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# Magsat Vector Magnetometer <br> Calibration Using Magsat Geomagnetic Field Measurements 

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#### Abstract

The Magnetic Field Satellite (Magsat) was launched on Oct. 30, 1979 into a nearly polar, sunsynchronous orbit, carrying a scalar magnetometer and a vector magnetometer. The satellite reentered the earth's atmosphere on Jui:e 11, 1980, having measured and transmitted more than thrse complete sets of global magne cic field data. The data obtained from the mission will be usec' primarily to compute a currently accurate model of the earth's main magnetic field, to update and refine world and regional magnetic charts, and to develop a global scalar and vector crustal magnetic anomaly map.

This report describes the in-flight calibration procedure used for 39 vector magnetometer system parameters and gives results obtained from some data sets and the numerical studies designed to evaluate the results.


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## MAGSAT VECTOR MAGNETOMETER CALIBRATION USING MAGSAT GEOMAGNETIC FIELD MEASUREMENTS

## INTRODUCTION

The Magnetic Field Satellite (Magsat) was launched from the Western Test Range at Vandenberg Air Force Base in California into a nearly polar, sun-synchronoui orbit on Cctober 30, 1979. The orbit initially had a perigee of 352 km ( 219 miles) and an apogee of 578 km ( 359 miles). The satellite reentered the earth's atmosphere on June 11, 1980, having measured and transmitted more than three complete sets of global magnetic field data.

Magsat carried a scalar magnetometer and a vector magnetometer. The scalar magnetometer was an optically pumped, atomic resonance, dual-cell, self-oscillating cesium vapor magnetometer. capable of measuring the magnitude of the earth's magnetic fieic with a resolution of about $\pm 0.6$ gamma ( 1 gamma $=1$ nanoTesla). The vector magnetometer consisted of three fluxgate sensing elements aligned along nearly orthogonal axes, capable of measuring each component of the earth's magnetic field wich a resolution of $\pm 0.5$ gamma.

The data obtained from the mission will be used primarily to compute a currently accurate model of the earth's main magnetic fieid, to update and refine world and regional magnetic charts. and to develop a global scalar and vector crustal magnetic anomaly map. Details concerning the mission and its objectives have been given by Langel, Reagan, and Murphy (1).

Prior to the launch of Magsat, the values of thirty-nine parameters associated with the vector magnetometer were determined in the laboratory at Goddard Space Flight Center. It could not be assumed, however, that these values would remain unchanged after launch and under space conditions. Because of this a statistical procedure was developed for estimating the system parameters from the data taken by the scalar and vector magnetometers in space. A computer program was written and the procedure was tested on measurements made by the magnetometers in the magnetic test facility at Goddard Space Flight Center. The procedure has been and is continuing to be appli d d to the real magnetic field data transmitted from Magsat.

This report describes the in-flight vector magnetometer calibration procedure and gives resuits obtze..ed from some data sets and the numerical studies designed to evaluate the results. A FORTRAN listing of the calibration program can be obtained upon request.

## the magsat vector magnetometer

The triaxial fluxgate magnetometer flown aboard the Magsat spacecraft was capable of measuring the earth's magnetic field vector components with a resolution of $\pm 0.5$ gamma. It was deveioped at NASA's Goddard Space Fligit Center by M. H. Acuna and co-workers (2). A basic triaxial tluxgate magretometer with i.-bit analog-to-digital converter had a dynamic range of $\pm 2000$ gammas. This range was increased to $\pm 64,000$ gammas in steps of approximately $\pm 1000$ gammas by using three 7-bit digita-to-analog converters as field offset generators, one for each sensor axis. Ground command could select either of two redundant configurations ( A or B ) for the magnetometer electronics; an additional ground command could select one of two configurations for analog-to-digital converter, field offset generators, digital interface, and power converter. A feedback coil nulled the ambient field independently along each sensor axis. The reader is referred to a rep irt by Acuna et al (2) for details of the design of the Magsat vector magnetometer.

The magnetic field values ( $y_{1}, y_{2}, y_{3}$ ) associated with the three sensor axes are computed from the equations

$$
\begin{aligned}
y_{j} & =u_{j} 7_{j 7}-\sum_{k=1}^{6} u_{j k} w_{j k}+w_{j 8}\left(64-K_{j}\right)^{3}+w_{j 9}\left(64-K_{j}\right)^{5}+a_{j}\left(f_{j}-b_{j}\right) ; j=1,2.3 \\
f_{j} & =(\text { integer given by A/D converter) minus (zero level value) } \\
& \left.=\text { ine measurement }\left(-2048 \leqslant f_{j} \leqslant 2047\right) \text { (zero level value }=2048\right) \\
K_{j} & =\text { integer from } 7 \text {-bit } D / A \text { converter } \\
& =\text { coarse measurement }\left(0<K_{j}<127\right) \\
b_{j} & =\text { bias in } f_{j} \\
a_{j} & =\text { scale factor }
\end{aligned}
$$

$w_{j \mathbf{k}}=$ value associated with $k$ th step of offset generator
$=1000 \times 2^{k-1}: k=1$ to 7
$u_{j k}=0$ or 1
$u_{j 7}=0$ if the most significant bit of $K_{j}$ is 1
$u_{j 7}=1$ if the most significant bit of $K_{;}$is 0
$u_{j k}=$ the $k$ th bit of $K_{j}, u_{j 0}$ being the least significant.
For a detailed explanation of equation (1). the reader is referred to Acuna (3). Nominal values were determined for the 33 parameters $w_{j k}, a_{j}, b_{j}(j=1,2,3 ; k=1,2, \ldots, 9)$ by various calibration procedures prior to laיnch $(2,3)$. The values for the Aconfiguration are given in Table 1. The cubic and quintic terms take into account small non-linearities in the vector magnetometer response funcsion.

An orthogonal reference frame was defined in the sensor assembly such that the angle between a magnetic sensor axis and the corresponding axis in the orthogonal frame was not more than 1 minute of arc. Let ( $x_{1}, x_{2}, x_{3}$ ) be the components of the magnetic field vector in the orthogonal frame and $\alpha_{i j}$ the cosine of the angle between the $i$ th sensor axis and the $j$ th axis of the orthogonal system at the common origin of the two systems. The vector component $\left(x_{1}, x_{2}, x_{3}\right)$ in the orthogonal system and the magnetic field values $\left(y_{1}, y_{2}, y_{3}\right)$ computed from (1) for the non-orthogonal axes of the sensors are related by the equations

$$
\begin{gather*}
y_{1}=a_{11} x_{1}+s_{1} a_{12} x_{2}+a_{13} x_{3}  \tag{2}\\
y_{2}=s_{2} a_{21} x_{1}+a_{22} x_{2}+a_{23} x_{3}  \tag{3}\\
y_{3}=a_{31} x_{1}+s_{3} a_{32} x_{2}+a_{33} x_{3}  \tag{4}\\
s_{1}=\sin v_{1} / v_{1}  \tag{5}\\
s_{2}=\sin v_{2} / v_{2}  \tag{6}\\
s_{3}=\sin v_{3} / v_{3}  \tag{7}\\
v_{1}=a_{12} x_{2} / r_{1} \tag{8}
\end{gather*}
$$

$$
\begin{align*}
& v_{2}=a_{21} x_{1} / r_{2}  \tag{9}\\
& v_{3}=a_{32} x_{2} / r_{3}  \tag{10}\\
& r_{1}=h_{11}+\delta_{2} h_{12} x_{1}+h_{13} \exp \left(\delta_{2} h_{14} x_{1}\right)  \tag{11}\\
& r_{2}=h_{21}+\delta_{1} h_{22} x_{2}+h_{23} \exp \left(\delta_{1} h_{24} x_{2}\right)  \tag{12}\\
& r_{3}=h_{31}+\delta_{2} h_{32} x_{3}+h_{33} \exp \left(\delta_{2} h_{34} x_{3}\right)  \tag{13}\\
& \delta_{1}=1 \text { if } x_{1}>0  \tag{14}\\
&=0 \text { if } x_{1}=0 \\
&=-1 \text { if } x_{1}<0 \\
& \delta_{2}=1 \text { if } x_{2}>0  \tag{15}\\
&=0 \text { if } x_{2}=0 \\
&=-1 \text { if } x_{2}<0 \\
& a_{11}=\left(1-a_{12}^{2}-a_{13}^{2}\right)^{1 / 2}  \tag{16}\\
& a_{22}=\left(1-a_{21}^{2}-a_{23}^{2}\right)^{1 / 2}  \tag{17}\\
& a_{33}=\left(1-a_{31}^{2}-a_{32}^{2}\right)^{1 / 2} \tag{18}
\end{align*}
$$

Nominal values were determined in the laboratory at GSFC prior to launch for the 6 parameters $a_{j k}$ ( $j \neq k$ ) and for the 12 parameters $h_{j k}$. The values for the A-configuration are given in Table 1. For further details see Acuna (2,3).

The $s_{1}, s_{2}$, and $s_{3}$ factors in equations (2), (3), and (4) and equations (5) through (15) by which they are computed are designed to model non-linearities in the vector magnetometer.

## PARAMETER ESTIMATION PROCEDURE

The parameters to be estimated are $w_{j k}(j=1,2,3 ; k=1,2 \ldots, 9), a_{j}, b_{j}(j=1,2.3), a_{12}$, $a_{13}, a_{21}, a_{23}, a_{31}, a_{32}$, and $\mathrm{x}_{\mathrm{ij}}(\mathrm{i}=1,2, \ldots, \mathrm{n}: \mathrm{j}=1,2,3)$ where $\mathrm{n}=$ the number of vector-scalar sets of measurements used in the estimation. If ( $\mathrm{x}_{\mathrm{il}}, \mathrm{x}_{\mathrm{i} 2}, \mathrm{x}_{\mathrm{i} 3}$ ) are the true vector components at time $\tau_{\mathrm{i}}$, then the corresponding true magnitude is given by

$$
\begin{equation*}
f_{i 4}=\left(x_{i 1}^{2}+x_{i 2}^{2}+x_{i 3}^{2}\right)^{1 / 2} . \tag{19}
\end{equation*}
$$

If $\left(f_{i 1}, f_{i 2}, f_{i 3}, f_{i 4}\right)$ is a true set of values at time $\tau_{i}$, we let $\left(\bar{f}_{i 1}, \overline{\mathrm{f}}_{\mathrm{i} 2}, \overrightarrow{\mathrm{f}}_{\mathrm{i} 3}, \overline{\mathrm{f}}_{\mathrm{i} 4}\right)$ be the corresponding set of field measurements at time $r_{i}$. If $\left(y_{i 1}, y_{i 2}, y_{i 3}\right)$ are the true values of the magnetic field along the sensor axes at time $\tau_{i}$, then from equation (1) we have

$$
\begin{align*}
f_{i j}= & {\left[y_{i j}-u_{i j 7} w_{j 7}+\sum_{k=1}^{6} u_{i j k} w_{j k}-w_{i j 8}\left(64-K_{i j}\right)^{3}\right.} \\
& \left.-w_{i, 9}\left(64-K_{i j}\right)^{5}\right] / a_{j}+b_{j} ; i=1 \text { to } n ; j=1,2,3 ; \tag{20}
\end{align*}
$$

where the $y_{i j}$ 's are computed from equations (2) through (18). We abbreviate equations [(20), (2) through (18)] by

$$
\begin{equation*}
f_{i j}=g_{j}\left(x_{i}, p\right) ; j=1,2,3 \tag{21}
\end{equation*}
$$

and equation (19) by

$$
\begin{equation*}
f_{i 4}=g_{4}\left(x_{i}\right) \tag{22}
\end{equation*}
$$

where $g_{j}(j=1,2,3,4)$ are function symbols, $x_{i}=\left(x_{i 1}, x_{i 2}, x_{i 3}\right)^{\prime}$, using' as a symbol for matrix transpose, and $p$ is a column vector with elements $w_{j k}(j=1,2,3 ; k=1$ to 9$), a_{1}, a_{2}, a_{3}, b_{1}, b_{2}, b_{3}$, $a_{12}, a_{13}, a_{21}, a_{23}, a_{31}, a_{32}$ in the order given. We further abbreviate for $\mathrm{i}=1$ to n

$$
\begin{align*}
& f_{i}=\left(f_{i 1}, f_{i 2}, f_{i 3}, f_{i 4}\right)^{\prime},  \tag{23}\\
& \vec{f}_{i}=\left(\bar{f}_{i 1}, \bar{f}_{i 2}, \bar{f}_{i 3}, \bar{f}_{i 4}\right)^{\prime},  \tag{24}\\
& \tilde{f}_{i}=\left(\tilde{f}_{i 1}, \tilde{f}_{i 2}, \tilde{f}_{i 3}, \tilde{f}_{i 4}\right)^{\prime},  \tag{25}\\
& \hat{f}_{i}=\left(\hat{f}_{i 1}, \hat{f}_{i 2}, \hat{f}_{i 3}, \hat{f}_{i 4}\right)^{\prime},  \tag{26}\\
& \tilde{f}_{i j}=g_{j}\left(\tilde{x}_{i}, \tilde{p}\right), j=1,2,3,  \tag{27}\\
& \tilde{f}_{i 4}=g_{4}\left(\tilde{x}_{i}\right),  \tag{28}\\
& \hat{f}_{i j}=g_{j}\left(\hat{x}_{i}, \hat{p}\right), j=1,2,3,  \tag{29}\\
& \hat{f}_{i 4}=g_{4}\left(\hat{x}_{i}\right), \tag{30}
\end{align*}
$$

where $\tilde{\mathbf{x}}_{\mathrm{i}}$ and $\tilde{\mathrm{p}}$ are guesses and $\hat{\mathbf{x}}_{\mathrm{i}}$ and $\hat{\mathrm{p}}$ are estimates for the true values of the parameters $\mathrm{x}_{\mathrm{i}}$ and p . By an estimate of a parameter we mean a value obtained with a statistical estimation procedure.

We assume that all sets of vector-scalar measurements $\vec{f}_{\text {: }}$ have the same error distribution with 0 means, 0 correlations, and standard deviations $\left(\sigma_{1}, \sigma_{2}, \sigma_{3}, \sigma_{4}\right)$, and that the a priori values of $p_{k}$ $(k=1$ to 39 ) are samples of size $!$ with error distributions whose means are all 0 and whose standard deviations are $\omega_{k}(k=1$ to 39$)$.

Suppose the true value associated with the ith measurement is $\psi_{i}$, that $\psi_{i}$ is given as a function of the complete set of true parameters $\theta=\left(\theta_{1}, \theta_{2}, \ldots \theta_{m}\right)$ by $\psi_{i}=g_{1}(\theta)$, and that errors in measurements of $\psi_{\mathrm{i}}$ have mean 0 and standard deviation $\sigma_{\mathrm{i}}$. Then the "weighted sensitivity matrix" $\Gamma$ is defined as the matrix whose element in the ith row and jth column is ( $\left.\partial \psi_{i} / \partial \theta_{j}\right) / \sigma_{i}$. Since we will have only a guess for $\theta, \tilde{\psi}_{\mathrm{i}}=\mathrm{g}_{\mathrm{i}}(\tilde{\theta})$ and $\Gamma$ is approximated by $\tilde{\Gamma}$. We see that $\Gamma$ for our estimation will have $4 n$ rows and $3 n+39$ columns.

Since many of the partial derivatives for our calibration problem are 0 , it will be convenient to partition $\Gamma$.

$$
\begin{gather*}
\Gamma=\left[\begin{array}{lll}
R & S
\end{array}\right],  \tag{31}\\
R=\left[\begin{array}{cccc}
R_{1} & 0 & \ldots & 0 \\
0 & R_{2} & \ldots & 0 \\
\vdots & \vdots & & \vdots \\
0 & 0 & \ldots & R_{n}
\end{array}\right], S=\left[\begin{array}{c}
S_{1} \\
S_{2} \\
\vdots \\
S_{n}
\end{array}\right], \tag{32}
\end{gather*}
$$

and $O$ is a $4 \times 3$ matrix all of whose elements are $0 . R_{i}=\left\{r_{i j k}\right](i=1$ to $n)$ are $4 \times 3$ matrices with $r_{i j k}(j=1$ to $4, k=1$ to 3$)$ as the element in the $j$ th row and $k$ th column with

$$
r_{i j k}=\left(\partial f_{i j} / \partial x_{i k}\right) / a_{j}
$$

The $O$ 's are zero submatrices of $R$ because $\partial f_{i j} / \partial x_{h k}=0$ unless $i=h . S_{i}=\left[s_{i j k}\right](i=1$ to $n)$ are $4 \times 39$ matrices with $s_{i j k}(j=1$ to $4, k=1$ to 39$)$ as the element in the $j$ th row and $k$ th column with

$$
\begin{equation*}
s_{i j k}=\left(\partial \mathrm{f}_{\mathrm{ij}} / \partial \mathrm{p}_{\mathrm{k}}\right) / \sigma_{\mathrm{j}} \tag{33}
\end{equation*}
$$

Note that all the elements in the 4 th row are 0 for $\mathrm{i}=1$ to n .

Let $z$ be a column vector of the true values of the estimated parameters and $\Omega$ a diagonal matrix whose diagonal elements are the variances ior the elements of $\bar{z}$, the a priori approximation of $z$. We define

$$
d=\left[\begin{array}{c}
d_{1}  \tag{34}\\
d_{2} \\
\vdots \\
d_{n}
\end{array}\right], d_{i}=\left[\begin{array}{c}
d_{i 1} \\
d_{i 2} \\
d_{i 3} \\
d_{i 4}
\end{array}\right], d_{i j}=\left(\vec{f}_{i j}-f_{i j}\right) / \sigma_{j} .
$$

Thus d is the weighted vector of residuals. Also

$$
\begin{equation*}
\tilde{\mathrm{d}}_{\mathrm{ij}}=\left(\overline{\mathrm{f}}_{\mathrm{ij}}-\mathrm{f}_{\mathrm{ij}}\right) / \boldsymbol{\sigma}_{\mathrm{j}} \tag{35}
\end{equation*}
$$

The weighted least squares estimate of $z$ is

$$
\begin{gather*}
\hat{z}=\tilde{z}+K^{-1}\left[\tilde{\Gamma}^{\prime} \tilde{d}+\Omega^{-1}(\bar{z}-\tilde{z})\right]  \tag{36}\\
K=\tilde{\Gamma}^{\prime} \tilde{\Gamma}+\Omega^{-1} . \tag{37}
\end{gather*}
$$

For the Magat calibration problem, we assume infinite standard deviations for a priori values of $x_{i j}$. This means the first $3 n$ diagonal elements of $\Omega^{-1}$ are 0 and the remaining 39 are $1 / \omega_{k}^{2}$ ( $k=1$ ts 39). Thus $\Omega^{-1}(\bar{z}-\bar{z})$ is a column vector whose first $3 n$ elements are 0 with the remaining elements given by

$$
\begin{equation*}
\bar{t}_{k}=\left(\bar{p}_{k}-\tilde{p}_{k}\right) / \omega_{k}^{2}: k=1 \text { to } 39 . \tag{38}
\end{equation*}
$$

We note that

$$
t_{k}=\left(\bar{p}_{k}-p_{k}\right) / \omega_{k}^{2}
$$

and define a column vector $t$ with 39 elements whose $k$ th element is given by (39). We also detine a $39 \times 39$ diagonal matrix $J$ with $k$ th diagonal element equal to $1 / \omega_{k}^{2}$. Thus

$$
\Omega^{-1}=\left[\begin{array}{ll}
O_{1} & O_{2}  \tag{40}\\
O_{2}^{\prime} & \mathrm{J}
\end{array}\right]
$$

where $\mathrm{O}_{1}$ and $\mathrm{O}_{2}$ are $3 \mathrm{n} \times 3 \mathrm{n}$ and $3 \mathrm{n} \times 39$ zero matrices. For our calibration problem we also have

$$
z=\left[\begin{array}{c}
x  \tag{41}\\
p
\end{array}\right], x=\left[\begin{array}{c}
x_{1} \\
x_{2} \\
\vdots \\
x_{n}
\end{array}\right], \quad x_{i}=\left[\begin{array}{c}
x_{i 1} \\
x_{i 2} \\
x_{i 3} \\
x_{i 4}
\end{array}\right]
$$

K can be written in partitioned form as

$$
\begin{align*}
K & =\left[\begin{array}{l}
R^{\prime} \\
S^{\prime}
\end{array}\right]\left[\begin{array}{ll}
R & S
\end{array}\right]+\left[\begin{array}{ll}
0 & 0 \\
0 & J
\end{array}\right] \\
& =\left[\begin{array}{ll}
R^{\prime} R & R^{\prime} S \\
S^{\prime} R & S^{\prime} S+J
\end{array}\right]=\left[\begin{array}{ll}
A & B \\
B^{\prime} & C
\end{array}\right], \tag{42}
\end{align*}
$$

where $A=R^{\prime} R, B=R^{\prime} S$, and $C=S^{\prime} S+J$. Note that $A, C$, and $K$ are symmetric. We aiso have

$$
\begin{gather*}
A=\left[\begin{array}{cccc}
A_{1} & 0 & \ldots & 0 \\
0 & A_{2} & \ldots & 0 \\
\vdots & \vdots & & \vdots \\
0 & 0 & \cdots & A_{n}
\end{array}\right], \quad B=\left[\begin{array}{c}
B_{1} \\
B_{2} \\
\vdots \\
B_{n}
\end{array}\right]  \tag{43}\\
A_{i}=R_{i}^{\prime} R_{i}, \quad B_{i}=R_{i}^{\prime} S_{i} \quad(i=1 \text { to } n)  \tag{44}\\
C=\sum_{i=1}^{n} S_{i}^{\prime} S_{i}+J \tag{45}
\end{gather*}
$$

$A_{i}$ is $3 \times 2, B_{i}$ is $3 \times 39, C$ is $39 \times 39, A$ is $3 n \times 3 n, B$ is $3 n \times 39, K$ is $(3 n+39) \times(3 n+39)$. Since $\mathbf{n}$ is typically several thousand, a direct inversion of $\mathbf{K}$ is impractical. The inversion problem is manageable, however, if we take advantage of the structure of $\mathbf{A}$ in a partitioned form of the inverse.

We can now write the bracketed part of equation (36) in the form

$$
\tilde{\Gamma}^{\prime} \tilde{d}+\Omega^{-1}(\tilde{z}-\tilde{z})=\left[\begin{array}{l}
\widetilde{R}^{1}  \tag{46}\\
\tilde{S}^{1}
\end{array}\right] \tilde{d}+\left[\begin{array}{l}
0 \\
\tilde{t}
\end{array}\right]=\left[\begin{array}{l}
\beta \\
\gamma
\end{array}\right]
$$

with the definitions

$$
\begin{gather*}
\beta=\tilde{R}^{\prime} \tilde{\mathrm{d}}  \tag{47}\\
\boldsymbol{\gamma}=\widetilde{\mathbf{S}}^{\prime} \tilde{\mathrm{d}}+\tilde{\mathfrak{t}} \tag{48}
\end{gather*}
$$

where $\beta$ is a column vector with $3 n$ components, $\gamma$ is a column vector with 39 components, and t is defined by equation (39).

Equation (36) becomes

$$
\left[\begin{array}{l}
\hat{x}  \tag{49}\\
\hat{p}
\end{array}\right]=\left[\begin{array}{l}
\tilde{x} \\
\bar{p}
\end{array}\right]+\left[\begin{array}{ll}
A & B \\
B^{\prime} & C
\end{array}\right]^{-1}\left[\begin{array}{l}
3 \\
\gamma
\end{array}\right]
$$

and we recall that $A^{\prime}=A$ and $C^{\prime}=C$.

If we express the inverse in the form

$$
\left[\begin{array}{ll}
A & B  \tag{50}\\
B^{\prime} & C
\end{array}\right]^{-1}=\left[\begin{array}{ll}
D & E \\
E^{\prime} & G
\end{array}\right]
$$

and recall the well-known formulas for the inverse of a partitioned matrix, we have

$$
\begin{gather*}
H=-A^{-1} B,  \tag{51}\\
G=\left(C+B^{\prime} H\right)^{-1},  \tag{52}\\
E=Y G  \tag{53}\\
D=A^{-1}+E H^{\prime} \tag{54}
\end{gather*}
$$

Partitioning $H, E$, and $D$ as

$$
H=\left[\begin{array}{c}
H_{1}  \tag{55}\\
H_{2} \\
\vdots \\
H_{n}
\end{array}\right], \quad E=\left[\begin{array}{c}
E_{1} \\
E_{2} \\
\vdots \\
E_{n}
\end{array}\right], \quad D=\left[\begin{array}{cccc}
D_{11} & D_{12} & \cdots & D_{1 n} \\
D_{21} & D_{22} & \cdots & D_{2 n} \\
\vdots & \vdots & & \\
D_{n 1} & D_{n 2} & \cdots & D_{n n}
\end{array}\right] \text {, }
$$

$D_{i j}^{\prime}=D_{j i}, i=1$ to $n, j=1$ to $n$, and using (43) we have

$$
\begin{gather*}
H_{i}=-A_{i}^{-1} B_{i} ; i=1 \text { to } n .  \tag{56}\\
G=\left(C+\sum_{i=1}^{n} B_{i}^{\prime} H_{i}\right)^{-1} .  \tag{57}\\
E_{i}=H_{i} G ; i=1 \text { to } n,  \tag{58}\\
D_{i i}=A_{i}^{-1}+E_{i} H_{i}^{\prime} ; i=1 \text { to } n,  \tag{59}\\
D_{i j}=E_{i} H_{i}^{\prime}: i=i \text { to } n-1: j=i+1 \text { to } n . \tag{50}
\end{gather*}
$$

Equation (49) can be written as two equations

$$
\begin{align*}
& \hat{\mathbf{x}}=\tilde{\mathbf{x}}+\mathrm{D} \beta+\mathrm{E}_{\boldsymbol{\gamma}}  \tag{61}\\
& \hat{\mathbf{p}}=\tilde{\mathbf{p}}+E^{\prime} \beta+\mathrm{G} \mathrm{\gamma} . \tag{62}
\end{align*}
$$

## From equation (47) we obtain

$$
\beta=\left[\begin{array}{c}
\beta_{1}  \tag{63}\\
\beta_{2} \\
\vdots \\
\beta_{n}
\end{array}\right], \quad \beta_{i}=\left[\begin{array}{c}
\beta_{i 1} \\
\beta_{i 2} \\
\beta_{i 3}
\end{array}\right]=\tilde{\mathrm{R}}_{\mathrm{i}}^{\prime} \tilde{\mathrm{d}}_{\mathrm{i}}
$$

We can now write $(61)$ and (62) in the forms

$$
\begin{gather*}
\hat{x}_{i}=\bar{x}_{i}+E_{i} \gamma+\sum_{j=1}^{n} D_{i j} \beta_{j}, i=1 \text { to } n  \tag{64}\\
\hat{p}=\tilde{p}+G \gamma+\sum_{i=1}^{n} E_{i}^{\prime} \beta_{i} \tag{65}
\end{gather*}
$$

We note that $E_{i}$ is $3 \times 39, D_{i j}$ is $3 \times 3, G$ is $39 \times 39, G^{\prime}=C$, and recall that $D_{i j}^{\prime}=D_{i j}$ for all $i$ 's and j's.

The covariance matrices associated with the estimates $\hat{\mathrm{x}}_{\mathrm{i}}(\mathrm{i}=1$ to n$)$ and $\hat{\mathrm{p}}$ are given by

$$
\begin{align*}
& \operatorname{cov}\left(\hat{x}_{i}, \hat{x}_{j}\right)=D_{i j},  \tag{66}\\
& \operatorname{cov}\left(\hat{x}_{i}, \hat{p}\right)=E_{i},  \tag{67}\\
& \operatorname{cov}(\hat{p}, \hat{p})=G \tag{68}
\end{align*}
$$

Correlation matrices can be obtained from the covariance matrices.

## RESULTS OF A CALIBRATION

The calibration results given below were obtained by exercising the computer program on a small sutset of the data recorded on Nov. 5, 1979. The number of vector-scalar sets used in the calibration was 1247, approximately equally spaced in time. Table 1 gives the ground-based calibration values for the system parameters along with the stardard deviations used to compute the weighte for these a priori values in the least-squares algorithm. Except for the $a_{i j}$ parameters, these standard deviations are larger than the realistic ones for the ground-based calibrations. This allows for the fact that the crue values of the parameters may be significantiy different after launch than before launch. The standard deviations for the a pnoti values of $a_{12}, a_{13}, a_{21}, a_{23}, a_{31}, a_{32}$ are held at the ground-based calibration values because of strong correlations between the errors in the least squares solutions for these parameters. Table 1 also shows the differences between the ground-based calibration values and the values estimated from the Nov. 5th ('79) data. The final column of Table 1 gives the standard deviations of the estimated parameters. The three parameters $w_{25}, w_{26}$, and $w_{27}$ enter the least squares procedure only as the linear combination $w_{27}-w_{26}-w_{25}$, due to the fact that the magnetic field components along the second sensor axis had maximum and minimum values of 10228.5 and -15246.1 . To obtain a solution, $w_{25}$ and $w_{26}$ were held fixed by giving them very small a priori standard deviations.

Table $\mathbf{2}$ gives 5 examples from the $\mathbf{1 2 4 7}$ sets of vector-scalar measurements used in the leastsquares calibration for Nov. $5,1979 . K_{i}$ and $\bar{f}_{i}(i=1,2,3)$ are the coarse and fine values recorded for the three magnetic field components and $\overline{\mathrm{f}}_{4}$ is the field magnitude measurement. The $\mathrm{K}_{\mathrm{i}}$ 's are error-free and are not considered as measurements in the least-squares estimation.procedure.

Table 1

| Parameter | Pre-iaunch Value | Standard Deviation | Estimated Correction | Standard Deviation |
| :---: | :---: | :---: | :---: | :---: |
| $w_{11}$ | 999.7 | 20 | -0.5 | . 05 |
| $w_{12}$ | 1998.7 | 20 | -0.3 | . 05 |
| $w_{13}$ | 3996.7 | 20 | -0.4 | . 06 |
| $\mathrm{w}_{14}$ | 7993.3 | 20 | -1.1 | . 08 |
| W15 | $15986 . \mathrm{i}$ | 20 | -1.9 | . 14 |
| $\mathrm{w}_{16}$ | 31973.8 | 20 | -4.5 | . 29 |
| $\mathrm{w}_{17}$ | 63947.5 | 20 | -10.2 | . 25 |
| ${ }^{\mathbf{w}} 18$ | . 963 E-5 | . $32 \mathrm{E}-5$ | -0.4 E-6 | . 31 E-5 |
| W19 | -. $603 \mathrm{E}-9$ | . $32 \mathrm{E}-9$ | -0.5 E-11 | . $32 \mathrm{E}-9$ |
| $w_{21}$ | 1004.3 | 20 | 0.7 | . 16 |
| $w_{22}$ | 2008. 7 | 20 | -0.3 | . 17 |
| $\mathrm{w}_{23}$ | 4016.8 | 20 | -0.5 | . 25 |
| ${ }^{\mathbf{w}}$ | 8037.6 | 20 | -2.9 | . 34 |
| $\mathrm{w}_{25}$ | 16069.6 | 1 E-12 | 0.0 | 1 E-12 |
| ${ }^{\mathbf{w}} \mathbf{2 6}$ | 32143.9 | 1 E-12 | 0.0 | 1 E-12 |
| $\mathbf{w}^{27}$ | 64287.9 | 20 | 0.1 | . 46 |
| ${ }^{\mathbf{w}} \mathbf{2 8}$ | . 436 E-4 | . 32 E 4 | -0.7 E-S | . 31 E-4 |
| $\mathbf{w}^{29}$ | -. $649 \mathrm{E}-9$ | . $32 \mathrm{E}-9$ | -0.3 E-12 | . $32 \mathrm{E}-9$ |
| $\mathbf{w}_{31}$ | 997.3 | 20 | 0.5 | . 03 |
| $\mathbf{w}_{32}$ | 1995.1 | 20 | 1.0 | . 03 |
| ${ }^{\mathbf{w}} \mathbf{3}$ | 3989.8 | 20 | 1.9 | . 04 |
| w34 | 7979.6 | 20 | 3.0 | . 05 |
| $\mathrm{w}_{35}$ | 15958.5 | 20 | 4.8 | . 09 |
| $\mathbf{w}_{36}$ | 31918.2 | 20 | 8.8 | . 14 |
| W37 | 63844.2 | 20 | 18.5 | . 12 |
| $\mathbf{w}_{38}$ | -. 508 E-5 | . $32 \mathrm{E}-6$ | 0.6 E-5 | . 31 E-6 |
| ${ }^{\mathbf{W}} 39$ | . $167 \mathrm{E}-8$ | .32 E-9 | 0.3 E-9 | . $26 \mathrm{E}-9$ |
| $\mathrm{b}_{1}$ | 4.0 | 20 | 6.8 | . 24 |
| $b_{2}$ | 13.0 | 20 | 4.6 | . 48 |
| $\mathrm{b}_{3}$ | 12.0 | 20 | 0.1 | . 19 |
| $\mathrm{a}_{1}$ | . 98248 | 1 E-4 | 0.4 E-4 | .5 E-4 |
| $\mathrm{a}_{2}$ | . 992226 | 1 E-4 | -2.1 E-4 | .9E4 |
| ${ }^{3}$ | . 98322 | 1 E-4 | -1.7 E-4 | . 4 E4 |
| $\left\llcorner a_{12}\right.$ | -228.5 | 6.5 | -16.8 | 4.2 |
| $\left\llcorner a_{13}\right.$ | 168.0 | 2.1 | 1.0 | 1.5 |
| $\mathrm{La}_{21}$ | -612.3 | 6.5 | 8.0 | 4.7 |
| $\stackrel{L a}{23}$ | 443.3 | 2.1 | 2.7 | 2.0 |
| $\mathrm{La}_{31}$ | 483.1 | 2.1 | 1.0 | 1.5 |
| La32 | 589.9 | 6.5 | 28.0 | 2.1 |

Table 1
(unestimated parameters)


Table 2

| $K_{1}$ | $\bar{f}_{1}$ | $K_{2}$ | $\bar{f}_{2}$ | $K_{3}$ | $\bar{f}_{3}$ | $\bar{f}_{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 55 | -900 | 62 | -655 | 14 | -563 | 51453.9 |
| 32 | -376 | 59 | -417 | 65 | 522 | 32740.9 |
| 74 | 855 | 49 | 812 | 99 | -462 | 38780.4 |
| 45 | 754 | 61 | 613 | 71 | 719 | 19981.9 |
| 54 | 136 | 73 | -1012 | 106 | 968 | 44726.4 |

In the remainder of this section will present some computer results obtained for these 5 vector-scalar sets as well as for all 1247 sets.

Table 3 gives the distribution of the absolute values of $64-K_{i}$ for the 1247 data set.
Table 3

| Number | $64-\mathrm{K}_{1}$ | $64 \mathrm{~K}_{2}$ | $64-\mathrm{K}_{3}$ |
| :---: | :---: | :---: | :---: |
| $=0$ | 21 | 46 | 11 |
| $=1$ | 40 | 177 | 14 |
| $>1 \&<3$ | 68 | 287 | 22 |
| $>3 \&<7$ | 132 | 437 | 65 |
| $>7 \&<15$ | 367 | 300 | 131 |
| $>15 \&<31$ | 603 | 0 | 339 |
| $>31 \&<63$ | 16 | 0 | 665 |

The values $\left(y_{1}, y_{2}, y_{3}\right)$ of the magnetic field along the three sensor axes are obtained by substituting the pre-launch values of 33 parameters and the data in Table 2 into equation (1). The results are given in the first 3 columns of Table 4. The fourth column of Table 4 gives $\bar{f}_{4}-f_{c}$, where $f_{c}$ is the approximate vector magnitude computed from the components $y_{1}, y_{2}, y_{3}$, using a nonPythagorean formula.

Table 4

| $y_{1}$ | $y_{2}$ | $y_{3}$ | $\bar{f}_{4}-f_{c}$ |
| :---: | :---: | :---: | :---: |
| -9881.2 | -2671.5 | -50437.0 | 15.1 |
| -32347.4 | -5447.8 | 1507.1 | 1.5 |
| 10828.3 | -14274.7 | 34452.8 | 2.5 |
| -18247.6 | -2417.6 | 7685.7 | 4.0 |
| -9862.3 | 8027.5 | 42840.6 | 10.0 |

When the values of $y_{1}, y_{2}$, and $y_{3}$ from Table 4 and the pretaunch values of $a_{12}, a_{13}, a_{21}, a_{23}$, $a_{31}$, and $a_{32}$ are substituted into equations (2) through (18), the vector components $\left(i_{1}, x_{2}, x_{3}\right)$ in the orthogonal system can be computed. They are given in the first three columns of Table 5. The fourth column of Table 5 gives $\bar{f}_{4}-\bar{f}$, where $\bar{f}$ is the vector magnitude computed from the components $\bar{x}_{1}, \bar{x}_{2}, \bar{x}_{3}$, using the Pythagorean formula (19). Differences between the fourtn columns of Table 4 and Table 5 are due to the fact that $s_{1}, s_{2}$, and $s_{3}$ differ slightiy from 1.

Table 5

| $\tilde{x}_{1}$ | $\tilde{x}_{2}$ | $\tilde{x}_{3}$ | $\bar{f}_{4}-\tilde{f}_{4}$ |
| :---: | :---: | :---: | :---: |
| -9843.0 | -2592.1 | -50421.7 | 15.1 |
| -32354.8 | -5537.8 | 1567.1 | 3.1 |
| 10784.8 | -14316.9 | 34387.2 | 2.0 |
| -13256.7 | -2486.7 | 7721.4 | 4.2 |
| -9888.6 | 7906.2 | 42886.6 | 10.1 |

The values of $\tilde{\mathrm{x}}_{1}, \tilde{\mathrm{x}}_{2}$, and $\tilde{\mathrm{x}}_{3}$ in Table 5 are used as first guesses in the ieast squares estimation procedure.

Tabie 6 gires the values of $\vec{x}_{1}-\hat{x}_{1}, \vec{x}_{2}-\hat{\mathbf{x}}_{2}$, and $\overline{\mathrm{x}}_{3}-\hat{\mathrm{x}}_{3}$ in the first three columns, where ( $\hat{\mathbf{x}}_{1}, \hat{\mathbf{x}}_{2}, \hat{\mathrm{x}}_{3}$ ) are sstimated values and ( $\tilde{\mathrm{x}}_{1}, \tilde{\mathrm{x}}_{2}, \tilde{\mathrm{x}}_{3}$ ) are the initial guesses in Table 5. Column 4 of Table 6 gives the values of $\bar{f}_{4}-\hat{f}_{4}$, where $\hat{f}_{4}$ was computed from $\hat{i}_{1}, \hat{x}_{2}, \hat{x}_{3}$ ) using the Pythagorean equation. We note that the estimated field vectors have magnitudes much closer to the measured magnitudes than the guesses ( $\tilde{\mathrm{x}}_{1}, \overline{\mathrm{x}}_{2}, \overline{\mathrm{x}}_{3}$ ) computed from the vector field measurements $\left(\bar{f}_{\mathrm{ii}}, \overline{\mathrm{f}}_{\mathrm{i}}, \overline{\mathrm{f}}_{\mathrm{iz}}\right.$ ) and the a priori parameter values. The means and root-mean-squares of the 1247 values of $\tilde{x}_{1}-\hat{x}_{1}$. $\tilde{x}_{2}-\hat{x}_{2}, \bar{x}_{3}-\hat{x}_{3}$, and $\bar{f}_{4}-\hat{f}_{4}$ are given in Table 7.

Let $\left(\check{x}_{1}, \check{x}_{2}, \check{x}_{3}\right)$ be the values computed by substituting the estimated 39 parameters in equation (1) through (18) and solving for ( $x_{1}, x_{2}, x_{3}$ ), and let $\check{f}_{4}=\left(\check{x}_{1}^{2}+\check{x}_{2}^{2}+\check{x}_{3}^{2}\right)^{1 / 2}$. Table 8 shows the differences between these estimates for $\left(\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}, \mathrm{f}_{4}\right)$ and the least squares estimates $\left(\hat{\mathrm{x}}_{1}, \hat{\mathrm{x}}_{2}, \hat{\mathrm{x}}_{3}\right.$, $\hat{f}_{4}$.

Table 6

| $\bar{x}_{1}-\hat{x}_{1}$ | $\tilde{x}_{2}-\hat{i}_{2}$ | $\tilde{x}_{3}-\hat{x}_{3}$ | $\bar{f}_{4}-\hat{f}_{4}$ |
| :---: | :---: | :---: | :---: |
| 5.1 | 5.1 | 14.6 | -0.5 |
| 2.0 | 5.6 | 2.5 | 0.3 |
| 8.9 | 0.6 | -5.9 | -1.0 |
| 3.7 | 5.6 | -0.7 | -0.2 |
| 5.6 | 4.1 | -11.0 | -1.0 |

Table 7

|  | $\tilde{x}_{1}-\hat{x}_{1}$ | $\tilde{x}_{2}-\hat{x}_{2}$ | $\dot{x}_{3}-\hat{x}_{3}$ | $\bar{f}_{4}-\hat{f}_{4}$ |
| :--- | :---: | :---: | :---: | :---: |
| MEAN | 7.0 | 3.2 | 1.7 | -0.001 |
| PMS | 3.0 | 1.8 | 9.8 | 0.52 |

Table 8

| $\hat{x}_{1}-\check{x}_{1}$ | $\hat{x}_{2}-\check{x}_{2}$ | $\hat{x}_{3}-\check{x}_{3}$ | $\hat{i}_{4}-\grave{f}_{4}$ |
| :---: | :---: | :---: | :---: |
| 0.1 | 0.0 | 0.4 | -0.4 |
| -0.2 | -0.1 | 0.0 | 0.2 |
| -0.2 | 0.3 | -0.6 | -0.7 |
| 0.1 | 0.0 | 0.0 | -0.1 |
| 0.1 | -0.1 | -0.7 | -0.7 |

The 1247 values of $\overline{\mathrm{f}}_{4}-\check{\mathrm{f}}_{4}$ have a mean of -.002 and an RMS of .90 , which compare very well with the values of -.001 and .52 in Table 7 for $\bar{f}_{4}-\hat{f}_{4}$. Thus least squares estimation is not needed for every day's data if a loss of about 0.5 gamma in accuracy can be accepted. We must consider that on the GSFC $360 / 91$, full estimation for 1247 data sets required 8 minutes, while computation of ( $x_{1}, x_{2}, x_{3}$ ) from equations (1) through (18) required 0.4 of a minute, a ratio of 30 to 1 .

The first three columns of Table 9 give the residuals $\overline{\mathrm{f}}_{\mathrm{j}}-\hat{\mathrm{f}}_{\mathrm{j}} \mathrm{j}=1,2,3$. where the $\overline{\mathrm{f}}_{\mathrm{j}}$ 's are the fine measurements given in Table 2 and the $\hat{\mathrm{f}}_{\mathrm{j}}$ are obtained by substituting the estimated parameters in equation (20) and computing ( $y_{1}, y_{2}, y_{3}$ ) for (20) by substituting ( $\hat{x}_{1}, \hat{x}_{2}, \hat{x}_{3}$ ), $\hat{a}_{12}, \hat{a}_{13}, \hat{a}_{21}$,
$\hat{a}_{23}, \hat{a}_{31}$, and $\hat{a}_{32}$ in equations (2) through (18). The fourth column of Table 9 repeats the fourth column of Table 6. The 1247 value sets of the numbers given for a sample of 5 in 7 : $: 9$ have means and RMS's given in Table 10. Table 10 is an indication of the goodness-of-fit of the solution to the measurements.

Table 9

| $\bar{f}_{1}-\hat{f}_{1}$ | $\bar{f}_{2}-\hat{f}_{2}$ | $\bar{f}_{3}-\hat{f}_{3}$ | $\bar{f}_{4}-\hat{\mathfrak{f}}_{4}$ |
| :---: | ---: | :---: | :---: |
| -0.1 | -0.0 | -0.4 | -0.5 |
| 0.2 | 0.0 | 0.0 | 0.3 |
| 0.2 | -0.3 | 0.6 | -1.0 |
| -0.1 | 0.0 | 0.0 | -0.2 |
| -0.2 | 0.1 | 0.7 | -1.0 |

Table 10

| $\bar{f}_{1}-\hat{f}_{1}$ | $\bar{f}_{2}-\hat{f}_{2}$ | $\bar{f}_{3}-\hat{f}_{3}$ | $\bar{f}_{4}-\hat{f}_{4}$ |
| :---: | :---: | :---: | :---: |
| -.001 | -.0009 | .0002 | -.001 |
| 0.19 | 0.06 | 0.32 | 0.52 |

## FURTHER CALIBRATION RESULTS

Knowing that under space conditions the calibration parameters determined pre-launch would vary, it was necessary to determine criteria for a first in-flight calibration, to be used as a control. to see any trends and shifts in subsequent calibrations typifying changes in the instrument. November 5th data was chosen for the control calibration under the criteria that (1) the instrument should have had time to shift mechanically, due to stresses in a space vacuum system, into some "permanent" position after three days in orbit, and (2) data were taken continuously throughout the day. producing a globally distributed data set for sampling.

The November 5th calibration (see Table 11; shows changes from pre-launch calibration (especially in the $\mathbf{z}$-axis), which illustrate the changes in the instrument. Calibrations were then made at four day intervals beginning November 2. 1979 and ending January 21, 1980 (excluding January 1,

Table 11

| Parameter | Pre-Launch Value | November 5th Value | Estimated Correction | Standard Deviation |
| :---: | :---: | :---: | :---: | :---: |
| $W_{11}$ | 999.7 | 999.6 | -0.1 | 0.14 |
| $w_{12}$ | 1998.7 | 1998.3 | 0.4 | 0.14 |
| $w_{13}$ | 3996.7 | 3997.0 | 0.3 | 0.15 |
| $W_{14}$ | 7993.3 | 7993.2 | -0.1 | 0.16 |
| $w_{15}$ | 15986.1 | 15985.9 | -0.2 | 0.24 |
| $w_{16}$ | 31973.8 | 31973.4 | -0.4 | 0.43 |
| ${ }^{W} 17$ | 63947.5 | 63940.0 | -7.4 | 0.39 |
| ${ }^{\mathbf{W}} 18$ | . 963 E.S | . 882 E-5 | -. 803 E-6 | . 98 E-6 |
| ${ }^{\mathbf{w}} 19$ | -. 603 E-9 | -.611 E-9 | -.829 E-11 | .10 E-9 |
| $w_{21}$ | 1004.3 | 1005.3 | 1.1 | 0.42 |
| ${ }^{\mathbf{w}} \mathbf{2}$ | 2008.7 | 2009.0 | 0.4 | 0.41 |
| ${ }^{\mathbf{w}}$ | 4016.8 | 4017.4 | 0.5 | 0.43 |
| ${ }^{*} 24$ | 8037.6 | 8036.7 | -0.9 | 0.43 |
| ${ }^{W}$ | 16069.6 | 16068.8 | -0.8 | 0.88 |
| $\mathrm{w}^{\mathbf{2}}$ | 32143.9 | 32143.2 | -0.8 | 0.88 |
| $w^{\mathbf{W}} \mathbf{}$ | 64287.9 .436 E. | 64288.7 | 0.8 $-910 \mathrm{E}-8$ | ${ }_{0}^{0.88}$ |
| ${ }^{*} 28$ | -436 E-649 E-9 | .436 E-4 -.649 E-9 | -. 910 E-8 | .10 E-5 |
| $W_{31}$ | 997.3 | 997.7 | 0.4 | 0.10 |
| W32 | 1995.1 | 1996.0 | 1.0 | 0.10 |
| W33 | 3989.8 | 3991.6 | 1.8 | 0.11 |
| $\mathbf{w}_{34}$ | 7979.6 | 7982.7 | 3.0 | 0.12 |
| $\mathbf{w}_{35}$ | 15958.5 | 15963.1 | 4.6 | 0.16 |
| w ${ }^{\text {w }}$ | 31918.2 | 31926.7 | 8.5 | 0.22 |
| $W_{37}$ | 63844.2 | 63862.6 | 18.4 | 0.24 |
| $W_{38}$ $W_{39}$ | $-.508 \mathrm{E}-5$ $.167 \mathrm{E}-8$ | -. 217 E-5 | 0.291 E-5 0.628 E-10 | .96 E-6 10 |
| $W_{39}$ $b_{1}$ | $.167 \mathrm{E}-8$ 4.0 | 173 E-8 7.8 | $0.628 \mathrm{E}-10$ 3.8 | $.10 \mathrm{E}-9$ 0.35 |
| $\mathrm{b}_{2}$ | 13.0 | 13.9 | 0.9 | 0.44 |
| $\mathrm{b}_{3}$ | 12.0 | 12.7 | 0.7 | 0.31 |
| $\mathrm{a}_{1}$ | . 98248 | . 98248 | 0.0 | . 50 E-5 |
| $\mathrm{a}_{2}$ | . 99226 | . 99226 | 0.0 | . 50 E-5 |
| $\mathrm{a}_{3}$ | . 98322 | . 98322 | 0.0 | . $50 \mathrm{E}-5$ |
| $L a_{12}$ | -228.5 | -228.3 | -0.2 | 0.99 |
| $L a_{13}$ | 168.0 | 168.0 | 0.01 | 0.033 |
| $L a_{21}$ | -612.3 | -612.2 | -0.1 | 1.0 |
| $L^{L} a_{23}$ | 443.3 | 443.3 | 0.01 | 0.033 |
| $L^{2} a_{31}$ | 483.1 | 483.1 | 0.01 | 0.033 |
| $L^{-32}$ | 589.9 | 595.6 | 5.7 | 0.95 |

1980, at this writing). The first 29 plots following this discussion show graphically the changes from the November 5th calibration for those parameters which changed more than 0.1 gamma (or 0.1 arcseconds, according to which parameter is being considered). A key to the symbols used in the plots can be located on the page before the plots. Two sets of parameters, $w_{16}-w_{17}$, and $w_{25}-w_{26}-w_{27}$ are piotted together since correlations between these parameters are so large that the parameters are believed to be mathematically inseparable.

Examination oit the calibration parameter plots shows trend-line changes with time, usually in one direction (i.e., non-oscillating), with the exceprion of several days calibrations. Upon further examination, these caibration exceptions are usually found common in several plots. One reason for the offset values is the status of the scalar sensors, $\mathbf{A}$ and $\mathbf{B}$. It is found that a different calibration will result for each of the following cases: (1) sensor A on only, (2) sensor B on only, or (3) both sensor $A$ and $B$ cn. Examination of the plots implies this to be the case in most instances. For those instances where the sensor status does not apply, other reasons such as diversion in the data distribution, or temperature variation of the sensors and platform may be part of an explanation, but no conclusions have been formalized at this time.

The second set of plots describe some statistics about the relationship between the scalar values and their associated vector magnitudes, for samples of data at four day intervals about the date of the calibration. The data samples had the calibration in question applied to them, and the mean, standard deviation about the mean, and the maximum of the differences between scalars and vector magnitudes were determined. Study of these plots shows conclusively the dependence of the calibration and the calibrated data on the scalar sensor status.

## Key to Symbols used in Plots

A: Scalar Sensor A on only
B: Scalar Sensor B on only
C: Scalar Sensors $A$ and $B$ both on
M: Mean of difference values between scalars and vector magnitudes

R: Standard deviation about the mean of difference values between scalars and vector magnitudes
deviation of wil estimate from november 5th estimate

deviation of wir estimate from november 5th estimate

deviation of wis estimate from november Sth estimate

23

deviation of wl4 estimate friol november 5th estimate

deviation cf his estimate from noverber 5th estimate

25

deviation of hig estimute from noverber 5th estimate

26

deviation of wit estimate from moverber Sth estimate


deviation of w2l estimate from novemeer 5th estimate

deviation of he2 estimate from noverger 5th estimate

deviation of w23 estimate from novereer 5th estimate

deviation of h24 estimate from noverber 5th estimate


deviation of wag estimate from noverber Sth estimate



deviation of w3i estimate from november 5th estimate

deviation of h32 estimate from noverber 5th estimate

deviation of wa3 estimate from noverber STh estimate


## deviation of h34 estimate from noverber 5Th estimate


deviation of w3s estimate from noverber Sth estimate

deviation of wbg estimate from noverber Sth estimate

deviation of h37 estimate from novereer 5th estimate

deviation of bi estimate from novereer 5th estimate

deviation of be estimate from noverber Sth estimute


FOUR DAY INTERVALS BEGINNING NOV 2. 1979
deviation of b3 estimate from november 5th estimate

deviation of ald estimate from noverger 5th estimate

deviation of ari estimute from noverger sth estimate


DEVIATION OF A32 ESTIMATE FROM NOVEIEER STH ESTIMATE

statistics for cal ibrayion of novereer 2. 1979


FOUR DAY INTERYNLS

Statistics for chlibration of noverber 6. 1979

statistics for cal igration of noverber 10. 1979


STATISTICS FOR CNLIBRATION OF NOVEMBER 14. 1979


FOUR DAY INTERVNLS

Statistics for chl ibration of noverber 18. 1979


STATISTICS FOR CALIERATION OF NOVEMEER 22. 1979


Statistics for caligration of noveraer 26. 1979


Statistics for caligration of noveteer 30. 1979


Statistics for chligration of december 4. 1979


STATISTICS FOR CNLIERATION OF DECDBER B. I979

statistics for calibration of decerber 12. 1979

statistics for cal ibration of decerger 16. 1979

61


STATISTICS FOR CNLIERATION OF DECEMBER 20. 1979


## STATISTICS FOR CALIBRATION OF DECERBER 24. 1979



STATISTICS FOR CALIBRATION OF DECEIBER 28. 1979

statistics for caligration of january 5. 1980


STATISTICS FOR CNLIERATION OF JMUNRY 9. 1980


Statistics for chlieration of maunay 13. 1980


STATISTICS FOR CNLIBRATION OF JANUARY 17. 1980


STATISTICS FOR CALIERATION OF JANUARY 21. 1980

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## SUMMARY

In summary, the results we have described indicate first that the calibration parameters of the MAGSAT vector magnetometer, particularly those associated with the third ( $\mathbf{z}$ ) axis, changed significantly between the pre-launch calibration and the first post-launch calibration. This can possibly be attributed to slight mechanical shifts due to the difference in stresses between a vacuum and nonvacuum envidonment. Second, some of the calibration parameters underwent small and slow systematic changes during the mission lifetime; and third, the calibration parameters determined from intercomparison with the scalar instrument are slightly different ( $\sim 1$ samma) between the cases when both sensor $A$ and sensor $B$ of the scalar magnetometer are operating, when only sensor $A$ is operating, and when only sensor B is operating. There are three possible causes for differences in vector calibration with respect to the scalar sensor configuration:

1. The change in null-zone configuration of the scalar instrument resuits in a bias in the distribution of vector directions visible in the calibration. This could cause imprecise determination of some calibration parameters, particularly those defining cross-coupling between axes of the vector instrument.
2. With only one sensor operabie, heading errors on the scalar magnetumeter are increased. Such errors will be geographically preferential, which means they will also occur at systematic directions of the field with respect to the vector magnetometer. That is, for certain field directions the scalar magnetometer would give an erroneous reading of up to 1.5 gammas, with a corresponding change in the determination of the vector magnetometer calibration parameters.
3. An interference of the scalar magnetometer with the vector magnetometer which changes with the scalar magnetometer configuration. Such an interference was experienced during pre-launch magnetic testing. In that case, the R. F. excitation signal from the scalar instrument caused a sensitivity shift in the vector instrument. This was most pronounced in the second ( $y$ ) axis. still noticeable in the third ( $\mathbf{z}$ ) axis, and least discernable in the first ( x ) axis. Subsequent radio frequency shielding reduced the effect to less than one gamma at full scale.

In addition to knowing that the calibration parameters differ with the scalar configuration, we know that the calibration constants determined under a specific scalar configuration result in a larger mean difference when applied to data with a different scalar configuration. This would seem to rule out the first hypothesized cause for the difference. We need, however, to distinguish between causes 2 and $\mathbf{3}$ because if cause $\mathbf{2}$ is correct then we should utilize the more accurate calibrations fr m times when both sensors $A$ and $B$ were operating. If, on the other hand, radio frequency interference differences are affecting the measurements, then the calibration actually does change and the calibration parameters should be those derived from the sca!ar data with the same configuration that was in effect at the time of the data being calibrated.

Examination of the calibration parameter plots indicates that all axes are affected by scalar configuration changes. This seems to indicate that cause $\mathbf{2}$ is operable. However, more analysis is required before a final verdict is in.

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## 16. Abutract

The Magnetic Field Satellite (Magsat) was launched on Oct. 30, 1979 into a nearly polar, sun-synchronous orbit, carrying a scalar magnetometer and a vector magne tometer. The satellite reentered the earth's atmosphere on June 11, 1980, having measured and transmitted more than three complete sets of global magnetic field data. The data obtained from the mission will be used primarily to compute a currently accurate model of the earth's main magnetic field, to update and refine world and regional magnetic charts, and to deveiop a global scalar and vector crustal magnetic anomaly map.

This report describes the in-flight calibration procedure used for 39 vector magnetometer system parameters and gives results obtained from some data sets and the numerical studies designed to evaluate the results.

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