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# Block IV Receiver Tracking Loop Performance in the Presence of a CW RFI

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This article is a part of a continuing effort to develop models to characterize the behavior of the Deep Space Network (DSN) Receiving System in the presence of a radio frequency interference (RFI). A model that allows one to predict the tracking performance of the Block IV receiver in the presence of a CW RFI is discussed. Experimental and analytical results are provided for a typical DSN operational mode. A curve of protection criteria that extends the current CCIR recommendation is also presented.

### I. Introduction

Communications systems such as the Deep Space Network (DSN) often are operated in a radio frequency interference (RFI) environment. The high sensitivity required to achieve planetary communications makes the DSN highly sensitive to RFI. To prevent RFI from causing detrimental effects on the DSN Receiving Systems, it is necessary to first determine their RFI susceptibility characteristics. This study is a part of a continuing effort to develop models to characterize the RFI susceptibility of various parts of the DSN Receiving System. When a very strong RFI signal is present at the DSN Receiving System input, it can saturate the maser and various parts of the receiver, producing harmonics, gain compression, and other nonlinearities. These characteristics have been studied in Ref. 1. In this article, we are dealing with CW RFIs that are close in frequency to the carrier frequency. Due to the high sensitivity of the carrier tracking loop, this type of RFI will degrade the carrier loop performance much before saturation effects become noticeable. It is the purpose of this study to develop an analytical model that will allow us to predict the tracking performance of the Block IV receiver when operated in the presence of a CW RFI in the receiver passband. It is hoped that this model together with the saturation models presented in Ref. 1 can provide information necessary for RFI protection and coordination.

## II. Effects of a CW RFI on a Phase Locked Loop

The effects of a CW RFI on a phase locked loop have been studied and documented by various authors (Refs. 2 through 7). The analysis on which the model to be developed is based is, however, from Refs. 2, 3, and 8. For the purpose of self-containment, necessary steps are repeated here. Basically when an RFI is present at the input of a phase locked loop with a fixed frequency offset from the desired signal, it creates a constant phase offset (static phase error) and a beat note that phase modulates the voltage-controlled oscillator (VCO) output. The beat frequency is the RFI offset frequency. Specifically, if we assume that the loop is initially locked to a desired signal with root-mean-square (rms) amplitude A and frequency  $\omega_c/2\pi$ , and that a CW RFI, with rms amplitude B and offset frequency  $\Delta \omega/2\pi$ , is present at the input, then the input to the loop is

$$x(t) = \sqrt{2}A\sin(\omega_c t) + \sqrt{2}B\sin(\omega_c + \Delta\omega)t \qquad (1)$$

Equation (1) can be rewritten as

$$x(t) = A(t)\sin(\omega_c t + \delta(t))$$
(2)

where:

$$A(t) = \sqrt{2}A \left(1 + R^2 + 2R \cos \Delta \omega t\right)^{1/2}$$
(3)

$$\delta(t) = \tan^{-1} \left( R \sin \left( \Delta \omega t \right) / (1 + R \cos \left( \Delta \omega t \right) ) \right)$$
(4)

and R = B/A is the interference-to-signal ratio (ISR).

Assuming that the phase locked loop remains in lock with the desired signal with a phase error  $\phi(t)$ , then the VCO output is

$$r(t) = \sqrt{2}\cos\left(\omega_{c}t - \phi(t)\right)$$
(5)

The phase detector output, i.e., the error signal, is then

$$\begin{aligned} \varrho(t) &= x(t) \cdot r(t) \\ &= \frac{\sqrt{2}}{2} A(t) \sin \left( \delta(t) - \phi(t) \right) \end{aligned} \tag{6}$$

where the double frequency term has been ignored.

The phase error,  $\phi(t)$ , is related to the error signal,  $\ell(t)$ , by Eq. (7)

$$\phi(t) = -K \frac{F(p)}{p} \ell(t) \tag{7}$$

where p = d/dt is the Heaviside operator, K is the open loop gain and F(s) is the loop filter transfer function. For the DSN Block IV receiver, the loop filter is a lead-lag filter having this form:

$$F(s) = \frac{1 + \tau_2 s}{1 + \tau_1 s}$$
(8)

where  $\tau_1$  and  $\tau_2$  are time constants.

Instead of solving the set of nonlinear differential equations (Eqs. 6 and 7) for  $\phi(t)$ , Bruno and Blanchard adopted a trial solution,

$$\phi(t) = \lambda + m \sin(\Delta \omega t + \nu) \tag{9}$$

which yields the following set of constraint equations (Ref. 4),

$$\sin(\lambda) = \frac{-m^2 \delta \cos \psi}{2 J_0(m)}$$
(10)

$$\sin(\lambda - \nu) = \frac{-m^2 \delta \cos \psi}{2 R J_1(m)}$$
(11)

$$R^{2} = \left[\frac{m \delta \sin \psi + 2J_{1}(m) \cos \lambda}{J_{0}(m) - J_{2}(m)}\right]^{2} + \left[\frac{m^{2} \delta \cos \psi}{2J_{1}(m)}\right]^{2}$$
(12)

where  $\psi$  is the phase angle of  $F(j\Delta\omega)$ ,  $J_0$ ,  $J_1$ , and  $J_2$  are Bessel functions of the first kind, and  $\delta$  is given by this equation,

$$\delta = \frac{\Delta\omega}{AK|F(j\Delta\omega)|} \tag{13}$$

The term  $\lambda$  in the above equations is the static phase error (SPE) and *m* is the modulation index or phase jitter.

Equations (9) through (12) represent the model of a phase locked loop operated in the presence of a CW RFI. This model assumes a noiseless condition that is approximately true when the loop is operated at strong signals. At weak signal levels where the noise effect dominates, this model is expected to fail. It is noted that  $\lambda$  in the above equations is restricted to the range of  $-\pi/2$  to  $\pi/2$  with  $|\lambda| = \pi/2$  representing the limiting condition beyond which the loop can no longer lock to the desired signal.

To evaluate the RFI effect on the loop, Eqs. (10), (11), and (12) must be solved simultaneously for  $\lambda$ , m, and  $\nu$  for a given R and  $\Delta\omega$ . This involves solving a set of nonlinear equations, which is not a trivial job. For small values of m, i.e.,  $m \ll \sqrt{2}$ , Levitt (Ref. 4) is able to reduce Eqs. (10), (11), and (12) to an explicit form

$$m^2 \cong R^2/(\delta^2 + 2\delta \sin \psi + 1) \tag{14}$$

$$\sin \lambda \cong \frac{-m^2 \delta \cos \psi}{2} \tag{15}$$

$$\sin(\lambda - \nu) \cong \frac{-m\delta\cos\psi}{R} \tag{16}$$

The explicit form makes it much easier to evaluate the RFI effect and is applicable to the Block IV phase locked loop in most cases.

#### III. The Effects of a Band Pass Limiter

The phase locked loop model presented in the above paragraph does not account for the effects of a band pass limiter (BPL). Since the Block IV receiver has a limiter in the 100-kHz module, the phase locked loop model must be modified to account for the limiter effects before it can be applied to the Block IV receiver. There are two parameters in the model needing modification. In the absence of a band pass limiter, the product AK in Eq. (13) is a constant for a given loop and a given input signal level, independent of the RFI power level. With a band pass limiter, AK must be replaced by  $\alpha_L K$  to account for the limiter suppression effects, where K now is the strong signal open loop gain and  $\alpha_L$  is the limiter suppression factor that is a function of both the signal-to-noise ratio and ISR. In addition, due to the suppression of the signal by RFI and vice versa, the interference-to-signal ratio that the loop sees, called the effective ISR, is not the same as that at the limiter input. This is the second parameter that needs to be modified.

The response of a BPL to two CW signals in noise was studied by J. J. Jones (Ref. 8). Again, only necessary steps are repeated here for clarity. If we assume that the input to an ideal BPL consists of two CW signals in additive narrowband gaussian noise and let W(t) denote the input signal, then

$$W(t) = W_1(t) + W_2(t) + n(t)$$
(17)

where  $W_1(t) = \sqrt{2}A \sin(\omega_c t)$  is the signal,  $W_2(t) = \sqrt{2}B \sin((\omega_c + \Delta \omega)t)$  is the RFI with offset frequency  $\Delta \omega/2\pi$ , and n(t) is the additive narrowband gaussian noise with total noise power N. The power in the signal is then  $S_1 = A^2$  and the power in the RFI is  $S_2 = B^2$ . The interference-to-signal ratio at the limiter input is  $R = B/A = (S_2/S_1)^{1/2}$ .

The output of the BPL consists of both discrete and continuous components due to signal, RFI, and noise. If we let Z(t) denote the output of the limiter near the carrier frequency, then the correlation function of Z(t) is

$$R_{z}(\tau) = \sum_{i=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{k=|i|,|i|+2}^{\infty} 2 \frac{b^{2}_{K|\ell||\ell-|i+1||}}{\left(\frac{K+|i|}{2}\right)! \left(\frac{K-|i|}{2}\right)!} \rho^{K}(\tau)$$
(18)

where  $\rho(\tau)$  is the correlation function of the base band components of n(t), normalized to unit power.

The output power for the signal and interference in the fundamental frequency band are:

$$(S_{1})_{0} = \frac{2}{\pi^{2}} \left( \frac{S_{1}}{N} \right) \left[ \sum_{i=0}^{\infty} \frac{\left( -\frac{S_{1}}{N} \right)^{i}}{i!(i+1)!} \times \Gamma \left( i+1/2 \right)_{2} F_{1} \left( -i, -i-1; 1; \frac{S_{2}}{S_{1}} \right) \right]^{2}$$
(19)

$$(S_{2})_{0} = \frac{2}{\pi^{2}} \left( \frac{S_{2}}{N} \right) \left[ \sum_{i=0}^{\infty} \frac{\left( -\frac{S_{1}}{N} \right)^{i}}{(i!)^{2}} \times \Gamma \left( i + 1/2 \right)_{2} F_{1} \left( -i, -i; 2; \frac{S_{2}}{S_{1}} \right) \right]^{2}$$
(20)

where  $\Gamma(\cdot)$  is the gamma function and  ${}_{2}F_{1}(a, b, c, d)$  is the Gauss hypergeometric function.

Assuming that signal components at high-frequency bands can be ignored, we can define a quantity called effective ISR as:

$$R_e = \left[\frac{(S_2)_0}{(S_1)_0}\right]^{1/2}$$
(21)

The limiter suppression factor  $\alpha_L$  is given by

$$\alpha_L = \left[\frac{(S_1)_0}{8/\pi^2}\right]^{1/2} = \frac{\pi (S_1)_0^{1/2}}{2\sqrt{2}}$$
(22)

Using Eqs. (21) and (22) and substituting  $R_e$  for R and K for AK in Eqs. (10) and (13), we now have a model that can

predict the performance of the Block IV receiver in the presence of RFI. The equations defining this model are:

$$\sin(\lambda) = \frac{-m^2 \delta \cos \psi}{2J_0(m)}$$
(23)

$$\sin(\lambda - \nu) = \frac{-m^2 \delta \cos \psi}{2R_e J_1(m)}$$
(24)

$$R_e^2 = \left[\frac{m\,\delta\,\sin\psi + 2\,J_1(m)\cos\lambda}{J_0(m) - J_2(m)}\right]^2$$

$$+\left[\frac{m^2\,\delta\,\cos\psi}{2\,J_1(m)}\right]^2\tag{25}$$

$$\delta = \frac{\Delta\omega}{\alpha_L K |F(j\Delta\omega)|}$$
(26)

where  $\psi$  as before, is the phase angle of the loop filter, and K is the strong signal open loop gain.

## **IV. Numerical Results**

The analytical model developed in the previous paragraphs has been applied to the DSN Block IV receiver for two tracking modes. These two modes are: (a) wide, Bn = 2 kHz, 2 BLO = 10 Hz, S-band (wide/2 kHz/10 Hz), and (b) wide, Bn =2 kHz, 2 BLO = 30 Hz, X-band (wide/2 kHz/30 Hz), where Bnis the predetection noise bandwidth and 2 BLO is the two sided threshold loop bandwidth. Three sets of curves have been generated for each of these two modes. They are:

- (1) Receiver drop lock level as a function of RFI offset frequency  $(\Delta F)$  with margin (M) as a parameter, where margin is defined as the ratio of signal to noise in 2 BLO.
- (2) Static phase error ( $\lambda$ ) vs ISR with  $\Delta F$  as a parameter.
- (3) Rms phase jitter  $(m/\sqrt{2})$  vs ISR with  $\Delta F$  as a parameter.

These curves are shown in Figs. 1 through 6. It is noted that as the signal or the RFI becomes too large, computation of  $(S_1)_0$ or  $(S_2)_0$  breaks down. It is thus necessary to employ asymptotic approximation when this happens (Ref. 8). This explains why the curves in Figs. 1 and 2 are not completely smooth.

#### V. Experimental Verification

Experimental verification of this model was performed in the Telecommunications Development Laboratory (TDL) using the Engineering Model Block IV receiver. The test set up is shown in Fig. 7. The RFI source was located in a screen room to minimize leakage that might affect the performance of the receiver or the calibration of the ISR. A doppler jitter program was used to measure the static phase error and rms phase jitter.

Experimental results are shown in Figs. 8 through 12. Figures 8 through 9 show the measured drop lock level as a function of RFI offset frequency. Experimental data agree very well with analytical results for strong signal conditions. At weak signal level (M = 10 dB), agreement is slightly poor. This is reasonable owing to the fact that the noise effects have been neglected in the development of the phase lock loop model. Figure 10 shows the measured static phase error as a function of ISR for some selected frequencies. The reason that these particular frequencies were chosen is because of the fact that the highest sampling rate of the Doppler Phase Jitter Program used in this test is 5 ms. These frequencies were chosen such that  $\lambda$  and m could be sampled correctly. Experimental data of the static phase error agrees very well with analytical data.

Figure 12 shows the measured phase jitter as a function of ISR. The measured values differ from theory by approximately a factor of two when  $\Delta F$  is less than 100 Hz and ISR is less than 0 dB. When  $\Delta F$  is larger than 100 Hz, or when ISR is larger than 0 dB, good agreement between experiment and theory is observed.

#### **VI.** Computer Simulation

A computer simulation program has been implemented on a Modcomp computer to further verify the analytical results. This program performs a block-by-block simulation of the operation of a phase locked loop preceded by a band pass filter. This loop is equivalent to the tracking loop of the Block IV receiver. Limited simulation results have been obtained and they show good agreement with experimental data (Figs. 13 through 15).

#### VII. CW RFI Protection Ratio

Whether one can perform RFI management effectively often depends on one's ability to accurately estimate the maximum acceptable interference power level, or the so called protection criterion. Current CCIR recommendation (Ref. 9) provides worst-case protection criteria based only on interference that may fall in the loop bandwidth. Based on the experimental and analytical results of this study, we have determined protection criteria as a function of frequency for the Block IV receiver under the influence of the type of RFI discussed in this article. A simplified straight-line approximation of the protection criteria as a function of RFI frequency offset is shown in Fig. 16. This protection is based on a maximum acceptable tracking loop degradation of 10 deg of static phase error or peak phase jitter, whichever is larger. This criterion is consistent with the current CCIR recommendation on protection for deep-space communications, which calls for a maximum acceptable degradation of 10 deg of phase modulation (Ref. 9). The maximum allowable absolute RFI power level in Fig. 16 is based on a 10-dB carrier margin, 10-Hz threshold loop bandwidth, and a -215.0-dBW/Hz noise spectral density. This noise spectral density corresponds to the nominal value expected for X-band operations. While the S-band and K<sub>n</sub>-band noise spectral densities are different from that of the X-band, the difference is small enough (1.6 dB, Ref. 9) that Fig. 16 may be considered adequate for all three frequency bands.

Based on Fig. 16, the maximum allowable power level for an in-band interference is -210 dBW. This value confirms the current CCIR recommendation of -220.0 dBW for a 1-Hz bandwidth, which corresponds to -210 dBW for a 10-Hz bandwidth.

It is noted that the protection criteria presented here are for receiver tracking only. Effects on telemetry, ranging, doppler, etc., have not yet been considered. Protection criteria for the receiving system as a whole would have to include all these effects. This will be examined in the future. Meanwhile, it is recommended that Fig. 16 be used for analysis of specific RFI coordination problems encountered by the DSN.

#### VIII. Conclusion

An analytical model that models the response of a phase locked loop preceded by a band pass limiter has been presented. This model can be used to predict the performance of the Block IV receiver carrier tracking loop in the presence of a CW RFI. Sets of curves have been generated for the static phase error, phase jitter, and out-of-lock values for the Block IV receiver using the often used tracking modes. Experimental verification and some computer simulation have been performed to verify the analytical results. Simulation and experimental results show good agreement with theoretical prediction for the static phase error and out-of-lock values. Predicted phase jitter is consistently lower than the experimental and simulated results by a factor of one-half for small ISR when the offset frequency is small. For large ISR, good agreement is observed. The analytical model assumes a noiseless condition, which is valid only when the loop is operated at strong signal levels. Experimental data indicate, however, that even at the minimum operating signal level of 10-dB carrier margin, reasonably good prediction can still be obtained.

Based on both analytical and experimental results, a protection criteria curve has been developed to protect the Block IV receiver tracking loop against CW RFI. This curve extends the current CCIR recommendation to allow calculation of out-ofband interference.

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Fig. 2. Block IV receiver droplock levels vs RFI offset frequency with margin as a parameter (wide/2 kHz/30 Hz)

















Fig. 8. Block IV receiver droplock level: comparison of analytical and experimental results (strong signal, 2 BLO = 10 Hz, wide mode).

Fig. 9. Block IV receiver droplock level: comparison of analytical and experimental results (moderate to weak signals, 2 BLO = 10Hz, wide mode)



Fig. 11. SPE vs ISR: comparison of analytical and experimental results (strong signal, 2 *BLO* = 10 Hz, wide mode)

INTERFERENCE/SIGNAL, dB

165



Fig. 12. Rms phase jitter vs ISR: comparison of analytical and experimental results (strong signal, 2 BLO = 10 Hz, wide mode)



Fig. 13. Comparison of analytical, experimental, and simulated data for the Block IV receiver droplock level (wide/2 kHz/10 Hz)



Fig. 14. Comparison of analytical, experimental, and simulated data for the Block IV receiver static phase error (wide/2 kHz/10 Hz)



Fig. 15. Comparison of analytical, experimental, and simulated data for the Block IV receiver rms phase jitter (wide/2 kHz/10 Hz)



Fig. 16. Recommended protection criteria for CW RFI for Block IV receiver (absolute power level based on 10-dB carrier margin and -205 dBW noise power in a 10-Hz bandwidth)