ALTERNATIVES FOR JET ENGINE CONTROL*

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SUMMARY

The general purpose of these studies has been to evaluate alternatives to linear quadratic regulator theory in the linear case and to examine nonlinear modelling and optimization approaches for global control. Context for the studies has been set by the DYNGEN digital simulator and by models generated for various phases of the F100 Multivariable Control Synthesis Program. With respect to the linear alternatives, studies have stressed the multivariable frequency domain. Progress has been made in both the direct algebraic approach to exact model matching, by means of stimulating work on the basic computational issues, and in the indirect generalized Nyquist approach, with the development of a new design idea called the CARDIAD method. (The acronym stands for Complex Acceptability Region for DIAgonal Dominance.) With respect to nonlinear modelling and optimization, the emphasis has been twofold: to develop analytical nonlinear models of the jet engine and to use these models in conjunction with techniques of mathematical programming in order to study global control over non-incremental portions of the flight envelope. A hierarchy of models has been developed, with present work focused upon the possibility of using tensor methods. A number of these models have been used in time optimal control studies involving DYNGEN.

INTRODUCTION

The decade of the 1970s has coincided with the beginning of yet another round of substantial development in the jet engine industry. A notable factor involved with this stage of modern engine evolution has been the inevitable growing interest in better and better performance, which in turn placed more and more demands upon the application of classical hydromechanical control technique as the primary base technology for engine design. Fortunately, milestone developments in digital hardware began to offer realistic opportunities for onboard computation in ways not heretofore possible. The combination of these two events pointed the way to a concept of increasing the role of electronics in engine control. In turn, this created a variety of new possibili-

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ties for application of recent theories of control design. The F100 Multivariable Control Synthesis Program (ref. 1) sponsored by the National Aeronautics and Space Administration, Lewis Research Center, and the Air Force Aero-Propulsion Laboratory, Wright-Patterson Air Force Base, is a major example. In the linear case, the primary tool employed was linear quadratic regulator (LQR) theory; in the nonlinear case, optimal control methods were not directly applied.

The purpose of these studies has been to evaluate alternatives to LQR in the linear case and to examine nonlinear modelling and optimization for global control in the nonlinear case.

CONTEXT OF THE STUDIES

Evaluation of various theories for control alternatives has taken place using linearized models related to the F100 Multivariable Control Synthesis Program and using the DYNGEN digital simulator (ref. 2). DYNGEN has the combined capabilities of GENENG (ref. 3) and GENENG II (ref. 4), together with an added capability for calculating transient performance. The DYNGEN digital simulation is particularized to a given situation by a process of loading data for the various maps associated with a given engine. The maps for these studies have been provided by engineering personnel at Lewis Research Center. These maps correspond to a hypothetical engine which is not closely identified with any current engine. But the data do correspond in a broad, general sense to realistic two spool turbofan engines. The simulation provides for two essential controls, main burner fuel flow and jet exhaust area. Portions of the envelope which can be used for linear or nonlinear experimentation are a function of the convergence properties of the DYNGEN algorithm as interfaced with the given engine data load.

MULTIVARIABLE FREQUENCY DOMAIN STUDIES

Modern studies of control in the multivariable frequency domain display various faces in various contexts. Here it is convenient to classify these as "direct" or "indirect".

The direct approach can usually be recognized by its attention to achieving completely specified dynamic performance. The idea is classical (refs. 5-6). In fact, some of the earliest attempts to expand the direct approach to the multi-input, multi-output case involved work with jet engines (refs. 7-8). As is apparent from reference 7, there is an unfailing tendency to call these methods algebraic in nature. That tendency persists to this day, when direct approaches in multivariable applications typically involve solution for compensations described by matrices of transfer functions, with the solutions often requiring the algebra of modules over rings of polynomials or stable rational functions.

The indirect approaches are usually recognizable by their relation to the classic work of Nyquist. Here the key equation is often written in the manner

$$p_{CL}(s) = |M(s)|\, p_{OL}(s),$$

where $p_{CL}(s)$ is the closed loop characteristic polynomial, $p_{OL}(s)$ is the open loop characteristic polynomial, $M(s)$ is the matrix return difference, and $|\cdot|$ denotes determinant. This very fundamental equation permits an essential generalization of the classical Nyquist idea, for $p_{CL}(s)$ can be used to characterize the exponentials involved in closed loop control. Basically, a Nyquist plot of $|M(s)|$ tends to contain the same type of information which proved so useful in classical design. A great deal of the design effort centers upon the way in which dynamical compensation affects the determinant which acts on $M(s)$. There are three well recognized ways to study this effect. These are (1) direct construction of $|M(s)|$ by any of the known methods for determinant calculation; (2) construction of the eigenvalues of $M(s)$ as a function of s, and use of the idea that the determinant is equal to the product of its eigenvalues (ref. 9); and (3) design of compensation so that $M(s)$ is approximately diagonal, with concomitant development of a relation between the Nyquist plot of $|M(s)|$ and plots of the diagonal elements of $M(s)$, as in reference 10.

THE DIRECT APPROACH

With regard to the direct approach, a substantial case study of exact model mathcing (ref. 11) has been carried out.

The exact model matching problem can be phrased as follows. Let $R(s)$ denote the field of rational functions in s and with coefficients from the real number field R. Further, let V_1, V_2, and V_3 be finite-dimensional vector spaces over the field $R(s)$. Finally, let

$$G_1 : V_2 \rightarrow V_3$$

and

$$G_2 : V_1 \rightarrow V_3$$

be given linear transformations on one vector space to another. Then the exact model matching problem is to find linear transformations

$$G : V_1 \rightarrow V_2$$

of vector spaces, if they exist, such that

$$G_1 G = G_2.$$

In a control problem, G_1 and G_2 are functions of the plant, the complete closed loop specifications, and the configuration chosen for the controller. The unknown G embodies the dynamics involved in the controller, relative to a fixed configuration of control.

The basic plant was a version of the F100 turbofan engine. Inputs were jet exhaust area and main burner fuel flow; states were fan inlet temperature, main burner pressure, fan speed, high compressor speed, and afterburner pressure; and outputs were thrust and high turbine inlet temperature. The linearized model approximated the small signal behavior of these engine variables in a neighborhood of 47° PLA.

Insofar as the authors are presently aware, this study represents one of the most elaborate exact model matching studies undertaken to date in the literature. Moreover, it is entirely in the spirit of the introductory work in references 7-8.

Technically, the mathematical framework was set up in terms of polynomial modules. The problem formulation itself has been recorded in reference 12, where it can serve as a comparison point for future algorithms. The computer algorithms implemented were those promulgated in the literature at that time (ref. 13).

These studies established several basic conclusions relative to the direct method:

(1) the direct method was of interest in jet engine control (indeed, had been proposed in industrial studies);

(2) the jet engine control problems typical of the 1970s were of sufficient size and complexity to overtax the routine solution procedures being mentioned in the literature at that time; and

(3) a substantial influx of ideas from the literature on numerical methods would be necessary before the direct method could be applied for jet engine control.

It is a pleasure to report that these results did indeed lead to the desired influx, so that computations of sufficient accuracy can now be made in seconds. Efforts involving the direct method are now being directed at the problem of making convenient specifications.

THE INDIRECT APPROACH

Though some efforts (ref. 14) were directed toward the evaluation of the eigenvalue approach (ref. 9) to $|M(s)|$, the major attention under the indirect approach classification in these studies was directed toward the idea of designing dynamical compensation so as to make $M(s)$ approximately diagonal in a way that would be useful in Nyquist studies.

Because of the indirect way in which compensation has an effect on $|M(s)|$, Nyquist analysis of $|M(s)|$ may be of little use to the designer for other than stability determination, for even the simplest systems. In the event that $M(s)$ is diagonal, design and stability considerations are reduced to a set of single input, single output problems, with net angular behavior of $|M(s)|$ being a consequence of summing the individual net behaviors of the diagonal entries.

Rosenbrock (ref. 10) has introduced the idea of diagonal dominance, which can be regarded as an approximate form of diagonality. An mxm matrix $Z(s)$ over $R(s)$ is said to be diagonally column dominant if for all $s \varepsilon D$ the Nyquist contour and for $i = 1,2,\ldots,m$

$$|z_{ii}(s)| > \sum_{\substack{j=1 \\ j\neq i}}^{m} |z_{ji}(s)|$$

Rosenbrock shows that, if a matrix $M(s)$ is diagonally column dominant, the net angular behavior of $|M(s)|$ on D can be inferred from that of $\{m_{ii}(s)\}$ on D. Thus the class of matrices for which design and stability analysis may be performed on only the diagonal entries is expanded from diagonal matrices to matrices which are diagonally dominant.

Efforts in these studies have focused upon methods to design compensation in order to achieve diagonal dominance.

The procedure which has been developed is called the CARDIAD method, where the acronym stands for Complex Acceptability Region for DIAgonal Dominance. The CARDIAD idea can be visualized as follows. Consider a unity negative feedback configuration with the mxm plant matrix $G(s)$ preceded by an mxm compensation matrix $K(s)$, both over $R(s)$. Except for renumbering of inputs, the design of $K(s)$ to achieve diagonal dominance may be restricted to $K(s)$ matrices having the unit transfer function 1 in each main diagonal position. This fact is an easy consequence of Rosenbrock's definition. In the CARDIAD approach, a sufficient condition for dominance in the ith column of

$$M(s) = I + G(s)K(s)$$

say, at a particular frequency $s \varepsilon D$, is expressed by a quadratic inequality of the type

$$f_i(v) = \langle v,Av\rangle + \langle v,b\rangle + c > 0$$

Here v is a vector in the real space R^{2m-2}, consisting of a list of the real and imaginary parts of the off-diagonal entries in the ith column of $K(s)$ at the particular frequency $s \varepsilon D$, $\langle\cdot,\cdot\rangle$ is the usual inner product, A is an Hermitian linear map, $b \varepsilon R^{2m-2}$, and $c \varepsilon R$. A, b, and c are functions of $G(s)$.

Several different approaches are used to choose v so that $f_i(v)$ is positive. These are described in detail by references 15-22. References 15-17 deal primarily with engine models having two inputs and two outputs; reference 18 focuses on a three input/output case; and references 6-8 treat four input/output situations.

The basic idea of a CARDIAD plot is easy to understand in the two input/output case. The compensation takes a form

$$\begin{bmatrix} 1 & x_2(s) + jy_2(s) \\ x_1(s) + jy_1(s) & 1 \end{bmatrix}$$

where for $i = 1,2$

$$x_i : D \rightarrow R$$

$$y_i : D \rightarrow R$$

are the functions defining the real and imaginary parts of the off-diagonal entries in column i. The quadratic inequality can be set equal to its limiting value

$$f_i(x_i(s),y_i(s)) = 0 ,$$

which defines a circle on R^2 with coordinates (x_i,y_i). For a particular $s \varepsilon D$, a solid circle is drawn on R^2 if (x_i,y_i) pairs inside the circle satisfy the inequality; and a dashed circle is drawn on R^2 if (x_i,y_i) pairs outside the circle satisfy the inequality. As s traverses D, these circles generate a CARDIAD "plot" on R^2. The plot is essentially a set of requirements, in graphical form, which are necessary and sufficient for compensator design to achieve dominance in the configuration described above.

When $m > 2$, various additional strategies are brought into play. These are described in some detail in the references.

NONLINEAR MODELLING AND OPTIMIZATION

With respect to nonlinear modelling and optimization, the emphasis has been twofold; to develop analytical nonlinear models of the jet engine deck and to use these models in conjunction with techniques of mathematical programming in order to study global control over non-incremental reaches of the flight envelope. The context for such studies has been established by DYNGEN, as described above.

The first method of modelling which was considered was that of analytical construction of the equations from the basic physical principles. In this case, there were sixteen nonlinear differential equations, as well as a large number of nonlinear static functions which provided additional coupling among the equations. Such a procedure then requires determination of parameters in the equations. A number of these parameters have very definite physical meanings, and these meanings were supplemented by simulation data when appropriate. Obtaining tractable models for the engine in this way, though promising from the point of view of physical insight, did not lead to very much mathematical insight. Subsequently, therefore, this method gave way to the following.

The second method of modelling placed increased emphasis upon the mathematical structure of the equations, with determination of parameters being done automatically from simulator data. A highlight of this part of the study was the development of the model class

$$\dot{x} = A(x)\,(x-g(u))$$

where $x \varepsilon R^n$, $u \varepsilon R^p$. The function g is arranged so as to satisfy the set-point or steady-state features of the engine deck, while the operator

$$A : R^n \rightarrow R^n$$

is useful to adjust the transient behavior of the model. The particulars of this idea were described in reference 23.

A number of possibilities exist for approaching the approximation of $A(x)$ and $g(u)$. One additional method and application has been presented in reference 24.

At this point in time, a new stage in the nonlinear modelling studies is being initiated. In this phase, extensive use will be made of the methods of multilinear algebra, specifically the theory of algebraic tensors.

Models of the types evolved in phases one and two have been used in time-optimal control studies. Results of these efforts have been written down in references 25-27.

CONCLUDING REMARKS

This brief paper has sketched a number of control alternatives which have been studied recently in the context of the DYNGEN digital engine simulator and of linear models deriving from the F100 Multivariable Control Synthesis Program. In the linear case, these studies have focused on alternatives to the linear quadratic regulator theory employed in that Program. In the nonlinear case, emphasis has been placed on nonlinear modelling and time-optimal control.

Principal results reported have been the case study on exact model matching, which has stimulated considerable new work in that problem area, the development of the CARDIAD plot as a design tool for generalized Nyquist work, and the introduction of a nonlinear model class which is proving to be helpful in recent engine design studies.

Present thrust in this work is toward the use of multilinear algebra for generalized nonlinear modelling.

Finally, the reader may be interested in the fact that the National Engineering Consortium sponsored an International Forum on Alternatives for Linear Multivariable Control in Chicago during October 1977. Authors in that meeting were asked to address a Theme Problem based upon F100 data. Two publications resulted, one a proceedings and one a hardbound book. Reference 23 is to the proceedings, while reference 18 is to the book. Much additional information may be found in those volumes.

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