINT 501,JJ1,JJ2,SUM1,SUM3                                       00058900
  162 CONTINUE                                                          00059000
      IF (KPRINT .EQ. 1) PRINT 499                                      00059100
  499 FORMAT(//)                                                        00059200
  180 CONTINUE                                                          00059300
  182 CONTINUE                                                          00059400
      TERROR = DSQRT(TERROR)                                            00059500
      RETURN                                                            00059600
      END                                                               00059700
```

```
      SUBROUTINE CROSS(LS,R,S,AA,BB,INDEX,TERM1,POWER,TERM2,
     1    TERM3)
      INTEGER TERM1,TERM2,TERM3,POWER,R
      DOUBLE PRECISION S(LS,R),AA(LS),BB(LS),FACT(7)
C
      DATA FACT/1.0D0,2.0D0,6.0D0,24.0D0,
     1    120.0D0,720.0D0,5040.0D0/
C
C
C


      IF (TERM2 .EQ. 0)  GO TO 20
C
C     COMPUTES S(INDEX+J,I)=S(TERM1,I)**POWER * S(TERM2+J,I)
C              FOR  J=0,1,...,TERM3-TERM2
C
      KK = -1
      DO 10 K = TERM2,TERM3
      KK = KK + 1
      INDX = INDEX + KK
      AA(INDX) = AA(TERM1)**POWER * AA(K)
      IPOW = POWER
      IF (TERM1 .EQ. K)  IPOW = IPOW + 1
      BB(INDX) = BB(TERM1)**POWER * BB(K) *FACT(IPOW)
      DO 10 I = 2,R
   10 S(INDX,I) = S(TERM1,I)**POWER * S(K,I)
      INDEX = INDEX + TERM3 - TERM2 + 1
C
C
      RETURN
C
C     COMPUTES S(INDEX,I) = S(TERM1,I)**POWER
C
   20 CONTINUE
      AA(INDEX) = AA(TERM1)**POWER
      BB(INDEX) = BB(TERM1)**POWER * FACT(POWER)
      DO 25 I = 2,R
   25 S(INDEX,I) = S(TERM1,I)**POWER
      INDEX = INDEX + 1
C
C
      RETURN
      END
```