N O T I C E

THIS DOCUMENT HAS BEEN REPRODUCED FROM MICROFICHE. ALTHOUGH IT IS RECOGNIZED THAT CERTAIN PORTIONS ARE ILLEGIBLE, IT IS BEING RELEASED IN THE INTEREST OF MAKING AVAILABLE AS MUCH INFORMATION AS POSSIBLE

(NASA-TM-82024) VECTOR ANALOGUES OF THE MAGGI-RUBINOWICZ THEORY OF EDGE DIFFRACTION (NASA) 27 p HC A03/MF A01 CSCL 20N

N81-12297

Unclas
G3/32 39809

NASA

Technical Memorandum **82024**

Vector Analogues of the Maggi-Rubinowicz Theory of Edge Diffraction

Robert Meneghini, Peter Shu and John Bay

OCTOBER 1980

National Aeronautics and
Space Administration

Goddard Space Flight Center
Greenbelt, Maryland 20771

VECTOR ANALOGUES OF THE MAGGI-RUBINOWICZ THEORY OF EDGE DIFFRACTION

Robert Meneghini
Code 946

Peter Shu
Code 727

John Bay
Systems and Applied Science Corporation
Bladensburg, Maryland

October 1980

GODDARD SPACE FLIGHT CENTER
Greenbelt, Maryland 20771

VECTOR ANALOGUES OF THE MAGGI-RUBINOWICZ THEORY OF EDGE DIFFRACTION

Robert Meneghini, Peter Shu and John Bay

ABSTRACT

The Maggi-Rubinowicz technique for scalar and electromagnetic fields can be interpreted as a transformation of an integral over an open surface to a line integral around its rim. Using this transformation, Maggi-Rubinowicz analogues are found for several vector physical optics representations. For diffraction from a circular aperture, a numerical comparison between these formulations shows the two methods are in good agreement. To circumvent certain convergence difficulties in the Maggi-Rubinowicz integrals that occur as the observer approaches the shadow boundary, a variable mesh integration is used. For the examples considered, where the ratio of the aperture diameter to wavelength is about ten, the Maggi-Rubinowicz formulation yields an 8 to 10 fold decrease in computation time relative to the physical optics formulation.

VECTOR ANALOGUES OF THE MAGGI-RUBINOWICZ THEORY OF EDGE DIFFRACTION

Introduction

The theory of the edge diffraction wave originated with Thomas Young, who observed that light diffracted through an aperture could be interpreted as an unperturbed incident wave combined with a wave disturbance arising at the aperture rim [1, 2]. Starting with the Helmholtz representation, the idea was formulated in a mathematically rigorous way by Maggi and later by Rubinwicz [3]. A comprehensive review article by Rubinowicz [1] traces the development of the theory up through the work of Miyamoto and Wolf.

Although the approaches of Maggi and Rubinowicz are different [1, 2, 4], both lead to the reduction of a surface integral to a line integral (usually corresponding to the physical edge of the scatterer) plus geometrical optics terms. Miyamoto and Wolf [5] were the first to demonstrate that such a transformation could be carried out exactly for an arbitrary incident field. Although their work centered on the transformation of the Kirchhoff integral, it was later shown that a similar procedure could be carried out for the Rayleigh and Sommerfeld integral representations [6]. In a more recent paper, expressions have been derived for the field behavior near the geometrical optics shadow boundary [7]. Apart from the importance of the results themselves, they provide information on the relationship of the Maggi-Rubinowicz technique to the geometrical theory of diffraction (GTD) and to the Braunbek approximation [8, 9].

Most of the papers cited above deal with the scalar formulation. For electromagnetic diffraction problems, vector analogues of the Maggi-Rubinowicz technique have been investigated by several authors [1, 10, 11]. The application of these ideas to antenna problems, however, does not seem to have attracted much attention and most of the practical results [12-15] have been presented for typical applications in optics. An exception is a paper by Gordon [16], who obtained scattered fields from a planar reflector. In this work, however, only scalar fields were considered.

In this paper we obtain the Maggi-Rubinowicz (M-R) analogues to three commonly used physical optics (P. O.) formulations. Two of these have been derived previously by somewhat different methods than the one given here [1, 11]. We next present some numerical examples comparing one of the M-R formulations with the corresponding P. O. representation. For the simple examples considered, we show that a variable mesh integration can be used to circumvent errors that normally occur when the observer approaches the geometrical optics (g. o.) shadow boundary.

Some Maggi-Rubinowicz Representations

Using an $e^{-i\omega t}$ time convention, we list below three P. O. representations for the scattered field,

$$\underset{\sim}{E}_F = \nabla \times \int_s (\hat{n}' \times \underset{\sim}{E}_a)\, G\, dS' - \frac{1}{i\omega\epsilon} (\nabla\nabla \cdot + k^2) \int_s (\hat{n}' \times \underset{\sim}{H}_a)\, G\, dS' \tag{1}$$

$$\underset{\sim}{E}_1 = 2\, \nabla \times \int_s (\hat{n}' \times \underset{\sim}{E}_a)\, G\, dS' \tag{2}$$

$$\underset{\sim}{E}_2 = - \frac{2}{i\omega\epsilon} (\nabla\nabla \cdot + k^2) \int_s (\hat{n}' \times \underset{\sim}{H}_a)\, G\, dS' \tag{3}$$

where G is the free space Green's function

$$G = \frac{e^{ik|\underset{\sim}{x} - \underset{\sim}{x}'|}}{4\pi|\underset{\sim}{x} - x'|}$$

and where $\underset{\sim}{x}$, $\underset{\sim}{x}'$ denote the coordinates of the observation point and source point respectively. For all three formulations, the surface of integration, S, is open. Denoting a closed surface S_c as the union of the surfaces S and $\overline{S}$, where the respective integrands are assumed to be zero on $\overline{S}$, we choose $\hat{n}'$ to be the inward normal to S_c. For the fields appearing in the integrands we have used a subscript 'a' to denote the approximate nature of the fields. To simplify the notation we will omit the subscripts in the subsequent equations.

Equations (2) and (3) apply to the problem of diffraction through an aperture in a perfectly conducting plane screen [17]. The observer is limited to the shadow half space, i.e., the half space excluding the sources of the incident field. The relevant approximations are $\underset{\sim}{E}_a = \underset{\sim}{E}_o$, $\underset{\sim}{H}_a = \underset{\sim}{H}_o$ where $\underset{\sim}{E}_o$, $\underset{\sim}{H}_o$ are the incident fields with the surface of integration being taken over the aperture. The complimentary problem of the screen can be found directly from $\underset{\sim}{E}_1$, $\underset{\sim}{E}_2$ by Babinet's principle [17].

Equation (1) is often referred to as the Franz formulation [18, 19]. It is identical to the expressions given by Stratton [20] and Kottler [21, 22] despite differences in appearance from these last two. For scattering from a perfectly conducting (and in general non-planar) object, $\hat{n}' \times \underset{\sim}{E} = 0$ and $\hat{n}' \times \underset{\sim}{H}_a = 2(\hat{n}' \times \underset{\sim}{H}_o)$ where the open surface S corresponds to the illuminated portion of the scatterer.

To find the M-R analog to $\underset{\sim}{E}_F$ we first relate $\underset{\sim}{E}_F$ to the vector Kirchhoff representation, $\underset{\sim}{E}_K$, where [23, 24]

$$\underset{\sim}{E}_K = \int_s \left| \underset{\sim}{E}\,(\hat{n}' \cdot \nabla' G) - G(\hat{n}' \cdot \nabla')\,\underset{\sim}{E} \right| dS' \tag{4}$$

the integrand can be rewritten as

$$2\underset{\sim}{E}(\hat{n}' \cdot \nabla' G) - (\hat{n}' \cdot \nabla')\, G\underset{\sim}{E}$$

Using $\nabla' \cdot \underset{\sim}{E} = 0$ and the vector identity

$$(\underset{\sim}{b} \cdot \nabla)\underset{\sim}{a} = \nabla(\underset{\sim}{a} \cdot \underset{\sim}{b}) - (\underset{\sim}{a} \cdot \nabla)\,\underset{\sim}{b} - \underset{\sim}{a} \times (\nabla \times \underset{\sim}{b}) - \underset{\sim}{b} \times (\nabla \times \underset{\sim}{a})$$

then

$$(\hat{n}' \cdot \nabla')\,(G\underset{\sim}{E}) = -G(\hat{n}' \times (\nabla' \times \underset{\sim}{E})) - \hat{n}' \times (\nabla' G \times \underset{\sim}{E}) + \hat{n}'(\underset{\sim}{E} \cdot \nabla' G) - \underset{\approx}{T} \cdot \hat{n}'$$

The quantity $\underset{\approx}{T}$ is a dyad, the ji component of which is

$$T_{ji} = \hat{x}_j \cdot \underset{\approx}{T} \cdot \hat{x}_i = \delta_{ji} \frac{\partial}{\partial x_i'}(GE_i) - \frac{\partial}{\partial x_j'}(GE_i)$$

where δ_{ji} is the Kronecker delta function.

Using the relationships

$$\nabla' \times \underset{\sim}{E} = i\omega\mu\underset{\sim}{H}$$

$$\hat{n}' \times (\nabla' G \times \underset{\sim}{E}) = (\hat{n}' \cdot \underset{\sim}{E})\ \nabla' G - (\hat{n}' \cdot \nabla' G)\,\underset{\sim}{E}$$

and the identity [1]

$$\int_s \underset{\approx}{T} \cdot \hat{n}'\, dS' = \oint (G\underset{\sim}{E} \times \hat{\ell}')\, d\ell'$$

ORIGINAL PAGE IS
OF POOR QUALITY

where $\hat{\ell}'$ is a unit vector along the closed contour C directed so that $\hat{\ell}'$ and $\hat{n}'$ are related via the right hand rule, then (4) becomes

$$\underset{\sim}{E}_K = \int_s [(\hat{n}' x \underset{\sim}{E}) x \nabla' G + i\omega\mu G(\hat{n}' x \underset{\sim}{H}) + (\hat{n}' \cdot \underset{\sim}{E}) \nabla' G] \, dS' \qquad (5)$$

$$+ \oint_c G(\underset{\sim}{E} x \hat{\ell}') \, d\ell'$$

The surface integral in this equation is often referred to as the Stratton-Chu integral, which we denote by $\underset{\sim}{E}_{s-c}$. It should be noted that the derivation of $\underset{\sim}{E}_{s-c}$ from $\underset{\sim}{E}_k$ is sometimes done under the assumption of a closed surface, e.g. [24], so that $\underset{\sim}{E}_{s-c} = \underset{\sim}{E}_k$. The rim integration in (5) is therefore a consequence of assuming the surface to be open.

The relationship between $\underset{\sim}{E}_{s-c}$ and $\underset{\sim}{E}_F$ is given by [17]

$$\underset{\sim}{E}_F = \underset{\sim}{E}_{s-c} - \frac{1}{i\omega\mu} \nabla \oint_c G\underset{\sim}{H} \cdot \hat{\ell}' \, d\ell' \qquad (6)$$

This equation has been derived by Stratton [20] and Kottler [21, 22] and is obviously equal to the Franz representation, (1).

In order to express $\underset{\sim}{E}_F$ solely in terms of line integrals we first write equation (4) in the form

$$\underset{\sim}{E}_k = \hat{x}_i \int_s \underset{\sim}{V}_i \cdot \hat{n}' \, dS'$$

where the implicit sum on i runs from 1 to 3 and where

$$\underset{\sim}{V}_i(\underset{\sim}{x}') = E_i(\underset{\sim}{x}') \nabla' G - G \nabla' E_i(\underset{\sim}{x}')$$

Following Miyamoto and Wolf [5] we notice that since $\nabla' \cdot \underset{\sim}{V}_i = 0$, a vector potential $\underset{\sim}{W}_i$ can be defined such that $\underset{\sim}{V}_i = \nabla' x \underset{\sim}{W}_i$. Assuming that $\underset{\sim}{W}_i$ has only a finite number of singularities that are excluded from the region of integration, then Stokes theorem yields

$$\underset{\sim}{E}'_k = \hat{x}_i \left\{ \oint_c \underset{\sim}{W}_i \cdot \hat{\ell}' \, d\ell' + \sum_j \oint_{c_j} (\underset{\sim}{W}_i \cdot \hat{\ell}') \, d\ell' \right\} \qquad (7)$$

where the notation $\underset{\sim}{E}'_k$ is used to distinguish the P. O. formulation of (4) from the corresponding M-R formulation.

The second term of (7) represents integrations around the singularities of $\underset{\sim}{W}_i$. The unit vector ℓ' around the contour c_j is directed in the opposite sense of $\hat{\ell}'$ around the rim c. The vector potential $\underset{\sim}{W}_i$ is given by [5]

$$\underset{\sim}{W}_i(\underset{\sim}{x},\underset{\sim}{x}') = -G\int_{\infty}^{0} e^{ik\mu}[\hat{r}\times\nabla' E_i(\underset{\sim}{x}'-\mu\hat{r})]\,d\mu + \underset{\sim}{W}_{\infty}$$

where, as before, $\underset{\sim}{x}$, $\underset{\sim}{x}'$ are the vectors from the origin to the observer and source point respectively and $\hat{r} = (\underset{\sim}{x}-\underset{\sim}{x}')/|\underset{\sim}{x}-\underset{\sim}{x}'|$. Miyamoto and Wolf have shown that $\underset{\sim}{W}_{\infty}$ is zero if the field satisfies the Sommerfeld radiation condition. These authors have shown, moreover, that if each component of the electric field can be written in the form $E_i = A_i e^{i\phi}$, then $\underset{\sim}{W}_i$ can be represented as an asymptotic power series in the wavenumber k,

$$\underset{\sim}{W}_i = -G\,E_i\,\frac{\hat{r}\times\hat{p}}{1-\hat{r}\cdot\hat{p}} + O(k^{-1})$$

with

$$\hat{p} = \nabla'\phi$$

$$\hat{r} = (\underset{\sim}{x}-\underset{\sim}{x}')/|\underset{\sim}{x}-\underset{\sim}{x}'|$$

For plane or spherical incidence waves all but the first term of the expansion for $\underset{\sim}{W}_i$ vanish.

Writing (7) in dyadic notation, with $\underline{\underline{W}} = \hat{x}_i\,\underset{\sim}{W}_i$,

$$\underset{\sim}{E}'_k = \oint_c \underline{\underline{W}}\cdot\hat{\ell}'\,d\ell' + \sum_j \oint_{c_j} \underline{\underline{W}}\cdot\hat{\ell}'\,d\ell'$$

then from (5) and (6), the M-R analogue of the Franz representation, E'_F, becomes,

$$\underset{\sim}{E}'_F = \oint_c \left[\underline{\underline{W}}\cdot\hat{\ell}' - G\underset{\sim}{E}\times\hat{\ell}' + \frac{1}{i\omega\mu}\nabla G(\underset{\sim}{H}\cdot\hat{\ell}')\right]d\ell' + \sum_j \oint_{c_j} \underline{\underline{W}}\cdot\hat{\ell}'\,d\ell' \qquad (8)$$

A similar formula has been derived by Rubinowicz [1].

The M-R analogues of (2) and (3) can be obtained in a straightforward manner by using a method described by Jones [17]. For an aperture in a perfectly conducting plane screen in the z=0 plane, we can write the scattered field in the diffraction half space by means of (1), i.e., $\underset{\sim}{E}_F(\underset{\sim}{x})$, with $\underset{\sim}{x} = (x,y,z)$.

For an observer at the image point, we employ the same formula obtaining $\underset{\sim}{E}_F(\underset{\sim}{x}_I)$, $\underset{\sim}{x}_I = (x,y,-z)$ where the Greens function in the integrand now becomes

$$G(\underset{\sim}{x}_I, \underset{\sim}{x}') = \frac{e^{ikr_I}}{4\pi r_I}$$

where $r_I = |\underset{\sim}{x}_I - \underset{\sim}{x}'|$ is the distance from the source to the image point. Since the observer at the image point is outside the original volume of integration then $\underset{\sim}{E}_F(\underset{\sim}{x}_I) = 0$. Since the source points are located in the z = 0 plane, then $G(\underset{\sim}{x}, \underset{\sim}{x}') = G(\underset{\sim}{x}_I, \underset{\sim}{x})$, $\hat{x} \cdot \nabla G(\underset{\sim}{x}, \underset{\sim}{x}') = \hat{x} \cdot \nabla G(\underset{\sim}{x}_I, \underset{\sim}{x}')$, $\hat{y} \cdot \nabla G(\underset{\sim}{x}, \underset{\sim}{x}') = \hat{y} \cdot \nabla G(\underset{\sim}{x}_I, \underset{\sim}{x}')$, $\hat{z} \cdot \nabla G(\underset{\sim}{x}, \underset{\sim}{x}') = -\hat{z} \cdot \nabla G(\underset{\sim}{x}_I, \underset{\sim}{x}')$. Using these facts it is not difficult to show [17] that by adding the z components of $\underset{\sim}{E}_F(\underset{\sim}{x})$ and $\underset{\sim}{E}_F(\underset{\sim}{x}_I)$ and subtracting the x, y components of $\underset{\sim}{E}_F(\underset{\sim}{x}_I)$ from $\underset{\sim}{E}_F(\underset{\sim}{x})$ we obtain equation (2). Equation (3) is obtained by adding the x, y components and subtracting the z components. We can express these operations through the combinations

$$\underline{\underline{I}} \cdot \underset{\sim}{E}_F(\underset{\sim}{x}) - (\underline{\underline{I}} - 2\hat{z}\hat{z}) \cdot \underset{\sim}{E}_F(\underset{\sim}{x}_I) \tag{9}$$

$$\underline{\underline{I}} \cdot \underset{\sim}{E}_F(\underset{\sim}{x}) + (\underline{\underline{I}} - 2\hat{z}\hat{z}) \cdot \underset{\sim}{E}_F(\underset{\sim}{x}_I) \tag{10}$$

$$\underline{\underline{I}} = \hat{x}\hat{x} + \hat{y}\hat{y} + \hat{z}\hat{z}$$

where (9) yields (2) and (10) yields (3). Now by replacing $\underset{\sim}{E}_F$ by $\underset{\sim}{E}'_F$ in the above equations (where $\underset{\sim}{E}'_F$ is the M-R analogue of $\underset{\sim}{E}_F$) we directly obtain the M-R analogues of $\underset{\sim}{E}_1$, $\underset{\sim}{E}_2$ that we denote by $\underset{\sim}{E}'_1$, $\underset{\sim}{E}'_2$:

$$\underset{\sim}{E}'_1 = \oint_c \left| \underline{\underline{D}}_1 \cdot \hat{\ell}' - 2\hat{z}G(\underset{\sim}{E} \times \hat{\ell}') \cdot \hat{z} \right| d\ell' + \sum_j \oint_{c_j} \underline{\underline{D}}_1 \cdot \hat{\ell}' \, d\ell' \tag{11}$$

$$\underset{\sim}{E}'_2 = \oint_c \left| \underline{\underline{D}}_2 \cdot \hat{\ell}' + 2G(\underline{\underline{I}} - \hat{z}\hat{z}) \cdot (\underset{\sim}{E} \times \hat{\ell}') + \frac{1}{i\omega\mu} \nabla G(\underset{\sim}{H} \cdot \hat{\ell}) \right| + \sum_j \oint_{c_j} \underline{\underline{D}}_2 \cdot \hat{\ell}' \, d\ell'$$

where

$$\underline{\underline{D}}_1 = \underline{\underline{I}} \cdot \underline{\underline{W}} - (\underline{\underline{I}} - 2\hat{z}\hat{z}) \cdot \underline{\underline{W}}'$$

$$\underline{\underline{D}}_2 = \underline{\underline{I}} \cdot \underline{\underline{W}} + (\underline{\underline{I}} - 2\hat{z}\hat{z}) \, \underline{\underline{W}}' \tag{14}$$

with

$$\underline{\underline{W}} = \underline{\underline{W}}(\underset{\sim}{x}, \underset{\sim}{x}') = \hat{x}_i \underset{\sim}{W}_i(\underset{\sim}{x}, \underset{\sim}{x}')$$

$$\underline{\underline{W}}' = \underline{\underline{W}}(\underset{\sim}{x}_I, \underset{\sim}{x}') = \hat{x}_i \underset{\sim}{W}_i(\underset{\sim}{x}_I, \underset{\sim}{x}')$$

Equation (11) has been obtained by Karczewski [11] by generalizing the work by Marchand and Wolf [6] to the vector case.

Results

To compare the M-R and the P. O. formulations we numerically compute the field diffracted from a circular aperture. To do this we use (2) for the P. O. and (11) for the corresponding M-R formulation. For plane and spherical wave incidence (the two cases that will be considered) the vector potentials simplify to

$$\underline{\underline{W}} = -GE_i \hat{x}_i \left(\frac{\hat{r} \times \hat{p}}{1 - \hat{r} \cdot \hat{p}} \right)$$

$$\underline{\underline{W}}' = -GE_i \hat{x}_i \left(\frac{\hat{r}_I \times \hat{p}}{1 - \hat{r}_I \cdot \hat{p}} \right)$$

Assuming the perfectly conducting plane screen to lie in the $z = 0$ plane with the sources of the incident field located in the $z < 0$ half space, then for observation points $z > 0$, (11) can be written

$$\underset{\sim}{E}_1' = \oint_c G \left| \underset{\sim}{E}_t (\underset{\sim}{A}_1 \cdot \hat{\ell}') + \hat{z} E_z (\underset{\sim}{A}_2 \cdot \hat{\ell}') - 2\hat{z} (\underset{\sim}{E} \times \hat{\ell}') \cdot \hat{z} \right| d\ell' + \text{g. o.} \tag{15}$$

where

$$\underset{\sim}{E}_t = \hat{x} E_x + \hat{y} E_y$$

$$\underset{\sim}{A}_{\frac{1}{2}} = \pm \frac{\hat{r}_I \times \hat{p}}{1 - \hat{r}_I \cdot \hat{p}} - \frac{\hat{r} \times \hat{p}}{1 - \hat{r} \cdot \hat{p}} \tag{16}$$

The term labeled g. o., which equals the second term of (11), is simply the vector geometrical optics field. This is determined by tracing the incident rays through the aperture, keeping track of the phase and amplitude, and preserving the vector nature of the incident field. That this term is a g. o. field follows from the work of Miyamoto and Wolf [5] and the fact that $\underline{\underline{W}}'$ has no singularities over the aperture since $1 - \hat{r}_I \cdot \hat{p}$ is never zero there [6].

As the observation point approaches the g. o. shadow boundary the singularity of $\underline{W}$ approaches the aperture rim. When the observer is directly on the shadow boundary a separation between the g.o. term and the rim integration is no longer possible. However, for the numerical computation we can use the fact that even with a high sampling rate in the observation space, it is highly improbable that the observer will lie so close to the shadow boundary that the value of the integrand will result in computer overflow. To properly account for the contribution to the integral near this singular point, the sampling rate along the rim must be increased. To accomplish this a variable sampling grid is used for the integration along the aperture rim. The sampling rate is determined so that the integrand changes by some small fraction of its value in moving from one point to the next. For the observer near the shadow boundary this procedure yields a fine sampling grid for that section of the aperture edge near the singularity and a more coarsely sampled grid along the remaining portion of the edge.

It is evident that this procedure circumvents rather than solves the underlying analytic problem. To obtain uniformly valid expressions as the observer approaches the shadow boundary it would be necessary to generalize the work of Otis, et al [7] to the vector case and to arbitrary aperture shapes.

In figures 1-5 are shown comparisons between the M-R (solid line) and the P. O. formulation (X). We have chosen throughout a frequency of 4 GHz and an aperture radius of 0.4 m. The screen and aperture are located in the z = 0 plane. The diffracted field has been computed for points in the x > 0, z > 0 quadrant of the x-z plane. The absissa of the graphs corresponds to the angle, in radians, measured from the z axis. The ordinate corresponds to the magnitude of a particular component of the field which is given in units of volts/meter or millivolts/meter. For the M-R results, a variable mesh integration around the aperture rim was used to generate all but the final figure.

For an incident monochromatic plane wave of the form

$$\underset{\sim}{E}_i = \hat{e}\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}}$$

where

$$\hat{e} = [(\hat{k}\cdot\hat{z})\,\hat{x} - (\hat{k}\cdot\hat{x})\,\hat{z}]$$

$$\hat{k}\cdot\hat{z} = \cos(\pi/8)$$

$$\hat{k}\cdot\hat{x} = \sin(\pi/8)$$

we plot in figures 1 and 2 the magnitude of the E_x, E_z components of the field. The distance, r_o, from the center of the aperture to the observer is 5 m. It can be seen that the results from the P. O. and M-R representations are in good agreement.

In figures 3 through 5 we assume an incident plane wave of the form

$$\underset{\sim}{E}^{i} = \hat{\theta}' f(\theta') e^{ikr}/r$$

where r is the distance measured from the source point. The distance from the source to the center of the aperture is taken to be 3 m. The polar angle θ' is measured with respect to the z' axis where $\hat{x} \cdot \hat{z}' = \cos(\pi/8)$, $\hat{y}\ \hat{z}' = 0$ $\hat{z} \cdot \hat{z}' = -\sin(\pi/8)$ and where the unit vectors $\hat{x}$, $\hat{y}$ correspond to the x, y axes located in the plane of the aperture. The unit vector associated with θ' is denoted by $\hat{\theta}'$. For the numerical computations we have assumed an isotropic source so that $f(\theta') = 1$.

In figure 3, the magnitude of E_x is plotted for a far field observer, $r_o = 5$ m. A fixed sampling grid was used to generate figure 5. The two discontinuities that occur near the opposite points of the shadow boundary in this figure are eliminated by using the variable mesh integration (figure 4).

The major advantage of the M-R relative to the P. O. representation is the smaller amount of computer time needed to calculate the diffracted field. For the parameters chosen here, with a diameter to wavelength ratio, D/λ, of about 10, the time needed for the M-R calculation is about a factor of 8 to 10 less than that required for the P. O. calculation. This factor in savings would increase for larger D/λ. For more complicated geometries, e.g., non-planar reflectors, the computational savings is not expected to be as great since the calculation of the g. o. terms would be more time consuming than in the simple aperture problems considered here.

It should be mentioned that for simple aperture shapes such as circular and rectangular, the Frauenhofer approximation can be used to obtain closed form expression for the M-R and P. O. representations (15) and (2). In particular, the results for a plane wave incident upon a circular aperture have been shown to be in good agreement for observation points outside the geometrically illuminated region (Appendix A). The M-R solution, however, is discontinuous near the shadow boundary. For this case the variable mesh integration is not applicable since the integration is performed prior to any numerical computation.

Applying the techniques used by Keller et al. [9] and Miyamoto and Wolf [5] to equations (2) and (15), it can be shown that the first terms from the asymptotic expansions in k are in agreement. This result holds for arbitrary aperture shapes and arbitrary fields of incidence. The expansions that were used, however, are not valid near the shadow boundary or near a caustic.

Acknowledgments

We wish to thank R. Schmidt for suggesting this problem to us and for his encouragement and assistance.

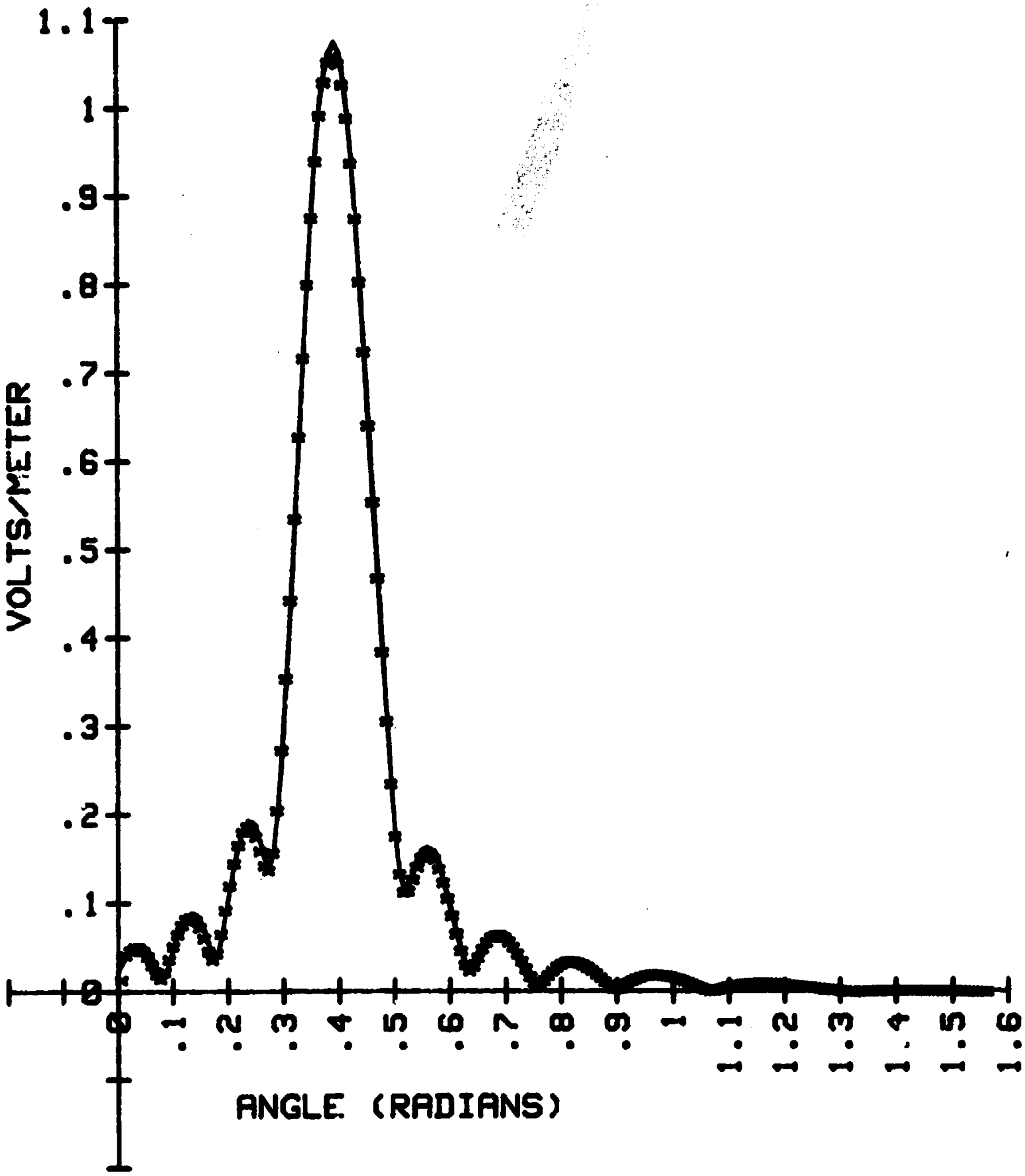

Figure 1. M-R (solid line) and P. O. (x) solutions for $|E_x|$, r_o = 5m; plane wave incidence.

ORIGINAL PAGE IS
OF POOR QUALITY

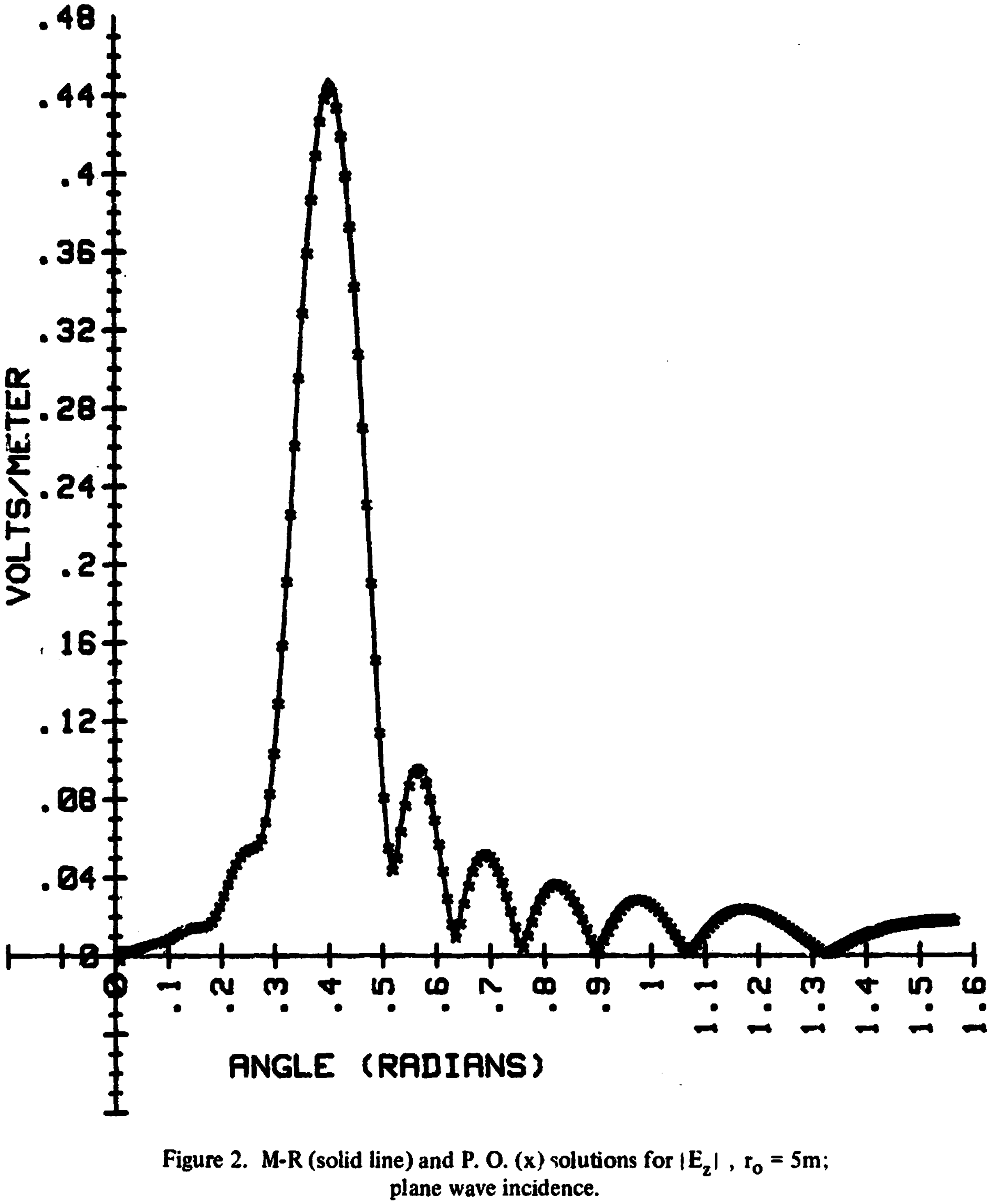

Figure 2. M-R (solid line) and P. O. (x) solutions for $|E_z|$, r_o = 5m; plane wave incidence.

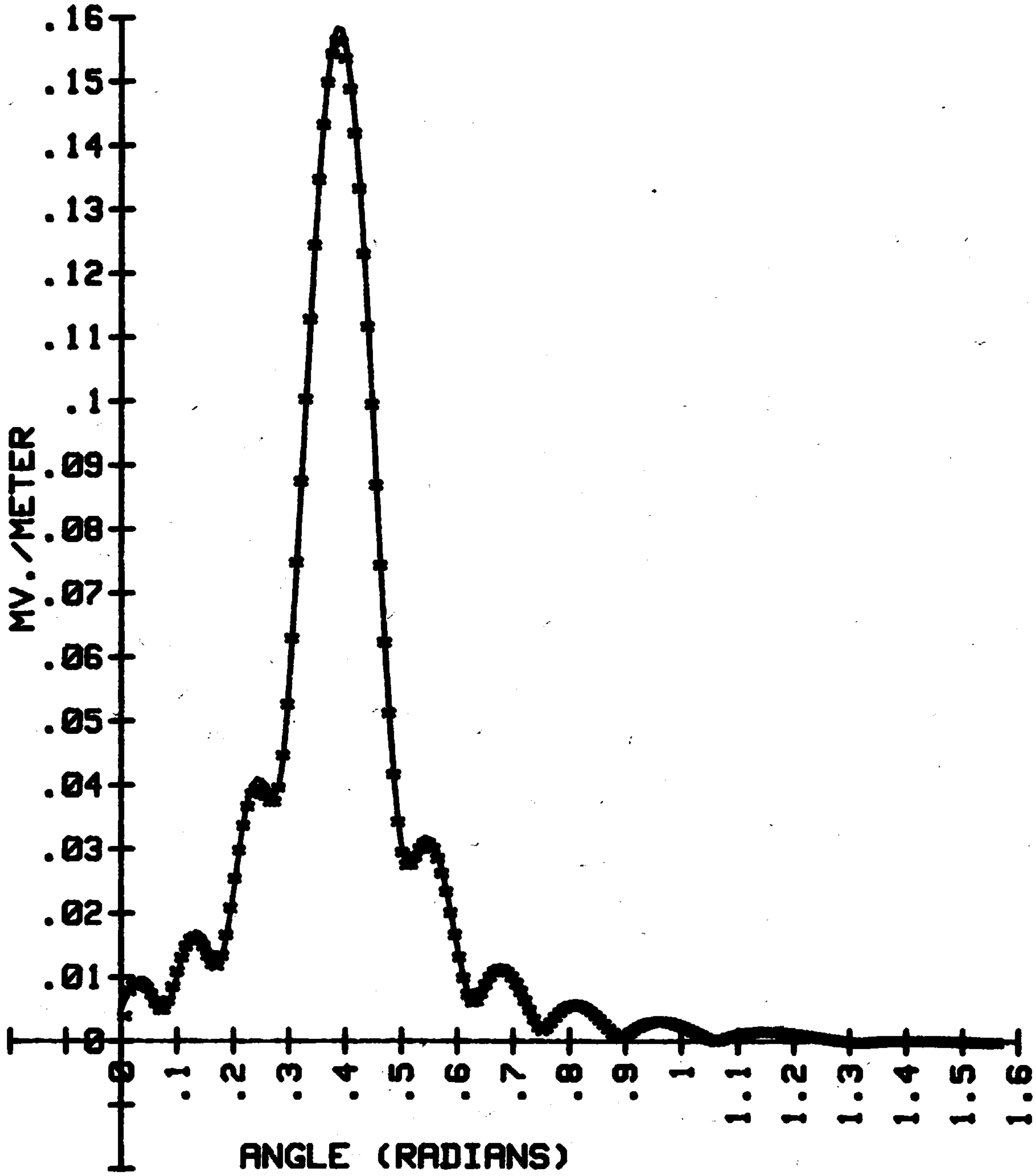

Figure 3. M-R (solid line) and P. O. (x) solutions for $|E_x|$, r_o = 10 km; spherical wave incidence.

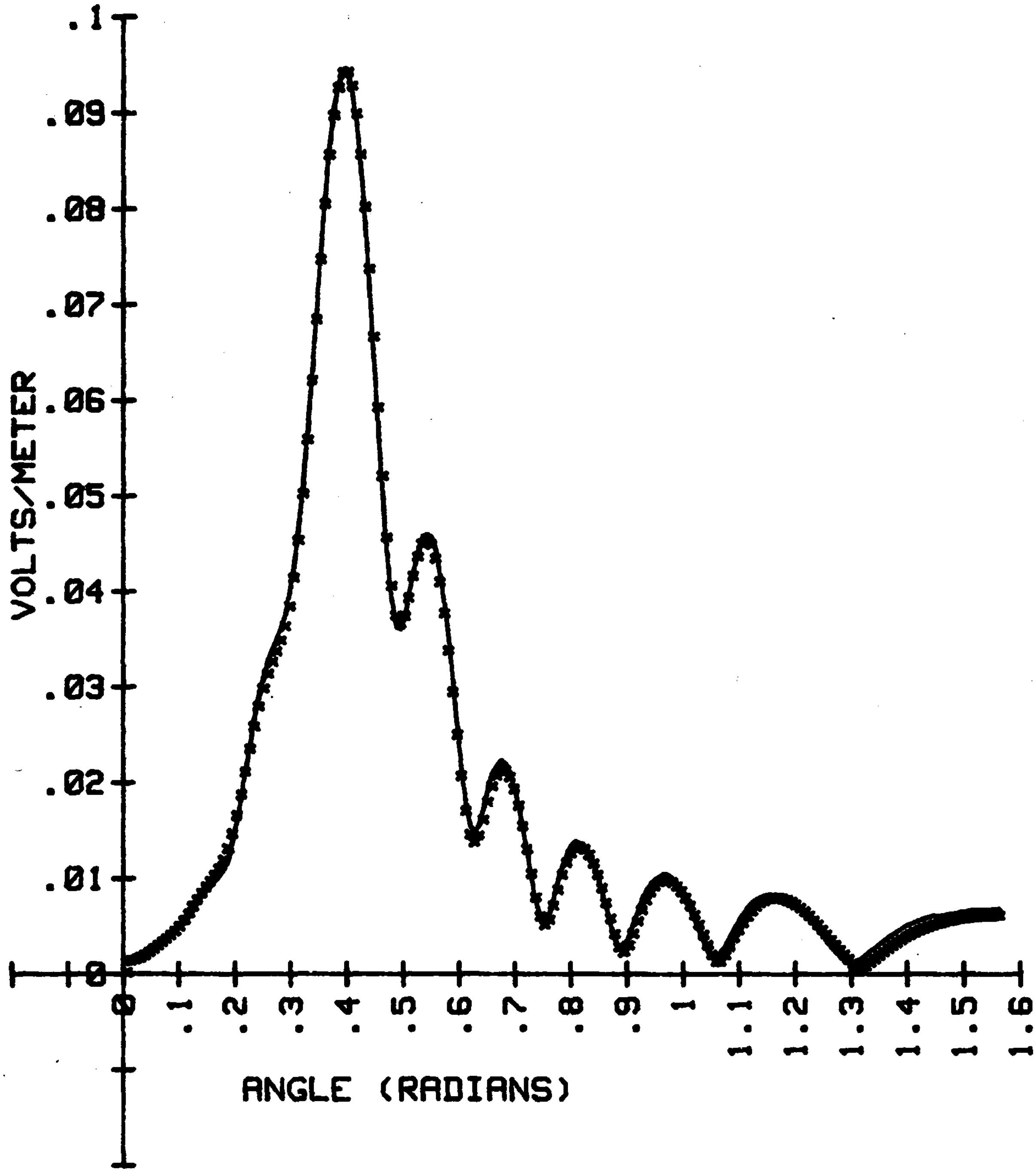

Figure 4. M-R (solid line) and P. O. (x) solutions for $|E_z|$, $r_o = 5m$; spherical wave incidence.

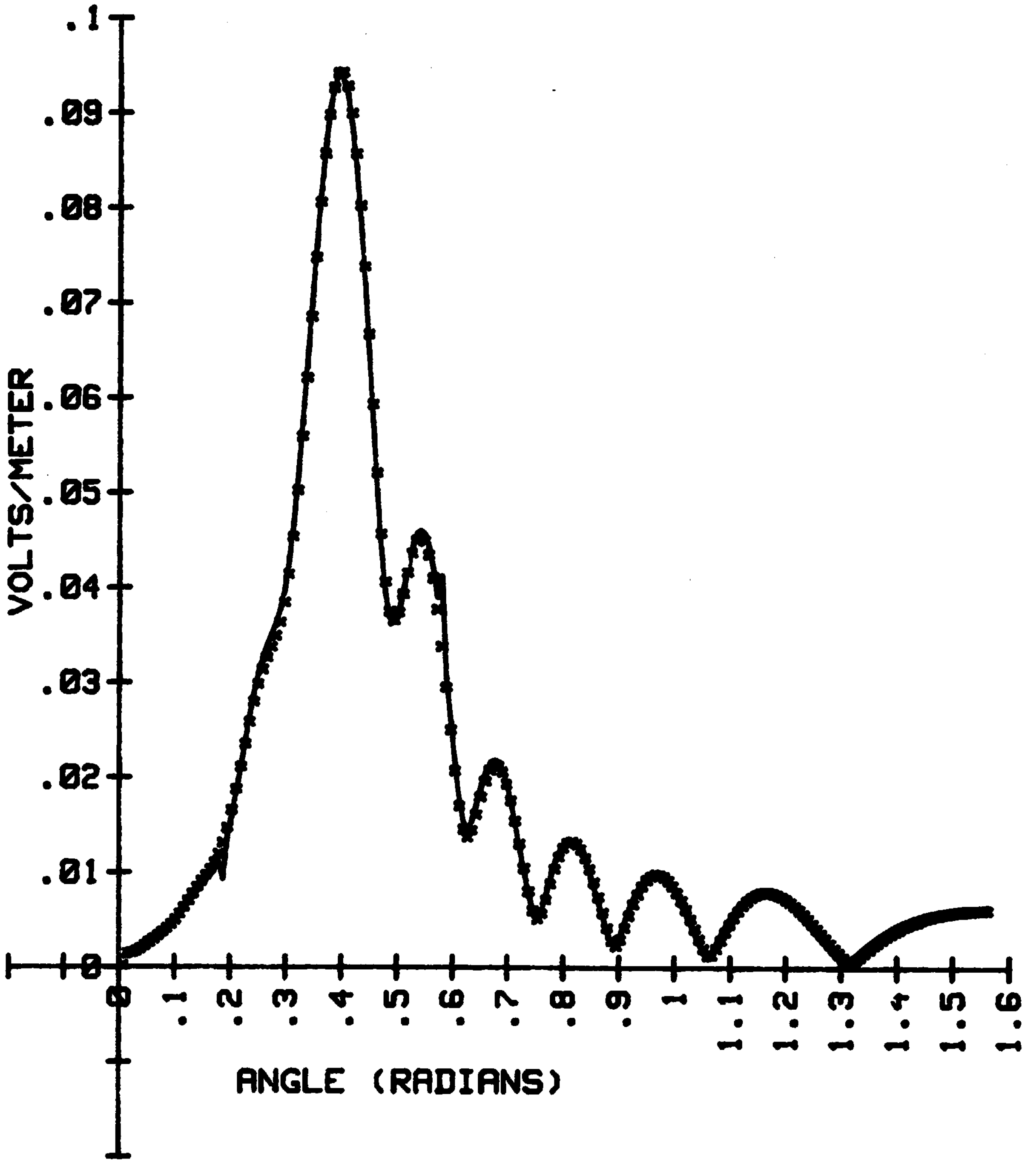

Figure 5. M-R (solid line) and P. O. (x) solutions for $|E_z|$, $r_o = 5m$; spherical wave incidence. Uniform sampling mesh along the aperture rim chosen for M-R solution.

REFERENCES

1. A. Rubinowicz, Progress in Optics, Ed. E. Wolf, New York, John Wiley and Sons, Vol. 4, 1965, pp 199-240.
2. F. Kottler, Progress in Optics, Ed. E. Wolf, New York, John Wiley and Sons, Vol 4, 1965, pp 281-314.
3. A. Rubinowicz, Die Beugungswelle in der Kirchhoffschen Theorie des Beugungsercheinungen, Ann. Physik, Vol 53, pp 257-278, 1917.
4. B. V. Baker and E. T. Copson, The Mathematical Theory of Huygens' Principle, New York: Oxford, 1950, pp 74-84.
5. K. Miyamoto and E. Wolf, Generalization of the Maggi-Rubinowicz Theory of the Boundary Diffraction Wave – Part I and II, J. Opt. Soc. Am., Vol 52, pp 615-637, 1962.
6. E. W. Marchand and E. Wolf, Boundary Diffraction Wave in the Domain of the Rayleigh-Kirchhoff Diffraction Theory, J. Opt. Soc. Am., Vol 52, pp 761-767, 1962.
7. G. Otis, J. L. Lachambre, J. W. Y. Lit and P. Lavigne, Diffracted Waves in the Shadow Boundary Region, J. Opt. Soc. Am., pp 551-553, 1977.
8. C. J. Bouwkamp, Diffraction Theory, Reports on Progress in Physics, Vol 17, pp 35-100, 1954.
9. J. B. Keller, R. M. Lewis and B. D. Seckler, Diffraction by an Aperture II, J. Appl. Phys., Vol. 28, pp 570-579, 1957.
10. O. Laporte and J. Meixner, Kirchhoff-Youngsche Theorie der Beugun Electromagnetischer Wellen, Z. Physik, Vol 153, pp 129-148, 1958.
11. B. Karczewski, Boundary Wave in Electromagnetic Theory of Diffraction, J. Opt. Soc. Am., Vol 53, pp 878-879, 1963.
12. G. Otis, Application of the Boundary-Diffraction Wave Theory to Gaussian Beams. J. Opt. Soc. Am.. Vol 64, pp 1545-1550, 1974.
13. E. W. Marchand and E. Wolf, Consistent Formulation of Kirchhoff's Diffraction Theory, J. Opt. Soc. Am., Vol 56, pp 1712-1722, 1966.

14. T. Suzuki, Extension of the Theory of the Boundary Diffraction Wave to Systems with Arbitrary Aperture-Transmittance Function, J. Opt. Soc. Am., Vol 61, pp 439-445, 1971.

15. J. W. Y. Lit and R. Tremblay, Boundary-Diffraction-Wave Theory of Cascaded-Apertures Diffraction, J. Opt. Soc. Am., Vol 59, pp 559-567, 1969.

16. W. B. Gordon, Far-Field Approximations to the Kirchhoff-Helmholtz Representations of Scattered Fields, IEEE Trans. Ant. and Prop., Vol AP-23, pp 590-592, 1975.

17. D. S. Jones, The Theory of Electromagnetism, New York: MacMillan, 1964, pp 53-56, 663-640.

18. A. Sommerfeld, Optics, New York: Academic Press, 1964, pp 325-328.

19. C. T. Tai, Kirchhoff Theory: Scalar, Vector or Dyadic? IEEE Trans. Ant. Prop., Vol AP-20, pp 114-115, 1972.

20. J. A. Stratton, Electromagnetic Theory, New York: McGraw-Hill, 1941, pp 460-470.

21. F. Kottler, Electromagnetische Theorie der Beugung an Schwarzen Schirmen, Ann. Phys., Vol 71, pp 457-508, 1923.

22. F. Kottler, Progress in Optics, Ed. E. Wolf, New York, John Wiley and Sons, Vol 6, 1967, pp 331-377.

23. R. E. Collin and F. J. Zucker, Antenna Theory, Pt. 1, New York: McGraw-Hill, 1969, pp 79-86.

24. J. D. Jackson, Classical Electrodynamics, New York: John Wiley and Sons, 1975, pp 427-438.

APPENDIX A

Comparison of P. O. and M-R in the Frauenhofer Region

For simple aperture shapes, closed form expressions can be obtained for the P. O. and M-R formulations in the Frauenhofer zone. In particular, we compute $\underset{\sim}{E}_1$, $\underset{\sim}{E}'_1$ and $\underset{\sim}{E}_k$ for the case of a plane wave incident upon a circular aperture. We place the screen in the z = 0 plane with the sources of the incident field in the $z < 0$ and the observer in the $z > 0$ half space. The incident electric field is assumed to be,

$$\underset{\sim}{E}_i = \hat{e}\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}'}$$

where

$$\hat{e} = [(\hat{k}\cdot\hat{z})\,\hat{x} - (\hat{k}\cdot\hat{x})\,\hat{z}]$$

$$\hat{k}\cdot\hat{z} = \cos(\pi/8)$$

$$\hat{k}\cdot\hat{x} = \sin(\pi/8)$$

$$\underset{\sim}{k} = k\hat{k}$$

To compute $\underset{\sim}{E}_1$ from the formula

$$\underset{\sim}{E}_1 = 2\ \nabla\text{x}\int_s (\hat{z}\text{x}\underset{\sim}{E})\,G\,dS' \qquad \text{(A1)}$$

we use the vector relations

$$\nabla\text{x}[(\hat{z}\text{x}\underset{\sim}{E})G] = \nabla G\text{x}(\hat{z}\text{x}\underset{\sim}{E}) + G\,\nabla\text{x}(\hat{z}\text{x}\underset{\sim}{E})$$

$$\nabla G\text{x}(\hat{z}\text{x}\underset{\sim}{E}) = (\nabla G\cdot\underset{\sim}{E})\hat{z} - (\nabla G\cdot\hat{z})\underset{\sim}{E}$$

Noticing that $\nabla\text{x}(\hat{z}\text{x}\underset{\sim}{E}) = 0$ since $\underset{\sim}{E}$ is a function only of the primed coordinates then

$$\underset{\sim}{E}_1 = 2\int_s \left|\hat{z}(\nabla G\cdot\underset{\sim}{E}) - (\nabla G\cdot\hat{z})\underset{\sim}{E}\right|\,dS'$$

Furthermore,

$$\nabla G = \hat{r}(ik - 1/r)G \sim ik\hat{r}_o\,G$$

$$G \sim \frac{e^{ikr_o}}{4\pi r_o}\,e^{-ik\underset{\sim}{x}'\cdot\hat{r}_o}$$

where $\hat{r}_o$ is the unit vector from the center of the aperture to the observer and r_o is the associated distance. For $\underset{\sim}{E}$ in the integrand we substitute $\underset{\sim}{E}_i$ (Kirchhoff approximation) so that

$$\underset{\sim}{E}_1 = \frac{ik\, e^{ikr_o}}{2\pi r_o} \int_s e^{ik(\hat{k}-\hat{r}_o)\cdot \underset{\sim}{x}'} \left[(\hat{r}_o \cdot \hat{e})\hat{z} - (\hat{r}_o \cdot \hat{z})\hat{e}\right] dS'$$

Using the fact that $(\hat{r}_o \cdot \hat{e})\hat{z} - (\hat{r}_o \cdot \hat{z})\hat{e} = (\hat{e} x \hat{z}) x\, \hat{r}_o = (\hat{k} \cdot \hat{z})(\hat{r}_o x \hat{y})$ then

$$\underset{\sim}{E}_1 = \frac{-ik\, e^{ikr_o}}{2\pi r_o} (\hat{k} \cdot \hat{z})(\hat{r}_o x \hat{y}) \int_s \exp\left[ik(\hat{k} - \hat{r}_o) \cdot \underset{\sim}{x}'\right] dS'$$

From equation (4) of the text, we find by a similar procedure that,

$$\underset{\sim}{E}_k = \frac{-ik\, e^{ikr_o}}{4\pi r_o} \left[(\hat{k} + \hat{r}_o) \cdot \hat{z}\right] \hat{e} \int_s \exp\left[ik(\hat{k} - \hat{r}_o) \cdot \underset{\sim}{x}'\right] dS' \qquad \text{(A2)}$$

To solve the integrals, let

$$\underset{\sim}{x}' = (r' \cos\phi', r' \sin\phi', 0)$$

$$\hat{r}_o = (\cos\phi \sin\theta, \sin\phi \sin\theta, \cos\theta)$$

then

$$(\hat{k} - \hat{r}_o) \cdot \underset{\sim}{x}' = r'\left[(\hat{k} \cdot \hat{x})\cos\phi' - \sin\theta\,(\cos\phi \sin\phi' + \cos\phi' \sin\phi)\right]$$

Using the transformation $\phi' = \psi + u$, where u is chosen so that the coefficient of $\sin\phi'$ in the above formula is zero, then

$$u = \mathrm{Tan}^{-1}(-c/d)$$

$$c = \sin\theta \sin\phi$$

$$d = (\hat{k} \cdot \hat{x}) - \sin\theta \cos\phi$$

and

$$(\hat{k} - \hat{r}_o) \cdot x' \rightarrow r' \zeta \cos\psi$$

$$\zeta = \left[(\hat{k} \cdot \hat{x})^2 + (\hat{r}_o \cdot \hat{y})^2 + (\hat{r}_o \cdot \hat{x})^2 - 2(\hat{k} \cdot \hat{x})(\hat{r}_o \cdot \hat{x})\right]^{1/2}$$

so

$$\int_s e^{ik(\hat{k}-\hat{r}_o)\cdot \underset{\sim}{x}'} dS' = \int_0^{2\pi} \int_0^{a} e^{ikr'\zeta\cos\psi}\, r'\, dr'\, d\psi = \frac{2\pi a\, J_1(ka\zeta)}{k\zeta}$$

Substituting this result into (A1) and (A2), then

$$\underset{\sim}{E}_1 = \frac{ia\, e^{ikr_o}}{\zeta r_o} (\hat{k} \cdot \hat{z})\, J_1(ka\zeta)\, (\hat{r}_o \times \hat{y}) \tag{A3}$$

$$\underset{\sim}{E}_2 = \frac{-ia\, e^{ikr_o}}{2\zeta r_o} (\hat{k} + \hat{r}_o) \cdot \hat{z}\, J_1(ka\zeta)\, \hat{e} \tag{A4}$$

where a is the radius of the aperture, and J_1 is the Bessel function of order 1.

To find the M-R analogue of $\underset{\sim}{E}_1$ we begin with equation (15) of the text,

$$\underset{\sim}{E}'_1 = \oint_c G \left| \underset{\sim}{E}_t (\underset{\sim}{A}_1 \cdot \hat{\ell}') + \hat{z}\, E_z (\underset{\sim}{A}_2 \cdot \hat{\ell}') - 2\hat{z}\, (\underset{\sim}{E} \times \hat{\ell}') \cdot \hat{z} \right| d\ell' + \text{g. o.}$$

where

$$\underset{\sim}{E}_t = \hat{x}\, E_x + \hat{y}\, E_y$$

$$\underset{\sim}{A}_{\substack{1\\2}} = \pm \frac{\hat{r}_I \times \hat{p}}{1 - \hat{r}_I \cdot \hat{p}} - \frac{\hat{r} \times \hat{p}}{1 - \hat{r} \cdot \hat{p}}$$

We again make use of the Kirchhoff assumption

$$\underset{\sim}{E}_t(\underset{\sim}{x}') = \hat{x}\, (\hat{k} \cdot \hat{z})\, e^{i\underset{\sim}{k} \cdot \underset{\sim}{x}'}$$

$$\underset{\sim}{E}_z(\underset{\sim}{x}') = -(\hat{k} \cdot \hat{x})\, e^{i\underset{\sim}{k} \cdot \underset{\sim}{x}'}$$

and the relations

$$\hat{p} = \hat{k}$$

$$\hat{\ell}' = (-\sin\phi', \cos\phi', 0)$$

$$(\hat{r} \times \hat{p}) \cdot \hat{\ell}' \sim -\sin\phi' \, [(\hat{k} \cdot \hat{z}) \sin\theta \cos\phi - (\hat{k} \cdot \hat{y}) \cos\theta)$$
$$+ \cos\phi' [(\hat{k} \cdot \hat{x}) \cos\theta - (\hat{k} \cdot \hat{z}) \sin\theta \cos\phi]$$

$$\hat{r} \cdot \hat{p} \sim (\hat{k} \cdot \hat{x}) \sin\theta \cos\phi + (\hat{k} \cdot \hat{z}) \cos\theta$$

where the observation point (x, y, z) has been expressed in terms of a spherical coordinate system so that $x = \cos\phi \sin\theta$, $y = \sin\phi \sin\theta$, $z = \cos\theta$. The expressions for $(\hat{r}_I \times \hat{p}) \cdot \hat{l}'$, $\hat{r}_I \cdot \hat{p}$ follow from the above formulas by replacing $\cos\theta$ with $-\cos\theta$.

After the resulting integrand is simplified we encounter the integral,

$$\int_0^{2\pi} e^{ika[(\hat{k}\cdot\hat{x})\cos\phi' - \cos(\phi-\phi')\sin\theta]} (a_1 \cos(\phi - \phi') + a_2 \cos\phi')\, d\phi'$$

Using the transformation $\phi' = \psi + u$, described above, the integral can be shown to reduce to

$$(a_1 \cos(\phi - u) + a_2 \cos u)\, 2\pi i\, J_1(ka\zeta)$$

where a_1, a_2 are constants and ϕ, u are defined above.

After rearranging terms, $\underset{\sim}{E}_1'$ can be expressed by the complicated formula,

$$\underset{\sim}{E}_1' = \frac{i\, a\, e^{ikr_o}}{r_o\, \zeta} J_1(ka\zeta)\, [-\hat{x}\, \beta_1 + \hat{z}(\beta_2 - (\hat{k} \cdot \hat{z})\, \sigma)] + \text{g. o.} \qquad \text{(A5)}$$

where

$$\sigma = (\hat{k} - \hat{r}_o) \cdot \hat{x}$$

$$\beta_1 = \frac{1}{\gamma} (\hat{k} \cdot \hat{z})(\hat{r}_o \cdot \hat{z}) \{ (\hat{r}_o \cdot \hat{z})^2 [(\hat{r}_o \cdot \hat{y})^2 - \sigma(\hat{r}_o \cdot \hat{x})] + (\hat{k} \cdot \hat{x})\, \sigma\, a \}$$

$$\beta_2 = \frac{1}{\gamma} (\hat{k} \cdot \hat{x}) \{ (\hat{r}_o \cdot \hat{z})^2 (\hat{k} \cdot \hat{z})(\hat{k} \cdot \hat{x})\sigma - a(\hat{k} \cdot \hat{z}) [\sigma(\hat{r}_o \cdot \hat{x}) - (\hat{r}_o \cdot \hat{y})^2] \}$$

$$a = 1 - (\hat{k} \cdot \hat{x})(\hat{r}_o \cdot \hat{x})$$

$$\gamma = (1 - (\hat{k} \cdot \hat{x})(\hat{r}_o \cdot \hat{x}))^2 - (\hat{r}_o \cdot \hat{z})^2 (\hat{k} \cdot \hat{z})^2$$

In figures A1, A2 we have plotted the magnitude of the E_x E_z components of the field for both the M-R (solid line) and the P. O. (x) formulations. The observer is taken to be at a distance of 10 km from the center of the aperture. We assume, in addition, an incident plane wave identical to that considered in the text. All other parameters are the same as those given in the text: f = 4 GHz and an aperture radius equal to 0.4 m. The geometrical optics term of (A5) has not been included in the numerical computation. In consequence, the M-R formulation, as plotted, is incorrect in the small angular region at the center of the main lobe.

However, even if the g. o. term were added to the solution, the discontinuity at the shadow boundary would persist. It therefore appears necessary to use a numerical integration with the variable grid method for those points within the g. o. illuminated region. For observation points outside this region it can be seen from the figures that the two solutions yield nearly the same results.

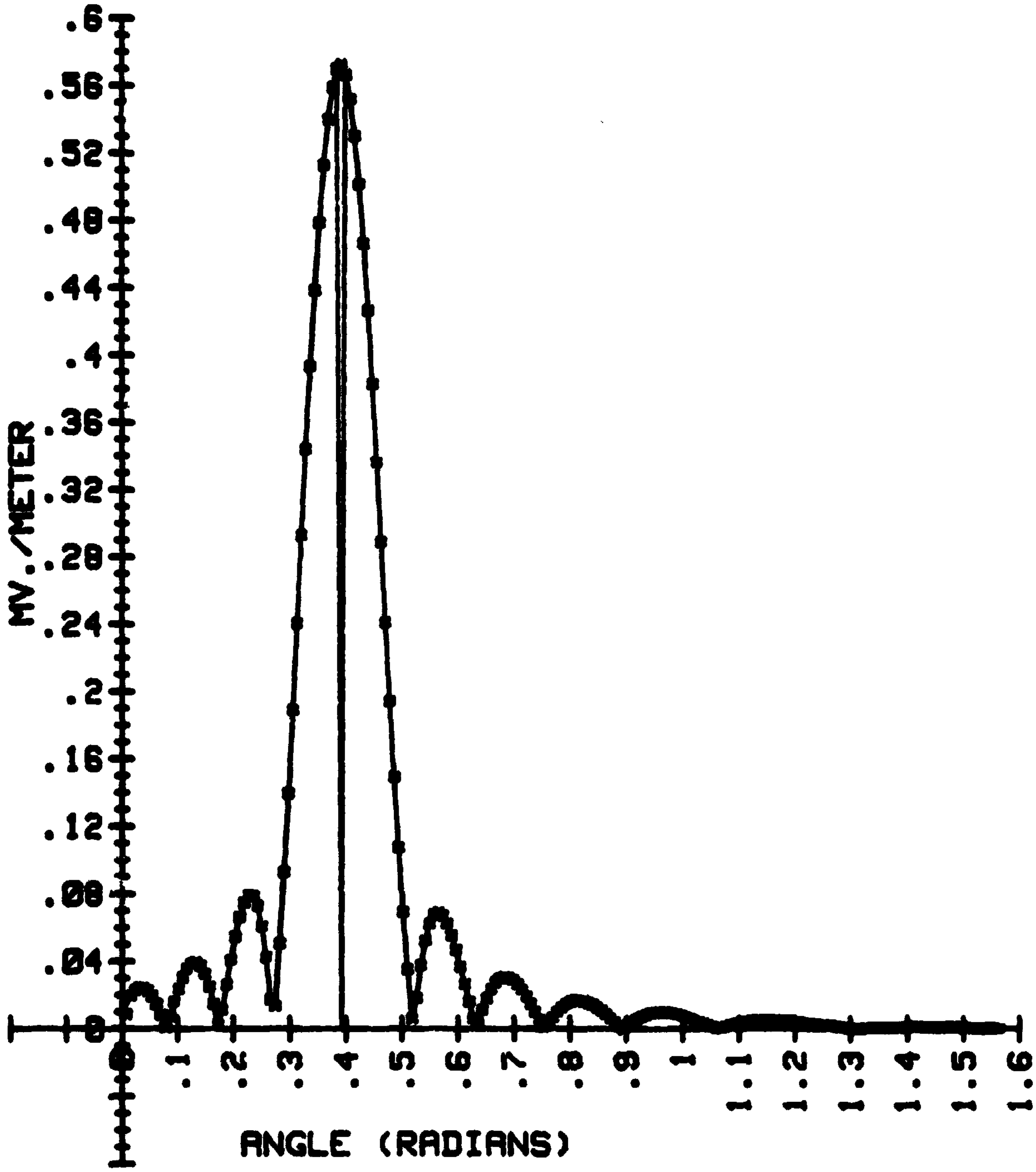

Figure A1. M-R (solid line) and P. O. (x) solutions for $|E_x|$, r_0 = 10 km: plane wave incidence; Frauenhofer solution.

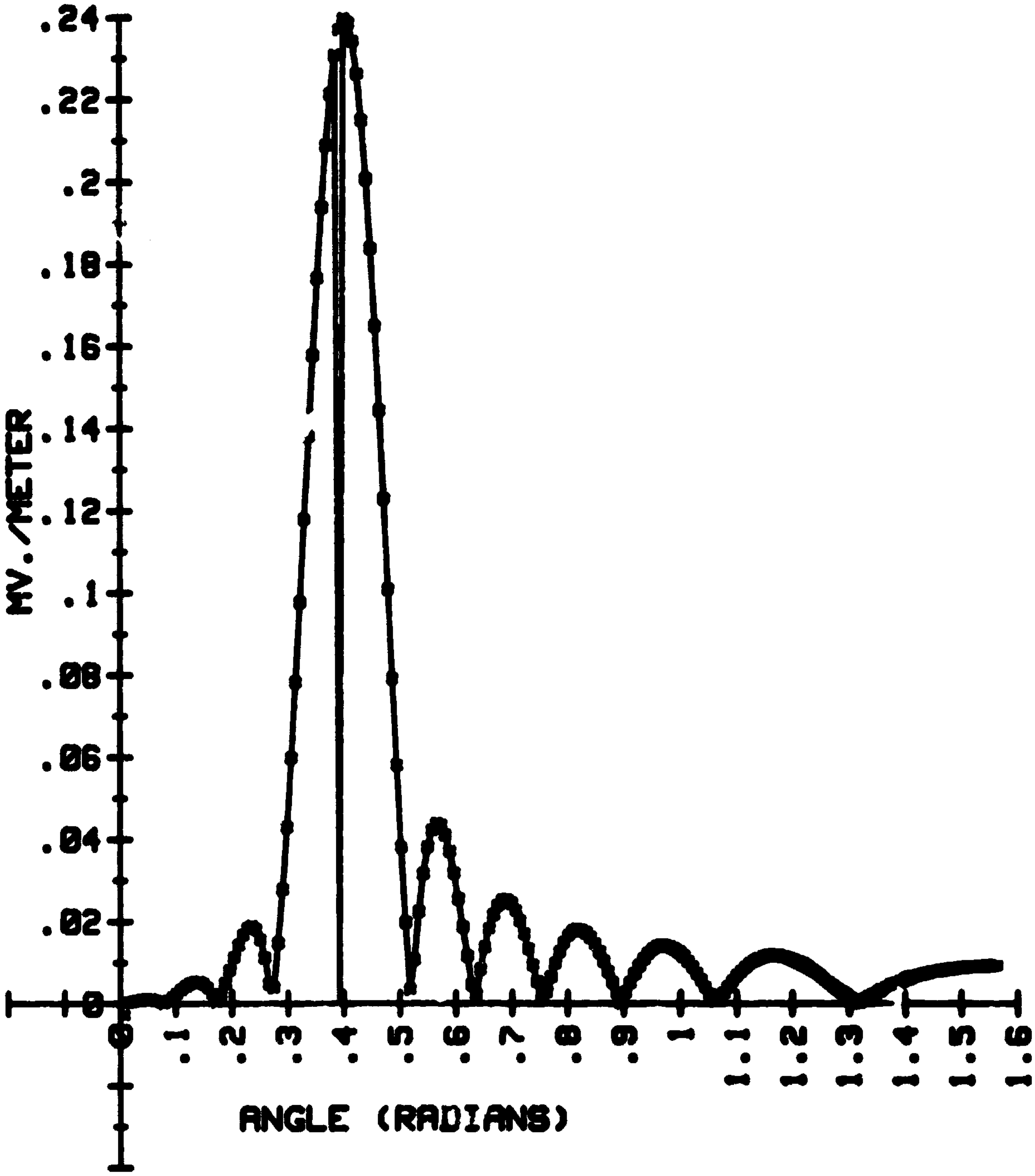

Figure A2. M-R (solid line) and P. O. (x) solutions for $|E_z|$, r_0 = 10 km; plane wave incidence; Fraunhofer solution.

BIBLIOGRAPHIC DATA SHEET

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
82024		

4. Title and Subtitle
Vector Analogues of the Maggi-Rubinowicz Theory of Edge Diffraction

5. Report Date

6. Performing Organization Code

7. Author(s)
Robert Meneghini, Peter Shu and John Bay

8. Performing Organization Report No.

9. Performing Organization Name and Address

10. Work Unit No.

11. Contract or Grant No.

12. Sponsoring Agency Name and Address
Goddard Space Flight Center
Greenbelt, Maryland 20771

13. Type of Report and Period Covered
Technical Memorandum

14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

The Maggi-Rubinowicz technique for scalar and electromagnetic fields can be interpreted as a transformation of an integral over an open surface to a line integral around its rim. Using this transformation, Maggi-Rubinowicz analogues are found for several vector physical optics representations. For diffraction from a circular aperture, a numerical comparison between these formulations shows the two methods are in good agreement. To circumvent certain convergence difficulties in the Maggi-Rubinowicz integrals that occur as the observer approaches the shadow boundary, a variable mesh integration is used. For the examples considered, where the ratio of the aperture diameter to wavelength is about ten, the Maggi-Rubinowicz formulation yields an 8 to 10 fold decrease in computation time relative to the physical optics formulation.

17. Key Words (Selected by Author(s))	18. Distribution Statement

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified		

*For sale by the National Technical Information Service, Springfield, Virginia 22151.

GSFC 25-44 (10/77)