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STUDY OF THE VORTEX CONDITIONS OF WINGS WITH LARGE SWEEPBACK BY EXTRAPOLATION OF THE JONES METHOD

P. Hirsch

Translation of "Essai d'étude du régime tourbillonnaire des ailes de forte flèche par extrapolation de la méthode de Jones," 16th Symposium of Applied Aerodynamics, Lille, November 13, 14 and 15 1979

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16. Abstract

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SUMMARY

The pockets of separation originating on the leading edges are surrounded by vortex sheets.

Their configuration and intensity are determined by four conditions with the JONES approximation, which is itself corrected by a simple logic.

Field pressures and stresses are computed for different cases and are compared with the test (pure deltas, swallow tails, truncations, "strakes", ~~ducks~~ canards, fuselage).

1 - INTRODUCTION /1

An increase in the angle of a strongly swept-back wing results in the formation of a pocket of twisting separation upon the leading edge.

The contact between sound fluid and the separated zone thus produces vortex surfaces which wind around in horn pattern upon the base of leading edges.

An attempt is made here to give an approximation of the phenomenon by using diagrams and simplified calculation assumptions based on the traditional JONES approximation.

Let us briefly review the principles:

Upon a strongly swept-back wing, the variation of flows along the longitudinal coordinate x is slow compared to their variation relative to the transverse coordinates y and z. In the

*Numbers in the margin indicate pagination in the foreign text.

condition of continuity, it follows that the derivative u'_x is smaller than derivatives v'_y, w'_z .

In a first approximation, the flows in v and w , in each transverse section, are almost independent from what occurs in the other sections and are influenced very little by the presence of longitudinal component u .

Here, under the same assumptions, the transverse diagram will correspond to that shown on figures 1 and 1-a which define the annotations of the configuration.

The pockets of separation are assumed to be of circular section. Inside the pocket, components v and w will be zero: the vortex boundary is therefore the line of jet and the angular vortex density $\partial\Gamma^*/\partial\theta$ of radius r which forms the boundary with the external flow will be constant.

Between the two pockets of separation, the linear circulation density of the vortex components connected to the wing will be $\partial\Gamma_1/\partial y$.

Γ^* and $\Gamma_1(y)$ will be functions of x which must be defined.

2 - CONDITIONS /2

The first condition to observe is that of the relative slip on the wing assumed here to be flat.

Since w is the component induced by the vortex system matching the conditions of JONES, and i the angle, we must have

$$i = w/V_0$$

with:

$$w(y) = \frac{1}{2\pi}\left[-\Gamma^*\left\{\frac{b/2-y}{(b/2-y)^2+r^2}+\frac{b/2+y}{(b/2+y)^2+r^2}\right\}+\sum_j \partial\Gamma_{1j}\frac{2y_j}{y^2-y_j^2}\right]$$

To define the parameters of the problem, let us set:

$$\frac{\delta\Gamma_{1j}}{2\pi V_0 b'_1/2\, i} = \delta\gamma_{1j}\ , \quad \frac{\Gamma^*}{2\pi V_0 b'_1/2\, i} = \gamma^*\ , \quad \zeta = \frac{y}{b'_1/2}\ , \quad \zeta_j = \frac{y_j}{b'_1/2}\ .$$

The configuration of the system will therefore depend on parameter

$$\mu = \frac{r}{b'_1/2}$$

where

$$b_1 = b'_1(1-\mu) \qquad \text{and} \qquad b_1 = b\,\frac{1-2\mu}{1-\mu}$$

The relationship above becomes:

$$-\gamma^*\left\{\frac{1-\mu-\zeta}{(1-\mu-\zeta)^2+\mu^2} + \frac{1-\mu+\zeta}{(1-\mu+\zeta)^2+\mu^2}\right\} + \sum_j \delta\gamma_{1j}(\zeta_j)\,\frac{2\zeta_j}{\zeta-\zeta_j} = 1 \qquad (1)$$

This is a functional relationship defining γ^* and $\delta\gamma_{1j}$ for μ given. The solution is easier by using a small computer, HP97, for example .

For $0<\mu<0.4$, we thus find that $\gamma_1(y)$ varies little relative to y and is expressed by the relationship of condensation:

Simultaneously:

$$\left.\begin{aligned} \gamma_1 &\cong 1.2\left[0.2+\mu^2\right] \\ \gamma^* &\cong 1.2\left[0.2+\mu(0.8-\mu)\right] \end{aligned}\right\} \qquad (2)$$

The slopes of x of quantities b_0, b, b_1, b'_1 define locally /3 the sweepback angles, such as:

$$\left(\frac{b_0}{2}\right)'_x = \frac{1}{tg\phi_0} \quad , \left(\frac{b}{2}\right)'_x = \frac{1}{tg\phi} \quad , \left(\frac{b_1}{2}\right)'_x = \frac{1}{tg\phi_1} \quad , \left(\frac{b'_1}{2}\right)'_x = \frac{1}{tg\phi'_1} \; .$$

The intensity of the vortex tube along its axis is $\Gamma = \Gamma^* / \sin\phi$.

The twist velocity at the contact of the tube in a plane perpendicular to the axis is $W_0 = \Gamma / 2\pi r$ at the contact of the wing.

Projected perpendicular to Ox it is $W = W_0 \sin\phi$ such that $W = \Gamma^* / 2\pi r$ (fig.2).

If we assume N is the ratio between W_0 and the potential incident component, here:

$$V_0 \cos\phi . \cos(\phi - \phi_0) \cong V_0 \cos\phi$$

Therefore: $\Gamma = 2\pi r N V_0 \cos\phi$, which will be expressed with

$$\frac{\Gamma}{2\pi \, b'_1/2 \, V_0 i} = \gamma = \gamma^* / \sin\phi$$

$$\frac{\gamma^* i}{\mu \sin\phi} = N \cos\phi_0$$

Let us now consider the distance δy between the leading edge of the wing and outline on the wing of the vortex tube axis, taken perpendicular to Ox . Its slope will be:

$$\delta y'_x = \frac{1}{tg\phi_0} - \frac{1}{tg\phi} \qquad (4)$$

A vortex component which follows the outline of this tube on the wing surface is subjected to a drag velocity, perpendicular to this outline is the half-resultant(*) between $V_0 \cos\phi$ coming from the external potential and:

$$-W_0 = -N V_0 \cos\phi_0$$

Therefore:

$$\delta y'_t \cong \frac{1}{2}\left[V_0 \cos\phi - N V_0 \cos\phi_0 \right]$$

In the axes related to Ox, $y = \delta y \sin\phi$,

$$\delta y'_t = \left(\frac{b_0}{2} - \frac{b_2}{2}\right)'_x x'_t = V_0 y'_x = V_0 \left[\frac{1}{tg\phi_0} - \frac{1}{tg\phi}\right]$$

This gives us the relationship:

$$\frac{1}{tg\phi_0} - \frac{1}{tg\phi} = \frac{1}{2}\sin\phi\left[\cos\phi - N\cos\phi_0\right]$$

i.e.:

$$N\cos\phi_0 = \cos\phi + \frac{2}{\sin\phi}\left(\frac{1}{tg\phi_0} - \frac{1}{tg\phi}\right) \quad (4)$$

These are the conditions which define the configuration ϕ, μ, and ϕ_1, and the vortex intensities Γ_1, Γ^*, when the angle i, sweepback ϕ_0 and factor N are defined.

The problem is solved easily by using mini-computers, such as the HP 97.

3 - CORRECTION FACTORS OF THE JONES APPROXIMATION

The approximation is in principle exactly applicable to the boundary condition where sweepback ϕ_0 reaches 90°.

It has two sources of error. The first results from the non-zero slope on the symmetry plane of the vortex lines. The second

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(*) The velocity of the vortex component is half the sum of the external and internal velocities at the sheet, with the latter being equal to zero.

one is because the vortex lines do not reach infinity with a constant circulation intensity upstream and downstream the section under consideration, which is implicitly assumed in the JONES approximation.

For the first cause of error, we have seen that the circulation Γ^* taken in a plane perpendicular to the plane of symmetry is deduced from the circulation around the vortex tube perpendicular to its axis by the relationship $\Gamma^* = \Gamma \sin\phi$ (fig.3)

A point M of section x is subjected to an induced velocity $w = \dfrac{\Gamma}{2\pi D}$ with $D = d \sin\phi$. Expressed with Γ^* and d :

$$w = \frac{\Gamma^*}{2\pi d \sin^2\phi}$$

The correction factor $\sin^2\phi$ thus appears in the condition of relative slip which defines $\Gamma^* = 2\pi d \dfrac{w}{V_0}$.

It may be compared with the results of the comparison between the calculation of lift performed at the ONERA with the rheoelectric tank and calculations with the JONES approximation for different sweepbacks:

ϕ_0	60°	70°	80°	15
ONERA correction factor	0.70	0.79	0.89	
$\sin^2\phi$	0.75	0.88	0.97	

Factor 1.2 is thus replaced in the expressions of γ^* and $\sum_j \delta\gamma_j$ by 1.2 $\sin^2\phi_0 = \nu$.

In order to evaluate the effects of the second error of the JONES approximation, the effects of each section of a vortex tube with increasing linear radius and circulation from the apex to the leading edge (beyond the latter $\partial\Gamma^*/\partial r$ is reduced to zero and has no more effect). The results will be compared with those of the preceding formulas.

By carrying out this calculation, we have shown that the correction factor required is very close to one, except in the region near the trailing edge of the wing (0.85 to 1 of the central chord) where it drops to a limit value 0.5, as suddenly as the sweepback is large.

4 - CHOICE of $N = \frac{W}{V_0 \cos\phi_0}$

If the leading edge is pointed, the separation must appear with very small incidences: we must therefore find the value $N = N_0$ for these incidences resulting in a lift at least equal to that of the JONES theory corrected in $\sin^2\phi$.

We thus find: $N_0 \cong 1{,}07 \quad 1{,}09$

for $55^\circ < \phi_0 < 80^\circ$

In the case of a wing which has a leading edge of curve $1/R$ of slope $y'_s(0)$, the twisting velocity will be higher than for /6
the previous case, hence: $N > N_0$.

It is possible to evaluate the variation $N - N_0$ by applying relationships which define the tangential velocity of a profile at the leading edge relative to the velocity of the perpendicular component $V_0 \cos\phi_0$ Therefore:

$$N - N_0 \cong 2\sqrt{R/\ell} - 1{,}1\, y'_{s_0}\, R/\ell$$

Here $\ell = \frac{b_0}{\sin\phi_0}$ and $R/\ell = \frac{1}{2}\left(e/\ell\right)^2$.

5 - PRESSURE FIELDS AND LOCAL STRESSES

They will be determined by applying the KUTTA law:

$$\frac{d^2F}{ds} = \rho V d\Gamma$$

V is the component of velocity perpendicular to the vortex stream on this point.

By assuming $q = \frac{\rho}{2} V_0^2$, the calculation leads to:

-The average pressure under the horn:

$$\frac{\Delta p^*_m}{q} = \frac{2\pi \gamma^* i}{\mu \, tg\, \phi}$$

-The pressure between horns:

$$\frac{\Delta p_1}{q} = \frac{8\pi \gamma_1 i}{tg\, \phi_1}.$$

-and local pressures (fig. 4) under the horn.

$$\frac{\Delta p^*}{q} = \frac{\Delta p^*_m}{q} \, x \left(\frac{1}{tg\, \phi_1} - \frac{1}{tg\, \phi_1} \right) f^*$$

The function f^* is defined by:

$$f^* = \int_{-\infty}^{\xi/r} \psi \cdot d(\xi/r)$$

ψ may be calculated point by point from the formulae of BIOT SA-VART and KUTTA as a function of coordinate ξ/r centered on the projection of the vortex tube axis (Fig. 4). /7

Accordingly, the local stress is:

$$\frac{1}{2q} \cdot \frac{dR_z}{dx} = \frac{\Delta p^*_m}{q} \, x \left[f^* \left(\frac{1}{tg\, \phi_1} - \frac{1}{tg\, \phi_1} \right) + \frac{1}{tg\, \phi_1} \cdot \frac{\Delta p_1}{\Delta p^*_m} \right].$$

For a pure triangular Δ, we thus have with S = wing surface:

$$\frac{R_z}{qS} = C_z = \frac{\Delta p_m^*}{q}\left[\rho^*\left(\frac{1}{\mathrm{tg}\,\phi_1'} - \frac{1}{\mathrm{tg}\,\phi_1}\right) + \frac{1}{\mathrm{tg}\,\phi_1} \cdot \frac{\Delta p_1}{\Delta p_m^*}\right] \mathrm{tg}\,\phi_0$$

The moment around the apex will be here:

$$\frac{1}{qS\ell} M(o) = h\, C_z$$

where: $0.6 < h < 0.63$ for $60° < \phi_0 < 75°$. (ℓ = central chord)

6 - RESULTS OF CALCULATIONS AND COMPARISON WITH THE TEST

6.1 - Pure Delta

Programs solved by using the small HP97 computer have resulted in numerous applications.

Pl. 1 shows the results of comparisons between calculations and tests found in various publications (ONERA - AGARD).

The calculated configurations of vortex tubes place the axes of the latter inside the wing in the vicinity of the leading edges. The pressure fields are satisfactorily shown; the test shows that the peak decreases as we move closer to the leading edge.

For a sweepback of 75°, the calculation of $C_z(\alpha)$ decreases the slope after 15° of incidence, which is not shown by the tests.

6.2 - Special Wing Shapes

We were able to calculate the cases of small truncated and swallow tail wings.

Pl. II provides the results and the comparison with the tests of BOBBIT and MONROE (wing of 71°2) on pressure fields, lift and moment.

The diagrams of figure 5 show the principle of the calculations and the regions where pressure integrations must be found.

In the region formed by the swallow tail, the distribution of components $\delta \Gamma_{\lambda}$ is the same as for the trailing edge of the wing. /8

Let us point out that the calculation also applies to a "negative" swallow tail: $\phi_0' < 0$.

6.3 - "Strake" and the Case of Multiple Sweepbacks

This is a difficult case because it must be shown that the horn-shaped vortices originating on the "strakes" and those on the downstream section of the wing are connected in position and intensity, which results in displacing the latter relative to their natural position. An additional intensity must be applied to them without modifying the condition of relative slip (Fig. 6).

Comparisons were possible only in cases where a fuselage was also present.

6.4 - ~~Duck~~ Canard

It was necessary to take into account the effects of inductance of the vortex tubes on the wake sheet of the duck downstream from the latter: the result was a deflection to the bottom of this sheet and a contraction of the tubes to the plane of symmetry. This effects the main wing and modifies, as for the case of the "strakes", the configuration of vortices of the main wing (Fig. 7).

By taking these precautions, it was possible to make satisfactory comparisons of the calculation - test (SAAB ~~duck~~ Canard - Pl. III).

6.5 - Fuselage

The first comparisons with the test showed that a fuselage with small diameter changed very little the results of a detached wing by giving it the surface of the total wing surface. On the other hand, in the case of a detached ~~duck~~ canard placed on a fuselage (i.e. a wing with small external surface compared to the total surface - ONERA Test of EHRLICH) - the calculation based on only the outside surface coincided more with the test result.

These observations have shown that there is an effect related to the ratio of the wing span divided by the diameter of the fuselage squared (aspect ratios). This explains the introduction of an operator in $(D/2\ell \cdot tg\phi_0)^2$, with ℓ the wing chord, D the diameter of the fuselage, taken in the form:

$$\left[1 - \exp\left(-\frac{K}{(D/2\ell \cdot tg\phi_0)^2}\right)\right]$$

It moves to zero when $D/2\ell \cdot tg\phi_0 \rightarrow 0$

and assumes the value making it possible to verify the EHRLICH test, for $K = 0.12$

Plates IV, V, VI, and VII result from the application of these different calculation methods.

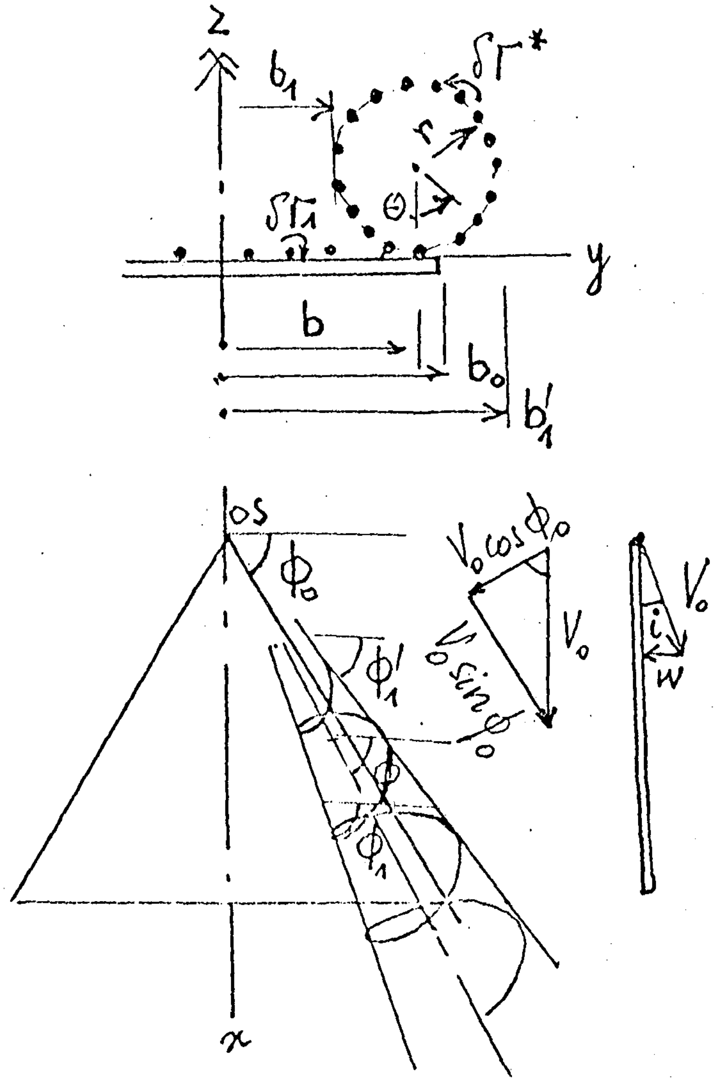


Figure 1

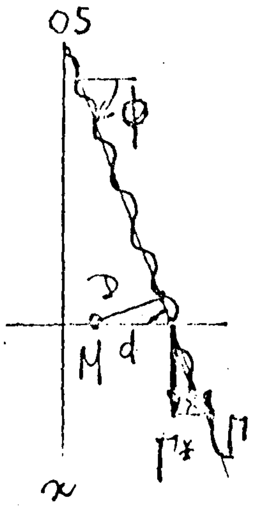


Figure 3

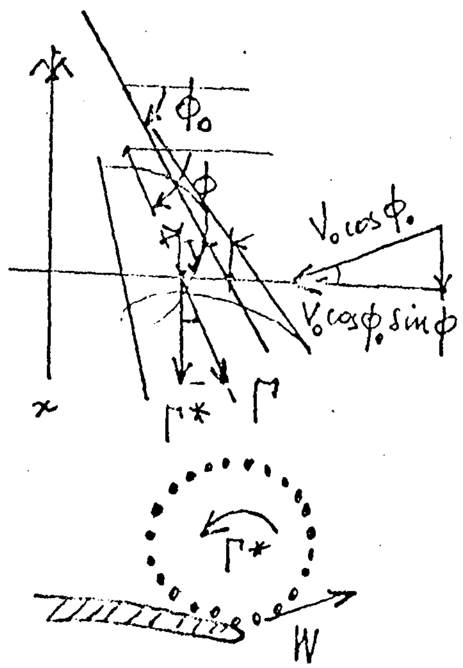


Figure 2

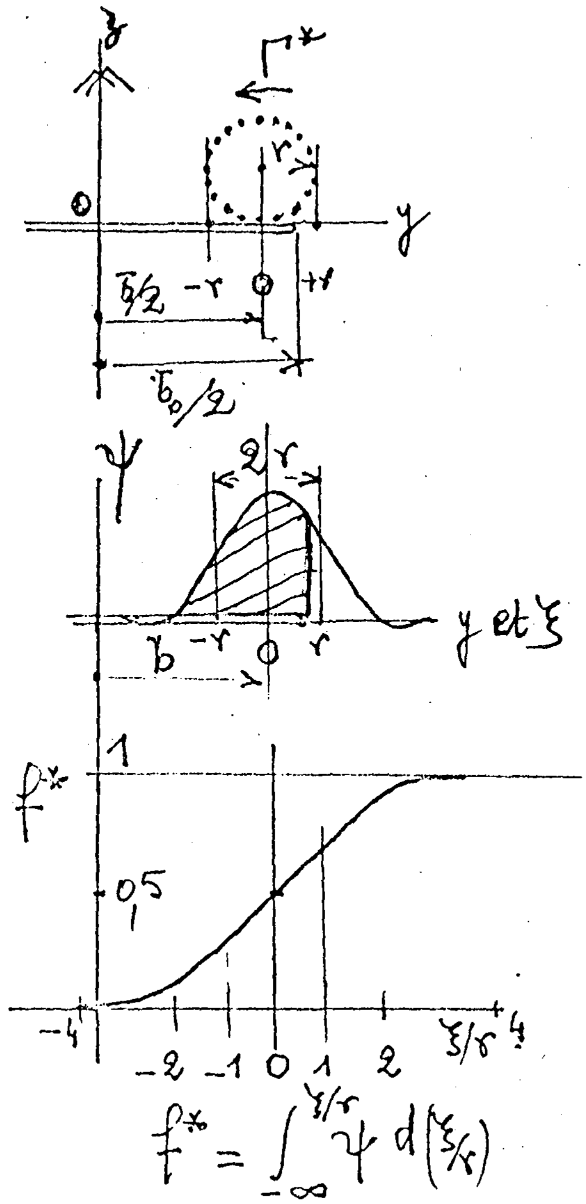


Figure 4

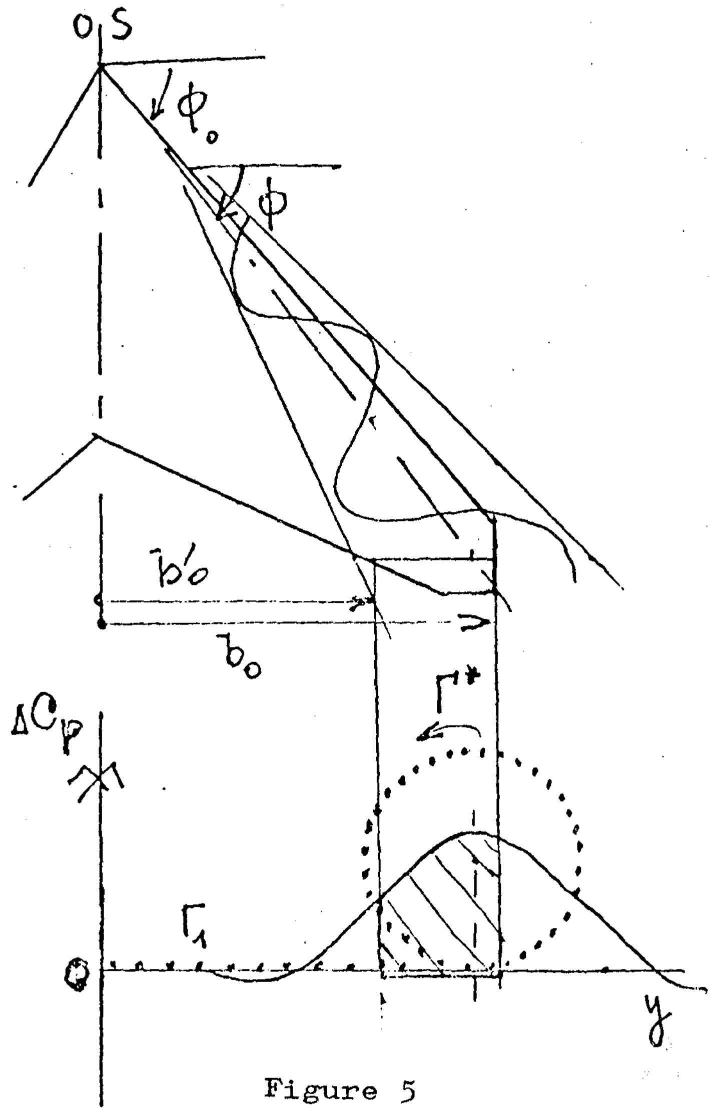


Figure 5

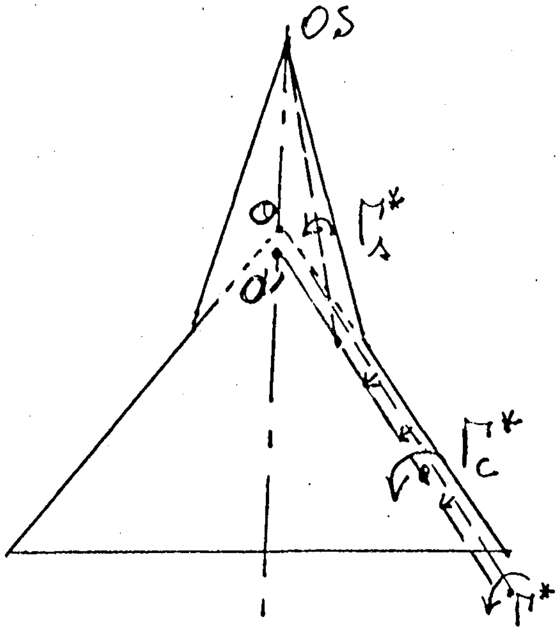


Figure 6

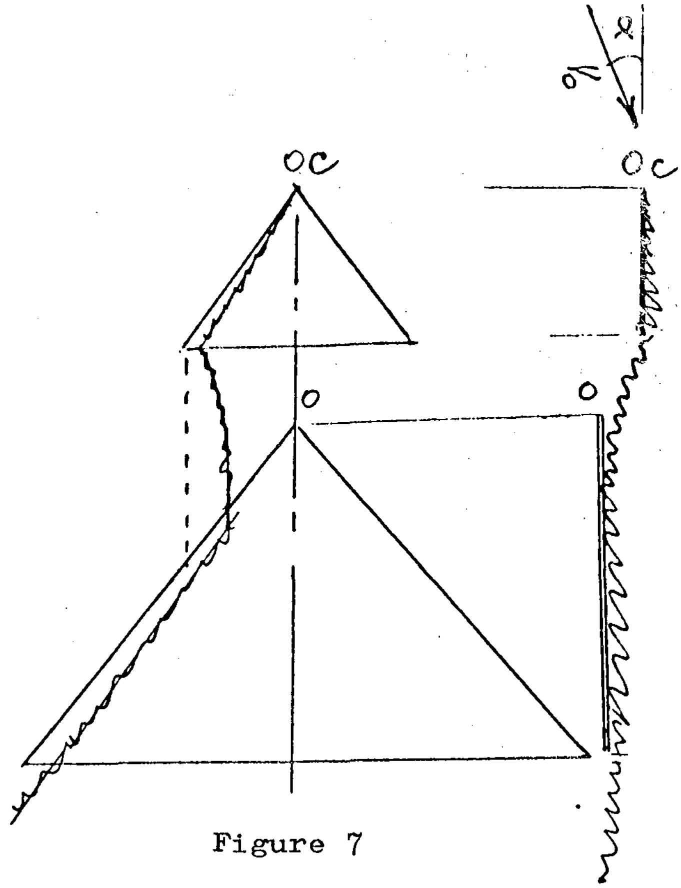
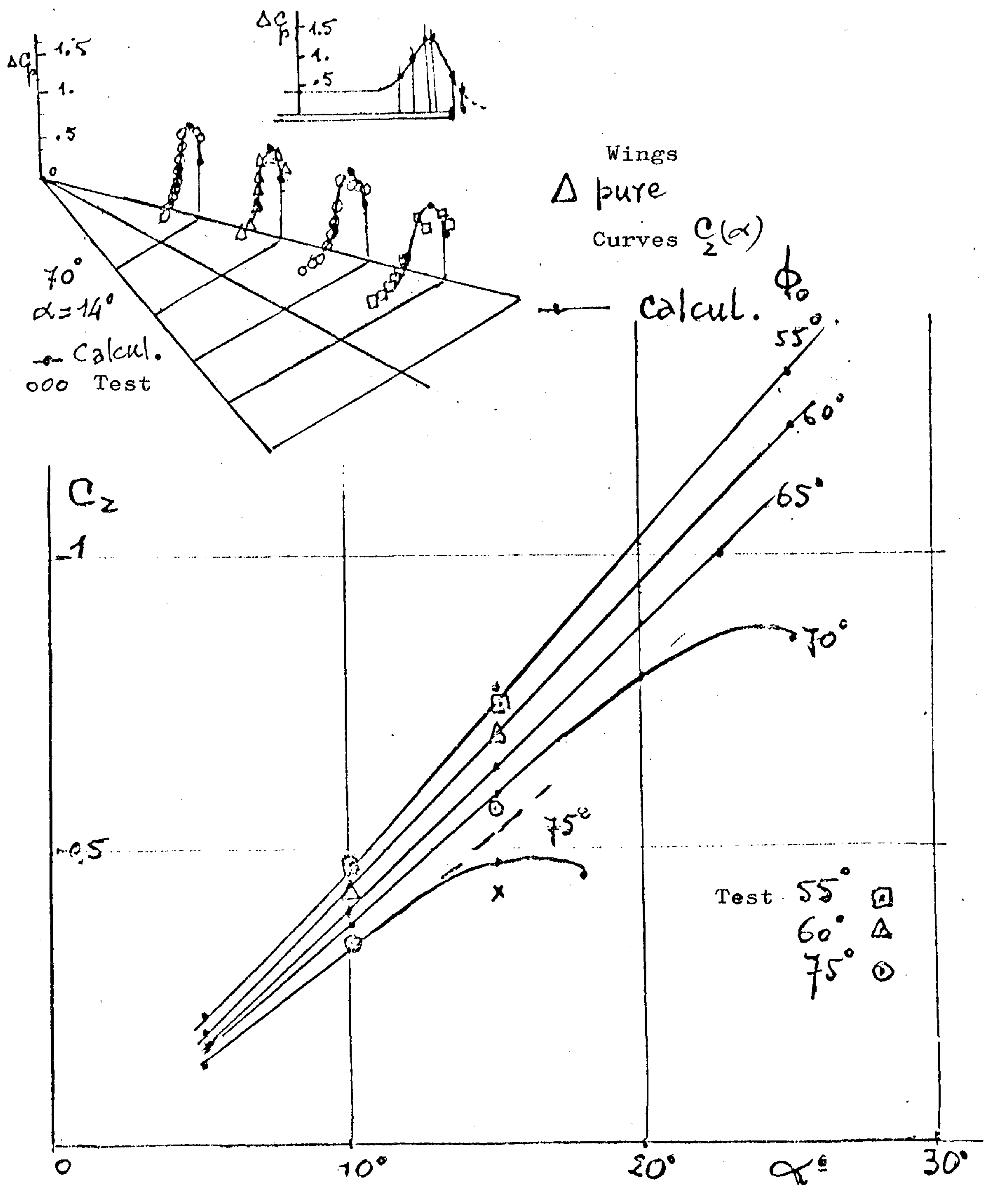
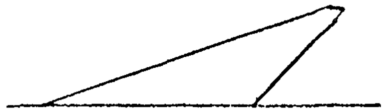


Figure 7



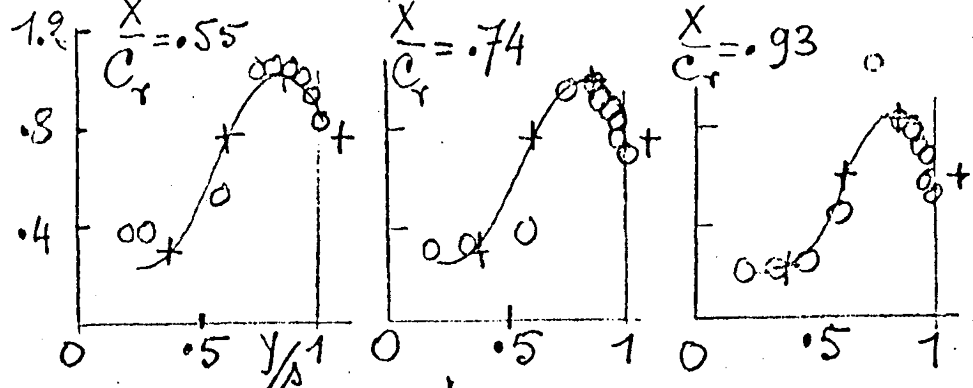
ΔCp 1.5 1. .5
ΔCp 1.5 1. .5 0
70°
α = 14°
Calcul.
ooo Test
Wings
Δ pure
Curves Cz(α)
Φ0
Calcul.
55°
60°
65°
70°
75°
Cz
1
.5
Test 55°
60°
75°
0
10°
20°
α°
30°

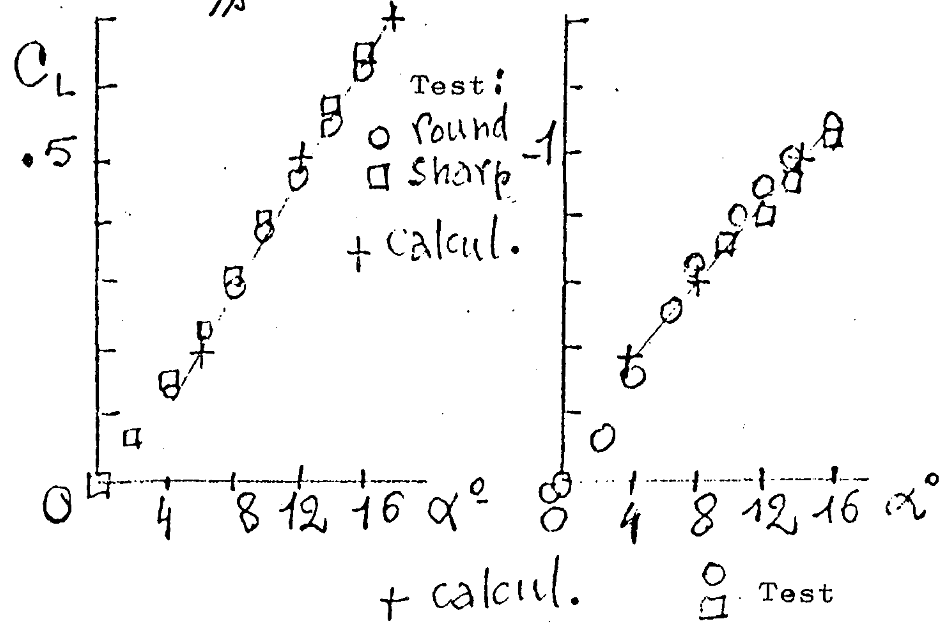
PERCY, Bobbitt & Monroe.

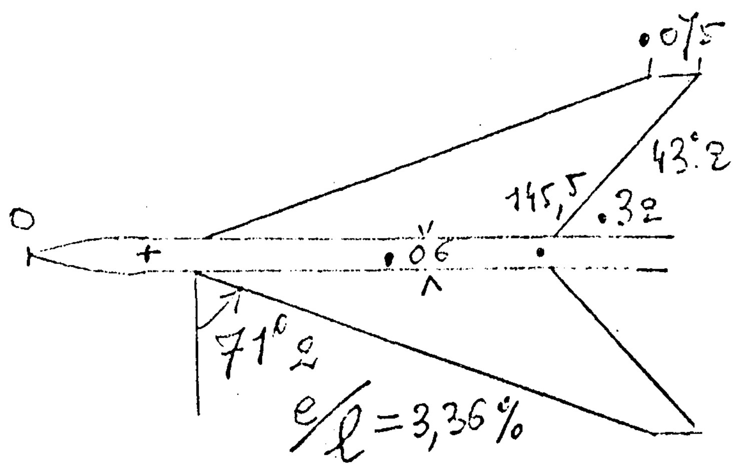


$\alpha = 12°$

Field Pressures





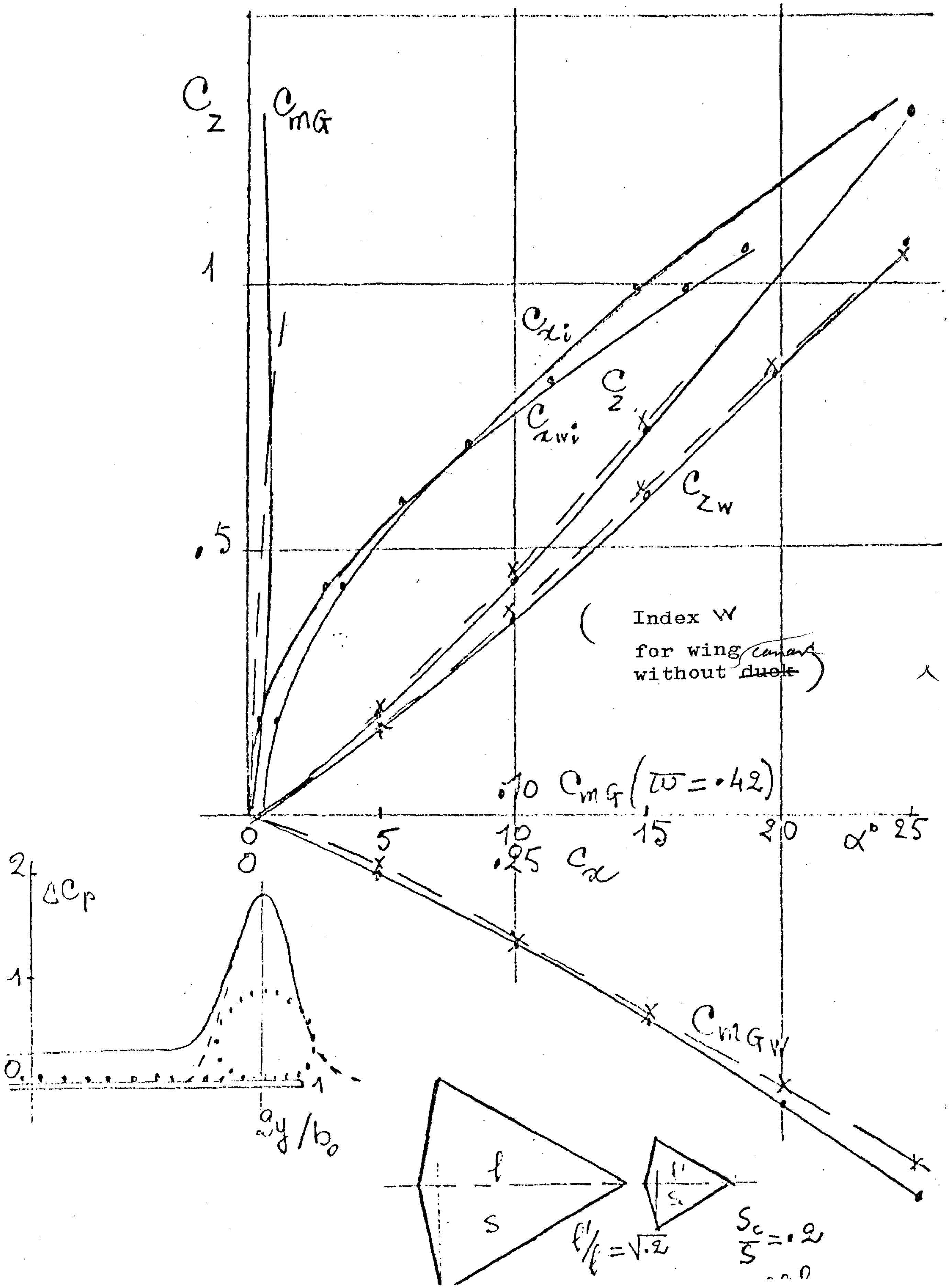


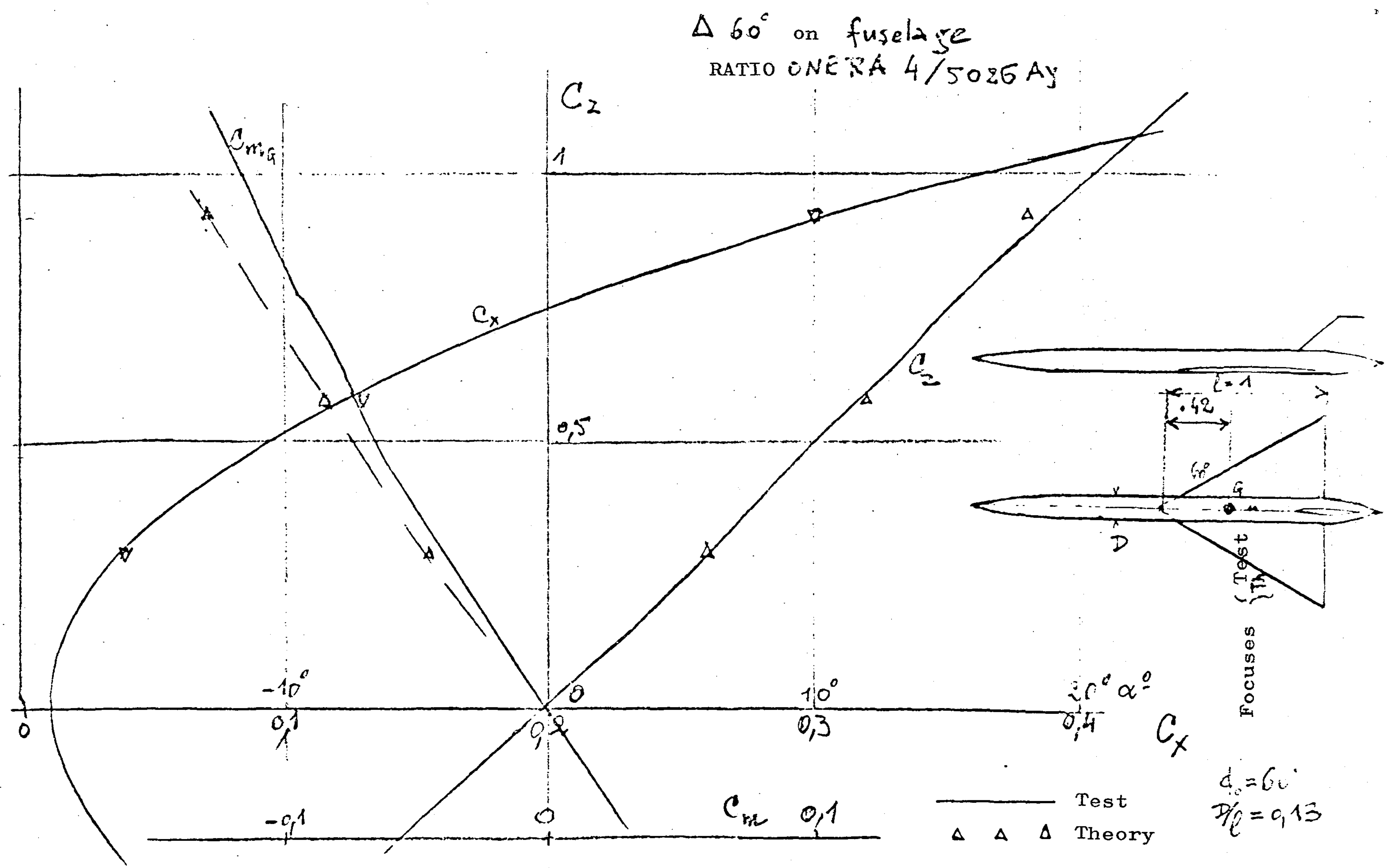
SAAB ~~DUCK~~ Canard (H. Behrbohm)

PL. III

—— Calcul.

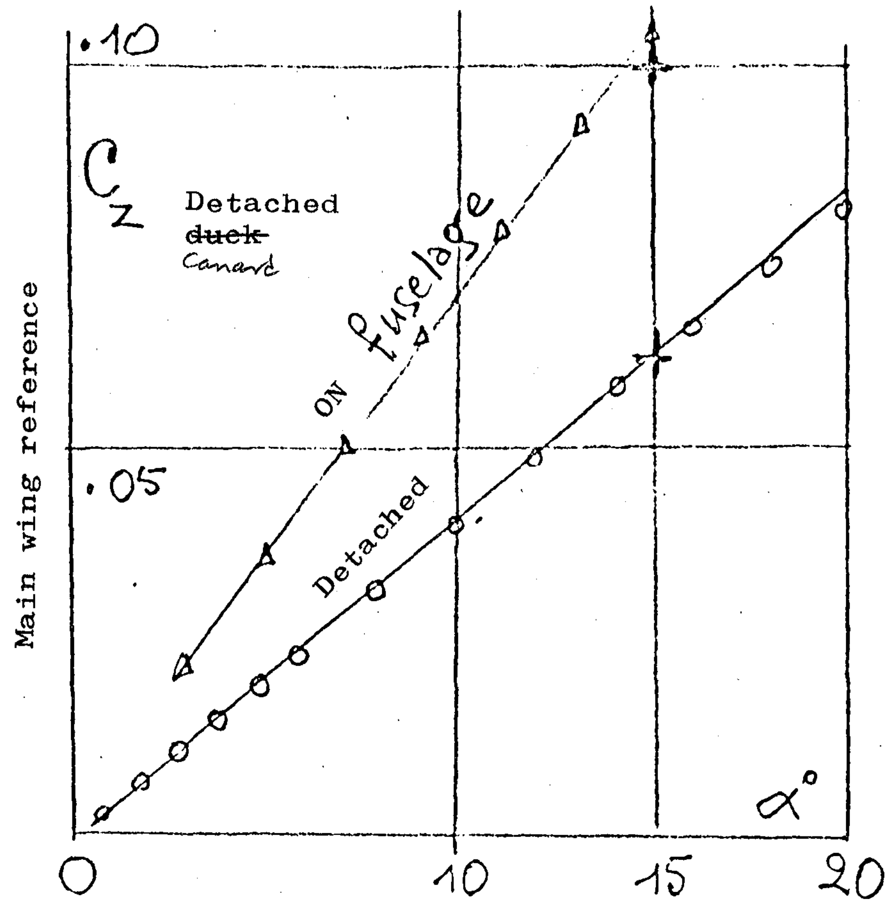
- - - Test

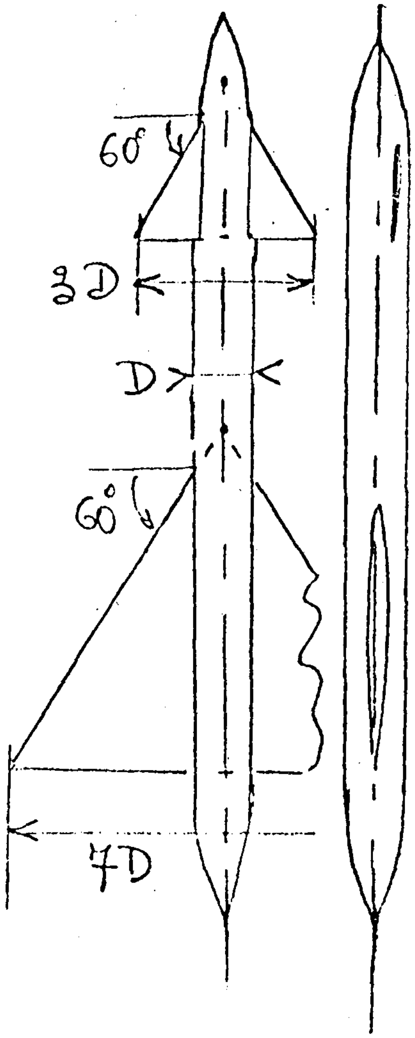




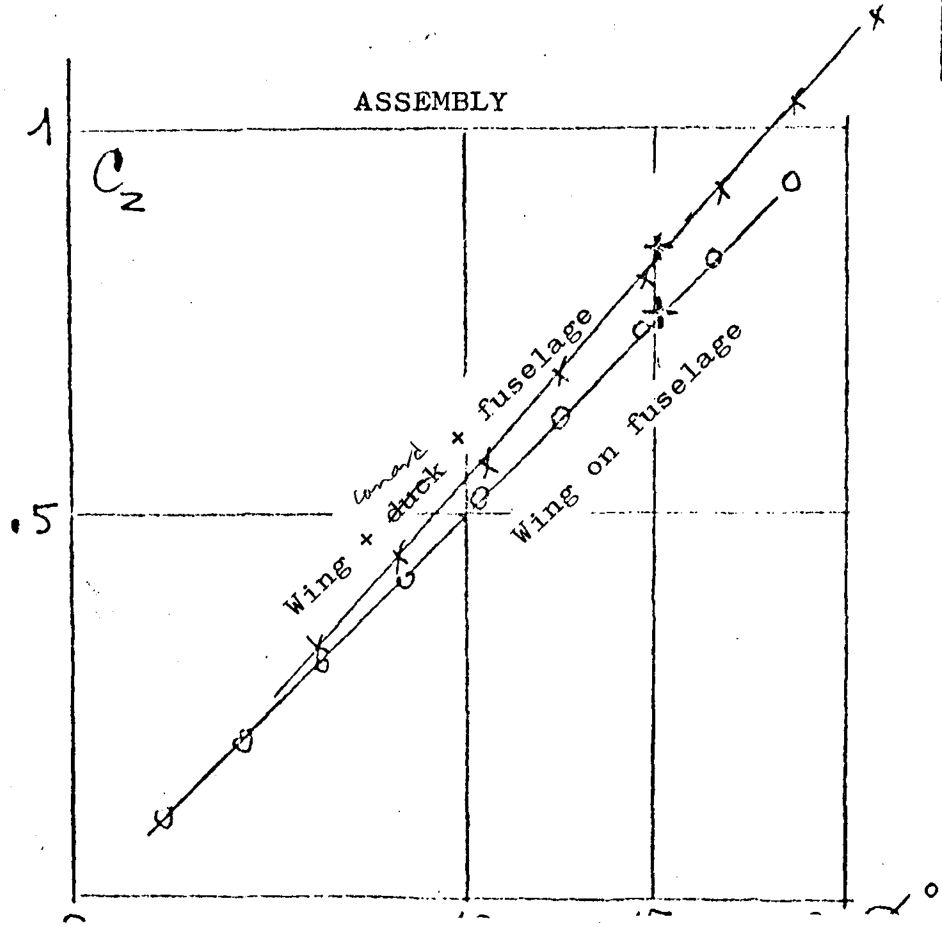
Δ 60° on fuselage
RATIO ONERA 4/5026 Ay
C_z
C_{m_G}
1
C_x
C_z
0,5
-10°
10°
20° α°
0
0,1
0,2
0,3
0,4
C_x
C_m
-0,1
0
0,1
Test
Δ Δ Δ Theory
l = 1
.42
60°
G
D
Focuses { Test Th
$d_0 = 60°$
$D/l = 0{,}13$

Pl. V





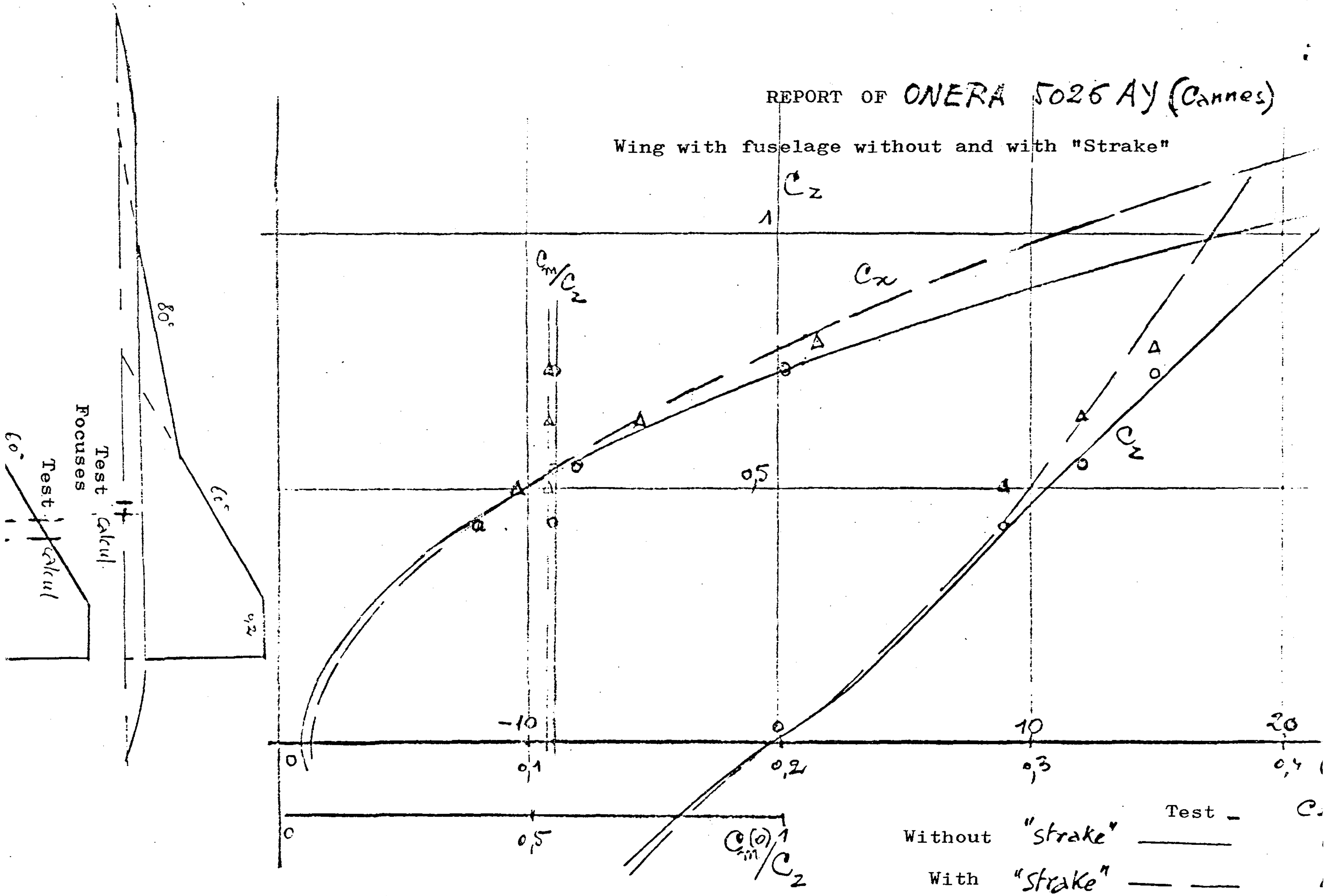
Airplane with ONERA ~~duck~~ canard

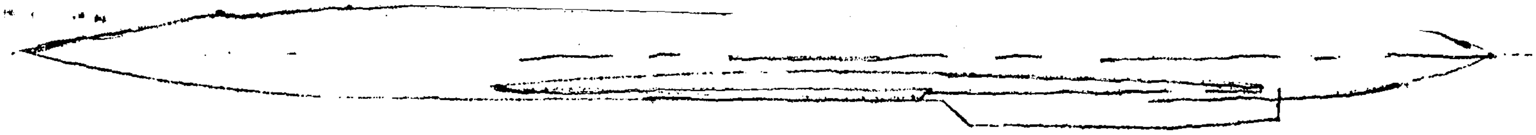


(Ehrlich)

+ calcul

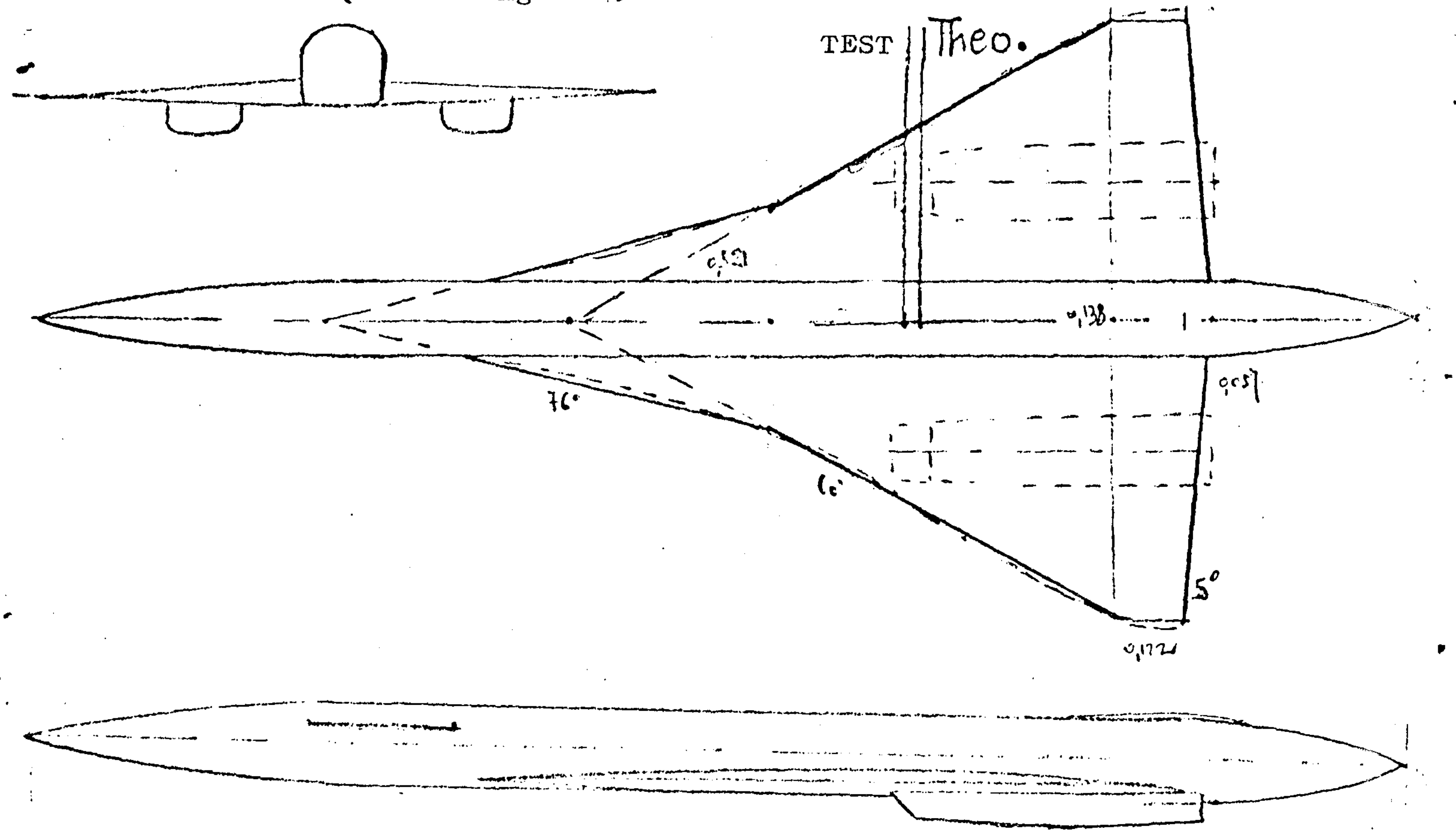
x, O, Δ TEST

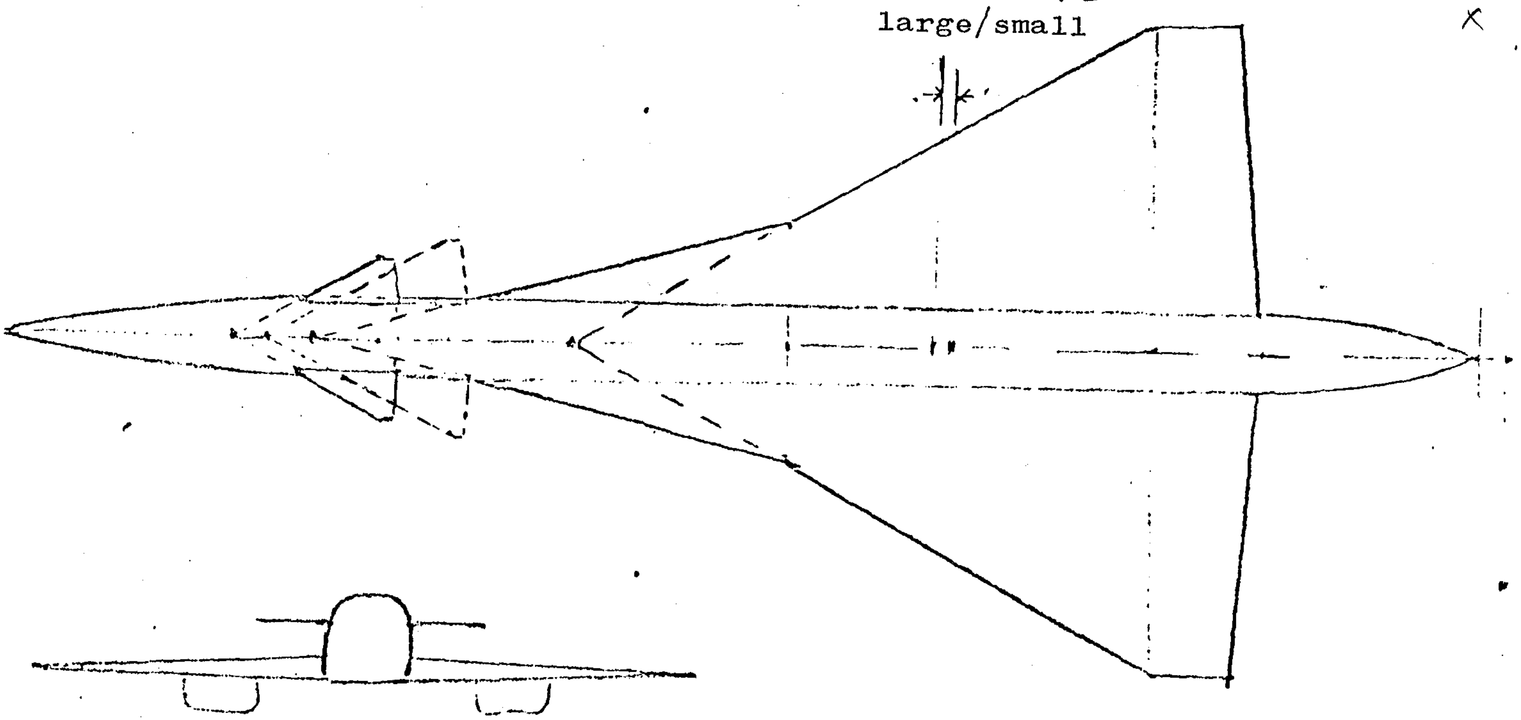

REPORT OF ONERA 5026 AY (Cannes)
Wing with fuselage without and with "Strake"
C_z
1
C_m/C_z
C_x
0,5
C_z
-10
10
20
0
0,1
0,2
0,3
0,4
0
0,5
$C_m^{(0)}/C_z$
1
Test
Without "strake"
With "strake"
80°
60°
0,2
Test
Focuses
Calcul
Test
Calcul
60°

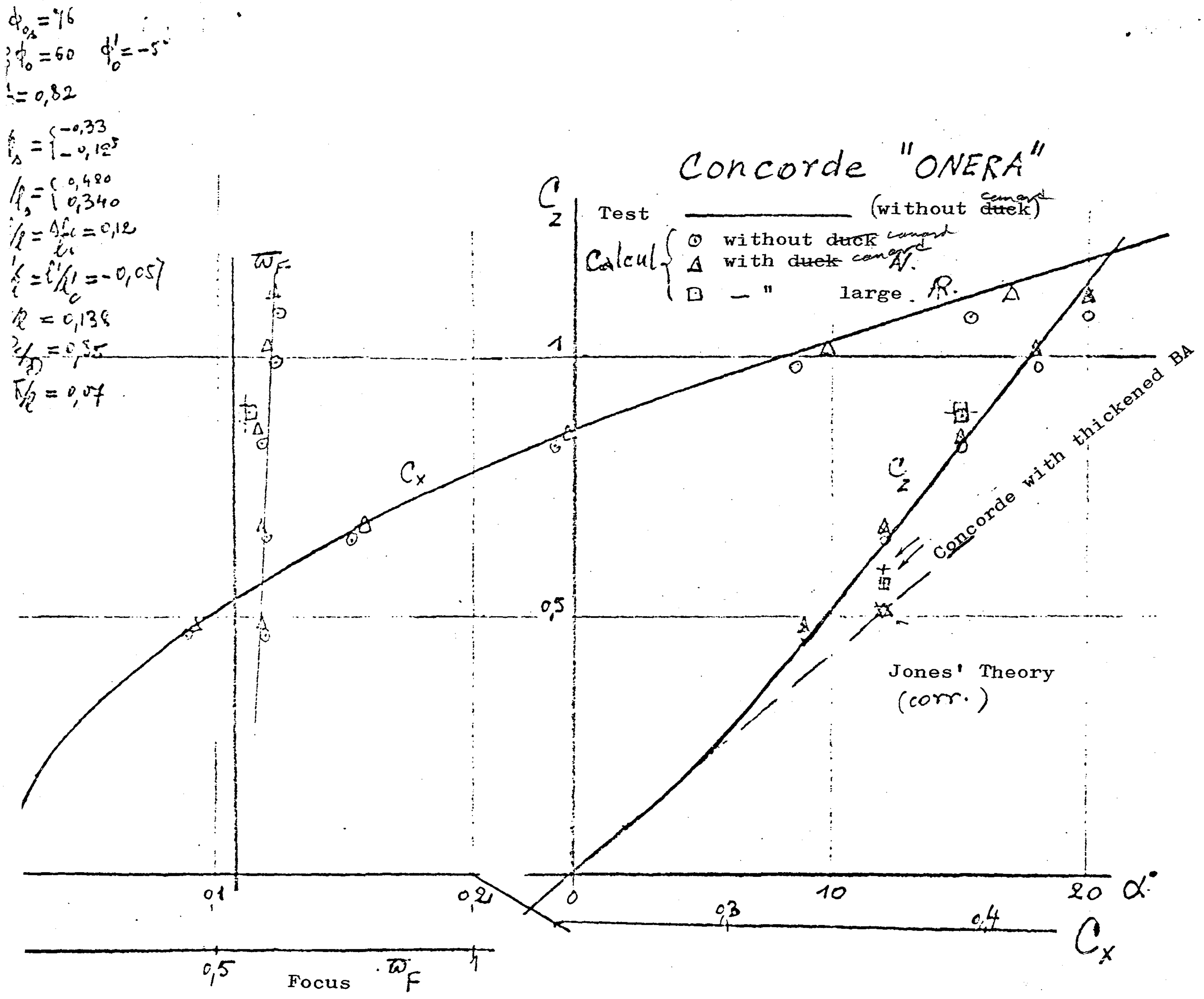


Concorde "ONERA"

(Flat Wing)






Concorde "ONERA"
Test (without canard)
Calcul
without canard
with canard AV.
— " large
Cz
Cx
1
0,5
0
10
20
α°
0,1
0,2
0,3
0,4
0,5
1
Focus ϖF
ϖF
Concorde with thickened BA
Jones' Theory
(corr.)
= 76
= 60
= -5°
= 0,82
-0,33
-0,125
0,480
0,340
= 0,12
= -0,057
= 0,138
= 0,55
= 0,07