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# ELECTRICAL TORQUES ON THE ELECTROSTATIC GYRO IN THE GYRO RELATIVITY EXPERIMENT

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## LIST OF ILLUSTRATIONS

## LIST OF SYMBOLS

| | |
|---|---|
| $\bar{T}$ | Torque vector |
| $\vec{i}, \vec{j}, \vec{k}$ | Orthogonal unit vectors in rotor axes |
| $\hat{F}_x, \hat{F}_y, \hat{F}_z$ | Orthogonal unit vectors in electrode axes |
| $V_i$ | Voltage on ith electrode (i = x, y, or z) |
| $\theta_o, \phi_o$ | Polar and azimuthal angles of rotor spin vector in electrode axes |
| $\theta'$ | Angle between rotor spin vector and integration point on electrodes |
| $d_o$ | Nominal rotor electrode gap |
| $\Delta d$ | Variation in rotor electrode gap |
| $\alpha, \beta, \gamma$ | Direction cosines of integration point on electrodes |
| $\alpha_o, \beta_o, \gamma_o$ | Direction cosines of rotor spin vector |
| $r_o$ | Nominal rotor radius |
| $\varepsilon_o$ | Permittivity constant |
| $r(\theta')$ | Rotor shape as function of $\theta'$ |
| $\theta_1$ | Electrode half angle |
| $M_i$ | $\frac{\varepsilon_o r_o^2}{2 d_o^2} (V_{i+}^2 - V_{i-}^2)$ (+ and - refer to opposite electrodes) |
| $P_i$ | $\frac{\varepsilon_o r_o^2}{2 d_o^2} (V_{i+}^2 + V_{i-}^2)$ |
| $h_i$ | Preload along ith axis |
| $f_i$ | Acceleration along ith axis |
| $\zeta$ | Difference in preloads for different axes |
| $t$ | Miscentering |
| $\theta$ | Misalignment angle |

TECHNICAL MEMORANDUM

# ELECTRICAL TORQUES ON THE ELECTROSTATIC GYRO IN THE GYRO RELATIVITY EXPERIMENT

## I. INTRODUCTION

The ultimate accuracy of an electrostatic gyro is determined by the level to which the Newtonian electrical torques can be reduced. An understanding of these torques is necessary for extrapolation of performance in a one-g environment to that expected in a nearly zero-g environment. The success of the Stanford Gyro Experiment depends on the reduction of all Newtonian gyro drifts to levels well below the relativity drifts, which are expected to be 7 arc sec/yr and 0.05 arc sec/yr for the geodetic and motional effects, respectively. It is important, first, to determine whether this is feasible and, second, to predict the accuracy to which the relativity effects can be measured, since this is important in assessing the relative desirability of performing the gyro experiment vis a vis other experimental tests of relativity theory. In addition, it is important to know the level of gyro performance on the ground at which a successful space experiment can be undertaken. For all these reasons, we give a general derivation of expressions for the torque on an electrostatic gyro from first principles. We also give a complete discussion of the subject, including numerical estimates of the torques for current state-of-the-art rotors. The various averaging effects are also discussed.

This report relies heavily on work done by Honeywell [1]. In Section II, we sketch the derivation of the basic torque equation derived in Reference 1. In Section III, we describe the sample rotors used for the purpose of our calculation. Section IV gives expressions for the torque in terms of the lower harmonics. These expressions are similar to those of Reference 1 but are those appropriate for circular electrodes. Section V gives the basic method of exact numerical integration we have developed and represents the main extension of the method of Reference 1. Our method includes the effect of all the higher harmonics up to the 20th. These harmonics are shown to be important and should be included in the calculation because some of the symmetry properties of the lower harmonics may not persist in the higher ones. Also, the Fourier series used in Reference 1 does not converge very rapidly. Numerical results for the three sample rotors are given in Section VI. Section VII gives results for the special case of a gimbaled gyro. This is not the actual configuration planned for the gyro experiments, but it illustrates the large reduction in torques for the gimbaled case. Section VIII describes how the method of Section V can be extended to secondary torques, i.e., those due to nonuniformity of the rotor electrode gap. The secondary torques are not calculated numerically in this report, but their magnitude is estimated. The remaining sections describe the three averaging effects: the orbital averaging (Section IX), the averaging due to spacecraft roll (Section X), and the averaging due to rotor spin (Section XI). The orbital averaging combined with spacecraft roll is shown to be effective for two of the three components of gravity gradient force, and the third component can be minimized by proper design of the spacecraft. The averaging due to spacecraft roll is shown to be effective for the primary torques and is crucial for the achievement of the goal of 1 milliarcsec/yr drift for the gyro. The averaging due to spin is described but not calculated, since a complete map of the rotor is not yet available. Section XII states the conclusions of the report.

## II. GENERAL DISCUSSION OF ELECTROSTATIC TORQUES

For a conductor in an external electric field $\vec{E}$, the force per unit area f is given by [1]

$$f = \frac{\varepsilon_o}{2} |\vec{E}|^2 \quad ,$$

where $\varepsilon_o = 8.85 \times 10^{-12}$ farads/m. The electric field is perpendicular to the surface of the conductor and zero inside the conductor. If we consider an irregularly shaped conductor, then the total force $\vec{F}$ and torque $\vec{T}$ are given by

$$\vec{F} = \frac{\varepsilon_o}{2} \oint_S |\vec{E}|^2 \vec{n} \, dS \tag{1}$$

$$\vec{T} = \frac{\varepsilon_o}{2} \oint_S |\vec{E}|^2 (\vec{r} \times \vec{n}) \, dS \quad , \tag{2}$$

where the integral is taken over the entire surface S, $\vec{n}$ is a unit vector normal to the surface, and $\vec{r}$ is a vector from some origin to the surface element dS. It is important to realize that $\vec{T}$ is dependent on our choice of origin. The evaluation of electric torques on a gyro is accomplished by choosing a suitable model for the actual shape of the gyro and calculating the integral in equation (2). For a perfectly spherical gyro $\vec{T} = 0$, as is obvious from equation (2).

To calculate $\vec{T}$ we need to make several assumptions, the validity of which we now discuss.

a) The rotor shape is symmetrical about its spin axis. This certainly appears reasonable if the rotor is spinning sufficiently fast, since the irregularities would then average out over many rotations of the rotor. Honeywell [1] has produced an argument which shows that this is, in fact, true if one has a sufficiently smooth rotor (see Section XI). There is one important exception, however, and that is for a rotor with radial mass unbalance perpendicular to the spin axis. In this case, the suspension forces can have an oscillatory component at the spin rate frequency and the radial mass unbalance torque does not average to zero. There is then present a torque component parallel to the spin axis which alters spin speed and can cause run-down.

b) The rotor shape can be represented by a cosine series of the form

$$r(\theta') = r_o + \sum_{n=2}^{\infty} a_n \cos (n\theta') \quad . \tag{3}$$

Almost any shape can be so represented, and the number of terms needed is related to the dimensions of the radial irregularities as discussed in Section III. A derivation that does not assume this is given in Section V.

c) The electric field at a given point in the rotor electrode gap is given by $|\vec{E}| = V/d$, where d is the rotor electrode distance at that point and V is the rotor electrode potential. The justification for this is that the rotor electrode gap is so small (1500 μin. typical) that one can treat the situation locally as a parallel plate in which the electric field is uniform. Honeywell [1] has made some calculations that bear this out.

d) Electrode edge effects can be neglected. It is difficult to estimate how good this assumption is, but it is probably much better for hexahedral electrodes than circular electrodes.

e) The variations in rotor electrode gap $\Delta d$ defined by $d = d_o + \Delta d$ satisfy the conditions

$$\frac{\Delta d}{d_o} << 1 \quad . \tag{4}$$

This is a good assumption, since the main contributor to $\Delta d$ is the centering errors which are of order 15 μin., giving

$$\frac{\Delta d}{d_o} \sim 0.01 \quad .$$

Given all these assumptions, one can write the expression for $\vec{T}$ in the form

$$\vec{T} = \frac{\epsilon_o r_o^2}{2 d_o^2} \sum_i V_i^2 \iint_{S_i} \left(1 - \frac{2\Delta d}{d_o}\right) \frac{dr(\theta')}{d\theta'} (\vec{i} \sin \phi'$$

$$- \vec{j} \cos \phi') \sin \theta' \, d\theta' \, d\phi' \quad , \tag{5}$$

where the sum is taken over the six electrodes and $\theta'$, $\phi'$ are spherical coordinates defined about the gyro spin axis. This follows directly from equation (2). We have used equation (4) and expanded the $1/d^2$ term, set $r = r_o$ in the term $dS = r^2 \sin \theta' \, d\theta' \, d\phi'$, and expressed the surface integral as a sum of integrals over electrodes at potentials $V_i$. The expression for $(\vec{r} \times \vec{n})$ is justified by noting that it must be tangent to a latitude line; i.e., in the direction given by the unit vector $\vec{i} \sin \phi' - \vec{j} \cos \phi'$ and of magnitude equal to the (small) angle between the radius vector and the normal vector to a curve $dr/d\theta'$ (this is worked out in elementary calculus).

The integral in equation (5) is in a spherical coordinate system defined by the rotor spin axis tipped at an arbitrary angle with respect to the electrode axes. Since we must calculate the surface integrals in a coordinate system defined by the electrode axes, we need to transform equation (5) to this coordinate system. If $(\theta_o, \phi_o)$ represents the orientation of the spin axis in these coordinates, then

$$\vec{i} = \cos \theta_o \cos \phi_o \hat{F}_x + \cos \theta_o \sin \phi_o \hat{F}_y - \sin \theta_o \hat{F}_z$$

$$\vec{j} = - \sin \phi_o \hat{F}_x + \cos \phi_o \hat{F}_y$$

$$\cos \phi' = \frac{1}{\sin \theta'} [\sin \theta \cos \theta_o \cos (\phi - \phi_o) - \cos \theta \sin \theta_o]$$

$$\sin \phi' = \frac{1}{\sin \theta'} [\sin \theta \sin (\phi - \phi_o)]$$

define the relationship between the primed and unprimed angles. The first two equations follow from the fact that the vector in the spin axis direction is

$$\vec{K} = \sin\theta_o \cos\phi_o \hat{F}_x + \sin\theta_o \sin\phi_o \hat{F}_y + \cos\theta_o \hat{F}_z$$

and $\vec{j}$ is defined to have no $\hat{F}_z$ component, while $\vec{i}$, $\vec{j}$, and $\vec{k}$ must be orthonormal. The second two equations follow from expressing ($\hat{F}_x$, $\hat{F}_y$, $\hat{F}_z$) in terms of ($\vec{i}$, $\vec{j}$, $\vec{k}$) and looking at the expansion of an arbitrary vector in the two bases. Substituting these relations into equation (5), we find

$$\vec{T} = \frac{\epsilon_o r_o^2}{2 d_o^2} \sum_i V_i^2 \int\int_{S_i} \left(1 - \frac{2\Delta d}{d_o}\right) \left[\hat{F}_x (\beta\gamma_o - \gamma\beta_o) + \hat{F}_y (\gamma\alpha_o - \alpha\gamma_o) + \hat{F}_z (\alpha\beta_o - \beta\alpha_o)\right] \frac{dr(\theta')/d(\theta')}{\sin\theta'} \sin\theta \, d\theta \, d\phi \quad . \tag{6}$$

where $\alpha = \sin\theta \cos\phi$, $\beta = \sin\theta \sin\phi$, and $\gamma = \cos\theta$ and similarly for $\alpha_o$, $\beta_o$, $\gamma_o$. This is the basic equation we will use in all further calculations. All that needs to be done is to take an expression for $\Delta d$ and $dr(\theta')/d\theta'$ and calculate the integrals. Also note that $\vec{T} \cdot \vec{K} = 0$. The important quantity in the expression is $dr(\theta')/d(\theta')$, which determines the magnitude of the torques. This completes the derivation of the basic torque equation. The remainder of the report is devoted to evaluating the equation.

## III. EXPANSION OF THE ROTOR SHAPE IN HARMONICS

The basic integral in equation (6) has been evaluated at great length by Honeywell [1] for hexahedral electrodes. This has been done by expanding $\Delta d$ and $dr(\theta')/d\theta'$ in a Fourier cosine series and calculating the integral separately for each harmonic. The results of this will be discussed in Section IV, but here we wish to discuss the convergence of such a series. To do this we have taken the latest roundness measurements of the best available rotors by Rank-Taylor-Hobson (December 20, 1976). These measurements are taken over three great circles on the rotor; and, since we are assuming azimuthal symmetry, we need only half of each great circle. The three curves we have selected are shown in Figures 1, 2, and 3, which give $\Delta r$ as a function of polar angle $\theta$. While some improvement may occur due to spin averaging (see Section XI), we believe this is a reasonable place to start in evaluating the harmonics for realistic rotors.

Figures 4, 5, and 6, show the Fourier coefficients for the shapes given in Figures 1, 2, and 3, respectively. These have been obtained numerically from the actual assumed shapes. (The first harmonic has no significance for these calculations.) These curves show that we do get convergence, but it may require up to 20 harmonics depending on the scale of the irregularities in the rotor. For example, in Figure 3 more harmonics are required because of the sharpness of the bumps. In fact, there is an inverse relationship between the number of harmonics required and the smallest angular scale on which the irregularities occur. A rotor with very sharp bumps will require many harmonics to faithfully approximate its shape. The contribution to the integral in equation (6) from $dr(\theta')/d\theta'$ is proportional to $n\sin(n\theta)$ for the nth harmonic, so we have the possibility of increasing contributions to the torque as a result of increasingly higher harmonics. In fact, if Reference 1 is studied, it will be found that for the higher harmonics the number of terms in each torque expression actually increases with harmonic number, as do the coefficients of each term.

These considerations illustrate the basic problem with this type of calculation. It appears that contributions resulting from higher harmonics will be significant when added up, but it becomes extremely tedious to calculate equation (6) for the higher harmonics. This is only feasible up to about the sixth harmonic and is extremely difficult even up to this level, as a perusal of Reference 1 will show. The only way out of this dilemma appears to be the method we use in Section V, where we evaluate equation (6) numerically for some actual rotor shapes.

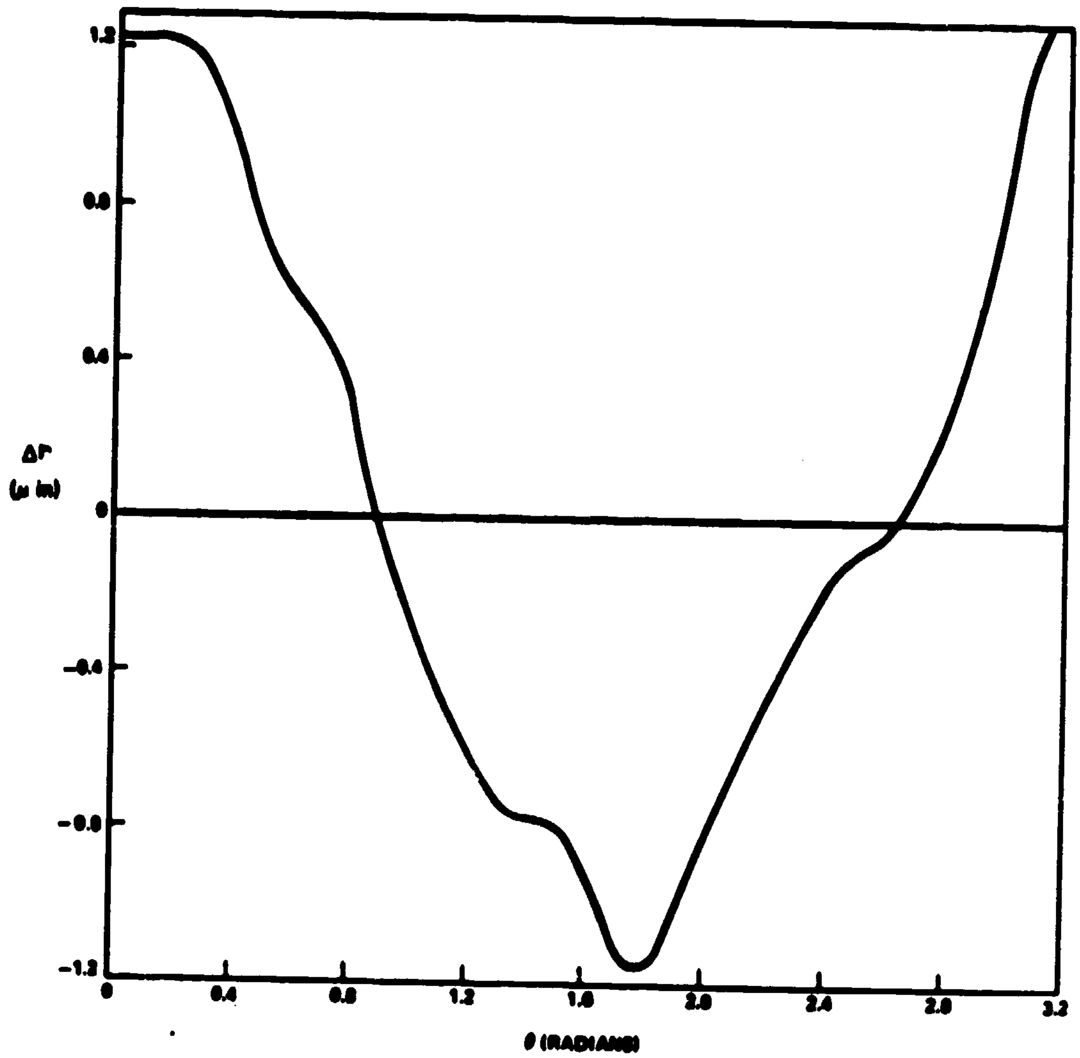


Figure 1. Deviation from sphericity,· Case 1.

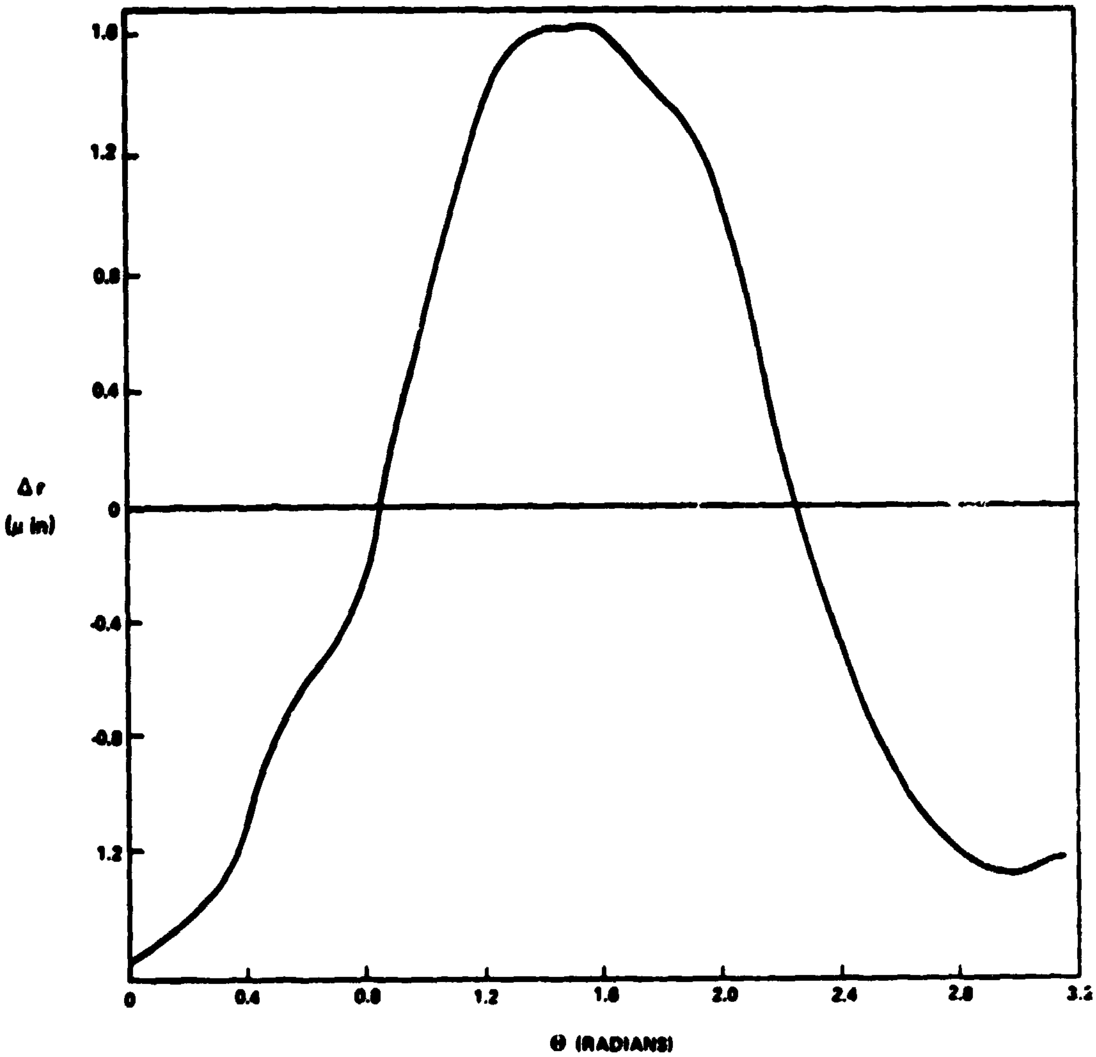


Figure 2. Deviation from sphericity, Case II.

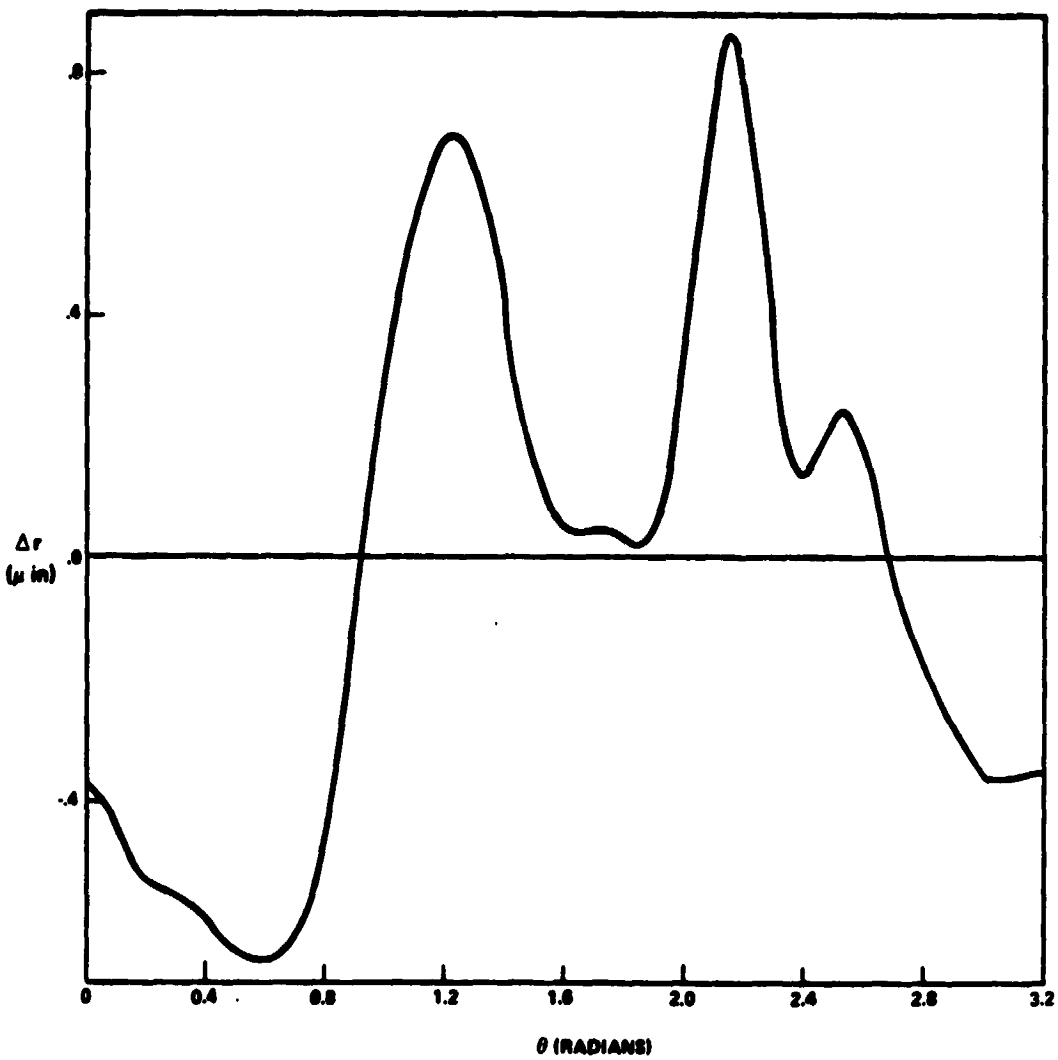


Figure 3. Deviation from sphericity, Case III.

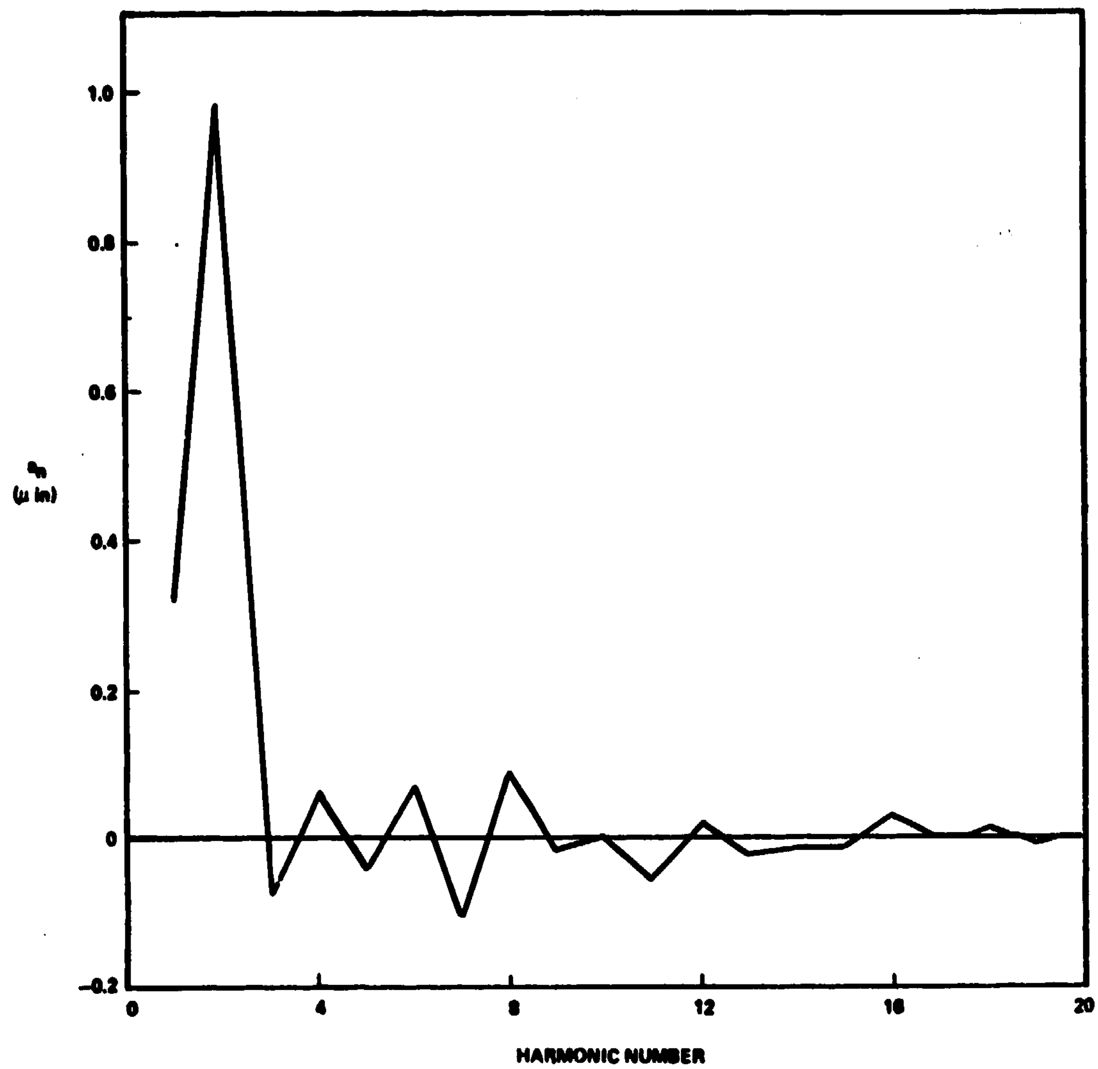


Figure 4. Size of harmonics, Case I.

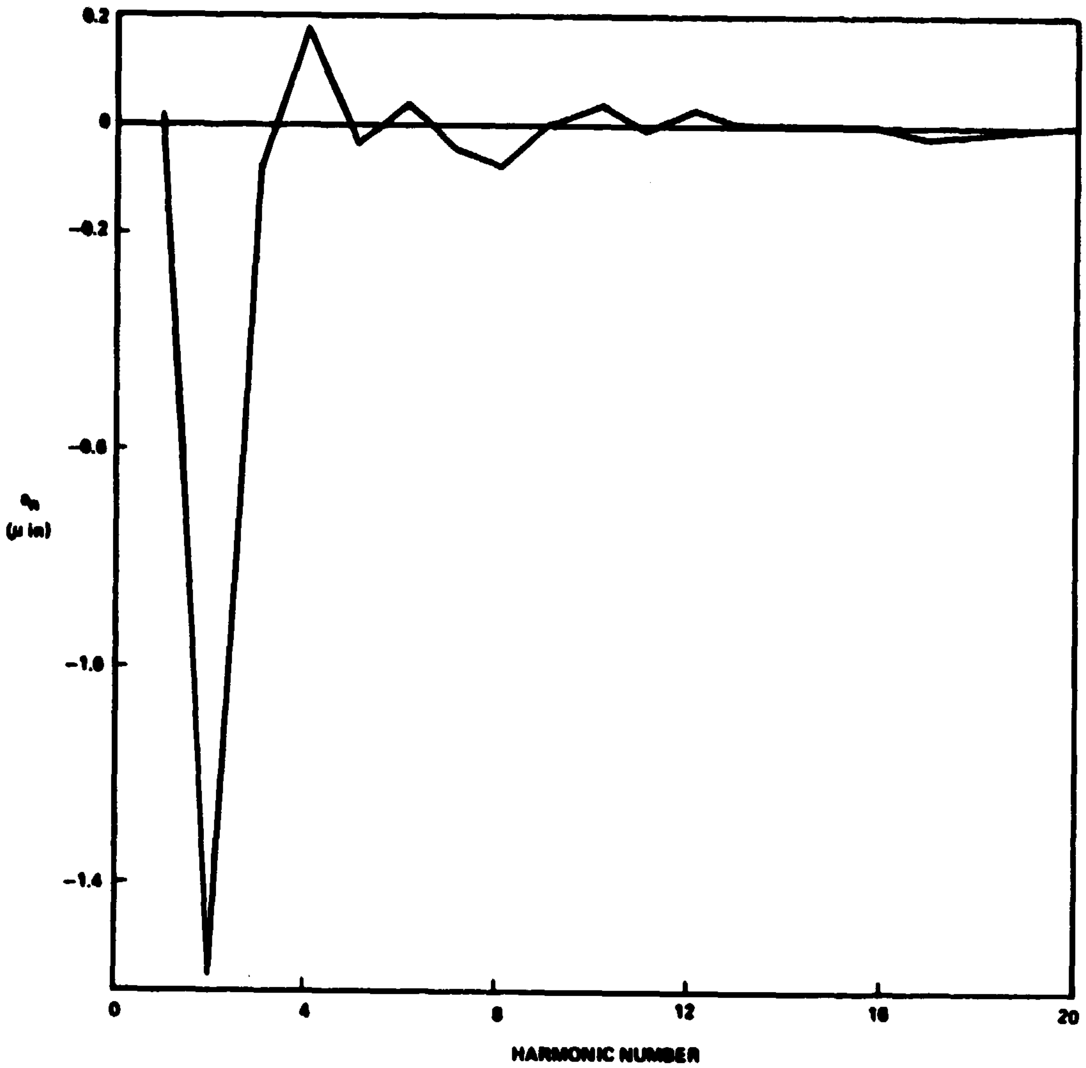


Figure 5. Size of harmonics, Case II.

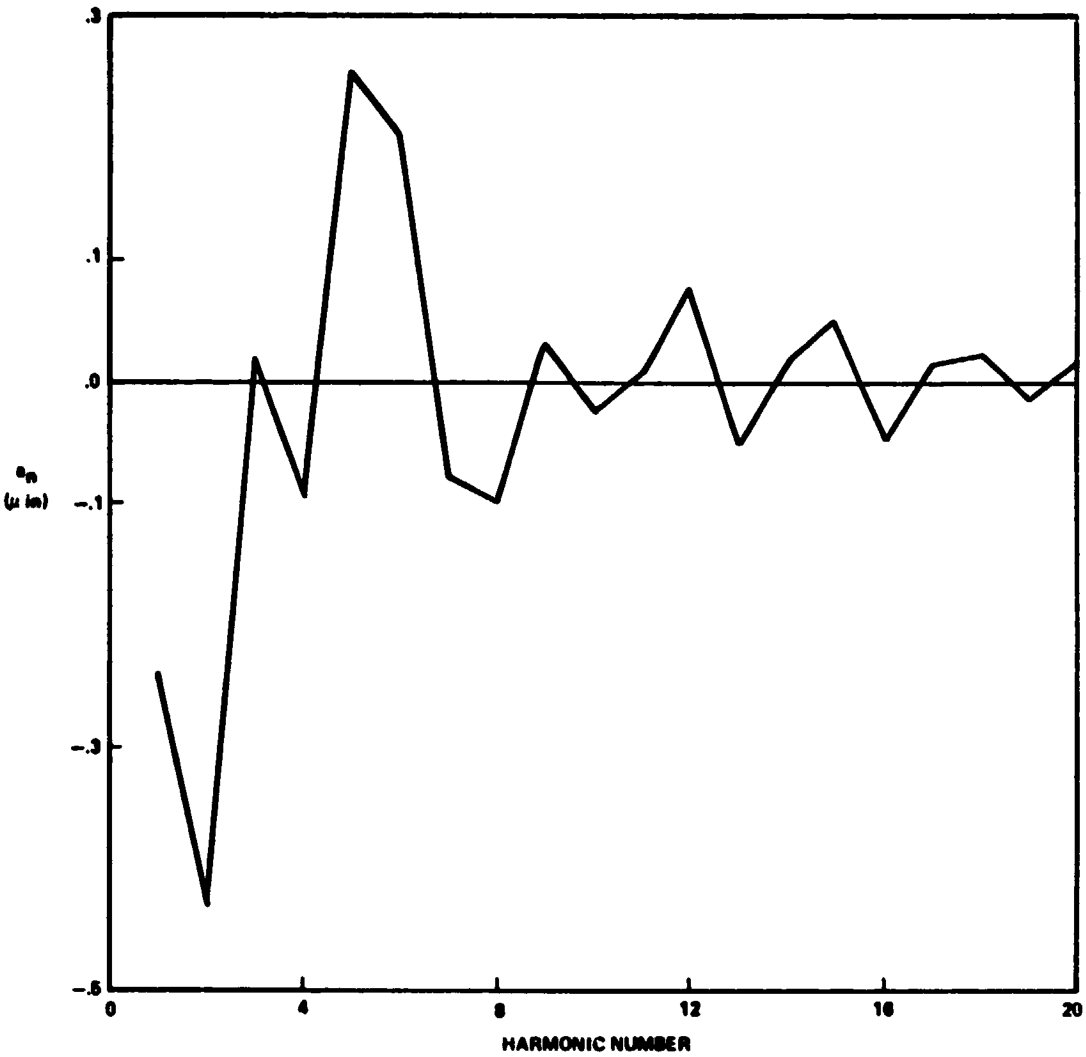


Figure 6. Size of harmonics, Case III.

## IV. TORQUES IN TERMS OF HARMONICS

The method used to evaluate equation (6) is to expand $dr(\theta')/d\theta'$ and $\Delta d$ in a Fourier series of the form

$$\Delta r = \sum_{n=2}^{\infty} a_n \cos(n\theta')$$

from which $dr(\theta')/d\theta'$ is obtained by differentiation, and the part of $\Delta d$ due to rotor nonsphericity is obtained by $\Delta d = -\Delta r$. The torques due to just the $dr(\theta')/d\theta'$ term ($\Delta d = 0$) are known as primary torques. We have calculated these for circular electrodes up to the fifth harmonic: the results are shown in Table 1. The y and z components of T are obtained by suitable permutation of $\alpha_o$, $\beta_o$, $\gamma_o$ and the $P_i$'s and $M_i$'s [1]. Honeywell has calculated these torques up to the sixth harmonic for hexahedral electrodes. We have developed a computer program to evaluate our expressions to see the angular behavior of the torques. This is necessary because heretofore the torques have been estimated from expressions valid only near the z-axis, and it is of interest to see how these torques behave for arbitrary orientations. Also, with our computer calculation we can compare the actual magnitudes of the second versus fourth and third versus fifth harmonics to see if convergence is occurring. Figures 7 through 10 show the relative magnitude of the torques for the second through the fifth harmonic in arbitrary units for typical values of the harmonic coefficients. The second harmonic is small because we have assumed equal preloads. It is clear that the fourth and fifth harmonics produce torques of the same order of magnitude as the second and third. A large number of calculations that are not shown here bear this out. This is also obvious from the inspection of Table 1. The expressions in Table 2 are computed from general expressions for the torque as in Table 1 by specializing the expressions to the case $\gamma_o \sim 1$, $\alpha_o << 1$, $\beta_o << 1$ and defining $\Omega = T/I\omega$, where T is the magnitude of the torque, $I = 2/5\ mr^2$ is the moment of inertia of the rotor and $\omega = v/r$ is the angular velocity of the rotor. The connection between the M's and P's in Table 1 and the acceleration f and preload h are derived as follows. From equation (1) we have for circular electrodes that the force on the rotor is

$$\vec{F} = \eta \sin^2 \theta_1 \ (M_x F_x + M_y F_y + M_z F_z) \quad ; \tag{7}$$

this determines the components of the acceleration as

TABLE 1

$$T_x = 2.727\, a_2\, (P_z - P_y)\, \beta_o\, \gamma_o + a_3 \Big\{ 7.078\, (\beta_o\, \gamma_o^2\, M_z$$

$$- \beta_o^2\, \gamma_o\, M_y) + 0.5890\, [(\beta_o^3 + \alpha_o^2 \beta_o)\, M_z - (\gamma_o^3 + \alpha_o^2\, \gamma_o)\, M_y]$$

$$+ 2.356\, (\gamma_o\, M_y - \beta_o\, M_z) \Big\} + a_4 \Big\{ \beta_o\, \gamma_o^3\, (16.37\, P_z + 3.74\, P_y)$$

$$- \beta_o^3\, \gamma_o\, (3.740\, P_z + 16.37\, P_y) + 3.740\, \alpha_o^2\, \beta_o\, \gamma_o\, (P_y - P_z)$$

$$+ 10.91\, \beta_o\, \gamma_o\, (P_y - P_z) \Big\} + 10\, \pi\, a_5 \Big\{ 0.5625\, \beta_o^2\, \gamma_o^2\, (\beta_o\, M_z$$

$$- \gamma_o\, M_y) + 0.5625\, \alpha_o^2\, \beta_o\, \gamma_o\, (\gamma_o\, M_z - \beta_o\, M_y)$$

$$+ 1.1273\, \beta_o\, \gamma_o\, (\gamma_o^3\, M_z - \beta_o^3\, M_y) + 0.03125\, \alpha_o^2\, (\beta_o^3\, M_z$$

$$- \gamma_o^3\, M_y) + 0.01562\, [\beta_o\, (\beta_o^4 + \alpha_o^4)\, M_z - \gamma_o\, (\gamma_o^4$$

$$+ \alpha_o^4)\, M_y] + 0.09375\, [\gamma_o\, (\gamma_o^2 + \alpha_o^2)\, M_y - \beta_o\, (\beta_o^2$$

$$+ \alpha_o^2)\, M_z] + 1.1265\, \beta_o\, \gamma_o\, (\beta_o\, M_y - \gamma_o\, M_z)$$

$$+ 0.125\, (\beta_o\, M_z - \gamma_o\, M_y) \Big\} \; .$$

$$P_i = \frac{\varepsilon_o\, r_o^2}{2\, d_o^2} \left(V_{i+}^2 + V_{i-}^2\right)$$

$$M_i = \frac{\varepsilon_o\, r_o^2}{2\, d_o^2} \left(V_{i+}^2 - V_{i-}^2\right)$$

$\theta_1 = 30^\circ$ = electrode half angle

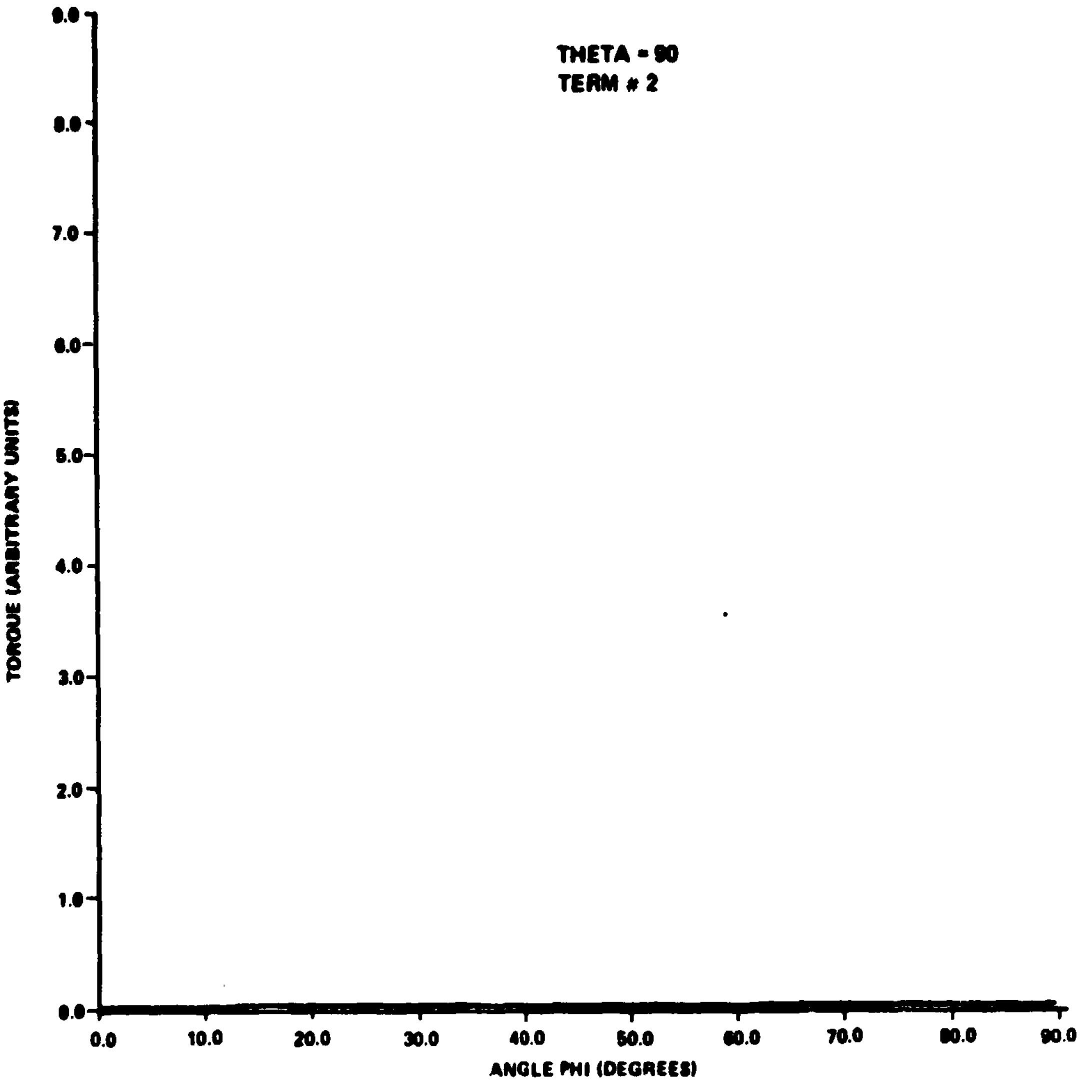


Figure 7. Second harmonic torque versus angle.

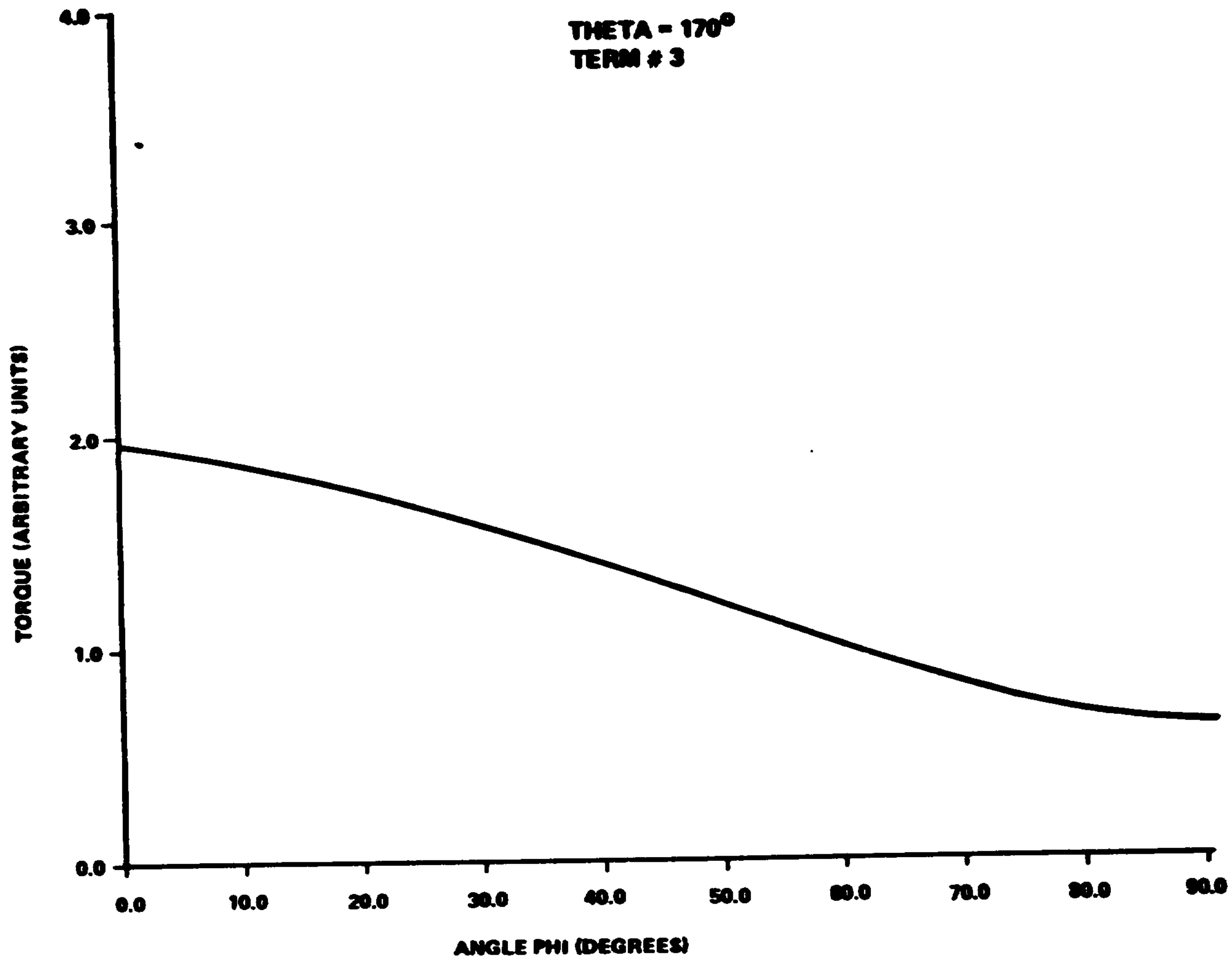


Figure 8. Third harmonic torque versus angle.

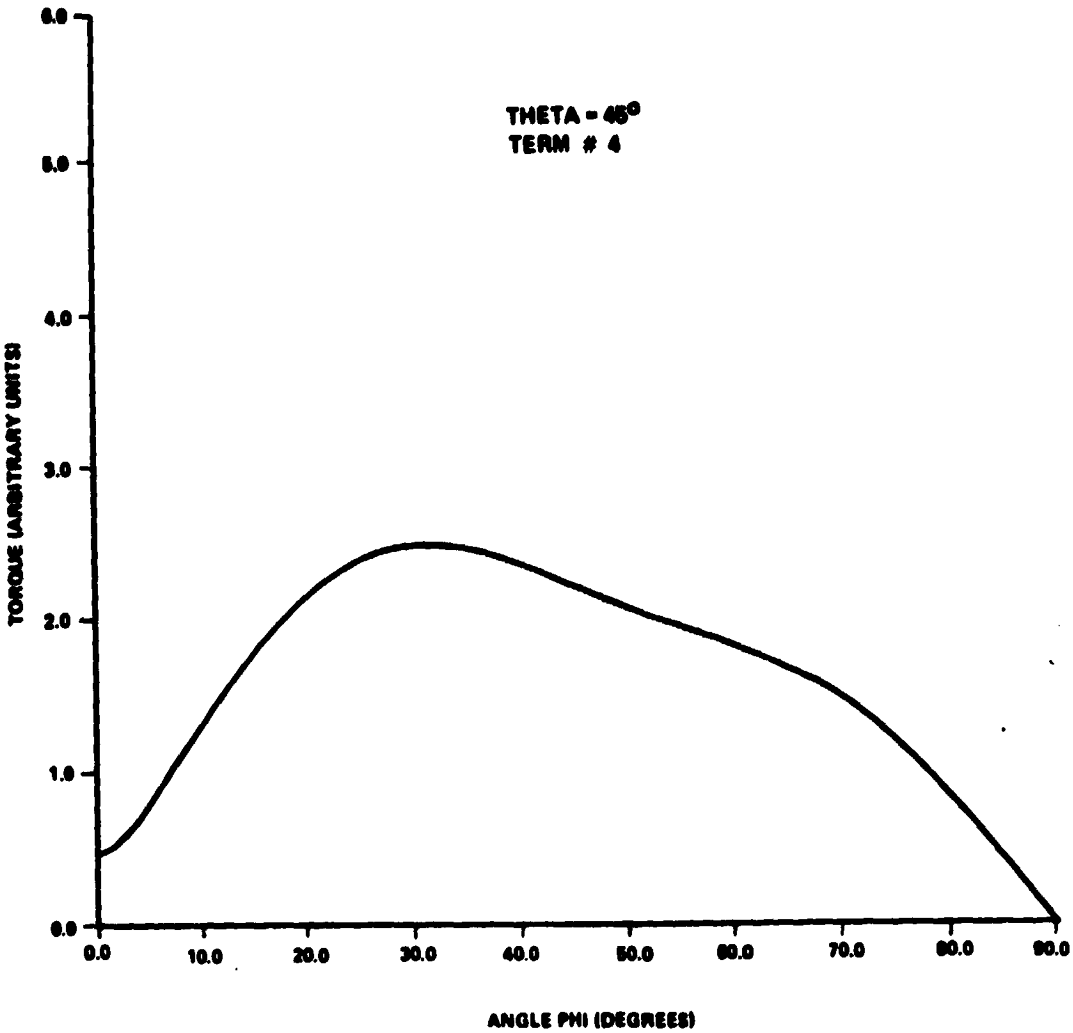


Figure 9. Fourth harmonic torque versus angle.

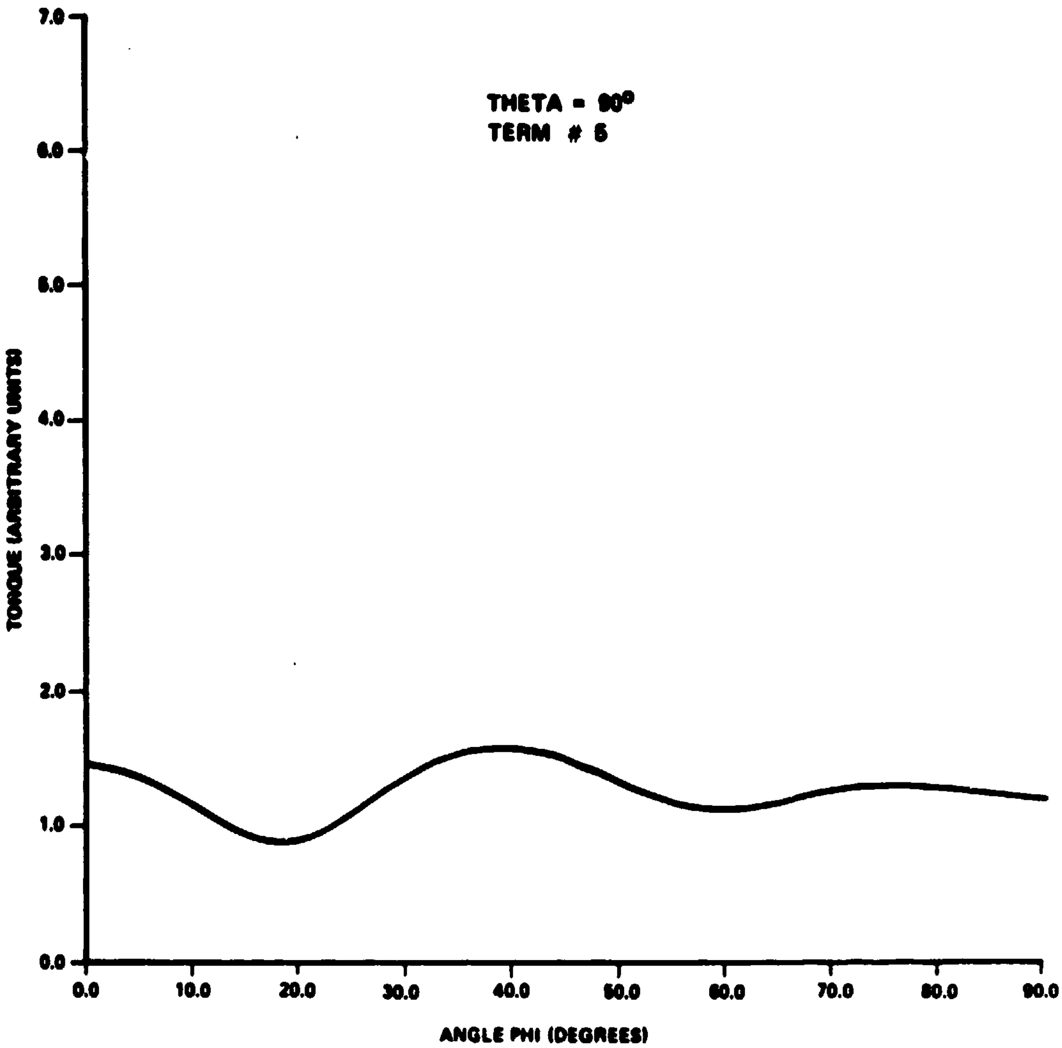


Figure 10. Fifth harmonic torque versus angle.

**TABLE 2**

| Drift Rates | Second Harmonic | Third Harmonic |
| --- | --- | --- |
| Aligned | 0 | $\frac{15}{2}\left(\cos^2\theta_1 - \frac{1}{5}\right)\left(\frac{a_3}{r_o}\right)\left(\frac{f}{V}\right)$ |
| Misaligned | $5\cos^2\theta_1\left(\frac{a_2}{r_o}\right)\left(\frac{\epsilon}{V}\right)\left(\Delta h + \frac{f^2}{h}\right)$ | $\frac{15}{2}\left(4\cos^2\theta_1 - 1\right)\left(\frac{a_3}{r_o}\right)\left(\frac{f}{V}\right)\epsilon$ |
| Miscentered | $5\sin^2\theta_1\left(\frac{a_2}{r_o}\right)\left(\frac{t}{d_o}\right)\left(\frac{f}{V}\right)$ | $\frac{3}{2}\left(\frac{2 - \cos\theta_1\left(7\cos^4\theta_1 - 10\cos^2\theta_1 + 5\right)}{\sin^2\theta_1}\right) \times \left(\frac{a_3}{r_o}\right)\left(\frac{t}{d_o}\right)\left(\frac{h}{V}\right)\left(1 + \frac{f^2}{h^2}\right)$ |

$\theta_1$ = electrode half angle

$r_o$ = rotor radius

$V$ = peripheral velocity

$h$ = preload

$\Delta h$ = difference in preload for different axes

$f$ = acceleration

$t$ = miscentering

$d_o$ = nominal rotor electrode gap

$\epsilon$ = misalignment angle

$$f_x = \frac{\pi \sin^2 \theta_1}{m} M_x \quad .$$

The expression in Table 2 for the aligned third harmonic then agrees with that of Table 1 except for the -1/5, which represents the displacement of the center of mass for a pear-shaped rotor of this type. Similarly, for the misaligned third harmonic, the terms in Tables 1 and 2 agree if we take $\beta_o = 0$ and neglect $\alpha_o^2$ and $\beta_o^2$. This brings out the fact that the expressions in Table 2 are only good for $\gamma_o \sim 1$ (gimbaled gyro) and shows why it is necessary to write a computer program to evaluate these torques for the nongimbaled case. This is described in Section V for all the harmonics.

The preload $h_i$ is defined here as the acceleration that will drive the voltage on one of the ith electrodes to zero and thus lose the suspension. (This definition differs from that of Reference 1.) For a linear suspension system we have $V_{i+} + V_{i-} = V_{io}$ = constant, so from equation (7)

$$mh_i = \frac{\epsilon_o r_o^2}{2 d_o^2} (V_{io}^2) (\pi \sin^2 \theta_1) \quad .$$

Using the identity

$$(V_{i-}^2 + V_{i+}^2) = \frac{1}{2} \left[ (V_{i+} + V_{i-})^2 + \frac{(V_{i+}^2 - V_{i-}^2)^2}{(V_{i+} + V_{i-})^2} \right]$$

$$= \frac{V_{io}^2}{2} \left[ 1 + \frac{(V_{i+}^2 - V_{i-}^2)^2}{V_{io}^4} \right]$$

and multiplying by $\epsilon_o r_o^2 / 2 d_o^2$, we obtain

$$P_i = \frac{m}{2 \pi \sin^2 \theta_1} \left[ h_i + \frac{f_i^2}{h_i} \right] \quad . \tag{8}$$

This is the basic equation connecting $P_i$ with $h_i$ and $f_i$. The preloads $h_i$ are adjusted to be larger than the maximum acceleration expected in a given environment. The misaligned second harmonic in Table 2 is seen to agree with Table 1 if $\beta_0$ is again identified with $\theta$ and $\epsilon$ is the difference in preloads for different axes.

Comparing these expressions with Reference 2 indicates that there are terms present for the aligned and miscentered third harmonic that are not included in Reference 2. These two terms put more severe restrictions on f and h than have previously been indicated. We postpone generating actual numerical tolerances until Section VI and refer to Reference 3, where numerical values for the parameters in Table 2 are given which allow the reduction of $\Omega$ to $\Omega_0 = 1.6 \times 10^{-16}$ rad/sec = 0.001 arc sec/yr, the accuracy necessary for a 2 percent measurement of the Lense Thirring effect.

Tables 3 and 4 contain miscentering torques for the second and third harmonics in terms of arbitrary electrode angle $\theta_1$. The corresponding expressions in Table 2 are obtained for the case $\gamma_0 \approx 1$ as before. These expressions illustrate the rapid increase in complexity for higher harmonics.

It is important to realize that the simplified expressions in Table 2 which have been used in previous drift estimates are only valid for near alignment of the rotor spin with the electrode axis. When this is not the case, the expressions in terms of harmonics become extremely tedious to evaluate for the higher harmonics, and it is best to resort to numerical integration of equation (6), as in Section V. The planned orientation for the final gyro experiment is not the gimbaled case but the case where the spin axis is midway between two electrodes. Thus, the expressions in Table 2 are not really relevant for this orientation, and what we really need is the case $\gamma_0 = \alpha_0 = 1/\sqrt{2}$. This is difficult to evaluate for all the terms in Table 1, but we immediately realize that the alignment factor $\theta$ for the second harmonic is no longer applicable. This is a very important point and increases the corresponding drifts by at least two orders of magnitude. This point is discussed further in Sections VI and VII.

**TABLE 3. SECOND HARMONIC MISCENTERING TORQUE (CIRCULAR ELECTRODES)**

$$\frac{\vec{T}_2}{a_2}\bigg|_{x_c} = \frac{x_c}{d_o}\Bigg[\hat{F}_x\left\{(2\pi \sin^4\theta_1)\,[\alpha_o(\beta_o M_z - \gamma_o M_y)]\right\} + \hat{F}_y\Big\{(2\pi\sin^4\theta_1)\,[\beta_o\gamma_o M_y$$

$$- (\alpha_o^2 - \gamma_o^2) M_z] + 2\pi \sin^4\theta_1 (1 + 2\cot^4\theta_1)\,[\alpha_o\gamma_o M_x]\Big\}$$

$$+ \hat{F}_z\Big\{(2\pi\sin^4\theta_1)\,[-\beta_o\gamma_o M_z - (\beta_o^2 - \alpha_o^2) M_y] - (2\pi\sin^4\theta_1)(1$$

$$+ 2\cot^4\theta_1)\,[\alpha_o\beta_o M_x]\Big\}\Bigg]$$

$x_c$ = miscentering error in x direction

**TABLE 4. THIRD HARMONIC MISCENTERING TORQUE (CIRCULAR ELECTRODES)**

$$\left.\frac{\hat{T}_3}{24\, a_3}\right|_{x_c} = \frac{x_c}{d_o}\Bigg\{ \hat{F}_x \left[\alpha_o \beta_o \gamma_o \left(-\frac{5}{2} A_2 - 2 A_1\right) (P_z - P_y)\right]$$

$$+ \hat{F}_y \Bigg[\alpha_o^2 \gamma_o \Bigg\{P_x \left(2 A_2 - 2 A_1 + 2\pi \frac{(1 - \cos^5 \theta_1)}{5}\right)$$

$$+ P_y \left(\frac{A_2}{4}\right) + P_z \left(\frac{11 A_2}{4} - 2 A_1\right)\Bigg\} + \beta_o^2 \gamma_o \Bigg\{(P_x$$

$$+ P_y)(A_1 - A_2) + P_z \left(\frac{A_2}{4}\right)\Bigg\} + \gamma_o^3 \Bigg\{(P_x + P_z)(A_1$$

$$- A_2) + P_y \left(\frac{A_2}{4}\right)\Bigg\} \quad \gamma_o \Bigg\{(P_y + P_z)\left(\frac{A_1}{4}\right)$$

$$+ P_x \left(\pi \frac{(1 - \cos^3 \theta_1)}{6}\right)\Bigg\}\Bigg] - \hat{F}_z \Bigg[\alpha_o^2 \beta_o \Bigg\{P_x \Bigg(2 A_2$$

$$- 2 A_1 + \frac{(1 - \cos^5 \theta_1)}{2}\Bigg) + P_z \left(\frac{A_2}{4}\right) - P_y \left(\frac{11 A_2}{4} - 2 A_1\right)\Bigg\}$$

$$+ \beta_o^3 \Bigg\{(P_x + P_y)(A_1 - A_2) + P_z \left(\frac{A_2}{4}\right)\Bigg\}$$

$$+ \beta_o \gamma_o^2 \Bigg\{(P_x + P_y)(A_1 - A_2) + P_y \left(\frac{A_2}{4}\right)\Bigg\}$$

$$- \beta_o \Bigg\{(P_y + P_z)\left(\frac{A_1}{4}\right) + P_x \left(\pi \frac{(1 - \cos^3 \theta_1)}{6}\right)\Bigg\}\Bigg]\Bigg\}$$

$$A_1 = \pi \int_o^{\theta_1} \sin^3 \theta \, d\theta$$

$$A_2 = \pi \int_o^{\theta_1} \sin^5 \theta \, d\theta$$

## V. EXACT NUMERICAL CALCULATION OF TORQUES

It is possible to evaluate equation (6) for arbitrary rotor spin orientation by doing the integrals numerically. To do this we use the substitutions in Reference 1 to include all the electrodes; that is:

| | |
|---|---|
| z- electrode | $\alpha \rightarrow -\alpha,\ \gamma \rightarrow -\gamma$ |
| x+ electrode | $\alpha \rightarrow \gamma,\ \gamma \rightarrow -\alpha$ |
| x- electrode | $\alpha \rightarrow -\gamma,\ \beta \rightarrow -\beta,\ \gamma \rightarrow -\alpha$ |
| y+ electrode | $\alpha \rightarrow -\beta,\ \beta \rightarrow \gamma,\ \gamma \rightarrow -\alpha$ |
| y- electrode | $\alpha \rightarrow \beta,\ \beta \rightarrow -\gamma,\ \gamma \rightarrow -\alpha$ . |

One notices then that permuting $\alpha$, $\beta$, and $\gamma$ is equivalent to permuting $\alpha_o$, $\beta_o$, and $\gamma_o$ in the expression for $\theta'$ and at the same time permuting the three basic integrals given by

$$I_1(\alpha_o, \beta_o, \gamma_o) = \int_0^{2\pi} d\phi \int_0^{\theta_1} \sin\theta\, d\theta \left[\alpha \frac{dr(\theta')/d\theta'}{\sin\theta'}\right]$$

$$I_2(\alpha_o, \beta_o, \gamma_o) = \int_0^{2\pi} d\phi \int_0^{\theta_1} \sin\theta\, d\theta \left[\beta \frac{dr(\theta')/d\theta'}{\sin\theta'}\right]$$

$$I_3(\alpha_o, \beta_o, \gamma_o) = \int_0^{2\pi} d\phi \int_0^{\theta_1} \sin\theta\, d\theta \left[\gamma \frac{dr(\theta')/d\theta'}{\sin\theta'}\right]$$

$$\cos\theta' = \alpha\alpha_o + \beta\beta_o + \gamma\gamma_o$$

$$\alpha = \sin\theta\cos\phi$$

$$\beta = \sin\theta\sin\phi$$

$$\gamma = \cos\theta\ .$$

Thus, it is possible to express equation (6) in terms of these three basic integrals with the arguments permuted in the appropriate fashion. The end result of this is as follows (where we have neglected the $\Delta d/d$ term):

$$T_x = \frac{\epsilon_o r_o^2}{2 d_o^2} \Big\{ \gamma_o \, [V_{z+}^2 \, I_2 \, (\alpha_o, \beta_o, \gamma_o) + V_{z-}^2 \, I_2 \, (-\alpha_o, \beta_o, -\gamma_o)$$

$$+ V_{x+}^2 \, I_2 \, (-\gamma_o, \beta_o, \alpha_o) - V_{x-}^2 \, I_2 \, (-\gamma_o, -\beta_o, -\alpha_o) + V_{y+}^2 \, I_3 \, (-\gamma_o, -\alpha_o, \beta_o)$$

$$- V_{y-}^2 \, I_3 \, (-\gamma_o, \alpha_o, -\beta_o)] + \beta_o \, [V_{z+}^2 \, I_3 \, (\alpha_o, \beta_o, \gamma_o)$$

$$+ V_{z-}^2 \, I_3 \, (-\alpha_o, \beta_o, -\gamma_o) + V_{x+}^2 \, I_1 \, (-\gamma_o, \beta_o, \alpha_o) + V_{x-}^2 \, I_1 \, (-\gamma_o, -\beta_o, -\alpha_o)$$

$$+ V_{y+}^2 \, I_1 \, (-\gamma_o, -\alpha_o, \beta_o) + V_{y}^2 \, I_1 \, ( \gamma_o, \alpha_o, -\beta_o)] \Big\}$$

$$T_y = \frac{\epsilon_o r_o^2}{2 d_o^2} \Big\{ \alpha_o \, [V_{z+}^2 \, I_3 \, (\alpha_o, \beta_o, \gamma_o) \quad V_{z-}^2 \, I_3 \, (-\alpha_o, \beta_o, -\gamma_o)$$

$$- V_{x+}^2 \, I_1 \, (-\gamma_o, \beta_o, \alpha_o) \quad V_{x-}^2 \, I_1 \, (-\gamma_o, -\beta_o, -\alpha_o) - V_{y-}^2 \, I_1 \, (-\gamma_o, -\alpha_o, \beta_o)$$

$$- V_{y-}^2 \, I_1 \, (-\gamma_o, \alpha_o, -\beta_o)] + \gamma_o \, [V_{z+}^2 \, I_1 \, (\alpha_o, \beta_o, \gamma_o)$$

$$+ V_{z-}^2 \, I_1 \, (-\alpha_o, \beta_o, -\gamma_o) \quad V_{x+}^2 \, I_3 \, (-\gamma_o, \beta_o, \alpha_o) + V_{x-}^2 \, I_3 \, (-\gamma_o, -\beta_o, -\alpha_o)$$

$$+ V_{y+}^2 \, I_2 \, (-\gamma_o, -\alpha_o, \beta_o) \quad V_{y}^2 \, I_2 \, (-\gamma_o, \alpha_o, -\beta_o)] \Big\}$$

$$T_z = \frac{\varepsilon_o r_o^2}{2 d_o^2} \Big\{ \beta_o [V_{z+}^2 I_1 (\alpha_o,\beta_o,\gamma_o) - V_{z-}^2 I_1 (-\alpha_o,\beta_o,-\gamma_o)$$

$$+ V_{x+}^2 I_3 (-\gamma_o,\beta_o,\alpha_o) - V_{x-}^2 I_3 (-\gamma_o,-\beta_o,-\alpha_o) - V_{y+}^2 I_2 (-\gamma_o,-\alpha_o,\beta_o)$$

$$+ V_{y-}^2 I_3 (-\gamma_o,\alpha_o,-\beta_o)] + \alpha_o [-V_{z+}^2 I_2 (\alpha_o,\beta_o,\gamma_o)$$

$$- V_{z-}^2 I_2 (-\alpha_o,\beta_o,-\gamma_o) - V_{x+}^2 I_2 (-\gamma_o,\beta_o,\alpha_o) + V_{x-}^2 I_2 (-\gamma_o,-\beta_o,-\alpha_o)$$

$$- V_{y+}^2 I_3 (-\gamma_o,-\alpha_o,\beta_o) + V_{y-}^2 I_3 (-\gamma_o,\alpha_o,-\beta_o)] \Big\} \quad .$$

This is the final exact expression for the primary torques (with no $\Delta d/d_o$ terms) in terms of an arbitrary rotor shape $r(\theta')$, and it does not rely on a harmonic expansion. The $V_i$'s can be expressed in terms of $f_i$ and $h_i$ directly from equations (7) and (8) as

$$V_{i\pm}^2 = \frac{d_o^2}{\varepsilon_o r_o^2} \left[ \pm f_i + \frac{1}{2} \left( h_i + \frac{f_i^2}{h_i} \right) \right] \frac{m}{\pi \sin^2 \theta_1} \quad . \qquad (9)$$

We have written a computer program (see Appendix) to evaluate these integrals numerically for an $r(\theta')$ which we can take directly from a roundness chart. The program does the double integrals using an 8-point Newton Coates quadrature formula with 97 divisions. It has been checked with the analytic expressions for the second and third harmonics in Table 1 and found to be accurate to four significant figures. The $r(\theta')$ is Fourier analyzed $(0 \le \theta' \le \pi)$, and the Fourier cosine series coefficients up to the 20th are used to reconstruct $dr(\theta')/d\theta'$. Thus, we expect harmonics up to the 20th to be accounted for.

## VI. NUMERICAL RESULTS

A number of computer runs were undertaken to establish the drift rates for various values of the parameters involved. The three sample rotors were used as previously described. To establish the preload dependence, a series of computations were made for a rotor orientation of $\theta_o = 45°$, $\phi_o = 0°$, where $\theta_o$ is the angle from the electrode axis and $\phi_o$ is the azimuthal angle.

The results are shown in Tables 5 through 8 for various levels of acceleration. The h's were taken to differ by 1 percent. The results indicate that the drifts are proportional to the preload for the lower values of f. The drifts indicate the general order of magnitude to be expected for arbitrary rotor orientation. The general level of drift was several hundred milliarc sec/yr for $h \sim 10^{-6}$ g.

TABLE 5. DRIFT RATES

($\theta_o = 45°$, $\phi_o = 0°$, $f \sim 10^{-10}$ g, $\Delta h = 0.01$ h)

| h(g's) | Case I | Case II (milliarc sec/yr) | Case III |
|---|---|---|---|
| $10^{-6}$ | 86.0 | 291.0 | 202.0 |
| $10^{-7}$ | 8.6 | 29.1 | 20.4 |
| $10^{-8}$ | 0.89 | 3.01 | 2.30 |

TABLE 6. DRIFT RATES

($\theta_o = 45°$, $\phi_o = 0°$, $f \sim 10^{-9}$ g, $\Delta h = 0.01$ h)

| h(g's) | Case I | Case II (milliarc sec/yr) | Case III |
|---|---|---|---|
| $10^{-6}$ | 86.0 | 291.0 | 204.0 |
| $10^{-7}$ | 8.9 | 30.0 | 23.0 |
| $10^{-8}$ | 2.5 | 8.3 | 8.5 |

**TABLE 7. DRIFT RATES**

($\theta_o$ = 45°, $\phi_o$ = 0°, f ∿ $10^{-8}$ g, Δh = 0.01 h)

| h (g's) | Case I | Case II | Case III |
|---|---|---|---|
| | (milliarc sec/yr) | | |
| $10^{-6}$ | 89.0 | 300.0 | 210.0 |
| $10^{-7}$ | 25.0 | 84.0 | 85.0 |
| $10^{-8}$ | 150.0 | 505.0 | 370.0 |

**TABLE 8. DRIFT RATES**

($\theta_o$ = 45°, $\phi_o$ = 0°, f ∿ $10^{-7}$ g, Δh = 0.01 h)

| h (g's) | Case I | Case II | Case III |
|---|---|---|---|
| | (milliarc sec/yr) | | |
| $10^{-6}$ | 254.0 | 839.0 | 850.0 |
| $10^{-7}$ | 1497.0 | 5053.0 | 3705.0 |

Next the relative effect of the higher harmonics was investigated by running the program for each harmonic separately normalized to a harmonic coefficient of $a_n$ = 1 μin. One sees from Table 9 that the higher harmonics do not decrease in importance very quickly. Combining this with Figures 4, 5, and 6, we see that the $a_n$ above the second do not drop off very quickly, so that the higher harmonics may be important and should be included in the calculation.

**TABLE 9. DRIFT RATES (f ∿ $10^{-9}$ g, h ∿ $10^{-6}$ g, $a_n$ = 1 μin.)**

| Harmonic Number | Δh = 0.01 h | Δh = 0 |
|---|---|---|
| | (milliarc sec/yr) | |
| 2 | 77.0 | 0.03 |
| 3 | 20.0 | 20.0 |
| 4 | 19.0 | 0.004 |
| 5 | 18.0 | 18.0 |
| 6 | 50.0 | 0.01 |
| 7 | 30.0 | 30.0 |
| 8 | 16.0 | 0.006 |
| 9 | 9.7 | 9.7 |
| 10 | 56.0 | 0.01 |
| 11 | 11.0 | 11.0 |
| 12 | 15.0 | 0.003 |
| 13 | 3.1 | 3.1 |

Finally, the factor $\zeta$ was set equal to zero for the higher harmonics, and this greatly reduced the torques for the even harmonics, as shown in Table 9. This means that the factor $\zeta$ is generally applicable and that the expression for fourth harmonic in Table 1 is probably wrong. We have not used it for computational purposes, and we include it in Table 1 to illustrate the complexity of the calculations involving higher harmonics.

To summarize, these results indicate that for the rotors considered, a preload $h \sim 10^{-6}$ g (determined primarily by roll rate), and the orientation planned for the final experiment ($\theta_o = 45°$, $\phi_o = 0$), the drifts are of order several hundred milliarc sec/yr, two orders of magnitude larger than the experiment goal of 1 milliarc sec/yr. To reach the design goal of the experiment one must do one or a combination of three things: (1) reduce the preload, (2) align the rotor with the electrode axis (see Section VI), or (3) rely on roll averaging of torque.

While item 2 is a valid solution, it is probably too difficult to implement given the increased difficulty of reading out the gyro due to the presence of electrodes in the readout loop. Item 3 is also probably a valid solution. This is discussed in Section X.

We notice that for the second harmonic term in Table 1 and $\alpha_o = \gamma_o = 1/\sqrt{2}$ we have

$$\Omega = \frac{T}{I\omega} \sim \frac{1}{2\pi \sin^2 \theta_1} \frac{(2.7)\ (a_2)}{(2/5)\ r_o} \frac{\zeta h}{V} \alpha_o \gamma_o \sim 150 \text{ milliarc sec/yr}$$

for $a_2 = 2\ \mu$in. (due to centrifugal distortion; see Reference 4), $h = 10^{-6}$ g, $\zeta = 0.01$, $\omega = 200$ cps. So just the second harmonic gives torques of the magnitude shown in Tables 5 through 10.

The current state of the art in electrostatic gyros is of order $3 \times 10^6$ milliarc sec/yr in a 1-g environment. Scaling up the results of this section by $10^6$, we obtain drift rates of $10^8$ milliarc sec/yr in a 1-g environment. Thus, the state-of-the-art rotors considered here will not have the 1-g performance of existing electrostatic gyros. This is because the current electrostatic gyros produced by Honeywell rely on gimbaling, ball balancing, centrifugal distortion compensation, and possibly computer modeling to reduce torques.

## VII. TORQUE ON A GIMBALED GYRO INCLUDING ALL THE HARMONICS

We can evaluate equation (6) without resorting to a Fourier expansion for the case $\gamma_o = 1$, $\alpha_o = \beta_o = 0$. In this case we have $\sin\theta = \sin\theta'$; so, neglecting the $\Delta d/d_o$ term, we obtain

$$\vec{T} = \frac{\epsilon_o r_o^2}{2 d_o^2} \sum_i V_i^2 \iint_{S_i} [\beta \hat{F}_x - \alpha \hat{F}_y] \frac{dr(\theta)}{d\theta}\, d\theta\, d\phi \quad .$$

For the z+ electrode we get

$$\vec{T}_{z+} = \frac{\epsilon_o r_o^2}{2 d_o^2} V_{z+}^2 \left[ \hat{F}_x \int_0^{\theta_1} d\theta \int_0^{2\pi} d\phi \sin\theta \sin\phi \frac{dr(\theta)}{d\theta} \right.$$

$$\left. - \hat{F}_y \int_0^{\theta_1} d\theta \int_0^{2\pi} d\phi \sin\theta \cos\phi \frac{dr(\theta)}{d\theta} \right] = 0 \quad .$$

Similarly, for the z- electrode, making the substitution $\alpha \to -\alpha$ $\gamma \to -\gamma$, we have

$$\vec{T}_{z-} = 0 \quad .$$

For the x+ electrode we have $\alpha \to \gamma$, $\gamma \to -\alpha$; so

$$\vec{T}_{x+} = \frac{\epsilon_o r_o^2}{2 d_o^2} V_{x+}^2 \left[ \hat{F}_x \int_0^{2\pi} d\phi \int_0^{\theta_1} \sin\theta\, d\theta \left( \frac{\sin\theta \sin\phi\, dr(\theta')/d\theta'}{\sqrt{\cos^2\theta + \sin^2\theta \sin^2\phi}} \right) \right.$$

$$\left. - \hat{F}_y \int_0^{2\pi} d\phi \int_0^{\theta_1} \sin\theta\, d\theta \left( \frac{\cos\theta\, dr(\theta')/d\theta'}{\sqrt{\cos^2\theta + \sin^2\theta \sin^2\phi}} \right) \right] .$$

where

$$\theta' = \sin^{-1}\sqrt{\cos^2\theta + \sin^2\theta\,\sin^2\phi} \quad .$$

This follows since the Jacobian of the transformation is $\sin\tilde{\theta}/\sin\theta$ [1] and

$$\sin\theta = \sqrt{\alpha^2 + \beta^2} \quad .$$

Therefore, making these substitutions gives us the radical in the denominator. Similarly, $\vec{T}_{x-} = -\vec{T}_{x+}$ so

$$\vec{T}_x = M_x\,[\hat{F}_x\,I_1 - \hat{F}_y\,I_2] \quad ,$$

where

$$I_1 = \int_0^{2\pi} d\phi \int_0^{\theta_1} \sin\theta\,d\theta \left(\frac{\sin\theta\,\sin\phi\,dr(\theta')/d\theta'}{\sqrt{\cos^2\theta + \sin^2\theta\,\sin^2\phi}}\right)$$

$$I_2 = \int_0^{2\pi} d\phi \int_0^{\theta_1} \sin\theta\,d\theta \left(\frac{\cos\theta\,dr(\theta')/d\theta'}{\sqrt{\cos^2\theta + \sin^2\theta\,\sin^2\phi}}\right) .$$

For the y+ electrode

$$\alpha \to -\beta \quad , \quad \beta \to \gamma \quad , \quad \gamma \to -\alpha$$

and

$$\vec{T}_{y+} = \frac{\varepsilon_o r_o^2}{2 d_o^2} V_{y+}^2 \left[ \hat{F}_x \int_0^{2\pi} d\phi \int_0^{\theta_1} \sin\theta \, d\theta \left( \frac{\cos\theta \, dr(\theta')/d\theta'}{\sqrt{\cos^2\theta + \sin^2\theta \sin^2\phi}} \right) \right.$$

$$\left. + \hat{F}_y \int_0^{2\pi} d\phi \int_0^{\theta_1} \sin\theta \, d\theta \left( \frac{\sin\theta \sin\phi \, dr(\theta')/d\theta'}{\sqrt{\cos^2\theta + \sin^2\theta \sin^2\phi}} \right) \right] .$$

For the y- electrode $\alpha \to \beta$ and $\beta \to -\gamma$; so clearly

$$\vec{T}_{y-} = -\vec{T}_{y+}$$

and

$$\vec{T}_y = M_y \left[ \hat{F}_x I_2 + \hat{F}_y I_1 \right] .$$

The $I_1$ integral can easily be shown to be zero by the following argument. The $\phi$ part of the integral can be written as

$$\int_0^{2\pi} f(\sin^2\phi) \sin\phi \, d\phi = \int_0^{\pi} f(\sin^2\phi) \sin\phi \, d\phi$$

$$+ \int_{\pi}^{2\pi} f(\sin^2\phi) \sin\phi \, d\phi \quad ;$$

letting $\phi = \pi + U$ in the second integral, then $\sin\phi = -\sin U$ and the latter integral becomes

$$\int_0^{\pi} f(\sin^2 U)(-\sin U) \, dU \quad ,$$

which exactly cancels the first integral. The final form of the torque is then

$$\vec{T} = I_2 \, [-M_x \hat{F}_y + M_y \hat{F}_x] \quad .$$

From Reference 1, page 137, this can be written

$$\vec{T} = \frac{I_2}{\pi \sin^2 \theta_1} \, [-\vec{K} \times \vec{F}] \quad ,$$

where $\vec{K} = \hat{F}_z$ is a unit vector in the direction of the spin axis and $\vec{F}$ is the force on the rotor given by equation (7). This is of the form of a mass unbalance torque for the component of unbalance along the spin axis. It includes all the harmonics through the integral $I_2$, which can be evaluated exactly for any given rotor. This will lead to a drift rate of magnitude

$$\Omega = \frac{|\vec{T}|}{I\omega} = \frac{I_2}{\pi \sin^2 \theta'} \; \frac{m f}{\frac{2}{5} m r_o V} = \frac{5}{2\pi \sin^2 \theta_1} \left(\frac{I_2}{r_o}\right) \frac{f}{V} \quad .$$

Using

$f = 10^{-9}$ g

$V = 2400$ cm/sec for $\omega = 200$ cps

$r_o = 0.75$ in. $\theta_1 = 30°$

we find

$$\Omega = (1.77 \times 10^{-9}) \, (I_2)$$

for $I_2$ in μin. $I_2$ has been evaluated numerically for the three sample rotors considered earlier, and the results are

| | $I_2$ |
|---|---|
| Case I | $0.963 \times 10^{-6}$ in. |
| Case II | $-0.710 \times 10^{-6}$ in. |
| Case III | $-0.995 \times 10^{-6}$ in. |

Case I $\Omega$ = 10.6 milliarc sec/yr

Case II $\Omega$ = 7.88 milliarc sec/yr

Case III $\Omega$ = 11.0 milliarc sec/yr.

The drifts are proportional to f and $1/\omega$. So for a perfectly aligned rotor we get drift rates of the order 10 milliarc sec/yr in a $10^{-9}$ g environment.

The large reduction in torques for the gimbaled case illustrates the great advantage for this orientation. This is, in fact, the orientation used for many existing electrostatic gyros and is responsible for their current excellent drift performance. These gyros use a rotor with a preferred spin axis so the axial mass unbalance torque can be minimized by balancing the ball. In fact, all the even harmonics vanish for this orientation [1]. As soon as the rotor spin axis is misaligned with the electrode axis, one picks up all the even harmonics, and these involve the relatively large preloads h. This accounts for the difference in the drifts calculated in this section and those calculated in Section VI. We can reduce the average f by drag-free control to the $10^{-10}$ g level, but h cannot be so reduced because of the roll of the spacecraft ($h \sim 10^{-6}$ g for 15 min roll period and 10 cm radius).

## VIII. SECONDARY TORQUES

The secondary torques are by definition the torques that arise from the $\Delta d/d_o$ term in equation (6). There are four sources of secondary torques [1]:

a) Rotor asphericity:

$$\Delta d = -\Delta r(\theta') \quad ,$$

where

$$\Delta r = r - r_o \quad .$$

$\theta'$ is given by

$$\theta' = \cos^{-1}(\alpha\alpha_o + \beta\beta_o + \gamma\gamma_o) \quad ,$$

so it is a complicated function of the direction cosines $\alpha, \beta, \gamma$ and $\alpha_o, \beta_o, \gamma_o$.

b) Rotor miscentering:

$$\Delta d = -\alpha x_c - \beta y_c - \gamma z_c \quad ,$$

where

$$\vec{r}_c = (x_c, y_c, z_c)$$

indicates the vector from the center of the housing to the center of mass of the rotor.

c) Electrode assembly errors:

$$\Delta d = \varepsilon \left\{ \frac{\Delta_\ell}{\sqrt{6}} (\alpha + \beta + 2\gamma) + \frac{\Delta_t}{\sqrt{2}} (\alpha - \beta) \right\} \quad .$$

where $\Delta_\ell$ is the translation of the upper hemisphere parallel to the projection of $\hat{F}_z$ on the separation plane and $\Delta_t$ is the translation perpendicular to this direction, $\varepsilon = +1$ for the upper half and $\varepsilon = -1$ for the lower half.

d) Electrode asphericity:

$$\Delta d = R(\alpha, \beta, \gamma) \quad ,$$

where R specifies the deviations from perfect sphericity of the electrodes. This requires a model for R. Honeywell [1] has computed the torques for the (a) and (b) cases up to the sixth harmonic and in case (c) for the second and third harmonics for hexahedral electrodes. Essentially no work has been done on case (d). Extracting numerical values from these formulas is difficult and, as we have seen previously, using only the lowest harmonics is misleading.

The secondary torques could be computed exactly using the method described in Section V. One defines the three integrals

$$I_1'(\alpha_o, \beta_o, \gamma_o) = \int_0^{2\pi} d\phi \int_0^{\theta_1} \sin\theta \, d\theta \left[\frac{\Delta d}{d_o} \alpha \frac{dr(\theta')/d\theta'}{\sin\theta'}\right]$$

$$I_2'(\alpha_o, \beta_o, \gamma_o) = \int_0^{2\pi} d\phi \int_0^{\theta_1} \sin\theta \, d\theta \left[\frac{\Delta d}{d_o} \beta \frac{dr(\theta')/d\theta'}{\sin\theta'}\right]$$

$$I_3'(\alpha_o, \beta_o, \gamma_o) = \int_0^{2\pi} d\phi \int_0^{\theta_1} \sin\theta \, d\theta \left[\frac{\Delta d}{d_o} \gamma \frac{dr(\theta')/d\theta'}{\sin\theta'}\right]$$

where $\Delta d$ is a function of $\alpha, \beta, \gamma$ and $\alpha_o, \beta_o, \gamma_o$. For the four previously mentioned cases, a complete expression for the torques could then be generated by suitable permutation of the direction cosines and the I's. Each case would have 36 terms. Note, however, that, except for case (a), upon carrying out the permutations listed in Section V, the I's would not transform into themselves; thus, one would not have just three basic integrals. For cases (b) and (c), one would have six basic integrals ($\Delta d$ proportional to either $\alpha, \beta$, or $\gamma$); and for case (d), one would have twelve basic integrals. This is a formidable computational task that has not been carried out as yet.

One important observation about the secondary torques is that none of them averages due to spacecraft roll. This is because they contain $\alpha$, $\beta$, and $\gamma$, and this destroys the averaging discussed in Section X even for the case of the rotor spin aligned perfectly with the spacecraft roll axis.

In lieu of a complete calculation of secondary torques, we can make the following observation. All the secondary torques are of order $\Delta d/d_o$ times the primary torques. So if we take the general level of primary torque drift to be of order 100 milliarc sec/yr, which would only require a slight rotor improvement, we see that $\Delta d/d_o$ need only be 1/100 to reduce the secondary torques to the milliarc sec/yr level. Since $d_o$ = 1500 $\mu$in., this requires a $\Delta d \sim 10$ $\mu$in. Therefore, this is the specification we will require on $\Delta d$. It is clearly satisfied for case (a). For case (b), a centering accuracy of 10 $\mu$in. is attainable, but it must be remembered that this is an absolute centering accuracy and not the stability of the centering point. The 10 $\mu$in. specification also applies to the housing centering errors and the electrode sphericity. A more definitive statement than this would require an enormous amount of numerical work. However, notice that these conclusions agree with the miscentering terms in Table 1, which are down by a factor $t/d_o$ from the primary torques for the lower harmonics. Also, the calculation in Reference 1 of electrode assembly errors bears out that they are down by a factor $\Delta/d_o$ for the lower harmonics.

There is, however, a qualification to this statement because of the fact that the secondary torques, while down by a factor $\Delta d/d_o$, may not contain the reductions due to symmetry that occur in the primary torques. This is illustrated by the miscentered third harmonic term in Table 2, which contains no $\zeta$ term and is of order 15 milliarc sec/yr for $a_3$ = 1 $\mu$in., $h = 10^{-6}$ g, $t/d_o \sim 10^{-2}$; and $\omega$ = 200 cps. The higher harmonics will add to this, and the total magnitude of the drift requires the numerical integration described previously. (The miscentered second harmonic gives an insignificant drift for these same parameters and $a_2$ = 2 $\mu$in. and $f \sim 10^{-9}$ g.) However, if the miscentering vector remains fixed with respect to the electrodes as the spacecraft rolls, then these miscentering torques will average to zero because they will cause no change in electrode voltages as the spacecraft rolls. These considerations indicate that a complete numerical integration for secondary torques is probably necessary.

## IX. ORBITAL AVERAGING OF GRAVITY GRADIENT FORCES

The purpose of this section is to compute the average gravity gradient acceleration as a function of distance from the center of mass of the satellite. The effect of roll averaging will then be computed.

The gravity gradient acceleration $\vec{f}$ is given by the equation

$$\vec{f} = \underline{V}\,\vec{\rho} \quad ,$$

where $\underline{V}$ is a matrix of the form

$$(V)_{ij} = \frac{\partial^2 V}{\partial x^i \, \partial x^j}$$

and V is the gravitational potential. $\vec{\rho}$ is the vector position of the point at which the acceleration is computed with respect to the center of mass. Taking $V = GM/r$, one easily finds that for a rectangular coordinate system

$$\underline{V} = \frac{GM}{r^5} \begin{pmatrix} 3x^2 - r^2 & 3xy & 3xz \\ 3xy & 3y^2 - r^2 & 3yz \\ 3xz & 3yz & 3z^2 - r^2 \end{pmatrix} \quad .$$

We now consider a circular orbit in the y-z plane with the roll axis in the z-direction. This implies the guide star is in the orbit plane, a situation that can be realized with a suitable choice of orbit. Then for a particular point in the spacecraft specified by $\vec{\rho}$ one has

$$\rho_z \doteq \text{const.}$$

$$\rho_y = \rho_o \cos \omega t$$

$$\rho_x = \rho_o \sin \omega t$$

where $\omega$ is the roll angular frequency. The orbital position is given by

$$y = R \cos \Omega t$$

$$z = R \sin \Omega t$$

$$x = 0$$

where $\Omega$ is the orbital angular frequency and R the orbital radius. Substituting this into the equation for $\underline{V}$ and $\vec{f}$, one obtains

$$\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \frac{GM}{R^5} \begin{bmatrix} - R^2 \rho_o \sin \omega t \\ R^2(3 \cos^2 \Omega t - 1) (\rho_o \cos \omega t) + (3 R^2 \cos \Omega t \sin \Omega t) \rho_z \\ (3 R^2 \cos \Omega t \sin \Omega t) (\rho_o \cos \omega t) + R^2(3 \sin^2 \Omega t - 1) \rho_z \end{bmatrix}$$

We can now compute the average values of these quantities, noting that

$$< \sin \omega t > = \frac{1 - \cos \omega T}{\omega T} ,$$

where T is the averaging time. Similarly,

$$< \cos^2 \Omega t \cos \omega t > = \frac{\sin \omega T}{2 \omega T} + \frac{\sin (2\Omega - \omega) T}{4 (2\Omega - \omega) T} + \frac{\sin (2\Omega + \omega) T}{4 (2\Omega + \omega) T}$$

$$< \sin \Omega t \cos \Omega t \cos \omega t > = \frac{1 - \cos (2\Omega - \omega) T}{4 (2\Omega - \omega) T} + \frac{1 - \cos (2\Omega + \omega) T}{4 (2\Omega + \omega) T}$$

$$< \cos \Omega t \sin \Omega t > = \frac{\sin^2 \Omega T}{2\Omega T}$$

$$< \cos \omega t > = \frac{\sin \omega T}{\omega T}$$

$$< \sin^2 \Omega t > = \frac{1}{2} - \frac{1}{4} \frac{\sin^2 \Omega T}{\Omega T}$$

We see that all components of $<\vec{f}>$ are of order $1/\Omega T = 2\pi/\#$ of orbits or $1/\omega T = 2\pi/\#$ revolutions and thus average to zero with the very important exception of the second term in $<f_z>$. Ignoring the term proportional to $1/\Omega T$, we have

$$<f_z> \sim \frac{GM}{2R^3} \rho_z \quad .$$

For $\rho_z \sim 10$ cm this term is of order $10^{-8}$ g. We see from Section VIII that this will make a large contribution to the torque; thus, every effort should be made to minimize it.

Notice that there are $<f_i^2>$ terms in the expression for torque, and none of these averages to zero. However, these occur in the combination $<f_i^2>/h_i$. For gravity gradient forces of order $10^{-8}$ g and $h_i \sim 10^{-6}$ g, these terms are of order $10^{-10}$ g, the same order of magnitude as the $<f_i>$ terms.

To summarize, we have shown that all gravity gradient accelerations average to zero under the combined effects of orbital and roll averaging with the exception of the component along the roll axis, and this can be minimized by placing all the gyros in the plane of the drag-free proof mass perpendicular to the roll axis.

For an elliptical orbit, one can write

$$\begin{bmatrix} <f_x>, \\ \\ <f_y> \\ \\ <f_z> \end{bmatrix} = GM \begin{bmatrix} -\rho_o < \frac{1}{r^3} \sin \omega t > \\ \\ \rho_o < \frac{3y^2-r^2}{r^5} \cos \omega t > + < \frac{yz}{r^5} > \rho_z \\ \\ \rho_o < \frac{yz}{r^5} \cos \omega t > + < \frac{3z^2-r^2}{r^5} > \rho_z \end{bmatrix},$$

where the average is now over an elliptical orbit. One can express y and z and thus r as a power series in the eccentricity e:

$$\frac{y}{a} = \cos \Omega t + \frac{1}{2} e (\cos 2 \Omega t - 3)$$

$$+ \frac{e^2}{8} (3 \cos 3 \Omega t - 3 \cos \Omega t) + \ldots .$$

$$\frac{z}{a} = \sin \Omega t + \frac{1}{2} e \sin 2 \Omega t$$

$$+ \frac{e^2}{24} (9 \sin 3 \Omega t - 15 \sin \Omega t) + \ldots$$

$$r = \sqrt{y^2 + z^2} \qquad a = \text{semi-major axis}$$

One can convince oneself then that

$$< \frac{1}{r^3} \sin \omega t > \to 0$$

$$< \frac{3y^2 - r^2}{r} \cos \omega t > \to 0$$

$$< \frac{yz}{r^5} \cos \omega t > \to 0,$$

just as in the circular orbit case. This is because one will have terms of the form

$$< \cos^m (k \Omega t) \sin^n (\ell \Omega t) \cos \omega t > ,$$

where m, n, k, and $\ell$ are integers and these terms average to zero. So, after averaging, we obtain

$$\begin{bmatrix} \langle f_x \rangle \\ \\ \langle f_y \rangle \\ \\ \langle f_z \rangle \end{bmatrix} = GM \begin{bmatrix} 0 \\ \\ \left\langle \frac{yz}{r^5} \right\rangle \rho_z \\ \\ \left\langle \frac{3z^2 - r^2}{r^5} \right\rangle \rho_z \end{bmatrix} .$$

But $\left\langle \frac{yz}{r^5} \right\rangle$ is of order $e^2/a^3$ for an elliptical orbit; so $\langle f_y \rangle$ is of order $e^2 GM \rho_z / a^3$, which is small for a nearly circular orbit. Also. $\left\langle \frac{3z^2 - r^2}{r^5} \right\rangle$ is different for a noncircular orbit. The corrections are of order e for small e. So by minimizing $\rho_z$ we essentially eliminate the the average force in a noncircular orbit also.

While the z component of gravity gradient acceleration does not average to zero, it will not cause substantial torque on the gyro aligned with the z-axis. This can be seen by the following argument. The voltages needed to cancel the z-component of acceleration are such that for each point on the rotor the voltage is the same as for a point opposite on a line drawn through the spin axis. The torque due to the original point thus precisely cancels the torque due to the opposite point. This can be seen explicitly for the third and fifth harmonics in Table 1 which vanish for $\alpha_o = \gamma_o = 1/\sqrt{2}$, $\beta_o = 0$, $M_x = M_z$, $M_y = 0$, the case appropriate for the acceleration and spin aligned with the z-axis. The even harmonics will still be there, but these are primarily due to asymmetries in the preload that would be there in any case. The non-zero $\langle f_y \rangle$ for an elliptical orbit will cause a torque on the gyro aligned with the z-axis. This acceleration is of order

$$\frac{GM}{R^3} e^2 \rho_z$$

and can be reduced to the $10^{-10}$ g level for $e \sim .1$. Thus, the non-zero average value of the z-component of gravity gradient acceleration will not significantly affect the gyro aligned with the z-axis from which the relativity information is extracted.

## X. AVERAGING OF TORQUES DUE TO SPACECRAFT ROLL

In the final version of the gyro experiment, the spacecraft will be rolled about an axis through the guide star. This is necessary to achieve the required readout accuracy, but it is also effective in reducing the primary electrical torques on the gyro. In this section, we evaluate the effect of averaging on the torques analytically.

Consider first the case in which the gyro spin axis is perfectly aligned with the roll axis. We know that the torque is perpendicular to the roll axis. Then, if everything else is constant, the tip of the torque vector will describe a circle, and the average of the torque over one revolution will be precisely zero.

To prove this analytically we note that equation (6) is also valid in a coordinate system with the roll axis as the z-axis. Now consider an element of electrode surface described by the direction cosines $\alpha$, $\beta$, $\gamma$. As the spacecraft rolls, these will vary in such a way that

$$\alpha = \sin\theta'' \cos \omega t$$

$$\beta = \sin\theta'' \sin \omega t$$

$$\gamma = \cos \theta''$$

where $\theta''$ remains constant. The rotor spin axis is at the point midway between two adjacent electrodes; in the present coordinates this means

$$\alpha_o = \beta_o = 0.$$

Hence $\theta' = \theta'' =$ constant. Thus, if the $V_i$'s remain constant in equation (6) then averaging over one roll we have

$$\langle\alpha\rangle = 0 \quad \langle\beta\rangle = 0 \quad .$$

Since this holds for each element of electrode area, we have (neglecting $\Delta d/d_o$ terms)

$$\langle\vec{T}\rangle = 0$$

for one complete revolution. At any given point the torque is reduced by a factor $\sim 1/n$ of its static value, so that for $10^4$ revolutions ($\sim$1 yr for 15 min roll period) the torques are reduced by four orders of magnitude. This holds only if the $V_i$'s remain constant, which is the case for the centrifugal accelerations due to the roll. It is not true for other accelerations, but these are taken care of by the drag-free proof mass and the gravity gradient acceleration averaging as described in Section IX. The averaging also does not hold for the secondary torques ($\Delta d/d_o$ terms), since these terms may contain, $\alpha$, $\beta$ and $\gamma$ and thus destroy the averaging. An important example of this is miscentering torques (see Section VIII).

If the rotor spin axis is not precisely aligned with the roll axis, then $\alpha_o \neq 0$, $\beta_o \neq 0$ and we get terms which do not average to zero, since $\gamma\alpha_o$ and $\gamma\beta_o$ terms are present, and also $\theta$ now varies as the spacecraft rolls. These terms are very difficult to compute, but we see they are of order $\alpha_o$ or $\beta_o$ times the primary torques. For example, for a 10 arc sec misalignment of the rotor spin axis, $\alpha_o$ and $\beta_o$ are of order $5 \times 10^{-5}$, so the nonaveraged part of the torques are reduced by this factor relative to the static primary torques.

Thus, we conclude that roll averaging is extremely effective in reducing torques provided the spin axis can be kept aligned closely to the roll axis. This statement applies only to the primary torques and requires that the preloads remain nearly constant during one roll period.

We can thus see that the effect of rolling the spacecraft is to eliminate the first $h_i$ term in equation (9); therefore, we have run our program with this term set equal to zero. The results are shown in Table 10. They indicate that the drift rates can be reduced to the milli-arc sec/yr level for $f \sim 10^{-9}$ g for the rotors considered here. We also note that the higher harmonics are not very significant. This was also the case for the torques before averaging when the relatively large second harmonic due to centrifugal distortion dominated the torque expression. This term is of order 2 $\mu$in. for $\omega$ = 200 cps and was included in the previous calculations of Tables 5 through 8. We conclude that milliarc sec/yr drifts are feasible for $f \sim 10^{-9}$ g after averaging due to roll is accounted for. Roll averaging of secondary torques could be accounted for also by setting $h_i = 0$ in equation (9) and performing the integrations described in Section VIII.

TABLE X. DRIFT RATES AFTER AVERAGING

| Average f (g's) | Case I | Case II | Case III |
|---|---|---|---|
| | | (milliarc sec/yr) | |
| $10^{-8}$ | 11.0 | 34.0 | 86.0 |
| $10^{-9}$ | 0.89 | 1.2 | 4.4 |
| $10^{-10}$ | 0.089 | 0.12 | 0.44 |

## XI. SPIN AVERAGING

Honeywell [1] has given a proof that if the rotor is spinning sufficiently fast, then one can assume that the rotor is axially symmetric with a shape given by

$$\tilde{r}(\theta) = \frac{1}{2\pi} \int_0^{2\pi} r(\theta,\phi)\, d\phi,$$

where $r(\theta,\phi)$ is the actual ball shape in terms of $\theta$ and $\phi$. This is valid provided

$$\frac{d\tilde{r}(\theta)}{d\theta} \gg \frac{1}{\pi d_o} \int_0^{2\pi} \Lambda(\theta',\zeta) \frac{\partial \Delta(\theta',\zeta)\, d\zeta}{\partial \theta'} \quad ,$$

where

$$\Delta(\theta,\phi) = r(\theta,\phi) - \tilde{r}(\theta).$$

Reference 1 refers to this as a "locally smooth" condition. To obtain the appropriate input for the torque calculation, one must compute $\tilde{r}(\theta)$. This requires a complete map of the ball in $\theta$ and $\phi$, which is not currently available. For the purposes of this report, we have used $r(\theta,\phi_o)$ for $0 < \theta < \pi$ and some $\phi_o$ instead of $\tilde{r}(\theta)$. Undoubtedly some averaging occurs in the integral of $\tilde{r}(\theta)$ that will improve the effective smoothness of the ball, but it is difficult to estimate how much until a complete map of the ball is available. A numerical integration to determine $\tilde{r}(\theta)$ will then be required.

## XII. CONCLUSION

The main conclusion of this report is that Newtonian electrical torques permit drift rates of order several hundred milliarc sec/yr with preloads of $10^{-6}$g and existing rotors. The preload setting is determined by safety considerations, and the roll rate, and correspondingly better drift rates are possible with lower preload settings. Roll averaging will reduce the primary torques to well below the 1 milliarc sec/yr level. The secondary torques are at the 1 milliarc sec/yr level for existing rotors, assuming a fractional uniformity of rotor electrode gap of 1/100. The gravity gradient forces average out when combined with the spacecraft roll except for the component along the roll axis. This component, however, does not produce significant torque on the gyro aligned with the roll axis from which the relativity information is extracted. The effects due to spin averaging of the rotor asphericity may lower the drift rates calculated here, but this calculation awaits a complete map of the rotor. The overall conclusion is that, from the point of view of electrical torque calculations, an accuracy of 1 milliarc sec/yr drift is a feasible goal for the gyro experiment.

## REFERENCES


1. Matchett, G.A.: Honeywell Report 20831FR, Vol. II, NASA Contract NAS-12-542, November 1968.
2. Everitt, C.W.F.: Conference on Experimental Tests of Gravitational Theories, November 1970, JPL TM33499.
3. Spencer, Tom: Ball Brothers Contract Report F7303.
4. Eby, P.: NASA TMX-64964, November 1975.

# APPENDIX

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```
*DIAGNOSTIC* THE NAME [illegible] APPEARS IN A DIMENSION OR TYPE STATEMENT BUT IS NEVER REFERENCED.
        10          DIMENSION [illegible]
        20          COMMON/NAME10/P(20)
        30          DIMENSION [illegible]
        40          DIMENSION [illegible]
        50          DIMENSION [illegible]
        60          COMMON/NAME2/[illegible]
        70          COMMON/NAME1/[illegible]
        80          COMMON/NAME11/[illegible]
        90          COMMON/NAME13/[illegible]
       100          COMMON/NAME14/[illegible]
       110          READ(5,103) [illegible]
       120      103 FORMAT(I2,[illegible])
       130          READ(5,100) [illegible]
       140      100 FORMAT(6F10.0)
       150          [illegible]
       160          PI=3.1415926
       170          [illegible]
       180          DO 75 N=1,12
       190          [illegible]
       200       75 CONTINUE
       210          DO 333 I=1,26
       220      333 [illegible]
       230          WRITE(6,290) [illegible]
       240          WRITE(6,290) [illegible]
       250          DO 80 I=1,26
       260       80 [illegible]
       270      290 FORMAT(8F10.5)
       280          DO [illegible] N=1,26
       290      [illegible] [illegible]
       300          DO 86 J=1,25
       310          [illegible]
       320          [illegible]
       330       86 CONTINUE
       340          [illegible]
       350          [illegible]
       360          [illegible]
       370          [illegible]
       380          DO 1333 N=1,12
       390          LOOP=0
       400          [illegible]
       410          DO 1000 [illegible]
       420          IF(LOOP.EQ.0) GO TO 2000
       430          [illegible]
       440     2000 [illegible]
       450          LOOP [illegible]
       460     1000 CONTINUE
       470          [illegible]
       480     1333 CONTINUE
       490          [illegible]
       500          [illegible]
       510          [illegible]
       520          [illegible]
       530          DO 1330 N=1,12
       540          LOOP=0
       550          [illegible]
       560          DO 1005 [illegible]
```

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OF POOR QUALITY

```
00235        IF(LPOP.EQ.0) GO TO 2445                                      003227
00237        IN(J)=IN(J-1)+DIL(K)                                           000233
00240   2445 J=J+1                                                          000236
00241        LPOP =1                                                        000240
00242   1445 CONTINUE                                                       000243
00244        J=J-1                                                          000243
00245   1334 CONTINUE                                                       000250
00247        THE=2.*PI*40. /360.                                            000250
00250        PHI=5.                                                         000255
00251        A=SIN(THE)*COS(PHI)                                            000257
00252        B=SIN(THE)*SIN(PHI)                                            000270
00253        C=COS(THE)                                                     000275
00254        DO 70 J=1,20                                                   000304
00257        F=J                                                            000304
00260        A1=0.                                                          000311
00261        DO 43 K=1,25                                                   000316
00264        AK=K                                                           000322
00265        A1   =A1    +(2./PI)*((A1(K)/(F *F  ))*(COS(F*XI(K+1))-COS(F*XI(K)) 003325
00265       *)+((YI(K+1)/F)*SIN(F*XI(K+1))-(YI(K)/F)*SIN(F*XI(K))))         003325
00266     43 CONTINUE                                                       003367
00270        P(J)=A1                                                        000367
00271     70 CONTINUE                                                       000372
00273        P(1)=0.                                                        000372
00274        P(2)=P(2)-2.54*2.1E-06                                         003373
00275    228 FORMAT(E20.6)                                                  003376
00276        FX=1.5E-07                                                     000376
00277        HX=1.0E-03                                                     003400
00300        FY=1.0E-07                                                     000402
00301        HY=.99E-03                                                     000404
00302        FZ=2.0E-07                                                     000406
00303        HZ=1.01E-03                                                    000410
00304        RM=63.                                                         000412
00305        FAC=RM/(2.*PI*SIN(THETA)**2)                                   000414
00306        VX1=FAC*(FX+(HX*FX*FX/HX)/2.)                                  000425
00307        VX2=FAC*(-FX+(HX*FX*FX/HX)/2.)                                 000436
00310        VY1=FAC*(FY+(HY*FY*FY/HY)/2.)                                  000442
00311        VY2=FAC*(-FY+(HY*FY*FY/HY)/2.)                                 003452
00312        VZ1=FAC*(FZ+(HZ*FZ*FZ/HZ)/2.)                                  000456
00313        VZ2=FAC*(-FZ+(HZ*FZ*FZ/HZ)/2.)                                 000465
00314        TX=C*(VZ1*F2(A,B,C)+VZ2*F2(-A,B,-C)+VX1*F2(-C,B,A)-VX2*F2(-C,-B,-A 000475
00314       1)+VY1*F3(-C,-A,B)-VY2*F3(-C,A,-B))                             000475
00314       2+B*(-VZ1*F3(A,B,C)+VZ2*F3(-A,B,-C)+VX1*F1(-C,B,A)+VX2*F1(-C,-B,-A) 003475
00314       3+VY1*F1(-C,-A,B)+VY2*F1(-C,A,-B))                              000475
00315        TX=TX/10                                                       000706
00316        WRITE(6,228) TX                                                000710
00321        TY=A*(VZ1*F3(A,B,C)-VZ2*F3(-A,B,-C)-VX1*F1(-C,B,A)-VX2*F1(-C,-B,-A) 000716
00321       1-VY1*F1(-C,-A,B)-VY2*F1(-C,A,-B))                              003716
00321       2+C*(-VZ1*F1(A,B,C)+VZ2*F1(-A,B,-C)-VX1*F3(-C,B,A)+VX2*F3(-C,-B,-A) 000716
00321       3+VY1*F2(-C,-A,B)-VY2*F2(-C,A,-B))                              000716
00322        TY=TY/10                                                       001126
00323        WRITE(6,228) TY                                                001130
00326        TZ=B*(VZ1*F1(A,B,C)-VZ2*F1(-A,B,-C)+VX1*F3(-C,B,A)-VX2*F3(-C,-B,-A 001136
00326       2)-VY1*F2(-C,-A,B)+VY2*F2(-C,A,-B))                             001136
00326       3+A*(-VZ1*F2(A,B,C)-VZ2*F2(-A,B,-C)-VX1*F2(-C,B,A)+VX2*F2(-C,-B,-A) 001136
00326       4-VY1*F3(-C,-A,B)+VY2*F3(-C,A,-B))                              001136
00327        TZ=TZ/10                                                       001350
00330        WRITE(6,228) TZ                                                001352
```

```
00333     1140        TOT=SQRT(TX*TX+TY*TY+TZ*TZ)                    001360
00379     1150        WRITE(6,220) TOT                               001375
00377     1160    667 CONTINUE                                       001406
00390     1170    668 CONTINUE                                       001406
00391     1180        DO77 J=1,97                                    001404
00399     1190        FNC(J)=COS(25.0*T(J))                          001406
00395     1200     77 CONTINUE                                       001415
00397     1210        CALL NEWCO                                     001415
00390     1220        WRITE(6,220) DSPTT                             001417
00393     1230        STOP                                           001425
00394     1240        END                                            001431

   ESD OF COMPILATION:        3 DIAGNOSTICS.
```

```
@FOR,IS NEWCO
MSA  E3 -01/22/79-17:22:39 (,0)

   SUBROUTINE NEWCO     ENTRY POINT 000066

   STORAGE USED: CODE(1) 000077; DATA(0) 000037; BLANK COMMON(2) 000000

    COMMON BLOCKS:

    0003   NAME13 000156
    0004   NAME16 000002

   EXTERNAL REFERENCES (BLOCK, NAME)

    0005   NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001   000005 1I06       0000 R 000003 AF1      0000 R 000004 AF2      0000 R 000005 AF3      0000 R 000006 AF4
    0000 R 000007 AF5        0004   000001 B262     0003 R 000142 DEL      0003 R 000000 DSDTT    0000 R 000002 FACT
    0003 R 000001 FNC        0000   000020 INJP$    0000 I 000000 JJ       0000 I 000001 K        0004   000000 TL

00101      1*         SUBROUTINE NEWCO                                                000000
00103      2*         COMMON/NAME13/DSDTT,FNC(97),DEL(12)                             000000
00104      3*         COMMON /NAME 16/ TL,B262                                        000000
00105      4*         JJ=1                                                            000000
00106      5*         DSDTT=0.                                                        000001
00107      6*         DO 1 K=1,12                                                     000005
00112      7*         FACT=4.*DEL(K)/19175.                                           000005
00113      8*         JJ=JJ+8                                                         000010
00114      9*         AF1=FNC(JJ-8) + FNC(JJ)                                         000014
00115     10*         AF2=FNC(JJ-7) + FNC(JJ-1)                                       000017
00116     11*         AF3=FNC(JJ-6) + FNC(JJ-2)                                       000022
00117     12*         AF4=FNC(JJ-5) + FNC(JJ-3)                                       000025
00120     13*         AF5=FNC(JJ-4)                                                   000030
00121     14*         DSDTT=DSDTT + FACT*(989.*AF1+5888.*AF2-928.*AF3+10496.*AF4      000032
00121     15*        1 -4540. *AF5)                                                   000032
00122     16*       1 CONTINUE                                                        000047
00124     17*         RETURN                                                          000047
00125     18*         END                                                             000076

       END OF COMPILATION:       NO  DIAGNOSTICS.
```

```
@FOR,IS F1
HSA  F1 -01/22/79-17:22:39 (,0)

   FUNCTION F1          ENTRY POINT 000221

   STORAGE USED: CODE(1) 000292; DATA(0) 000209; BLANK COMMON(2) 000000

    COMMON BLOCKS:

    0003   NAME1   000161
    0004   NAME10  000024
    0005   NAME13  000172
    0006   NAME11  044602
    0007   NAME2   000155
    0010   NAME14  000014

   EXTERNAL REFERENCES (BLOCK, NAME)

    0011   NEWCO
    0012   COS
    0013   SIN
    0014   ACOS
    0015   NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001     000002 1136     0001     000011 1206     0001     000022 1236     0001     000113 1336     0001     000144 1506
    0001     000171 1556     0000 R 000152 AJ         0010 R 000000 DAL        0005 R 000192 DEL        0007 R 000191 DIL
    0005 R 003000 DSDTT      0000 R 000147 E          0005 R 000001 FNC        0000 R 000000 F1         0000 R 000001 GNC
    0000 I 000145 I          0000   000157 14JPS      0000 I 000151 J          0000 I 000144 JM         0000 I 000143 JL
    0000 I 000142 M          0000 I 000154 KM         0000 I 000146 L          0000 I 000153 LR         0004 R 000000 P
    0006   000000 R1         0000 R 000150 S          0006 R 022301 S1         0003 R 000000 TM         0007 R 000000 TN
    0005   000156 VSS

00101      10*            FUNCTION F1(A,B,C)                                              000002
00103      20*            COMMON/NAME1/TM(97)                                             000002
00104      30*            COMMON/NAME10/P(20)                                             000002
00105      40*            COMMON/NAME13/DSDTT,FNC(97),DEL(12),VSS(12)                     000002
00106      50*            COMMON/NAME11/R1(97,97),S1(97,97)                               000002
00107      60*            COMMON/NAME2/TN(97),DIL(12)                                     000002
00110      70*            COMMON/NAME14/DAL(12)                                           000002
00111      80*            DIMENSION GNC(97)                                               000002
00112      90*            DO 878 M=1,12                                                   000002
00115     100*       878  DEL(M)=DAL(M)                                                   000002
00117     110*            DO 90 JL=1,97                                                   000011
00122     120*            DO80 JM=1,97                                                    000022
00125     130*            I=JM                                                            000030
00126     140*            L=JL                                                            000040
00127     150*            E=A*SIN(TM(I))*COS(TN(L))+B*SIN(TM(I))*SIN(TN(L))+C*COS(TM(I))  000042
00130     160*            E=ACOS(E)                                                       000077
```

```
00131        S=0.                                                                003103
00132        DO 70 J=1,20                                                        000113
00135        AJ=J                                                                003113
00136        S    =S    -J*P(J)*SIN(AJ*E)                                        003116
00137     70 CONTINUE                                                            000136
00141        S1(I,L)=S                                                           003136
00142     80 FNC(JM)=S1(JK,JL)*SIN(TM(JK))*COS(TM(JL))*SIN(TM(JK))/SIN(E)       000140
00144        CALLNEWCO                                                           000153
00147     90 GNC(JL)=DSDTT                                                       000155
00150        DO97 LM=1,12                                                        000164
00152     97 DFL(LM)=DIL(LK)                                                     000164
00155        DO 98 MM=1,97                                                       003171
00157     98 FNC(MM)=GNC(KM)                                                     000171
00161        CALL NEWCO                                                          000173
00162        F1=DSDTT                                                            000175
00163        RETURN                                                              003177
00164        END                                                                 000241
```

END OF COMPILATION:        NO DIAGNOSTICS.

```
@FOR,IS F2
MSA  E3 -01/22/79-17:22:43 1,0)

FUNCTION F2          ENTRY POINT 000721

STORAGE USED: CODE(1) 000242; DATA(0) 000204; BLANK COMMON(2) 000000

COMMON BLOCKS:

0003   NAME1   000141
0004   NAME10  000024
0005   NAME13  000172
0006   NAME11  044602
0007   NAME2   000155
0010   NAME14  000014

EXTERNAL REFERENCES (BLOCK, NAME)

0011   NERCO
0012   SIN
0013   COS
0014   ACOS
0015   NERR3$

STORAGE ASSIGNMENT   (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0001     000002 113G      0001     000011 120G      0001     000022 123G      0001     000113 133G      0001     000164 150G
0001     000171 155G      0000 R 000152 AJ        0010 R 000000 DAL       0005 R 000142 DEL       0007 R 000141 DIL
0005 R 000000 DSDTT       0000 R 000147 E         0005 R 000001 FNC       0000 R 000000 F2        0000 R 000001 GNC
0000 I 000145 I           0000     000157 INJP$   0000 I 000151 J         0000 I 000144 JK        0000 I 000143 JL
0000 I 000142 K           0000 I 000154 KM        0000 I 000146 L         0000 I 000153 LK        0004 R 000000 P
0006     000000 R1        0000 R 000150 S         0006 R 022301 S1        0003 R 000000 TM        0007 R 000000 TN
0005     000156 VSS

00101     1*          FUNCTION F2(A,B,C)                                        000002
00103     2*          COMMON/NAME1/TM(97)                                       000002
00104     3*          COMMON/NAME10/P(20)                                       000002
00105     4*          COMMON/NAME13/DSDTT,FNC(97),DFL(12),VSS(12)               000002
00106     5*          COMMON/NAME11/R1(97,97),S1(97,97)                         000002
00107     6*          COMMON/NAME2/TN(97),DIL(12)                               000002
00110     7*          COMMON/NAME14/DAL(12)                                     000002
00111     8*          DIMENSION GNC(97)                                         000002
00112     9*          DO 878 K=1,12                                             000002
00115    10*      878 DEL(K)=DAL(K)                                             000002
00117    11*          DO 90 JL=1,97                                             000011
00122    12*          DO80 JK=1,97                                              000022
00125    13*          I=JK                                                      000030
00126    14*          L=JL                                                      000040
00127    15*          E=A*SIN(TM(I))*COS(TN(L))+B*SIN(TM(I))*SIN(TN(L))+C*COS(TM(I))   000042
00130    16*          E=ACOS(E)                                                 000077
```

```
00131          S=0.                                                              000103
07132          DO 70 J=1,20                                                      000113
00135          AJ=J                                                              000113
00136          S   =S    -J*P(J)*SIN(AJ*E)                                       003116
00137       70 CONTINUE                                                          000136
00191          S1(I,L)=S                                                         003136
00147       80 FNC(JK)=S1(JK,JL)*SIN(TM(JK))*SIN(TM(JL))*SIN(TM(JK))/SIN(E)      000140
00199          CALLNEWCO                                                         000153
07145       90 GNC(JL)=DSDTT                                                     000155
00147          DO97 LK=1,12                                                      000164
00152       97 DEL(LK)=DIL(LK)                                                   000164
00154          DO 98 KM=1,97                                                     003171
00157       98 FNC(KM)=GNC(KM)                                                   000171
00161          CALL NEWCO                                                        000173
00162          F2=DSDTT                                                          000175
00163          RETURN                                                            000177
00164          END                                                               003241

      END OF COMPILATION:       NO  DIAGNOSTICS.
```

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```
@FOR,IS F3
MSA  F3 -01/22/79-17:22:49 (,0)

   FUNCTION F3            ENTRY POINT 000216

   STORAGE USED: CODE(1) 000236; DATA(0) 000201; BLANK COMMON(2) 000000

    COMMON BLOCKS:

    0003   NAME1  000191
    0004   NAME10 000024
    0005   NAME13 000172
    0006   NAME11 044602
    0007   NAME2  000155
    0010   NAME14 000014

   EXTERNAL REFERENCES (BLOCK, NAME)

    0011   NERCD
    0012   SIN
    0013   COS
    0014   ACOS
    0015   NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001   000002 113G       0001   000010 120G       0001   000013 123S       0001   000076 133G       0001   000162 150S
    0001   000167 155G       0000 R 000152 AJ         0010 R 000000 DAL        0005 R 000142 DEL        0007 R 000141 DTL
    0005 R 000000 DSOTT      0000 R 000147 E          0005 R 000001 FNC        0000 R 000000 F3         0000 R 000001 GNC
    0000 I 000145 I          0000   000157 INJP$      0000 I 000151 J          0000 I 000144 JK         0000 I 000143 JL
    0000 I 000142 K          0000 I 000154 KM         0000 I 000146 L          0000 I 000153 LN         0004 R 000000 P
    0006   000000 R1         0000 R 000150 S          0006 R 022301 S1         0003 R 000000 TH         0007 R 000000 TN
    0005   000156 VSS

00101      1*            FUNCTION F3(A,B,C)                                                000002
00103      2*            COMMON/NAME1/TH(97)                                               000002
00104      3*            COMMON/NAME10/P(20)                                               000002
00105      4*            COMMON/NAME13/DSOTT,FNC(97),DEL(12),VSS(12)                       000002
00106      5*            COMMON/NAME11/R1(97,97),S1(97,97)                                 000002
00107      6*            COMMON/NAME2/TN(97),DTL(12)                                       000002
00110      7*            COMMON/NAME14/DAL(12)                                             000002
00111      8*            DIMENSION GNC(97)                                                 000002
00112      9*            DO 878 K=1,12                                                     000002
00115     10*        878 DEL(K)=DAL(K)                                                     000002
00117     11*            DO 90 JL=1,97                                                     000013
00122     12*            DO80 JK=1,97                                                      000013
00125     13*            I=JK                                                              000013
00126     14*            L=JL                                                              000023
00127     15*            E=A*SIN(TM(I))*COS(TN(L))+B*SIN(TH(I))*SIN(TN(L))+C*COS(TM(I))    000025
00130     16*            E=ACOS(E)                                                         000062
```

```
00131      170              S=0.                                                     000066
00132      180              DO 70 J=1,20                                             000076
00135      190              AJ=J                                                     000076
00136      200              S    =S    -J*P(J)*SIN(AJ*E)                             000101
00137      210          70  CONTINUE                                                 000121
00141      220              SI(I,L)=S                                                000121
00147      230          90  FNC(JK)=SI(JK,JL)*COS(T*(JK))*SIN(TH(JK))/SIN(E)         000123
00144      240              CALLNEWCO                                                000151
00145      250          90  GNC(JL)=DSOTT                                            000153
00147      260              DO97 LK=1,12                                             000162
00152      270          97  DTL(LK)=DIL(LK)                                          000162
00154      280              DO 98 KK=1,97                                            000167
00157      290          98  FNC(KK)=GNC(KK)                                          003167
00161      300              CALL NEWCO                                               000172
00162      310              F3=DSOTT                                                 000173
00163      320              RETURN                                                   000175
00164      330              END                                                      003235

      END OF COMPILATION:        NO  DIAGNOSTICS.
```

# APPROVAL

## ELECTRICAL TORQUES ON THE ELECTROSTATIC GYRO IN THE GYRO RELATIVITY EXPERIMENT

By Peter Eby and Wesley Darbro

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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