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DETERMINATION OF PRINCIPAL STRESSES IN BIREFRINGENT COMPOSITES BY HOLE-DRILLING METHOD

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ABSTRACT

A complete stress analysis and reliable failure criteria are essential for important structural applications of composites in order to fully utilise their unique properties. The inhomogeneity, anisotropy and inelasticity of many composites make the use of experimental methods indispensable. Among the experimental techniques, transmission photoelasticity has been extended to birefringent composites in recent years. The extension is not straight-forward, in view of the complex nature of the photoelastic response of such model materials. This paper very briefly reviews the important developments in the subject and then describes the theoretical basis for a new method of determining the individual values of principal stresses in composite models. The method consists in drilling very small holes at points where the state of stress has to be determined. Experiments are then described which verify the theoretical predictions. The limitations of the method are pointed out and it is concluded that valuable information concerning the state of stress in a composite model can be obtained through the suggested method.

INTRODUCTION

Considerable progress has been achieved in the application of transmission photoelastic techniques to composite orthotropic model materials in recent years. The developments in the subject have been reviewed by the author (1). The interpretation of the photoelastic response is not straight-forward, in terms of stresses or strains. The isochromatic fringe order is a complex function of the principal stresses (or strains), their orientations, etc. When the in-plane elastic or photoelastic properties are equal in two mutually perpendicular directions, the fringe order is proportional to the difference in principal stresses (or strains); when the in-plane properties in the two perpendicular directions are unequal, the fringe order is a linear function of the principal stresses (or strains) if the principal stress (or strain) directions coincide with the material symmetry axes. The isoclinic fringes give the directions of the principal birefringence components according to a Mohr circle of birefringence; the principal strain directions are a better approximation to the isoclinic parameter than are the principal stress directions.

It is, therefore, clear that for the transmission photoelastic analysis of an orthotropic model to yield useful information, methods must be developed to determine the individual values of principal stresses (or strains). Several methods have already been proposed, such as shear difference, numerical solution of the compatibility equation and holography. These methods have been reviewed by the author (2). Some of these proposed techniques suffer from the disadvantage that they use the photoelastic response partially and rely on analytical procedures which either give rise to error or are cumbersome. The holographic method of combining isochromatics and isopachics is not feasible for composites because of the complex nature of both families of fringes. There is consequently a need for a simple and completely experimental method of determining the individual values of principal stresses or strains.

A new method, consisting of perforating the composite photoelastic model with small circular holes and determining the photoelastic response on the boundaries of the holes, is described in this paper. The method, as applied to isotropic models, is reviewed and its extension to orthotropic models is explained. Then experimental results are presented to verify the proposed technique.

HOLE-DRILLING METHOD APPLIED TO ISOTROPIC MODELS

In order to determine the magnitudes and directions of the principal stresses at a given point in the interior of an isotropic photoelastic model, Tesa (3) suggested making a very small circular hole at the point. Thus an artificial free boundary is created for which the knowledge of isochromatics will be sufficient to establish the values of the principal stresses and their directions. Referring to Fig. 1, the hole is represented by the circle of radius a. The stresses at the points A and B are determined from the isochromatic fringe orders at these points. Then it is possible to show that the principal stress magnitudes are given by

$$\sigma_1 = (\sigma_A + 3\sigma_B)/8 \qquad (1)$$

$$\sigma_2 = (3\sigma_A + \sigma_B)/8 \qquad (2)$$

This method has the disadvantage of depending on the precise determination of the boundary stresses at the edge of a small hole. Durelli and Murray (4) have overcome this disadvantage by determining the principal stresses (corresponding to the center of the hole) from the principal stress differences measured at interior points. These interior points are represented as C and D on a circle of radius 2a in Figure 1. If the principal stress differences at the points C and D are denoted by σ_C and σ_D, then it is possible to show that

$$\sigma_1 = \left(15\sigma_D + 7\sigma_C\right) /11 \tag{3}$$

$$\sigma_2 = \left(15\sigma_C + 7\sigma_D\right) /11 \tag{4}$$

HOLE-DRILLING METHOD APPLIED TO COMPOSITE MODELS

The state of stress around a circular hole in a composite plate subjected to a biaxial loading is quite complex, in the general case. Simplifications can be made if the composite plate is considered to be subjected to stresses which act along the material symmetry axes, as shown in fig. 2.

When only the stress parallel to the reinforcement, σ_1, is acting, the tangential stress on the boundary of the hole is given by

$$\sigma_\theta = \sigma_1 \frac{E_\theta}{E_L} \left[-K\cos^2\theta + (n+1)\sin^2\theta \right] \tag{5}$$

where

$$K = \sqrt{\frac{E_L}{E_T}}$$

$$n = \sqrt{2\left(\frac{E_L}{E_T} - \nu_{LT}\right) + \frac{E_L}{G_{LT}}}$$

E is the Young's modulus, G the shear modulus, ν the Poisson's ratio, L and T the material symmetry axes and θ is the angle measured from the σ_1 direction. At the points A and B ($\theta = 0, \pi$)

$$\sigma_{A,B} = -\sigma_1/K \tag{6}$$

and at the points C and D ($\theta = \frac{\pi}{2}, \frac{3\pi}{2}$)

$$\sigma_{C,\, D} = \sigma_1(1+n) \tag{7}$$

when the stress perpendicular to the reinforcement, σ_2, is acting alone, the tangential stress on the hole boundary is given by

$$\sigma_\theta = \sigma_2 \frac{E_\theta}{E_L} \left[(K+n)\, K\cos^2\theta - K^2\sin^2\theta\right] \tag{8}$$

At the points A and B

$$\sigma_{A,\, B} = \sigma_2 \frac{K+n}{K} \tag{9}$$

and at the points C and D

$$\sigma_{C,\, D} = -K\sigma_2 \tag{10}$$

Superposing the stresses σ_1 and σ_2 and solving for them,

$$\sigma_1 = \frac{\sigma_B + K^2\sigma_A}{n(n+K+1)} \tag{11}$$

$$\sigma_2 = \frac{\sigma_B + K(1+n)\,\sigma_A}{n(n+K+1)} \tag{12}$$

While the measurement of the isochromatic fringe order is difficult on the hole boundary and it would be preferable to make the measurement at interior points, the analytical expressions for stresses at interior points in an orthotropic model are not available in closed form. Therefore, the stresses have to be determined from Eqs. (11) and (12).

DESCRIPTION OF TESTS AND RESULTS

To verify the proposed experimental method, a circular disk of 7.6 cm. diameter was tested in diametral compression. The disk was machined from a unidirectionally reinforced E-glass-polyester laminate. The elastic and photoelastic constants for the material were determined by following standard calibration procedures and are given in Table 1.

Table 1 ELASTIC AND PHOTOELASTIC PROPERTIES OF BIREFRINGENT COMPOSITE MODEL

Property	Value
E_L	28.8 GPa
E_T	9.4 GPa
G_{LT}	3.17 GPa
ν_{LT}	0.3
f_L	156 KPa - m/fringe
f_T	78 KPa - m/fringe
f_{LT}	69 KPa - m/fringe

Circular holes of 0.32 cm. diameter were drilled on radial lines parallel and perpendicular to the reinforcement, at locations 1.27 cm. and 2.54 cm. from the center. Electrical resistance strain gages were mounted at similar points diametrically across from the holes.

The disk was loaded under diametral compression and the isochromatic fringe pattern was photographed at a load of 2225N. Light-field and dark-field isochromatic fringe patterns when the load was applied parallel to the reinforcement direction are shown in Figs. 3 and 4. Strain gage readings were recorded and converted into stresses by making use of the measured elastic constants. The test was repeated with the load perpendicular to the reinforcement direction. The corresponding light-field and dark-field isochromatic finge patterns are shown in Figs. 5 and 6. Again the stresses were computed from the strain gage readings. The fringe orders at the boundaries of the holes were determined in each case by photoelastic compensation and the stresses were computed from Eqs. (11) and (12).

Typical results obtained from the various methods are shown in Fig. 7. The non-dimensional principal stresses, for transverse loading, on the horizontal diameter are shown as a function of the distance from the disk center. The open circles represent strain gage results and the results from the proposed hole-drilling technique are shown by closed circles. Results from a combined reflection-transmission technique described in Ref. 5 are shown by the continuous line. The stress values given by Eqs. (11) and (12) are seen to agree quite well with the strain gage results and also with the reflection-transmission results.

CONCLUSIONS

It has been demonstrated in this paper that the individual values of the principal stresses in orthotropic photoelastic models can be determined by drilling very small holes at the points of interest and measuring the isochromatic fringe orders at the boundaries of these holes. The necessary equations have been developed and the method has been verified by comparison with strain gage results for a circular disk under diametral compression.

The method was originally proposed for and applied to isotropic models. For such models the method can be improved by measuring the fringe order at interior points. This improvement can not be applied to orthotropic models because analytical expressions for the state of stress away from a hole are not readily available. Also, the method has been demonstrated only for the special case when the principal stress directions and material (and model)- symmetry axes coincide. Further work is required to make the method more general.

A simple, reliable method of determining the complete state of stress in a composite structure (by photoelastic modelling) will contribute to an efficient utilisation of the fibers, resins and fabrication methods which are constantly being improved. It is hoped that this paper can contribute to working together for strength.

ACKNOWLEDGMENT

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REFERENCES

1. R. Prabhakaran, Developments in Photo-Orthotropic-Elasticity, Proceedings of the IUTAM Symposium on Optical Methods in Solid Mechanics, Poitiers, France, September 1979.

2. R. Prabhakaran, Use of Transmission Photoelasticity to Composite Materials, a chapter in Developments in Composite Materials, Vol. 2, Edited by G. S. Holister, to be published by Applied Science Publishers, Ltd., England.

3. V. Tesar, La Photoelasticimetrie et ses Applications dans les Constructions Aeronautiques, La Science Aerienne, Paris, p. 372, September-October 1932.

4. A. J. Durelli and W. M. Murray, Stress Distribution Around a Circular Discontinuity in Any Two-Dimensional System of Combined Stress, Proceedings of 14th Semi-Annual Eastern Photoelasticity Conference, Yale University, pp. 21-36, 1941.

5. R. Prabhakaran, Separation of Principal Stresses in Photo-Orthotropic-Elasticity, Fibre Science and Technology, Vol. 13, pp. 245-254, 1980.

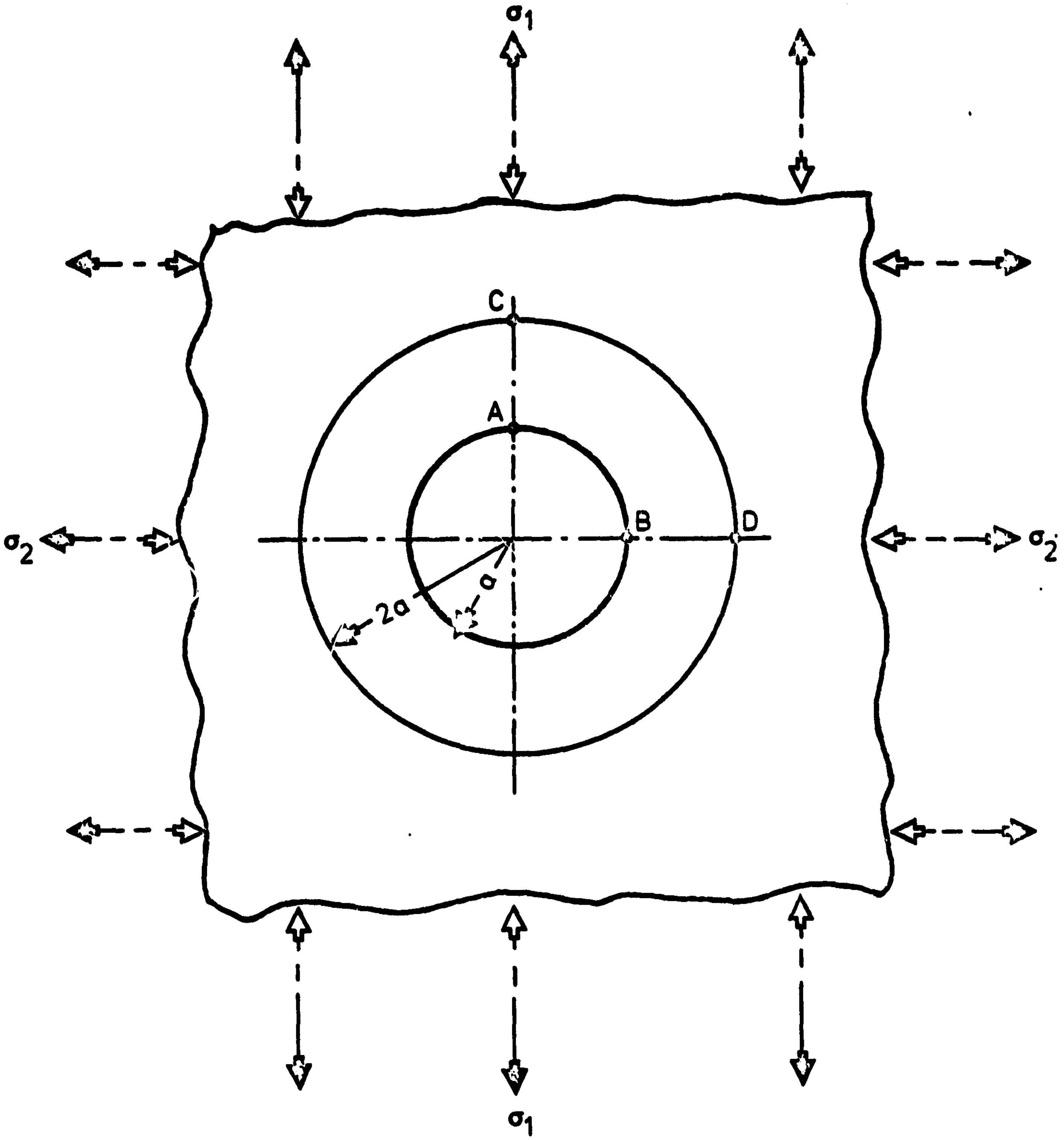

Fig. 1 DETERMINATION OF σ_1 AND σ_2 IN TERMS OF σ_A AND σ_B OR σ_C AND σ_D IN ISOTROPIC MODELS

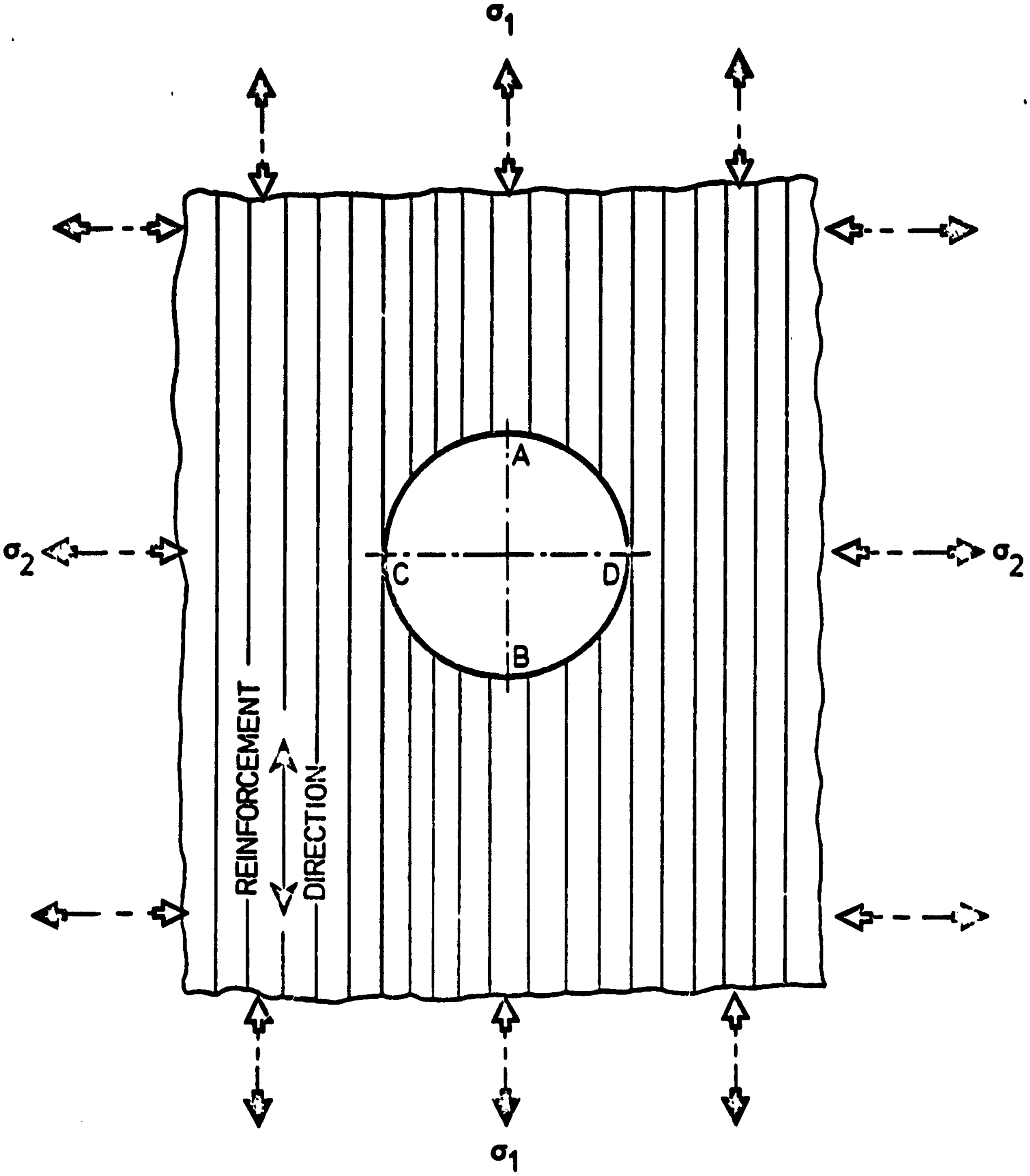

Fig. 2 DETERMINATION OF σ_1 AND σ_2 IN TERMS OF σ_A AND σ_D IN ORTHOTROPIC MODELS

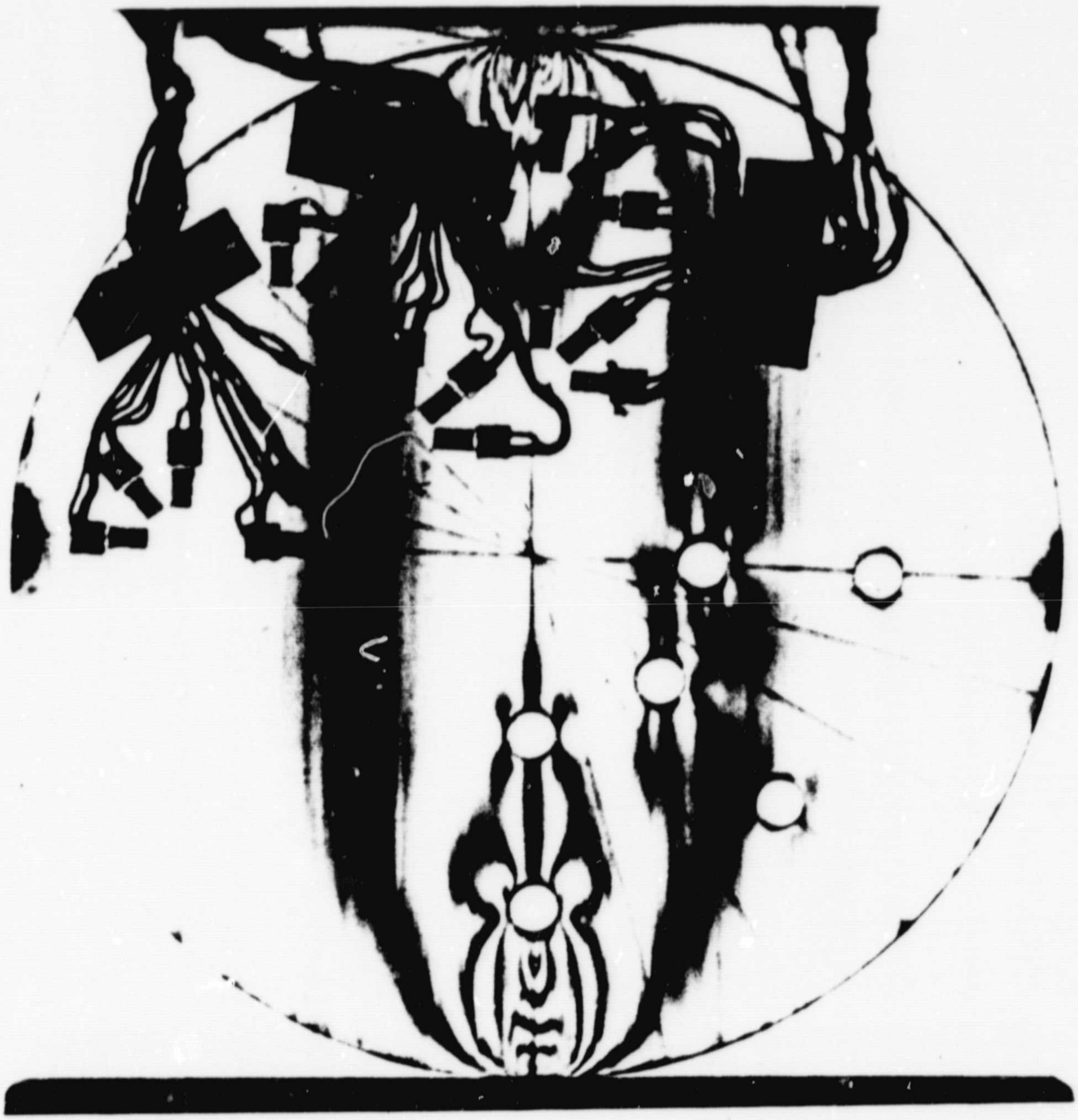

Fig. 3 LIGHT-FIELD ISOCHROMATIC FRINGE PATTERN FOR PARALLEL LOADING

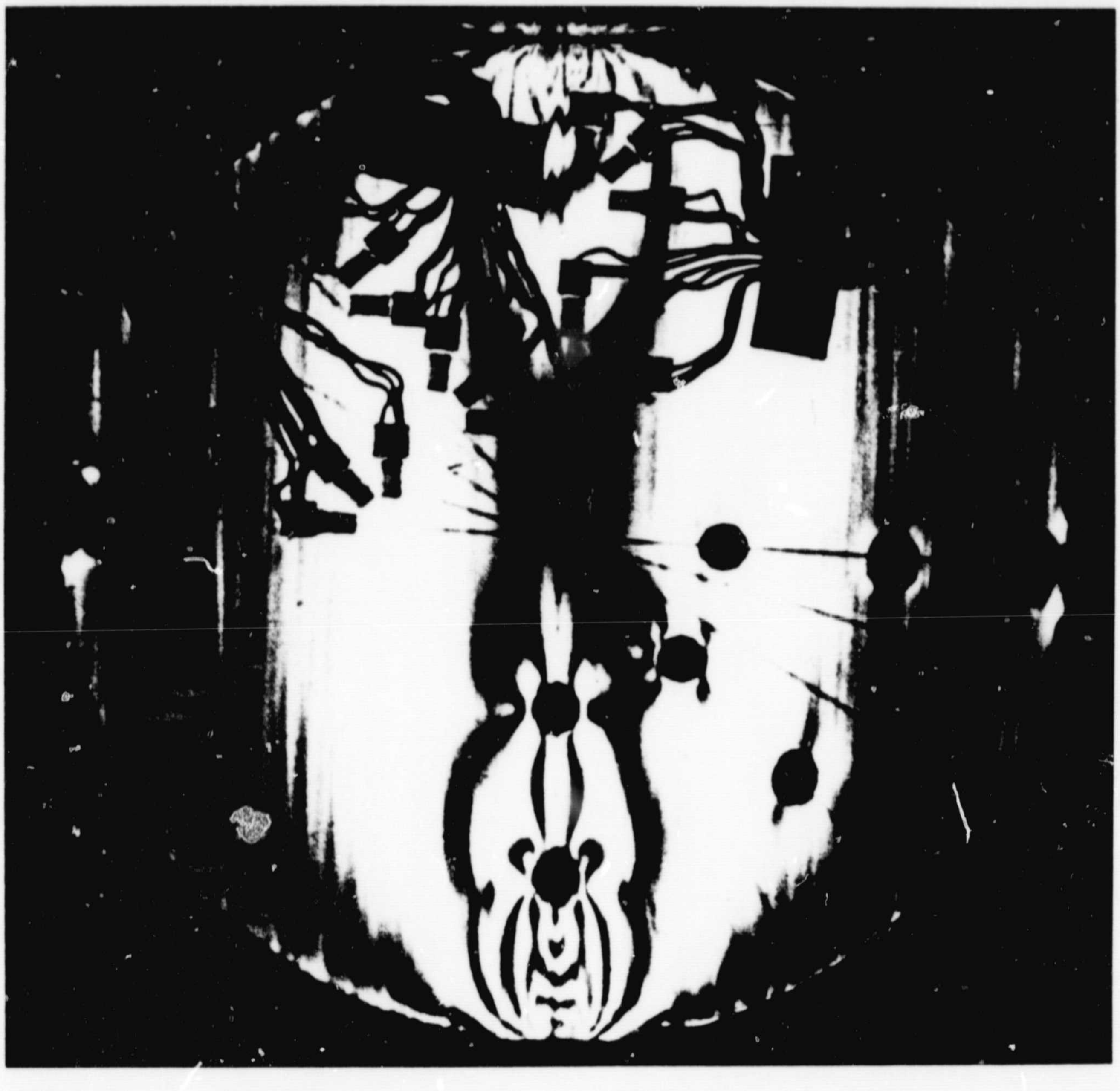

Fig. 4 DARK-FIELD ISOCHROMATIC FRINGE PATTERN FOR PARALLEL LOADING

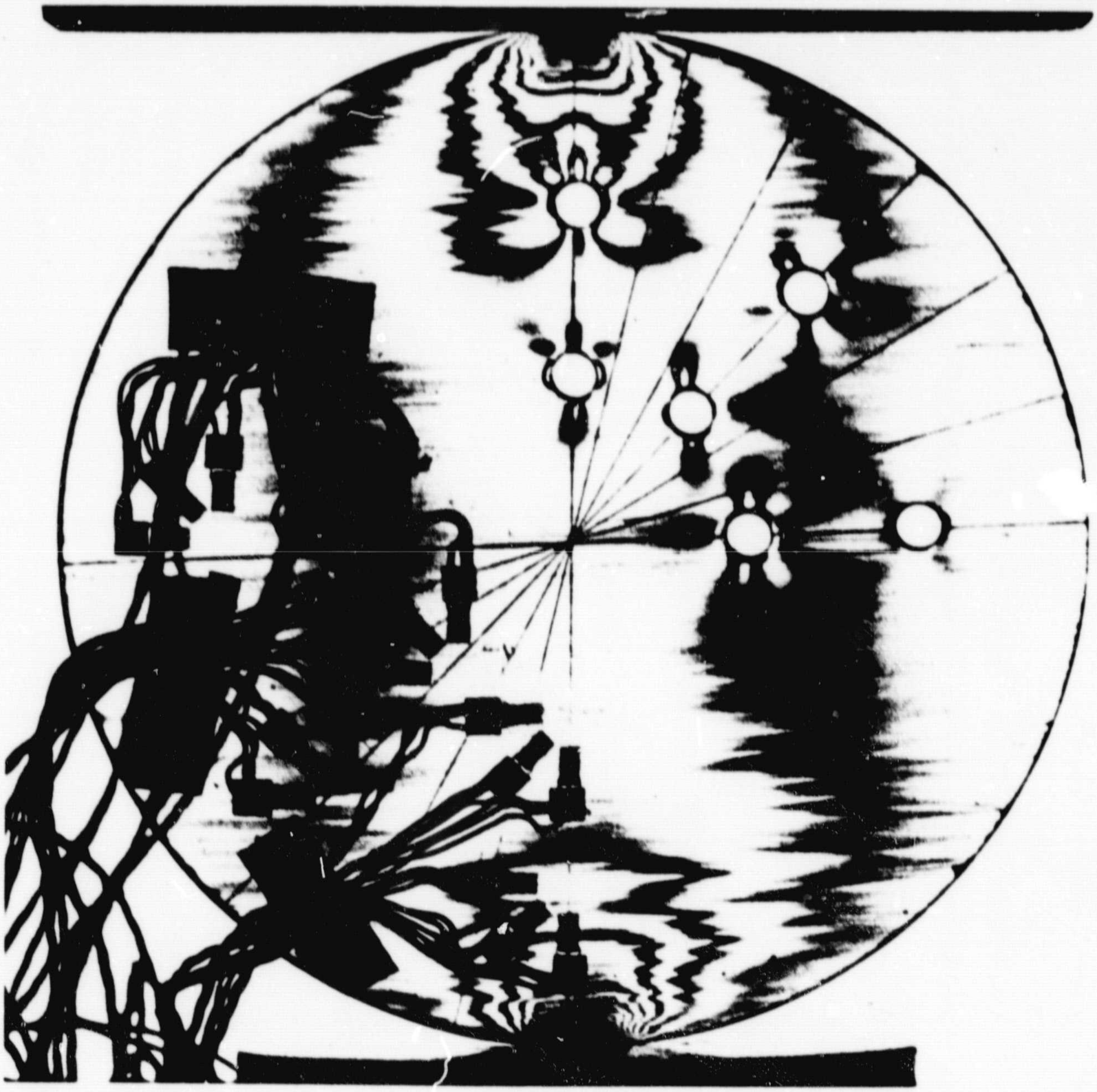

Fig. 5 LIGHT-FIELD ISOCHROMATIC FRINGE PATTERN FOR TRANSVERSE LOADING

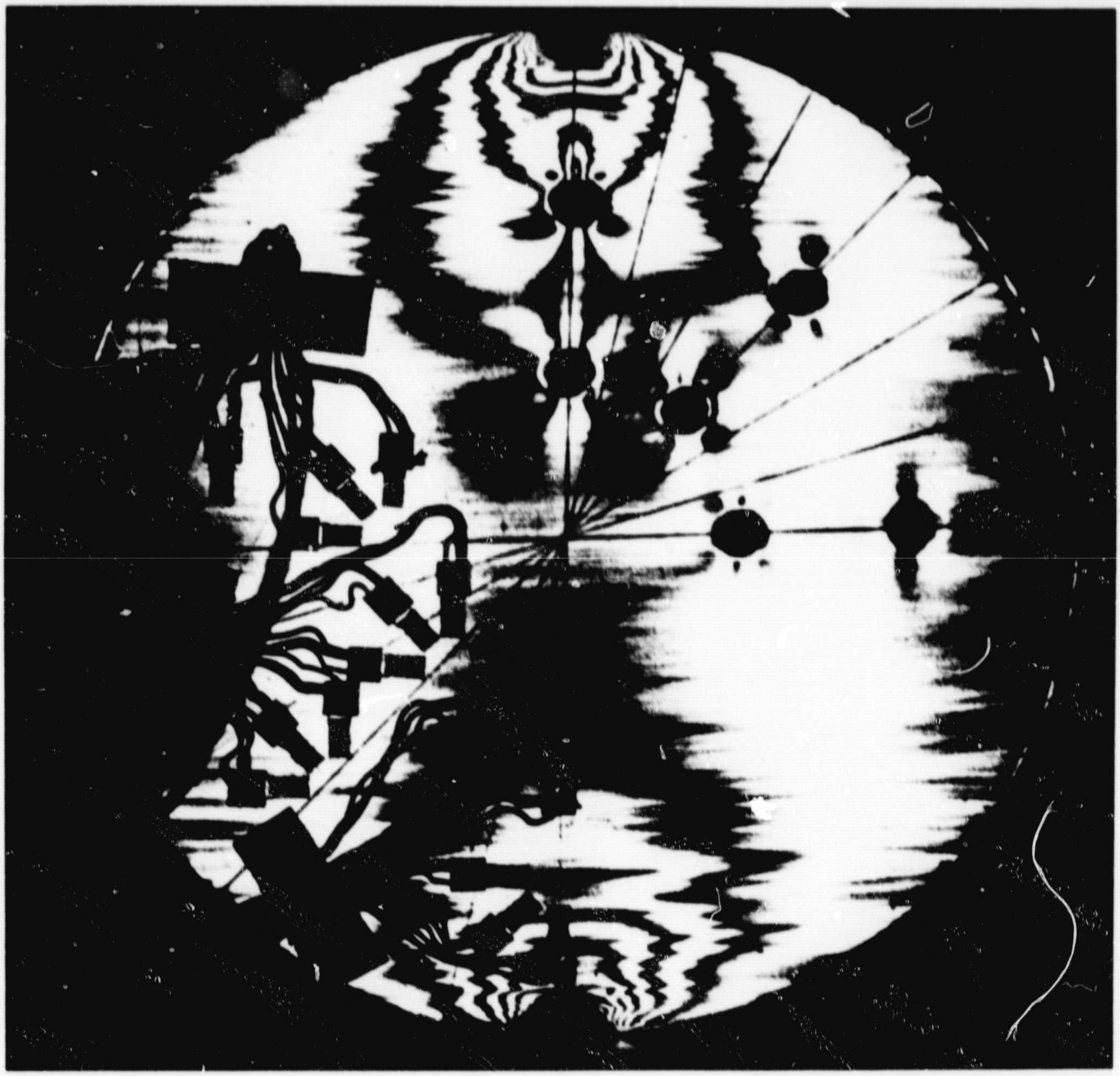

Fig. 6 DARK-FIELD ISOCHROMATIC FRINGE PATTERN FOR TRANSVERSE LOADING

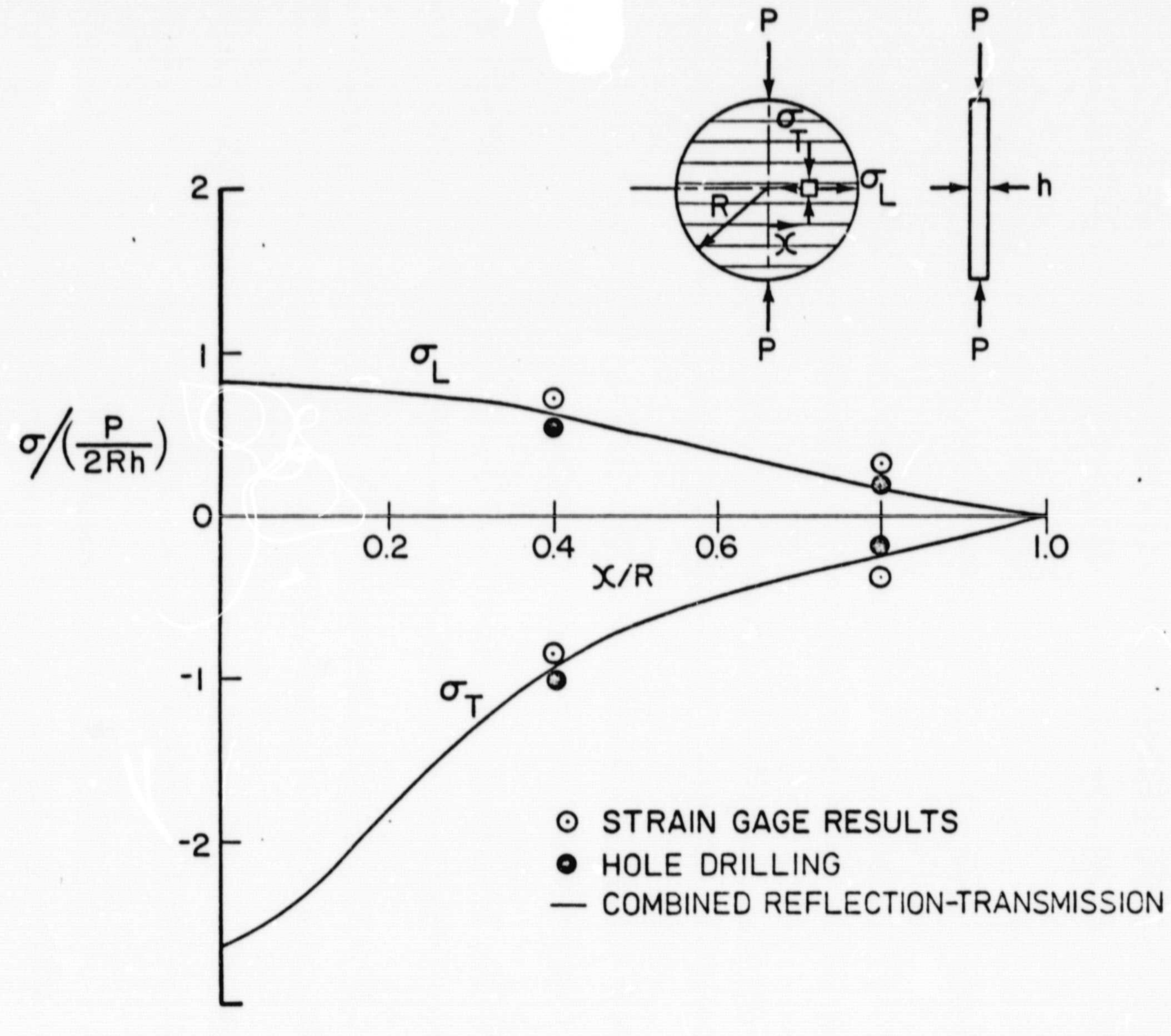

Fig. 7 STRESSES ALONG THE HORIZONTAL DIAMETER FOR TRANSVERSE LOADING

Biographical Sketch of the Author

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Author obtained his B. Tech. degree from Indian Institute of Technology, Madras, M. E. degree from Indian Institute of Science, Bangalore and Ph.D. in Mechanics from Illinois Institute of Technology, Chicago. Author has taught at the University of Iowa, Indian Institute of Technology, Kanpur and is at present an Associate Professor at Old Dominion University, Norfolk. Author's interests include mechanical properties of polymers and composites and experimental stress analysis. Author has over fifty papers in technical journals and has presented several papers at professional meetings including invited papers at meetings sponsored by the International Union of Theoretical and Applied Mechanics and the Polish Academy of Sciences.