Grid Generation for Time Dependent Problems: Criteria and Methods

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Abstract

We consider the problem of generating local mesh refinements when solving time dependent partial differential equations. We first discuss the problem of creating an appropriate grid, given a mesh function h defined over the spatial domain. A data structure which permits efficient use of the resulting grid is described. Secondly, we show that a good choice for h is an estimate of the local truncation error, and we discuss several ways to estimate it. We conclude by comparing the efficiency and implementation problems of these error estimates.

<u>WHAT</u> ADAPTIVE MESH GENERATION FOR TIME DEPENDENT PDE'S

<u>OBJECTIVES</u> REDUCE # MESH PTS

MINIMIZE OVERHEAD

TRADEOFF: EXTRA PTS. VS. EXTRA LOGIC

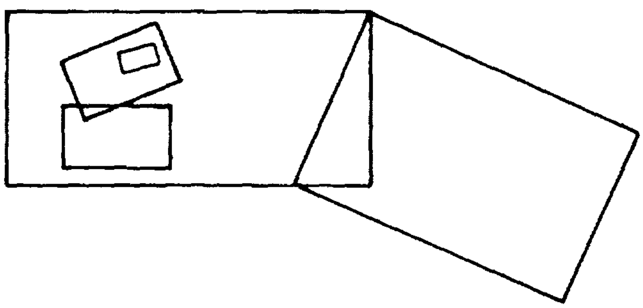
<u>REQUIREMENTS</u>

- MARCHING ALGORITHMS WILL BE USED
- COMPUTING TRANSIENT SOLN BY FINITE DIFF.
- TIMESTEP SMALLER ON FINER GRIDS, MESH RATIO CONSTANT
- GRIDS MUST CHANGE WITH TIME
- COARSEST GRID DOES <u>NOT</u> CHANGE WITH TIME

DESCRIPTION OF GRIDS

- LOCALLY UNIFORM
- RECTANGLES OF ARBITRARY ORIENTATION. EXTENSIONS TO CURVILINEAR GRIDS FITS INTO SAME FRAMEWORK
- SUPPOSE BASE GRID $G_0 = \bigvee_j G_{0,j}$ FORM HIERARCHY OF NESTED GRIDS WHERE EACH REFINED GRID IS WHOLLY CONTAINED IN A SINGLE COARSER GRID

$$G_l = \bigvee_j G_{l,j}$$



- REFINED GRIDS CAN BE CONSTRUCTED AUTOMATICALLY AT $t = 0$ FROM INITIAL DATA.

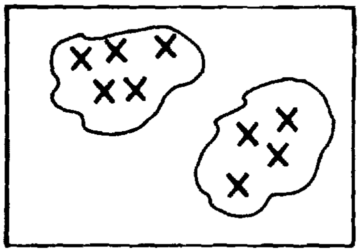
HOW GRIDS ARE FORMED

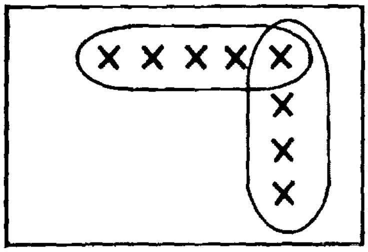
GIVEN A "MESH FUNCTION" $h(s, y)$ USED TO DETERMINE WHERE TO PLACE REFINED GRIDS.

FLAG GRID PTS. WHERE $h(x, y) > \varepsilon$.

- CLUSTER
- ORIENTATION
- GOOD FIT ?

CLUSTERING





- NEARREST NEIGHBOR

  d(PT., CLUSTER) < dmax $\Longrightarrow$
  PT. ε CLUSTER

- SPANNING TREES

  CONNECT ALL PTS. ACCORDING TO SOME CRITERIA.
  BREAK LONGEST LINKS.

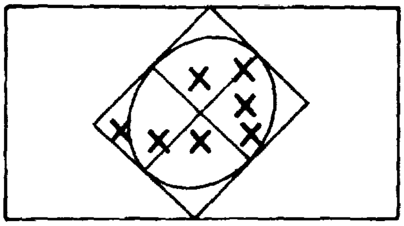
  - MINIMAL SPANNING TREES
  - MINIMUM DIAMETER TREES

ORIENTATION

- FIT ELLIPSE TO FLAGGED PTS. OF A CLUSTER USING 1ST AND 2ND MOMENTS.
- USE MAJOR AND MINOR AXES OF THE ELLIPSE TO GET RECTANGLE ORIENTATION.

(REF: D. GENNERY, "OBJECT DETECTION AND MEASUREMENT USING STEREO VISION") PROCS. IJCAI, 1979, pp 320-327

- FIT MIN. BOX TO INCLUDE FLAGGED PTS. + SMALL BUFFER ZONE FOR SAFETY.



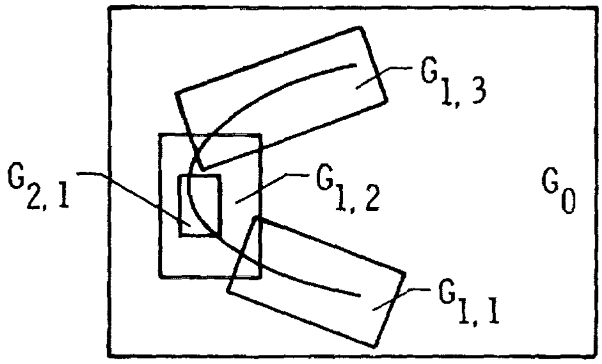
GOODNESS OF FIT

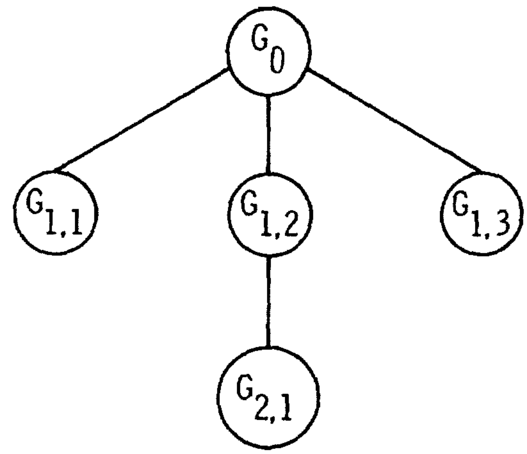
- RATIO OF FLAGGED TO UNFLAGGED PTS.
- IF TOO LOW, RECLUSTER AND REFIT.

KEEPING TRACK OF GRIDS

NESTING SUGGESTS USE OF TREE STRUCTURE

(REF: KNVTH, "ART OF COMPUTER PROGRAMMING", VOL. 1)





INFORMATION FOR EACH GRID

1) GRID LOCATION
2) SPATIAL AND TEMPORAL STEP SIZES
3) SIZE OF GRID
4) 3 TREE LINKS
5) PTR. TO INTERSECTING GRIDS
6) MAIN STORAGE AREA PTR.

POINTS TO NOTE

1) EASY TO HANDLE FAIRLY GENERAL REGIONS.

   ALL THE WORK IN SETTING UP THE PROBLEM IS IN SPECIFYING THE LOCATION OF THE COARSE GRID AND ITS CONSTITUENT RECTANGLES. THE REST IS AUTOMATIC.

2) EASY TO USE DIFFERENT METHODS ON DIFFERENT GRIDS.

WHAT IS h(x,y)?

WOULD LIKE TO EQUIDISTRIBUTE THE GLOBAL ERROR.

1D LINEAR THEORY SAYS IF CONTROL

(1) INITIAL ERROR
(2) BOUNDARY ERROR
(3) LOCAL TRUNCATION ERROR

AND METHOD IS STABLE FOR IBVP THEN THE METHOD CONVERGES.

(1) AND (2) CONTROLLED BY STD. MEANS

(3) CONTROLLED BY REFINING MESHES

USE LOCAL TRUNCATION ERROR FOR h(x, y).

REQUIREMENTS FOR LOCAL ERROR ESTIMATOR

- ACCURATELY MIMIC ERROR BEHAVIOR
- REASONABLY ACCURATE ESTIMATE - NOT NEC. A BOUND
- CHEAP TO COMPUTE
  FLEXIBLE - EASY TO SWITCH INTEGRATORS
- THE FEWER TIME LEVELS THE BETTER.

POSSIBLE ESTIMATORS

DIRECT ESTIMATION OF TRUNC. ERROR

- FIND LEADING TERM

  (e.g. $\frac{k^2}{6} V_{ttt} + \frac{h^2}{6} V_{xxx}$)

- ESTIMATE BY DIVIDED DIFFERENCES

PROBLEMS

- HARD TO FIND LEADING TERM
- HARD TO CHANGE INTEGRATORS
- NO CHEAPER THAN OTHER ESTIMATES

LOWER ORDER ESTIMATES

(V_t, V_{tt})

- ESTIMATE SOLN. GROWTH IN TIME
- PROS - CHEAP, BETTER THAN GRADIENT ESTIMATES
- CONS - ACCURATE TRENDS BUT INACCURATE ESTIMATE OF MAGNITUDE.

GRADIENTS

- USE U_X

PROBLEMS

- EASY TO FOLL (e.g. FORCING FN.)
- NO CHEAPER THAN V_t
- GOOD ONLY FOR SHOCKS

DEFERRED CORRECTION

- USES 2 METHODS
- COMPUTE ERROR ESTIMATE AS A FUNCTION OF THE 2 SOLUTIONS

PROS	CONS
MOST ACCURATE ESTIMATES TESTED	EXTRA TIME LEVELS FOR 2ND METHOD
	DIFFICULT TO FIND 2ND METHOD AND ERROR RELATION

SPECIAL CASE (2h, 2k)

- 2ND METHOD USES SAME INTEGRATOR WITH DOUBLE THE STEP SIZES
- ERROR $\dfrac{V_{h,k} - \hat{V}_{2h,2k}}{2^{P+1} - 1}$

USE OF DIFFERENTIAL EQ. TO ELIMINATE TIME DERIV.

- USE $U_t = f(u,x,t)_x$ TO REPLACE TIME DERIVS. IN TRUNCATION ERROR

PROBLEMS

- MESSY TO FIND V_{ttt}
- VERY PROBLEM AND METHOD DEPENDENT
- USEFUL ONLY IF EXTREME PENALTY FOR USING EXTRA TIME LEVELS.

CONCLUSION

AUTOMATIC REFINED GRID GENERATION

- ARBITRARY ORIENTATION OF RECTANGLES
- LOW OVERHEAD OF GRID REPRESENTATION
- REFINEMENTS BASED ON (2h, 2k) ESTIMATES OF LOCAL TRUNCATION ERROR