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OZONE DATA AND MISSION SAMPLING ANALYSIS

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SUMMARY

Techniques have been developed to analyze global data sets of atmospheric constituents and to evaluate mission sampling strategies using these global data sets. Mathematical formulations and computer programs were developed to reduce and model global data fields and to perform statistical analyses of results.

The grouping scheme used to reduce data into a global grid network is shown and data storage methods are discussed. Procedures for modeling these data with spherical harmonic functions and empirical orthogonal functions (EOF) are detailed mathematically and numerical computer solutions are developed. Eigenanalysis techniques in conjunction with these EOF models are illustrated for reducing the dimensionality of large data sets.

The seemingly ever-present "missing data" problem is examined using the sample autocorrelation function. A linear regression technique is demonstrated which generates a "corrected" ozone satellite data set based on Dobson spectrophotometer (land based) measurements.

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I. INTRODUCTION

Defining the temporal and spatial variability of atmospheric constituents requires a sampling strategy and sensing technique that is consistent with the nature of the species being studied. Measurements of the global ozone field have been made for years from the ground[1], from aircraft and balloons[2], and more recently from satellites[3]. This information can be examined to determine something about the statistical nature of these data and, generally, the types of sampling schemes that should be considered.

The objective of this sampling study is to evaluate various sampling schemes which are based on the current understanding of the global ozone field and on other mission related constraints. To accomplish this, representative data must be acquired and reduced into a usable form. A model of the global ozone field must be developed. Computer simulated missions can be generated by "measuring" the global ozone field as represented by this model as viewed by selected sampling schemes. How well these sampling missions "recover" the model is determined by statistical analysis techniques which serve in the mission evaluation process.

This report addresses itself not so much to the overall sampling evaluation problem but to the techniques that have been thus far developed and utilized toward that end, especially in the areas of preliminary data manipulation and reduction, model development, computer simulations of sampling missions, and the associated statistical analysis techniques used throughout the work.

Appendix A shows the primary computer programs mentioned during the discussion.

II. PRELIMINARY DATA ANALYSIS - DATA MANIPULATION AND REDUCTION

Global ozone data utilized in this study are primarily from the Backscattered Ultraviolet (BUV) experiment aboard the Nimbus 4 satellite. These data have been received from the National Space Science Data Center in the form of IBM unformatted binary magnetic tapes. To some lesser extent ozone measurements from Dobson spectrophotometers are used. These data will be mentioned later in this report. This section is concerned with the BUV data. Particular items discussed include:

1. Conversion of the data tapes from IBM internal format to NOS-CDC internal format

2. Preliminary data analysis

3. Data grouping
   (a) Global grid system
   (b) Statistical analysis
   (c) Data retrieval technique

1. IBM Format to CDC Format Conversion

The BUV ozone data used for this study have been received on magnetic tape written in IBM internal format. In order to generate a NOS compatible set of data tapes the 32 bit IBM words must be unpacked into 60 bit CDC words, and the IBM internal format must be converted to CDC internal format.

A computer program (BUVCOP2) was written to accomplish this task. This program has successfully generated a set of NOS tapes containing global ozone data from April 10, 1970 through May 6, 1977. Table B-1 shows the time coverages and designations of the various magnetic tapes involved in this process. Appendix B discusses the data tape structure and the IBM to NOS-CDC internal format conversion in more detail.

2. Preliminary Data Analysis

Once a set of usable data tapes have been acquired, they must be carefully reviewed to ensure that their general format and content are consistent with the user's understanding and that there are no apparent problems with the data. A computer program (BUV3) has been written to look for particular problems associated with the data. These include:

1. Out of sequence (OOS) data - data that are chronologically out of order.
2. Out of range (OOR) data - a data record containing a latitude or longitude value out of its realistic range (-90° > latitude > 90°, 0° > longitude > 360°), or a measured observable whose value is inconsistent with accompanying user information.

3. Inconsistent local time (ILT) data - since Nimbus 4 is in a Sun-synchronous orbit, the satellite should cross the equator at approximately the same local solar time each orbit. This is the case for both the ascending and descending portions of the orbit. However, only the ascending portion of the orbit is of concern here since the descending portion of the orbit is on the dark side of the Earth and the BUV experiment only works in the sunlight. Local solar times can readily be calculated by the expression,

$$t_e = t_g + \phi/15°,$$

where t_e is the local time, t_g is the Greenwich mean solar time (GMT) of the observation, and ϕ is the longitude of the observation measured eastward from the prime meridian (PM). The analysis program calculates t_e for observations within 5° of the equator and compares them to the known local crossing time, t_k. If the difference $t_e - t_k$ is less than some predetermined acceptable Δt, agreement in the two is assumed to be good. Due to orbital considerations t_k may change slightly as a function of time over several years.

4. Repetitive data - two or more data records that occurred at either the same time or same position (the latter being consecutive measurements) but that differ in the values of other parameters.

5. Duplicate data - a data record or records that exactly duplicates another data record.

6. Reversed ground track - a series of data records that show the satellite ground track moving the wrong direction latitudinally.

Such problems need to be identified and, where practical, eliminated. If known or suspected problem areas remain, one must be mindful of their potential impact in further analyses.

Table 1 is representative of the information one may expect from the BUV3 analysis program. This particular analysis table is for the third set of BUV data (BUV III). Items such as the number of files, number of records (observations), and mission duration give information useful for future analyses as well as confirming the data tapes' general structure and content. The diagnostics such as the quantity and nature of abnormal ozone values, help in determining what, if any, data editing must be performed. For example, an observation near the equator whose calculated local time, t_e, disagrees with the known local time, t_k, significantly could mean that either the GMT or longitude are incorrect. However, if several observations near the equator for a given orbit show disagreement, the entire orbit is suspect and requires more careful scrutiny. Appendix C describes a linear approximation for calculating local time as a function of latitude that is used between ±60° for this purpose.

3. Data Grouping Scheme

The most recent set of BUV data tapes covers the period from April 10, 1970 through May 6, 1977 and contains 1,034,456 total ozone observations. There are 20 parameters associated with each observation as described in Table B-2. This amounts to 20,689,120 computer words of data that are contained on these tapes. In order to work with such large quantities of data they must be grouped in a manageable form and stored in such a way as to be easily retrievable.

It was decided to group the data according to a global grid system each element of which would be 5° in latitude by 15° in longitude. This arrangement lends itself nicely to the format of a data array dimensioned 36 x 24 where there are 36 rows representing the 5° latitudinal zones and 24 columns representing the 15° longitudinal sectors. This global grid system is illustrated in Figure 1. The indices shown in the figure follow from the expressions,

$$
i = \begin{cases} (\theta/5°) + 1, & \text{for } 0° \le \theta < 90° \\ (\theta/5°) + 19, & \text{for } -90° < \theta < 0° \end{cases} \tag{1}
$$

and

$$
j = (\phi_W/15°) + 1, \quad \text{for} \quad 0° \le \phi_W < 360°, \tag{2}
$$

where θ is the latitude and ϕ_W is the longitude measured westward from the PM.

As the data are being grouped into this grid, it is convenient to compile a set of elementary statistics describing the data's behavior. Useful quantities that can be readily calculated for a given time period include:

1. Sampling distribution
2. Data means
3. Data variances.

The basis for this analysis is the grid format described. Each obervation is placed in a grid block based on its latitude (θ) and longitude (ϕ_W) according to equations (1) and (2). The global sampling distribution can readily be determined by counting the accumulation of observations into each block (i,j). The zonal data distribution is found by summing this result over j ($1 \le j \le 24$) for each individual zone ($1 \le i \le 36$).

For each grid block the mean ozone value, the mean position of observations, and the mean time of observations are calculated as shown below:

$$
X_{ij} = \sum_{\ell=1}^{k_{ij}} d_\ell / k_{ij} \tag{3}
$$

where the d_ℓ represent the ℓth data record of either latitude, longitude, time, or ozone value contained in block (i,j); k_{ij} is the number of observations contained in block (i,j); and X_{ij} is the block mean for whichever of the above quantities is represented by d_ℓ.

Zonal means are calculated by

$$X_i = \sum_{j=1}^{24} \left(\sum_{\ell=1}^{k_{ij}} d_\ell \right)_j \Big/ \sum_{j=1}^{24} k_{ij}, \tag{4}$$

or

$$X_i = \sum_{j=1}^{24} X_{ij}\, k_{ij}/h_i, \tag{5}$$

where

$$h_i = \sum_{j=1}^{24} k_{ij}. \tag{6}$$

Associated variance calculations follow from

$$\mathrm{VAR}(\chi) \equiv \langle(\chi - \langle\chi\rangle)^2\rangle. \tag{7}$$

Then the variance of the data contributing to the grid block mean becomes

$$\sigma^2_{k_{ij}} = \frac{1}{k_{ij}-1}\left[\sum_{\ell=1}^{k_{ij}} d_\ell^2 - k_{ij}\, X_{ij}^2\right], \tag{8}$$

and the variance of the data contributing to the zonal mean becomes

$$\sigma^2_{h_i} = \frac{1}{h_i-1}\left[\sum_{j=1}^{24} \left(\sum_{\ell=1}^{k_{ij}} d_\ell^2\right)_j - h_i\, X_i^2\right]. \tag{9}$$

The subscripts on σ^2 show the number of ozone observations in the sample being considered.

Finally, a "data mean" and variance are calculated which include all available data from the global grid. The data mean is

$$X = \sum_{i=1}^{36} \sum_{j=1}^{24} \left(\sum_{\ell=1}^{k_{ij}} d_\ell \right)_{ij} \Big/ \sum_{i=1}^{36} \sum_{j=1}^{24} k_{ij}, \tag{10}$$

or

$$X = \sum_{i=1}^{36} \sum_{j=1}^{24} X_{ij}\, k_{ij}/M, \tag{11}$$

where

$$M = \sum_{i=1}^{36} \sum_{j=1}^{24} k_{ij}. \tag{12}$$

This "data mean", X, is not referred to as a global mean since the spatial distribution of the BUV data is non-uniform and, therefore, X is necessarily area biased. In addition, these data do not provide global coverage due to orbit and sensor design. In fact, BUV annual coverage extends only from approximately 80° south latitude to 80° north latitude. Otherwise, the extent to which the global grid is filled depends on the length of the time interval being considered and upon the actual portion of the BUV mission being examined. The latter is due to the fact that the data density per unit time decreases in the later years of the mission.

The variance of the data contributing to the data mean is

$$\sigma_M^2 = \frac{1}{M-1} \left[\sum_{i=1}^{36} \sum_{j=1}^{24} \left(\sum_{\ell=1}^{k_{ij}} d_\ell^2 \right)_{ij} - MX^2 \right]. \tag{13}$$

The computer program OZSTAT2 was written to perform these analysis tasks. Graphics capabilities included in the OZSTAT2 program provide for each case a plot of the zonal means with $\pm 1\sigma_{X_i}$ error bars, a scatter diagram of the ozone distribution as a function of latitude, and histograms of the data sampling distribution as a function of latitude or longitude. Examples of the graphics output are shown in Figures 2 through 4. A listing of this computer program and accompanying subroutines is included in Appendix I.

A means of storing and accessing these reduced data for specified time intervals is required. Typical time periods examined in this study include seasonal (90 days), monthly (30 days), weekly (7 days), and, less frequently, daily intervals. It was, therefore, decided to store this information on a daily basis in such a way that data for larger time intervals can conveniently be generated by accumulating the appropriate daily values.

Specific quantities that must be accessible on a daily basis per grid block are,

1. Sampling Distribution
2. Ozone Mean
3. Average Latitude of Observations
4. Average Longitude of Observations
5. Average Time of Observations

6-9. Variances Associated with Items 2-5 above.

Rather than storing these specific nine pieces of information per grid block, it was decided to save the sums and the sums of the squares of the ozone, time, latitudinal, and longitudinal values along with the sampling distribution from which the required means and variances are readily calculable by

$$\bar{\chi}_\ell = \sum_{ij} \chi_{ij\ell}/k_{ij} \tag{14}$$

and

$$\sigma^2_{\chi_\ell} = \frac{1}{k_{ij}-1}\left[\sum_{ij} \chi^2_{ij\ell} - (\sum_{ij} \chi_{ij\ell})^2/k_{ij}\right] \tag{15}$$

where $\bar{\chi}_\ell$ is the mean, $\sigma^2_{\chi_\ell}$ is the associated variance, $\sum_{ij} \chi_{ij\ell}$ is the sum, and $\sum_{ij} \chi^2_{ij\ell}$ is the sum of the squares of the ℓth quantity for grid block (i,j). The ℓ's signify the following:

$\ell = 1$ Ozone,
$\ell = 2$ Time,
$\ell = 3$ Latitude,
$\ell = 4$ Longitude.

The number of samples per block (i,j) is k_{ij}.

It was further decided to store this information on a mass storage random access (MSRA) file, primarily because this approach minimizes the computer storage problem inherent with these large data sets and also because of the convenience associated with utilizing the MSRA file for this kind of storage and retrieval process. A set of subroutines have been designed to access this MSRA file returning to the calling program a data array in the form of the standard 36 x 24 global grid system containing one of the nine quantities mentioned above for a given day or collection of days. These subroutines can be easily incorporated into computer programs requiring these grided ozone data without drastically affecting the program's storage requirement. Details concerning the MSRA file, its creation and its access are contained in Appendix D.

The preliminary data analysis concepts discussed above are beneficial for the following reasons:

1. Setting up a standard grid network as outlined establishes a basis for data analysis and lends itself nicely to making preliminary statistical calculations.
2. The preliminary statistical analysis shows how the data distribution varies as a function of latitude and longitude which helps in the development of mission sampling strategies.
3. Large data sets become more easily manageable when described by a global grid network which can be put into the form of a data array in the computer and saved on MSRA files.

III. STATISTICAL MODELING AND ANALYSIS TECHNIQUES

An essential part of this mission sampling study is the development of models which describe the variability of the global ozone field and the statistical analysis techniques which can be used to evaluate these models and the sampling schemes that they represent. The model primarily used in this work has been the Spherical Harmonic model, though the modeling of data with Empirical Orthogonal Functions has also been investigated and used to some extent throughout the effort. These models and certain statistical analysis techniques have been incorporated into computer programs which will be discussed.

Cases arise where it is desirable to have a completely filled global grid system. The BUV data does not provide this required global coverage. A computer program has been prepared to handle this missing data problem using either a Spherical Harmonic model or a "model" based on autocorrelation functions. The data fill problem is discussed later in this section.

4. Spherical Harmonic Model - Parameter Estimation and Evaluation

The form of the spherical harmonic model chosen for this study is,

$$y(\theta_i,\phi_i) = \sum_{m=o}^{M} \sum_{n=m}^{M} [A_{mn} Y^e_{mn}(\theta_i,\phi_i) + D_{mn} Y^o_{mn}(\theta_i,\phi_i)] + \varepsilon_i \qquad (16)$$

where

$$Y^e_{mn}(\theta,\phi) = \cos(m\phi)\, F^s_{mn} P^m_n(\cos\theta), \qquad (17)$$

$$Y^o_{mn}(\theta,\phi) = \sin(m\phi)\, F^s_{mn} P^m_n(\cos\theta), \qquad (18)$$

$$F^s_{mn} = \begin{cases} 1, & \text{for } m=o \\ \left[\dfrac{2(n-m)!}{(n+m)!}\right]^{1/2}, & \text{for } m>o \end{cases}, \qquad (19)$$

$P_n^m(\cos\theta)$ are the associated Legendre functions of degree n and order m, and A_{mn} and D_{mn} are the coefficients associated with the functions $Y_{mn}^e(\theta,\phi)$ and $Y_{mn}^o(\theta,\phi)$, respectively. F_{mn}^s is the Adolf Schmidt seminormalization constant.[4] ε_i is the error associated with the ith observation at colatitude θ_i and longitude ϕ_i.

For a given data set coefficients for a spherical harmonic model of specified degree and order are determined by a least squares solution that minimizes the sum of the squares of the residuals $\underline{\varepsilon}^T\underline{\varepsilon}$.

Equation (16) above can be rewritten as

$$y(\theta_i,\phi_i) = \sum_{n=1}^{N} f_n(\theta_i,\phi_i)\, b_n + \varepsilon_i \tag{20}$$

where both the odd and even functions, $Y_{mn}^o(\theta,\phi)$ and $Y_{mn}^e(\theta,\phi)$, are included in $f(\theta,\phi)$, and, similarly, the coefficients A_{mn} and D_{mn} are included in b. Some care must be exercised in maintaining the proper ordering of the terms in equation (20). Note that N in equation (20) is the number of coefficients (and therefore the number of functions) contained in the model and not the degree of the model. Generally, the order and degree of the spherical harmonic models used in this study are equal. If a specified model is of order M and degree M, then

$$N = (M + 1)^2. \tag{21}$$

Equation (20) can be written in matrix form as

$$\underline{Y} = \underline{\underline{F}}\ \underline{B} + \underline{E}. \tag{22}$$

The double underline signifies a matrix quantity while a single underline denotes a vector. To minimize the sum of the squares of the residuals the quantity

$$SS = \underline{E}^T\underline{E} = (\underline{Y} - \underline{\underline{F}}\ \underline{B})^T(\underline{Y} - \underline{\underline{F}}\ \underline{B}) \tag{23}$$

must be differentiated with respect to $\underline{B}$. This leads to the so-called "normal equations" which can be solved such that

$$\hat{\underline{B}} = (\underline{\underline{F}}^T \underline{\underline{F}})^{-1} \underline{\underline{F}}^T \underline{Y}. \quad [5] \tag{24}$$

The estimated coefficients contained in the $\hat{\underline{B}}$ vector are unbiased since

$$\hat{\underline{B}} = \underline{B}. \quad [6]$$

Information regarding the sampling can also be gained from equation (24). To that end calculate the covariance of $\hat{\underline{B}}$ as follows. Rewrite equation (24) as

$$\hat{\underline{B}} = \underline{\underline{G}}\, \underline{Y}, \tag{25}$$

where $\underline{\underline{G}} = (\underline{\underline{F}}^T \underline{\underline{F}})^{-1} \underline{\underline{F}}^T$ is a function of sampling position only and is therefore treated here as a constant. The covariance matrix for $\hat{\underline{B}}$ can be found by the law of propagation of errors[6] such that

$$\underline{\underline{Covar}}(\hat{\underline{B}}) = \underline{\underline{G}}\, \underline{\underline{Covar}}\, (\underline{Y})\, \underline{\underline{G}}^T. \tag{26}$$

It is assumed that all components of $\underline{Y}$ are independent,

$$Covar(y_i, y_j) = 0 \qquad \text{for } i \neq j, \tag{27-a}$$

and have the same variance,

$$Var(y_i) = \sigma^2 , \tag{27-b}$$

so that

$$\underline{\underline{Covar}}(\underline{Y}) = \sigma^2\, \underline{\underline{I}} \tag{27-c}$$

where $\underline{\underline{I}}$ is the identity matrix.

Then equation (26) may be rewritten as

$$\underline{\underline{Covar}}(\hat{\underline{B}}) = \underline{\underline{G}}\, \sigma^2\, \underline{\underline{I}}\, \underline{\underline{G}}^T \tag{28-a}$$

$$= [(\underline{\underline{F}}^T \underline{\underline{F}})^{-1} \underline{\underline{F}}^T]\, [(\underline{\underline{F}}^T \underline{\underline{F}})^{-1} \underline{\underline{F}}^T]^T \sigma^2 \tag{28-b}$$

$$\underline{\underline{Covar}}(\hat{\underline{B}}) = [\underline{\underline{F}}^T \underline{\underline{F}})^{-1} \underline{\underline{F}}^T F[(F^T F)^{-1}]^T \sigma^2. \tag{28-c}$$

Now consider some symmetric matrix $\underline{\underline{z}}$. Since the operations TRANSPOSE(T) and invserse (-1) commute[7],

$$(\underline{\underline{z}}^{-1})^T = (\underline{\underline{z}}^T)^{-1} . \tag{29}$$

But $\underline{\underline{Z}}$ is also symmetric; therefore,

$$\underline{\underline{Z}} = \underline{\underline{Z}}^T , \tag{30}$$

and

$$(\underline{\underline{Z}}^{-1})^T = \underline{\underline{Z}}^{-1} . \tag{31}$$

As $\underline{\underline{F}}^T\underline{\underline{F}}$ is also a symmetric matrix, by applying equation (31), equation (28-c) may be written as

$$\underline{\underline{Covar}}(\hat{\underline{B}}) = (\underline{\underline{F}}^T\underline{\underline{F}})^{-1} \sigma^2, \tag{32}$$

where the variances associated with the estimated coefficients $\hat{b}_n$ are the corresponding diagonal elements of the covariance matrix. The off-diagonal elements are, of course, the covariance terms. In this study σ^2 has typically been set equal to one, so that

$$\underline{\underline{Covar}}(\hat{\underline{B}}) = (\underline{\underline{F}}^T\underline{\underline{F}})^{-1}. \tag{33}$$

This is an important result that statistically describes how well the model can be fitted to the sample space being considered. Recall that this result is independent of the observation vector $\underline{Y}$.

An interesting, if only heuristic, illustration is the case where only one sample position is contained in the sampling scheme for a spherical harmonic model. Consider the product of the observation matrix $\underline{\underline{F}}$ and its transpose written as

$$\underline{\underline{S}} = \underline{\underline{F}}^T\underline{\underline{F}} = \begin{bmatrix} \sum_{i=1}^{P} f_{i1}^2 & \sum_{i=1}^{P} f_{i1}f_{i2} & \cdots & \sum_{i=1}^{P} f_{i1}f_{iN} \\ \sum_{i=1}^{P} f_{i2}F_{i1} & \sum_{i=1}^{P} f_{i2}^2 & \cdots & \cdot \\ \vdots & & & \\ \sum_{i=1}^{P} f_{iN}f_{i1} & & \cdots & \sum_{i=1}^{P} f_{iN}^2 \end{bmatrix} , \tag{34}$$

where P is the number of observations and the function f is the same as it was in equation (20). Then

$$S_{kj} = \sum_{i=1}^{P} f_k(\theta_i,\phi_i)\ f_j(\theta_i,\phi_i), \tag{35}$$

or

$$S_{kj} = f_k(\theta_1,\phi_1)\ f_j(\theta_1,\phi_1) + f_k(\theta_2,\phi_2)\ f_j(\theta_2,\phi_2) + \ldots + f_k(\theta_p,\phi_p)\ f_j(\theta_p,\phi_p)\ . \tag{36}$$

The one sample position occurs at

$$\theta = \theta_1 = \theta_2 = \ldots = \theta_p$$

and

$$\phi = \phi_1 = \phi_2 = \ldots = \phi_p.$$

Equation (36) then becomes

$$S_{kj} = P\ f_k(\theta,\phi)\ f_j(\theta,\phi), \tag{37}$$

and equation (34) becomes

$$\underline{\underline{S}} = \underline{\underline{F}}^T\underline{\underline{F}} = P \begin{bmatrix} f_1^2 & f_1f_2 \ldots & f_1f_N \\ f_2f_1 & f_2^2 \ldots & \\ \vdots & & \\ f_Nf_1 & \ldots & f_N^2 \end{bmatrix} . \tag{38}$$

Now if for some matrix $\underline{\underline{z}}$ all elements of a row (or column) may be obtained from the elements of another row (or column) by multiplication by a constant, that is, if z_{ij} = (constant) $z_{\ell j}$ for all j or z_{ij} = (constant) $z_{i\ell}$ for all i, then det $\underline{\underline{z}} = 0$. [6]

Also the inverse of a matrix $\underline{\underline{z}}$ can be calculated element by element according to

$$(z^{-1})_{ij} = \frac{\text{cofactor } (z_{ji})}{\det(\underline{\underline{z}})} , \tag{39}$$

where $\det(\underline{\underline{z}})$ is the determinant of the $\underline{\underline{z}}$ matrix.

It can then be seen from equations (38) and (39) that

$$\det(\underline{\underline{s}}) = 0 , \tag{40-a}$$

hence the covariance matrix

$$\underline{\underline{S}}^{-1} \to \infty , \tag{40-b}$$

or the variance associated with estimated coefficient values would be infinite.

This result demonstrates the inability of the least squares technique to accurately estimate the required coefficients of a model with N functions ($N > 1$) when the sample space consists of only one point. A more general comment that may be inferred from this example is that the variance in the estimated coefficient vector is a function of the sampling distribution and is not necessarily dependent on the number of samplings.

5. Statistical Analysis of Spherical Harmonic Model

The data variance σ_d^2 of the observations contained in the vector $\underline{Y}$ is calculated by

$$\sigma_d^2 = \frac{1}{(P-1)} \left[\sum_{i=1}^{P} y_i^2 - \left(\sum_{i=1}^{P} y_i \right)^2 / P \right] , \tag{41}$$

which comes simply from the definition of variance as in equation (7), where P is, as above, the number of observations.

The RMS residual between the measurement and the spherical harmonic model is,

$$\text{RMS} = \left[\frac{1}{P} \underline{\underline{E}}^T \underline{\underline{E}} \right]^{1/2} . \tag{42}$$

By equation (23),

$$\text{RMS} = \left[\frac{1}{P} (\underline{Y} - \underline{\underline{F}}\, \underline{\hat{B}})^T (\underline{Y} - \underline{\underline{F}}\, \hat{B}) \right]^{1/2} \tag{43}$$

where $\hat{\underline{B}}$ is the vector of estimated coefficients that minimizes the RMS. Equation (43) can be expanded so that

$$\mathrm{RMS} = \left[\frac{1}{P}\left(\underline{Y}^T\underline{Y} - \underline{Y}^T\underline{\underline{F}}\,\hat{\underline{B}} - \hat{\underline{B}}^T\underline{\underline{F}}^T\underline{Y} + \hat{\underline{B}}^T\underline{\underline{F}}^T\underline{\underline{F}}\,\hat{\underline{B}}\right)\right]^{1/2} . \tag{44}$$

Note the term

$$\underline{Y}^T\underline{\underline{F}}\,\hat{\underline{B}} = \underline{Y}^T(1 \times P) \times \underline{\underline{F}}(P \times N) \times \hat{\underline{B}}(N \times 1)$$

is a scalar. Therefore,

$$\underline{Y}^T\underline{\underline{F}}\,\hat{\underline{B}} = (\underline{Y}^T\underline{\underline{F}}\,\hat{\underline{B}})^T = \hat{\underline{B}}^T\underline{\underline{F}}^T\underline{Y} , \tag{45}$$

and

$$\mathrm{RMS} = \left[\frac{1}{P}\left(\underline{Y}^T\underline{Y} - 2\hat{\underline{B}}^T\underline{\underline{F}}^T\underline{Y} + \hat{\underline{B}}^T\underline{\underline{F}}^T\underline{\underline{F}}\,\hat{\underline{B}}\right)\right]^{1/2}. \tag{46}$$

Substitution of equation (24) into the last term of equation (46) leads to

$$\mathrm{RMS} = \left[\frac{1}{P}\left(\underline{Y}^T\underline{Y} - 2\hat{\underline{B}}^T\underline{\underline{F}}^T\underline{Y} + \hat{\underline{B}}^T\underline{\underline{F}}^T\underline{Y}\right)\right]^{1/2} . \tag{47}$$

In this manipulation it must be remembered that

$$(\underline{\underline{F}}^T\underline{\underline{F}})(\underline{\underline{F}}^T\underline{\underline{F}})^{-1} = \underline{\underline{I}},$$

where $\underline{\underline{I}}$ is the identity matrix.

Equation (47) quickly simplifies to

$$\mathrm{RMS} = \left[\frac{1}{P}\left(\underline{Y}^T\underline{Y} - \hat{\underline{B}}^T\underline{\underline{F}}^T\underline{Y}\right)\right]^{1/2}, \tag{48-a}$$

or

$$\mathrm{RMS}^2 = \frac{1}{P}\left(\underline{Y}^T\underline{Y} - \hat{\underline{B}}^T\underline{\underline{F}}^T\underline{Y}\right). \tag{48-b}$$

The RMS^2 value above is also known as the error variance, σ_e^2. This is the portion of the data variance not explained by the model. The model variance is then

$$\sigma_m^2 = \sigma_d^2 - \sigma_e^2 .$$

The ratio

$$R^2 = \sigma_m^2/\sigma_d^2$$

is often used as a criteria to judge the adequacy of the assumed model where

$$0 \le R^2 \le 1 \ .$$

R^2 must approach unity for the model to account for the data variability. The importance of terms in a given model can be measured by the relative power of their coefficients. Of specific interest is the power of the coefficients of degree n. This quantity is referred to as the degree variance, σ_n^2, and is defined as the average square of the spherical harmonic solution, $\hat{y}_n(\theta,\phi)$, for degree n,[8] or

$$\sigma_n^2 \equiv \frac{\int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \hat{y}_n^2(\theta,\phi)da}{\int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} da}, \tag{49}$$

where da is the differential area $\sin\theta d\theta d\phi$, and

$$\hat{y}_n(\theta,\phi) = \sum_{m=0}^{n} [A_{mn}Y_{mn}^e(\theta,\phi) + D_{mn}Y_{mn}^o(\theta,\phi)] \tag{50}$$

The $Y_{mn}^e(\theta,\phi)$ and $Y_{mn}^o(\theta,\phi)$ are spherical harmonic functions as defined in equations (17) and (18). The A_{mn} and D_{mn} are the coefficients associated with $Y_{mn}^e(\theta,\phi)$ and $Y_{mn}^o(\theta,\phi)$, respectively. Letting the notation $\int_{\theta,\phi}$ denote the double integral $\int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi}$, the numerator of equation (49) may be written as

$$I_1 = \sum_{m=0}^{n} \int_{\theta,\phi} [A_{mn}Y_{mn}^e(\theta,\phi) + D_{mn}Y_{mn}^o(\theta,\phi)]^2 \, da, \tag{51}$$

or

$$I_1 = \sum_{m=0}^{n} [A_{mn}^2 \int_{\theta,\phi} (Y_{mn}^e)^2 \, da + 2 A_{mn} D_{mn} \int_{\theta,\phi} Y_{mn}^e Y_{mn}^o \, da +$$

$$D_{mn}^2 \int_{\theta,\phi} (Y_{mn}^o)^2 \, da]. \quad (52)$$

These integrals are evaluated in Appendix E so that

$$I_1 = \frac{4\pi}{2n+1} \sum_{m=0}^{n} [A_{mn}^2 + D_{mn}^2 \; \delta_{mo}^*]. \quad (53)$$

The denominator of equation (49) is

$$I_2 = \int_{\theta,\phi} da = 4\pi,$$

such that,

$$\sigma_n^2 = \sum_{m=o}^{n} (A_{mn}^2 + D_{mn}^2 \; \delta_{mo}^*)/(2n+1),$$

or acknowledging the fact that $D_{mn} \equiv o$ for $m = o$,

$$\sigma_n^2 = \frac{1}{2n+1} \sum_{m=o}^{n} (A_{mn}^2 + D_{mn}^2). \quad (54\text{-a})$$

The total power in the model coefficients can be found by summing over the M degree variances such that

$$\text{Total Power} = \sum_{n=o}^{M} \sigma_n^2 = \sum_{n=o}^{M} \frac{1}{2n+1} \sum_{m=o}^{n} (A_{mn}^2 + D_{mn}^2). \quad (54\text{-b})$$

Also of interest is the integral

$$I_3 = \int_{\theta,\phi} \hat{y}_n(\theta,\phi) \; \hat{y}_\ell(\theta,\phi) da, \quad (55)$$

or

$$I_3 = \sum_{m=o}^{n} [A_{mn}A_{m\ell} \int_{\theta,\phi} Y^e_{mn}Y^e_{m\ell}\ da + A_{mn}D_{m\ell} \int_{\theta,\phi} Y^e_{mn}Y^o_{m\ell}\ da +$$

$$+ D_{mn}A_{m\ell} \int_{\theta,\phi} Y^o_{mn}Y^e_{m\ell}\ da + D_{mn}D_{m\ell} \int_{\theta,\phi} Y^o_{mn}Y^o_{m\ell}\ da] \ . \qquad (56)$$

These integrals are also evaluated in Appendix E, so that

$$I_3 = 0.$$

Then the degree covariances,

$$\sigma^2_{n\ell} = \frac{\int_{\theta,\phi} \hat{y}_n(\theta,\phi)\ \hat{y}_\ell(\theta,\phi)\ da}{\int_{\theta,\phi} da}, \qquad (57)$$

are zero.

The contribution of the zonal coefficients to σ^2_n is easily determined from

$$P_{zn} = \frac{Z_n}{\sigma^2_n} \times 100\%, \qquad (58)$$

where P_{zn} is the percentage of the zonal contribution to the degree variance at degree n, and where by equation (54-a) for m = o,

$$Z_n = \frac{A^2_{on} + D^2_{on}}{2n + 1} \qquad (59)$$

is the zonal contribution for degree n.

But D_{mn} does not exist for m = o since by equation (18) $Y^o_{m=o,n}(\theta,\phi) = 0$, so

$$Z_n = \frac{A^2_{on}}{2n + 1} \ . \qquad (60)$$

Substituting this result along with equation (54-a) into equation (58) gives

$$P_{zn} = \frac{A_{on}^2}{\sum\limits_{m=o}^{n} (A_{mn}^2 + D_{mn}^2)} \times 100\%, \tag{61}$$

again remembering that $D_{mn} = o$ when $m = o$.

This result is useful in determining the relative importance of the zonal contribution to the nth degree variance.

The model statistics discussed above, P_{zn}, degree variance, and total power, explain the distribution of power in the spherical harmonic model.

Computer program GLSRAN2 performs the various calculations mentioned thus far in this section. Comments concerning the associated Lengendre function recurrence relations utilized in GLSRAN2 are given in Appendix F. Specific details concerning file manipulations, calculation methods, and output are elaborated on in Appendix G.

Further statistical analyses are performed utilizing the results of computer program GLSRAN2 mentioned above. These are the zonal and global means and variances as based on the least squares fit to the spherical harmonic model.

First, it is desired to derive an expression for the ozone value as a function of colatitude only which will serve as an estimate of the zonal mean, $\bar{z}(\theta)$. To accomplish this the model estimate as given in equation (16) is integrated with respect to longitude such that

$$\bar{z}(\theta) = \frac{\int_{\phi=o}^{2\pi} \hat{y}(\theta,\phi)d\phi}{\int_{\phi=o}^{2\pi} d\phi} . \tag{62}$$

The numerator may be written as

$$\int_{\phi=o}^{2\pi} \hat{y}(\theta,\phi)d\phi = \sum_{m=o}^{M} \sum_{n=m}^{M} F_{mn}^{s} P_n^m(\cos\theta) \int_{\phi=o}^{2\pi} [\hat{A}_{mn}\cos(m\phi) + \hat{D}_{mn}\sin(m\phi)]d\phi, \tag{63}$$

or

$$\int_{\phi=0}^{2\pi} \hat{y}(\theta,\phi)d\phi = 2\pi \sum_{n=0}^{M} \hat{A}_{on}P_n(\cos\theta) \ .$$

Then

$$\bar{z}(\theta) = \sum_{n=0}^{M} \hat{A}_{on}P_n(\cos\theta) \ . \tag{64}$$

The estimated global mean is found from

$$\bar{g} = \frac{\int_{\theta,\phi} \hat{y}(\theta,\phi) \, da}{\int_{\theta,\phi} da} \ . \tag{65}$$

The numerator may be written as

$$\int_{\theta}\int_{\phi} \hat{y}(\theta,\phi) \, da = \sum_{m=0}^{M} \sum_{n=m}^{M} F^{s}_{mn} \int_{\theta=0}^{\pi} P^{m}_{n}(\cos\theta) \sin\theta d\theta \int_{\phi=0}^{2\pi} [\hat{A}_{mn}\cos(m\phi)$$

$$+ \hat{D}_{mn}\sin(m\phi)] \, d\phi. \tag{66}$$

Evaluation of the integration over ϕ gives

$$\int_{\phi=0}^{2\pi} [\hat{A}_{mn}\cos(m\phi) + \hat{D}_{mn}\sin(m\phi)] \, d\phi = \begin{cases} 0 & , \text{ for } m \neq o \\ 2\pi\hat{A}_{on}, & \text{for } m = o \end{cases} \ . \tag{67}$$

The integral over θ in equation (66) may be written as

$$\int_{\theta=0}^{\pi} P^{m}_{n}(\cos\theta) \sin\theta d\theta = \int_{x=-1}^{1} P_n(x) \, dx \tag{68}$$

using the substitution $x = \cos\theta$ and where because of equation (67) there is only reason to evaluate the integral for $m = o$.

From Appendix E

$$\int_{x=-1}^{1} P_{\ell}(x) \, P_n(x) \, dx = \frac{2}{2n+1} \delta_{n\ell};$$

then, if $\ell = 0$,

$$\int_{x=-1}^{1} P_o(x)\, P_n(x)\, dx = 2\delta_{no}. \tag{69}$$

Recalling that $P_o(x) = 1$, equation (69) may be written as

$$\int_{x=-1}^{1} P_n(x)\, dx = 2\delta_{no}, \tag{70}$$

and the numerator of equation (65) becomes

$$\int_{\theta,\phi} \hat{y}(\theta,\phi)\, da = \begin{cases} 4\pi\hat{A}_{oo}, & \text{for } m = n = 0 \\ 0 & \text{, otherwise} \end{cases} . \tag{71}$$

Finally, since $\int_{\theta,\phi} da = 4\pi$, the estimated global mean is

$$\overline{g} = \hat{A}_{oo} , \tag{72-a}$$

the variance of which is

$$\text{Var}(\overline{g}) = \text{Var}(\hat{A}_{oo}), \tag{72-b}$$

where $\text{Var}(\hat{A}_{oo})$ is calculated by equation (33).

The global mean can also be calculated in terms of area weighted zonal means. Another representation of the variance of the global mean can be estimated from this result in terms of $\underline{\underline{C}}\text{ovar}(\underline{\hat{B}})$ elements. This technique is developed below.

In terms of zonal means the global mean may be written as

$$\overline{g}_z = \sum_{i=1}^{I} a_i \, \overline{z}_i / A \tag{73}$$

where $\overline{z}_i$ is the estimated mean for the $i\underline{th}$ zone, the constant weighting factor a_i is the surface area of the $i\underline{th}$ zone, and

$$A = \sum_{i=1}^{I} a_i$$

is the global surface area. Equation (73) may be rearranged as

$$A\overline{g}_z = \sum_{i=1}^{I} a_i \, \overline{z}_i \ , \tag{74}$$

and, if the variance is taken of both sides, it becomes

$$\mathrm{Var}(A\overline{g}_z) = \mathrm{Var}\left(\sum_{i=1}^{I} a_i \, \overline{z}_i \right) \tag{75}$$

where I is the number of zones and the $\overline{z}_i$ are to be treated as random variables. By the definition of variance (equation 7) the right hand side of equation (75) becomes

$$\mathrm{Var}\left(\sum_{i=1}^{I} a_i \, \overline{z}_i \right) = \left\langle \left(\sum_{i=1}^{I} a_i \, \overline{z}_i - \left\langle \sum_{i=1}^{I} a_i \, \overline{z}_i \right\rangle \right)^2 \right\rangle$$

$$= \left\langle \left(\sum_{i=1}^{I} a_i \, \overline{z}_i - \sum_{i=1}^{I} a_i \langle \overline{z}_i \rangle \right)^2 \right\rangle$$

$$\mathrm{Var}\left(\sum_{i=1}^{I} a_i \, \overline{z}_i \right) = \left\langle \left(\sum_{i=1}^{I} a_i \, Z_i \right)^2 \right\rangle , \tag{76}$$

where

$$Z_i = (\overline{z}_i - \langle \overline{z}_i \rangle) . \tag{77}$$

Equation (76) may be expanded such that

$$\mathrm{Var}\left(\sum_{i=1}^{I} a_i \bar{z}_i\right) = \langle (a_1 Z_1)^2 + (a_2 Z_2)^2 + \ldots$$

$$+ (a_I Z_I)^2 + 2a_1 Z_1 a_2 Z_2 + \ldots + 2a_{I-1} Z_{I-1} a_I Z_I \rangle$$

or

$$\mathrm{Var}\left(\sum_{i=1}^{I} a_i \bar{z}_i\right) = \left\langle \sum_{i=1}^{I} a_i^2 Z_i^2 + 2 \sum_{j=1}^{I-1} \sum_{k=j+1}^{I} a_j a_k Z_j Z_k \right\rangle. \tag{78}$$

Notice that

$$\langle Z_i^2 \rangle = \langle (\bar{z}_i - \langle \bar{z}_i \rangle)^2 \rangle = \mathrm{Var}(\bar{z}_i) \tag{79}$$

and

$$\langle Z_j Z_k \rangle = (\bar{z}_j - \langle \bar{z}_j \rangle)(\bar{z}_k - \langle \bar{z}_k \rangle) = \mathrm{Covar}(\bar{z}_j, \bar{z}_k); \tag{80}$$

then equation (78) becomes

$$\mathrm{Var}\left(\sum_{i=1}^{I} a_i \bar{z}_i\right) = \sum_{i=1}^{I} a_i^2 \mathrm{Var}(\bar{z}_i) + 2 \sum_{j=1}^{I-1} \sum_{k=j+1}^{I} a_j a_k \mathrm{Covar}(\bar{z}_j, \bar{z}_k) . \tag{81}$$

The left side of equation (75) is

$$\mathrm{Var}(A\bar{g}_z) = A^2 \mathrm{Var}(\bar{g}_z) . \tag{82}$$

Equating equations (81) and (82) it is found that

$$\mathrm{Var}(\bar{g}_z) = \frac{\sum_{i=1}^{I} a_i^2 \mathrm{Var}(\bar{z}_i) + 2 \sum_{j=1}^{I-1} \sum_{k=j+1}^{I} a_j a_k \mathrm{Covar}(\bar{z}_j, \bar{z}_k)}{A^2} . \tag{83}$$

$\mathrm{Var}(\bar{z}_i)$ is the variance of the mean for zone i. The colatitude position of zone i is taken to be at θ_i so that from equation (64)

$$\mathrm{Var}(\bar{z}_i) = \mathrm{Var}[\sum_{n=o}^{M} \hat{A}_{on} P_n(\cos\theta_i)] \tag{84}$$

or

$$\mathrm{Var}(\bar{z}_i) = \mathrm{Var}(\sum_{n=o}^{M} P_{ni} \hat{A}_{on}) \tag{85}$$

where P_{ni} is the $n\underline{th}$ degree Legendre function evaluated for θ_i. Equation (85) may be evaluated in an analogous fashion to the technique used in the evaluation of equation (75) where $\hat{A}_{on}$ is to be treated as the random variable. Then equation (85) may be rewritten by comparison with equation (83) as

$$\mathrm{Var}(\bar{z}_i) = \sum_{n=o}^{M} P_{ni}^2 \mathrm{Var}(\hat{A}_{on}) + 2 \sum_{n=o}^{M-1} \sum_{\ell=n+1}^{M} P_{ni} P_{\ell i} \mathrm{Covar}(\hat{A}_{on}, \hat{A}_{o\ell}). \tag{86}$$

In order to complete the evaluation of equation (83) $\mathrm{Covar}(\bar{z}_j, \bar{z}_k)$ must be written in terms of known quantities. Recall that the covariance is defined as

$$\mathrm{Covar}(x,y) \equiv \langle (x - \langle x \rangle)(y - \langle y \rangle) \rangle . \tag{87}$$

Then substituting

$$\bar{z}_i = \sum_{n=o}^{M} P_{ni} \hat{A}_{on} \tag{88}$$

into the covariance definition, equation (87),

$$\mathrm{Covar}(\bar{z}_j, \bar{z}_k) = \langle (\sum_{n=o}^{M} P_{nj}\hat{A}_{on} - \langle \sum_{n=o}^{M} P_{nj}\hat{A}_{on} \rangle)(\sum_{n=o}^{M} P_{nk}\hat{A}_{on} - \langle \sum_{n=o}^{M} P_{nk}\hat{A}_{on} \rangle)$$

$$= \langle (\sum_{n=o}^{M} P_{nj}\hat{A}_{on} - \sum_{n=o}^{M} P_{nj}\langle \hat{A}_{on} \rangle)(\sum_{n=o}^{M} P_{nk}\hat{A}_{on} - \sum_{n=o}^{M} P_{nk}\langle \hat{A}_{on} \rangle) \rangle . \tag{89}$$

Let

$$W_n = \hat{A}_{on} - \langle \hat{A}_{on} \rangle, \tag{90}$$

then

$$\text{Covar}(\bar{z}_j, \bar{z}_k) = \langle (\sum_{n=o}^{M} P_{nj} W_n) (\sum_{n=o}^{M} P_{nk} W_n) \rangle ,$$

and

$$\text{Covar}(\bar{z}_j, \bar{z}_k) = \langle \sum_{n=o}^{M} \sum_{\ell=o}^{M} P_{nj} P_{\ell k} W_n W_\ell \rangle ,$$

or

$$\text{Covar}(\bar{z}_j, \bar{z}_k) = \sum_{n=o}^{M} \sum_{\ell=o}^{M} P_{nj} P_{\ell k} \langle W_n W_\ell \rangle . \tag{91}$$

However, by equation (90),

$$\langle W_n W_\ell \rangle = \langle (\hat{A}_{on} - \langle \hat{A}_{on} \rangle) (\hat{A}_{o\ell} - \langle \hat{A}_{o\ell} \rangle) \rangle = \text{Covar}(\hat{A}_{on}, \hat{A}_{o\ell}). \tag{92}$$

Then substituting equation (92) into equation (91) the required covariance becomes

$$\text{Covar}(\bar{z}_j, \bar{z}_k) = \sum_{n=o}^{M} \sum_{\ell=o}^{M} P_{nj} P_{\ell k} \text{ Covar}(\hat{A}_{on}, \hat{A}_{o\ell}). \tag{93}$$

With the help of a little algebra equation (91) may also be written as

$$\text{Covar}(\bar{z}_j, \bar{z}_k) = \sum_{n=o}^{M} P_{nj} P_{nk} \langle W_n^2 \rangle + \sum_{n=o}^{M-1} \sum_{\ell=n+1}^{M} (P_{nj} P_{\ell k} + P_{\ell j} P_{nk}) \langle W_n W_\ell \rangle . \tag{94}$$

By equation (90)

$$\langle W_n^2 \rangle = \langle (\hat{A}_{on} - \langle \hat{A}_{on} \rangle)^2 \rangle = \text{Var}(\hat{A}_{on}). \tag{95}$$

Substituting equations (92) and (95) into equation (94) gives

$$\text{Covar}\,(\bar{z}_j,\bar{z}_k) = \sum_{n=o}^{M} P_{nj}P_{nk}\,\text{Var}(\hat{A}_{on})$$

$$+ \sum_{n=o}^{M-1} \sum_{\ell=n+1}^{M} (P_{nj}P_{\ell k} + P_{\ell j}P_{nk})\,\text{Covar}(\hat{A}_{on},\hat{A}_{o\ell})\,. \qquad (96)$$

This is a reassuring result since it reduces to equation (86) for the zonal variance when $j = k$.

Computer programs GLOBZON and ZONVAR have been prepared to perform these calculations based on the results of the least squares fit to the spherical harmonic model, specifically the model's zonal coefficients, $\hat{A}_{on}$, and the zonal elements of the covariance matrix, $\text{Covar}(\hat{A}_{on},\hat{A}_{o\ell})$. When the model is written in the form of equation (22) these quantities are calculated from equations (24) and (33), respectively.

To summarize these results the mean, $\bar{z}_i$, for zone i as found by computer program GLOBZON is

$$\bar{z}_i = \sum_{n=o}^{M} P_{ni}\hat{A}_{on}$$

where M is the degree of the model, P_{ni} is the nth degree Legendre function for the colatitude θ_i at the center of the zone, and $\hat{A}_{on}$ is the nth degree zonal coefficient. This result was shown in equation (64) and further developed and used in equation (84).

The global mean, $\bar{g}$, was shown by equation (72) simply to be the first spherical harmonic model coefficient, or

$$\bar{g} = \hat{A}_{oo}.$$

In order to calculate the global variance, $Var(\bar{g}_z)$, the global mean was written out in terms of area weighted zonal means as shown in equation (73). The global variance was found to be

$$Var(\bar{g}_z) = \frac{\sum_{i=1}^{I} a_i^2 \, Var(\bar{z}_i) + 2 \sum_{j=1}^{I-1} \sum_{k=j+1}^{I} a_j a_k \, Covar(\bar{z}_j, \bar{z}_k)}{A^2}$$

as shown in equation (83). A favorable comparison of this result with equation (72-b),

$$Var(\bar{g}) = Var(\hat{A}_{oo}) \ ,$$

tends to confirm the accuracy of the zonal variance calculation as used in the $Var(\bar{g}_z)$ calculation. The zonal variance, $Var(\bar{z}_i)$, is

$$Var(\bar{z}_i) = \sum_{n=o}^{M} P_{ni}^2 \, Var(\hat{A}_{on}) + 2 \sum_{n=o}^{M-1} \sum_{\ell=n+1}^{M} P_{ni} P_{\ell i} \, Covar(\hat{A}_{on}, \hat{A}_{o\ell}),$$

and

$$Covar(\bar{z}_j, \bar{z}_k) = \sum_{n=o}^{M} P_{nj} P_{nk} \, Var(\hat{A}_{on}) + \sum_{n=o}^{M-1} \sum_{\ell=n+1}^{M} (P_{nj} P_{\ell k} + P_{\ell j} P_{nk}) \, Covar(\hat{A}_{on}, \hat{A}_{o\ell}).$$

6. Eigenanalysis - Empirical Orthogonal Functions

The subject of eigenanalysis may best be introduced by means of a simple, if not trivial, illustration. Consider the data shown in the table below.

Table. Data Set as Viewed in the x_1 - x_2 Coordinate System.

Observation No. i	x_{i1}	x_{i2}
1	1	1
2	2	2
3	3	3

The data means and covariance matrix can be calculated as

$$\bar{x}_1 = 2, \tag{97-a}$$

$$\bar{x}_2 = 2, \tag{97-b}$$

and

$$\underline{\underline{C}}\text{OVAR}_x = \begin{pmatrix} \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} \end{pmatrix}, \tag{97-c}$$

where

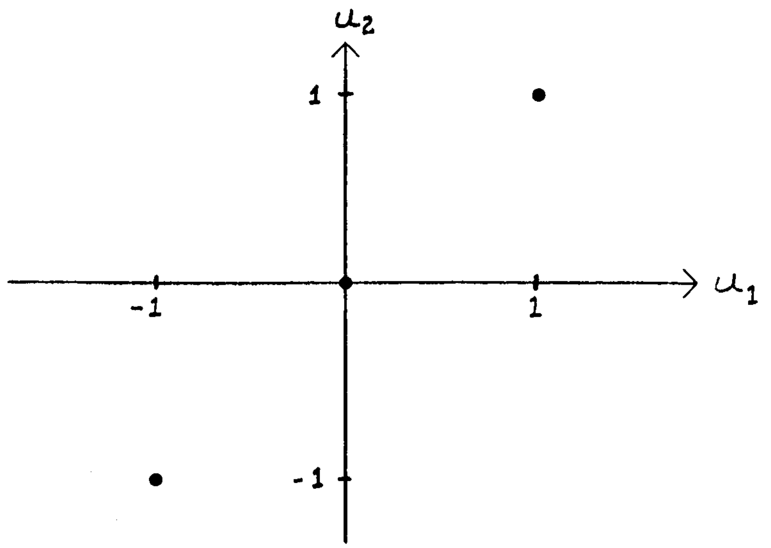
$$\text{COVAR}_{ij} = \frac{1}{3} \sum_{k=1}^{3} (x_{ki} - \bar{x}_i)(x_{kj} - \bar{x}_j). \tag{98}$$

The data variance is the sum of the diagonal terms in the covariance matrix or the "trace" of that matrix and is written as

$$\sigma^2 = \text{Tr}(\underline{\underline{C}}\text{OVAR}) = 4/3. \tag{99}$$

Now with the data mean ($\bar{x}_1 = \bar{x}_2 = 2$) taken to be the origin of a new coordinate system with axes u_1 and u_2, the data in the table above are distributed as shown in the figure below.

Figure. The u_1 - u_2 Coordinate System shows the "mean centered" data representation.



The origin of this new coordinate system may be thought of as being displaced by some mean vector, $\underline{m}$, where

$$\underline{m} = 2\hat{x}_1 + 2\hat{x}_2. \tag{100}$$

$\hat{x}_1$ and $\hat{x}_2$ are unit vectors along the x_1 and x_2 axes, respectively.

Define another coordinate axis such that it is colinear with the data. Call this axis ψ_1. The third coordinate system is completed by placing the coordinate axis ψ_2 through $u_1 = u_2 = 0$ and perpendicular to ψ_1 in the direction shown in Figure 5. The coordinates of the data in the ψ_1 - ψ_2 coordinate system are tabulated below.

Table. Coordinates of Data in ψ_1 - ψ_2 System.

Observation No.	ψ_{i1}	ψ_{i2}
1	$-\sqrt{2}$	0
2	0	0
3	$\sqrt{2}$	0

The covariance matrix for the data as represented in this coordinate frame is

$$\underline{\underline{C}}OVAR = \begin{pmatrix} \frac{4}{3} & 0 \\ 0 & 0 \end{pmatrix} . \qquad (101)$$

This analysis is of interest since it shows that the data set initially represented by two coordinate axes, x_1 and x_2, can be represented, with the proper translation and rotation of these axes, by only one axis, ψ_1, as the ψ_2 component of all three observations is zero. This result effectively cuts in half the amount of information required to describe this set of data. It follows then that the data variance must all be accounted for along the ψ_1 axis as is shown in equation (101) in accordance with equation (99). Because of this, and since the data mean is zero, the variance may be found from the mean of the sum of the squares of the ψ_1 axis data coordinates, or

$$\sigma^2 = \frac{1}{3} \sum_{i=1}^{3} \psi_{i1}^2 , \qquad (102\text{-a})$$

and

$$\sigma^2 = \frac{4}{3} . \qquad (102\text{-b})$$

Also, by equation (101), ψ_{i1} and ψ_{i2}, i = 1, 2, 3, are uncorrelated since COVAR(ψ_1, ψ_2) = 0.

Now consider the case where the data set is in the form of a matrix $\underline{\underline{X}}$(MxN). $\underline{\underline{X}}$ may be thought of as containing M measurements of an observable vector dimensioned by N or as N coordinate vectors dimensioned by M. The objective is to reduce the number of coordinate vectors required to accurately represent $\underline{\underline{X}}$ and at the same time to keep account of the data variability explained by this representation. Though more computationally involved, this problem is fundamentally the same as the preceding example. That is, by the proper selection of another coordinate system, the data may be arranged with respect to its coordinate axes so that the data variance is maximized along a smaller number of its coordinate vectors and so that the various coordinate vectors are uncorrelated with each other, i.e.,

$$\mathrm{COVAR}(\psi_i, \psi_j) = 0, \text{ for } i = 1, 2, \ldots, N \text{ and } i \neq j.$$

To this end the covariance matrix describing the data set must be diagonalized (all off diagonal terms are required to be zero). The covariance matrix is

$$\underline{\underline{\mathrm{COVAR}}}(\underline{\underline{X}}) = \frac{1}{M}\, \underline{\underline{U}}^T \underline{\underline{U}} \tag{103}$$

where the U matrix is defined such that

$$u_{ij} \equiv x_{ij} - \bar{x}_j \,, \tag{104-a}$$

and

$$\bar{x}_j = \frac{1}{M} \left(\sum_{i=1}^{M} x_{ij} \right) \tag{104-b}$$

is the data average for the jth column of $\underline{\underline{X}}$.

Diagonalizing the covariance matrix defined by equation (103) results in a new covariance matrix statistically describing the data in a new coordinate system or "eigenspace". This covariance matrix is of the form

$$\underline{\underline{\Lambda}} = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & \\ \vdots & & \ddots & \\ 0 & & & \lambda_N \end{bmatrix} . \tag{105}$$

All off-diagonal elements are zero. The diagonal elements of $\underline{\underline{\Lambda}}$ are eigenvalues, or characteristic values as they are sometimes called. Associated with each eigenvalue is a principal axis, a coordinate axis in eigenspace. Any vector $\underline{\psi}$, as defined in equation (106) below, that is parallel to a principal axis is called an eigenvector. The eigenvalue equation is

$$(\underline{\underline{C}}OVAR)\underline{\psi} = \lambda\ \underline{\psi}\ , \tag{106}$$

which may be rewritten as

$$[(\underline{\underline{C}}OVAR) - \underline{\underline{I}}\lambda]\underline{\psi} = 0\ , \tag{107}$$

where $\underline{\underline{I}}$ is the identity matrix.

It is necessary to find non-trivial solutions for equation (107), that is, solutions where $\underline{\psi} \neq 0$. Since equation (107) is a representation of N homogeneous simultaneous equations, it can only be solved if the determinant of the coefficients vanishes, or

$$|\underline{\underline{C}}OVAR - \underline{\underline{I}}\lambda| = 0\ . \tag{108}$$

This is often referred to as the secular equation. Values for the scalar constant λ which come from the solution of the secular equation are the sought eigenvalues. These eigenvalues are arranged in decreasing magnitude along the diagonal of $\underline{\underline{\Lambda}}$ in equation (105).

Once the eigenvalues are known, the associated eigenvectors can be found by equation (107). The N eigenvectors that pass through the origin are the coordinate axes in the eigenspace coordinate frame. The coordinates of the data in eigenspace are given by

$$\underline{\underline{C}} = \underline{\underline{U}}\ \underline{\underline{\psi}}^T \tag{109}$$

where $\underline{\underline{U}}$ is defined by equations (104) and $\underline{\underline{\psi}}$ is a square matrix containing the N eigenvectors by row. The coordinates of the data in the original coordinate system can be found by

$$\underline{\underline{X}} = \underline{\underline{U}} + \underline{\underline{a}} \tag{110-a}$$

where

$$\underline{\underline{U}} = \underline{\underline{C}}\ \underline{\underline{\psi}}\ , \tag{110-b}$$

and the matrix a, containing the N column means of $\underline{\underline{X}}$, is given by

$$a_{kj} = \frac{1}{M} \left(\sum_{i=1}^{M} x_{ij} \right) , \tag{110-c}$$

for k = 1, 2, . . ., M.

It will now be of interest to return to the earlier illustrative example solving it from the point of view of an eigenvalue problem as developed above. From the data in the table (Data Set as Viewed in the x_1 - x_2 Coordinate System) and with equations (97) and (104) the U matrix may be written as,

$$\underline{\underline{U}} = \begin{pmatrix} -1 & -1 \\ 0 & 0 \\ 1 & 1 \end{pmatrix}. \tag{111}$$

This is the "mean centered" or "zero mean" data representation as shown in the figure above. Then by equation (103)

$$\underline{\underline{COVAR}}(\underline{\underline{X}}) = \begin{pmatrix} \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} \end{pmatrix}, \tag{112}$$

which is in agreement with equation(97-c). The required eigenvalues can be found with a little algebra and equation (108) as follows:

$$\left| \begin{pmatrix} \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0$$

$$\frac{2}{3} \left| \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} \frac{3}{2}\lambda & 0 \\ 0 & \frac{3}{2}\lambda \end{pmatrix} \right| = 0. \tag{113}$$

Let

$$\lambda' = \frac{3}{2}\lambda \ , \tag{114}$$

so that

$$\begin{vmatrix} 1 - \lambda' & 1 \\ 1 & 1 - \lambda' \end{vmatrix} = 0 \ . \tag{115}$$

The determinant on the left hand side is readily evaluated giving

$$\lambda'^2 - 2\lambda' = 0 \ . \tag{116}$$

The solution of this quadratic equation is

$$\lambda_1' = 2, \tag{117-a}$$

and

$$\lambda_2' = 0 \ , \tag{117-b}$$

or, by equation (114),

$$\lambda_1 = 4/3, \tag{118-a}$$

and

$$\lambda_2 = 0 \ . \tag{118-b}$$

Substituting this result into equation (107) yields

$$\psi_{11} = \psi_{12} \tag{119}$$

for the first eigenvalue, and

$$\psi_{21} = -\psi_{22} \tag{120}$$

for the second eigenvalue. Here ψ_{ij} is the component of the i<u>th</u> eigenvector along the u_j axis.

The eigenvector associated with the first eigenvalue is any vector which has equal components along the u_1 and u_2 axes. Then the principal axis can be taken as the eigenvector that passes through the origin of the u_1 - u_2 coordinate system, such that the unit vector along this principal axis is

$$\hat{e}_1' = \frac{\hat{e}_1 + \hat{e}_2}{\sqrt{2}} , \tag{121}$$

where $\hat{e}_1$ and $\hat{e}_2$ are unit vectors along the u_1 and u_2 axes, respectively, and the $1/\sqrt{2}$ is a normalization constant.

Similarly, the unit vector along the second principal axis is

$$\hat{e}_2' = \frac{\hat{e}_2 - \hat{e}_1}{\sqrt{2}} . \tag{122}$$

Equations (121) and (122) can be combined and represented in matrix form as

$$\underline{\underline{\psi}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{123}$$

Here the matrix ψ contains in rows the two eigenvectors that represent the principal axes in eigenspace.

It can quickly be shown that ψ_1 and ψ_2 form an orthonormal set since

$$\underline{\underline{\psi}}^T \underline{\underline{\psi}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

From this result it follows that

$$\underline{\psi}_1 \cdot \underline{\psi}_1 = \underline{\psi}_2 \cdot \underline{\psi}_2 = 1$$

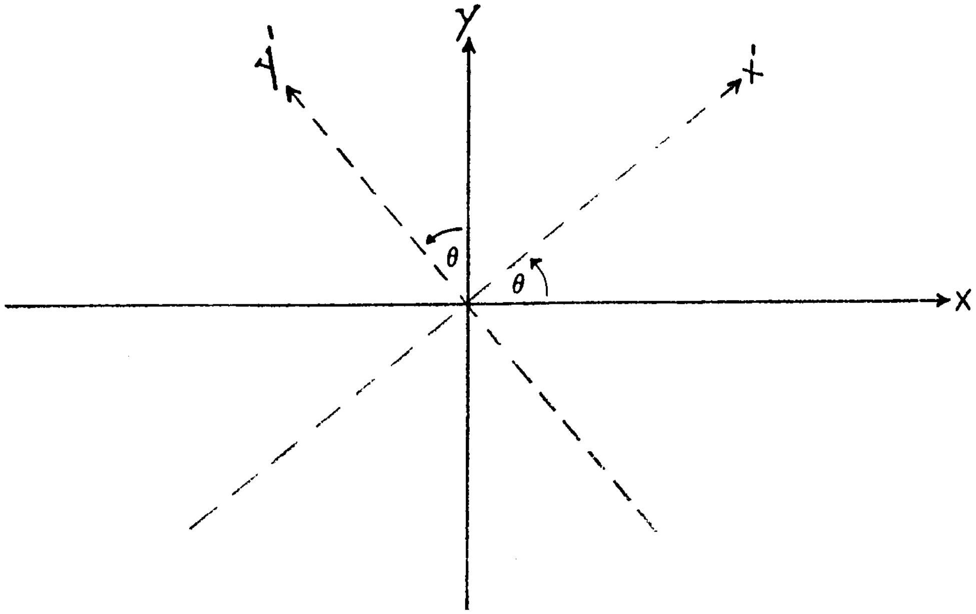
demonstrating that $\underline{\psi}_1$ and $\underline{\psi}_2$ are normalized and that

$$\underline{\psi}_1 \cdot \underline{\psi}_2 = \underline{\psi}_2 \cdot \underline{\psi}_1 = 0$$

showing that $\underline{\psi}_1$ and $\underline{\psi}_2$ are orthogonal to each other.

To digress a bit it is interesting to note that the matrix ψ in equation (123) is, in fact, the transpose of a rotational transformation and can be calculated by the perhaps more conventional technique illustrated in the figure below.

Figure. Coordinate Axes Rotation



In the figure the primed axes, x' and y', have been rotated as shown through an angle θ. They can be represented with respect to the original axes as

$$x' = x\cos\theta + y\sin\theta \tag{124-a}$$

and

$$y' = y\cos\theta - x\sin\theta\ , \tag{124-b}$$

or written in matrix form

$$(x' \quad y') = (x \quad y)\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}. \tag{125}$$

Notice that for $\theta = 45°$ the rotational transformation in equation (125) becomes

$$\underline{\underline{\psi}}^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} . \tag{126}$$

Now, by equations (109), (111), and (123), the coordinates of the original data (Table. Data Set as Viewed in the $x_1 - x_2$ Coordinate System) in eigenspace can be calculated as

$$\underline{\underline{C}} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} ,$$

or

$$\underline{\underline{C}} = \begin{bmatrix} -\sqrt{2} & 0 \\ 0 & 0 \\ \sqrt{2} & 0 \end{bmatrix} . \tag{127}$$

The original data coordinates can be reconstructed by equations (110) combined as

$$\underline{\underline{X}} = \underline{\underline{a}} + \underline{\underline{C}}\,\underline{\underline{\psi}} . \tag{128}$$

Substituting equations (97), (123), and (127) into equation (128) gives

$$\underline{\underline{X}} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} -\sqrt{2} & 0 \\ 0 & 0 \\ \sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{129}$$

$$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} -1 & -1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\underline{\underline{X}} = \begin{pmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{pmatrix} . \tag{130}$$

However, as previously stated, the $\underline{\underline{X}}$ data matrix should be retrievable by utilizing data along only the ψ_1 axis. Equation (129) for only the first eigenvector becomes

$$\underline{\underline{X}} = \begin{pmatrix} 2 & 2 \\ 2 & 2 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} -\sqrt{2} \\ 0 \\ \sqrt{2} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{131}$$

$$= \begin{pmatrix} 2 & 2 \\ 2 & 2 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} -1 & -1 \\ 0 & 0 \\ 1 & 1 \end{pmatrix}$$

$$\underline{\underline{X}} = \begin{pmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{pmatrix} . \tag{132}$$

To calculate individual elements of $\underline{\underline{X}}$ equation (128) may be written as

$$x_{ij} = a_j + \sum_{k=1}^{N} C_{ik} \psi_{kj} \tag{133}$$

where the i subscript on "$\underline{a}$" has been dropped since the column index, j, determines the value of $\underline{a}$, now treated as a vector.

These eigenanalysis techniques have been used to some extent in the ozone sampling study. Empirical orthogonal functions (EOF) have been used in the development of a global ozone model. These empirical orthogonal functions are the eigenvectors of the covariance matrix associated with a set of gridded ozone data. The coefficients associated with these functions are the coordinate vectors of the gridded ozone data represented in eigenspace as found in the C matrix defined above.

This EOF ozone model will be discussed below according to the following four development stages.

1. Establish an appropriate data grid system.
2. Calculate data base for model.
3. Develop model.
4. Test model.

The EOF model development is based on the assumption that there will be no missing data blocks in the grid system. This assumption eliminates from consideration polar regions where there is no BUV data coverage. A further consideration is whether latitudinal or longitudinal variability is being investigated. For latitudinal variability studies data are arranged as shown in Figure 6-A. For longitudinal variability studies data are arranged as shown in Figure 6-B. The elements of the grids are found from equation (3) where the i and j indices are now defined as in Figures 6-A and B.

Three data arrays constitute the minimum data base requirement for the EOF model. One of these arrays contains the eigenvector matrix, another contains the matrix of coordinate vectors in eigenspace or the coefficient matrix, and the last contains the N column averages of the gridded data (Figures 6-A and B). With one set of these three arrays the EOF model can reconstruct the original data grid for some specified time period. Though the EOF model is time independent, by supplying eigenvector, coefficient, and column average arrays for several time periods a model which is effectively time dependent can be formulated. The EOF model data base as generated during this study of the BUV-I data consists of one such set of arrays per week for 50 weeks.

Data base array sets are calculated by computer program EOFA2. In this program the column averages are found by equation(104-b), the eigenvectors, $\underline{\psi}$, defined by equation (107) are computed by subroutine SYMQL[9], and the coefficient matrix is calculated by equation (109). These data base arrays are saved and maintained on a MSRA file such that model data is accessible on a weekly basis.

For purposes of this discussion, it will be assumed that the source data for the EOF model is arranged as in Figure 6-A. The fundamental model representation of an ozone value in grid block (i,j) is

$$x_{ij} = a_j + \sum_{k=1}^{n} C_{ik} \psi_{kj} \tag{134}$$

as shown in equation (133). In equation (134)

$$1 \leq n \leq N$$

where n is the number of eigenvalues to be used by the model and is determined by the percentage of the total variability, P(%), to be explained or accounted for. The expression showing the relationship between n and P(%) is given below as

$$P(\%) = \frac{1}{\sigma^2} \sum_{k=1}^{n} \lambda_k \times 100\% , \tag{135}$$

where

$$\sigma^2 = \sum_{\ell=1}^{N} \lambda_\ell . \tag{136}$$

Also, as has been demonstrated above (equations 99 and 102), the data variance may be written as

$$\sigma^2 = Tr(\underline{\underline{C}}OVAR) = \frac{1}{M} \sum_{j=1}^{N} \sum_{i=1}^{M} C_{ij}^2 . \tag{137}$$

The model's time dependency is incorporated by the proper selection of the $\underline{a}$ vector and the $\underline{\underline{C}}$ and $\underline{\underline{\psi}}$ matrices from the MSRA file as discussed above.

The model development thus far makes available only the somewhat limited capability of calculating discrete ozone values associated with grid block (i,j). This capability must be extended so that ozone values for specified positions on the Earth's surface, within the geographic boundary limitations of the model's data base, can be computed. This would result in a model of the form

$$OZONE = X[\psi(\theta), C(\phi), t, P(\%)]. \tag{138}$$

To this end Fourier series representations are calculated for the required eigenvectors and column means as a function of latitude, θ, and for the required coefficients as a function of longitude, ϕ. Appendix H gives a brief development of the Fourier series representation that will be utilized below.

First consider approximating an eigenvector "curve" composed of discrete values. These values are equally spaced along a latitudinal axis and are located at latitude zone centers as shown on the bottom scale of Figure 7. The BUV-I data modeled by this technique generally has good latitudinal coverage, depending on the season, from the latitude zone centered at -77.5° to the zone centered at 77.5°. As can be seen from Figure 7 this corresponds to an eigenvector of 32 discrete components. For the purpose of representing an eigenvector by a Fourier series this figure also shows certain transition scales. The "Fictitious Latitude Scale" simply shows the latitudinal data range where zero degrees corresponds to the gridded data's southern extreme. The "Fourier Scale Range" shows the domain of the periodical Fourier functions which will be used to represent the eigenvector.

Notice that the Fourier scale range extends slightly beyond the discrete data scale. As far as the Fourier scale is concerned there are 33 pieces of data, but due to the periodic nature of the Fourier representation the functional value of the first discrete data point must equal the functional value of the last, or

$$f(0°) = f(360°) \; . \qquad (139)$$

Then over the Fourier scale range there are 32 intervals between the equally spaced data so that

$$\frac{\text{Fourier Scale Range}}{\text{No. of Intervals over Scale}} = \frac{360°}{32} = 11.25°/\text{interval}. \qquad (140)$$

Both latitude scales contain 31 intervals so that the length of either latitude scale in terms of the Fourier scale is

$$11.25°/\text{interval} \times 31 \text{ intervals} = 348.75°.$$

Let κ_1 be the conversion factor from the fictitious latitude to the Fourier scale such that

$$\kappa_1 = \frac{348.75°}{155°} = 2.25 \; . \qquad (141)$$

Also let θ be the actual latitude value, θ_2 be the fictitious latitude value, θ_1 be the Fourier scale value, and d_θ be the discrete data point number including any fractional part. Then,

$$\theta_1 = \kappa_1\theta_2. \quad (142)$$

But

$$\theta_2 = \theta + 77.5°, \quad (143)$$

so

$$\theta_1 = \kappa_1(\theta + 77.5°) . \quad (144)$$

This expression shows the relationship between the actual latitude, θ, and the corresponding Fourier angle, θ_2.

The relationship between d_θ and θ_2 may be written as

$$\theta_2 = \kappa_2(d_\theta - 1), \quad (145)$$

where

$$\kappa_2 = 155°/31 \text{ intervals}. \quad (146)$$

Then by equation (143)

$$\theta = \kappa_2(d_\theta - 1) - 77.5°, \quad (147)$$

and by equation (144)

$$\theta_1 = \kappa_1\kappa_2(d_\theta - 1), \quad (148)$$

which gives the Fourier angle in terms of the discrete data point number scale.

From the development in Appendix H the required eigenvector may be approximated by a Fourier series expansion

$$\psi(\theta) = A_0 + \sum_{\ell=1}^{Q} [A_\ell\cos(\ell\kappa_1(\theta + 77.5°)) + B_\ell\sin(\ell\kappa_1(\theta + 77.5°))] \quad (149)$$

where Q = 16, since 2Q = 32 is the number of independent discrete pieces of data, and where equation (144) was substituted into equation (H-6) for the Fourier angle.

The procedure for finding an approximation for the mean vector "curve" is quite the same and leads to

$$a(\theta) = E_0 + \sum_{\ell=1}^{Q} [E_\ell \cos(\ell\kappa_1(\theta + 77.5°)) + J_\ell \sin(\ell\kappa_1(\theta + 77.5°))], \quad (150)$$

where only the Fourier coefficients are changed. They are found as outlined in equations (H-4) and (H-5).

The Fourier representation for the coefficients is similar to that above except for certain scaling differences. The BUV-I gridded data ranges on the longitude scale from the block centered on 7.5° to the block centered on 352.5°. The Fourier angle may be written as

$$\phi_1 = \phi - 7.5° , \quad (151)$$

from which by equation (H-6)

$$C(\phi) = R_0 + \sum_{\ell=1}^{Q} [R_\ell \cos(\ell(\phi - 7.5)) + S_\ell \sin(\ell(\phi - 7.5))] \quad (152)$$

where Q = 12, since 2Q = 24 is the number of discrete independent pieces of data, corresponding in this case to longitudinal sectors, and where the coefficients are found again by equations (H-4) and (H-5) using the already known discrete values of $\underline{C}$.

Computer program EAMOD1 was prepared to implement this model and to briefly analyze the results. To summarize the model development above as incorporated into the computer model consider the problem of finding an ozone value for some point on the Earth's surface (θ', ϕ') at time t. Further, P'(%) of the data variability is to be accounted for.

First, a set of data arrays for the appropriate time period t are accessed from MSRA file. Recall these three data arrays contain the eigenvector matrix, $\underline{\underline{\psi}}$, the coefficient matrix, $\underline{\underline{C}}$, and the column average vector, $\underline{a}$. The number of eigenvectors required to achieve the specified data variability can be determined from equation (135).

Since eigenvalues are not saved on the MSRA file, elements from the coefficient matrix may be used for this task. As shown above, the k<u>th</u> eigenvalue may be written as

$$\lambda_k = \frac{1}{M} \sum_{i=1}^{M} C_{ik}^2, \tag{153}$$

and equation (135) becomes

$$P(\%) = \frac{\sum_{k=1}^{n} [\frac{1}{M} \sum_{i=1}^{M} C_{ik}^2]}{\sigma^2} \times 100\% . \tag{154}$$

Equation (154) is summed iteratively over k until

$$P(\%) \geq P'(\%) \tag{155}$$

at which point n = k is taken to be the required number of eigenvectors.

The eigenvectors are arranged by row, and the associated coefficients are arranged by column as stored in their respective arrays. Each of the first n eigenvectors are fitted according to equation (149), and each of the first n coefficients (column vectors) are fitted according to equation (152). Similarly the mean vector is represented by equation (150). The required ozone value can now be computed by rewritting equation (134) in terms of actual model parameters, instead of the global grid indices (i,j), as

$$X[\psi(\theta'), C(\phi'), t, P'(\%)] = a_t(\theta') + \sum_{k=1}^{n[P'(\%)]} C_t(\phi')_k \; \psi_t(\theta')_k \tag{156}$$

where the notation n[P'(%)] signifies that the number of eigenvectors used is a function of the explained data variability and where the subscript t indicates that the arrays come from the data base for the time interval t.

The ozone modeling technique using empirical orthogonal functions has been implemented and briefly analyzed by computer program EAMOD1. The modeling aspect has been described above. As a quick evaluation of the model's usefulness, selected BUV-I sampling data was used to generate the required model data base. Then the BUV-I ozone values associated with this sampling were compared with the model predictions for those values. Within the latitudinal limits of the model, the errors ranged from 0% to 10%.

7. Data Fill Technique by Autocorrelation Functions

The autocorrelation function typically thought of as being associated with time series analysis has been somewhat modified here and has been engineered into a data fill technique on a spatial basis.

$$R(k) \equiv E[x_n \, x_{n+k}] \tag{157}$$

is the definition of the autocorrelation function where E is the expectation operator and the set of x_i, i = 0, 1, 2, . . ., N, is "zero mean" data.[10] The sample autocorrelation function

$$R_N(k) = \frac{1}{N} \sum_{i=o}^{N-|k|-1} x_i \, x_{i+|k|} \tag{158}$$

is the estimate of the autocorrection function, where k, the lag, is representative of time separation.[10]

Consider the case of global spatial distribution instead of time distribution. Let k represent a lag of 5° latitudinally and ℓ represent a lag of 15° longitudinally. The number of samples with respect to latitude is

$$N_k = \frac{180°}{5°} = 36, \tag{159}$$

and with respect to longitude is

$$N_\ell = \frac{360°}{15°} = 24. \tag{160}$$

Then by analogy to equation (158)

$$R_{N(k,\ell)}(k,\ell) = \frac{1}{N(k,\ell)} \sum_{j=1}^{24-|\ell|} \sum_{i=1}^{36-|k|} x_{ij} \, x_{i+|k|,j+|\ell|} . \tag{161}$$

However, in accordance with the latitudinal index convention as shown in Figure 1, equation (161) is written as

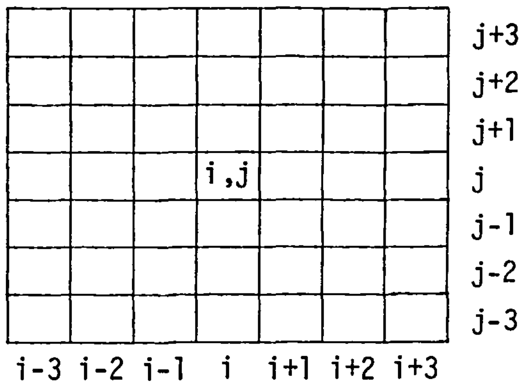
$$R_{N(k,\ell)}(k,\ell) = \frac{1}{N(k,\ell)} \sum_{j=1}^{24-|\ell|} \sum_{i=1}^{18-|k|} \sum_{i=19}^{36-|k|} x_{ij} \, x_{i+|k|,j+|\ell|} , \tag{162}$$

where

$$N(k,\ell) = N_k N_\ell - N_d$$

and N_d is the number of grid blocks containing no data.

Now consider the data block (i,j), shown in the figure below, containing no data.



In a sense the objective is to find a weighted mean of the 48 blocks surrounding (i,j) which will serve as the "fill-in" value for the block (i,j).

In general a weighted mean may be written as

$$\bar{x} = \sum_{i=1}^{n} \alpha_i x_i / \sum_{i=1}^{n} \alpha_i , \qquad (163)$$

where α_i is the weighting factor associated with x_i. Should some x_i have no value, indicated by $x_i = 0$, over the range $1 \leq i \leq n$, then equation (163) is written as

$$\bar{x} = \sum_{i=1}^{n} \alpha_i x_i / \sum_{i=1}^{n} \delta_i \alpha_i \qquad (164)$$

where

$$\delta_i = \begin{cases} 1, & \text{for } x_i \neq 0 \\ 0, & \text{for } x_i = 0 \end{cases} . \qquad (165)$$

Finding the value for the block (i,j) is a two-dimensional problem requiring summation over latitude and longitude. Let Y_{ij} be the required weighted mean. Then by equation (164)

$$Y_{ij} = \frac{\sum_{k=-3}^{3} \sum_{\ell=-3}^{3} R_{|k||\ell|} Y_{i+k,j+\ell}}{\sum_{k=-3}^{3} \sum_{\ell=-3}^{3} R_{|k||\ell|} \delta_{i+k,j+\ell}} \qquad (166)$$

where $R_{|k||\ell|}$ is now treated as a weighting factor and from equation (162)

$$\underset{N(k,\ell)}{R\ (k,\ell)} = \frac{1}{N(k,\ell)} \sum_{j=1}^{24-|\ell|} \sum_{i=1}^{18-|k|} \sum_{i=19}^{36-|k|} Y_{ij} Y_{i+|k|,j+|\ell|}. \qquad (167)$$

The technique briefly discussed above is currently being used as implemented in computer program OZFILL1 on two levels, partial fill and complete fill. Using the partial fill technique 1/2 of the total surrounding 48 data blocks must contain non-zero ozone values (an ozone value of zero implies no data). Also previously filled blocks are not included in this count. The complete fill technique is used without regard to the above restrictions.

IV. BUV CORRECTION TECHNIQUE - DOBSON DATA

Ozone data as measured by the Dobson spectrophotometer have been investigated and analyzed in conjunction with the BUV sampling analysis.[11] These data were obtained from the World Ozone Data Centre in Ontario, Canada and subsequently have been used to adjust the BUV data as will be briefly explained below.

For a given Dobson station certain BUV measurements are selected based on temporal and spatial considerations in order to calculate a linear least squares fit between the Dobson, y_d, and the BUV, y_b, data. The great circle distance, s, between the Dobson station (θ_1, ϕ_1) and the BUV subsatellite point (θ_2, ϕ_2) is given by

$$s = R_{\cos}{}^{-1}(\sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2 \cos|\phi_2 - \phi_1|) , \qquad (168)$$

where R = 6367.3951 kilometers is the average earth radius based on the Clarke spheroid of 1866.[12] The least squares fit is of the form

$$y_d = \beta_o + \beta_1 y_b \qquad (169)$$

where β_o and β_1 are the resulting regression coefficients.

A sufficient number of Dobson stations are utilized so that the range in latitudinal coverage is from approximately 75° to -45°. Both β_o and β_1 may be fit as a function of latitude, θ, by the least squares method so that

$$\beta_o = \alpha_{01} + \alpha_{02} \cos 2\theta \qquad (170a)$$

and

$$\beta_1 = \alpha_{11} + \alpha_{12} \cos 2\theta . \qquad (170b)$$

Then the "corrected" BUV ozone measurements, y_c, as "adjusted" by the Dobson data may be calculated from

$$y_c = \alpha_{01} + \alpha_{02} \cos 2\theta + (\alpha_{11} + \alpha_{12} \cos 2\theta) y_b . \qquad (171)$$

Table 1. Preliminary Analysis of the BUV-III Data

FTN FILE #	1ST DAY	LAST DAY	TOT. DAYS	1ST REC.#	LAST REC. #	TOTAL REC.	ABNORM. OZ -999.	ABNORM. OZ -77.	ABNORM. OZ OTHER	IEQC
1	99	126	28	1	23,591	23,591	6	818	0	0
2	126	153	28	23,592	47,373	23,782	0	889	0	10
3	154	182	29	47,374	72,143	24,770	2	1,000	0	1
4	182	210	29	72,144	97,309	25,166	10	995	0	0
5	210	238	29	97,310	122,760	25,451	11	993	0	0
6	238	266	29	122,761	147,467	24,707	0	917	0	0
7	266	293	28	147,468	171,742	24,275	17	893	0	0
8	294	322	29	171,743	198,572	26,830	86	937	0	0
9	322	349	28	198,573	226,539	27,967	0	982	1	0
10	350	364	15	226,540	240,933	14,394	0	508	2	0
11	365	392	28	240,934	264,729	23,796	2	745	2	0
12	393	420	28	264,730	284,198	19,469	1	495	7	0
13	421	448	28	284,199	302,873	18,675	1	586	0	0
14	449	490	42	302,874	326,854	23,981	13	680	0	0
EOF										
15	491	518	28	326,855	345,651	18,797	7	495	1	0
16	519	546	28	345,652	367,473	21,822	3	701	0	0
17	547	575	29	367,474	386,978	19,505	0	623	2	0
18	575	603	29	386,979	406,530	19,552	2	681	0	0
19	603	623	21	406,531	420,244	13,714	0	449	0	0
20	631	658	28	420,245	441,153	20,909	4	710	0	0
21	659	686	28	441,154	462,056	20,903	7	685	0	0
22	687	714	28	462,057	483,204	21,148	1	667	1	0
23	715	729	15	483,205	495,325	12,121	0	388	0	0
24	730	757	28	495,326	520,757	25,432	2	814	3	0
25	758	785	28	520,758	546,946	26,189	32	871	8	6
26	786	813	28	546,947	571,915	24,969	8	954	2	0
27	814	841	28	571,916	594,889	22,974	4	671	0	0
28	842	855	14	594,890	607,974	13,085	0	334	1	0
EOF										
TOTALS						607,974	219	20,481	30	17

SUMMARY:

NO. OF FTN FILES	28
NO. OF RECORDS	607,974
ABNORMAL OZONE	
OZ = -999.	219
OZ = - 77.	20,481
OTHER	30, NOT INCLUDED IN THE INTERVAL $.200 \leq$ ABSOLUTE OZONE VALUE $\leq .650$

RECORDS SUCH THAT ABSOLUTE LATITUDE $\leq 5°$	34,698
BAD CROSSING TIMES (IEQC)	17
OBSERVATIONS ON EQUATOR	0
TOTAL DAYS ON TAPE	757

Figure 1. Global Grid System as Developed in Computer Program OZSTAT2's Data Grouping Scheme

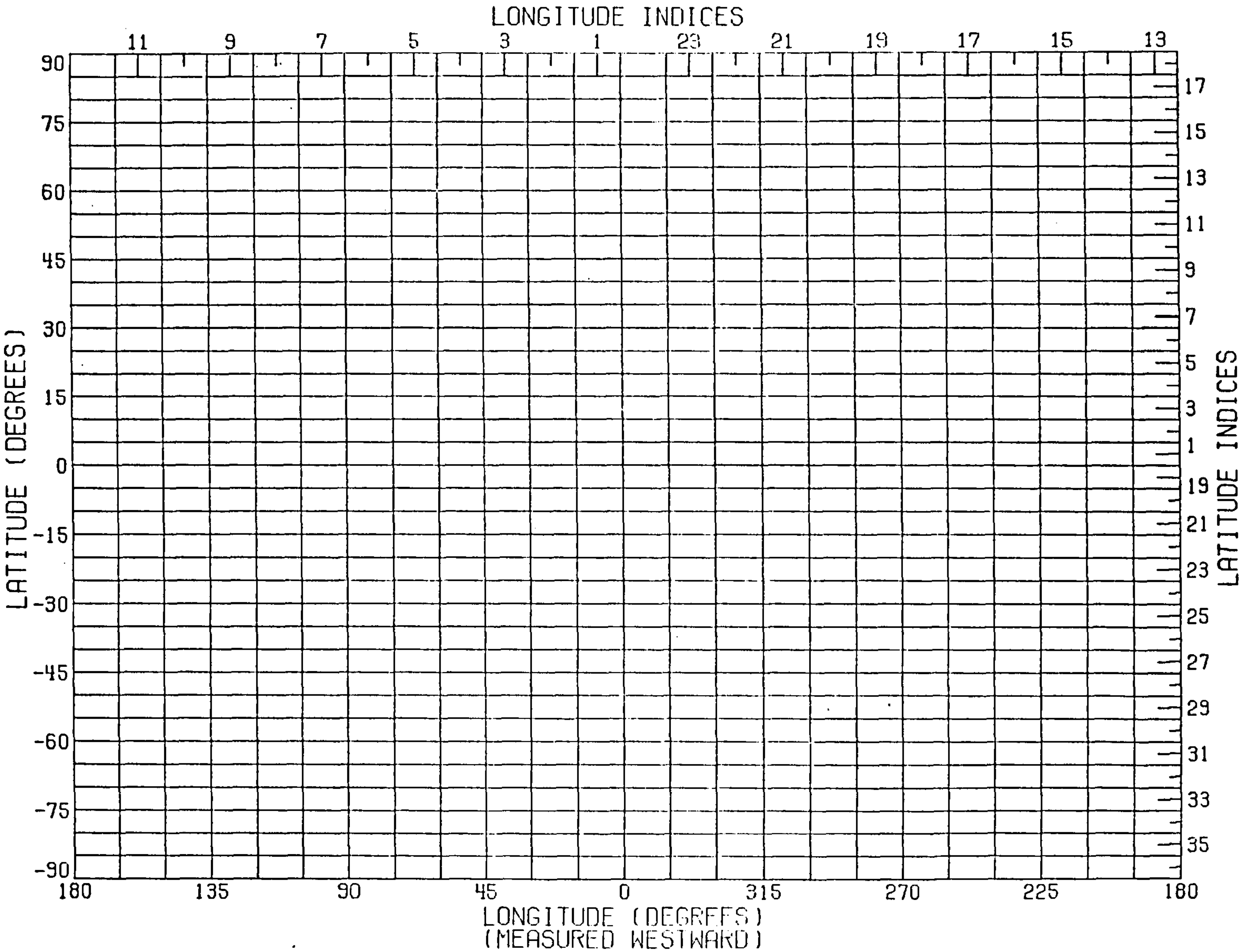


Figure 2. Example of Program OZSTAT2 Graphics Capability
This plot shows BUV Zonal Means for June 22, 1970.

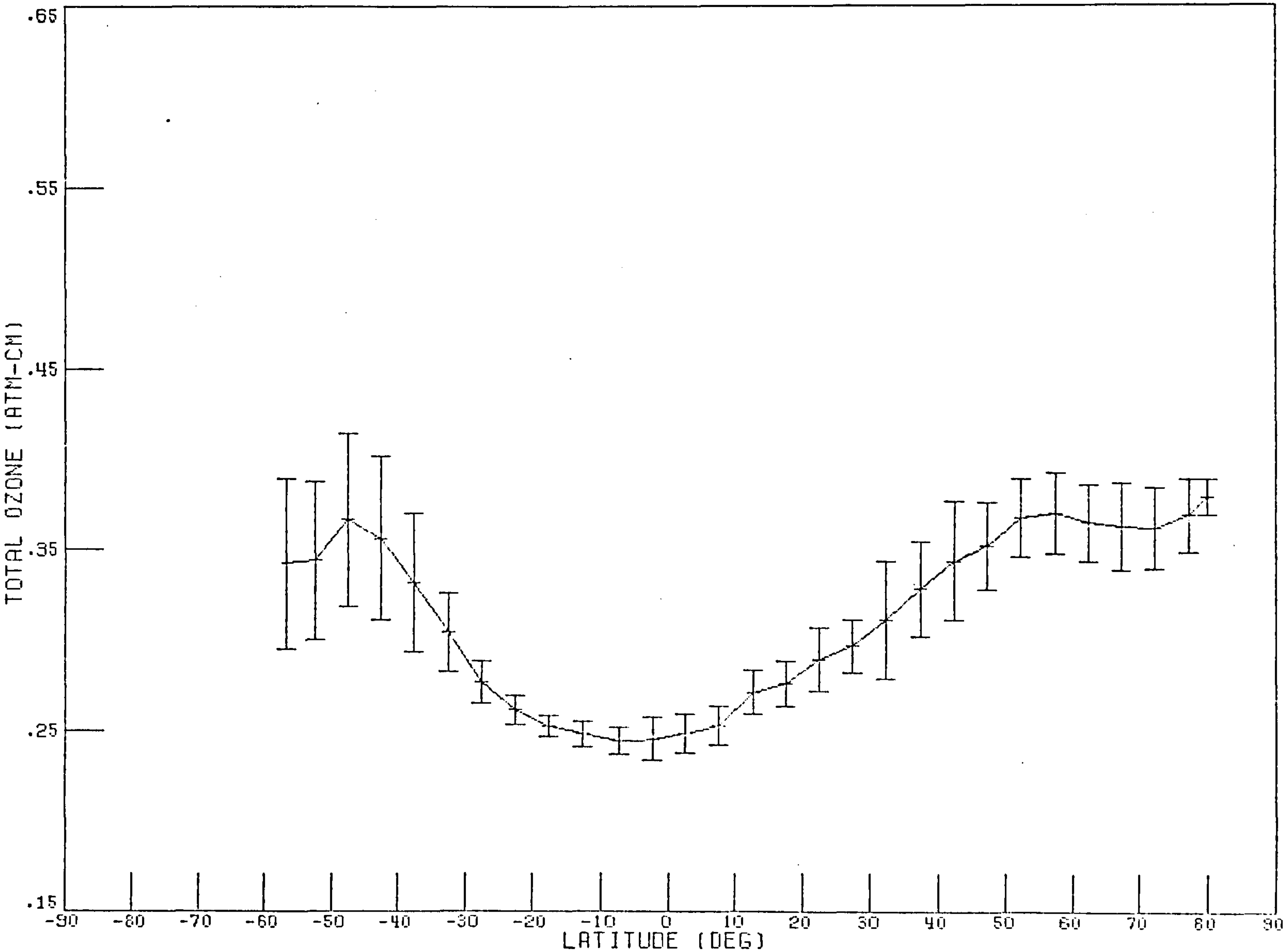


Figure 3. Example of Program OZSTAT2 Graphics Capability

This scatter diagram shows the BUV ozone data distribution for June 22, 1970.

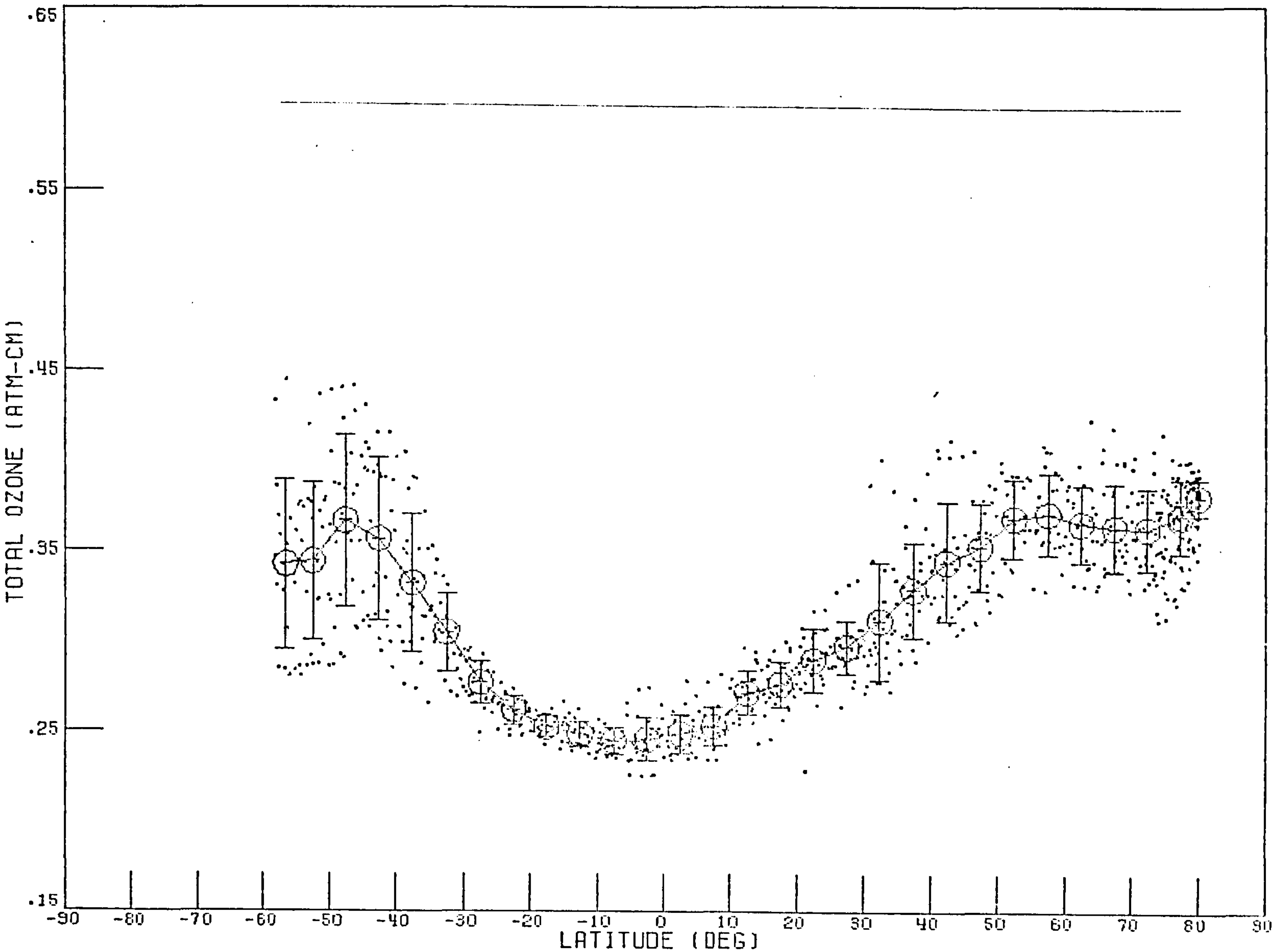


Figure 4. Example of Program OZSTAT2 Graphics Capability

This histogram shows the latitudinal sampling distribution of BUV ozone data for June 22, 1970. Actual Number of Data Points = Normalized Number of Data Points x 103.

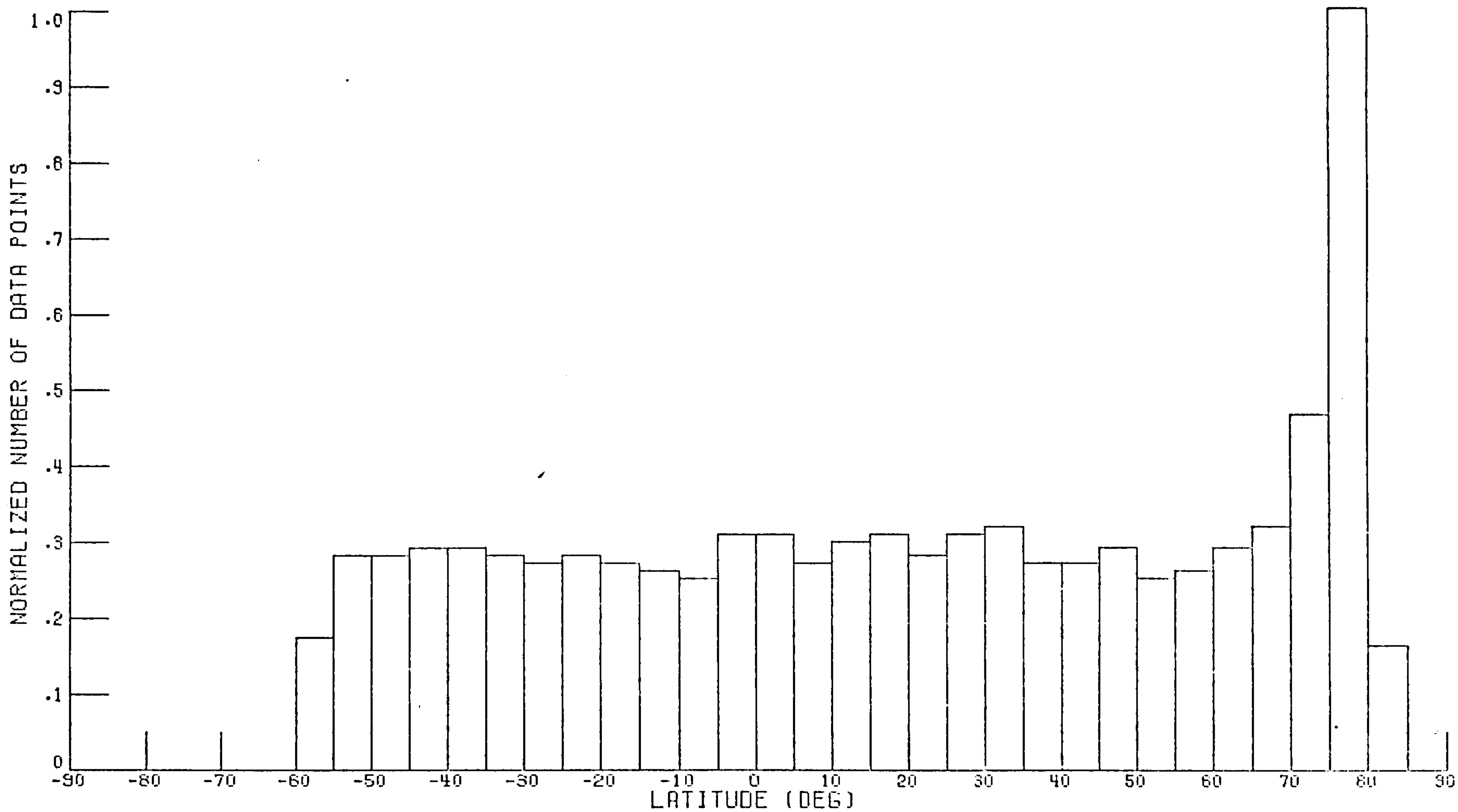


Figure 5. Relationship of the Three Coordinate Systems $x_1 - x_2$, $u_1 - u_2$, and $\psi_1 - \psi_2$

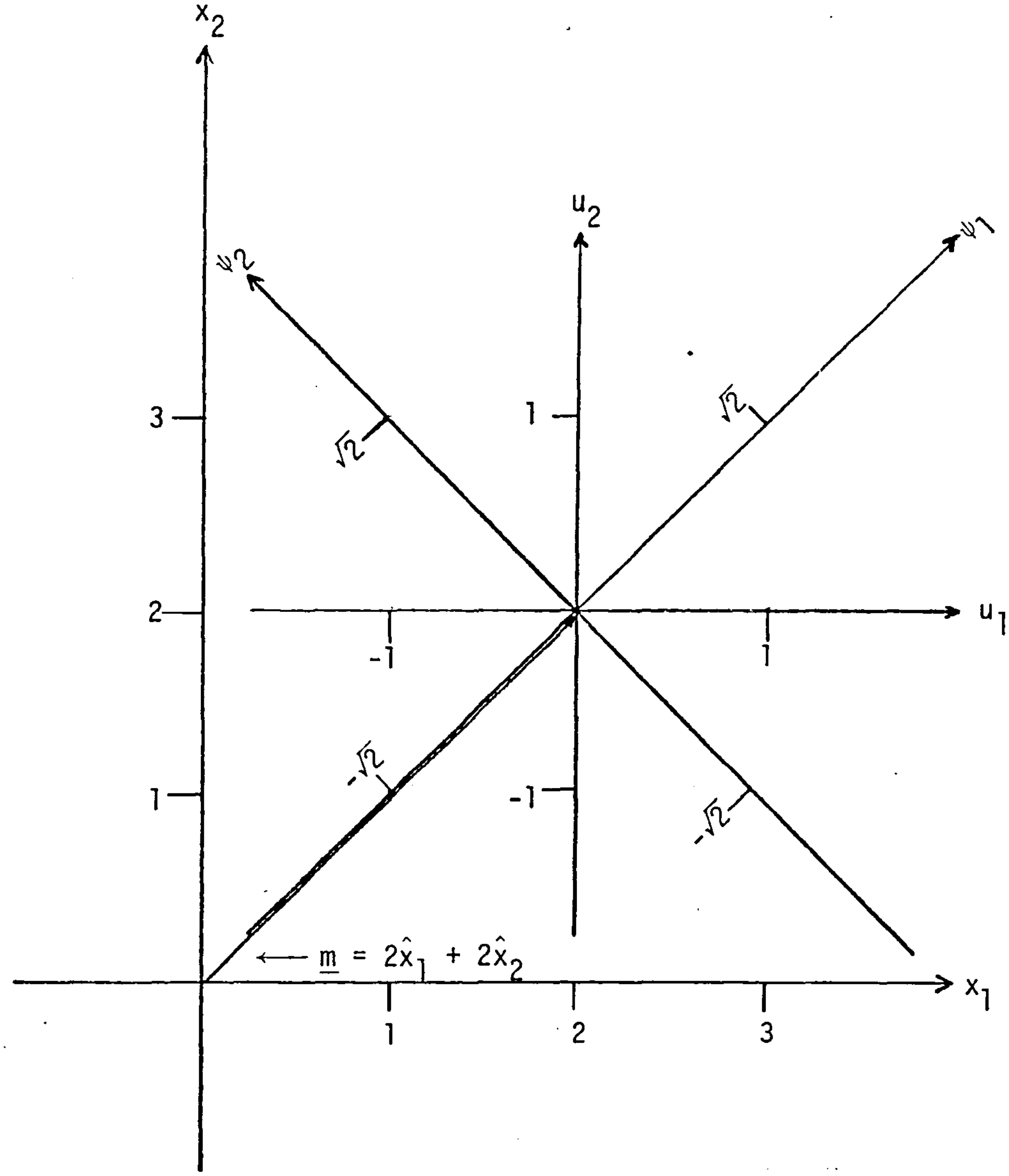


Figure 6-A. EOF Model Arrangement for Latitudinal Variability Studies

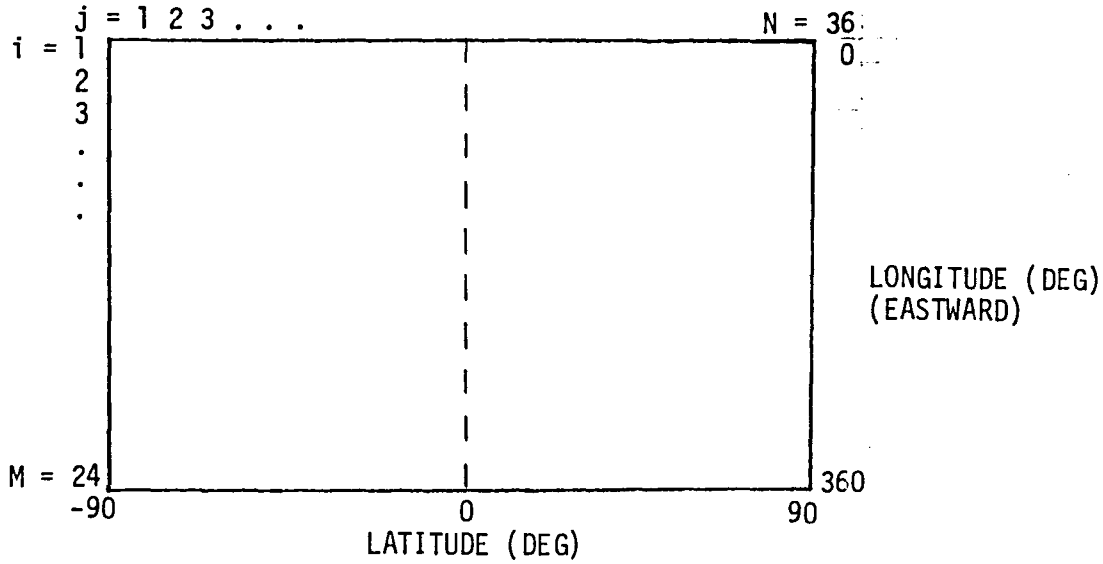


Figure 6-B. EOF Model Arrangement for Longitudinal Variability Studies

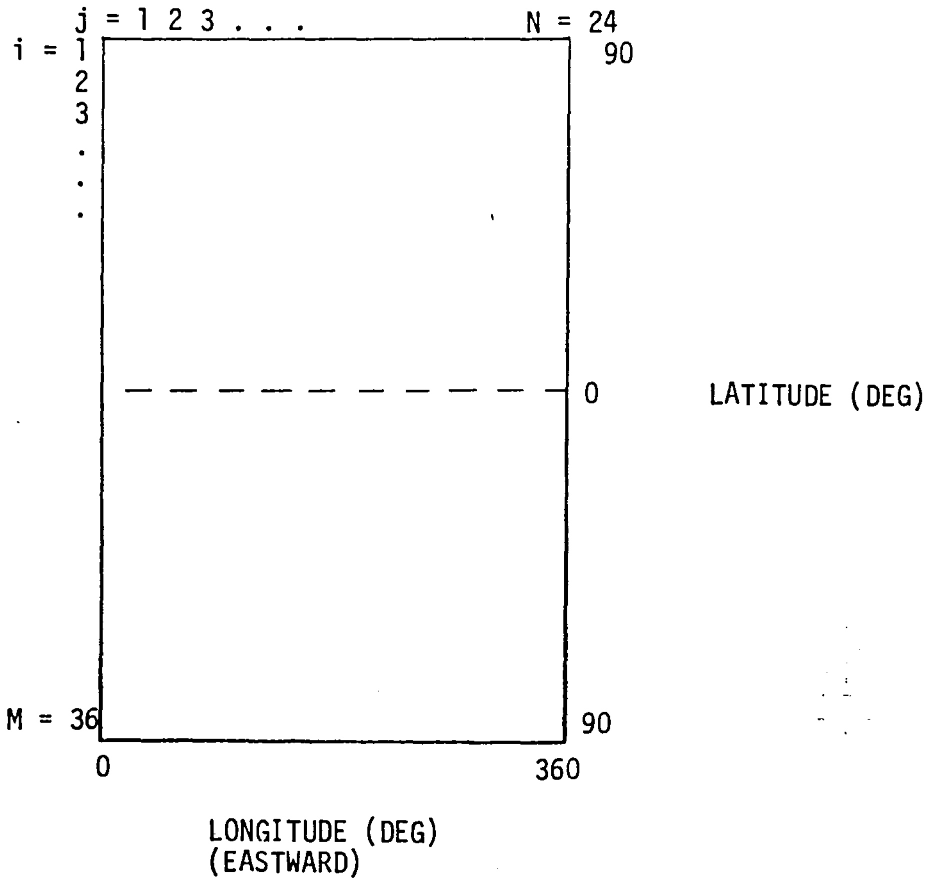
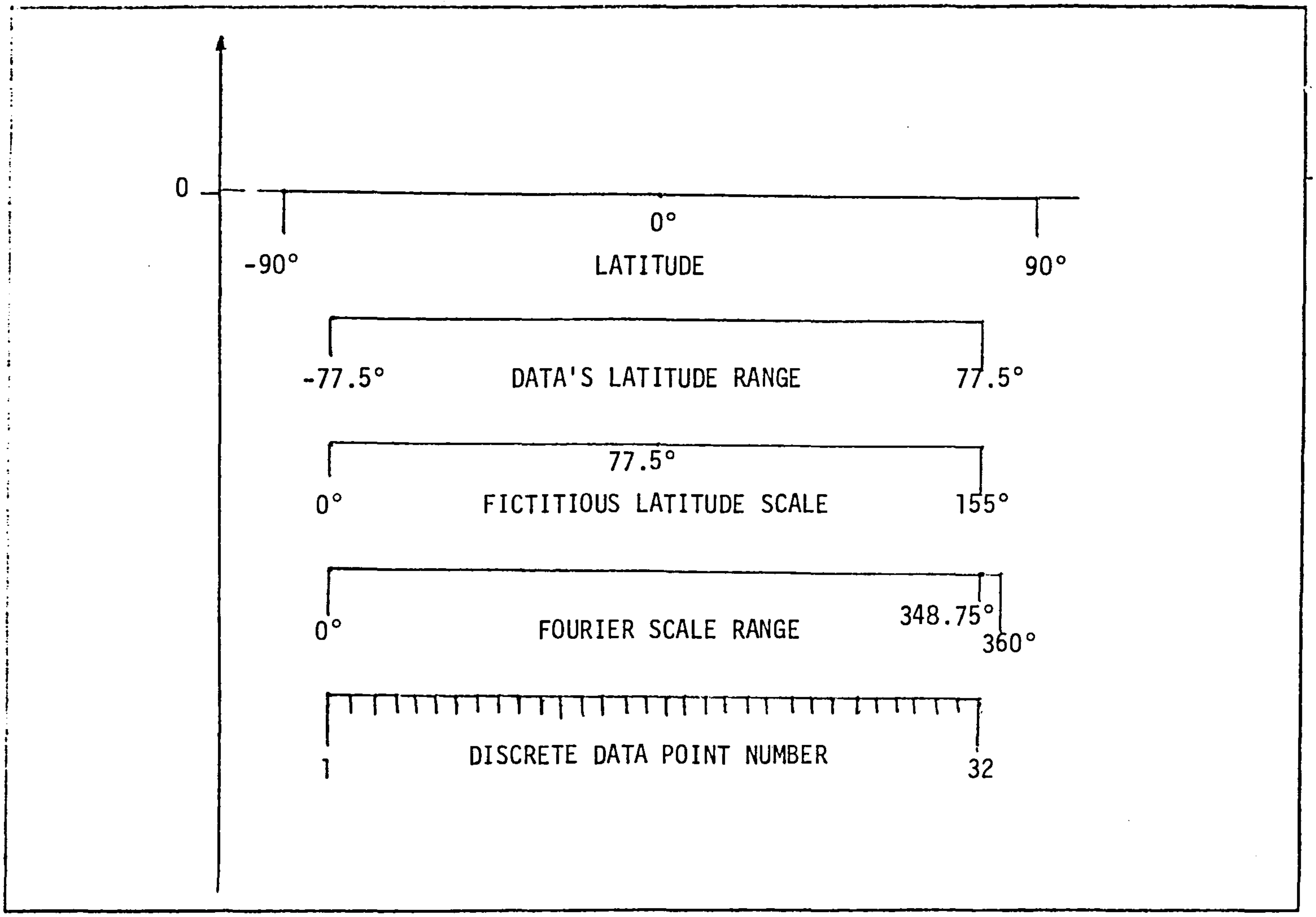


Figure 7. Transition Scales from the Latitude Scale to the Fourier Scale for Eigenvector Representation.



APPENDIX A - PRIMARY COMPUTER PROGRAMS MENTIONED THROUGHOUT MEMORANDUM

	Program Name	Purpose	Reference Section
1.	BUVCOP2	Convert magnetic tapes from IBM to NOS-CDC internal format.	IBM Format to CDC Format Conversion (section 1) and APPENDIX B.
2.	BUV3	Preliminary data analysis program.	Preliminary Data Analysis (section 2).
3.	OZSTAT2	Group data into global grid system. Perform elementary statistical calculations. Generate statistical graphics describing global grid grouping.	Data Grouping Scheme (section 3).
4.	GLSRAN2	Regression and statistical analysis for polynomial expansions, spherical harmonic functions, Fourier functions, and other specified functions.	Spherical Harmonic Model (section 4). Statistical Analysis of Spherical Harmonic Model (section 5).
5.	GLOBZON	Calculate global and zonal means based on spherical harmonic model coefficients.	Statistical Analysis of Spherical Harmonic Model (section 5).
6.	ZONVAR	Calculate global and zonal variances based on zonal elements of covariance matrix describing spherical harmonic coefficients.	Statistical Analysis of Spherical Harmonic Model (section 5).
7.	EOFA2	Eigenanalysis program calculates data base arrays for EOF model.	Eigenanalysis - Empirical Orthogonal Functions (section 6).
8.	EAMOD1	EOF model and analysis.	Eigenanalysis - Empirical Orthogonal Functions (section 6).
9.	OZFILL1	Implements data fill techniques by autocorrelation and spherical harmonic functions.	Data Fill Technique by Autocorrelation Functions (section 7).

APPENDIX B

To illustrate the IBM to NOS-CDC conversion process, the most recent set of data received will be considered. These data are contained on three IBM 9-track magnetic tapes. The following information comes from documentation received with these data tapes.

1. Tape density - 1600 BPI
2. Mode - Binary
3. Parity - Odd
4. Block (PRU) size - 8000 bytes
5. Logical record length - 80 bytes

All three tapes were generated on an IBM 360, and each tape contains 14 files.

With the technique used, one physical record unit (PRU, 8000 bytes) or block of data is buffered into the central processor at a time. This is the equivalent of 2000 IBM words or 1067 CDC words. Figure B-1 illustrates what shall be referred to in the subsequent discussion as a sub-block, that is, 15, 32-bit words arranged as eight packed 60-bit words. Sub-blocks are 480 bits long since this is the smallest common multiple of 32 and 60. The conversion process is accomplished with one sub-block at a time. The procedure as coded in Program BUVCOP2 is described below.

The first block of data is buffered into an array A dimensioned by 1100. Unused storage locations of this array contain the value of zero. The first sub-block (eight words) from A is placed into the array C dimensioned by eight. The 15, 32-bit words in the sub-block are unpacked and arranged into 15 right justified 60-bit words in Subroutine IBMWRDS. These 15, 60-bit words are stored in a temporary array B dimensioned by 15 and subsequently into the first 15 locations of an array D dimensioned by 2200. This process continues until all words in the data block have been stored in D. Subroutine IBMFPC from the READIBM subroutine package can now convert these numbers to CDC internal format floating point numbers which are stored in an array E dimensioned by 2010. In general, the E array contains 2,010 words. That is, Number of words in E =

Number of sub-blocks x Number of 32 bit words/sub-block,

= 134 sub-blocks x 15 words/sub-blocks

= 2,010 words.

The number of complete 20 word logical records in E is the integer part of 2,010/20 or 100 records. The elements of the E array are finally written 20 words (one logical record) at a time onto an output file which is stored on NOS 9-track tapes. This procedure for converting a block of data from IBM internal format to CDC internal format is shown schematically in Figure B-2.

This process is repeated with the next block until the end of the tape is reached. A listing of Program BUVCOP2 and Subroutine IBMWRDS follows this appendix.

A final comment regarding this conversion concerns the actual storage of data on magnetic tapes. The above technique converts one tape at a time. The program must therefore be run three times since three IBM tapes were received containing these data. The minimum amount of data contained on any one of these three tapes is 278,259 logical records or 5,565,180 words. Since a standard NOS 9-track tape, 2,400 feet in length, will hold only 3,880,421 60-bit words, two of these tapes are required. Three NOS tapes were required to hold the data from the IBM tape with the most data. Tape designations, and associated coverage periods, are shown in Table B-1.

These NOS tapes have been prepared to be read with an unformatted binary READ, one logical record (20 words) at a time. These 20 words are listed in Table B-2. The six of these words stored per record on the condensed tapes, generated to minimize storage and reduce computer time, are indicated with an asterisk (*).

Table B-1. Magnetic Tape Designations and Their Corresponding Time Coverages

Time Period	IBM Reel Designation (1)	NOS TAPE Designation (1)	NOS TAPE Designation (2)
April 10, 1970 - May 6, 1971	30906	NV0738 NV0739	
May 7, 1971 - May 5, 1972	34037	NN1004 NV0103	NV0740 NV0104
May 6, 1972 - May 7, 1977	32701	NV0333 NV0334 NV0335	

1 - Contains 20 words per logical record.
2 - Contains 6 words per logical record - condensed tape.

Table B-2. The Twenty Words that Constitute a Logical Record on the BUV Data Tapes

1.	Logical Sequence Number
2.	Orbit Number
3.	Year*
4.	Day of Year*
5.	Seconds of Day*
6.	Latitude*
7.	Longitude (westward)*
8.	Solar-Zenith Angle
9-12.	Monochromator N Values, (312.5 - 339.8)nm
13-16.	Photometer N Values, (312.5 - 339.8)nm
17.	A Channel Total Ozone Value
18.	B Channel Total Ozone Value
19.	Recommended Reflectivity
20.	Recommended Total Ozone*

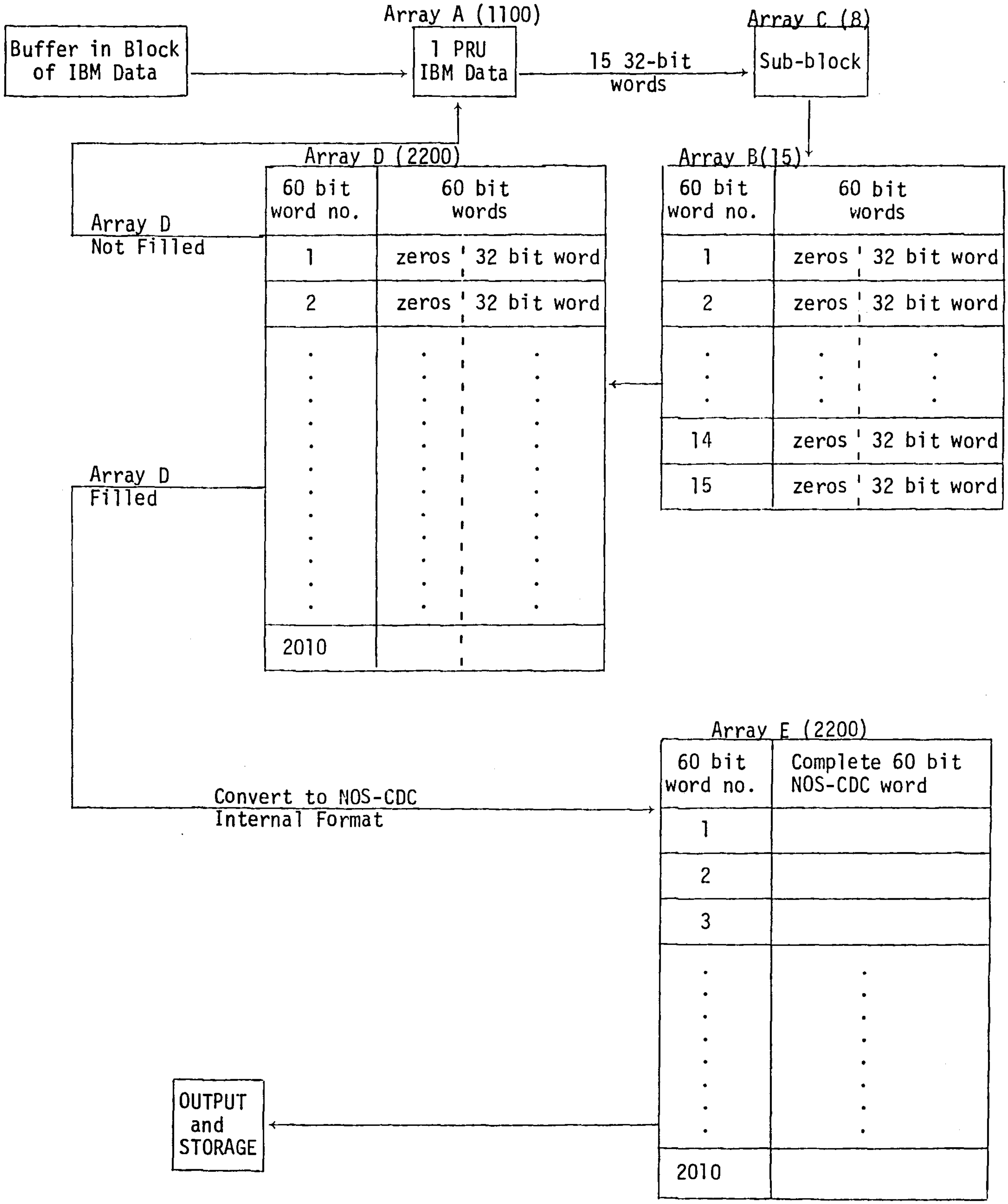
* Designates those six words maintained on condensed data tapes.

Figure B-1. Sub-Block Structure.*

Packed 60-bit Word Number	32 BIT WORD ARRANGEMENT
1	32 bit word || 28 bits
2	4 bits || 32 bit word || 24 bits
3	8 bits || 32 bit word || 20 bits
4	12 bits || 32 bit word || 16 bits
5	16 bits || 32 bit word || 12 bits
6	20 bits || 32 bit word || 8 bits
7	24 bits || 32 bit word || 4 bits
8	28 bits || 32 bit word

* Sub-block contains 480 bits of information. This is the equivalent of 8 60-bit words or 15 32-bit words.

Figure B-2. Schematic Showing the Procedure Used to Convert a Block of IBM to NOS-CDC Internal Format



```
                    BUVCOP2
      PROGRAM [redacted] (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1,TAPE2)
      DIMENSION A(1100)
      DIMENSION B(15)
      DIMENSION D(2200),E(2200),C(8)
      DIMENSION OUT(20)
C     C CONTAINS 8 CDC 60 BIT WRDS PACKED WITH IBM 32 BIT WRDS
C     B CONTAINS 15 CDC 60 BIT WRDS FILLED WITH RIGHT JUSTIFIED IBM 32 BIT WRDS
C     D CONTAINS 2000-60 BIT CDC WRDS FILLED WITH RIGHT JUSTIFIED IBM 32 BIT WRD
C     E CONTAINS 2000 60 BIT CDC WRDS EQUIV. TO THE 32 BIT IBM WRDS
      NEUF=0
      ICNT=0
      IREC=0
    4 CONTINUE
      IF (NEUF.GE.14) GO TO 300
      DO 2 I=1,100000
      DO 13 MRL=1,1100
   13 A(MRL)=0.
      BUFFER IN (1,1) (A(1),A(1100))
      IF(UNIT(1)) 1,3,5
    1 NWDS=LENGTH(1)
      IC=0
      IDDT=15
      IDYT=8
      IDB=0
      DO 10 IDX=1,NWDS,8
      DO 11 IDY=1,IDYT
      ID=IDY-1
   11 C(IDY)=A(IDX+ID)
      CALL IBMWRDS(C,B)
      DO 12 IDD=1,IDDT
      IC=IC+1
   12 D(IDB+IDD)=B(IDD)
      IDB=IDB+15
   10 CONTINUE
   14 CONTINUE
      CALL IBMFPC (D,E,IC)
COMMENT -- IC IS THE NUMBER OF 60 BIT WORDS CONTAINED IN THE E ARRAY.
C              THIS NUMBER IS EQUIVALENT TO THE NUMBER OF 32 BIT WORDS
C              IN SOME BLOCK ON THE IBM TAPE.

C              IC1 IS THE TOTAL NUMBER OF 20 WORD LOGICAL RECORDS/PRU.
      IC1=IC/20
```

PROGRAM RLT   74/74   OPT=1   FTN 4.6+452   79/09/17. 13.33

```
C          SUBTRACT 1 SINCE RECORD PRIOR TO THE END OF BLOCK CONTAINS
C          BAD DATA.
      IC1=IC1-1
C          IC2 IS THE NUMBER OF GOOD NON-ZERO WORDS IN BLOCK.
      IC2=IC1*20
      IF(NEOF.EQ.0) PRINT 910, IC,IC1,IC2
  910 FORMAT (I50,3(I5,3X))
      ICNT=ICNT+IC1
      IREC=IREC+IC1
      IV=0
      DO 250 II=1,IC1
      DO 200 III=1,20
      IV=IV+1
      OUT(III)=E(IV)
  200 CONTINUE
      WRITE (2) OUT
      IF (IREC.LE.101.AND.II.EQ.1) PRINT 905, IREC,OUT(3),OUT(4),
     1OUT(5)/3600.
  905 FORMAT (5X,*IREC= *,I6,5X,*YEAR,DAY,HOUR*,3(2X,F8.3))
  250 CONTINUE
    2 CONTINUE
    3 NEOF=NEOF+1
      PRINT 101, NEOF,1,IREC,OUT(3),OUT(4),OUT(5)/3600.
  101 FORMAT (* END FILE *I2* BLOCKS READ = *I6,*  IREC= *,I6,
     15X,*YEAR,DAY,HOUR*,3(2X,F8.3))
      IREC=0
      GO TO 4
  300 CONTINUE
      PRINT 900, ICNT
  900 FORMAT (/////I10,*ICNT= *,I8)
      STOP
    5 PRINT 103
  103 FORMAT (* PARITY ERROR*)
      END
```

SYMBOLIC REFERENCE MAP (R=1)

```
      SUBROUTINE IBMWRDS (A,B)
      DIMENSION A(8),B(15)
      INTEGER TAIL,HEAD
      IBM=77777777776000000000B
      J=1
      DO 10 I=1,8
      LASTB=(I-1)*4
      IF(LASTB.EQ.0) GO TO 1
C     MASK=2**(LASTB+1)-1
      MASK=2**(LASTB)-1
      AA=SHIFT(A(I),LASTB)
      TAIL=MASK.AND.AA
      B(J-1)=B(J-1).OR.TAIL
    1 IF(I.EQ.1) AA=A(1)
      B(J)=SHIFT(AA.AND.IBM,32)
      J=J+1
      IF(I.EQ.8) GO TO 2
      IFST=32-LASTB-4
C     MASK=2**( IFST+1)-1
      MASK=2**( IFST)-1
      HEAD=A(I).AND.MASK
      B(J)=SHIFT(HEAD,32-IFST)
      J=J+1
   10 CONTINUE
    2 RETURN
      END
```

SYMBOLIC REFERENCE MAP (R=1)

```
ENTRY POINTS
     3  IBMWRDS

VARIABLES       SN   TYPE             RELOCATION
     0  A            REAL       ARRAY      F.P.            75  AA        REAL
     0  B            REAL       ARRAY      F.P.            67  HEAD      INTEGER
    72  I            INTEGER                               70  IBM       INTEGER
    76  IFST         INTEGER                               71  J         INTEGER
    73  LASTB        INTEGER                               74  MASK      INTEGER
```

APPENDIX C - LINEAR APPROXIMATION FOR CALCULATING LOCAL TIME AS A FUNCTION OF LATITUDE

A straight line approximation to the ascending portion of the local time variation for a Sun-synchronous orbit curve from -60° to +60° latitude was calculated and is shown in Figure C-1. The relationship between the local solar time, t_ℓ, and the latitude, θ, for the observation was originally estimated to be

$$t_\ell = \frac{629.45 - \theta}{53.57} \quad . \qquad \text{(C-1)}$$

Since, selected BUV-III data, closely corroborated by TRACK2 computer program simulations, have led to what is thought to be a better estimate, that is

$$t_\ell^\circ = \frac{604.54 - \theta}{50.93} \quad . \qquad \text{(C-2)}$$

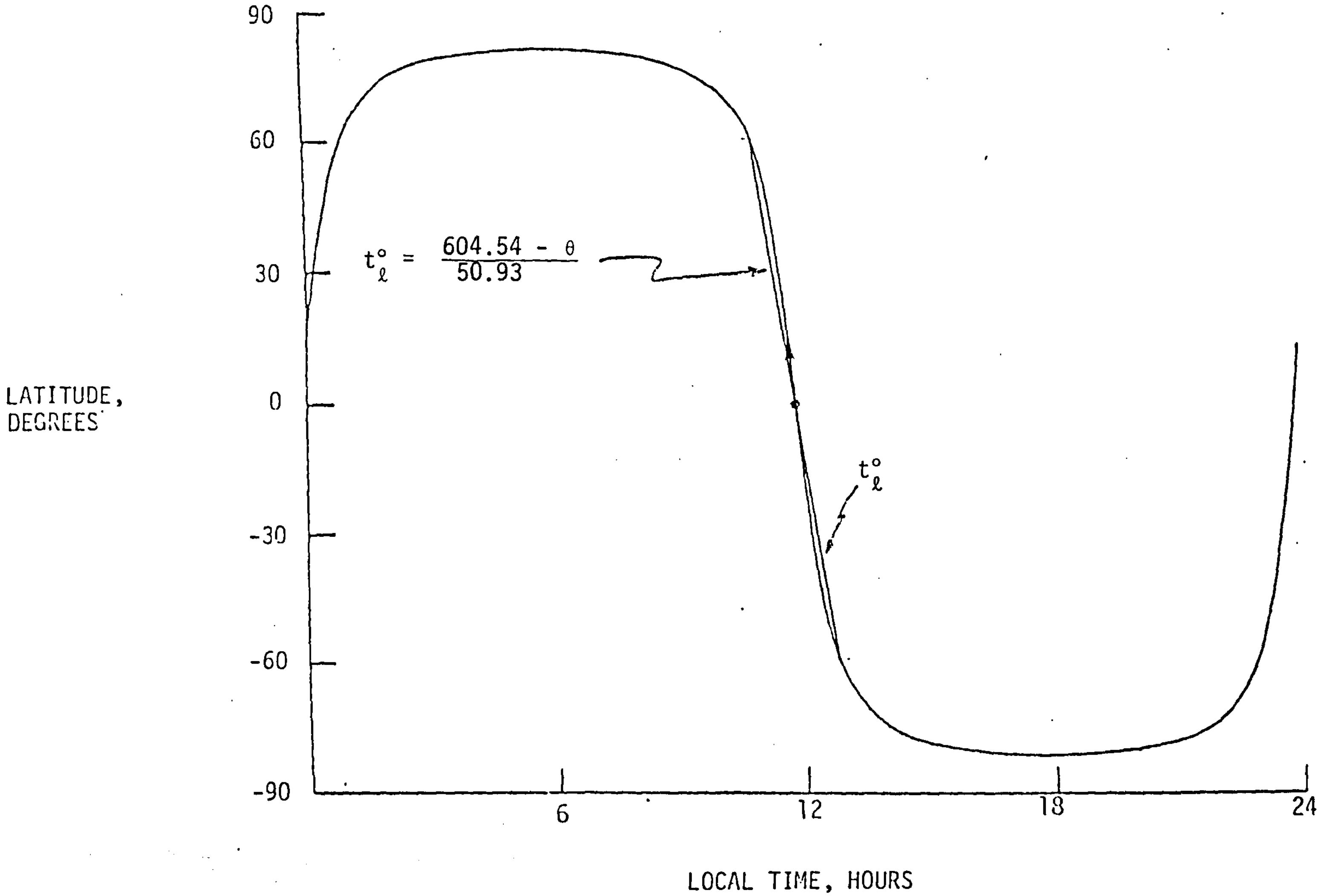
In any case, the error table shown below shows the maximum difference between equations (C-1) and (C-2) to be 0.1780 hours (10.68 minutes) where

$$\Delta t = t_\ell^\circ - t_\ell \quad . \qquad \text{(C-3)}$$

Table. Error Analysis

θ	Δt (hours)
60°	0.0619
30°	0.0909
0°	0.1200
-30°	0.1490
-60°	0.1780

Figure C-1. Approximation to the ascending portion of the local time variation of the Nimbus 4 Sun-synchronous orbit curve.



APPENDIX D - STORAGE OF GRIDDED OZONE DATA ON A MASS STORAGE RANDOM ACCESS FILE

A global grid system in the form of an array dimensioned 36 x 24 has been selected to represent the BUV ozone data. Each of the 36 rows corresponds to a 5° latitudinal zone while each of the 24 columns corresponds to a 15° longitudinal sector. Associated with each of the 864 blocks of the global grid are nine values that must be saved and stored such that they will be readily accessible when needed. For each of these values there is a separate array identified by the parameter ISET as shown in the table below.

Table. Global Arrays Saved on Mass Storage Random Access File

ISET	Array Name	Description
1	KK	Sampling Distribution
2	SUMX	Sum of ozone observations for each block
3	SUMXSQ	Sum of squares of ozone observations for each block
4	SUMT	Sum of observation times for each block
5	SUMTSQ	Sum of squares of the observation times for each block
6	SUMLT	Sum of the observed latitude for each block
7	SUMLTSQ	Sum of squares of the observed latitude for each block
8	SUMLG	Sum of the observed longitude for each block
9	SUMLGSQ	Sum of squares of the observed longitude for each block

It was decided that these arrays should be accessible on a daily basis for the 392 days beginning April 10, 1970 and ending May 6, 1971 or according to the time convention adopted during this study, NIMDAYS 100-491.

Making use of a mass storage random access (MSRA) file for this purpose is quite suitable. As can be seen, the actual data storage requirement here is

$$\frac{9 \text{ arrays}}{\text{days}} \times \frac{864 \text{ words}}{\text{array}} \times 392 \text{ days} = 3{,}048{,}192 \text{ words.}$$

However, by specifying a particular array for a given day, or several days, the computer storage requirement is reduced to that needed for only one array plus an INDEX array mentioned below.

This is illustrated in the following figure.

Figure. Mass Storage Random Access File Arrangement of Global Data Arrays

	ISET								
MSRA Day No.	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	10	11	12	13	14	15	16	17	18
3									
4									
. . .									
390									
391									
392	3520	3521	3522	3523	3524	3525	3526	3527	3528

Each of the blocks (1-3528) shown in the figure represent a data array. Let "NDEX" be the number that specifies a particular array, and "IDAY" be the MSRA day number specification. Then

$$\text{NDEX} = 9 \times (\text{IDAY} - 1) + \text{ISET} . \qquad \text{(D-1)}$$

Since

$$IDAY = NIMDAY - 99, \qquad (D\text{-}2)$$

expression (D-1) may be written in terms of NIMDAY as

$$NDEX = 9 \times (NIMDAY - 100) + ISET. \qquad (D\text{-}3)$$

For example, if the array SUMXSQ(ISET = 3) were required for NIMDAY 101, then

$$NDEX = 12,$$

and the 12th array would be accessed from the mass storage file.

The INDEX array mentioned earlier must be present and must be dimensioned by (A + 1) where A is the total number of arrays on the MSRA.

Listings of the subroutines GETDAT1 and GETDAT2 which access the BUV MSRA file follow this appendix.

```
                  SUBROUTINE GETDAT1 (XDATA,NDATA,NIMDAY,ISET)
                  DIMENSION XDATA(36,24),NDATA(36,24)
            C
            COMMENT -- MUST "CALL OPENMS (1,INDEX,3529,0)" IN MAIN OR CALLING
            C          PROGRAM.
            C
            COMMENT -- IDEX=1,392 CORRESPONDS TO NIMDAY=100,491, WHERE NIMDAY=100
            C          IS APRIL 10, 1970.
            C
            COMMENT -- FOR ISET=1, THE DATA DISTRIBUTION ARRAY CORRESPONDING TO
            C          NIMDAY IS RETURNED AS NDATA.
            C
            COMMENT -- OTHERWISE, ONE OF THE FOLLOWING ARRAYS IS RETURNED IN XDATA.
            C          ISET=2,  SUM
            C          ISET=3,  SUMXSQ
            C          ISET=4,  SUMT
            C          ISET=5,  SUMTSQ
            C          ISET=6,  SUMLT
            C          ISET=7,  SUMLTSQ
            C          ISET=8,  SUMLG
            C          ISET=9,  SUMLGSQ
            C
            C
                  NDEX=(NIMDAY-100)*9+ISET
                  IF (ISET.EQ.1) GO TO 75
                  CALL READMS (1,XDATA,864,NDEX)
                  RETURN
               75 CONTINUE
                  CALL READMS (1,NDATA,864,NDEX)
                  RETURN
                  END
```

SYMBOLIC REFERENCE MAP (R=1)

RY POINTS
3 GETDAT1

```
      SUBROUTINE  GETDAT2 (A,AVAR,KDATA,IT1,IT2,ISPEC,ICODE)
      DIMENSION A(36,24),AVAR(36,24),KDATA(36,24),XDATA(36,24)
      DIMENSION NDATA(36,24)
C
COMMENT -- SUBROUTINE GETDAT2 FINDS THE MEAN VALUE AND VARIANCE, IF
C          REQUESTED (SEE "ICODE" BELOW), OVER SOME SPECIFIED TIME
C          INTERVAL FROM SUMX AND SUMXSQ TYPE DATA WHICH HAS BEEN
C          STORED IN THE FORM OF A RANDOM ACCESS FILE, ACCESSIBLE
C          ON LOCAL FILE TAPE1.
C
C          THIS SUBROUTINE REQUIRES SUBROUTINE GETDAT1.
C
COMMENT -- DEFINITIONS OF FORMAL PARAMETERS.
C          THE FOLLOWING ARE INPUT PARAMETERS.
C          IT1   = IS THE FIRST DAY OF THE TIME INTERVAL OVER WHICH
C                  CALCULATIONS ARE MADE.
C          IT2   = IS THE LAST DAY OF THE TIME INTERVAL OVER WHICH
C                  CALCULATIONS ARE MADE.
C          ISPEC=1,  FIND MEAN OZONE VALUES.
C          ISPEC=2,  FIND MEAN TIME OF OBSERVATION VALUES.
C          ISPEC=3,  FIND MEAN LATITUDE.
C          ISPEC=4,  FIND MEAN LONGITUDE.
C          ICODE=0,  FIND ONLY MEAN VALUES.
C          ICODE=1,  FIND VARIANCE ASSOCIATED WITH ABOVE MEAN.
C
C          THE FOLLOWING ARE OUTPUT ARRAYS.
C          A     = CONTAINS MEAN VALUES.
C          AVAR  = CONTAINS VARIANCE VALUES(SEE "ICODE" BELOW).
C          KDATA = CONTAINS DATA DISTRIBUTION
C
C
COMMENT -- THE FOLLOWING STATEMENT REQUIRES THAT
C          "PRESET=INDEF" IN THE LDSET CARD.
      IF (IT1.EQ.KCODE1.AND.IT2.EQ.KCODE2) GO TO 20
      DO 15 I=1,36
      DO 15 J=1,24
      KDATA(I,J)=0
   15 CONTINUE
   20 CONTINUE
      DO 25 I=1,36
      DO 25 J=1,24
      A(I,J)=AVAR(I,J)=0.
```

```
   25 CONTINUE
C
      ISET=ISPEC*2
      DO 150 NIMDAY=IT1,IT2
C
COMMENT -- THE FOLLOWING STATEMENT REQUIRES THAT
C             "PRESET=INDEF" IN THE LDSET CARD.
      IF (IT1.EQ.KCODE1.AND.IT2.EQ.KCODE2) GO TO 35
      CALL GETDAT1 (XDATA,NDATA,NIMDAY,1)
      DO 30 I=1,36
      DO 30 J=1,24
      KDATA(I,J)=KDATA(I,J)+NDATA(I,J)
   30 CONTINUE
   35 CONTINUE
      CALL GETDAT1 (XDATA,NDATA,NIMDAY,ISET)
      DO 40 I=1,36
      DO 40 J=1,24
      A(I,J)=A(I,J)+XDATA(I,J)
   40 CONTINUE
C
      IF (ICODE.EQ.0) GO TO 150
      ISET=ISET+1
      CALL GETDAT1 (XDATA,NDATA,NIMDAY,ISET)
      DO 55 I=1,36
      DO 55 J=1,24
      AVAR(I,J)=AVAR(I,J)+XDATA(I,J)
   55 CONTINUE
      ISET=ISET-1
C
  150 CONTINUE
C
      IF (ICODE.EQ.0) GO TO 300
      DO 175 I=1,36
      DO 175 J=1,24
      IF (KDATA(I,J).LE.1) GO TO 165
      AVAR(I,J)=(AVAR(I,J)-A(I,J)*A(I,J)/KDATA(I,J)) /(KDATA(I,J)-1)
      GO TO 175
  165 CONTINUE
      AVAR(I,J)=0.
  175 CONTINUE
  300 CONTINUE
      DO 350 I=1,36
```

```
      DO 350 J=1,24
      IF (KDATA(I,J).EQ.0) GO TO 350
      A(I,J)=A(I,J)/KDATA(I,J)
  350 CONTINUE
      KCODE1=IT1     $     KCODE2=IT2
      RETURN
      END
```

```
SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
    3  GETDAT2

VARIABLES    SN  TYPE                 RELOCATION
    0  A         REAL        ARRAY      F.P.          0   AVAR      REAL      ARRAY     F.P.
  241  I         INTEGER                              0   ICODE     INTEGER             F.P.
  243  IRET      INTEGER                              0   ISPEC     INTEGER             F.P.
    0  IT1       INTEGER                F.P.          0   IT2       INTEGER             F.P.
  242  J         INTEGER                            237   KCODE1    INTEGER
  240  KCODE2    INTEGER                              0   KDATA     INTEGER   ARRAY     F.P.
 2005  NDATA     INTEGER     ARRAY                  244   NIMDAY    INTEGER
  245  XDATA     REAL        ARRAY

EXTERNALS          TYPE   ARGS
       GETDAT1              4

STATEMENT LABELS
    0  15                                40  20                            0  25
    0  30                               105  35                            0  40
    0  55                               150  150                         173  165
  174  175                              201  300                         212  350

LOOPS  LABEL     INDEX      FROM-TO     LENGTH    PROPERTIES
  25   15      * I          35 38        13B                 NOT INNER
  32   15        J          36 38         2B       INSTACK
  41   25      * I          40 43        14B                 NOT INNER
  47   25        J          41 43         3B       INSTACK
  60   190     * NIMDAY     46 72        73B                 EXT REFS  NOT INNER
```

APPENDIX E - ORTHONORMALITY PROPERTY OF SPHERICAL HARMONIC FUNCTIONS

The functions $\psi_k(x)$ for k = 1, 2, 3, . . . , are orthogonal over the interval (a,b) and, therefore, have the property that

$$\int_a^b \psi_i(x)\ \psi_j(x)dx = 0, \text{ for } i \neq j. \qquad \text{(E-1)}$$

If i = j, and if

$$\int_a^b [\psi_i(x)]^2 dx = 1, \qquad \text{(E-2)}$$

then the functions are also normal, or normalized, and form an orthonormal set of functions over the interval (a,b). Equations (E-1) and (E-2) can be written as

$$\int_a^b \psi_i(x)\ \psi_j(x)dx = \delta_{ij}, \qquad \text{(E-3)}$$

where δ_{ij}, the Kronecker delta, has the property that

$$\delta_{ij} \equiv \begin{cases} 0, \text{ for } i \neq j \\ 1, \text{ for } i = j \end{cases} . \qquad \text{(E-4)}$$

This concept can be expanded to include spherical harmonic functions over the surface of a unit sphere. Let $y(\theta,\phi)$ be a function on the surface of a unit sphere, such that

$$y(\theta,\phi) = \sum_{m=0}^{M} \sum_{n=m}^{M} [A_{mn} Z^e_{mn}(\theta,\phi) + D_{mn} Z^o_{mn}(\theta,\phi)], \qquad \text{(E-5)}$$

where

$$Z^e_{mn}(\theta,\phi) = \cos(m\phi)\ P^m_n(\cos\theta), \qquad \text{(E-6a)}$$

and

$$Z^o_{mn}(\theta,\phi) = \sin(m\phi)\ P^m_n(\cos\theta). \qquad \text{(E-6b)}$$

The $P^m_n(\cos\theta)$ are associated Legendre functions.

It can be shown that

$$\int_{x=-1}^{1} P_n^m(x)\ P_\ell^m(x)dx = \frac{(n+m)!}{(n-m)!}\frac{2}{2n+1}\delta_{n\ell}, \quad \text{[13]} \tag{E-7}$$

from which it follows that

$$\int_{x=-1}^{1} P_n(x)\ P_\ell(x) = \frac{2}{2n+1}\delta_{n\ell} \tag{E-8}$$

where $P_n(x)$ and $P_\ell(x)$ are associated Legendre functions for $m = 0$ or simply Legendre functions.

Now consider the following integral equations which must be evaluated:

$$I_1 = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} Z_{mn}^{e}(\theta,\phi)\ Z_{k\ell}^{o}(\theta,\phi)da, \tag{E-9}$$

$$I_2 = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} Z_{mn}^{e}(\theta,\phi)\ Z_{k\ell}^{e}(\theta,\phi)da, \tag{E-10}$$

$$I_3 = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} Z_{mn}^{o}(\theta,\phi)\ Z_{k\ell}^{o}(\theta,\phi)da. \tag{E-11}$$

The first may be written as

$$I_1 = \int_{\theta,\phi} P_n^m(\cos\theta)\ P_\ell^k(\cos\theta)\ \cos(m\phi)\ \sin(k\phi)da \tag{E-12}$$

where

$$da = \sin\theta d\theta d\phi \tag{E-13}$$

is the differential surface area of a unit sphere and the notation $\int_{\theta,\phi}$ is equivalent to $\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}$.

Consider the integration over ϕ.

$$\int_{\phi=0}^{2\pi} \cos(m\phi)\ \sin(k\phi)d\phi = 0 \qquad \text{(E-14)}$$

for $m = k$ or $m \neq k$. Substituting this result into equation (E-12) leads to

$$I_1 = 0$$

or

$$\int_{\theta,\phi} Z^e_{mn}(\theta,\phi)\ Z^o_{k\ell}(\theta,\phi)da = 0. \qquad \text{(E-15)}$$

The next integral can be written as

$$I_2 = \int_{\theta,\phi} P^m_n(\cos\theta)\ P^k_\ell(\cos\theta)\ \cos(m\phi)\ \cos(k\phi)da \qquad \text{(E-16)}$$

or by (E-13) as

$$I_2 = \int_{\theta,\phi} P^m_n(\cos\theta)\ P^k_\ell(\cos\theta)\ \sin\theta\ \cos(m\phi)\ \cos(k\phi)d\phi d\theta. \qquad \text{(E-17)}$$

Again integrating over ϕ yields

$$\int_{\phi=0}^{2\pi} \cos(m\phi)\ \cos(k\phi)d\phi = \begin{cases} 0, & \text{for } m \neq k \text{ and } m \geq 0 \\ \pi, & \text{for } m = k \text{ and } m \neq 0 \end{cases} \qquad \text{(E-18)}$$

and $I_2 = 0$ for $m \neq k$. Otherwise, equation (E-17) becomes

$$I_2 = \pi\int_{x=-1}^{1} P^m_n(x)\ P^m_\ell(x)dx \qquad \text{(E-19)}$$

where the substitutions $x = \cos\theta$ and $dx = -\sin\theta d\theta$ have been made along with corresponding changes in the limits of integration. Substituting equation (E-7) into equation (E-19) leads to

$$\pi \int_{x=-1}^{1} P_n^m(x)\, P_\ell^m(x)\,dx = \frac{2\pi}{2n+1} \frac{(n+m)!}{(n-m)!} \delta_{n\ell} \tag{E-20}$$

for $m \neq 0$.

If $m = k = 0$, the integral in equation (E-18) becomes

$$\int_{\phi=0}^{2\pi} \cos(m\phi)\, \cos(k\phi)\, d\phi = \int_{\phi=0}^{2\pi} d\phi = 2\pi, \tag{E-21}$$

and

$$I_2 = \frac{4\pi}{2n+1} \delta_{n\ell} \tag{E-22}$$

for $m = k = 0$. Then the integral in equation (E-10) has been evaluated and can be written as

$$\int_{\theta,\phi} Z_{mn}^e(\theta,\phi)\, Z_{k\ell}^e(\theta,\phi)\,da = = \begin{cases} \dfrac{4\pi}{2n+1} \delta_{n\ell}, & \text{for } m = k = 0 \\ \\ \dfrac{2\pi}{2n+1} \dfrac{(n+m)!}{(n-m)!} \delta_{n\ell}\delta_{mk}, & \text{otherwise} \end{cases} \tag{E-23}$$

The final integral may be written as

$$I_3 = \int_{\theta,\phi} P_n^m(\cos\theta)\, P_\ell^k(\cos\theta)\, \sin(m\phi)\, \sin(k\phi)\,da. \tag{E-24}$$

By inspection, if $m = 0$, $I_3 = 0$.

For $m \neq 0$ the integration over ϕ gives

$$\int_{\phi=0}^{2\pi} \sin(m\phi)\ \sin(k\phi) d\phi = \begin{cases} 0, & \text{for } m \neq k \\ \pi, & \text{for } m = k \end{cases} . \tag{E-25}$$

Equation (E-24) then becomes for $m = k$

$$I_3 = \pi \int_{\theta=0}^{\pi} P_n^m(\cos\theta)\ P_\ell^m(\cos\theta)\ \sin\theta d\theta , \tag{E-26}$$

which as before can be written as

$$I_3 = \pi \int_{x=-1}^{1} P_n^m(x)\ P_\ell^m(x) dx = \frac{2\pi}{2n+1} \frac{(n+m)!}{(n-m)!} \delta_{n\ell} . \tag{E-27}$$

Finally, the integral in equation (E-11) is

$$\int_{\theta,\phi} Z_{mn}^o(\theta,\phi)\ Z_{k\ell}^o(\theta,\phi) da = \frac{2\pi}{2n+1} \frac{(n+m)!}{(n-m)!} \delta_{n\ell} \delta_{mk} \delta_{mo}^* , \tag{E-28}$$

where δ_{ab}^* is defined such that

$$\delta_{ab}^* \equiv \begin{cases} 0, & \text{for } a = b \\ 1, & \text{for } a \neq b \end{cases} . \tag{E-29}$$

Now define

$$Y_{mn}^e(\theta,\phi) \equiv F_{mn}^s Z_{mn}^e(\theta,\phi) , \tag{E-30}$$

and

$$Y_{mn}^o(\theta,\phi) \equiv F_{mn}^s Z_{mn}^o(\theta,\phi) , \tag{E-31}$$

where

$$F^s_{mn} = \begin{cases} 1, \text{ for } m = 0 \\ \left[\frac{2(n-m)!}{(n+m)!}\right]^{1/2}, \text{ for } m > 0 \end{cases} . \tag{E-32}$$

It is necessary to evaluate the three integrals

$$I_1' = \int_{\theta,\phi} Y^e_{mn}(\theta,\phi)\ Y^o_{k\ell}(\theta,\phi)da, \tag{E-33}$$

$$I_2' = \int_{\theta,\phi} Y^e_{mn}(\theta,\phi)\ Y^e_{k\ell}(\theta,\phi)da, \tag{E-34}$$

$$I_3' = \int_{\theta,\phi} Y^o_{mn}(\theta,\phi)\ Y^o_{k\ell}(\theta,\phi)da. \tag{E-35}$$

The right-hand sides of equations (E-33) through (E-35) may be written as

$$\int_{\theta,\phi} Y^e_{mn}(\theta,\phi)\ Y^o_{k\ell}(\theta,\phi)da = F^{s^2}_{mn} \int_{\theta,\phi} Z^e_{mn}(\theta,\phi)\ Z^o_{k\ell}(\theta,\phi)da, \tag{E-36}$$

$$\int_{\theta,\phi} Y^e_{mn}(\theta,\phi)\ Y^e_{k\ell}(\theta,\phi)da = F^{s^2}_{mn} \int_{\theta,\phi} Z^e_{mn}(\theta,\phi)\ Z^e_{k\ell}(\theta,\phi)da, \tag{E-37}$$

$$\int_{\theta,\phi} Y^o_{mn}(\theta,\phi)\ Y^o_{k\ell}(\theta,\phi)da = F^{s^2}_{mn} \int_{\theta,\phi} Z^o_{mn}(\theta,\phi)\ Z^o_{k\ell}(\theta,\phi)da, \tag{E-38}$$

Substituting equation (E-15) into equation (E-36) yields

$$\int_{\theta,\phi} Y^e_{mn}(\theta,\phi)\ Y^o_{k\ell}da = 0. \tag{E-39}$$

Similarly by equations (E-23) and (E-37)

$$\int_{\theta,\phi} Y^e_{mn}(\theta,\phi)\ Y^e_{k\ell}(\theta,\phi)da = \frac{4\pi}{2n+1}\delta_{n\ell}\delta_{mk}. \tag{E-40}$$

Finally, equation (E-38) may be evaluated by equation (E-28) as

$$\int_{\theta,\phi} Y^o_{mn}(\theta,\phi)\ Y^o_{k\ell}(\theta,\phi)da = \frac{4\pi}{2n+1}\delta_{n\ell}\delta_{mk}\delta^*_{mo}. \tag{E-41}$$

The results required for arriving at equation (53) can be found from equations (E-39) through (E-41), respectively. That is,

$$\int_{\theta,\phi} Y^e_{mn}(\theta,\phi)\ Y^o_{mn}(\theta,\phi)da = 0, \tag{E-42}$$

$$\int_{\theta,\phi} [Y^e_{mn}(\theta,\phi)]^2 da = \frac{4\pi}{2n+1}, \tag{E-43a}$$

and

$$\int_{\theta,\phi} [Y^o_{mn}(\theta,\phi)]^2 da = \frac{4\pi}{2n+1}\ \delta^*_{mo}. \tag{E-43b}$$

Though incidental to this discussion, it should be noted that the functions $Y^e_{mn}(\theta,\phi)$ and $Y^o_{mn}(\theta,\phi)$ are orthogonal over the unit sphere since

$$\int_{\theta,\phi} Y^e_{mn}(\theta,\phi)\ Y^e_{k\ell}(\theta,\phi)da = 0 \tag{E-44a}$$

and

$$\int_{\theta,\phi} Y^o_{mn}(\theta,\phi)\ Y^o_{k\ell}(\theta,\phi)da = 0 \tag{E-44b}$$

for $n \neq \ell$, $m \neq k$, or both, and

$$\int_{\theta,\phi} Y^e_{mn}(\theta,\phi)\ Y^o_{k\ell}(\theta,\phi)da = 0 \tag{E-44c}$$

in any case.

However, these functions are not normalized over the sphere as can be seen by equations (E-43) but are said to be semi-normalized according to Adolf Schmidt[4] by the constant F^S defined in equation (E-32).

APPENDIX F - RECURRENCE RELATIONS FOR ASSOCIATED LEGENDRE POLYNOMIALS

In the modeling of atmospheric constituents with spherical harmonic functions it is useful to have the capability of calculating the required associated Legendre functions using recurrence relations. Many such relations exist for associated Legendre polyonomial functions.

Subroutine LEGNDR4 has been written to calculate the associated Legendre functions up to and including those of some specified order, m, and degree, n, for a given colatitude, θ. This subroutine is listed in Appendix G with the GLSRAN2 program.

If $P_n^m(x)$ is the associated Legendre function of order m and degree n, then the first two functions are defined as[7],

$$P_0^0(x) = 1, \qquad \text{(F-1)}$$

and

$$P_1^0(x) = x, \qquad \text{(F-2)}$$

where

$$x = \cos(\theta). \qquad \text{(F-3)}$$

The functions of higher order and degree are evaluated by two recurrence relations. Consider the recurrence relation[7]

$$P_{n+1}^m(x) = \frac{1}{n-m+1} [(2n + 1) \times P_n^m(x) - (n+m)\ P_{n-1}^m(x)]. \qquad \text{(F-4)}$$

This expression is used to calculate zero order (m=0) functions of degree n+1 from the two preceding zero order terms. Setting m=0, equation (F-4) becomes,

$$P_{n+1}^0(x) = \frac{1}{n+1} [(2n+1) \times P_n^0(x) - nP_{n-1}^0(x)]. \qquad \text{(F-5)}$$

Equation (F-5) is the first recurrence relation used in subroutine LEGNDR4. The second recurrence relation used in LEGNDR4 comes from[7]

$$(2n+1)(1-x^2)^{1/2} P_n^m(x) = P_{n+1}^{m+1}(x) - P_{n-1}^{m+1}(x). \qquad \text{(F-6)}$$

Replacing n+1 with n and m+1 with m equation (F-6) may be rewritten as

$$P_n^m(x) = P_{n-2}^m(x) + (2n-1)(1-x^2)^{1/2} P_{n-1}^{m-1}(x). \qquad \text{(F-7)}$$

Consider the first term on the right-hand-side of equation (F-7). Since the order must be equal to or less than the degree of the function (see equation (16)),

$$m \leq n-2, \qquad \text{(F-8a)}$$

or

$$n \geq m+2, \qquad \text{(F-8b)}$$

and the required recurrence relation for the higher order ($m>0$) associated Legendre functions becomes,

$$P_n^m(x) = PQ + (2n-1)(1-x^2)^{1/2} P_{n-1}^{m-1}(x) \qquad \text{(F-9)}$$

where

$$PQ = \begin{cases} P_{n-2}^m(x), & \text{for } n \geq m+2 \\ 0, & \text{otherwise} \end{cases} . \qquad \text{(F-10)}$$

The numerical technique described above as utilized in subroutine LEGNDR4 has been verified up through m = n = 12 on the NOS-CDC computer system at NASA/LaRC.

APPENDIX G - THE GLSRAN2 PROGRAM

The primary purposes of the GLSRAN2 program as used in the ozone sampling study are to generate global stratospheric ozone models in terms of surface spherical harmonic functions by performing least squares fits to sets of BUV data and to perform certain statistical analyses as have been outlined in this report ("Spherical Harmonic Model" and "Statistical Analysis of Spherical Harmonic Model"). The spherical harmonic model representation as shown in equation (20) is used by GLSRAN2. The table below shows the relationship between the functions, f_i, as used in this representation and those, Y^e_{mn} and Y^o_{mn}, as shown in equation (16).

Table. Relationship Between Spherical Harmonic Function Representations

Zonal Functions	Sectoral Functions	Tesseral Functions
$F_1 = P^o_o$	$F_{M+2} = Y^e_{11}$	$F_{(3M+1)+1} = Y^e_{12}$
$F_2 = P^o_1$	$F_{M+3} = Y^o_{11}$	$F_{(3M+1)+2} = Y^o_{12}$
$F_3 = P^o_2$	$F_{M+4} = Y^e_{22}$	$F_{(3M+1)+3} = Y^e_{13}$
.	$F_{M+5} = Y^o_{22}$	$F_{(3M+1)+4} = Y^o_{13}$
.	.	.
.	.	.
$F_{M+1} = P^o_M$	.	.
	$F_{3M} + Y^e_{MM}$	$F_{(3M+1)+NT-1} = Y^e_{M-1,M}$
	$F_{3M+1} = Y^o_{MM}$	$F_{(3M+1)+NT} = Y^o_{M-1,M}$

In the table the functions P^o_i, for i = 0, 1, . . ., M, are the zonal associated Legendre functions, or simply Legendre functions. M is the order and degree of the model.

$$NT = M(M - 1) \tag{G-1}$$

is the number of tesseral functions. There are M + 1 zonal functions and 2M sectoral functions. The number of terms in a model of order and degree M is

$$N = (M + 1)^2. \qquad \text{(G-2)}$$

Model coefficients are computed according to equation (24) which may be written as

$$\underline{B} = \underline{\underline{S}}^{-1} \, \underline{R} \qquad \text{(G-3)}$$

where $\underline{\underline{S}}$, the "information" matrix, is defined by equation (34) and

$$\underline{R} = \underline{\underline{F}}^T \, \underline{Y}. \qquad \text{(G-4)}$$

The S matrix, dimensioned N x N, is strickly a function of the sampling. As S is a symmetric matrix only its upper full triangle--the diagonal elements and those above the diagonal--is used in GLSRAN2. This implementation reduces computer time as well as the storage requirement. Since solving for the model coefficients requires that the inverse of S be computed, these time and storage savings become even more noteworthy.

The upper full triangle of S is "packed" into a vector. This vector, called $\underline{V}$ to avoid confusion, contains

$$e = \frac{N}{2}(N + 1) \qquad \text{(G-5)}$$

elements. The correspondence between S matrix elements and V vector elements is given by

$$V(i) = S(m,n) \qquad \text{(G-6a)}$$

where

$$i = m + n(n - 1)/2. \qquad \text{(G-6b)}$$

The GLSRAN2 program is set up to either calculate the V vector based on input sample data or to access a previously calculated V vector through a local file. This is also the case for the R vector though to calculate $\underline{R}$ actual ozone observations must also be available.

Once these data are contained on working local files, GLSRAN2 makes available several options regarding which model coefficients or set of coefficients can be computed. The S matrix elements contained on local file are associated with a "master" model. The most obvious option is to compute the N coefficients for this master model. Three other options exist as listed below.

1. Coefficients may be calculated for a model of order L ($L < M$). To do this the program selects the required "subset" of the packed S matrix elements contained on local file and forms a new set of packed S matrix elements. The same is done for the R vector.

2. Model coefficients may be calculated based on a specified number of independent sampling observations (for example, a certain number of Dobson stations). When this option is selected the program determines the size of the model such that the number of model terms is equal to or less than the number of independent observations and then proceeds to find the S matrix elements required to form the new S matrix for the subset model.

3. Particular model coefficients may be specified according to degree, n, order, m, and whether they are to be associated with an odd ($i = 1$), $Y^{o}_{mn}(\theta,\phi)$, or even ($i = 0$), $Y^{e}_{mn}(\theta,\phi)$, spherical harmonic function (see equations 17 and 18). Identification of required coefficients by this option follows from the expression:

$$k = \begin{cases} n + 1, & \text{for } m = 0, \\ (M + 1) + 2m - 1 + 1, & \text{for } m = n, \\ 3M + m(2n - m - 1) + i, & \text{for } m \neq 0 \\ & \text{and } m \neq n. \end{cases} \qquad \text{(G-7)}$$

This technique is illustrated below since the idea is fundamental to the three options as used to determine spherical harmonic function indices or the master S matrix elements required to form the subset S matrix. Assume the S matrix is associated with a master model of degree and order $M = 5$ and that the coefficients specified in the table below are sought.

Table. Y^i_{mn} Functional Form Indices with Corresponding F_k Functional Form Indices

	m	n	i	k
1	0	0	-	1
2	0	3	-	4
3	2	2	0	9
4	1	2	1	18

From the table it can be seen that for a 5th degree spherical harmonic model

$$Y^e_{00} = F_1 ,$$
$$Y^e_{03} = F_4 ,$$
$$Y^e_{22} = F_9 , \quad \text{(G-8)}$$

and

$$Y^o_{12} = F_{18}.$$

Also in terms of master S matrix elements the subset S matrix for this example is

$$SS = \begin{bmatrix} S_{11} & S_{14} & S_{19} & S_{1,18} \\ S_{41} & S_{44} & S_{49} & S_{4,18} \\ S_{91} & S_{94} & S_{99} & S_{9,18} \\ S_{18,1} & S_{18,4} & S_{18,9} & S_{18,18} \end{bmatrix} .$$

The following discussion pertains to the input/output (I/O) requirements and capabilities of GLSRAN2. As a complete listing of GLSRAN2 and its subroutines is included in this appendix the discussion is limited to I/O items involving the spherical harmonic model.

Four NAMELIST input lists control the program's operation. These are named below along with their associated parameters.

1. DATA
   - (a) NDATA - number of observations in data set.
   - (b) MORD - order of master model.
2. JOB
   - (a) IDATA = 1 - simulate a data set based on an input sampling scheme and model coefficients.
     - = 2 - data set is an input quantity.
   - (b) IFUNC = 2 - spherical harmonic model fit to be performed.
   - (c) IOPT = 0 - do not calculate S matrix. S matrix is already on local file TAPE4.
     - = 1 - calculate S matrix and store it on local file TAPE4.
     - = 2 - calculate S matrix, store it on local file TAPE4, and STOP program execution.
   - (d) JOPT - same description as IOPT above except that JOPT pertains to the R vector.
   - (e) ITAPE = 1
   - (f) ICASE - number of cases to be run requiring a new data set.
   - (g) JCASE - number of "sub-model" cases to be run per data set.
3. PARAMTR
   - (a) BETA - input coefficients used for data simulation.
4. JOB2
   - (a) METHOD = 1 - calculate coefficients for specified subset model.
     - = 2 - determine number of coefficients to calculate based on specified number of independent observations.
     - = 3 - particular coefficients to be calculated are specified.
     - = 4 - calculate coeffcients for complete master model.
   - (b) NFUNC - number of coefficients in subset model.
   - (c) MMORD - order of subset model.
   - (d) ICODE = 0 - do not compute coefficients.
     - = 1 - compute coefficients.

GLSRAN2 uses the FORTRAN variable dimensions source statement preprocessor program PRE. Variables input by this program control the size of GLSRAN2 arrays. These variables are:

1. N - the number of coefficients in the master model.
2. NN - the maximum value of NFUNC for a given run such that NN $\leq$ N.
3. NV - the number of element in the packed S matrix array such that NV = N(N + 1)/2.

Local files used by GLSRAN2 include:

1. TAPE1 - used for input data that must be rearranged by a user supplied subroutine to meet TAPE2 input file requirements.
2. TAPE2 - standard format input data file read by subroutine REALDAT.
3. TAPE3 - used to store such items as model coefficients and covariance matrix elements for future use.
4. TAPE4 - contains elements of packed master S matrix.
5. TAPE7 - contains master R vector.

```
      PROGRAM GLSRAN2 (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,          GLSRAN2
     1TAPE1,TAPE2,TAPE3,TAPE4,TAPE7)                                   GLSRAN2
      COMMON PI,X1,X2,YVAR,DX,X,ER,IFUNC,Y,Z1,Z2,Z,MORD,NDATA
      DIMENSION F(169),S(14365),R(169),B(169)                          PREPROCS
      DIMENSION INDEX (169)                                            PREPROCS
      DIMENSION BETA(169)                                              PREPROCS
      DIMENSION NAME(15)
      NAMELIST/DATA/X1,X2,Y1,Y2,YVAR,NDATA,Z1,Z2,MORD
      NAMELIST/JOB/IDATA,IFUNC,IOPT,JOPT,ITAPE,ICASE,JCASE
      NAMELIST/PARAMTR/BETA
      NAMELIST/JOB2/METHOD,NFUNC,MMORD,ICODE
      DATA NAME/7HS      ,7HR      ,7HCOVAR  ,7HB      ,7HCOV    ,
     X7HCOR    ,7HZBAR   ,7HSY     ,7HRY     ,7HA      ,7HB1     ,
     X7HCORI   ,7HRCOVAR ,7HSI     ,7HCOVARI /
C                                                                      GLSRAN2
COMMENT -- GLSRAN2 - PARAMETER LIST.                                   GLSRAN2
C    N         IS THE TOTAL NUMBER OF COEFFICIENTS(AND THEREFORE THE NUMBER GLSRAN2
C              OF FUNCTIONS) THAT MAKE UP THE "MASTER" MODEL.          GLSRAN2
C              N IS A VARDIM INPUT PARAMETER.                          GLSRAN2
C              (IF A NEW MASTER MODEL IS NOT BEING CALCULATED, N MAY BE SET GLSRAN2
C              EQUAL TO NN, SEE BELOW).                                GLSRAN2
C    NFUNC     IS THE TOTAL NUMBER OF COEFFICIENTS(AND THEREFORE THE NUMBER GLSRAN2
C              OF FUNCTIONS) THAT MAKE UP THE 'SUBSET' MODEL FOR A     GLSRAN2
C              PARTICULAR CASE.                                        GLSRAN2
C              NFUNC IS DETERMINED AS FOLLOWS,                         GLSRAN2
C              METHOD=1,   ORDER AND DEGREE, MMORD AND NNDEG, RESPECTIVELY, GLSRAN2
C                          ARE BOTH KNOWN FOR THE DESIRED "SUBSET" MODEL.  GLSRAN2
C                          THEN,                                       GLSRAN2
C                          NFUNC=1+NNDEG+MMORD(MMORD+1).               GLSRAN2
C              METHOD=2,   NFUNC=NUMBER OF INDEPENDENT OBSERVATIONS    GLSRAN2
C                          TO BE MODELED.                              GLSRAN2
C              METHOD=3,   NFUNC=NUMBER OF COEFFICIENTS SPECIFIED      GLSRAN2
C                          TO BE MODELED.                              GLSRAN2
C              METHOD=4,   USE ENTIRE "MASTER" MODEL.                  GLSRAN2
C                          NFUNC=N.                                    GLSRAN2
C    NN        IS THE MAXIMUM VALUE OF NFUNC DURING A GIVEN RUN,       GLSRAN2
C              BUT NOT TO BE LARGER THAN N.                            GLSRAN2
C              NN IS A VARDIM INPUT PARAMETER.                         GLSRAN2
C    NV        IS THE NUMBER OF ELEMENTS IN THE V-VECTOR(PACKED FORM OF    GLSRAN2
C              THE UPPER FULL TRIANGLE OF THE S-MATRIX).               GLSRAN2
C              NV IS A VARDIM INPUT PARAMETER.                         GLSRAN2
C              NV=N(N+1)/2.                                            GLSRAN2
```

```
C   NVV    IS THE NUMBER OF ELEMENTS IN THE VV-VECTOR(PACKED FORM OF     GLSRAN2
C          THE UPPER FULL TRIANGLE OF SS-MARTRIX, S-MATRIX FOR THE       GLSRAN2
C          "SUBSET" MODEL).                                              GLSRAN2
C          NVV=NFUNC(NFUNC+1)/2.                                         GLSRAN2
C   NOTE --  IN GLSRAN2, BOTH VECTORS V AND VV WILL USE THE STORAGE      GLSRAN2
C            SPACE IN THE S ARRAY(ONE AT A TIME).  THEREFORE,            GLSRAN2
C            THE S-ARRAY MUST BE DIMENSIONED BY NV(THE LARGER OF NV AND  GLSRAN2
C            NVV) WHEN THE V VECTOR IS TO BE CALCULATED.                 GLSRAN2
C                                                                        GLSRAN2
COMMENT -- GLSRAN2 - LOCAL FILE REQUIREMENTS.                            GLSRAN2
C   TAPE1 -- DATA FOR SUBROUTINE INPUT WHICH IS TO BE REARRANGED         GLSRAN2
C            AND PUT ONTO TAPE2.                                         GLSRAN2
C   TAPE2 -- LOCAL FILE CONTAINING DATA TO BE READ IN BY                 GLSRAN2
C            SUBROUTINE REALDAT.                                         GLSRAN2
C   TAPE3 -- RESERVED FOR RESULTS SUCH AS CALCULATED MODEL COEFFICIENTS  GLSRAN2
C            SO THAT THEY MAY BE SAVED ON PERMANENT FILE OR              GLSRAN2
C            ON MAGNETIC TAPE SUBSEQUENT TO PROGRAM EXECUTION.           GLSRAN2
C   TAPE4 -- LOCAL FILE TO CONTAIN UPPER FULL TRIANGLE OF S-MATRIX       GLSRAN2
C            WHICH IS STORED THERE IN "PACKED" FORM.  THIS DATA MAY      GLSRAN2
C            ALREADY EXIST OR MAY BE CALCULATED IN THE PROGRAM.          GLSRAN2
C   TAPE7 -- LOCAL FILE TO CONTAIN R-MATRIX ASSOCIATED WITH THE SAME     GLSRAN2
C            SAMPLING SCHEME DEFINED BY THE S-MATRIX BEING USED.         GLSRAN2
C                                                                        GLSRAN2
C                                                                        GLSRAN2
      N      =169                                                        PREPROCS
      NN     =169                                                        PREPROCS
      NV     =14365                                                      PREPROCS
      READ (5,DATA)                                                      GLSRAN2
      READ (5,JOB)                                                       GLSRAN2
      READ (5,PARAMTR)                                                   GLSRAN2
      WRITE (6,DATA)                                                     GLSRAN2
      WRITE (6,JOB)                                                      GLSRAN2
      WRITE (6,PARAMTR)                                                  GLSRAN2
C                                                                        GLSRAN2
COMMENT -- SET SEED FOR RANF AS PI=ARC COS (-1)                          GLSRAN2
      PI=ACOS (-1.)                                                      GLSRAN2
      CALL RANSET (PI)                                                   GLSRAN2
C                                                                        GLSRAN2
      DO 999 LLL=1,ICASE                                                 GLSRAN2
      PRINT 175, LLL                                                     GLSRAN2
C                                                                        GLSRAN2
      IF (ITAPE.EQ.1) READ(5,401) NDATA                                  DATMOD2
```

```
      401 FORMAT (I5)                                                     DATMOD2
          CALL SECOND(TIME)                                               GLSRAN2
          PRINT 300, NDATA,TIME                                           GLSRAN2
      300 FORMAT (1X,*NDATA=*,I8,5X,*TIME=*,F10.3)                        GLSRAN2
    COMMENT -- IF IOPT=0, NO NEW V-ARRAY IS REQUIRED.                     GLSRAN2
    C            IF JOPT=0, NO NEW R-ARRAY IS REQUIRED.                   GLSRAN2
          IF (JOPT.EQ.0.AND.IOPT.EQ.0) GO TO 58                           GLSRAN2
    C                                                                     GLSRAN2
    COMMENT -- INITIALIZE INPUT PARAMETERS TO GLSCOR1.                    GLSRAN2
          W=1.                                                            GLSRAN2
          F(1)=1.                                                         GLSRAN2
    C                                                                     GLSRAN2
          CALL GLSCOR1 (F,S,R,W,B,Y,N,NV,JOPT,SUMY,YSQSUM,IERR)           GLSRAN2
    C                                                                     GLSRAN2
    C     *     *     *     *     *     *     *     *                     GLSRAN2
    C                                                                     GLSRAN2
    COMMENT -- PROGRAM CHOSES EITHER TO USE REAL DATA OR TO SIMULATE      GLSRAN2
    C            ITSOWN DATA SUCH THAT,                                   GLSRAN2
    C            IDATA=1 --- DATA IS SIMULATED                            GLSRAN2
    C            IDATA=2 --- REAL DATA IS READ IN                         GLSRAN2
    C                                                                     GLSRAN2
    C            SUBSEQUENTLY THE REQUIRED MODEL FUNCTIONS ARE CALCULATED. GLSRAN2
          ICOUNT=0                                                        GLSRAN2
       25 CONTINUE                                                        GLSRAN2
          IF (IDATA.EQ.1) CALL SIMDAT1 (BETA,N,F)                         GLSRAN2
          IF (IDATA.EQ.2) CALL REALDAT (F,N)                              GLSRAN2
          IF (IFUNC.EQ.999) STOP2                                         GLSRAN2
          IF (IFUNC.EQ.998) STOP3                                         GLSRAN2
    C                                                                     GLSRAN2
          CALL GLSSUM1 (F,S,R,W,B,Y,N,NV,JOPT,SUMY,YSQSUM,IERR)           GLSRAN2
          ICOUNT=ICOUNT+1                                                 GLSRAN2
          IF (ICOUNT.EQ.NDATA) GO TO 50                                   GLSRAN2
          GO TO 25                                                        GLSRAN2
    C                                                                     GLSRAN2
    C                                                                     GLSRAN2
       50 CONTINUE                                                        GLSRAN2
          IF (JOPT.EQ.0) GO TO 54                                         GLSRAN2
          REWIND 7                                                        GLSRAN2
          DO 52 I=1,N                                                     GLSRAN2
          WRITE (7) R(I)                                                  GLSRAN2
       52 CONTINUE                                                        GLSRAN2
          IF (JOPT.EQ.2) STOP5                                            GLSRAN2
```

```
   54 CONTINUE                                                               GLSRAN2
      IF (IOPT.EQ.0) GO TO 58                                                GLSRAN2
      REWIND 4                                                               GLSRAN2
      DO 56 I=1,NV                                                           GLSRAN2
      WRITE (4) S(I)                                                         GLSRAN2
   56 CONTINUE                                                               GLSRAN2
      IF (IOPT.EQ.2) STOP5                                                   GLSRAN2
   58 CONTINUE                                                               GLSRAN2
C                                                                            GLSRAN2
C                                                                            GLSRAN2
COMMENT -- FOLLOWING STATEMENT 58, THE REQUIRED R AND S ARRAYS ARE           GLSRAN2
C          STORED ON LOCAL FILES TAPE7 AND TAPE4, RESPECTIVELY.              GLSRAN2
C                                                                            GLSRAN2
C          NOW USING THESE ARRAYS OR SPECIFIED SUBSETS OF THEM               GLSRAN2
C          (SPECIFIED ACCORDING TO THE VALUE OF "METHOD" -SEE TABLE          GLSRAN2
C          BELOW), CALCULATE THE COEFFICIENTS OF THE REQUIRED FUNCTION       GLSRAN2
C          (ACCORDING TO "IFUNC") AND STORE RESULTS IN THE ARRAY "B".        GLSRAN2
C          THE JCASE PARAMETER SPECIFIES THE NUMBER OF CASES TO BE           GLSRAN2
C          RUN WITH CURRENT LOCAL FILE DATA.                                 GLSRAN2
C                                                                            GLSRAN2
COMMENT --                                                                   GLSRAN2
C     METHOD    DESCRIPTION                 NFUNC            MMORD           GLSRAN2
C     ******    ***********                 *****            ****            GLSRAN2
C                                                                            GLSRAN2
C     1         SPECIFY SUBSET MODEL        NUMBER OF        ORDER           GLSRAN2
C                                           SUBSET MODEL                     GLSRAN2
C                                           FUNCTIONS                        GLSRAN2
C                                                                            GLSRAN2
C     2         SPECIFY NUMBER OF           NUMBER OF        N/A             GLSRAN2
C               INDEPENDENT OBSERVATIONS    OBSERVATIONS                     GLSRAN2
C                                                                            GLSRAN2
C     3         SPECIFY PARTICULAR          NUMBER OF        N/A             GLSRAN2
C               COEFFICIENTS                COEFFICIENTS                     GLSRAN2
C                                                                            GLSRAN2
C     4         USE COMPLETE MASTER                                          GLSRAN2
C               MODEL DATA AS REQUIRED                                       GLSRAN2
C                                                                            GLSRAN2
C     NOTE --   NN IS VARIABLY DIMENSIONED BY THE                            GLSRAN2
C               PREPROCESSOR PROGRAM, PRE, AS THE                            GLSRAN2
C               LARGEST NUMBER OF SUBSET FUNCTIONS                           GLSRAN2
C               REQUIRED IN A GIVEN RUN.                                     GLSRAN2
C                                                                            GLSRAN2
```

```
            DO 950 JJJ=1,JCASE                                              GLSRAN2
            READ (5,JOB2)                                                   GLSRAN2
            WRITE (6,JOB2)                                                  GLSRAN2
            IF (METHOD.EQ.4) GO TO 60                                       GLSRAN2
            IF (METHOD.EQ.1) CALL SUBS1 (INDEX,NFUNC,MMORD,N,MORD)          GLSRAN2
            IF (METHOD.EQ.2) CALL SUBS2 (INDEX,NFUNC,MORD)                  GLSRAN2
            IF (METHOD.EQ.3) CALL SUBS3(INDEX,NFUNC,NDEG,MORD)              GLSRAN2
            IF (INDEX(1).EQ.-999) GO TO 60                                  GLSRAN2
            GO TO 65                                                        GLSRAN2
         60 CONTINUE                                                        GLSRAN2
            NFUNC=N                                                         GLSRAN2
      C          NFUNC IS SET EQUAL TO N HERE FOR THE CASE OF USING THE FULL GLSRAN2
      C          MASTER MODEL WHEN METHOD=2(IE. INDEX(1)=-999 WAS RETURNED  GLSRAN2
      C          FROM SUBROUTINE SUBS2).                                    GLSRAN2
      C          THEREFORE NFUNC WILL NOT HAVE TO BE DEFINED FOR CASES      GLSRAN2
      C          WHERE METHOD=4.                                            GLSRAN2
            DO 62 I=1,N                                                     GLSRAN2
            INDEX(I)=I                                                      GLSRAN2
         62 CONTINUE                                                        GLSRAN2
         65 CONTINUE                                                        GLSRAN2
            REWIND 4                                                        GLSRAN2
            REWIND 7                                                        GLSRAN2
            KCOUNT=0                                                        GLSRAN2
            JCOUNT=0                                                        GLSRAN2
            DO 70 II=1,NFUNC                                                GLSRAN2
            I=INDEX(II)                                                     GLSRAN2
      COMMENT -- IF ICODE=0(ACCORDING TO 'JOB2' NAMELIST INPUT), DO NOT     GLSRAN2
      C          COMPUTE COEFFICIENTS.  THEREFORE, TAPE7 IS NOT REQUIRED.   GLSRAN2
            IF (ICODE.EQ.0) GO TO 67                                        GLSRAN2
         66 CONTINUE                                                        GLSRAN2
            IF (KCOUNT.GT.INDEX(NFUNC)) GO TO 90                            GLSRAN2
            KCOUNT=KCOUNT+1                                                 GLSRAN2
            READ(7) RR                                                      GLSRAN2
            IF (KCOUNT.NE.I) GO TO 66                                       GLSRAN2
            R(II)=RR                                                        GLSRAN2
         67 CONTINUE                                                        GLSRAN2
            DO 70 JJ=1,II                                                   GLSRAN2
      COMMENT -- IVV IS THE INDEX FOR THE VECTOR VV, TO BE STORED IN THE    GLSRAN2
      C          S-ARRAY.                                                   GLSRAN2
            IVV=(II*(II-1))/2+JJ                                            GLSRAN2
            J=INDEX(JJ)                                                     GLSRAN2
      COMMENT -- IV IS THE INDEX FOR THE VECTOR V, NOW CONTAINED ON TAPE4.  GLSRAN2
```

```
      IV=(I*(I-1))/2+J                                                  GLSRAN2
   68 CONTINUE                                                          GLSRAN2
      IF (JCOUNT.GT.IV) GO TO 90                                        GLSRAN2
      JCOUNT=JCOUNT+1                                                   GLSRAN2
      READ(4) V                                                         GLSRAN2
      IF (JCOUNT.NE.IV) GO TO 68                                        GLSRAN2
      S(IVV)=V                                                          GLSRAN2
   70 CONTINUE                                                          GLSRAN2
COMMENT -- NOW HAVE THE REQUIRED S AND R ARRAYS.                        GLSRAN2
C                                                                       GLSRAN2
      NVV=NFUNC*(NFUNC+1)/2                                             GLSRAN2
      CALL GLSCOV1 (F,S,R,W,B,Y,NFUNC,NVV,ICODE,SUMY,YSQSUM,IERR)       GLSRAN2
      IEND=MMORD+1                                                      DATMOD2
      IEND=IEND*(IEND+1)/2                                              DATMOD2
      DO 71 I=1,IEND                                                    DATMOD2
      WRITE (3,789) S(I)                                                DATMOD2
   71 CONTINUE                                                          DATMOD2
C                                                                       GLSRAN2
      PRINT 201                                                         GLSRAN2
      NDEG=MORD                                                         GLSRAN2
      NZM=NDEG+1                                                        GLSRAN2
      NSM=NZM+2*MORD                                                    GLSRAN2
      DO 80 I=1,NFUNC                                                   GLSRAN2
      IF (INDEX(I).LE.NZM) GO TO 74                                     GLSRAN2
      IF (INDEX(I).LE.NSM) GO TO 75                                     GLSRAN2
      NOTF=INDEX(I)-NSM                                                 GLSRAN2
      NOTF1=NOTF+1                                                      GLSRAN2
      NTG=0                                                             GLSRAN2
      DO 72 J=1,MORD                                                    GLSRAN2
      LDEG=(NOTF1-NTG)/2+J                                              GLSRAN2
      IF (LDEG.LE.NDEG) GO TO 73                                        GLSRAN2
      NTG=NTG+(MORD-J)*2                                                GLSRAN2
   72 CONTINUE                                                          GLSRAN2
   73 CONTINUE                                                          GLSRAN2
      LORD=J                                                            GLSRAN2
      LEO=MOD(NOTF1,2)                                                  GLSRAN2
      GO TO 76                                                          GLSRAN2
   74 CONTINUE                                                          GLSRAN2
      LORD=0                                                            GLSRAN2
      LEO=0                                                             GLSRAN2
      LDEG=INDEX(I)-1                                                   GLSRAN2
      GO TO 76                                                          GLSRAN2
```

```
   75 CONTINUE                                                             GLSRAN2
      NOSF=INDEX(I)-NZM                                                    GLSRAN2
      LDEG=(NOSF+1)/2                                                      GLSRAN2
      LEO=MOD(NOSF+1,2)                                                    GLSRAN2
      LORD=LDEG                                                            GLSRAN2
   76 CONTINUE                                                             GLSRAN2
      K=I+(I*(I-1))/2                                                      GLSRAN2
      WRITE (6,200) B(I),S(K),SQRT(S(K)),I,R(I),INDEX(I),LORD,LDEG,LEO     GLSRAN2
   80 CONTINUE                                                             GLSRAN2
C                                                                          GLSRAN2
      GO TO 95                                                             GLSRAN2
   90 CONTINUE                                                             GLSRAN2
      PRINT 110, KCOUNT,JCOUNT,NFUNC,INDEX(NFUNC),II,I,JJ,J,IVV,IV         GLSRAN2
      STOP6                                                                GLSRAN2
C                                                                          GLSRAN2
   95 CONTINUE                                                             GLSRAN2
      DO 900 I=1,NFUNC                                                     GLSRAN2
      K=I+(I*(I-1))/2                                                      DATMOD2
      WRITE(3,789) B(1),S(K)                                               DATMOD2
C     WRITE (3,789) B(I),S(K)                                              DATMOD2
  900 CONTINUE                                                             GLSRAN2
      NNDEG=NFUNC-1-MMORD*(MMORD+1)                                        DATMOD2
      VARDATA=(YSQSUM-SUMY*SUMY/NDATA)/(NDATA-1)                           DATMOD2
      BR1=R(1)*B(1)                                                        DATMOD2
      BR=0.                                                                DATMOD2
      DO 515 I=1,NFUNC                                                     DATMOD2
      BR=BR+B(I)*R(I)                                                      DATMOD2
  515 CONTINUE                                                             DATMOD2
C                                                                          DATMOD2
C                                                                          DATMOD2
      A1=(B(1)*SUMY)/(NDATA-1)                                             DATMOD2
      A2=-(SUMY*SUMY)/(NDATA*(NDATA-1))                                    DATMOD2
      PRINT 1005, NDATA,NFUNC,NNDEG,SUMY,SUMY*SUMY,YSQSUM,BR,BR1,A1,A2     DATMOD2
 1005 FORMAT (*1*,*FUNDAMENTAL STATISTICAL PARAMETERS*//                   DATMOD2
     11X,*TOTAL MEASUREMENTS(NDATA)*,T40,*= *,I6/                          DATMOD2
     21X,*NUMBER OF MODEL COEFFICIENTS(NFUNC)*,T40,*= *,I4/                DATMOD2
     31X,*DEGREE OF MODEL(NNDEG)*,T40,*= *,I3/                             DATMOD2
     41X,*SUMY*,T40,*= *,E15.8/                                            DATMOD2
     51X,*SUMY SQUARED(SUMY X SUMY)*,T40,*= *,E15.8/                       DATMOD2
     61X,*YSQSUM*,T40,*= *,E15.8/                                          DATMOD2
     71X,*YEXPSQSUM(BR)*,T40,*= *,E15.8/                                   DATMOD2
     81X,*B(1) X R(1)   (BR1)*,T40,*= *,E15.8/                             DATMOD2
```

```
     91X,*A1*,T40,*= *,E15.8/                                         DATMOD2
     *1X,*A2*,T40,*= *,E15.8)                                         DATMOD2
C                                                                     DATMOD2
C                                                                     DATMOD2
      VARERR=(YSQSUM-BR)/(NDATA-1)                                    DATMOD2
      VARMOD=VARDATA-VARERR                                           DATMOD2
C                                                                     DATMOD2
      MDF=NFUNC-1                                                     DATMOD2
      XMSM=VARMOD/MDF                                                 DATMOD2
      IERDF=NDATA-NFUNC                                               DATMOD2
      XMSE=VARERR/IERDF                                               DATMOD2
C                                                                     DATMOD2
C                                                                     DATMOD2
COMMENT -- CALCULATE THE DEGREE VARIANCES(AVERAGE SQUARE) OF THE      DATMOD2
C          SPHERICAL HARMONIC MODEL.                                  DATMOD2
COMMENT -- LET THE FIRST FIVE(5) ELEMENTS OF THE ARRAY R CONTAIN      DATMOD2
C          VARDEG(1 THROUGH 5), FOR 5-TH DEGREE MODEL.                DATMOD2
      DO 525 I=1,MMORD                                                DATMOD2
      F(I)=0.                                                         DATMOD2
      R(I)=0.                                                         DATMOD2
COMMENT -- INN IS THE DEGREE OF THE COEFFICIENT.                      DATMOD2
      INN=I                                                           DATMOD2
      IJ=I+1                                                          DATMOD2
      DO 524 J=1,IJ                                                   DATMOD2
COMMENT -- JMM IS THE ORDER OF THE COEFFICIENT.                       DATMOD2
      JMM=J-1                                                         DATMOD2
COMMENT -- IF JMM=0, COEFFICIENT IS ZONAL.                            DATMOD2
C          IF JMM=INN, COEFFICIENT IS SECTORAL.                       DATMOD2
C          OTHERWISE THE COEFFICIENT IS TESSERAL.                     DATMOD2
      IF (JMM.EQ.0) GO TO 518                                         DATMOD2
      IF (JMM.EQ.INN) GO TO 520                                       DATMOD2
C                                                                     DATMOD2
COMMENT -- CALCULATE B INDEX FOR TESSERAL COEFFICIENTS.               DATMOD2
      NT=INN-JMM                                                      DATMOD2
COMMENT -- JMMM IS THE NUMBER OF PRECEDING ROWS CONTAINING TESSERAL   DATMOD2
C          FUNCTIONS.                                                 DATMOD2
      JMMM=JMM-1                                                      DATMOD2
      IF (JMMM.EQ.0) GO TO 517                                        DATMOD2
      DO 516 II=1,JMMM                                                DATMOD2
      NT=NT+(MMORD-II)                                                DATMOD2
  516 CONTINUE                                                        DATMOD2
  517 CONTINUE                                                        DATMOD2
```

```
      KEVEN=3*MMORD+2*NT                                                DATMOD2
      KODD=1+KEVEN                                                      DATMOD2
      GO TO 521                                                         DATMOD2
  518 CONTINUE                                                          DATMOD2
C                                                                       DATMOD2
COMMENT -- CALCULATE B INDEX FOR ZONAL COEFFICIENTS.                    DATMOD2
      KEVEN=INN+1                                                       DATMOD2
      KODD=-99                                                          DATMOD2
      GO TO 521                                                         DATMOD2
  520 CONTINUE                                                          DATMOD2
C                                                                       DATMOD2
COMMENT -- CALCULATE B INDEX FOR SECTORAL COEFFICIENTS.                 DATMOD2
      KEVEN=MMORD+2*JMM                                                 DATMOD2
      KODD=KEVEN+1                                                      DATMOD2
  521 CONTINUE                                                          DATMOD2
C                                                                       DATMOD2
COMMENT -- CALCULATE THE "SQUARE ROOT" OF THE EVEN AND ODD TERM         DATMOD2
C     CONTRIBUTIONS OF THE DEGREE VARIANCES.                            DATMOD2
      EVEN=B(KEVEN)                                                     DATMOD2
      REVEN=R(KEVEN)                                                    DATMOD2
      IF (KODD.EQ.-99) GO TO 522                                        DATMOD2
      ODD=B(KODD)                                                       DATMOD2
      RODD=R(KODD)                                                      DATMOD2
      GO TO 523                                                         DATMOD2
  522 CONTINUE                                                          DATMOD2
      ODD=0.                                                            DATMOD2
      RODD=0.                                                           DATMOD2
  523 CONTINUE                                                          DATMOD2
      F(I)=F(I)+EVEN*REVEN+ODD*RODD                                     DATMOD2
      R(I)=R(I)+EVEN*EVEN+ODD*ODD                                       DATMOD2
      PRINT 1001, I,J,KODD,ODD,RODD,KEVEN,EVEN,REVEN,F(I)               DATMOD2
 1001 FORMAT (1X,*I=*,I3,* J=*,I3,                                      DATMOD2
     1* KODD=*,I3,* ODD=*,E15.8,                                        DATMOD2
     2* RODD=*,E15.8,* KEVEN=*,I3,                                      DATMOD2
     3* EVEN=*,E15.8,* REVEN=*,E15.8,                                   DATMOD2
     4* F(I)=*,E15.8)                                                   DATMOD2
  524 CONTINUE                                                          DATMOD2
      BR1=BR1+F(I)                                                      DATMOD2
COMMENT -- F(I) CONTAINS VALUES FOR THE MEAN SQUARE DUE TO              DATMOD2
C            COEFFICIENTS PER DEGREE.                                   DATMOD2
COMMENT -- F(I+NNDEG) CONTAINS VALUES OF DEGREES OF FREEDOM             DATMOD2
C            FOR THESE MEAN SQUARE CALCULATIONS.                        DATMOD2
```

```
            F(I)=F(I)/(NDATA-1)                                          DATMOD2
            F(I+NNDEG)=2.*INN+1.                                         DATMOD2
            F(I)=F(I)/F(I+NNDEG)                                         DATMOD2
      COMMENT -- S(I) CONTAINS VALUES OF THE ZONAL POWER PER DEGREE.     DATMOD2
            S(I)=      B(I+1)*B(I+1)/R(I)*100.                           DATMOD2
            R(I)=R(I)/(2*INN+1)                                          DATMOD2
        525 CONTINUE                                                     DATMOD2
            PRINT 1002, BR,BR1                                           DATMOD2
       1002 FORMAT (1X,*BR=*,E15.8,*BR1=*,E15.8)                         DATMOD2
            TPOWER=0.                                                    DATMOD2
            DO 550 I=1,NNDEG                                             DATMOD2
            TPOWER=TPOWER+R(I)                                           DATMOD2
        550 CONTINUE                                                     DATMOD2
      C                                                                  DATMOD2
            PRINT 590, VARDATA,VARMOD,XMSM,MDF,VARERR,XMSE,IERDF,        DATMOD2
           1VARMOD/VARDATA,TPOWER,A1,A1+A2,A1+A2                         DATMOD2
        590 FORMAT (///T37,*PERCENT.*,T49,*VARMOD*,T66,*DEG. CONTRIB.*,  DATMOD2
           *T83,*ACCUMULATIVE*,T100,*MEAN SQUARE*,T117,*DEG. FREEDOM*/   DATMOD2
           *T37,*ZON. POWER*,T49,*DEG. CONTRIB.*,T69,*+ A2*/             DATMOD2
           *1X,*VARDATA= *,E15.8/                                        DATMOD2
           11X,*VARMOD= *,E15.8,T98,E15.8,T117,I5/                       DATMOD2
           21X,*VARERR= *,E15.8,T98,E15.8,T117,I5/                       DATMOD2
           31X,*RATIO = *,E15.8/                                         DATMOD2
           41X,*TPOWER= *,E15.8//                                        DATMOD2
           5T43,*A1=*, E15.8,T64,E15.8,T81,E15.8)                        DATMOD2
      C                                                                  DATMOD2
            ACCUM=A1+A2                                                  DATMOD2
            DO 535 I=1,NNDEG                                             DATMOD2
            CAPPA=F(I)*F(I+NNDEG)                                        DATMOD2
            ACCUM=ACCUM+CAPPA                                            DATMOD2
            PRINT 595, I,R(I),S(I),CAPPA,CAPPA+A2,ACCUM,F(I),F(I+NNDEG)  DATMOD2
        595 FORMAT (1X,*VARDEG(*,I2,*)= *,E15.8,T36,F10.5,T47,E15.8,T64,E15.8,DATMOD2
           1T81,E15.8,T98,E15.8,T117,F5.0)                               DATMOD2
      COMMENT -- LET R(I) NOW CONTAIN PERCENTAGE POWER.                  DATMOD2
            R(I)=(R(I)/TPOWER)*100.                                      DATMOD2
        535 CONTINUE                                                     DATMOD2
            DO 545 I=1,NNDEG                                             DATMOD2
            PRINT 585, I,R(I)                                            DATMOD2
        585 FORMAT(1X,I2,5X,*PERCENT. POWER= *,F10.5)                    DATMOD2
        545 CONTINUE                                                     DATMOD2
        950 CONTINUE                                                     GLSRAN2
        999 CONTINUE                                                     GLSRAN2
```

```
          STOP                                                              GLSRAN2
C                                                                           GLSRAN2
C                                                                           GLSRAN2
      100 FORMAT (1X,*Z1= *,E15.8,5X,*Z2= *,E15.8,5X,*Y= *,E15.8,5X,*ER= *, GLSRAN2
         1E15.8)                                                            GLSRAN2
      105 FORMAT (T10,*T= *,E15.8)                                          GLSRAN2
      110 FORMAT (*1*,*STOP6 INDICATES A POTENTIAL RUN-AWAY LOOP SITUATION EGLSRAN2
         1XISTS IN LOOP - 70 IN GLSRAN2.*/1X,*THE FOLLOWING PARAMETERS ARE PGLSRAN2
         2RINTED AS DIAGNOSTIC AIDS -- KCOUNT,JCOUNT,NFUNC,INDEX(NFUNC),II,IGLSRAN2
         3,JJ,J,IVV,IV*/1X,10(I5,5X))                                       GLSRAN2
      175 FORMAT (*1*,//////////T25,*BEGIN PRINT FOR CASE NUMBER *,I3///////GLSRAN2
         1///)                                                              GLSRAN2
      200 FORMAT (T10,E15.8,T30,E15.8,T50,E15.8,T2,I3,T72,E15.8,T90,I4,T101,GLSRAN2
         1I2,T110,I2,T120,I1)                                               GLSRAN2
      201 FORMAT (//T14,*EST. COEF.*,T34,*BETA VAR.*,T53,*ST. DEVIATION*,   GLSRAN2
         1T75,*R-VECTOR*,T90,*INDEX*,T99,*ORDER*,T108,*DEGREE*,T118,        GLSRAN2
         2*EVEN=0*/                                                         GLSRAN2
         3T17,*B *,T34,*COVAR(I,I)*,T53,*SQRT(COVAR(I,I))*,T90,*ARRAY*,T119,GLSRAN2
         4*ODD=1*/)                                                         GLSRAN2
      215 FORMAT (///1X,A7)                                                 GLSRAN2
      789 FORMAT(2(5X,E15.8))                                               DATMOD2
          END                                                               GLSRAN2
```


SYMBOLIC REFERENCE MAP (R=1)

```
TRY POINTS
502   GLSRAN2

RIABLES       SN  TYPE            RELOCATION
356   ACCUM       REAL                               20332  A1        REAL
333   A2          REAL                               55137  B         REAL      ARRAY
661   BETA        REAL        ARRAY                  20331  BR        REAL
330   BR1         REAL                               20357  CAPPA     REAL
  4   DX          REAL                   / /             6  ER        REAL                 / /
351   EVEN        REAL                               20360  F         REAL      ARRAY
276   I           INTEGER                            20256  ICASE     INTEGER
263   ICODE       INTEGER                            20275  ICOUNT    INTEGER
252   IDATA       INTEGER                            20313  IEND      INTEGER
```

```
      SUBROUTINE GLSCOR1 (F,S,R,W,B,Y,N,NV,ICODE,SUMY,YSQSUM,IERR)
      DIMENSION F(N),S(NV),R(N),B(N)
C
COMMENT -- INITIALIZATION LOOP FOR THE R,B, AND S ARRAYS.
      DO 25 I=1,N
      R(I)=0.
      B(I)=0.
   25 CONTINUE
      DO 35 I=1,NV
      S(I)=0.
   35 CONTINUE
      SUMY=0.
      YSQSUM=0.
      RETURN
C
C    *     *     *     *     *     *     *     *     *     *     *
C    *     *     *     *     *     *     *     *     *     *     *
C
      ENTRY GLSSUM1
COMMENT -- SUMMATION OF THE R AND S ARRAYS.
      IF (ICODE.EQ.0) GO TO 150
      DO 125 I=1,N
      R(I)=R(I)+F(I)*W*Y
  125 CONTINUE
C
C
COMMENT -- CALCULATE THE SUM OF THE Y"S AND Y SQUARED SUMMED.
      SUMY=SUMY+Y
      YSQSUM=YSQSUM+Y*Y
  150 CONTINUE
COMMENT -- CALCULATE THE UPPER FULL TRIANGLE OF THE S-MATRIX AND STORE
C            THE RESULT IN THE ONE-DIMENSIONAL ARRAY, S, DIMENSIONED
C            BY NV -- SEE PARAMETER LIST IN GLSRAN2.
      DO 175 J=1,N
      DO 175 I=1,J
      K=I+(J*(J-1))/2
      S(K)=S(K)+F(I)*F(J)*W
  175 CONTINUE
      RETURN
C
C    *     *     *     *     *     *     *     *     *     *     *
C    *     *     *     *     *     *     *     *     *     *     *
```

```
C
      ENTRY GLSCOV1
COMMENT -- FIND INVERSE OF S AND STORE IN S.
C           FIND ESTIMATION PARAMETERS, B=SR WHERE S IS NOW THE
C                 COVARIANCE MATRIX OF PARAMETRIC ESTIMATION.
      IOP=1
      CALL SPDIMF (N,S,IOP,DET,ISCALE,IERR)
      IF (ICODE.EQ.0) GO TO 250
      DO 225 I=1,N
      B(I)=0.
      DO 225 J=1,N
      IO=I
      JO=J
      IF (I.LE.J) GO TO 200
      IO=J
      JO=I
  200 CONTINUE
      K=IO+(JO*(JO-1))/2
      B(I)=B(I)+S(K)*R(J)
  225 CONTINUE
  250 CONTINUE
      RETURN
      END
```



```
   SYMBOLIC REFERENCE MAP (R=1)

NTRY POINTS
  3  GLSCOR1        101  GLSCOV1        30  GLSSUM1

ARIABLES      SN  TYPE           RELOCATION
  0  B           REAL      ARRAY    F.P.          165  DET       REAL
  0  F           REAL      ARRAY    F.P.          161  I         INTEGER
  0  ICODE       INTEGER            F.P.            0  IERR      INTEGER               F.P.
167  IO          INTEGER                          164  IOP       INTEGER
166  ISCALE      INTEGER                          162  J         INTEGER
170  JO          INTEGER                          163  K         INTEGER
  0  N           INTEGER            F.P.            0  NV        INTEGER               F.P.
  0  R           REAL      ARRAY    F.P.            0  S         REAL       ARRAY      F.P.
```

```
      SUBROUTINE SIMDAT1 (BETA,N,F)
      COMMON PI,X1,X2,YVAR,DX,X,ER,IFUNC,Y,Z1,Z2,Z,MORD
      DIMENSION BETA(N),F(N)
      CALL SIMDAT2
COMMENT -- SEE REALDAT FOR EXPLANATION OF DATA TO BE SIMULATED AS INPUT.
      IF (MORD.NE.0) CALL SIMDAT3
      IF (IFUNC.EQ.1) GO TO 25
      IF (IFUNC.EQ.2) GO TO 125
      IF (IFUNC.EQ.4) GO TO 250
      IFUNC=999
      RETURN
C
C
COMMENT -- CALCULATE THE LINEAR POLYNOMIAL FUNCTIONS OF DEGREE "NP".
   25 CONTINUE
      CALL POLY (F,BETA,N)
   50 CONTINUE
      Y=Y+ER
      RETURN
C
C
COMMENT -- CALCULATE THE FIRST "NP" ZONAL SPHERICAL HARMONIC FUNCTIONS.
  125 CONTINUE
      CALL SPHARM (F,N)
  135 CONTINUE
      Y=0.
      DO 150 I=1,N
      Y=Y+BETA(I)*F(I)
  150 CONTINUE
      GO TO 50
COMMENT -- END SPHERICAL HARMONIC CALCULATIONS.
C
C
COMMENT -- CALCULATE THE "NP" FOURIER FUNCTIONS, OTHER THAN F(1)=1.
  250 CONTINUE
      CALL FORFNC1 (F,N)
      GO TO 135
COMMENT -- END FOURIER CALCULATIONS.
C
C
      END
```

```
              SUBROUTINE REALDAT (F,N)
              COMMON PI,X1,X2,YVAR,DX,X,ER,IFUNC,Y,Z1,Z2,Z,MORD
              DIMENSION F(N)
        C
        C
        COMMENT -- EXPLANATION OF INPUT.
        C                    X                    Z                    Y
        C     IFUNC=1,       INDEPENDENT          NONE                 DEPENDENT
        C                    VARIABLE                                  VARIABLE
        C
        C     IFUNC=2,       CO-LATITUDE          LONGITUDE            DEPENDENT
        C                    (RADIANS)            (RADIANS)            VARIABLE
        C                                                              (OZONE)
        C
        C     IFUNC=3,       LATITUDE             NONE                 DEPENDENT
        C                    (RADIANS)                                 DEPENDENT
        C                                                              VARIABLE
        C                                                              (BUV-GRIDED
        C                                                              MODEL DATA
        C                                                              CORRECTED
        C                                                              FOR DOBSON)
        C
        C     IFUNC=4,       SCALED FOURIER       NONE                 APPROPRIATE
        C                    ANGLE(RADIANS)                            DEPENDENT
        C                                                              VARIABLE
        C
        C
              IF (MORD.EQ.0) GO TO 15
              READ (2) X,Z,Y
              IF (EOF(2)) 25,50
           15 CONTINUE
              READ(2) X,Y
              IF (EOF(2)) 25,50
        C
        C
           25 CONTINUE
              IFUNC=998
              RETURN
        C
        C
           50 CONTINUE
              IF (IFUNC.EQ.1) GO TO 75
```

```
      IF (IFUNC.EQ.2) GO TO 150
      IF (IFUNC.EQ.3) GO TO 250
      IF (IFUNC.EQ.4) GO TO 350
      IFUNC=999
      RETURN
C
C
COMMENT -- CALCULATE LINEAR POLYNOMIAL FUNCTIONS THROUGH DEGREE "NP".
   75 CONTINUE
      DO 125 I=2,N
      F(I)=F(I-1)*X
  125 CONTINUE
C
      RETURN
C
C
COMMENT -- CALCULATE THE "NP" SPHERICAL HARMONIC FUNCTIONS.
  150 CONTINUE
      CALL SPHARM (F,N)
      RETURN
C
C
COMMENT -- CALCULATE F(2)=COS(2*LAT), WHERE X=LAT.
  250 CONTINUE
      F(2)=COS(2*X)
      RETURN
C
C
COMMENT -- CALCULATE THE "NP" FOURIER FUNCTIONS, OTHER THAN F(1)=1.
  350 CONTINUE
      CALL FORFNC1 (F,N)
      RETURN
C
C
      END
```

SYMBOLIC REFERENCE MAP (R=1)

```
      SUBROUTINE POLY (F,A,N)
      COMMON PI,X1,X2,YVAR,DX,X,ER,IFUNC,G
      DIMENSION F(N),A(N)
      NP=N-1
      G=A(NP+1)
      IF (NP.EQ.0) RETURN
      DO 15 M=1,NP
      J=NP+1-M
      F(M+1)=F(M)*X
      G=A(J)+X*G
   15 CONTINUE
      RETURN
      END
```

```
     SYMBOLIC REFERENCE MAP (R=1)

TRY POINTS
  3  POLY

RIABLES      SN  TYPE              RELOCATION
  0  A           REAL        ARRAY      F.P.        4  DX      REAL                 / /
  6  ER          REAL                   / /         0  F       REAL        ARRAY     F.P.
 10  G           REAL                   / /         7  IFUNC   INTEGER              / /
 31  J           INTEGER                           30  M       INTEGER
  0  N           INTEGER                F.P.       27  NP      INTEGER
  0  PI          REAL                   / /         5  X       REAL                 / /
  1  X1          REAL                   / /         2  X2      REAL                 / /
  3  YVAR        REAL                   / /

ATEMENT LABELS
  0  15

IOPS  LABEL    INDEX     FROM-TO     LENGTH    PROPERTIES
 20   15       M           7 11          5B     INSTACK

IMMON BLOCKS    LENGTH
     / /             9
```

```
      SUBROUTINE SPHARM (F,N)
      COMMON PI,X1,X2,YVAR,DX,X,ER,IFUNC,Y,Z1,Z2,Z,MORD
      DIMENSION F(1)
      DIMENSION Q(13,13)
C
C
C     *      *      *      *      *      *      *      *
COMMENT -- CALCULATE THE SECOND THROUGH THE (NDEG+1) - TH F-FUNCTIONS.
C
      MPLUS=MORD+1
      NP=N-1
      NDEG=NP-MORD*MPLUS
      CALL LEGNDR4 (MPLUS,NDEG,X,Q)
      M=0
      DO 25 I=1,NDEG
      F(I+1)=Q(1,I+1)
   25 CONTINUE
C     *      *      *      *      *      *      *      *
C
C
C     *      *      *      *      *      *      *      *
COMMENT -- CALCULATE THE 2*MORD SECTORAL FUNCTIONS.
C            STORE RESULTS IN THE F ARRAY, ELEMENTS (NDEG+2) THROUGH
C            (NDEG+1+2*MORD).
C
      NN1=NDEG+3
      NN2=2*MORD+NN1-2
      M=0
      DO 50 I=NN1,NN2,2
      M=M+1
      FS=SQRT(2./RFAC(M+M))
      FN=Q(M+1,M+1)*FS
      F(I-1)=FN*COS (M*Z)
      F(I)=FN*SIN(M*Z)
   50 CONTINUE
      IF (MORD.EQ.1) RETURN
C     *      *      *      *      *      *      *      *
C
C
C     *      *      *      *      *      *      *      *
COMMENT -- CALCULATE THE NUMTES=N-NN2 TESSERAL FUNCTIONS.
C
```

```
            NN1=NN2+2
            NN2=N
            M=0
            NN=NDEG
            DO 75 I=NN1,NN2,2
            IF (NN.LT.NDEG) GO TO 70
            M=M+1
            NN=M
         70 CONTINUE
            NN=NN+1
            FS=SQRT(2.*RFAC(NN-M)/RFAC(NN+M))
            FN=Q(M+1,NN+1)*FS
            F (I-1)=FN*COS (M*Z)
            F(I)=FN*SIN(M*Z)
         75 CONTINUE
            RETURN
            END
```


SYMBOLIC REFERENCE MAP (R=1)

```
NTRY POINTS
   3  SPHARM

ARIABLES      SN  TYPE             RELOCATION
   4  DX          REAL                  / /            6  ER         REAL                  / /
   0  F           REAL        ARRAY      F.P.        153  FN         REAL
 152  FS          REAL                                147  I          INTEGER
   7  IFUNC       INTEGER               / /          146  M          INTEGER
  14  MORD        INTEGER               / /          143  MPLUS      INTEGER
   0  N           INTEGER                F.P.        145  NDEG       INTEGER
 154  NN          INTEGER                            150  NN1        INTEGER
 151  NN2         INTEGER                            144  NP         INTEGER
   0  PI          REAL                  / /          155  Q          REAL        ARRAY
   5  X           REAL                  / /            1  X1         REAL                  / /
   2  X2          REAL                  / /           10  Y          REAL                  / /
   3  YVAR        REAL                  / /           13  Z          REAL                  / /
  11  Z1          REAL                  / /           12  Z2         REAL                  / /
```

```
      DOUBLE FUNCTION RFAC (NO)
      RFAC=1.
      IF(NO.LT.2) GO TO 11
      DO 9 I=1,NO
      RFAC=RFAC*I
    9 CONTINUE
 11   RETURN
      END
```

```
     SYMBOLIC REFERENCE MAP (R=1)

TRY POINTS
  5  RFAC

RIABLES     SN  TYPE            RELOCATION
 30  I          INTEGER                              0   NO        INTEGER         F.P.
 26  RFAC       DOUBLE

ATEMENT LABELS
  0  9                              24  11

OPS  LABEL    INDEX     FROM-TO    LENGTH    PROPERTIES
 16  9        I            4 6        5B      INSTACK

ATISTICS
PROGRAM LENGTH                   31B       25
        52000B CM USED
```

```
      SUBROUTINE FORFNC1 (F,N)
      COMMON PI,X1,X2,YVAR,DX,X
      DIMENSION F(N)
C     ***********************************************************
C
COMMENT -- NP=N-1 FOURIER FUNCTIONS ARE CALCULATED PER       *
C            CALL TO SUBROUTINE FORFNC1.                     *
C         -- THESE NP FUNCTIONS ARE OF THE FORM,             *
C            F(2*I)=COS (I*X)                                *
C    AND                                                     *
C            F(2*I+1)= SIN (I*X),                            *
C    FOR I=1,M                                               *
C    WHERE M=NP/2.                                           *
C
C     ***********************************************************
C
C
      M=(N-1)/2
      DO 25 I=1,M
      F(2*I)=COS(I*X)
      F(2*I+1)=SIN(I*X)
   25 CONTINUE
C
      RETURN
      END
```


SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
   3 FORFNC1

VARIABLES	SN	TYPE	RELOCATION						
4	DX	REAL	/ /	0	F	REAL	ARRAY	F.P.
24	I	INTEGER		23	M	INTEGER		
0	N	INTEGER	F.P.	0	PI	REAL		/ /
5	X	REAL	/ /	1	X1	REAL		/ /
2	X2	REAL	/ /	3	YVAR	REAL		/ /

```
      SUBROUTINE LEGNDR4 (NORD,NDEG,COLAT,P)
      DIMENSION P(13,1)
      DOUBLE Q(13,13)
      DOUBLE X,SINE
      MORD=NORD-1
      Q(1,1)=1.
      P(1,1)=Q(1,1)
      IF (NDEG.EQ.0) RETURN
      X=COS(COLAT)
      SINE=DSQRT(1.-X*X)
      Q(1,2)=X
      P(1,2)=Q(1,2)
      IF (NDEG.EQ.1.AND.MORD.EQ.0) RETURN
      N=0
   50 CONTINUE
      IF (MORD.NE.0) GO TO 150
      N=N+1
   75 CONTINUE
COMMENT -- CALCULATE ZERO ORDER TERM OF DEGREE N+1 WITH THE TWO
C               PREVIOUS ZERO ORDER TERMS.
      Q(1,N+2)=((2*N+1)*X*Q(1,N+1)-N*Q(1,N))/(N+1)
      P(1,N+2)=Q(1,N+2)
      IF (MORD.EQ.0.AND.NDEG.EQ.N+1) RETURN
      GO TO 50
  150 CONTINUE
      N=N+1
      M=0
  225 CONTINUE
COMMENT -- CALCULATE HIGHER THAN ZERO ORDER TERMS OF DEGREE N.
      M=M+1
      IF (X.EQ.1..OR.X.EQ.-1.) GO TO 250
      TQ=0.
      IF (N.GE.M+2) TQ=Q(M+1,N-1)
      Q(M+1,N+1)=TQ+(2*N-1)*SINE*Q(M,N)
      GO TO 300
  250 CONTINUE
      Q(M+1,N+1)=0.
  300 CONTINUE
      P(M+1,N+1)=Q(M+1,N+1)
      IF (M.EQ.MORD.AND.N.EQ.NDEG) RETURN
      IF (M.EQ.N.OR.M.EQ.MORD) GO TO 75
      GO TO 225
```

```
               END


RD NR. SEVERITY  DETAILS    DIAGNOSIS OF PROBLEM

    12    1      P         ARRAY REFERENCE OUTSIDE DIMENSION BOUNDS.



      SYMBOLIC REFERENCE MAP (R=1)

NTRY POINTS
   3  LEGNDR4

ARIABLES      SN  TYPE             RELOCATION
    0  COLAT       REAL                   F.P.         163  M          INTEGER
  161  MORD        INTEGER                             162  N          INTEGER
    0  NDEG        INTEGER                F.P.           0  NORD       INTEGER               F.P.
    0  P           REAL        ARRAY      F.P.         165  Q          DOUBLE      ARRAY
  157  SINE        DOUBLE                              164  TO         REAL
  155  X           DOUBLE

XTERNALS           TYPE    ARGS
       COS         REAL       1 LIBRARY                     DSQRT      DOUBLE      1 LIBRARY

TATEMENT LABELS
   35  50                                40  75                                    76  150
  101  225                              132  250                                  136  300

TATISTICS
  PROGRAM LENGTH                 707B      455
          52000B CM USED
```

```
      SUBROUTINE SUBS1 (INDEX,NN,MMORD,N,MORD)
      DIMENSION INDEX (NN)
COMMENT -- DEFINE REQUIRED PARAMETERS.
      NNDEG=NN-MMORD*(MMORD+1)-1
      NSZON=NNDEG+1
      NZS=NSZON
      ISS=1+NZS
      NSSEC=2*MMORD
      NSS=NZS+NSSEC
      ITS=1+NSS
      NSTES=NN-(NSZON+NSSEC)
      NTS=NSS+NSTES
      NDEG=N-MORD*(MORD+1)-1
      NMZON=NDEG+1
      NZM=NMZON
      NMSEC=2*MORD
      NSM=NZM+NMSEC
COMMENT -- CALCULATE THE ELEMENTS OF THE INDEX ARRAY.
      DO 50 KZON=1,NZS
      INDEX(KZON)=KZON
   50 CONTINUE
      K=NZM
      DO 100 KSEC=ISS,NSS
      K=K+1
      INDEX(KSEC)=K
  100 CONTINUE
      IF (NSTES.EQ.0) GO TO 200
COMMENT -- IROWS IS THE TOTAL NUMBER OF ROWS OF TESSERAL FUNCTIONS
C             IN THE SUBSET MODEL.
      IROWS=MMORD-1
      K=NSM
COMMENT -- LEFT IS THE NUMBER OF 'MASTER' SPHERICAL HARMONIC TESSERAL
C             FUNCTIONS REMAINING IN ROW IROW.
      LEFT=0
      KTES=NSS
      DO 175 IROW=1,IROWS
      K=K+LEFT
COMMENT -- NOMTR IS THE NUMBER OF 'MASTER' MODEL TESSERALS IN ROW IROW.
C             NOMMTR IS THE NUMBER OF 'SUBSET' MODEL TESSERALS IN ROW IROW.
      NOMTR=(MORD-IROW)*2
      NOMMTR=(MMORD-IROW)*2
            DO 150 KSTEP=1,NOMMTR
```

```
                  KTES=KTES+1
                  K=K+1
                  INDEX(KTES)=K
          150 CONTINUE
               LEFT=NOMTR-NOMMTR
          175 CONTINUE
          200 CONTINUE
              RETURN
              END
```

SYMBOLIC REFERENCE MAP (R=1)

```
NTRY POINTS
   3  SUBS1

ARIABLES      SN  TYPE            RELOCATION
   0  INDEX       INTEGER   ARRAY     F.P.          125  IROW      INTEGER
 122  IROWS       INTEGER                           104  ISS       INTEGER
 107  ITS         INTEGER                           120  K         INTEGER
 121  KSEC        INTEGER                           130  KSTEP     INTEGER
 124  KTES        INTEGER                           117  KZON      INTEGER
 123  LEFT        INTEGER                             0  MMORD     INTEGER         F.P.
   0  MORD        INTEGER             F.P.            0  N         INTEGER         F.P.
 112  NDEG        INTEGER                           115  NMSEC     INTEGER
 113  NMZON       INTEGER                             0  NN        INTEGER         F.P.
 101  NNDEG       INTEGER                           127  NOMMTR    INTEGER
 126  NOMTR       INTEGER                           116  NSM       INTEGER
 106  NSS         INTEGER                           105  NSSEC     INTEGER
 110  NSTES       INTEGER                           102  NSZON     INTEGER
 111  NTS         INTEGER                           114  NZM       INTEGER
 103  NZS         INTEGER

TATEMENT LABELS
   0  50                    0  100                    0  150
   0  175                 100  200
```

```
      SUBROUTINE SUBS2 (INDEX,M,MORD)
COMMENT -- GIVEN A NUMBER, M, OF INDEPENDENT OBSERVATIONS, SUBROUTINE
C          SUBS2 CALCULATES THE ARRAY INDEX WHICH CONNECTS THE INDICES
C          OF THE S-MATRICES(UPPER FULL TRIANGLES ONLY) OF THE MASTER
C          MODEL(ORDER=MORD) AND THE SUBSET MODEL(SIZE TO BE
C          DETERMINED IN THIS SUBROUTINE BASED ON M).
C
COMMENT -- NOTE      M MUST BE .GE. 1.
C
      DIMENSION INDEX(M)
      NMSEC=2*MORD
      IMAX=MORD+1
COMMENT -- NMSEC IS THE MAXIMUM NUMBER OF SECTORAL FUNCTIONS IN THE
C          MASTER MODEL.
C          IMAX IS THE MAXIMUM NUMBER OF ZONAL FUNCTIONS IN THE
C          MASTER MODEL -- REFERRED TO AS NMZON IN SUBROUTINE SUBS1.
      IMAXSQ=IMAX*IMAX
COMMENT -- IMAXSQ IS THE MAXIMUM NUMBER OF FUNCTIONS AVAILABLE IN THE
C          MASTER MODEL.
      IF (M-IMAXSQ) 15,225,250
   15 CONTINUE
      DO 25 I=2,IMAX
      IF (I*I.GT.M) GO TO 50
   25 CONTINUE
COMMENT -- THIS LOOP SHOULD NOT FINISH NORMALLY.  IF IT DOES A
C          DIAGNOSTIC WILL BE PRINTED AND EXECUTION STOPPED.
      PRINT 925
  925 FORMAT (*1*,*EXECUTION STOPPED IN SUBROUTINE SUBS2*)
      STOP4
C
C     *     *     *     *     *     *     *     *     *     *     *
C
   50 CONTINUE
      NISSEC=2*(I-2)
      NISZON=I-1
COMMENT -- NISSEC AND NISZON ARE THE NUMBER OF SECTORAL AND ZONAL
C          FUNCTIONS, RESPECTIVELY, IN THE INITIAL SUBSET MODEL OF
C          ORDER= I-2.
      MDIFF =M-NISZON*NISZON
COMMENT -- MDIFF IS THE DIFFERENCE BETWEEN M AND THE NUMBER OF FUNCTIONS
C          IN A MODEL OF ORDER=I-2, WHICH CONTAINS (I-1)*(I-1) FUNCTIONS
      MSEC=MDIFF/2
```

```
      MZON=MDIFF-MSEC
      MTES=0
COMMENT -- MSEC AND MZON ARE THE NUMBER OF EXTRA SECTORAL AND ZONAL
C           FUNCTIONS, RESPECTIVELY.  MTES IS THE NUMBER OF EXTRA
C           TESSERAL FUNCTIONS REQUIRED, IF ANY.
      ITEST=NISSEC+MSEC-NMSEC
      IF (ITEST) 75,75,70
   70 CONTINUE
COMMENT -- MSEC+NISSEC IS LARGER THAN NMSEC, THE MAXIMUM NUMBER OF
C           SECTORAL FUNCTIONS AVAILABLE IN THE MASTER MODEL.
      MSEC=NMSEC-NISSEC
      MTES=ITEST
   75 CONTINUE
      ITEST=MZON+NISZON-IMAX
      IF (ITEST) 85,85,80
   80 CONTINUE
      MZON=IMAX-NISZON
      MTES=MTES+ITEST
   85 CONTINUE
COMMENT -- THE CALCULATION OF MTES IS NOT ACTUALLY REQUIRED IN ORDER
C           TO FIND NSTES -- HOWEVER, AS A DEBUGGING AID IT IS A
C           USEFUL PARAMETER.
      NZS=NISZON+MZON
      ISS=NZS+1
      NSS=NZS+NISSEC+MSEC
      ITS=NSS+1
      NSSEC=NSS-NZS
      NSTES=M-NSSEC-NZS
      NTS=NSS+NSTES
C
C     *     *     *     *     *     *     *     *     *     *     *
C     *     *     *     *     *     *     *     *     *     *     *
C
      DO 125 KZON=1,NZS
      INDEX(KZON)=KZON
  125 CONTINUE
      IF (NSSEC.LT.1) GO TO 155
      K=IMAX
      DO 150 KSEC=ISS,NSS
      K=K+1
      INDEX(KSEC)=K
  150 CONTINUE
```

```
      155 CONTINUE
          IF (NSTES.LT.1) GO TO 180
COMMENT -- IROWS IS THE TOTAL NUMBER OF ROWS OF TESSERAL FUNCTIONS
C                IN THE SUBSET MODEL.
          IROWS=I-2
C                NOTE -- RECALL THAT NISZON=I-1
          IF (NISZON*NISZON.EQ.M) IROWS=IROWS-1
          K=3*MORD+1
COMMENT -- LEFT IS THE NUMBER OF 'MASTER' SPHERICAL HARMONIC TESSERAL
C                FUNCTIONS REMAINING IN ROW IROW.
          LEFT=0
          KTES=NSS
          DO 175 IROW=1,IROWS
          K=K+LEFT
COMMENT -- NOMTR IS THE NUMBER OF 'MASTER' MODEL TESSERALS IN ROW IROW.
C                NOMMTR IS THE NUMBER OF 'SUBSET' MODEL TESSERALS IN ROW IROW.
          NOMTR=(MORD-IROW)*2
          NOMMTR=(NISZON-IROW)*2
          IF (NISZON*NISZON.EQ.M) NOMMTR=NOMMTR-2
               DO 160 KSTEP=1,NOMMTR
               KTES=KTES+1
          IF (KTES.GT.M) GO TO 180
               K=K+1
               INDEX(KTES)=K
      160 CONTINUE
           LEFT=NOMTR-NOMMTR
      175 CONTINUE
      180 CONTINUE
          RETURN
C
     ■225 CONTINUE
          PRINT 950, M
      950 FORMAT (/////1X,*M= *,I4,*, IS THE TOTAL NUMBER OF FUNCTIONS AVAIL
         1ABLE IN THE SPECIFIED MASTER MODEL.*/1X,*THEREFORE, THE UPPER FULL
         2TRIANGLE OF THE MASTER MODEL WILL BE READ DIRECTLY FROM TAPE  AND
         3USED IN THE FOLLOWING CALCULATIONS.*)
          GO TO 300
      250 CONTINUE
          PRINT 975, M,IMAXSQ
      975 FORMAT (/////1X,*THE NUMBER OF INDEPENDENT OBSERVATIONS, *,I4,*, I
         1S GREATER THAN THE NUMBER OF FUNCTIONS, *,I4,*, CONTAINED IN THE S
         2PECIFIED MASTER MODEL.*/1X,*THEREFORE, THE UPPER FULL TRIANGLE OF
```

```
               3THE MASTER MODEL WILL BE READ DIRECTLY FROM TAPE  AND USED IN THE
               4FOLLOWING CALCULATIONS.*)
           300 CONTINUE
   130         INDEX(1)=-999
               RETURN
               END
```

RD NR. SEVERITY DETAILS   DIAGNOSIS OF PROBLEM

SYMBOLIC REFERENCE MAP (R=1)

ITRY POINTS
3 SUBS2

ARIABLES		SN	TYPE	RELOCATION	
114	I		INTEGER		
113	IMAXSQ		INTEGER		
141	IROW		INTEGER		
125	ISS		INTEGER		
127	ITS		INTEGER		
135	KSEC		INTEGER		
140	KTES		INTEGER		
137	LEFT		INTEGER		
117	MDIFF		INTEGER		
120	MSEC		INTEGER		
121	MZON		INTEGER		
116	NISZON		INTEGER		
143	NOMMTR		INTEGER		
126	NSS		INTEGER		
112	IMAX		INTEGER		
0	INDEX		INTEGER	ARRAY	F.P.
136	IROWS		INTEGER		
123	ITEST		INTEGER		
134	K		INTEGER		
144	KSTEP		INTEGER		
133	KZON		INTEGER		
0	M		INTEGER		F.P.
0	MORD		INTEGER		F.P.
122	MTES		INTEGER		
115	NISSEC		INTEGER		
111	NMSEC		INTEGER		
142	NOMTR		INTEGER		
130	NSSEC		INTEGER		

```
      SUBROUTINE SUBS3 (INDEX,NFUNC,NDEG,MORD)
COMMENT -- PARTICULAR COEFFICIENTS OF INTEREST ARE SPECIFIED AS CARD
C          INPUT TO BE READ FROM THIS SUBROUTINE.  THE NUMBER OF
C          FUNCTIONS, NFUNC, TO BE READ IS A FORMAL PARAMETER OF THIS
C          SUBROUTINE AND DEFINES THE NUMBER OF ELEMENTS CONTAINED
C          IN THE ARRAY INDEX.
      DIMENSION INDEX(NFUNC)
      DO 100 I=1,NFUNC
      READ (5, 915) LORD,LDEG,LEO
COMMENT --             LORD -- ORDER OF FUNCTION.
C                      LDEG -- DEGREE OF FUNCTION.
C                      LEO --     =0,  FUNCTION IS EVEN.
C                                 =1,  FUNCTION IS ODD.
      IF (LORD.EQ.0) GO TO 25
      IF (LORD.EQ.LDEG) GO TO 50
      K=(NDEG+1)+2*MORD+LEO+LORD*(2*LDEG-LORD-1)-1
      GO TO 75
   25 CONTINUE
      K=LDEG+1
      GO TO 75
   50 CONTINUE
      K=(NDEG+1)+2*LORD-1+LEO
   75 CONTINUE
      PRINT 925, I,LORD,LDEG,LEO,K
      INDEX(I)=K
  100 CONTINUE
      RETURN
  915 FORMAT (I2,I2,I1)
  925 FORMAT (1X,*FUNCTION NUMBER *,I4,* HAS BEEN SPECIFIED AS ORDER= *,
     1I2,*, DEGREE= *,I2,*, AND LEO= *,I1,*.  K= *,I4)
      END
```


SYMBOLIC REFERENCE MAP (R=1)

```
NTRY POINTS
  3  SUBS3
```

```
      SUBROUTINE PRTSYM1 (S,NVV,NFUNC,ICODE)
COMMENT -- S IS A VECTOR WHICH CONTAINS THE UPPER FULL TRIANGLE OF
C           SOME SYMMETRIC MATRIX Z.  S IS PACKED AS FOLLOWS,
C           K=0
C           DO 20 J=1,NFUNC
C           DO 10 I=1,J
C           K=K+1
C           S(K)=Z(I,J)
C        10 CONTINUE
C        20 CONTINUE
C           WHERE NFUNC IS THE ORDER OF THE MATRIX Z.
C
C           PRTSYM1 WILL EITHER,
C           A.  PRINT THE VECTOR S AS THE UPPER FULL TRIANGLE OF Z,
C               ICODE=0, OR,
C           B.  PRINT THE CORRELATION FORM OF Z(SAME FORMAT AS ABOVE)
C               FOR ICODE=1.
C
C           NFUNC AS DEFINED IN GLSRAN2 IS THE NUMBER OF COLUMNS IN THE
C           Z-MATRIX.
C
C           MCOL = NUMBER OF COLUMNS/LINE OF PRINT AND MUST BE IN
C           AGREEMENT WITH FORMAT STATEMENT 900 AND 950.
C
C           THE ARRAY A DIMENSIONED AS A(MCOL) IS USED FOR PRINTING
C           SO THAT S WILL NOT BE DESTROYED WHEN ICODE=1.
C
C           Z IS DIVIDED INTO IR SECTIONS FOR PRINTING.
C
C           JIDX = J INDEX OF FIRST COLUMN
C           FOR A PARTICULAR SECTION, KSEC.
C           JNDX = J INDEX OF LAST COLUMN
C           FOR A PARTICULAR SECTION, KSEC.
C           JNDX ALSO = I INDEX OF LAST ROW
C           FOR A PARTICULAR SECTION, KSEC.
C           KSEC = NUMBER OF SECTION BEING PRINTED.
      DIMENSION S(NVV),A(12)
      MCOL=12
      JNDX=0
      IRATIO=NFUNC/MCOL
      IR=IRATIO+1
      IDIFF=NFUNC-IRATIO*MCOL
```

```
      IF (IDIFF.NE.0) GO TO 25
      IR=IRATIO
      IDIFF=MCOL
   25 CONTINUE
      DO 100 KSEC=1,IR
      JIDX=JNDX+1
      JNDX=JNDX+MCOL
      IF (KSEC.EQ.IR) JNDX=(JNDX-MCOL)+IDIFF
      JEND=JNDX-(KSEC-1)*MCOL
      PRINT 950, (J,J=JIDX,JNDX)
      DO 100 I=1,JNDX
      DO 30 J=JIDX,JNDX
      J1=J-(KSEC-1)*MCOL
      A(J1)=0.
   30 CONTINUE
      IF (I.GT.JIDX) JIDX=JIDX+1
      DO 50 J=JIDX,JNDX
      J1=J-(KSEC-1)*MCOL
      K=(J*(J-1))/2+I
      IF (ICODE.EQ.0) GO TO 35
      K1=(I*(I+1))/2
      K2=(J*(J+1))/2
      A(J1)=S(K)/SQRT(S(K1)*S(K2))
      GO TO 50
   35 CONTINUE
      A(J1)=S(K)
   50 CONTINUE
      PRINT 900, I,(A(J1),J1=1,JEND)
  100 CONTINUE
      RETURN
  900 FORMAT (T4,I3,12(E10.3))
  950 FORMAT (//T11,12(I3,7X)/)
      END
```

SYMBOLIC REFERENCE MAP (R=1)

Appendix H - FOURIER SERIES REPRESENTATION OF A DISCRETE DATA SET

The Fourier series approximation takes the form

$$g(x) = A_o + \sum_{\ell=1}^{q} [A_\ell \cos(\ell x) + B_\ell \sin(\ell x)], \qquad \text{(H-1)}$$

where $g(x)$ is periodic over 2π and $q \leq Q$. If now $f(x)$ is a function for $2Q + 1$ discrete equally spaced values of x over the same period as $g(x)$ above, a set of Fourier coefficients may be found that satisfies equation (H-1) by using the least-squares criterion. Let ε_r be the error associated with the rth value of x, then

$$\varepsilon_r = f(x_r) - g(x_r),$$

and

$$\varepsilon_r^2 = [f(x_r) - A_o - \sum_{\ell=1}^{q} (A_\ell \cos(\ell x_r) + B_\ell \sin(\ell x_r))]^2. \qquad \text{(H-2)}$$

The least-squares technique as discussed earlier in this report requires that

$$\sum_r [f(x_r) - A_o - \sum_{\ell=1}^{q} (A_\ell \cos(\ell x_r) + B_\ell \sin(\ell x_r))]^2$$

be minimized. This leads to

$$A_o = \frac{1}{2Q} \sum_{r=-Q+1}^{Q} f(x_r), \qquad \text{(H-3a)}$$

$$A_\ell = \frac{1}{Q} \sum_{r=-Q+1}^{Q} f(x_r)\cos(\ell x_r), \qquad \text{(H-3b)}$$

for $\ell \neq 0, Q$,

$$A_Q = \frac{1}{2Q} \sum_{r=-Q+1}^{Q} f(x_r)\cos(Qx_r), \qquad \text{(H-3c)}$$

and

$$B_\ell = \frac{1}{Q} \sum_{r=-Q+1}^{Q} f(x_r)\sin(\ell x_r).^{14} \qquad \text{(H-3d)}$$

Equations (H-3) may be rewritten as

$$A_o = \frac{1}{Q} [\frac{1}{2} H_o + H_1 + H_2 + . . . + H_{Q-1} + H_Q], \qquad \text{(H-4a)}$$

$$A_\ell = \frac{2}{Q} [\frac{1}{2} H_o + H_1 \cos(\ell x_1) + H_2 \cos(\ell x_2) + . . . +$$

$$H_{Q-1} \cos(\ell x_{Q-1}) + \frac{1}{2} H_Q \cos(\ell x_Q)], \qquad \text{(H-4b)}$$

$$A_Q = \frac{1}{Q} [\frac{1}{2} H_o - H_1 + H_2 - . . . + (-1)^{Q-1} H_{Q-1}, \qquad \text{(H-4c)}$$

and

$$B_\ell = \frac{2}{Q} [G_1 \sin(\ell x_1) + G_2 \sin(\ell x_2) + . . . + G_{Q-1} \sin(\ell x_{Q-1})], \qquad \text{(H-4d)}$$

where

$$H(x) = \frac{1}{2} [f(x) + f(-x)] \qquad \text{(H-5a)}$$

and

$$G(x) = \frac{1}{2} [f(x) - f(-x)].^{14} \qquad \text{(H-5b)}$$

Equations (H-4) and (H-5) then give the required Fourier coefficients which will satisfy equation (H-1).

Since f(x) is periodic over 2π,

$$f(\pi) = f(-\pi)$$

so that there are 2Q independent pieces of data. Then, as the objective of this Fourier series representation is data interpolation, rather than say "smoothing", the q in equation (H-1) takes on the value Q so that all 2Q possible terms are used. Equation (H-1) may now be written as

$$f(x) = A_o + \sum_{\ell=1}^{Q} [A_\ell \cos(\ell x) + B_\ell \sin(\ell x)]. \qquad \text{(H-6)}$$

APPENDIX I - THE OZSTAT2 PROGRAM

This appendix contains a listing of the OZSTAT2 program and its subroutines.

The OZSTAT2 program has three basic capabilities:

a. Data Grouping
b. Statistical Analysis
c. Computer Graphics

The program is designed to read the BUV data formatted as described in Appendix B and to group the data into a global grid system. Based on this grid system means and variances are calculated for individual grid blocks, latitudinal zones, and for that part of the grid system that contains data. These calculations are described in more detail in section 3.

Graphics capabilities included in the OZSTAT2 program provide for each case a plot of the zonal means with ±1 sigma error bars, a scatter diagram of the ozone distribution as a function of latitude, and histograms of the data sampling distribution as a function of latitude or longitude.

```
      PROGRAM OZSTAT2 (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE10,    OZSTAT2    1
     1TAPE11,TAPE12,TAPE13,TAPE14,TAPE15,TAPE1,TAPE2)                   OZSTAT2    2
      COMMON/DD/DATE                                                    OZSTAT2    3
      DIMENSION A(4),K(5),ALAT(100),AOZ(100),DATE(2)                    OZSTAT2    4
      DIMENSION SUMXSQ(36,24),KK(36,24),GP(36,24),SUMX(36,24)           OZSTAT2    5
      DIMENSION KCOUNT(36,24),SUMGPSQ(36,24),VARGP(36,24)               OZSTAT2    6
      DIMENSION ILAT(36),RLAT(38),RKK(36,24),RLATBND(38),NAME(36)       OZSTAT2    7
      DIMENSION RLNGBK(26),RK(26)                                       OZSTAT2    8
      DIMENSION VARLATB(36),AVGLATB(36),RAT(36),STDEVP(36),STDEVN(36)   OZSTAT2    9
      DIMENSION XDATA(6)                                                OCT79      1
      DIMENSION SUMT(36,24),SUMTSQ(36,24),SUMLT(36,24),SUMLG(36,24)     OCT79      2
      DIMENSION SUMLTSQ(36,24),SUMLGSQ(36,24)                           OCT79      3
      DATA NAME/3HI=1,3HI=2,3HI=3,3HI=4,3HI=5,3HI=6,3HI=7,3HI=8,3HI=9,4HOZSTAT2   12
     1I=10,4HI=11,4HI=12,4HI=13,4HI=14,4HI=15,4HI=16,4HI=17,4HI=18/     OZSTAT2   13
C     SET UP PLOT VECTOR FILE                                           OZSTAT2   14
C     SUBROUTINE PSEUDO (FN), FN = FILENAME, CAN BE FOUND IN SECTION    OZSTAT2   15
C     1.4.1 OF THE GRAPHICS MANUAL                                      OZSTAT2   16
C     IF LEROY IS NOT SPECIFIED, LIQUID INK PEN, BALL POINT PEN         OZSTAT2   17
C       IS AUTOMATICALLY CALLED, IF REQUIRED, BY DEFAULT.               OZSTAT2   18
C       LEROY IS ONLY USED FOR THE CALCOMP POSTPROCESSOR.               OZSTAT2   19
C     FIRST FRAME MUST CONTAIN AT LEAST FIVE PLOT VECTORS WHEN USING    OZSTAT2   20
C       CALCOMP.                                                        OZSTAT2   21
      CALL PSEUDO(6LMYSAV1)                                             OZSTAT2   22
      CALL LEROY                                                        OZSTAT2   23
      DO 10 I=1,6                                                       OZSTAT2   24
   10 CALL CALPLT (0.,0.,-3)                                            OZSTAT2   25
C     *     *     *     *     *     *     *                             OZSTAT2   26
C     INITIALIZE PARAMETERS FOR SUBROUTINE SELECT OPTIONS               OZSTAT2   27
C     IF JSELECT =0  DO GRID BLOCKS ONLY                                OZSTAT2   28
C     IF JSELECT =1  DO GRID POINTS ONLY                                OZSTAT2   29
C     IF JSELECT =2  DO BOTH                                            OZSTAT2   30
      JSELECT=1                                                         OZSTAT2   31
      JSELECT=2                                                         OZSTAT2   32
      JSELECT=0                                                         OZSTAT2   33
C     MSELECT =0, SKIPS BOTH PLOT ROUTINES                              OZSTAT2   34
C     MSELECT=1, CALL AVARPLT ONLY                                      OZSTAT2   35
C     MSELECT=2, CALL HISTPLT ONLY                                      OZSTAT2   36
C     MSELECT =3, CALLS BOTH                                            OZSTAT2   37
      MSELECT =0                                                        OZSTAT2   38
      MSELECT =1                                                        OZSTAT2   39
      MSELECT =2                                                        OZSTAT2   40
      MSELECT =3                                                        OZSTAT2   41
```

```
      C     *     *     *     *     *     *     *                  OZSTAT2     42
      C     INITIALIZE PARAMETERS FOR SELECTING GRID BLOCK SIZE       OZSTAT2     43
      C          ISIZE IS THE LATITUDE DIMENSION IN DEGREES           OZSTAT2     44
      C          JSIZE IS THE LONGITUDE DIMENSION IN DEGREES          OZSTAT2     45
            ISIZE=5                                                   OZSTAT2     46
            JSIZE=15                                                  OZSTAT2     47
      C          NLAT IS THE NUMBER OF ISIZE LATITUDE ZONES           OZSTAT2     48
      C          NLONG IS THE NUMBER OF JSIZE LONGITUDE BANDS         OZSTAT2     49
      C     DIMENSION STATEMENTS MUST BE ADJUSTED FOR EACH RUN ACCORDING OZSTAT2  50
      C          TO NLAT AND NLONG.                                   OZSTAT2     51
            NLAT=180/ISIZE                                            OZSTAT2     52
            NLONG=360/JSIZE                                           OZSTAT2     53
            PI=ACOS(-1.)                                              OCT79        4
      C     NCALC IS THE NUMBER OF TIME PERIODS OVER WHICH CALCULATIONS OZSTAT2   54
      C       WILL BE EXECUTED.                                       OZSTAT2     55
            NCALC=8                                                   OCT79        5
            NCALC=1                                                   OCT79        6
      C                                                               OZSTAT2     57
            DO 200 L=1,NCALC                                          OZSTAT2     58
            READ (5,225) K4,NDAY,DATE                                 OZSTAT2     59
      C     K4 FROM TIME INTERVAL NCALC+1 MUST BE GREATER THAN NDAY   OZSTAT2     60
      C       FROM TIME INTERVAL NCALC                                OZSTAT2     61
      C     ********************************************              OZSTAT2     62
      C     ************************************************************ OZSTAT2  63
      C     TOTAL OZONE DATA IS AVERAGED.  MEAN AND VARIANCE ARE PUT INTO OZSTAT2 64
      C          PARTICULAR ELEMENTS OF AN NLAT X NLONG GRID SYSTEM.    *** OZSTAT2 65
      C     ******************************************               OZSTAT2     66
      C     IN THE FOLLOWING STATEMENTS, CERTAIN PARAMETERS ARE INITIALIZED OZSTAT2 67
      C     IJ IS USED AT STAEMENT 57                                 OZSTAT2     68
            IJ=-1                                                     OZSTAT2     69
      C     IEOF IS USED AT STATEMENT 21.  IEOF=1 INDICATES THAT THE  OZSTAT2     70
      C          END OF FILE HAS BEEN REACHED.                        OZSTAT2     71
            IEOF=0                                                    OZSTAT2     72
            IDAY=K4                                                   OZSTAT2     73
            IISUM=0                                                   OZSTAT2     74
            GSUM=0.                                                   OZSTAT2     75
            DO 15 I=1,NLAT                                            OZSTAT2     76
            DO 15 J=1,NLONG                                           OZSTAT2     77
            KK(I,J)=0                                                 OZSTAT2     79
            SUMXSQ(I,J)=0.                                            OZSTAT2     80
            SUMX(I,J)=0.                                              OZSTAT2     81
            SUMT(I,J)=0.                                              OCT79        7
```

```
            SUMTSQ(I,J)=0.                                                  OCT79        8
            SUMLT(I,J)=0.                                                   OCT79        9
            SUMLG(I,J)=0.                                                   OCT79       10
            SUMLTSQ(I,J)=0.                                                 OCT79       11
            SUMLGSQ(I,J)=0.                                                 OCT79       12
         15 CONTINUE                                                        OZSTAT2     82
      C     ****************************************************************OZSTAT2     83
            IF((MSELECT.EQ.0).OR.(MSELECT.EQ.2)) GO TO 18                   OZSTAT2     84
      C     CALL AVARPLT TO CONSTRUCT AXES AND LABELS FOR SCATTER DIAGRAM,  OZSTAT2     85
      C       MEAN CURVE AND STANDARD DEVIATION PLOT.  DEFINE M AS ANYTHING.OZSTAT2     86
            M=100                                                           OZSTAT2     87
            CALL AVARPLT (VARLATB,AVGLATB,ALAT,AOZ,RAT,STDEVP,STDEVN,NLAT)  OZSTAT2     88
      C                                                                     OZSTAT2     89
         18 CONTINUE                                                        OZSTAT2     90
            DO 35 M=1,100                                                   OZSTAT2     91
         20 READ(1) (XDATA(J),J=1,6)                                        OCT79       13
            IF (EOF(1)) 21,22                                               OZSTAT2     93
         21 IEOF=1                                                          OZSTAT2     94
            GO TO 23                                                        OZSTAT2     95
         22 CONTINUE                                                        OZSTAT2     96
            K(4)=XDATA(2)                                                   OCT79       14
            K(4)=(XDATA(1)-1970.)*365.+K(4)                                 OCT79       15
            IF (XDATA(1).EQ.1973..OR.XDATA(1).EQ.1974..OR.XDATA(1).EQ.1975..OCT79       16
           1OR.XDATA(1).EQ.1976.) K(4)=K(4)+1                               OCT79       17
            IF (XDATA(1).EQ.1977.) K(4)=K(4)+2                              OCT79       18
            RK5=XDATA(3)/3600.                                              OCT79       19
            K(5)=RK5                                                        OCT79       20
            A(1)=(RK5-K(5))*60.                                             OCT79       21
            A(2)=XDATA(4)                                                   OCT79       22
            A(3)=XDATA(5)                                                   OCT79       23
            A(4)=ABS(XDATA(6))                                              OCT79       24
            IF (A(4).EQ.999..OR.A(4).EQ.77.) GO TO 20                       OCT79       25
      C     K(4) IS THE DAY NUMBER ********************                     OZSTAT2     97
            IF (A(4).LT.0.200.OR.A(4).GT.0.65)          PRINT 905, XDATA    OCT79       26
        905 FORMAT (*0*,T10,6(E15.8,5X)/)                                   OCT79       27
            IF (K(4).LT.IDAY) GO TO 20                                      OZSTAT2     98
            IF (K(4).LE.NDAY) GO TO 30                                      OZSTAT2     99
            BACKSPACE 1                                                     OZSTAT2    100
         23 IF (M.EQ.1) GO TO 25                                            OZSTAT2    101
            IF((MSELECT.EQ.0).OR.(MSELECT.EQ.2)) GO TO 25                   OZSTAT2    102
            CALL SCAT (VARLATB,AVGLATB,ALAT,AOZ,M-1,RAT,STDEVP,STDEVN,NLAT) OZSTAT2    103
         25 IF (IEOF.EQ.1) GO TO 75                                         OZSTAT2    104
```

```
      GO TO 50                                                            OZSTAT2    105
C     A(2) IS THE LATITUDE                                                OZSTAT2    106
   30 ALAT(M)=A(2)                                                        OZSTAT2    107
      AOZ(M)=A(4)                                                         OZSTAT2    108
      K4=K(4)                                                             OZSTAT2    109
C     ******************************************************************  OZSTAT2    110
C     DATA RECORDS FOR WHICH THE LATITUDE = 0 DEGREES WILL BE ASSIGNED    OZSTAT2    111
C     TO THE NORTHERN HEMISPHERE.  HOWEVER, IT IS BELIEVED THAT NO        OZSTAT2    112
C     SUCH RECORDS OCCUR BETWEEN DAYS 101 AND 465.                        OZSTAT2    113
C     ******************************************                          OZSTAT2    114
      LAT=ABS(A(2))                                                       OZSTAT2    115
      I=LAT/ISIZE+1                                                       OZSTAT2    116
      IF (A(2).LT.0) I=I+NLAT/2                                           OZSTAT2    117
C     A(3) IS THE LONGITUDE                                               OZSTAT2    118
      LONG=A(3)                                                           OZSTAT2    119
      J=LONG/JSIZE+1                                                      OZSTAT2    120
      KK(I,J)=KK(I,J)+1                                                   OZSTAT2    129
      XSQ=A(4)*A(4)                                                       OZSTAT2    130
      SUMXSQ(I,J)=SUMXSQ(I,J)+XSQ                                         OZSTAT2    131
      SUMX(I,J)=SUMX(I,J)+A(4)                                            OZSTAT2    132
      TM=K(4)+XDATA(3)/86400.                                             OCT79       28
      TMSQ=TM*TM                                                          OCT79       29
      SUMTSQ(I,J)=SUMTSQ(I,J)+TMSQ                                        OCT79       30
      SUMT(I,J)=SUMT(I,J)+TM                                              OCT79       31
      SUMLT(I,J)=SUMLT(I,J)+A(2)                                          OCT79       32
      SUMLG(I,J)=SUMLG(I,J)+A(3)                                          OCT79       33
      SUMLTSQ(I,J)=SUMLTSQ(I,J)+A(2)*A(2)                                 OCT79       34
      SUMLGSQ(I,J)=SUMLGSQ(I,J)+A(3)*A(3)                                 OCT79       35
   35 CONTINUE                                                            OZSTAT2    133
      IF((MSELECT.EQ.0).OR.(MSELECT.EQ.2)) GO TO 18                       OZSTAT2    134
      CALL SCAT (VARLATB,AVGLATB,ALAT,AOZ,100,RAT,STDEVP,STDEVN,NLAT)     OZSTAT2    135
      GO TO 18                                                            OZSTAT2    136
C     ******************************************************************  OZSTAT2    137
C     *********************************                                   OZSTAT2    138
C     BEGIN STATISTICS CALCULATIONS FOR GRID SYSTEMS.                     OZSTAT2    139
   50 IF (JSELECT.EQ.1) GO TO 97                                          OZSTAT2    140
      PRINT 115, NLAT,ISIZE,JSIZE,NLAT/2,NLAT/2+1,NLAT,NLONG              OZSTAT2    141
      PRINT 116, DATE                                                     OZSTAT2    142
      PRINT 117, L                                                        OCT79       36
      GVAR=0.                                                             OCT79       37
      KBLK=0                                                              OCT79       38
      DO 65 I=1,NLAT                                                      OZSTAT2    143
```

```
            SUM=0.                                                     OZSTAT2   144
            ISUM=0                                                     OZSTAT2   145
            SSQLATB=0.                                                 OZSTAT2   146
            TSUM=0.                                                    OCT79      39
            TSQLATB=0.                                                 OCT79      40
            PRINT 120                                                  OZSTAT2   147
C           **********************************                         OZSTAT2   148
            DO 55 J=1,NLONG                                            OZSTAT2   149
            IF (KK(I,J).EQ.0) GO TO 51                                 OZSTAT2   150
            AX=SUMX(I,J)/KK(I,J)                                       OZSTAT2   158
            AT=SUMT(I,J)/KK(I,J)                                       OCT79      41
            AVLAT=SUMLT(I,J)/KK(I,J)                                   OCT79      42
            AVLONG=SUMLG(I,J)/KK(I,J)                                  OCT79      43
            IF (KK(I,J).EQ.1) GO TO 52                                 OZSTAT2   159
            VARX=(SUMXSQ(I,J)-KK(I,J)*AX*AX)/(KK(I,J)-1.)              OZSTAT2   160
            VART=(SUMTSQ(I,J)-KK(I,J)*AT*AT)/(KK(I,J)-1.)              OCT79      44
            GO TO 53                                                   OZSTAT2   161
         51 AX=0.                                                      OZSTAT2   162
            AT=0.                                                      OCT79      45
            AVLAT=0.                                                   OCT79      46
            AVLONG=0.                                                  OCT79      47
         52 VARX=0.                                                    OZSTAT2   163
            VART=0.                                                    OCT79      48
         53 CONTINUE                                                   OZSTAT2   164
C           SUM IS ACCUMULATIVE OZONE CONTENT/LATITUDE BAND            OZSTAT2   165
            SUM=SUM+SUMX(I,J)                                          OZSTAT2   166
            TSUM=TSUM+SUMT(I,J)                                        OCT79      49
C           GSUM IS THE GLOBAL ACCUMULATIVE OZONE LAYER THICKNESS      OZSTAT2   167
            GSUM=GSUM+SUMX(I,J)                                        OZSTAT2   168
C           ISUM IS THE NUMBER OF DATA POINTS AVERAGED/LATITUDE BAND   OZSTAT2   169
            ISUM=ISUM+KK(I,J)                                          OZSTAT2   170
            SSQLATB=SSQLATB+SUMXSQ(I,J)                                OZSTAT2   171
            TSQLATB=TSQLATB+SUMTSQ(I,J)                                OCT79      50
            PRINT 100, AX,VARX,KK(I,J),I,J,AVLAT,AVLONG,AT,VART        OCT79      51
            IF (KK(I,J).EQ.0) GO TO 55                                 OCT79      52
            KBLK=KBLK+1                                                OCT79      53
            WRITE (10) (90.-AVLAT)*PI/180.,AVLONG*PI/180.,AX*1000.     OCT79      54
C           *    *    *    *    *    *    *    *    *                  OCT79      55
C           *    *    *    *    *    *    *    *    *                  OCT79      56
C           XLAT(I,J)=AVLAT                                            OCT79      57
C           XLONG(I,J)=AVLONG                                          OCT79      58
C           XOZ(I,J)=AX                                                OCT79      59
```

```
C     *     *     *     *     *     *     *     *     *                    OCT79      60
C     *     *     *     *     *     *     *     *     *                    OCT79      61
C     GRID BLOCK DATA FOR NCALC WEEKS IS STORED IN ZDATA.                  OZSTAT2   174
C     NUMBER OF DATA POINTS/GRID BLOCK FOR NCALC WEEKS                     OZSTAT2   175
C       IS STORED IN NDATA.                                                OZSTAT2   176
   55 CONTINUE                                                             OZSTAT2   179
C     **********************************                                   OZSTAT2   180
C     IISUM IS THE TOTAL NUMBER OF OZONE RECORDS USED                      OZSTAT2   181
C     IN THESE CALCULATIONS.                                               OZSTAT2   182
      IISUM=IISUM+ISUM                                                     OZSTAT2   183
C     *********************************************                        OZSTAT2   184
C     ILAT(I), I=1,NLAT, IS THE NUMBER OF DATA POINTS/LATITUDE BAND,       OZSTAT2   185
C        SUCH THAT I=1,NLAT CORRESPONDS TO LATITUDE BANDS FROM SOUTH       OZSTAT2   186
C       TO NORTH.  ILAT IS AN INPUT PARAMETER OF HISTPLT.                  OZSTAT2   187
      IF (I.GE.NLAT/2+1) GO TO 57                                          OZSTAT2   188
      ILAT(I+NLAT/2)=ISUM                                                  OZSTAT2   189
      GO TO 58                                                             OZSTAT2   190
C     IJ IS INITIALIZED AS -1.                                             OZSTAT2   191
   57 IJ=IJ+2                                                              OZSTAT2   192
      ILAT(I-IJ)=ISUM                                                      OZSTAT2   193
   58 CONTINUE                                                             OZSTAT2   194
C     *********************************************                        OZSTAT2   195
      IF (ISUM.EQ.0) GO TO 60                                              OZSTAT2   196
      AX=SUM/ISUM                                                          OZSTAT2   197
      AT=TSUM/ISUM                                                         OCT79      62
      GVAR=GVAR+SSQLATB                                                    OCT79      63
      IF (ISUM.EQ.1) GO TO 61                                              OZSTAT2   198
      VARX=(SSQLATB-AX*AX*ISUM)/(ISUM-1.)                                  OZSTAT2   199
      VART=(TSQLATB-AT*AT*ISUM)/(ISUM-1.)                                  OCT79      64
      GO TO 64                                                             OZSTAT2   200
   60 AX=0.                                                                OZSTAT2   201
      AT=0.                                                                OCT79      65
   61 VARX=0.                                                              OZSTAT2   202
      VART=0.                                                              OCT79      66
   64 VARLATB(I)=VARX                                                      OZSTAT2   203
      AVGLATB(I)=AX                                                        OZSTAT2   204
C     *     *     *     *     *     *     *     *     *     *              OCT79      67
C     WRITE (10,910) AX*1000.,SQRT(VARX*1000000.)                          OCT79      68
C 910 FORMAT (1X,2(F7.3,2X))                                               OCT79      69
C     *     *     *     *     *     *     *     *     *     *              OCT79      70
   65 PRINT 110, I,AX,ISUM,VARX,AT,VART                                    OCT79      71
C     *     *     *     *     *     *     *     *     *                    OCT79      72
```

```
C     *     *     *     *     *     *     *     *     *                   OCT79      73
C     WRITE(10) KK                                                        OCT79      74
C     WRITE(10) XLAT                                                      OCT79      75
C     WRITE(10) XLONG                                                     OCT79      76
C     WRITE(10) XOZ                                                       OCT79      77
C     *     *     *     *     *     *     *     *     *                   OCT79      78
C     *     *     *     *     *     *     *     *     *                   OCT79      79
C     ********************************                                    OZSTAT2   206
C                                                                         OZSTAT2   207
      PRINT 125, IISUM,IDAY,K4                                            OZSTAT2   208
      GAV=GSUM/IISUM                                                      OZSTAT2   209
      PRINT 130,GAV                                                       OZSTAT2   210
      GVAR=(GVAR-GAV*GAV*IISUM)/(IISUM-1.)                                OCT79      80
      PRINT 135, GVAR,SQRT(GVAR)                                          OCT79      81
  135 FORMAT (1X,*GLOBAL VARIANCE= *,E15.8/                               OCT79      82
     11X,*STANDARD DEVIATION= *,E15.8)                                    OCT79      83
      PRINT 925, KBLK                                                     OCT79      84
  925 FORMAT (*0*,*A TOTAL OF *,I6,* BLOCKS ARE FILLED.*)                 OCT79      85
C     ****************************************************************   OZSTAT2   211
      IF (MSELECT.EQ.0) GO TO 71                                          OZSTAT2   212
      IF (MSELECT.EQ.2) GO TO 70                                          OZSTAT2   213
      CALL ASTD (VARLATB,AVGLATB,ALAT,AOZ,M,RAT,STDEVP,STDEVN,NLAT)       OZSTAT2   214
      IF (MSELECT.EQ.1) GO TO 71                                          OZSTAT2   215
   70 CALL HISTPLT (ILAT,KK,NLAT,NLONG,RLAT,RKK,RLATBND,RLNGBK,RK,NAME,  OZSTAT2   216
     1NLAT+2,NLONG+2)                                                     OZSTAT2   217
   71 CONTINUE                                                            OZSTAT2   218
C     *******************************************                         OZSTAT2   219
      IF (JSELECT.EQ.2) GO TO 97                                          OZSTAT2   220
      GO TO 200                                                           OZSTAT2   221
   75 PRINT 145, K(4)                                                     OZSTAT2   222
      IF(K(4).EQ.173) GO TO 50                                            OCT79      86
      PRINT 160                                                           OZSTAT2   224
      STOP 1                                                              OZSTAT2   225
   97 CALL GRID (SUMX,SUMXSQ,KK,GP,NLAT,NLONG,KCOUNT,SUMGPSQ,VARGP)       OZSTAT2   226
C     ******************************************                          OZSTAT2   227
C     ******************************************                          OZSTAT2   228
C     ******************************************                          OZSTAT2   229
C     ******************************************                          OZSTAT2   230
  200 CONTINUE                                                            OZSTAT2   231
      CALL NFRAME                                                         OZSTAT2   232
      CALL CALPLT (0.,0.,999)                                             OZSTAT2   233
      STOP                                                                OZSTAT2   237
```

```
       100 FORMAT (1X,2(E16.8,5X),I3,5X,2(I2,5X),T70,F7.3,T88,F8.3,T105,F8.3,OCT79       87
          1T119,E15.8)                                                       OCT79       88
       110 FORMAT (*0*,*THE AVERAGE OZONE DENSITY FOR THE LATITUDE BAND*/1X,*OZSTAT2    239
          1CORRESPONDING TO I= *,I2,* IS *,E16.8,* THIS CALCULATION IS BASED OZSTAT2    240
          2ON *,I6,* RECORDS OF DATA*/1X,*THE VARIANCE OF THE MEAN IS *,E16.8OZSTAT2    241
          1/1X,*TIME AVERAGE= *,F8.3,10X,*TIME VARIANCE= *,E15.8)            OCT79       89
       115 FORMAT (*1*,*IN THE FOLLOWING *,I2,* TABLES, OZONE DENSITY HAS BEEOZSTAT2    243
          1N AVERAGED BY GRID BLOCKS.*/1X,*THESE BLOCKS REPRESENT AN AREA ON OZSTAT2    244
          2 THE EARTH"S SURFACE THAT IS *,I2,* DEGREES LATITUDE BY *,I2,* DEGOZSTAT2    245
          3REES LONGITUDE.*/1X,*LATITUDE BANDS CORRESPONDING TO LATITUDE INDIOZSTAT2    246
          4CES 1 THROUGH *,I2,* ARE IN THE NORTHERN HEMISPHERE*/1X,*WHILE LATOZSTAT2    247
          5ITUDE INDICES *,I2,* THROUGH *,I2,* ARE IN THE SOUTHERN HEMISPHEREOZSTAT2    248
          6.  LONGITUDE INDICES (1-*,I2,*) RANGE FROM*/1X,*THE GREENWICH MERIOZSTAT2    249
          7DIAN WESTWARD THROUGH 360 DEGREES.*)                              OZSTAT2    250
       116 FORMAT (1X,*THESE CALCULATIONS ARE FOR THE TIME PERIOD *,A10,A4)  OZSTAT2    251
       117 FORMAT (11X,*AND ARE FOR CASE NUMBER *,I2)                        OCT79       90
       120 FORMAT (*0*,T8,*MEAN*,T27,*VARIANCE*,T43,*K(I,J)*,T53,*I*,T60,*J*,OZSTAT2    252
          1T70,*MEAN LATITUDE*,T88,*MEAN LONGITUDE*,T105,*AVERAGE TIME*,     OCT79       91
          2T119,*VARIANCE*)                                                  OCT79       92
       125 FORMAT (*0*,I7,* RECORDS OF DATA ARE USED IN THE ABOVE CALCULATIONOZSTAT2    254
          1S*/1X,*THIS DATA INCLUDES RECORDS FROM DAYS *,I4,* THROUGH *,I4,*,OCT79       93
          2 INCLUSIVE.*)                                                     OZSTAT2    256
       130 FORMAT (*0*,*THE GLOBAL OZONE LAYER THICKNESS AVERAGE FOR THIS TIMOZSTAT2    257
          1E PERIOD IS *,E15.8)                                              OZSTAT2    258
       145 FORMAT (*0*,*REACHED END OF FILE.  DAY NUMBER = *,I5)             OCT79       94
       160 FORMAT (///*0*,*REACHED EOF PRIOR TO LAST DAY ON FILE*)           OZSTAT2    260
       225 FORMAT (2I4,A10,A4)                                               OCT79       95
           END                                                               OZSTAT2    262
```

```
RD NR. SEVERITY  DETAILS    DIAGNOSIS OF PROBLEM

  13      I      NAME      DATA VARIABLE LIST EXCEEDS ITEM LIST, EXCESS VARIABLES NOT INITIALIZED.
```

SYMBOLIC REFERENCE MAP (R=1)

```
      SUBROUTINE GRID (S,SS,K,GP,NLAT,NLONG,KCOUNT,SUMGPSQ,VARGP)
      COMMON/DD/DATE
      DIMENSION S(NLAT,NLONG),K(NLAT,NLONG),GP(NLAT,NLONG),KCOUNT(NLAT,N
     1LONG),SS(NLAT,NLONG),SUMGPSQ(NLAT,NLONG),VARGP(NLAT,NLONG)
      DIMENSION DATE (2)
      M=NLAT/2-1
      M1=NLAT/2+1
      M2=NLAT/2
      M3=NLAT/2+2
      PRINT 105, NLAT,M2,M1,M3,NLAT,NLONG
      PRINT 110,DATE
C     *******************************************
C     GRID POINTS IN THE NORTHERN HEMISPHERE ARE CALCULATED BELOW
      DO 25 I=1,M
      DO 25 J=1,NLONG
      L=J+1
      IF (L.EQ.NLONG+1) L=1
      N=I+1
      SUMGPSQ (I,J)=SS(I,J)+SS(I,L)+SS(N,J)+SS(N,L)
      KCOUNT(I,J)=K(I,J)+K(I,L)+K(N,J)+K(N,L)
      IF (KCOUNT(I,J).EQ.0) GO TO 20
      GP(I,J)=(S(I,J)+S(I,L)+S(N,J)+S(N,L))/KCOUNT(I,J)
      IF (KCOUNT(I,J).EQ.1) GO TO 21
      VARGP (I,J)=(SUMGPSQ(I,J)-KCOUNT(I,J)*GP(I,J)*GP(I,J))/(KCOUNT(I,J
     1)-1.)
      GO TO 25
   20 GP(I,J)=0.
   21 VARGP (I,J) =0.
   25 CONTINUE
C     *******************************************
C     GRID POINTS ALONG THE EQUATOR ARE CALCULATED BELOW
      DO 30 J=1,NLONG
      L=J+1
      IF (L.EQ.NLONG+1) L=1
      SUMGPSQ(M1,J)=SS(1,J)+SS(1,L)+SS(M1,J)+SS(M1,L)
      KCOUNT (M1,J)=K(1,J)+K(1,L)+K(M1,J)+K(M1,L)
      IF (KCOUNT(M1,J).EQ.0) GO TO 27
      GP(M1,J)=(S(1,J)+S(1,L)+S(M1,J)+S(M1,L))/KCOUNT(M1,J)
      IF (KCOUNT(M1,J).EQ.1) GO TO 28
      VARGP (M1,J)=(SUMGPSQ(M1,J)-KCOUNT(M1,J)*GP(M1,J)*GP(M1,J))/(KCOUN
     1T(M1,J)-1.)
      GO TO 29
```

```
   27 GP (M1,J)=0.
   28 VARGP (M1,J)=0.
   29 GP (M2,J)=0.
      VARGP (M2,J)=0.
      KCOUNT (M2,J)=0
   30 CONTINUE
C     ********************************************
C     GRID POINTS IN THE SOUTHERN HEMISPHERE ARE CALCULATED BELOW
      DO 35 I=M3,NLAT
      DO 35 J=1,NLONG
      L=J+1
      IF (L.EQ.NLONG+1) L=1
      N=I-1
      SUMGPSQ (I,J)=SS(I,J)+SS(I,L)+SS(N,J)+SS(N,L)
      KCOUNT(I,J)=K(N,J)+K(N,L)+K(I,J)+K(I,L)
      IF (KCOUNT(I,J).EQ.0) GO TO 33
      GP(I,J)=(S(I,J)+S(I,L)+S(N,J)+S(N,L))/KCOUNT(I,J)
      IF (KCOUNT(I,J).EQ.1) GO TO 34
      VARGP (I,J)=(SUMGPSQ(I,J)-KCOUNT(I,J)*GP(I,J)*GP(I,J))/(KCOUNT(I,J
     1)-1.)
      GO TO 35
   33 GP(I,J)=0.
   34 VARGP (I,J)=0.
   35 CONTINUE
C     ********************************************
C     KC IS A COUNTER FOR THE TOTAL NUMBER OF DATA POINTS USED IN THESE
C     CALCULATIONS
C     OZDEN IS AN ACCUMULATIVE SUM OF TOTAL OZONE DENSITY, SUMMED IN A
C     PARTICULAR LATITUDE BAND.
C     KC1 IS A COUNTER FOR THE NUMBER OF DATA POINTS USED IN THE
C     CALCULATIONS FOR A PARTICULAR LATITUDE BAND.
C     ********************************************
      KC=0
      DO 55 I=1,NLAT
      OZDEN=0.
      KC1=0
      PRINT 100
      DO 50 J=1,NLONG
      KC=KC+KCOUNT(I,J)
      KC1=KC1+KCOUNT(I,J)
      OZDEN=OZDEN+GP(I,J)*KCOUNT(I,J)
   50 PRINT 125, GP(I,J),KCOUNT (I,J),I,J,VARGP (I,J)
```

```
                AVDEN=OZDEN/KC1                                                  GRID
             55 PRINT 150, I,AVDEN,KC1                                           GRID
                PRINT 175,KC                                                     GRID
            100 FORMAT (*0*,T4,*MEAN OZONE DENSITY*,T27,*VARIANCE*,T43,*KCOUNT*,T5GRID
               13,*I*,T60,*J*)                                                   GRID
            105 FORMAT (*1*,*IN THE FOLLOWING *,I2,* TABLES, OZONE DENSITY HAS BEEGRID
               1N AVERAGED BY GRID POINTS.  LATITUDE INDICES 1 THROUGH *,I2      GRID
               2/1X,*CORRESPOND TO THE NORTHERN HEMISPHERE, LATITUDE INDEX *,I2,* GRID
               3CORRESPONDS TO THE EQUATOR, AND LATITUDE INDICES *,I2,* THROUGH *,GRID
               4I2/1X,*CORRESPOND TO THE SOUTHERN HEMISPHERE.  LONGITUDE INDICES AGRID
               5RE SIMILAR TO THOSE ABOVE WITH J= *,I2,* CORRESPONDING TO LONGITUDGRID
               6E=0*/1X,*DEGREES OR THE MERIDIAN THROUGH GREENWICH*)             GRID
            110 FORMAT (1X,*THESE CALCULATIONS ARE FOR THE TIME PERIOD *,A10,A4)  GRID
            125 FORMAT (1X,E16.8,T44,I4,T53,I2,T60,I2,T25,E16.8)                 GRID
            150 FORMAT (*0*,*THE AVERAGE OZONE DENSITY CORRESPONDING TO LATITUDE IGRID
               1NDEX *,I2,* IS *,E16.8,*.  THIS MEAN IS BASED ON *,I6,* DATA POINTGRID
               2S.*)                                                             GRID
            175 FORMAT (*0*,I7,* DATA RECORDS ARE USED IN THE ABOVE CALCULATIONS.*GRID
               1)                                                                GRID
                RETURN                                                           GRID
                END                                                              GRID
```


SYMBOLIC REFERENCE MAP (R=1)

```
TRY POINTS
  3  GRID
```

RIABLES	SN	TYPE	RELOCATION						
500	AVDEN	REAL			0	DATE	REAL	ARRAY	DD
0	GP	REAL	ARRAY	F.P.	471	I	INTEGER		
472	J	INTEGER			0	K	INTEGER	ARRAY	F.P.
475	KC	INTEGER			0	KCOUNT	INTEGER	ARRAY	F.P.
477	KC1	INTEGER			473	L	INTEGER		
465	M	INTEGER			466	M1	INTEGER		
467	M2	INTEGER			470	M3	INTEGER		
474	N	INTEGER			0	NLAT	INTEGER		F.P.
0	NLONG	INTEGER		F.P.	476	OZDEN	REAL		
0	S	REAL	ARRAY	F.P.	0	SS	REAL	ARRAY	F.P.

```
      SUBROUTINE HISTPLT (ILAT,KK,NLAT,NLONG,RLAT,RKK,RLATBND,RLNGBK,RK,
     1NAME,K,L)
      COMMON/DD/DATE
      DIMENSION ILAT(NLAT),RLAT(K),KK(NLAT,NLONG),RKK(NLAT,NLONG)
      DIMENSION RLATBND(K),RLNGBK(L),RK(L),NAME(NLAT)
      DIMENSION DATE(2)
      K1=K-1     $     L1=L-1
      ICOUNT=0
      M=-17
      NLAT1=0
      RLATBND(K1)=0.
      RLATBND(K)=1.
      RLAT(K1)=0.
      RLAT(K)=1.
      RLNGBK(L1)=0.
      RLNGBK(L)=2.
      RK(L1)=0.
      RK(L)=0.1
C     ILAT CONTAINS THE NUMBER OF DATA POINTS/LATITUDE BAND
C     KK CONTAINS THE NUMBER OF DATA POINTS /GRID BLOCK
C     FIND MAX VALUES OF ILAT AND KK, ILATMAX AND KKMAX, RESPECTIVELY
      ILATMAX=0   $   KKMAX=0
      DO 15 I=1,NLAT
      IF (ILAT(I).GT.ILATMAX) ILATMAX=ILAT(I)
      DO 15 J=1,NLONG
      IF (KK(I,J).GT.KKMAX) KKMAX=KK(I,J)
   15 CONTINUE
      PRINT 101
      PRINT 100, ILAT
      PRINT 103, DATE
      PRINT 104
      PRINT 106, DATE
      IF (NLAT.LE.18) GO TO 21
   19 ICOUNT=ICOUNT+1
      M=M+18
      NLAT1=NLAT1+18
      PRINT 105, (I,I=M,NLAT1)
      DO 20 J=1,NLONG
   20 PRINT 110, J,(KK(I,J),I=M,NLAT1)
      IF (NLAT-ICOUNT*18.GT.18) GO TO 19
      M=M+18
   21 IF (M.LT.0)  M=1
```

```
      PRINT 105, (I,I=M,NLAT)
      DO 22 J=1,NLONG
   22 PRINT 110, J,(KK(I,J),I=M,NLAT)
C     FIND NORMALIZED VALUES OF ILAT AND KK
      DO 25 I=1,NLAT
C     RLAT IS NORMALIZED VALUE OF ILAT
      RLAT(I)=ILAT(I)/FLOAT(ILATMAX)
      DO 25 J=1,NLONG
C     RKK IS NORMALIZED VALUE OF KK
      RKK(I,J)=KK(I,J)/FLOAT(KKMAX)
   25 CONTINUE
      PRINT 125, ILATMAX,KKMAX
      PRINT 126
      DO 26 I=1,NLAT
   26 PRINT 127, RLAT(I)
      PRINT 103, DATE
      PRINT 128
      PRINT 106, DATE
      ICOUNT=0
      M=-17
      NLAT1=0
      IF (NLAT.LE.18) GO TO 31
   29 ICOUNT=ICOUNT+1
      M=M+18
      NLAT1=NLAT1+18
      PRINT 105, (I,I=M,NLAT1)
      DO 30 J=1,NLONG
   30 PRINT 135, J,(RKK(I,J),I=M,NLAT1)
      IF (NLAT-ICOUNT*18.GT.18) GO TO 29
      M=M+18
   31 IF (M.LT.0) M=1
      PRINT 105, (I,I=M,NLAT)
      DO 32 J=1,NLONG
   32 PRINT 135, J,(RKK(I,J),I=M,NLAT)
      XL=9.
      YL=5.
C     FOR PROPER SCALING MULTIPLY RLAT(I) BY THE LENGTH OF THE Y-AXIS
      DO 35 I=1,NLAT
      RLAT(I)=RLAT(I)*YL
   35 RLATBND(I)=(XL/NLAT)/2.+(I-1)*XL/NLAT
      DO 36 J=1,NLONG
   36 RLNGBK(J)=J
```

```
      C     DX AND DY ARE X AND Y AXES SCALE FACTORS                            HISTPLT    85
            DX=180./XL                                                          HISTPLT    86
            DY=1./YL                                                            HISTPLT    87
      C     T(X OR Y) = FREQUENCY OF TIC MARKS/SCALE FACTOR, WHERE SCALE        HISTPLT    88
      C       FACTOR(DX OR DY) IS A CONVERSION FACTOR BETWEEN AXES UNITS        HISTPLT    89
      C       AND REAL LENGTH                                                   HISTPLT    90
            TX=-10./DX                                                          HISTPLT    91
            TY=-.1/DY                                                           HISTPLT    92
      C     THE INPUT PARAMETERS REQUIRED FOR SUBROUTINE BARPLT HAVE BEEN       HISTPLT    93
      C     CALCULATED.                                                         HISTPLT    94
      C     *    *    *    *    *    *    *                                     HISTPLT    95
      C     PLOT HISTOGRAMS                                                     HISTPLT    96
      C                                                                         HISTPLT    97
      C     DATA DISTRIBUTION PER LATITUDE ZONE                                 HISTPLT    98
            CALL NFRAME                                                         HISTPLT    99
            CALL AXES(0.,0., 0.,XL,-90.,DX  , TX,0.,14HLATITUDE (DEG),.10,-14)HISTPLT   100
            CALL AXES(0.,0.,90.,YL,0.,DY,TY,0.,32HNORMALIZED NUMBER OF DATA POHISTPLT   101
           1INTS,.10,32)                                                        HISTPLT   102
      C     WBAR IS THE BAR WIDTH WHICH IS ISIZE(SEE OZSTAT PARAMETER LIST)     HISTPLT   103
      C         DEGREES WIDE.                                                   HISTPLT   104
            WBAR=(180./NLAT)/DX                                                 HISTPLT   105
            CALL BARPLT (RLATBND,RLAT,NLAT,1,1,WBAR,0)                          HISTPLT   106
            CALL NOTATE (2.,6.00,.15,30HNUMBER OF DATA POINTS/LAT BAND,0.,30) HISTPLT   107
            CALL NOTATE (2.,5.75,.15,32HHISTOGRAM INCLUDES DATA FOR DAYS,0.,32HISTPLT   108
           1)                                                                   HISTPLT   109
            CALL NOTATE (2.,5.50,.15,DATE,0.,14)                                HISTPLT   110
            ISELECT =0                                                          HISTPLT   111
            IF (ISELECT.EQ.0) GO TO 90                                          HISTPLT   112
      C                                                                         HISTPLT   113
      C     DATA DISTRIBUTION PER GRID BLOCK                                    HISTPLT   114
            DO 50 I=1,NLAT                                                      HISTPLT   115
            DO 45 J=1,NLONG                                                     HISTPLT   116
         45 RK(J)=RKK(I,NLONG+1-J)                                              HISTPLT   117
      C                                                                         HISTPLT   118
      C                                                                         HISTPLT   119
      C     NO MODIFICATIONS FOR VARIABLE BLOCK SIZE BELOW THIS POINT.          HISTPLT   120
      C                                                                         HISTPLT   121
      C                                                                         HISTPLT   122
            CALL NFRAME                                                         HISTPLT   123
            CALL AXES (0.,0.,0.,18.,0.,2.,TX,0.,33HLONGITUDE INDICES FOR GRID HISTPLT   124
           1BLOCKS,.15,-33)                                                     HISTPLT   125
            CALL AXES(0.,0.,90.,YL,0.,DY,TY,0.,32HNORMALIZED NUMBER OF DATA POHISTPLT   126
```

```
           1INTS,.15,32)                                                        HISTPLT   127
            CALL NOTATE  (4.,12.,.15,32HNUMBER OF DATA POINTS/GRID BLOCK,0.,32HISTPLT   128
           1)                                                                   HISTPLT   129
            CALL NOTATE (4.,11.75,.15,14HLATITUDE INDEX,0.,14)                  HISTPLT   130
            CALL NOTATE (6.,11.75,.15,NAME(I),0.,10)                            HISTPLT   131
            CALL NOTATE (4.,11.5,.15,32HHISTOGRAM INCLUDES DATA FOR DAYS,0.,32HISTPLT   132
           1)                                                                   HISTPLT   133
            CALL NOTATE (4.,11.25,.15,DATE,0.,14)                               HISTPLT   134
            CALL BARPLT (RLNGBK,RK,36,1,1,0.25,0)                               HISTPLT   135
         50 CONTINUE                                                            HISTPLT   136
         90 CONTINUE                                                            HISTPLT   137
            RETURN                                                              HISTPLT   138
        100 FORMAT (1X,*ILAT= *,I5)                                             HISTPLT   139
        101 FORMAT (*1*, *THE NUMBER OF DATA POINTS/LATITUDE BAND FROM SOUTH THISTPLT   140
           10 NORTH ARE*)                                                       HISTPLT   141
        103 FORMAT (//*0*,*FOR THE TIME INTERVAL *,A10,A4)                      HISTPLT   142
        104 FORMAT (*1*,*NUMBER OF DATA POINTS/GRID BLOCK*)                     HISTPLT   143
        105 FORMAT (*0*,8X,18(I2,5X)/)                                          HISTPLT   144
        106 FORMAT (1X,A10,A4)                                                  HISTPLT   145
        110 FORMAT (1X,I2,4X,18(I4,3X))                                         HISTPLT   146
        125 FORMAT (*0*,*LATMAX= *,I5,10X,*KKMAX= *,I5)                         HISTPLT   147
        126 FORMAT (*1*, *NORMALIZED NUMBER OF DATA POINTS/LATITUDE BAND*)      HISTPLT   148
        127 FORMAT (1X,*RLAT= *,F7.4)                                           HISTPLT   149
        128 FORMAT (*1*,*NORMALIZED NUMBER OF DATA POINTS/GRID BLOCK*)          HISTPLT   150
        135 FORMAT (1X,I2,4X,18(F5.2,2X))                                       HISTPLT   151
            END                                                                 HISTPLT   152
```

SYMBOLIC REFERENCE MAP (R=1)

```
RY POINTS
 3  HISTPLT

IABLES     SN  TYPE           RELOCATION
 0  DATE       REAL      ARRAY    DD          1140  DX        REAL
141  DY        REAL                           1134  I         INTEGER
127  ICOUNT    INTEGER                           0  ILAT      INTEGER   ARRAY     F.P.
132  ILATMAX   INTEGER                        1145  ISELECT   INTEGER
135  J         INTEGER                           0  K         INTEGER             F.P.
```

```
      SUBROUTINE AVARPLT (V,A,U,T,M,RAT,STDEVP,STDEVN,NLAT)             AVARPLT   1
      COMMON/DD/D                                                       AVARPLT   2
      DIMENSION V(NLAT),A(NLAT),RAT(NLAT),STDEVP(NLAT),STDEVN(NLAT)     AVARPLT   3
      DIMENSION U(M),T(M)                                               AVARPLT   4
      DIMENSION D(2)                                                    AVARPLT   5
C     V AND A ARE THE VARIANCE AND AVERAGE, RESPECTIVELY, OF THE OZONE  AVARPLT   6
C     DENSITY/LATITUDE BAND                                             AVARPLT   7
C     U AND T ARE THE LATITUDE AND OZONE DENSITY INFORMATION/DATA RECORDAVARPLT   8
C     USED TO PLOT THE SCATTER DIAGRAM.                                 AVARPLT   9
C     M IS THE DIMENSION OF U AND T.                                    AVARPLT  10
C     STDEVP IS THE STANDARD DEVIATION + MEAN, PROPERLY SCALED TO PLOT  AVARPLT  11
C     STDEVN IS THE STANDARD DEVIATION - MEAN, PROPERLY SCALED TO PLOT  AVARPLT  12
C     SUBROUTINE CALPLT (X,Y,IPEN) IS LOCATED IN SECTION 1.4.3 OF       AVARPLT  13
C     THE GRAPHICS MANUAL                                               AVARPLT  14
C     IPEN=2     PEN DOWN                                               AVARPLT  15
C     IPEN=3     PEN UP                                                 AVARPLT  16
C     IPEN LESS THAN ZERO WILL ASSIGN X=0, Y=0 AS THE LOCATION OF       AVARPLT  17
C     THE PEN AFTER MOVING THE X,Y (CREATE A NEW REFERENCE POINT).      AVARPLT  18
C     SUBROUTINE PNTPLT (X,Y,ISYM,IS) CAN BE FOUND IN SECTION 1.4.70    AVARPLT  19
C     OF THE GRAPHICS MANUAL.                                           AVARPLT  20
C     SUBROUTINE PSEUDO (FN), FN = FILENAME, CAN BE FOUND IN SECTION    AVARPLT  21
C     1.4.1 OF THE GRAPHICS MANUAL                                      AVARPLT  22
C                                                                       AVARPLT  23
C     INITIALIZE PARAMETERS                                             AVARPLT  24
      XL=8.                                                             AVARPLT  25
      YL=6.0                                                            OCT79     1
      DX=180./XL                                                        AVARPLT  27
      YMAX=0.65                                                         OCT79     2
      YMIN=0.15                                                         OCT79     3
      DY=(YMAX-YMIN)/YL                                                 OCT79     4
      TX=-10./DX       $      TY=-.1/DY                                 AVARPLT  29
      XT=ABS(TX)                                                        AVARPLT  30
      UMAX=0.                                                           AVARPLT  31
      UMIN=0.                                                           AVARPLT  32
      TMAX=-1.                                                          OCT79     5
      TMIN=1.                                                           OCT79     6
C                                                                       AVARPLT  33
C     CONSTRUCT PLOT LABELS AND AXES.                                   AVARPLT  34
      CALL NFRAME                                                       AVARPLT  35
      CALL NOTATE (2.,5.00,.15,15HSCATTER DIAGRAM,0.,15)                AVARPLT  36
      CALL NOTATE (2.,4.75,.15,36HINCLUDES MEAN AND STANDARD DEVIATION,0AVARPLT  37
     1.,36)                                                             AVARPLT  38
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          CALL NOTATE (2.00,4.50,.15,42HDATA TAKEN FROM NIMBUS IV BUV MEASURAVARPLT      39
         1EMENTS,0.,42)                                                   AVARPLT      40
          CALL NOTATE (2.00,4.25,.15,35HTHIS DIAGRAM INCLUDES DATA FOR DAYS,AVARPLT      41
         10.,35)                                                          AVARPLT      42
          CALL NOTATE (2.00,4.00,.15,D,0.,14)                             AVARPLT      43
          CALL AXES (0.,0.,0.,XL,-90.,DX,TX,0.,14HLATITUDE (DEG),.10,-14) AVARPLT      44
          CALL AXES (0.,0.,90.,YL,.15,DY,TY,0.,20HTOTAL OZONE (ATM-CM),.10, OCT79       7
         120)                                                             OCT79         8
          CALL CALPLT (0.,YL,3)                                           AVARPLT      47
          CALL CALPLT (XL,YL,2)                                           AVARPLT      48
          CALL CALPLT (XL,0.,2)                                           AVARPLT      49
          CALL CALPLT (0.,0.,3)                                           AVARPLT      50
          RETURN                                                          AVARPLT      51
    C     ********************************************                    AVARPLT      52
          ENTRY SCAT                                                      AVARPLT      53
    C     PLOT SCATTER DIAGRAM - ALSO FIND MAXIMUM AND MINIMUM LATITUDES  AVARPLT      54
          DO 30 I=1,M                                                     AVARPLT      55
    C     FIND MAXIMUM AND MINIMUM LATITUDE VALUES ************************AVARPLT      56
          IF (U(I).GT.UMAX) UMAX=U(I)                                     AVARPLT      57
          IF (U(I).LT.UMIN) UMIN=U(I)                                     AVARPLT      58
    COMMENT -- FIND MAXIMUM AND MINIMUM OZONE VALUES ********************* OCT79         9
          IF (T(I).GT.TMAX) TMAX=T(I)                                     OCT79        10
          IF (T(I).LT.TMIN) TMIN=T(I)                                     OCT79        11
    C     ********************************************                    AVARPLT      59
    C     PLOT SCATTER DIAGRAM                                            AVARPLT      60
          X=(U(I)+90.)/DX                                                 AVARPLT      61
          Y=(T(I)-YMIN)/DY                                                OCT79        12
       30 CALL PNTPLT(X,Y,-21,1)                                          AVARPLT      63
          RETURN                                                          AVARPLT      64
    C     ********************************************                    AVARPLT      65
          ENTRY ASTD                                                      AVARPLT      66
          XS=XL/NLAT                                                      AVARPLT      67
          NLAT1=NLAT/2                                                    AVARPLT      68
          NLAT2=NLAT1+1                                                   AVARPLT      69
          DO 35 I=1,NLAT                                                  AVARPLT      70
          STDEVP(I)=(A(I)+SQRT(V(I)))/DY                                  AVARPLT      71
          STDEVN(I)=(A(I)-SQRT(V(I)))/DY                                  OCT79        13
          STDEVP(I)=STDEVP(I)-YMIN/DY                                     OCT79        14
          STDEVN(I)=STDEVN(I)-YMIN/DY                                     OCT79        15
       35 CONTINUE                                                        OCT79        16
          DO 40 I=1,NLAT1                                                 AVARPLT      73
          J=I+NLAT1                                                       AVARPLT      74
```

```
      C      RAT IS THE SCALED DATA MATRIX FOR SPACING PLOTTED AVERAGES AND     AVARPLT      75
      C      STANDARD DEVIATION ALONG THE X - AXIS                              AVARPLT      76
             RAT(I)=XS/2.+(NLAT/2-1+I)*XS                                       AVARPLT      77
             RAT(J)=XS/2.+(NLAT-J)*XS                                           AVARPLT      78
          40 CONTINUE                                                           AVARPLT      79
      COMMENT -- FIND LATITUDE INDEXES, IMAX AND IMIN, CORRESPONDING TO         AVARPLT      80
      C            UMAX AND UMIN.  THEN, CALCULATE AN ADJUSTED VALUE OF         AVARPLT      81
      C            RAT(IMAX) AND RAT(IMIN) SUCH THAT THE EXTREME MEANS WILL     AVARPLT      82
      C            BE PLOTTED IN THE END LATITUDE ZONES HALF-WAY BETWEEN THE    AVARPLT      83
      C            ZONE"S BEGINNING AND THE EXTREMUM LATITUDE VALUES.           AVARPLT      84
             IMAX=UMAX/(180/NLAT)                                               AVARPLT      85
             LMAX=IMAX*(180/NLAT)                                               AVARPLT      86
             IMAX=IMAX+1                                                        AVARPLT      87
      COMMENT -- RATMAX IS THE HALF-WAY POINT FOR THE EXTREME MAXIMUM           AVARPLT      88
      C            LATITUDE ZONE.                                               AVARPLT      89
             RATMAX=(UMAX-LMAX)/(2.*DX)                                         AVARPLT      90
             RAT(IMAX)=RAT(IMAX)-XS/2.                                          AVARPLT      91
             RAT(IMAX)=RAT(IMAX)+RATMAX                                         AVARPLT      92
             IMIN=UMIN/(180/NLAT)                                               AVARPLT      93
             LMIN=IMIN*(180/NLAT)                                               AVARPLT      94
             IMIN=(NLAT/2+1)-IMIN                                               AVARPLT      95
      COMMENT -- RATMIN IS THE HALF-WAY POINT FOR THE EXTREME MINIMUM           AVARPLT      96
      C            LATITUDE ZONE.                                               AVARPLT      97
             RATMIN=(UMIN-LMIN)/(2.*DX)                                         AVARPLT      98
             RAT(IMIN)=RAT(IMIN)+XS/2.                                          AVARPLT      99
             RAT(IMIN)=RAT(IMIN)+RATMIN                                         AVARPLT     100
      C      ********************************************                       AVARPLT     101
             DO 41 I=1,NLAT                                                     AVARPLT     102
      C      A  NOW BECOMES THE PROPERLY SCALED AVERAGE TO BE PLOTTED           AVARPLT     103
      C      ALONG THE Y-AXIS                                                   AVARPLT     104
          41 A(I)=(A(I)-YMIN)/DY                                                OCT79        17
      C      ********************************************                       AVARPLT     106
      C      NOW PLOT MEANS AND DRAW IN CONNECTING "CURVE"                      AVARPLT     107
              IF (A(1).EQ.-YMIN/DY) GO TO 42                                    OCT79        18
             CALL PNTPLT (RAT(1),A(1),-11,2)                                    AVARPLT     108
          42 CONTINUE                                                           OCT79        19
             DO 45 I=2,IMAX                                                     AVARPLT     109
             IF (A(I-1).EQ.-YMIN/DY) GO TO 43                                   OCT79        20
             CALL CALPLT (RAT(I),A(I),2)                                        AVARPLT     110
          43 CONTINUE                                                           OCT79        21
             IF (A(I).EQ.-YMIN/DY) GO TO 45                                     OCT79        22
             CALL PNTPLT (RAT(I),A(I),-11,2)                                    OCT79        23
```

```
   45 CONTINUE                                                           OCT79        24
      CALL CALPLT (RAT(1),A(1),3)                                        AVARPLT     112
      DO 50 I=NLAT2,IMIN                                                 AVARPLT     113
      IF (A(1).EQ.-YMIN/DY.AND.I.EQ.NLAT2) GO TO 46                      OCT79        25
      IF (A(I).EQ.-YMIN/DY.OR.A(I-1).EQ.-YMIN/DY.AND.I.GT.NLAT2) GOTO 46OCT79         26
      CALL CALPLT (RAT(I),A(I),2)                                        AVARPLT     114
   46 CONTINUE                                                           OCT79        27
      IF (A(I).EQ.-YMIN/DY) GO TO 50                                     OCT79        28
      CALL PNTPLT (RAT(I),A(I),-11,2)                                    OCT79        29
   50 CONTINUE                                                           OCT79        30
C     PLOT STANDARD DEVIATION                                            AVARPLT     116
      DO 55 I=1,IMIN                                                     AVARPLT     117
      IF ((I.GT.IMAX).AND.(I.LT.NLAT2)) GO TO 55                         AVARPLT     118
      IF (A(I).EQ.-YMIN/DY) GO TO 55                                     OCT79        31
      CALL CALPLT ((RAT(I)-0.06),STDEVN(I),3)                            AVARPLT     119
      CALL CALPLT ((RAT(I)+0.06),STDEVN(I),2)                            AVARPLT     120
      CALL CALPLT (RAT(I),STDEVN(I),3)                                   AVARPLT     121
      CALL CALPLT (RAT(I),STDEVP(I),2)                                   AVARPLT     122
      CALL CALPLT ((RAT(I)-0.06),STDEVP(I),3)                            AVARPLT     123
      CALL CALPLT ((RAT(I)+0.06),STDEVP(I),2)                            AVARPLT     124
   55 CONTINUE                                                           AVARPLT     125
C     PLOT MEANS AND STANDARD DEVIATIONS AS A SEPERATE FRAME.            AVARPLT     126
      CALL NFRAME                                                        AVARPLT     127
      CALL NOTATE (2.00,5.00,.15,27HMEAN AND STANDARD DEVIATION,0.,27)   AVARPLT     128
      CALL NOTATE (2.00,4.75,.15,42HDATA TAKEN FROM NIMBUS IV BUV MEASURAVARPLT      129
     1EMENTS,0.,42)                                                      AVARPLT     130
      CALL NOTATE (2.00,4.50,.15,35HTHIS DIAGRAM INCLUDES DATA FOR DAYS,AVARPLT      131
     10.,35)                                                             AVARPLT     132
      CALL NOTATE (2.00,4.25,.15,D,0.,14)                                AVARPLT     133
      CALL AXES (0.,0.,0.,XL,-90.,DX,TX,0.,14HLATITUDE (DEG),.10,-14)    AVARPLT     134
      CALL AXES (0.,0.,90.,YL,.15,DY,TY,0.,20HTOTAL OZONE (ATM-CM),.10,  OCT79        32
     120)                                                                OCT79        33
      CALL CALPLT (0.,YL,3)                                              AVARPLT     137
      CALL CALPLT (XL,YL,2)                                              AVARPLT     138
      CALL CALPLT (XL,0.,2)                                              AVARPLT     139
      CALL CALPLT (0.,0.,3)                                              AVARPLT     140
C     NOW PLOT MEANS AND DRAW IN CONNECTING "CURVE"                      AVARPLT     141
      IF (A(1).EQ.-YMIN/DY) GO TO 556                                    OCT79        34
      CALL PNTPLT (RAT(1),A(1),-22,2)                                    OCT79        35
  556 CONTINUE                                                           OCT79        36
      DO 56 I=2,IMAX                                                     AVARPLT     143
      IF (A(I-1).EQ.-YMIN/DY) GO TO 557                                  OCT79        37
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```
            CALL CALPLT (RAT(I),A(I),2)                                   AVARPLT    144
        557 CONTINUE                                                      OCT79       38
            IF (A(I).EQ.-YMIN/DY) GO TO 56                                OCT79       39
            CALL PNTPLT (RAT(I),A(I),-22,2)                               OCT79       40
         56 CONTINUE                                                      OCT79       41
            CALL CALPLT (RAT(1),A(1),3)                                   AVARPLT    146
            DO 57 I=NLAT2,IMIN                                            AVARPLT    147
            IF (A(1).EQ.-YMIN/DY.AND.I.EQ.NLAT2) GO TO 558                OCT79       42
            IF (A(I).EQ.-YMIN/DY.OR.A(I-1).EQ.-YMIN/DY.AND.I.GT.NLAT2) GOTO558OCT79   43
            CALL CALPLT (RAT(I),A(I),2)                                   AVARPLT    148
        558 CONTINUE                                                      OCT79       44
            IF (A(I).EQ.-YMIN/DY) GO TO 57                                OCT79       45
            CALL PNTPLT (RAT(I),A(I),-22,2)                               OCT79       46
         57 CONTINUE                                                      OCT79       47
C           PLOT STANDARD DEVIATION                                       AVARPLT    150
            DO 58 I=1,IMIN                                                AVARPLT    151
            IF ((I.GT.IMAX).AND.(I.LT.NLAT2)) GO TO 58                    AVARPLT    152
            IF (A(I).EQ.-YMIN/DY) GO TO 58                                OCT79       48
            CALL CALPLT ((RAT(I)-0.06),STDEVN(I),3)                       AVARPLT    153
            CALL CALPLT ((RAT(I)+0.06),STDEVN(I),2)                       AVARPLT    154
            CALL CALPLT (RAT(I),STDEVN(I),3)                              AVARPLT    155
            CALL CALPLT (RAT(I),STDEVP(I),2)                              AVARPLT    156
            CALL CALPLT ((RAT(I)-0.06),STDEVP(I),3)                       AVARPLT    157
            CALL CALPLT ((RAT(I)+0.06),STDEVP(I),2)                       AVARPLT    158
         58 CONTINUE                                                      AVARPLT    159
            PRINT 110                                                     AVARPLT    160
            PRINT 115, D                                                  AVARPLT    161
            DO 60 I=1,NLAT                                                AVARPLT    162
         60 PRINT 100, I,RAT(I),A(I),V(I),STDEVP(I),STDEVN(I)             AVARPLT    163
            PRINT 105, M,UMAX,UMIN                                        AVARPLT    16
            PRINT 120,TMIN,TMAX                                           OCT79       49
        120 FORMAT (1X,*TMIN= *,E15.8,5X,*TMAX= *,E15.8)                  OCT79       50
            RETURN                                                        AVARPLT    165
        100 FORMAT (*0*,*I= *,I2,4X,*RAT= *,F6.2,4X,*A= *,F6.2,4X,*V= *,E11.4,AVARPLT 166
          14X,*STDEVP= *,F6.2,4X,*STDEVN= *,F6.2)                         AVARPLT    167
        105 FORMAT (*0*,*M= *,I6,5X,*UMAX= *,F7.3,5X,*UMIN= *,F7.3)       AVARPLT    168
        110 FORMAT (*1*,*X-AXIS SCALE -RAT-,AVERAGES,VARIANCES,AND STANDARD DEAVARPLT 169
          1VIATIONS USED IN AVARPLT*)                                     AVARPLT    170
        115 FORMAT (T9,*FOR THE TIME PERIOD *,A10,A4)                     AVARPLT    171
            END                                                           AVARPLT    172
```

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15. Supplementary Notes

Langley technical monitor: Joseph W. Drewry

16. Abstract

A methodology has been developed to analyze discrete data obtained from the global distribution of ozone. Statistical analysis techniques are applied to describe the distribution of data variance in terms of empirical orthogonal functions and components of spherical harmonic models. The effects of uneven data distribution and missing data are considered. Data fill based on the autocorrelation structure of the data is described. Computer coding of the analysis techniques is included.

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