# NASA TECHNICAL MEMORANDUM

NASA TM-82390

# STEERING LAW FOR PARALLEL MOUNTED DOUBLE-GIMBALED CONTROL MOMENT GYROS – REVISION A

By H. F. Kennel
Systems Dynamics Laboratory

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*Marshall Space Flight Center, Alabama*

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16. ABSTRACT

Mounting of double-gimbaled control moment gyros (CMG's) of unlimited outer gimbal angle freedom with all their outer gimbal axes parallel allows drastic simplification of the CMG steering law development in the redundancy management and failure accommodation and in the mounting hardware. The advantages of the parallel mounting for the CMG steering law development are such that a law could be developed which is applicable to any number of CMG's with arbitrary angular momentum. Parallel mounting of the CMG's in conjunction with the steering law can therefore be considered a "CMG kit" suitable for many missions of differing momentum requirements. It also means that increasing momentum demands during the design phase of a space vehicle can be easily met by the addition of one or more CMG's of the original momentum capacity rather than a redesign to a larger momentum capacity. Another advantage of the parallel mounting is that the failure of any CMG can be treated like any other, i.e., only one failure mode is possible. The CMG steering law distributes the CMG momentum vectors such that all inner gimbal angles are equal which reduces the rate requirements on the outer gimbal axes. The steering law also spreads the outer gimbals which ensures avoidance of singularities internal to the angular momentum envelope.

This report is an extensive revision of Reference 1 to the extent that a different outer gimbal angle distribution function is used, significantly reducing the software requirements.

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| Symbol | Definition |
|---|---|
| [B] | 2 × n torque matrix (for outer rates), Nms |
| $b_{ij}$ | components of [B]; i = 1,2; j = 1,2, ..., n; Nms |
| c | cos (before Greek symbol) |
| DIV | divisor for rate limiting |
| $H_i$ | angular momentum magnitude of the i-th CMG; i = 1,2, ... , n; Nms |
| $\underline{H}_G$ | total angular momentum of the CMG system; Nms |
| $H_{G1}$, $H_{G2}$, $H_{G3}$ | components of $\underline{H}_G$; Nms |
| $\dot{\underline{H}}_P$ | angular momentum change for power generation; Nm |
| i | index |
| j | index |
| $K_a$ | inner distribution gain; 1/s |
| $K_b$ | outer distribution gain; 1/s |
| $K_i$ | $T_{Limi}/H_i$; CMG torque constant; 1/s |
| n | number of double-gimbaled CMGs |
| s | sin (before Greek symbol) |
| $\underline{T}_C$ | control torque command; Nm |
| $T_{C1}$, $T_{C2}$, $T_{C3}$ | components of $\underline{T}_C$; Nm |
| $T_{CM2}$, $T_{CM3}$ | modified torque commands; Nm |
| $\underline{T}_G$ | total CMG torque on the vehicle; Nm |
| $T_{G1}$, $T_{G2}$, $T_{G3}$ | components of $\underline{T}_G$; Nm |
| $T_{LIM}$ | maximum CMG gimbal torquer torque; Nm |
| $T_{1i}$, $T_{2i}$, $T_{3i}$ | torque components in the W-coordinate system (with $\beta_i = 0$); i = 1,2,3 ... n; Nm |

## DEFINITION OF SYMBOLS (Concluded)

| Symbol | Definition |
| --- | --- |
| $T_2'$, $T_3'$ | torque components due to $\dot{\alpha}_i$'s and $\dot{\beta}_{Di}$'s; Nm |
| $W_1$, $W_2$, $W_3$ | CMG coordinate system axes |
| $\alpha_o$ | inner gimbal reference angle; rad |
| $\alpha_i$, $\beta_i$ | inner and outer gimbal angle; i = 1,2, ..., n; rad |
| $\alpha_R$ | ideal inner gimbal reference angle; rad |
| $\dot{\alpha}_i$, $\dot{\beta}_i$ | total inner and outer rates; i = 1,2, ..., n; rad/s |
| $\dot{\alpha}_{Di}$, $\dot{\beta}_{Di}$ | inner and outer distribution rate; i = 1,2, ..., n; rad/s |
| $\dot{\alpha}_L$, $\dot{\beta}_L$ | inner and outer rate limit; rad/s |
| $\dot{\alpha}_{LIM}$, $\dot{\beta}_{LIM}$ | inner and outer rate limit due to hardware; rad/s |
| $\dot{\alpha}_{TLIM}$, $\dot{\beta}_{TLIM}$ | inner and outer rate limit due to $T_{LIM}$; rad/s |
| $\underline{\dot{\beta}}_C$ | outer rate command vector (due to $T_{CM2}$, $T_{CM3}$); rad/s |
| $\dot{\beta}_{Ci}$ | components of $\underline{\dot{\beta}}_C$; i = 1,2, ..., n, rad/s |
| $\dot{\beta}_{DMAX}$ | maximum possible outer distribution rate; rad/s |
| $\delta_{ij}$ | $\left.\begin{matrix} = 0 \text{ for } i \neq j \\ = 1 \text{ for } i = j \end{matrix}\right\}$ Kronecker delta |
| $\_$ | a bar under a quantity denotes a vector |
| $[\ ]^T$ | a superscript T denotes a transpose on a vector or a matrix |
| $\{\ \}^{-1}$ | a superscript -1 denotes the matrix inverse |
| $\cdot$ | a dot over a quantity indicates time derivative |

TECHNICAL MEMORANDUM

# STEERING LAW FOR PARALLEL MOUNTED DOUBLE-GIMBALED CONTROL MOMENT GYROS – REVISION A

## INTRODUCTION

As space vehicles become larger and the orbital mission times become longer, double-gimbaled control moment gyros (CMGs) will be used more and more for the attitude control and angular momentum storage functions. There is, therefore, a need for a simple, but universally applicable CMG steering law, which will take the control torque commands and generate gimbal rate commands in such a fashion, that, inspite of the highly non-linear CMG behavior, the resulting torques on the vehicle are ideally equal to the commanded ones. Furthermore, this steering law should allow the use of any number of CMGs, which allows the tailoring to the angular momentum requirement while at the same time making the failure accommodation a built-in feature. It should also allow the mixing of CMGs of different angular momentum magnitudes to the extent that some or all CMGs could be used also for energy storage, eliminating or greatly reducing the need for batteries.

In this report, a steering law is presented, which has all these features, due mainly to the idea of mounting all the outer gimbal axes parallel to each other. This allows the decomposition of the steering law problem into a linear one for the inner gimbal angle rates and a planar one for the outer gimbal angle rates. Since the inner rates are not affected by the outer ones, they are generated first and are then known. An outer gimbal angle distribution function (to avoid singularities internal to the angular momentum envelope) next generates distribution rates, and finally the pseudoinverse method is used to insure that the desired total torque is delivered.

This report is an extensive revision of Reference 1 to the extent that a different distribution function has been used, reducing the software requirements significantly.

## PARALLEL MOUNTING ARRANGEMENT

The proposed mounting, with all outer gimbal axes parallel, is shown in Figure 1 (for convenience only, the outer gimbal axes are also shown colinear). This mounting arrangement has many advantages. The mounting interfaces can be identical, i.e., the mounting brackets and hardware, cable harnesses, etc. There is no need to individually identify the CMGs, and the on-board computer can assign an arbitrary label to

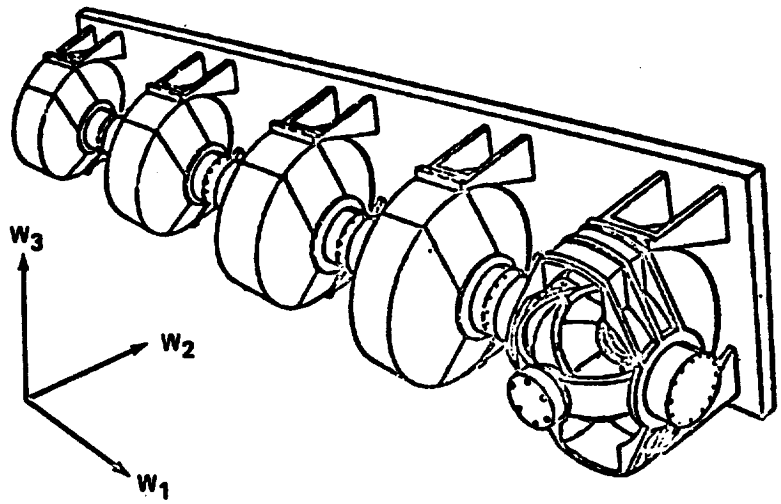



Figure 1. Parallel mounting arrangement.

any CMG, which could be changed from one computation cycle to the next. This simplifies the steering law and the redundancy management. The parallel mounting in conjunction with a steering law that accepts any number of CMGs makes failure accommodation a built-in feature. However, if increasing angular momentum requirements during the design of a space vehicle demand it (and the moments of inertia always tend to increase), additional CMGs can be added with minimum impact on the hardware and almost none on the software. The parallel mounting also makes the visualization of the system operation exceedingly simple (especially when compared with the momentum envelopes and saturation surfaces of skewed mounted single-gimbaled CMGs).

## TOTAL ANGULAR MOMENTUM AND TORQUE COMMANDS

Given a CMG coordinate system W, Figure 2 shows the inner and outer gimbal angles $\alpha_i$ and $\beta_i$ of the ith CMG (note that the inner gimbal angle is defined mathematically negative). With this definition, the angular momentum of n CMGs is:

$$\underline{H}_G = \begin{bmatrix} H_{G1} \\ H_{G2} \\ H_{G3} \end{bmatrix} = \begin{bmatrix} \sum_i H_i \, s\alpha_i \\ \sum_i H_i \, c\alpha_i \, c\beta_i \\ \sum_i H_i \, c\alpha_i \, s\beta_i \end{bmatrix} \quad . \qquad (1)$$

where $i = 1, 2, \ldots, n$ and $H_i$ is the angular momentum magnitude of the ith CMG.

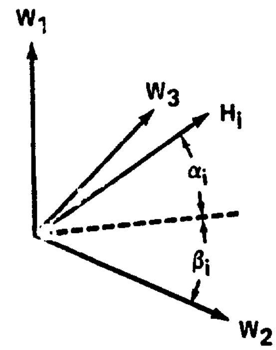



Figure 2. CMG gimbal angles.

The CMG angular momentum change is

$$\underline{\dot{H}}_G = \begin{bmatrix} \sum_i H_i \, c\alpha_i \, \dot{\alpha}_i \\ \sum_i (-H_i \, s\alpha_i \, c\beta_i \, \dot{\alpha}_i - H_i \, c\alpha_i \, s\beta_i \, \dot{\beta}_i ) \\ \sum_i (-H_i \, s\alpha_i \, s\beta_i \, \dot{\alpha}_i + H_i \, c\alpha_i \, c\beta_i \, \dot{\beta}_i ) \end{bmatrix} + \underline{\dot{H}} \quad , \tag{2}$$

where

$$\underline{\dot{H}} = \begin{bmatrix} \sum_i \dot{H}_i \, s\alpha_i \\ \sum_i \dot{H}_i \, c\alpha_i \, c\beta_i \\ \sum_i \dot{H}_i \, c\alpha_i \, s\beta_i \end{bmatrix} \quad .$$

We can assume that $\underline{\dot{H}}$ is known (from power demand or the difference between present and past value of $\underline{H}$) and it is accounted for in the torque command:

$$\underline{T} = \dot{\underline{H}}_G - \dot{\underline{H}} = \begin{bmatrix} \sum_i H_i \, c\alpha_i \, \dot{\alpha}_i \\ \sum_i (-H_i \, s\alpha_i \, c\beta_i \, \dot{\alpha}_i - H_i \, c\alpha_i \, s\beta_i \, \dot{\beta}_i) \\ \sum_i (-H_i \, s\alpha_i \, s\beta_i \, \dot{\alpha}_i + H_i \, c\alpha_i \, c\beta_i \, \dot{\beta}_i) \end{bmatrix} . \qquad (3)$$

The task is now to find a set of gimbal angle rate commands $\dot{\underline{\alpha}}$ and $\dot{\underline{\beta}}$ which, when inserted into equation (3) will satisfy the torque command, while at the same time utilizing the excess degrees of freedom to distribute the gimbal angles such that the singularities internal to the angular momentum envelope are avoided.

## DESIRABLE INNER GIMBAL ANGLE DISTRIBUTION

The n double gimbaled CMGs have 2n degrees of freedom. Three are needed to satisfy the torque command; the excess of 2n-3 degrees of freedom is utilized to achieve a desirable gimbal angle distribution. Before one can decide on a desirable distribution, the characteristics of a double-gimbaled CMG have to be considered.

The inner gimbal angle rate needed to produce a given torque perpendicular to the inner gimbal axis is independent of the inner and the outer gimbal angles. However, the outer gimbal angle rate needed to produce a given torque perpendicular to the outer gimbal axis is inversely proportional to the cosine of the inner gimbal angle. Therefore, it is desirable to keep the cosine of the inner gimbal angle high, i.e., it is desirable to minimize the maximum inner gimbal angle. This then reduces the outer gimbal angle rate requirements. The maximum inner gimbal angle is minimized when all inner gimbal angles are equal to the inner gimbal reference angle, c.f. equation (1),

$$\alpha_R = \sin^{-1} \left\{ \frac{\sum_i H_i \, s\alpha_i}{\sum_i H_i} \right\} . \qquad (4)$$

Inner gimbal angle redistribution should be made without resulting in a torque along the $W_1$ axis. This implies [cf. equation (3)],

$$T_{D1} = \sum_i H_i \, c\alpha_i \, \dot{\alpha}_{Di} = 0 \quad .$$

We select the distribution rate such that

$$\dot{\alpha}_{Di} = K_a (\alpha_o - \alpha_i) \quad .$$

The $W_1$ torque is now

$$T_{D1} = \sum_i H_i c\alpha_i K_a (\alpha_o - \alpha_i) \quad .$$

Setting this torque to zero results in

$$\alpha_o = \frac{\sum_i H_i c\alpha_i \alpha_i}{\sum_i H_i c\alpha_i} \quad . \tag{5}$$

When all $\alpha_i = \alpha_o$, the latter has converged to its ideal value of equation (4).

However, a torque does result in the $W_2$-$W_3$ plane and it will be treated in conjunction with the outer gimbal angle distribution.

## DESIRABLE OUTER GIMBAL ANGLE DISTRIBUTION

The situation is not as clear-cut with respect to the desirable outer gimbal angle distribution. However, for double-gimbaled CMGs a singular condition inside the total angular momentum envelope can only occur when some of the momentum vectors (at least one) are antiparallel and the rest parallel to their resultant total angular momentum vector. Maintaining adequate and more or less equal spacing between the vectors will therefore eliminate the possibility of a singularity. Many distribution functions are possible and the goal is to find one which minimizes the software requirements. Compared to the outer gimbal angle distribution function of Reference 1, a much simpler one has been developed which has none of the disadvantages (no modified outer gimbal angles, no sorting, no extra logic for failed CMGs), but retained all the advantages. The new distribution function introduces repulsion between all possible CMG pairs, i.e., proportional to the product of the angular momentum magnitudes and proportional to the supplement of the outer gimbal angle differences.

Since the repulsion is generated by pairs (and all possible pairs are treated equally), no sorting is necessary. However, due to the stronger repulsion of the immediate neighbors, the outer gimbal angles act as if sorting had been done. A failed CMG is simply ignored (its angular momentum magnitude is set to zero) since it does not generate a repulsion. Any number of CMGs can be failed or resurrected at the beginning of any computation cycle. The rates due to this distribution function are

$$\dot{\beta}_{Di} = -K_b \sum_j \{H_i H_j [\beta_i - \beta_j - \pi \operatorname{sgn} (\beta_i - \beta_j)] (1 - \delta_{ij})\} \quad . \tag{6}$$

Without any momentum constraint the CMG outer gimbals would come to rest when they are separated by an angle of $2\pi/n$, assuming identical angular momentum magnitudes. (This would not make any sense for two CMGs since they would be antiparallel. However, two CMGs have only one excess degree of freedom and it is taken up by the inner distribution, and no distribution is possible for the outer gimbals).

## INNER GIMBAL RATE COMMANDS

The change of the $W_1$ angular momentum component is not a function of the outer gimbal rates [equation (3)],

$$\dot{H}_{G1} = T_{G1} = \sum_i H_i \, c\alpha_i \, \dot{\alpha}_i \quad . \tag{3a}$$

Since all inner gimbal angles should be equal (enforced by the inner gimbal distribution), an inner gimbal rate command for all CMGs of

$$\dot{\alpha}_c = T_{C1} / \sum_i H_i \, c\alpha_i \quad , \tag{7}$$

will result in $T_{G1} = T_{C1}$, if it is assumed that the actual and the commanded gimbal rates are equal. For the total inner gimbal angle rate command we have then [cf. equation (5)],

$$\dot{\alpha}_i = K_a (\alpha_o - \alpha_i) + T_{C1} / \sum_i H_i \, c\alpha_i \quad , \tag{8}$$

where it should be remembered that the distribution rates are non-torque-producing with respect to the $W_1$ axis. The effect of the total inner gimbal angle rates on the $W_2$ and $W_3$ momentum components is treated later. Gimbal rate limiting will be discussed after the outer gimbal rate commands have been treated.

## OUTER GIMBAL RATE COMMANDS

The outer distribution rates of equation (6) as well as the total inner gimbal angle rate commands of equation (8) generate a torque in the $W_2$-$W_3$ plane. This torque is

$$\begin{bmatrix} T_2' \\ T_3' \end{bmatrix} = \begin{bmatrix} -\sum_i H_i \, (c\alpha_i \, s\beta_i \, \dot{\beta}_{Di} + s\alpha_i \, c\beta_i \, \dot{\alpha}_i) \\ \sum_i H_i \, (c\alpha_i \, c\beta_i \, \dot{\beta}_{Di} - s\alpha_i \, s\beta_i \, \dot{\alpha}_i) \end{bmatrix} ,$$

and the $W_2$-$W_3$ torques must be modified as follows:

$$\begin{bmatrix} T_{CM2} \\ T_{CM3} \end{bmatrix} = \begin{bmatrix} T_{C2} - T_2' \\ T_{C3} - T_3' \end{bmatrix} .$$

To generate the modified torque we now apply the pseudo-inverse method to get a unique set of outer gimbal rate commands

$$\begin{bmatrix} \dot{\beta}_{C1} \\ \dot{\beta}_{C2} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \dot{\beta}_{Cn} \end{bmatrix} = [B]^T \, \{[B][B]^T\}^{-1} \begin{bmatrix} T_{CM2} \\ T_{CM3} \end{bmatrix} ,$$

where

$$[B] = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \end{bmatrix} ,$$

with

$$b_{1i} = -H_i \, c\alpha_i \, s\beta_i$$

$$b_{2i} = H_i \, c\alpha_i \, c\beta_i \quad .$$

The total outer gimbal rate commands are then

$$\dot{\beta}_i = \dot{\beta}_{Di} + \dot{\beta}_{Ci} \quad .$$

PROPORTIONAL GIMBAL RATE LIMITING

The gimbal torques will have a definite torque limit, $T_{LIM}$. In order not to exceed this limit, we have to establish the torque demand on the torquers due to the gimbal rate commands. Setting the outer gimbal angle to zero we get for the torque of the ith CMG [equation (3)],

$$\begin{bmatrix} T_{1i} \\ T_{2i} \\ T_{3i} \end{bmatrix} = H_i \begin{bmatrix} c\alpha_i \, \dot{\alpha}_i \\ -s\alpha_i \, \dot{\alpha}_i \\ c\alpha_i \, \dot{\beta}_i \end{bmatrix} .$$

The torque $T_{1i}$ is the torque load on the outer gimbal torquer and $T_{3i}$ is the torque load on the inner gimbal torquer. Assuming the same torque limits for both inner and outer gimbal torquers, we get for the gimbal rate limits

$$\dot{\alpha}_{TLIM} = \dot{\beta}_{TLIM} = K_i / c\alpha_i \quad ,$$

with $K_i = T_{LIMi}/H_i$ a CMG design constant, more or less the same for all double gimbaled CMGs. There are also other rate limits ($\dot{\alpha}_{LIM}$, $\dot{\beta}_{LIM}$) due to hardware limits like gimbal rate tachometer limits, voltage limits, etc.

To reduce the magnitude of the actual torque only, but keep the same direction as the commanded torque, a proportional scaling of all gimbal rates by dividing by DIV is done, where:

$$\mathrm{DIV} = \mathrm{MAX}\left\{1, \frac{|\dot{\alpha}_1|}{\dot{\alpha}_L}, \frac{|\dot{\alpha}_2|}{\dot{\alpha}_L}, \ldots, \frac{|\dot{\alpha}_n|}{\dot{\alpha}_L}, \frac{|\dot{\beta}_1|}{\dot{\beta}_L}, \frac{|\dot{\beta}_2|}{\dot{\beta}_L}, \ldots, \frac{|\dot{\beta}_n|}{\dot{\beta}_L}\right\},$$

with

$$\dot{\alpha}_L = \mathrm{MIN}\ (\dot{\alpha}_{TLIM}, \dot{\alpha}_{LIM}) \quad ,$$

and

$$\dot{\beta}_L = \mathrm{MIN}\ (\dot{\beta}_{TLIM}, \dot{\beta}_{LIM}) \quad .$$

## PERFORMANCE DISCUSSION

As implied by the discussion of the desirable gimbal angle distribution, there is no need for a strict adherence to the ideal distribution. For the inner distribution gain $K_a$ this means that its value can be chosen over a wide range up to $1/\Delta t$ where $\Delta t$ is the computer cycle time, especially since the torque producing portion of the inner gimbal rate command tends to keep the inner gimbal angles on their distribution. To select the outer gimbal distribution gain we have to remember that the maximum magnitude of the outer distribution rate can be as large as [cf. equation (6)],

$$\dot{\beta}_{DMAX} = K_b H_i \left(\sum_{j=0}^{n} H_j - H_i\right)\pi \quad .$$

For a desired maximum outer distribution rate of 0.02 rad/s and with 5 CMGs of an angular momentum magnitude of 3000 Nms we get for example,

$$K_b \approx 1.8E\text{-}10 \ .$$

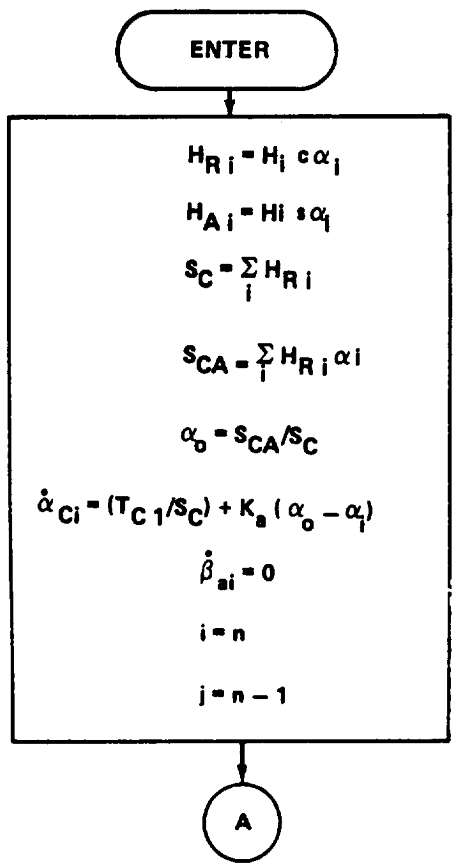
If the outer distribution rates are limited before $\beta_{Ci}$'s are calculated, a higher gain value could be used.

No special effort has to be made to "unhook" the CMG momentum vectors when, after bunching up due to saturation, the torque command is such that the momentum magnitude is reduced. In the real world, the gimbal angle readouts are noisy enough to introduce unhooking. Therefore, if a simulation is too ideal and problems are encountered, some noise should be added to the gimbal angles. The same considerations apply for starting up from an internal singularity (the distribution functions prevent any later internal singularities from occurring).

While any number of double-gimbaled CMGs can be accommodated (two CMGs are the minimum), the cycle time might become a problem for very large numbers of CMGs. Since any angular momentum magnitude is allowed, there is then the possibility to group several CMGs, add their angular momentum, and consider the group as one CMG as far as the steering law is concerned. The group is then commanded to its combined outer and inner gimbal angle. Setting the angular momentum magnitude of any CMG to zero is a convenient way of signaling that the CMG has failed.

The general logic flow, including the input and output parameters for the steering law is shown in Figure 3. The implementation for the high level language APL is shown in Figure 4 and a detailed flow in Figure 5.

| | | | |
|---|---|---|---|
| INPUTS | $\underline{T}_C$ | Nm | TORQUE COMMAND ($T_{C1}$, $T_{C2}$, $T_{C3}$) |
| | $\alpha_i$ | rad | INNER GIMBAL ANGLES |
| | $\beta_i$ | rad | OUTER GIMBAL ANGLES |
| | $s\alpha_i$, $c\alpha_i$ | | SINES & COSINES OF THE $\alpha_i$' S |
| | $s\beta_i$, $c\beta_i$ | | SINES & COSINES OF THE $\beta_i$' S |
| | $H_i$ | Nms | CMG ANGULAR MOMENTUM MAGNITUDES |
| OUTPUTS | $\dot{\alpha}_{Ci}$ | rad/s | INNER GIMBAL RATE COMMANDS |
| | $\dot{\beta}_{Ci}$ | rad/s | OUTER GIMBAL RATE COMMANDS |




n = MAXIMUM NUMBER OF CMG'S

$K_a$ = INNER GIMBAL ANGLE DISTRIBUTION GAIN

ALL i's RANGE FROM 1 to n

Figure 3. Steering law logic flows.

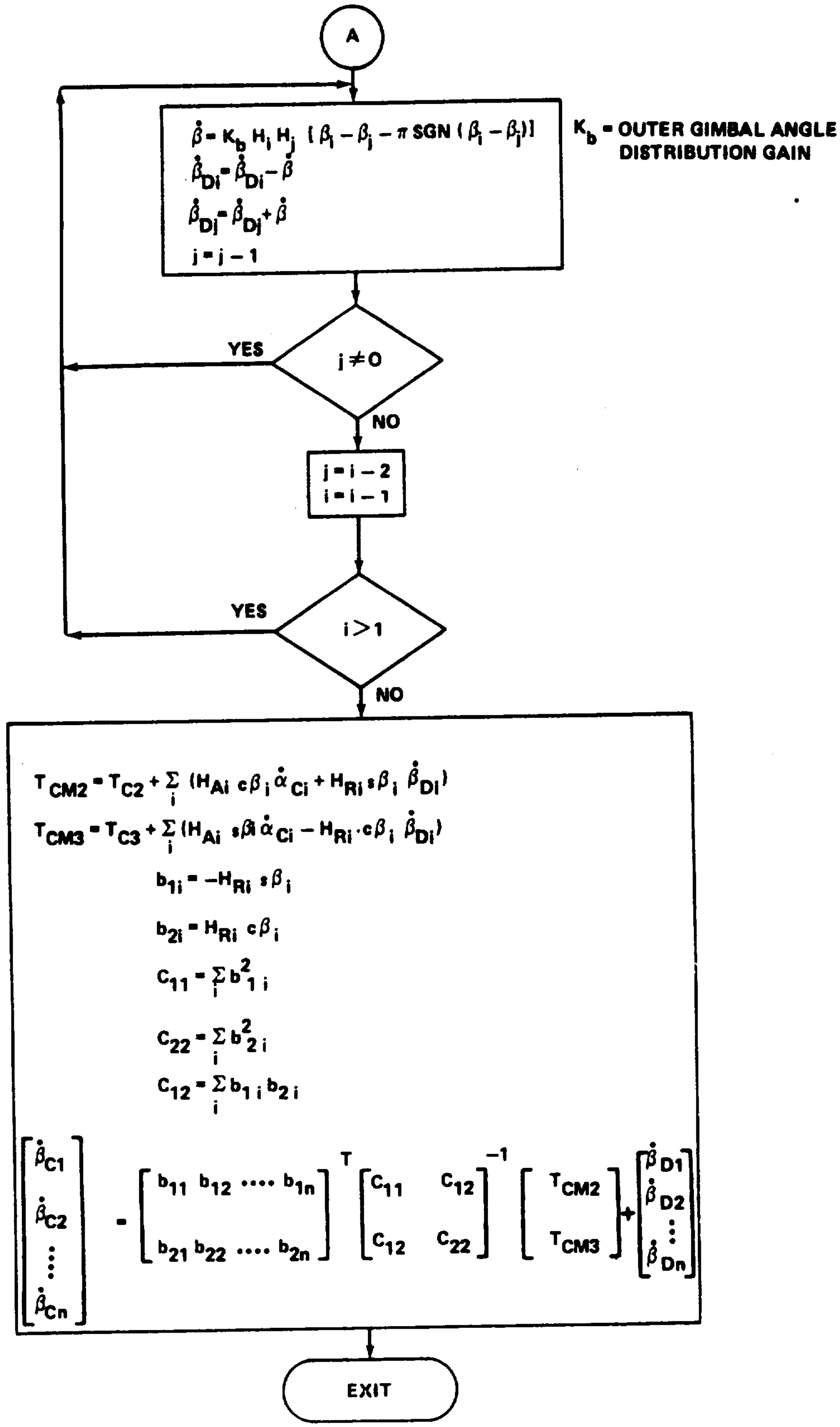



Figure 3. (Concluded).

```
     ∇ STLAW;HA;HR;SC;T ;BD;B;M
[1]    ⍝
[2]    ⍝
[3]    ⍝
[4]    ⍝
[5]    ⍝             *****   STEERING LAW FOR N DOUBLE    *****
[6]    ⍝             ***** GIMBALED CONTROL MOMENT GYROS ******
[7]    ⍝
[8]    ⍝
[9]    ⍝ INPUT_PARAMETERS:  TC  =  TC1  TC2  TC3
[10]   ⍝
[11]   ⍝                    AL  =  α1   α2    ....  αn
[12]   ⍝
[13]   ⍝                    BE  =  β1   β2    ....  βn
[14]   ⍝
[15]   ⍝                     H  =  H1   H2    ....  Hn
[16]   ⍝
[17]   ⍝
[18]   ⍝ LOCAL_VARIABLES:   HA, HR, SC, T, BD, B, M
[19]   ⍝
[20]   ⍝
[21]   ALD←(TC[1]+SC)-KA×AL-(+/HR×AL)÷SC←+/HR←H×2○AL
[22]   BD←KB×+/(M×⍉M←(N,N)⍴H)×B-PI××B←B-⍉B←(N,N)⍴BE
[23]   T←TC[2 3]+(+/(HA×ALD×2○BE)+HR×BD×1○BE),+/((HA←H×1○AL)×ALD×1○BE)-HR×BD×2○BE
[24]   BED←BD+(⍉B)+.×(⌹B+.×⍉B←(2,N)⍴(-HR×1○BE),HR×2○BE)+.×T
[25]   ⍝
[26]   ⍝
[27]   ⍝ OUTPUT_PARAMETERS:  ALD  =  α̇1   α̇2   ....  α̇n
[28]   ⍝
[29]   ⍝                     BED  =  β̇1   β̇2   ....  β̇n
     ∇
```

Figure 4.  Steering law for n double-gimbaled control moment gyros.

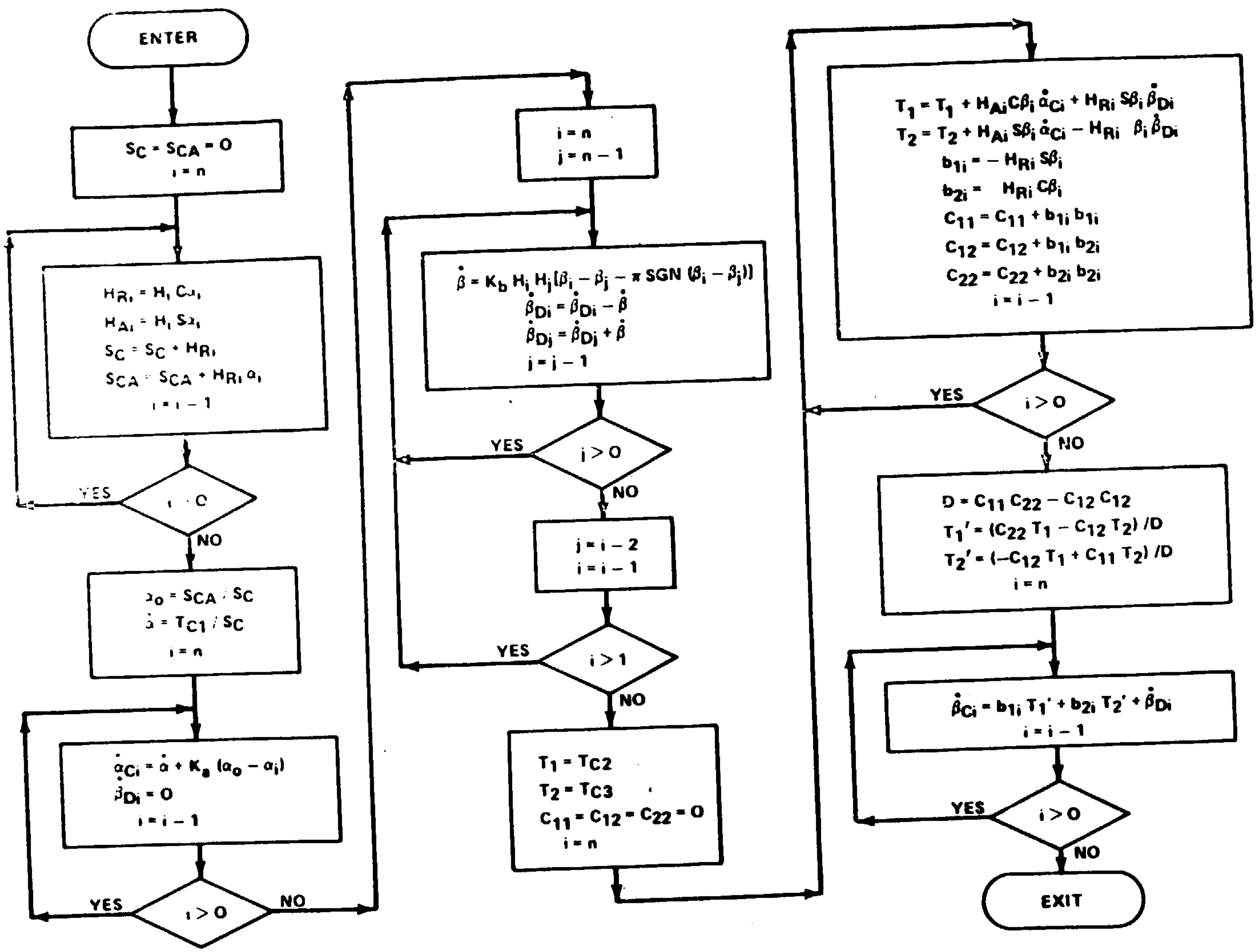



Figure 5. Detailed logic flow.

## REFERENCE

1. Kennel, H. F.: Steering Law for Parallel Mounted Double-Gimbaled Control Moment Gyros. NASA TM X-64930, February 1975.

# APPROVAL

## STEERING LAW FOR PARALLEL MOUNTED DOUBLE-GIMBALED CONTROL MOMENT GYROS – REVISION A

By Hans F. Kennel

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

George D. Hopson

GEORGE D. HOPSON
Director, Systems Dynamics Laboratory

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