STUDY OF A PRECISE POSITIONING SYSTEM OF THE COMSS ALTIMETRIC SATELLITE/RADAR

F. Nouel, M. Berge

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16. Abstract

This study attempts to reconstruct a satellite trajectory at an accuracy level of 5-10 centimeters. The reconstruction of the orbit at a level of 1 to 2 meters, and comparison of all results for the two altitudes, 650 and 850 km, are also considered.

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TABLE OF CONTENTS

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STUDY OF A PRECISE POSITIONING SYSTEM
OF THE COMSS ALTIMETRIC SATELLITE/RADAR
(HASP)

(High Accuracy System Positioning)

F. Nouel* M. Berge**

1. OBJECTIVES OF THE STUDY /6***

If a satellite is equipped with an altimeter whose measurements have an accuracy on the order of between 5 to 10 centimeters (a noise), it is apparent that the satellite orbit must be calculated with an identical accuracy level or better, and this is practically indispensable in the case of the SEAST. In order to solve calibration problems of the altimeter, the exactness with which the trajectory is reconstructed also is involved.

Therefore the proposed system from this study will respond to the first objective: reconstruction of the satellite trajectory at an accuracy level of 5-10 centimeters.

For this purpose, two other objectives have to be taken into account: possibility of reconstruction of the orbit at a level of 1 to 2 meters, but during a shorter time interval, i.e., between 1 to 2 days; comparison of the entire results for two different altitudes, 650 and 850 km.

2. HOW CAN WE PROCEED WITH THE STUDY?

The first difficulty which one encounters is to know how such a study should be carried out. In effect, how can one

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estimate 5 centimeters (Figure 1) in the position of a satellite which is at an altitude of 600 km or more, and which traverses a distance of more than 40,000 km during one orbit around the Earth?

On the other hand, the HASP system does not exist, and thus no constraint has to be imposed on the methods considered. The study will be based on tracking techniques which now exist, or which probably will exist very shortly, and this will be associated with processing techniques. This association has rarely been taken into account in the past.

Considering the experience acquired and the enormous effort expended for calculating the SEASAT trajectory, the directions of research are relatively well defined.

It is within this context that the study was carried out. The uniqueness of the system will depend only on the initial choice of the research method.

3. GENERAL REMARKS ABOUT THE TRAJECTORY CALCULATION

3.1. Vector representation

No matter what type of measurements are considered, the satellite station vector $(\vec{\rho})$ is involved (the two instruments at each extreme point are assumed to be points) (Figure 2):

. either in terms of direction (photograph against a star background, interferometry),
. or by its modulus (distance measurement)
. or variations in its modulus (Doppler measurements)

This vector will be calculated in an inertial reference /8

Figure 1. 5 centimeters!

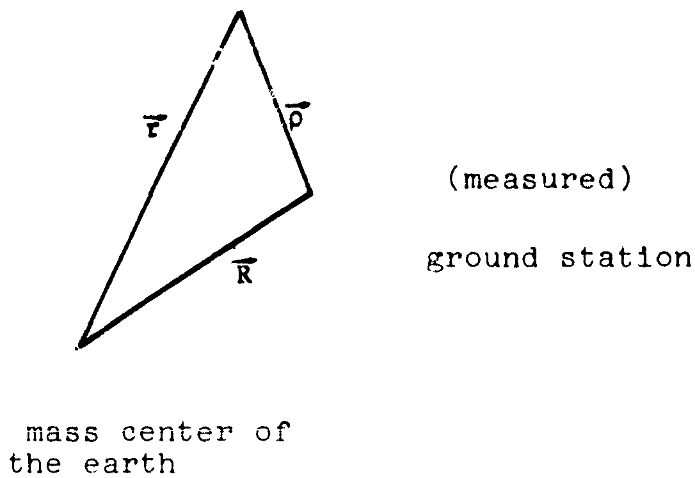


Figure 2

system in order to write the equations of dynamics.

Therefore, at the station extremity it is necessary to know the position of the station $(\vec{R})$ in a coordinate system, and this system has to be referred to an inertial system. It is obvious that the accuracy with which one can transfer from one system to another will evolve from this study.

As far as the satellite extremity is concerned $(\vec{r})$, its motion is due to a certain number of forces for which models exist, but which are more or less representative. As an example, we have the following:

- from the attraction due to the Earth's potential. The Earth's potential is usually extended in a series of spherical harmonics,
- gravitational attraction forces from the Moon and the Sun which are sufficiently well known for satellites close to the Earth,
- attraction forces caused by the potential of terrestrial, oceanic, and atmospheric tides, which introduces long term perturbations,
- the force of atmospheric drag which obviously is limited to a certain altitude range,
- forces caused by pressure and direct radiation of the Sun and the pressure re-emitted by the Earth, in particular, in the case of a helio-synchronous orbit, the direct pressure will have a relatively constant and continuous effect. The re-emitted pressure becomes variable for shadow-Sun passages, and for cloud covers. This is difficult to model. /9

3.2. Calculation techniques

At the present time, calculation techniques for following a satellite consist of calculating an orbit which represents the measurements the best. This orbit is then used to interpolate or predict the satellite positions. Considering the perturbations and the required accuracy, this evaluation may not be done to a great extent: this is because the trajectory arc lasts for several days.

In the following study, we shall use the notation below:

<u>short arc</u>-

This is one part of the satellite orbit which can extend up to one complete revolution around the Earth.

Long arc:

If we consider several successive revolutions and even orbits which extend one or several days, we will be dealing with a long arc. Considering the desired accuracy for space geodesy for a close satellite, in general the duration will not exceed 5 days.

4. EVALUATION OF THE FORCES ACTING ON A SATELLITE FOR A SHORT ARC

Approximately, a close satellite of the Earth traverses 7,000 km in 20 minutes (about 1,000 seconds). Thus an acceleration of 10^{-7} m/s^2 will result in a displacement on the order of 5 centimeters in 20 minutes, which is within the range of the present study. Consequently, the accelerations from which the satellite motion will be calculated must be known to 10^{-8} m/s^2, so that the error remains smaller than 1 centimeter over a period of 20 minutes. /10

Non-gravitational forces

In order to order the orders of magnitudes as a function of altitude for atmospheric friction, for the same satellite (example taken from D5B), we will have an acceleration (in m/s^2) of 10^{-5} at 300 km, 10^{-7} at 600 km, and $5 \cdot 10^{-10}$ at 1,000 km.

The following are accelerations due to pressure forces:
- $3 \cdot 10^{-8}$ for direct solar pressure
- $3 \cdot 10^{-9}$ for reflected solar pressure
- $4 \cdot 10^{-9}$ for the inrared.

The problem arises because one encounters variations by a factor of 100 of these accelerations during an orbit (passage into the shadow), and even more so over a day.

Forces caused by terrestrial potential

When a new geodesic satellite is launched, the first factor encountered when one determines the orbit is that the potential use is inadequate.

For example, we can mention GEOS III which led to determination of the models GEM 9 and 10. When SEASAT was launched, it was necessary to calculate the models PGS-S1 and PGS-S2, and we not only did not have an improvement in the arcs from one to three days, but also for arcs lasting several hours. Nevertheless, the geographic distribution of observation stations is an important factor in determining most of the coefficients of the potential model. For various reasons (isolation, political) certain regions of the globe will remain inaccessible. /11

5. PRESENT ACCURACY OF ORBIT CALCULATIONS: SEASAT EXAMPLE

From known results for several geodesic satellites, i.e., where it was desired to obtain the best accuracy in the calculations, we can make the following estimates:

The radial component is calculated with an error between 1 to 2 meters in general. If particular efforts are expended, we can reach an accuracy of 50 centimeters.

This accuracy is valid for arcs varying between 1 to 2 revolutions up to 1 to 2 days in the more precise case.

The SEASAT results in addition show the following predominant error sources due to our lack of knowledge:

- terrestrial potential 30%
- atmospheric friction 20%
- solar pressure 20%
- propagation phenomena 10%
- station positions 10%
- calculation procedures 10%

6. METHOD OF "SHORT ARCS"

In accordance with a goal of space geodesy, new methods of calculation have been proposed in order to calculate the positions relative to the stations on the ground.

The general philosophy of these methods consists of observing the same satellite trajectory arc from several stations, so that the errors introduced by the poor knowledge of the satellite position are common to all of the stations, and thus their effects are reduced. /12

For example, with the satellites TRANSIT, whose accuracy of the trajectory is on the order of 25 meters, if one uses ephermerides which are called operational and which are transmitted by the satellite, we can calculate the station positions with an accuracy on the order of 50 centimeters to 1 meter.

Another very characteristic example is the estimation of the quality of measurement apparatus: the apparatus noise can be estimated by using measurements collected over a pass above the observation station (case of laser with an accuracy between 2 and 5 centimeters) with data processing whose satellite motion model is simple.

This method of short arcs has one drawback: this increases the number of measurement apparatus, but the measurement techniques have improved accuracy, and as a result long wave radio waves have to be used.

7. EXISTENCE OF TRACKING MEANS

At the present time, laser techniques give the required accuracy (2 cm) in distance, but in spite of the efforts to reduce their size (mobile stations and very mobile stations), the construction, installation, and operation have increased payoffs.

On the other hand, we have seen a very important improvement in radio techniques, for example:

- the Transit system: Doppler receivers now have a small size
- automation of measurements: Eole or Argos. /13
- appearance of distance measurements with an accuracy and exactness of several centimeters (Dialogue model) or systems with real time exploitation (GPS system).
- VLBI stations which are mobile and are still in the experimental stage and are still quite large

The operational problems and cost problems have partially been solved, and at the present we can look upon tracking equipment in a different light.

A network which is more dense could be considered with quasi-automatic collection of data and centralization of calculations.

/15

1. SELECTION OF MEASUREMENT INSTRUMENT

Present systems

Laser

The measurement accuracy is on the order of 10 to 20 cm, and for certain stations it is only a few centimeters, for any measured distance.

The calibration problems, i.e., the measurement of the systematic effects which affect the measurements, are relatively easier to resolve than for the radio-location systems. Nevertheless, on a centimeter level the laser method encounters the same problem. There is the delay due to the electronics, the delay which varies with temperature, the sensitivity of the photomultiplier, etc...

From the satellite point of view, the laser requires several on-board reflectors, which have minimum cost. On the other hand, the ground stations are substantial, even for the new generation of mobile stations.

Finally, their operating cost is very high. Thus they have been considered in numerous programs.

NASA (including SAO), Interkosmos, and Europe are almost the only organizations which have them. Only NASA and Interkosmos have deployed a global network. This implies international cooperation which is difficult to implement.

There is still the problem of operations during cloud cover which over the year amounts to almost 30% success rate for the number of programmed passes: this number varies from one geographic location to another. /16

RADIO LOCATION

Transit System

This is based on measuring the Doppler effect alone, and is used everywhere throughout the world. It was developed in 1960, and therefore a long time ago from a technological point of view, in spite of important improvements, especially concerning the oscillator stability. It has been successful especially in space geodesy, and has been reliable. The costs of operation are relatively low, and operating it is relatively simple.

Obviously, the satellite segment is more complex than in the case of the laser.

This system operates practically for all times and the success rate is between 70 to 80% when one uses the automatic mode, i.e., when all passes are observed, even those having a maximum elevation less than 10°, which is included in 20 to 30% of rejects.

GPS System (Global Positioning System)

It is being installed at the present time with 6 satellites. The GPS will give access to distance and Doppler measurements.

For reliability reasons, the USAF has turned its attention towards radio-locating techniques. For the Transit system, the most important features which were adapted for defining the GPS can be summarized as follows:

- use of a distance-Doppler pair which gives a better geometric definition of the measured point
- visibility of several satellites from one point on the Earth so as to incrase the geometric factor in localizing the point /17
- utilization of higher frequencies in order to reduce the influence of the ionosphere
- stability of oscillators in order to increase the measurement accuracy

EOLE, ARGOS Systems

Here, the desired localization accuracy is not comparable with that of the laser or the GPS. This was not the principal objective. In the two cases, it was desired to automate the localization of a large number of objects (balloons, buoys, etc.) which are distributed over the globe. It was desired to rapidly give the users the results of the position determinations by using the satellite itself as a data transmission vehicle. Even though the concept is relatively old, it was not widely accepted by users which wanted to be autonomous, often a fictitious factor. Information processing and telemetry has changedthis point of view more and more. Centralized data bases have appeared, which are acceptable to many users.

Interferometer systems

These are based on the phase comparison of signals coming from the same source and received at two different points. This technique allows one to determine the orientation of the satellite (DIANE system). The distance between the stations on the ground has been determined better, because of more precise oscillators and a better synchronization of the local clocks. Phase /18 comparison is carried out at different times on computers. Let us recall that, by increasing the distance of the base, we can obtain a better angular resolution. Radio astronomers have used this method.

Such a system is now being developed, and tested, but the station is still costly and substantial, at least in its initial design.

Radio interferometry as a means of localizing close satellites will develop in the near future, but within a different context from the present time. Comparing the phase of a signal to another phase, of course, leads to a measurement of distance, Doppler measurements, and interferometry.

CONCEPT OF THE HASP MEASUREMENT INSTRUMENT

Definition

For reliability reasons, accessibility, cost, we will not discuss lasers. We wish to develop a new instrument without being compromised by previous commitments (such as NASA with the laser). We will have to draw on previous experience of radio tracking, and use the lastest techniques available in electronics and calculation methods.

It is always possible to place several laser reflectors on the satellite, in order to enlarge international participation and in order to verify the accuracy of the proposed sytem using another technique. /19

Orbit coverage

It is imperative to have the best possible orbit coverage: example, use of "S-band" measurements for calculating the SEASAT trajectory.

This leads to a radio tracking system.

Distance-Doppler pair

The measurements of distance and Doppler measurements complement each other perfectly: the position of the satellite is defined by six orbital parameters, or in an equivalent manner by its position and velocity.

The measurement of distance is performed along the station-satellite direction, and the Doppler measurement (or radial velocity) is done in the direction along the line perpendicular to the distance.

Example: identifical selection in GPS

Measurement of double trajectory distance

In GPS, distance is defined along a simple trajectory: this is only for military reasons, because the ground station has to remain quiet. Consequently, the satellite and the station have to both have frequency references and very stable time bases, which are set into phase before they are used.

In a simple trajectory D (measured) = d (geometric) $+ c\,\Delta\tau$ where c is the speed of light and $\Delta\tau$ is the shift between the two clocks. This correction term disappears for a double trajectory.

Double trajectory Doppler measurement /20

This problem is similar to the previous one. TRANSIT or GPS carry out simple trajectory Doppler measurements.

One could believe that, in contrast to the distance which gives a measurement scale related to frequency, the effect of a Doppler frequency deviation does not have any effect. This is true if one considers a single measurement, but as soon as one wants to mix measurements from several receivers it is necessary to know that the frequency deviations are known.

Simple Doppler $$f_D = f_r - f_c \frac{v}{c}$$

Double Doppler $$f_D = f_c \frac{2v}{c}$$

where f_D is the measured Doppler frequency
f_r is the receiver frequency
f_c is the satellite frequency
v is the speed of the satellite relative to the station
c is the speed of light

Shipping of the measurement apparatus

The double trajectory principle in distance and Doppler can be conceived along the station-satellite-station line: the satellite segment will be simplified.

In this case on the ground we multiply the number of reference oscillators, and it is necessary to provide synchronization (on the order of one microsecond) between all clocks. This is the last problem, the problem of transmitting measurements to the calculation center. This problem is costly, and will result in substantial delays in the processing center. /21

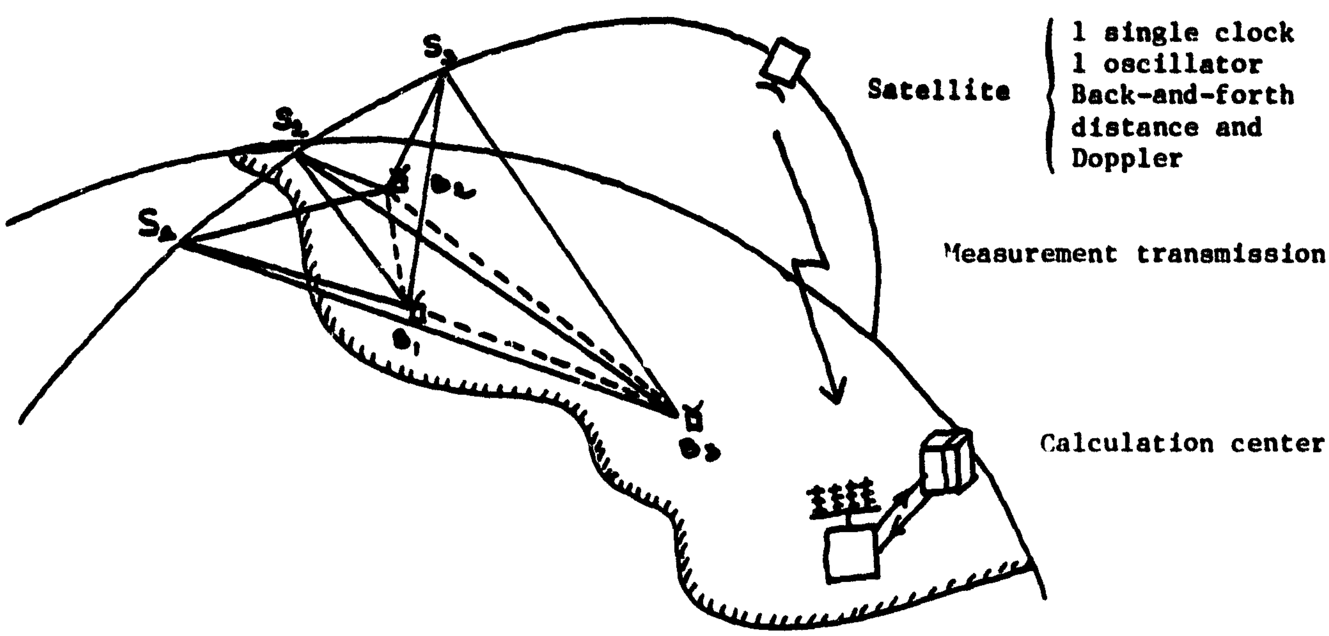
Shipping of the measurement apparatus implies miniaturization in terms of weight and volume, and reduced power consumption. On the other hand, for all of the distance and Doppler measurements we have a single frequency reference, and a single time reference. This eliminates an important source of errors which are present in present systems. (Even for lasers, where incorrect dating leads to rejection of measurements which often have an accuracy of several centimeters!, and this comes about because of poor synchronization).

The last, and not least important, advantage is the rapid centralization of measurements.

The groud equipment is simplified, and the number of stations can be increased.

Summarizing: the following diagram shows the system which could be used in HASP:

- Doppler and distance double trajectory measurements
- shipping of measurement apparatus



S_1, S_2, S_3 : geometric satellite positions

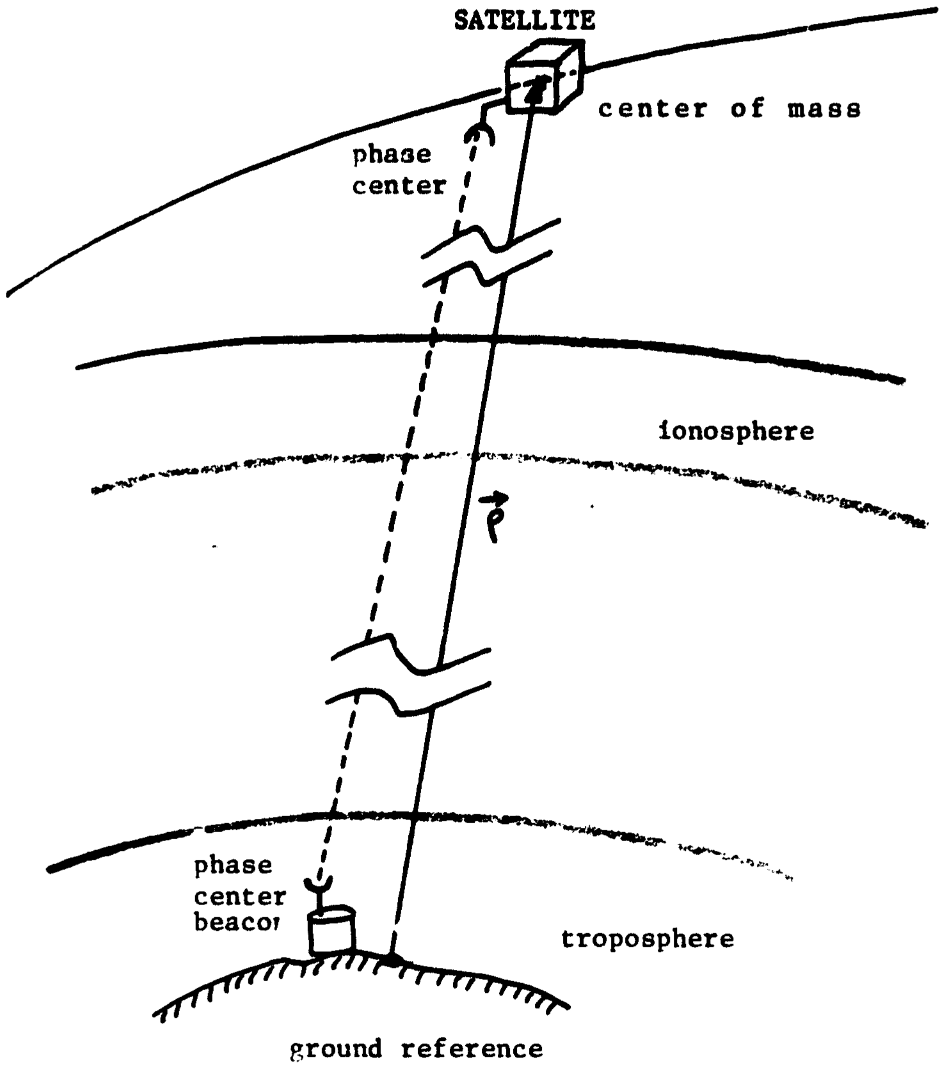
B_1, B_2, B_3 : beacons

/22

CORRECTIONS TO BE MADE TO MEASUREMENTS

In effect, the station-satellite radius vector $\vec{\rho}$ goes from a station point which is geometrically well defined to the satellite center of mass, to which the dynamic equations will be applied.

For each measurement it is necessary to correct either geometric effects, or propagation effects with an accuracy greater than the 5-10 centimeters which one desires to reconstruct.



The list of corrections which we mentioned is not exhaustive and we cannot give a solution to all of these problems here. /23 Each one has to be dealt with by a certain specialist. This list is only intended to provide the corrections which can be predicted and which have to be studied. Certain responses have already been made during development.

Geometric corrections

- Studies on the behaviour of the phase centers during a passage above a station as a function of geometric conditions.

- Attitude of the satellite which allows one to relate the phase center to the center of mass for the satellite which serves as a reference

- Identical problems for the beacon, which has to be tied to a specified material point on the line: the possible change of a beacon may not change the tracking network references

Calibration corrections

These are indispensable and fundamental. Thus they have to be studied with the same accuracy and have to be available for processing: for example, it could happen that a correction to be applied will evolve over time.

Corrections for propagations

Ionosphere

The methods of reducing ionosphere errors are of two kinds, if we eliminate the model which appears to still be too uncertain:

- increase the frequency of operation, because the error is proportional to $1/f^2$,

- operate with two frequencies which have a known ratio /24
so they can be combined to eliminate the ionosphere effect. It is required that the particular pair selected does not result in an amplification of the instrument errors, if one wishes to have a reduction in the ionosphere error.

We can assume that it is possible to reduce this error to a level of one centimeter for distance and 10^{-2} mm/s for speed.

Troposphere

The problem here is more delicate, and will require a detailed in-depth study in order to evaluate the present possibilities of correction, because this could have important consequences on the complexity and the cost of the measurement apparatus.

The troposphere error is independent of frequency up to 15 GHz, and is divided into two components:

- the dry component which is modeled relatively well as a function of temperature measurements and pressure measurements at the station (d'Hopfield model)

- the humid component which is poorly modeled. For example, it would be necessary to measure the water vapor content along the wave trajectory, for example, with a radiometer (proposed by MacDoran).

For this troposphere correction, one has to admit that one cannot carry out distance and velocity measurements below an elevation of about 10 to 15°, or even 20°, if the apparatus complexity is increased excessively, considering the benefit of these low elevation measurements. /25

2. SELECTION OF THE CALCULATION TECHNIQUE

Since the errors caused by the Earth potential model are assumed to be predominant, we have made an in-depth study of these errors.

EVALUATION OF PERTURBATIONS CAUSED BY THE EARTH'S POTENTIAL

One classical method consists of evaluating the influence on osculating elements of terms of the Earth's potential (Kaula formalism summarized in the appendix) using a given satellite.

The existing program has been modified:

- in order to take into account the most complete model at the present time, GEM 10B (36 degrees, order 36 in the expansion of spherical harmonics),

- in order to calculate the perturbations up to a limit of 10 cm.

It would not serve any purpose to reproduce all these perturbations, but one can summarize the results.

For a satellite of eccentricity e = 0.001 and inclination of 98° 5, we obtain - depending on altitude - the following number of perturbations:

at 650 km, 4,100 terms
at 800 km, 3,100 terms
at1,000 km, 2,400 terms

These terms are to be taken into account in the model in order to drop down to 10 cm.

These are results which could be expected, and which confirm the problems of a precise orbit calculation.

Another important point is the period with which these perturbations are produced.

Approximately, a 5 minute duration corresponds to a trajectory length of 2,100 km, and thus one will have sinusoidal orbital perturbations with wavelength of 2,000 km, distances comparable with that of the oceans. /26

One finds that the number for which the period is less than

15 minutes (6,000 km for the satellite track over the ocean) decreases by about 30% when the altitude changes from 650 km to 1,000 km.

The greater the altitude, the longer the periods become, and in the case of 650 km all of the periods between 5 and 15 minutes are sampled.

As an example, Table 1 gives the perturbations in meters (for a 17^{o} term and order 1) for each of the orbital elements with the corresponding period expressed in days or minutes. For this term, the summation over p goes from 0 to 17 and the summation for q goes from $-\infty$ to $+\infty$. Only the significant p and q values are maintained. Thus one can compare the number of terms to be taken into account for the three different altitudes (650 km, 800 and 1,000).

FUNDAMENTAL CONSEQUENCES

It seems to us to be almost impossible at the present to have available terrestrial potential models which could translate all these perturbations. Thus it is necessary to reconstruct the satellite position in a geometric manner, i.e., to have a sufficient variation of measurements in order to localize the satellite for a geometric figure.

Then it is necessary to accumulate these individual locations over time so as to describe the satellite trajectory.

Using this method, one is tempted to increase substantially the number of observation stations of the satellite. There is never the possibility of locating the stations on the oceans. For these parts of the trajectories, it is impossible to make a geometric determination. Thus it is necessary to use satellite motion models for describing it.

TABLE 1

ALTITUDE	L (order)	M (degree)	P	Q	PERIODE (J)	(MIN)	A	E	I	G	P+M
650 KM	17	1	0	0	.004	5	.1				.1
	17	1	0	-1	.004	6		.1			
	17	1	0	1	.004	5		.2			
	17	1	1	1	.004	6		.1			
	17	1	6	0	.014	19				.1	
	17	1	7	0	.023	33				.1	
	17	1	8	0	.073	105				.3	
	17	1	8	-1	.986	1419		.2			
	17	1	9	0	.064	91				.3	
	17	1	9	1	1.002	1443		.6			
	17	1	10	0	.022	31				.1	
	17	1	11	0	.013	19				.1	.1
	17	1	12	0	.010	13					.1
	17	1	12	-1	.008	12		.1			
	17	1	13	0	.007	10					.1
	17	1	13	-1	.007	9		.1			
	17	1	14	0	.006	8					.1
	17	1	14	-1	.006	8		.1			
	17	1	15	0	.005	7					.1
	17	1	15	-1	.005	6		.1			
	17	1	16	-1	.004	6		.1			
	17	1	17	0	.004	5	.1				.1
	17	1	17	-1	.004	5		.1			
800 KM	17	1	0	0	.004	5	.1				.1
	17	1	0	1	.004	5		.1			
	17	1	7	0	.024	34				.1	
	17	1	8	0	.075	108				.2	
	17	1	8	-1	.987	1420		.1			
	17	1	9	0	.056	94				.2	
	17	1	9	1	1.002	1442		.4			
	17	1	10	0	.023	32				.1	
	17	1	14	0	.006	9					.1
	17	1	14	-1	.006	8		.1			
	17	1	15	-1	.005	7		.1			
	17	1	17	0	.004	5	.1				.1
	17	1	17	-1	.004	5		.1			
1 000 KM	17	1	0	0	.004	6	.1				.1
	17	1	0	1	.004	5		.1			
	17	1	8	0	.079	113				.2	
	17	1	8	-1	.988	1422		.1			
	17	1	9	0	.068	98				.1	
	17	1	9	1	1.001	1442		.3			
	17	1	17	-1	.004	5		.1			

Part of the study thus consisted of evaluating the arc length which can be modeled without losing a fixed accuracy. A certain number of factors play a role, and it is necessary to evaluate them: for example, for networks which observe the relative positioning errors, the accuracy, and the measurement type, etc. /28

GEOMETRIC SOLUTION

Constraints on simultaneous observation

- A distance measurement ρ gives a relation in the form

$$\rho = \sqrt{(X - x)^2 + (Y - y)^2 + (Z - z)^2}$$

where X, Y, Z are the position coordinates of the satellite and x, y, z are the observation station coordinates.

In order to have the satellite position, three simultaneous measurements of distance are necessary for three different stations with common visibility of the satellite.

- In order to obtain the speed of the satellite, we require three distinct Doppler effect measurements. In reality, the Doppler effect will be translated by a relation of the form

$$\rho_1 - \rho_2$$

where ρ_1 and ρ_2 are, respectively, the distance at the beginning and end of the integration of the measured Doppler effect.

Observation relationships

Overall, for each instant we will have the following:
6 observation relationships
3 distance measurements for the 3 position unknowns
3 Doppler measurements for the 3 velocity unknowns

Obtaining simultaneous measurements /29

Application of the geometric method requires groups of measurements made with 3 beacons or stations, but which correspond to a single satellite position.

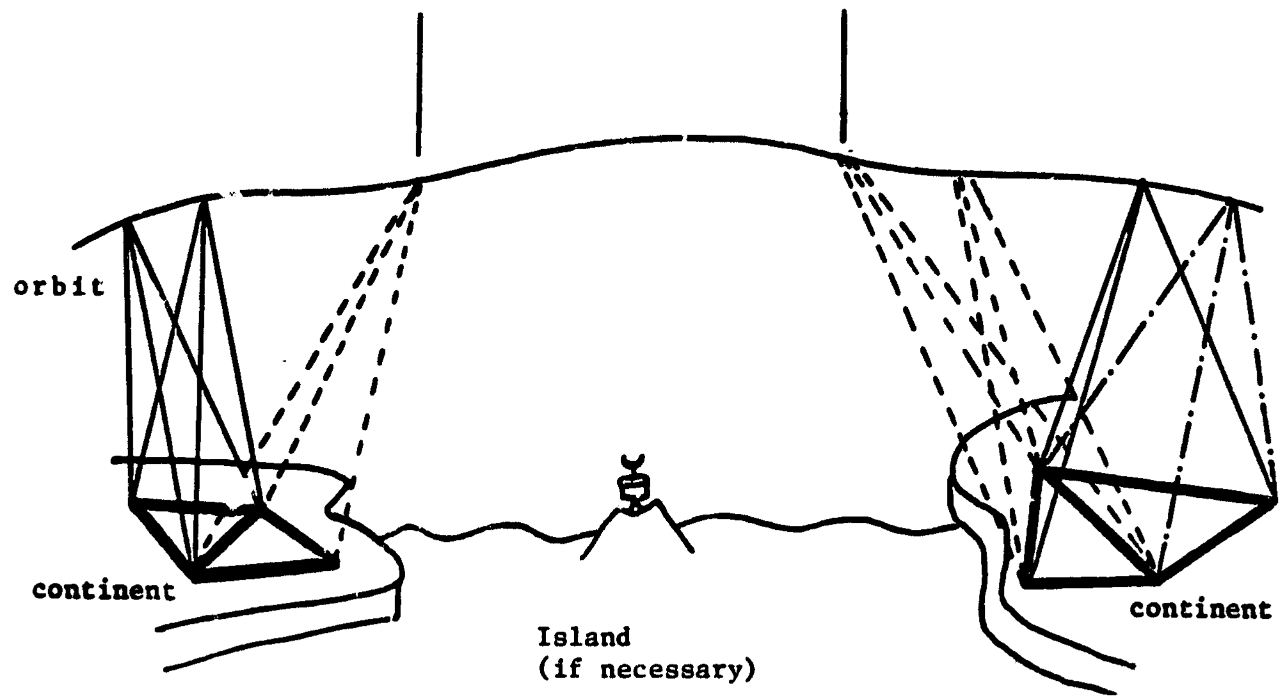
In reality, it is quite probable that the interrogation of the beacons will be made in sequence. Thus it is necessary to interpolate among the measurements collected over the entire pass above the three beacons.

Experience has shown that this "smoothing" cannot be made with a polynomial with the desired accuracy. It is preferable to select a single motion model for the satellite over the duration of the pass.

The algorithm still has to be defined in order to assure that the methodological errors remain smaller than 5 cm, and this kind of treatment has to have convergence of the method in most cases. /30

INTERPOLATION USING SHORT ARCS

Principle Trajectory observed and geometrically reconstructed	Part of trajectory connected with short arcs	Trajectory observed and geometrically reconstructed



/30

We shall assume that for each continent and for the ocean coast we have more or less dense observation station networks (to be evaluated). These networks will allow the reconstruction of the part of the trajectory which is visible to all stations (geometric solution). If we assume that for a given arc we can reconstruct the two extremities of the arc under consideration, the critical point is now to evaluate whether one can connect them by calculating a trajectory obtained from satellite motion models, while maintaining the desired accuracy. Is it necessary to have an instrument on an island? /31

The correction between the two extremities will be made using the short arc method.

VALIDATION OF THE HASP METHOD

1. PRINCIPLE

We shall attempt to determine the perturbations introduced in the orbit when one uses incorrect dynamic models for calculating it. This will be done using an ideal measurement distribution, and then we shall evaluate the influence of a degradation in the orbit coverage by the measurements. /33

For this purpose, we shall start by generating a satellite orbit as a function of a certain model (often called SIGEDI in the illustrations). From this orbit, we can simulate distance and Doppler measurements for a given collection of stations.

After this, these simulated measurements will be used as observations for a trajectory calculation which will be made either using a different dynamic model (referenced often by the word TRAJEDI in the illustrations), or by introducing error models in the station coordinates and/or the measurements themselves.

We can divide into two classes the errors which will influence the accuracy with which one can determine the satellite position:

- errors in measurements
- errors in the force models

The errorsin the measurements influence the individual position of the satellite, whereas the errors in the models will introduce errors when one attempts to relate individual positions.

The various steps of the simulation chain and the processing chain are described in Tables 3 and 4 respectively.

TABLE 3

SCHEMATIC OF SIMULATION CHAIN

/34

Orbit prediction

- determination of the new simulation period
- tabulation of the orbit using numerical integration

creation of a file

- determination of the collection of visibility periods for each station
- programming of the interrogation instants

Generation of measurements, distance and Doppler

- perfect measurements

Introduction of noise to the perfect measurements (possibly)

- real measurements

Regrouping of the real measurements by station and pass

Creation of a data bank

- CHRONOS file (chronological)
- list of passes
- data base (measurements)

Calculation of the station-satellite vector $\vec{\rho}_0$ which will be used as a reference

TABLE 4

SCHEMATIC OF DATA PROCESSING CHAIN /35

- Creation of a chronological measurement file and a station sub-file
- Initiation of constants
- Reading of data
- Organization of effective parameters (which will be estimated)
- Calculation of the partial derivative matrix measurements with respect to the effective orbital parameters: A
- Calculation of the contribution of each measurement to the second term matrix: B (observed quantity - theoretical quantity).
- Calculation of the matrix $C = A^T \pi A$ (π = weight) and storing of this matrix
- Inversion of the matrix C and storage of C^{-1}
- Possible change of the potential model with respect to the simulation
- Selection of a line of A for each measurement
- Calculation of the contribution of these measurements to the second term matrix $D = A^T \pi B$.
- Solving of the system $CX = D \Longrightarrow X = C^{-1}D$,
- Test of the convergence of the solution in the sense of the least squares
- Adjustment of the effective parameters
- Calculation of the vector ρ as a function of the new reconstructed orbit
- Comparison of ρ and ρ_0 and plot of $\rho - \rho_0$

2. NUMERICAL VALUES OF THE PARAMETERS /36

The description of the curves which show the result of this study first requires several definitions about the initial elements selected.

For all of the simulations which are used as a reference in all studies:

- the Earth potential model is GEM 10B
- the measurements are created for 15 stations

Orbital parameters

* A 650 km :
  - $a = 7\ 028\ 140$ m
  - $e = 0{,}001$
  - $i = 98{,}5$ °
  - $\omega = \Omega = M = 0$

* A 850 km :
  - $a = 7\ 228\ 140$ m
  - $e = 0{,}001$
  - $i = 98{,}7$ °
  - $\omega = \Omega = M = 0$

STATION NETWORK /37

			latitude	longitude
			in degrees	
North	101	**Neskaupstadhur**	65,5	- 14
	102	**Jan Mayen**	71,3	- 8,5
	103	**Scresbysund**	71	- 22
	104	**Longyearbyen**	78,3	16
	105	**Hammerfest**	70,5	24
	106	**Glasgow**	56	- 4
South	107	**Ushuaia**	- 55	- 68
	108	**Rawson**	- 43,5	- 65
	109	**South Georgia**	- 54,5	- 37
	110	**Joinville**	- 63,5	- 55,5
	111	**Montevideo**	- 35	- 56
	112	**Stanley**	- 57,5	- 52
Center	113	**Flores**	- 39,5	- 31,3
	114	**Canaries**	28	- 18
	115	**Natal**	- 5,92	- 35,16

/38

Noise and Bias in the Stations and Measurements

In order to approach the real observation conditions, it is possible to introduce random and systematic errors during processing. The numerical values used are given below.

At 650 and 850 km

*In the stations:

. Noise of 20 cm (in this case, we do not desire to improve the positioning of the stations)

. Bias of $-1 \cdot 10^{-5}$ degrees in the stations of the northern hemisphere and Natal (in this case, we will attempt to determine new coordinates for the group of stations which have a bias)

At 650 km

*in the measurements

. noise of 0.02 m in distance

. noise of 0.02 mm/s in velocity

At 850 km

*In the measurements

. noise of 0.02 m in distance
0.02 mm/s in velocity

. bias of 0.05 m in distance
-0.1 mm/s in velocity

*In the stations

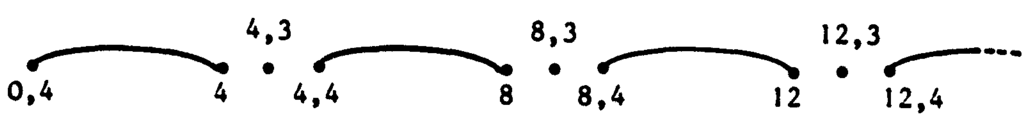
. noise of 0.02m

. bias of $-1-10^{-5} * \cos(\phi)$, where ϕ is the latitude for the southern hemisphere stations and Natal

General data /39

- Simulation step = 60 seconds
- Interrogation period from one station to another: (interrogation step) = 10 seconds
- Interrogation period for the same station = 60 seconds
- Spacing of measurement times:
  - distance date = 4.3 seconds
  - Doppler beginning = 0.4 seconds
  - Doppler end = 4 seconds

Stated otherwise:



- Emission frequency (reference) = 2 000 000 000 Hz
- Diffference frequency = 3 750 000 Hz
- Friction model used is the model derived at 1,000 km, 1965 with the Barlier constants (Jacckia Table).

Thus the surface to mass ratio is $0{,}25 \cdot 10^{-2}$ m^2/kg.

/40

Force models

Model GEM 10B

- Constant used

  GM = 0,398600640 D + 15 (m^3/s^2)

  Earth radius = 0.637814000 D + 07 (m)

  Flattening = 0.298255000 D + 03

  Speed of light = 0.299792500 D + 09 (m/s)
- Maximum degrees for the terrestrial terms and the zonal terms: 36

Model GEM 8

- Constants used

  GM = 0,398600800 E + 15

  Earth radius = 0.637814500 E + 07

  Flattening = 0.299792500 E + 09

  Speed of light = 0.299792500 E + 09

- Maximum degrees of the harmonics = 30

Compromise between GEM 10B and GEM 8

In order to make the models GEM 10B and 8 compatible, we also modified GEM 8 by introducing the GEM 10B constants given above: in the following this is given as model "GEM 8" with the constants of GEM 10B.

3. NUMERICAL RESULTS

Description of curves /41

The following curves are representative for a selected orbit arc:

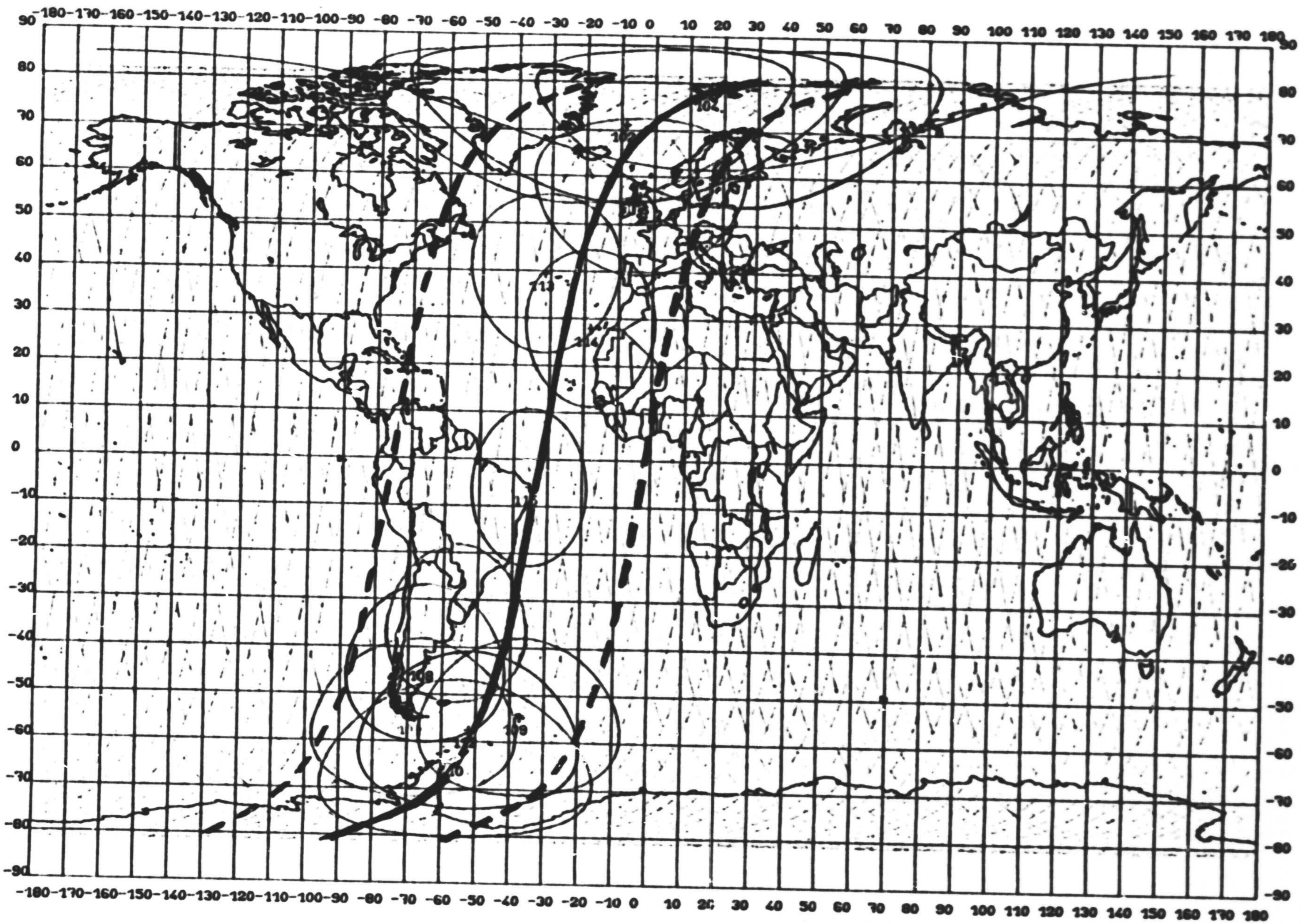
$$R = \rho - \rho_0 \quad \text{as a function of time}$$

where ρ is the center Earth-satellite vector determined from measurements and stations during processing, and ρ_0 is the vector used as a reference which was calculated during simulation.

$\rho - \rho$ is expressed in centimeters

The time is expressed in days, in the Julian calendar.

RESULTS FOR AN ALTITUDE OF
650 KM



Satellite altitude: 650 km

Visibility circles: 10°

Solid lines: main arc studied

Dashed lines: secondary arcs also studied

1 - SIMULATION OF PERFECT MEASUREMENTS
- TREATMENT WITH THE MODEL GEM 10B

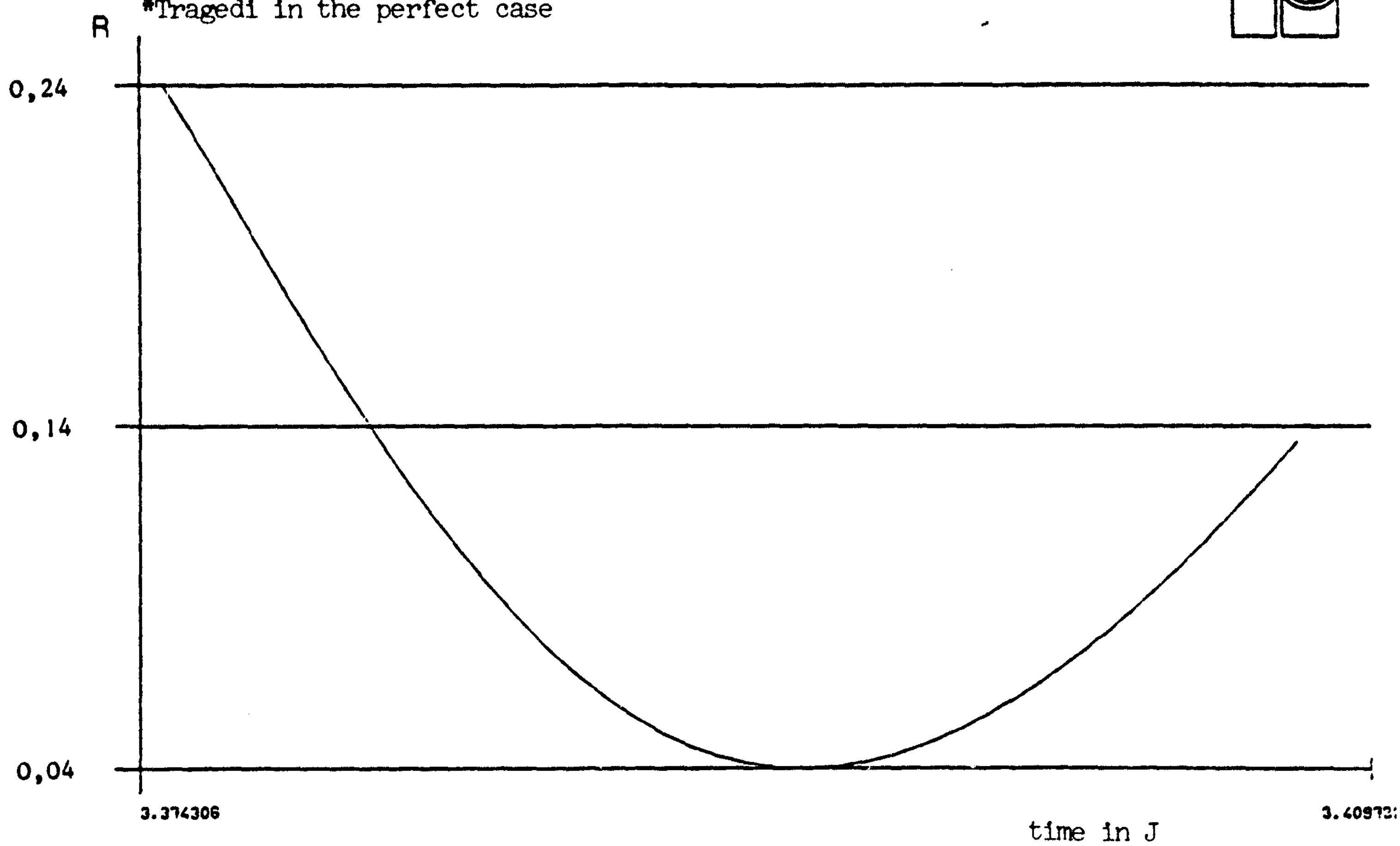
The first concern was to provide compatibility of /44
simulation programs and processing programs.

$R = o - \rho_0$ is less than 3 millimeters

/45

Distance difference (earth center/satellite) between the orbit tabulated during the s'mulation and the orbit tabulated in an imperfect case, i.e., satellite at 650 km
* initial orbital parameters at 7763.374305556

*Tragedi in the perfect case



2.3.4. SIMULATION OF PERFECT MEASUREMENTS TREATMENT WITH THE MODEL GEM 10B

- 2 . Coordinates of station with noise

-3 . Coordinates of station with noise

. Bias in the station coordinates in the South and Natal

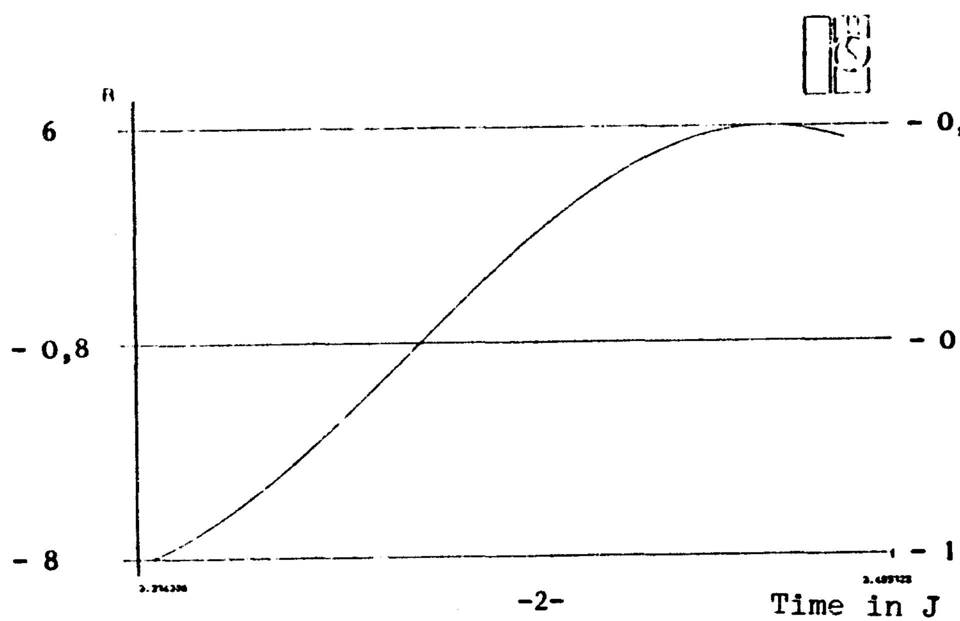
-4 . Bias in the station coordinates in the South and Natal

In general, for bias in the coordinates of part of the network where the noise over the complex of coordinates has an error with a characteristic feature: it is not the local geometric effect which could degrade the interpretation of altitude measurements over a short distance. A noise of 20 centimeters is found almost completely again in $\rho - \rho_0$ (curve 2).

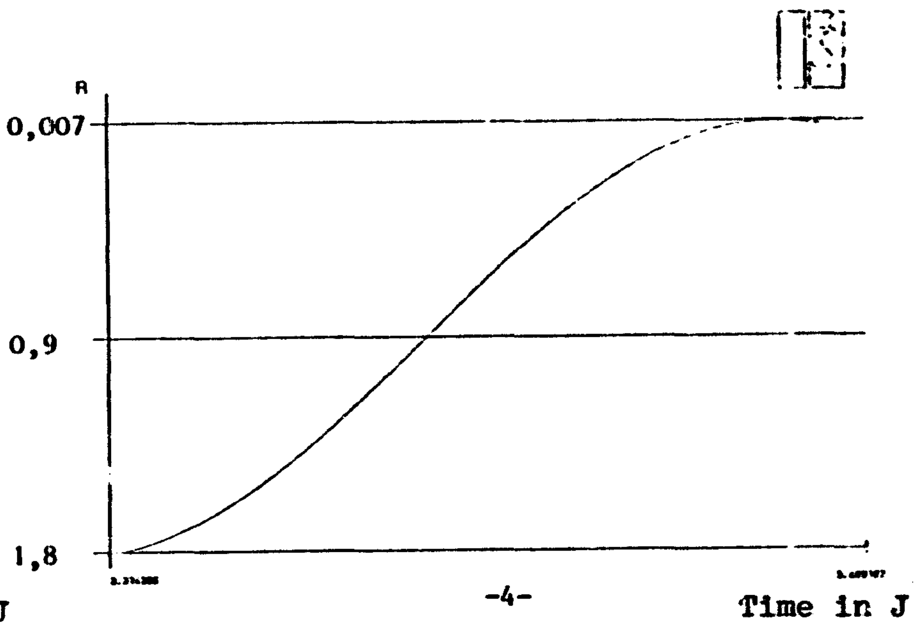
On the other hand, it seems necessary to have a homogeneous precise station network, because a bias over one part of the network will lead to a substantial error (curve 3).

The bias over one part of the network can be reabsorbed, which is indicated and shown in curve 4.

Distance difference (Earth center/satellite) between orbit tabulated during simulations and orbit tabulated for an imperfect case, i.e., satellite at 650 km
*initial parameters of the orbit 7763.374305556
*coordinates of station with noise at 0.2 m, (illegible)

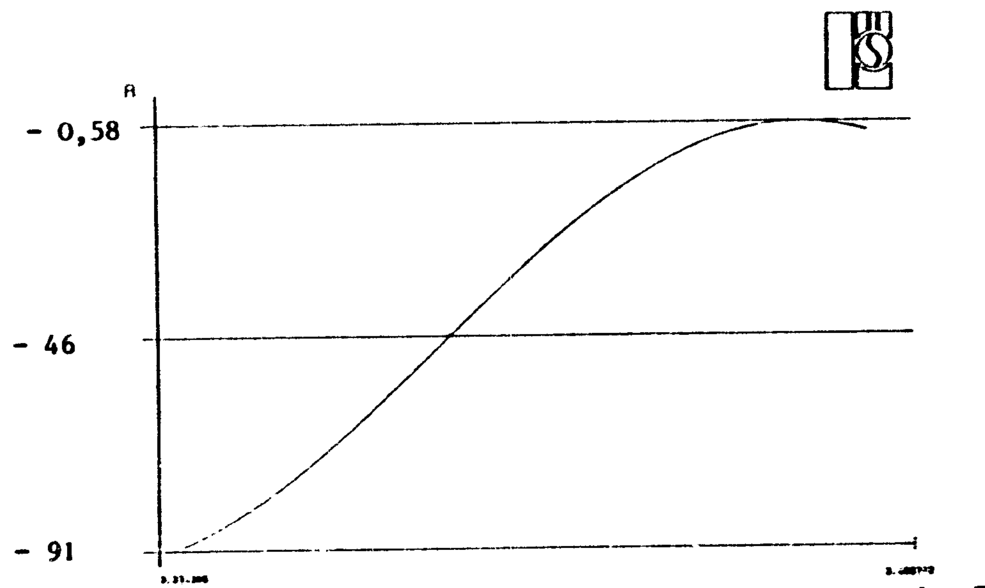


Distance difference (Earth center/satellite) between orbit tabulated during simulations and orbit tabulated for an imperfect case, i.e., satellite at 650 km
*initial parameters of the orbit 7763.37405556
*systematic error in the stations of -0.00001 degrees



/47

Distance difference (Earth center/satellite)
between orbit tabulated during simulations
and orbit tabulated for an imperfect case, i.e.,
satellite at 650 km
*initial parameters of the orbit 7763.374305556
*coordinates of stations with noise at 0.2 m,
*systematic error in the stations of -0.00001 degrees



-3-

5.6.7 SIMULATION OF PERFECT MEASUREMENTS

TREATMENT WITH THE MODEL GEM 10B BY
ELIMINATING THE MEASUREMENTS FROM THE TOP

-5 . Coordinates of stations with noise

. Bias in the coordinates of the southern stations

-6 . Bias in the coordinates of the southern station

-7 . Coordinates of the stations with noise

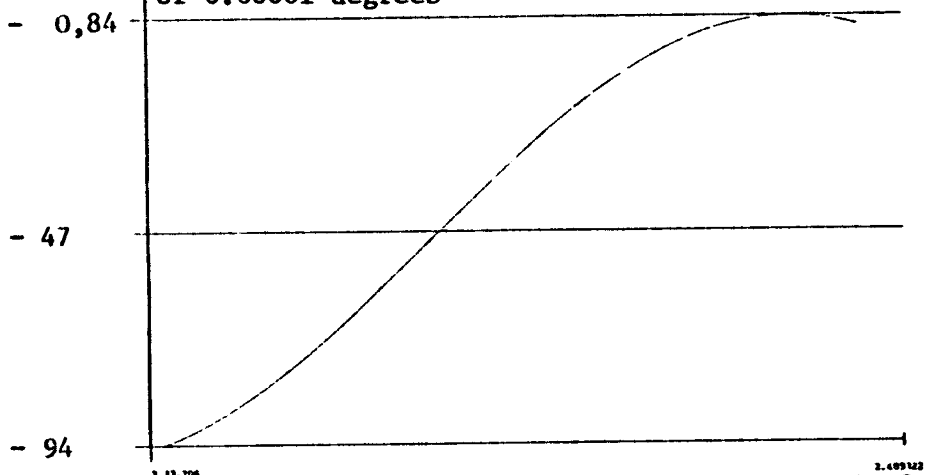
. Bias in the coordinates of the southern stations
(Free coordinates except height)

If a station is missing in the center of an arc (Natal in this case), we find a slight degradation:

- comparison between curves 3 and 5, and between curves 6 and 4

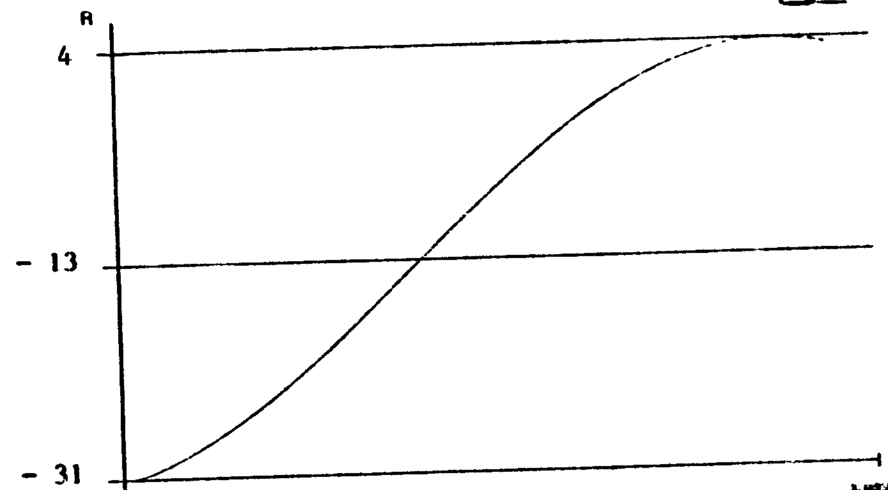
Nevertheless, it should be realized that for a single pass it is difficult to want to simultaneously determine the satellite orbit and a part of the station coordinates: the algorithm to be selected is certainly not the one above, and we clearly see the important role of a preliminary network calculation. Curve 7 was calculated by not determining the height of the stations, and should be compared with curve 5 where the three components were calculated.

Distance difference (Earth center--satellite/
Earth) between
- orbit tabulated during simulation and
- orbit tabulated in an imperfect case,
  i.e., satellite at 650 km
- *initial parameters of the orbit 7763.
  374305556
- *Natal station eliminated
- *coordinates of stations with noise 0.2m
- *systematic air in stations of the P.S.
  of 0.00001 degrees



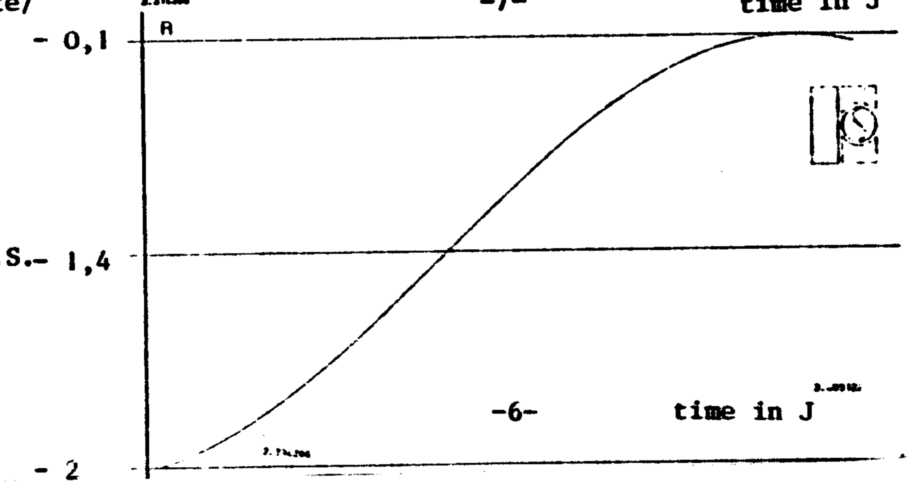
-5-

Distance difference (Earth center--satellite/
Earth) between
- orbit tabulated during simulation and
- orbit tabulated in an imperfect case,
  i.e., satellite at 650 km
- *initial parameters of the orbit 7763.
  374305556
- *Natal station eliminated
-*coordinates of stations with noise 0.2m
-*systematic air in stations of the P.S.
  of 0.00001 degrees



-7-

Distance difference (Earth center--satellite/
Earth) between
- orbit tabulated during simulation and
- orbit tabulated in an imperfect case,
  i.e., satellite at 650 km
- *initial parameters of the orbit 7763.
  374305556
- *Natal station eliminated
- *systematic air in the stations of the P.S.
  is -0.00001 degrees



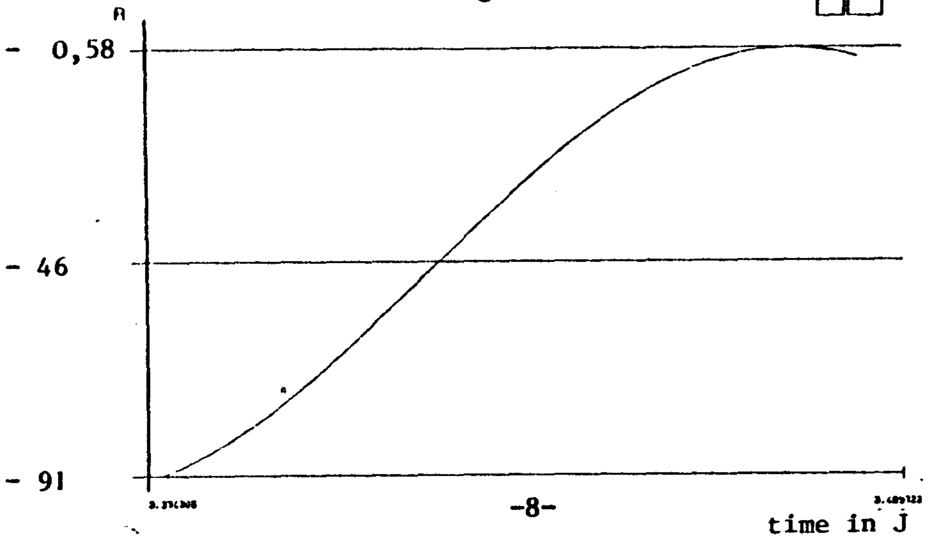
-6-

/49

8-9-10 - SIMULATION OF PERFECT MEASUREMENTS
- PROCESSING WITH THE MODEL GEM 10B
- COORDINATES OF STATIONS WITH NOISE
- BIAS IN THE COORDINATES OF THE SOUTHERN STATIONS AND NATAL
- 9 . MEASUREMENTS OF NATAL ARE ELIMINATED
-10 . MEASUREMENTS OF NATAL AND CANARY ISLANDS ARE ELIMINATED

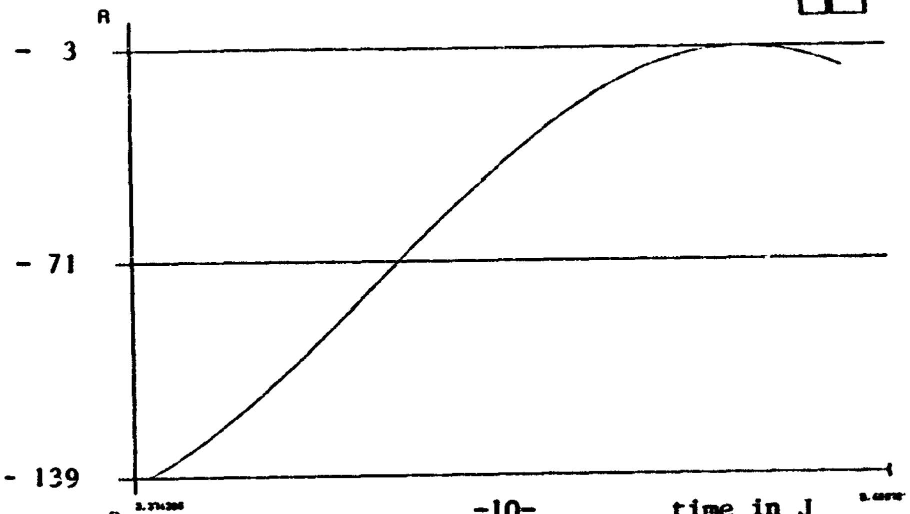
The total coverage from the orbit studied is also an important parameter as can be seen by comparing curves 8, 9 and 10.

Distance difference (Earth center/
satellite) between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case
  i.e., satellite at 650 km
- *initial parameters of the orbit
  at 7763.374305556
- *coordinates of the stations with noise
  0.21 m
- *systematic air in the stations of the
  P.S. of -0.00001 degrees



-8-

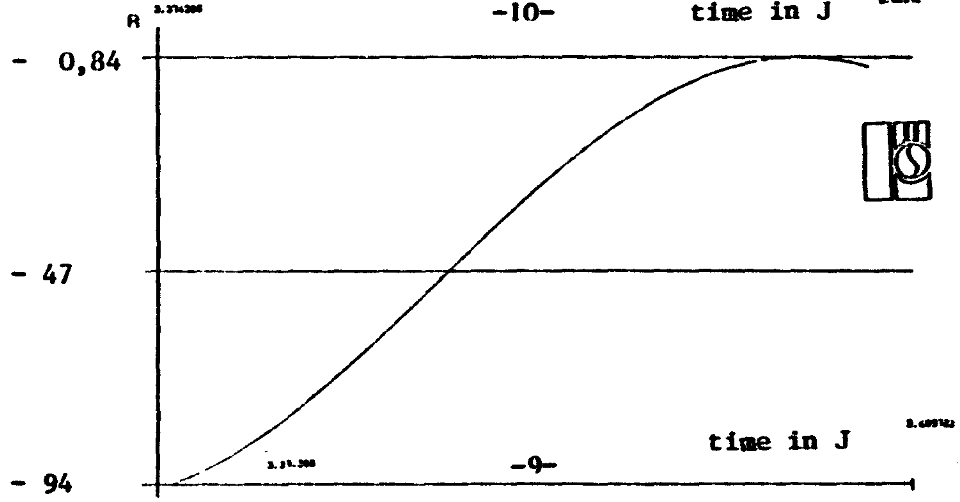
Distance difference (Earth center/
satellite) between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case
  i.e., satellite at 650 km
- *initial parameters of the orbit
  at 7763.374305556
- *Natal station and Canary Island station
  eliminated
-*coordinates of the stations with noise 0.2 m
-*systematic air in the stations of the P.S.
  of -0.00001 degrees



-10-

Distance difference (Earth center/
satellite) between
- orbit tabulated during simulation
- orbit tabulated in an imperfenct case
  i.e., satellite at 650 km
- *initial parameters of the orbit
  at 7763.374305556
- *Natal station eliminated
- *coordinates of the stations with noise
  0.2 m
- systematic air in the stations of the
  P.S. of -0.00001 degrees

/51



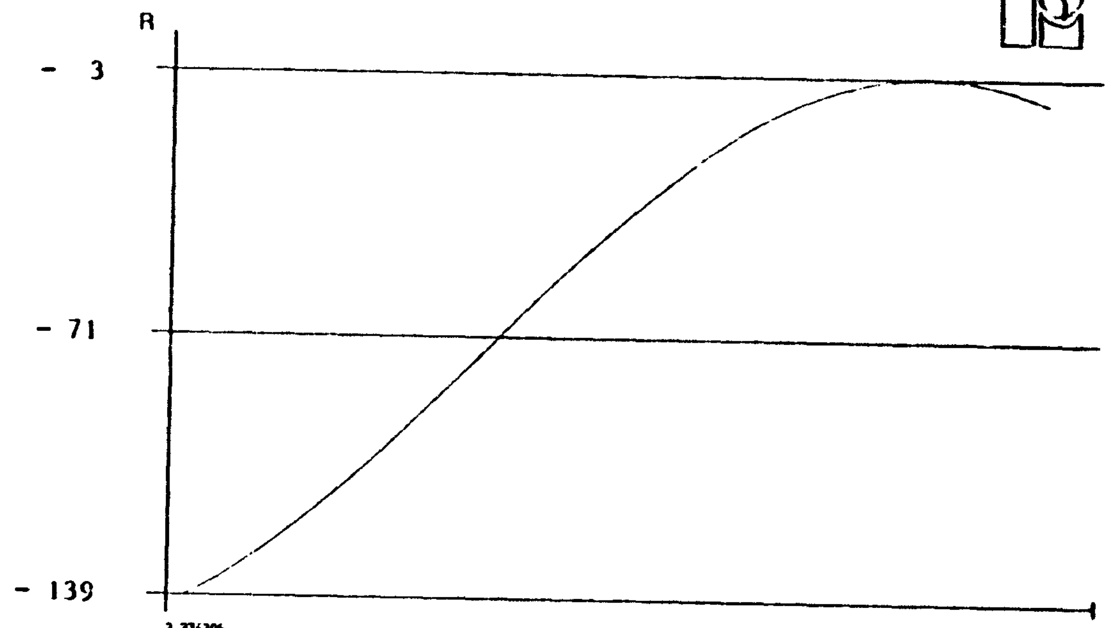
-9-

11-12 - SIMULATION OF PERFECT MEASUREMENTS
- TREATMENT WITH THE MODEL GEM 10B
- COORDINATES OF STATIONS WITH NOISE
- BIAS IN THE COORDINATES OF SOUTHERN STATIONS

  - 11 . MEASUREMENTS OF NATAL AND CANARY ISLANDS ARE ELIMINATED
  - 12 . MEASUREMENTS OF NATAL AND CANARY ISLANDS ARE ELIMINATED AND THE STATION HEIGHT OF SOUTHERN STATIONS IS A FIXED PARAMETER

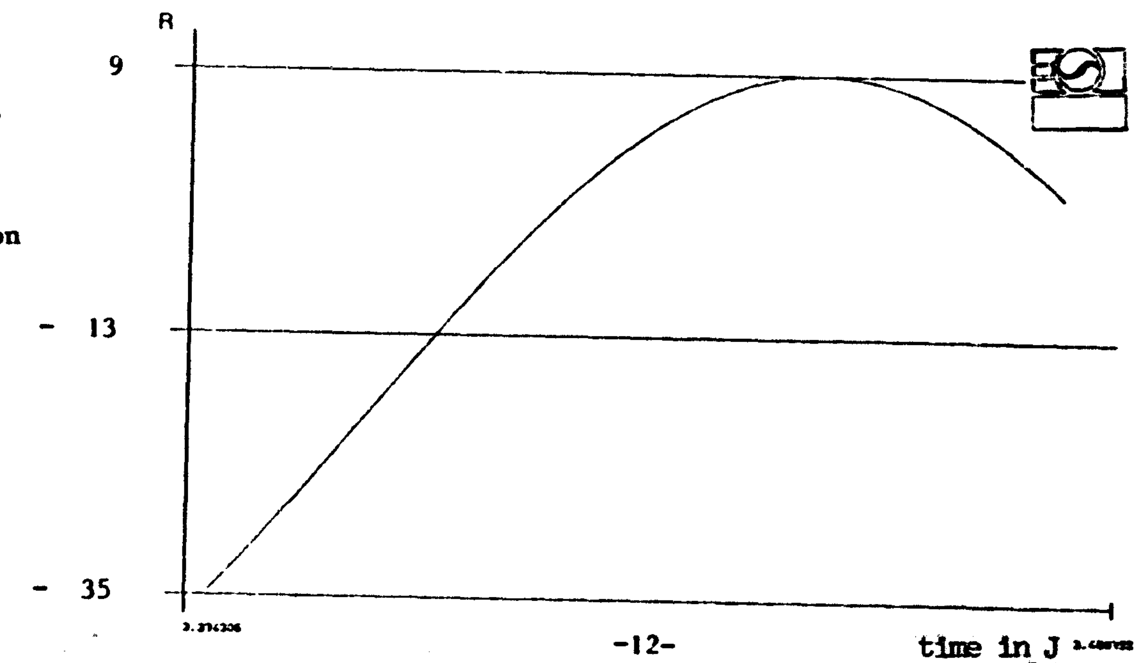
Another illustration of the algorithm which can be used. Here, we examine the influence of the station altitude during processing.

Distance difference (Earth center/
satellite) between
- orbit tabulated during simulation
- orbit tabulated in the imperfect case,
  i.e., satellite at 650 km
- *initial parameters of the orbit at
  7763.374305556
- *Natal station and Canary Island station
  eliminated
- *station coordinate noise 0.2m
- *systematic air in the stations of the
  P.S. is -0.00001 degrees



-11-

Distance difference (Earth center/
satellite) between
- orbit tabulated during simulation
- orbit tabulated in the imperfenct case,
  i.e., satellite at 650 km
- *initial parameters of the orbit at
  7763.374305556
- *Natal station and Canary Island station
  eliminated
- *station coordinate noise 0.2
- *systematic air in the stations of the
  P.S. is -0.00001 degrees



-12-

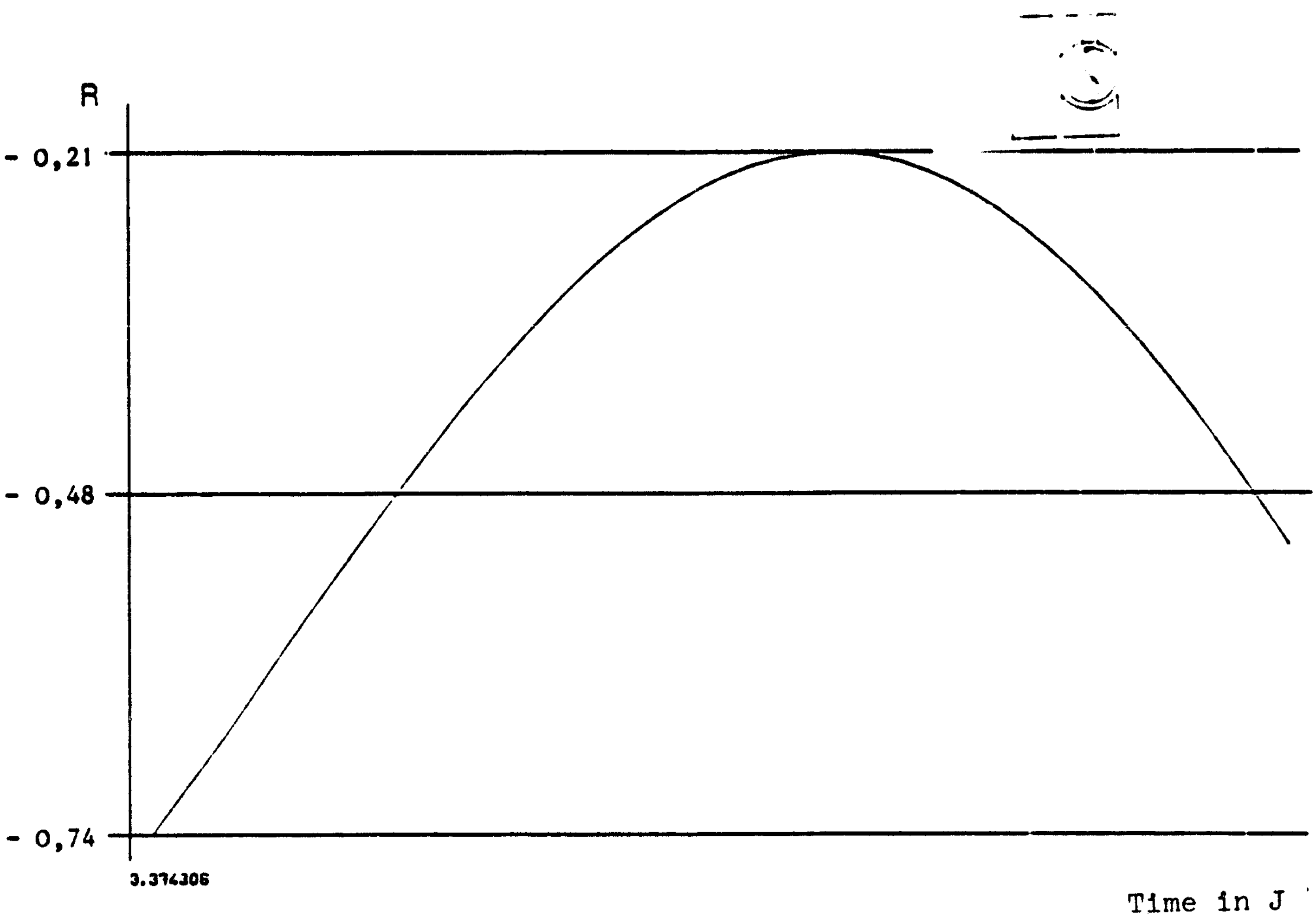
/53

- 13 - SIMULATION OF MEASUREMENTS WITH NOISE

- TREATMENT WITH THE MODEL GEM 10B

Influence of the measurement accuracy does not produce any short period perturbations and has a characteristic signature.

Distance difference (Earth center/
satellite) between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case
  i.e., satellite at 650 km
- *initial parameters of the orbit
  at 7763.374305556
- *Sigedi measurements with noise
- *Tragedi in the perfect case

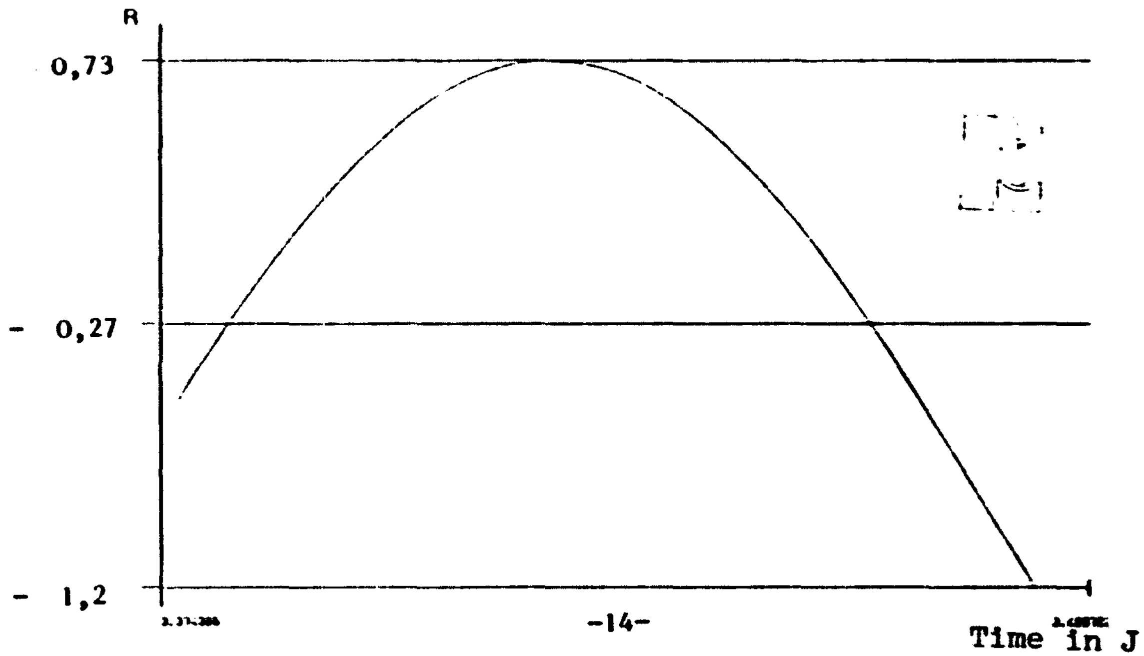


-13-

14-15 - SIMULATION OF MEASUREMENT WITH NOISE
- TREATMENT WITH THE MODEL GEM 10B
- BIAS IN THE COORDINATE OF THE SOUTHERN STATIONS AND NATAL
- 15 . NOISE IN THE STATIONS COORDINATES

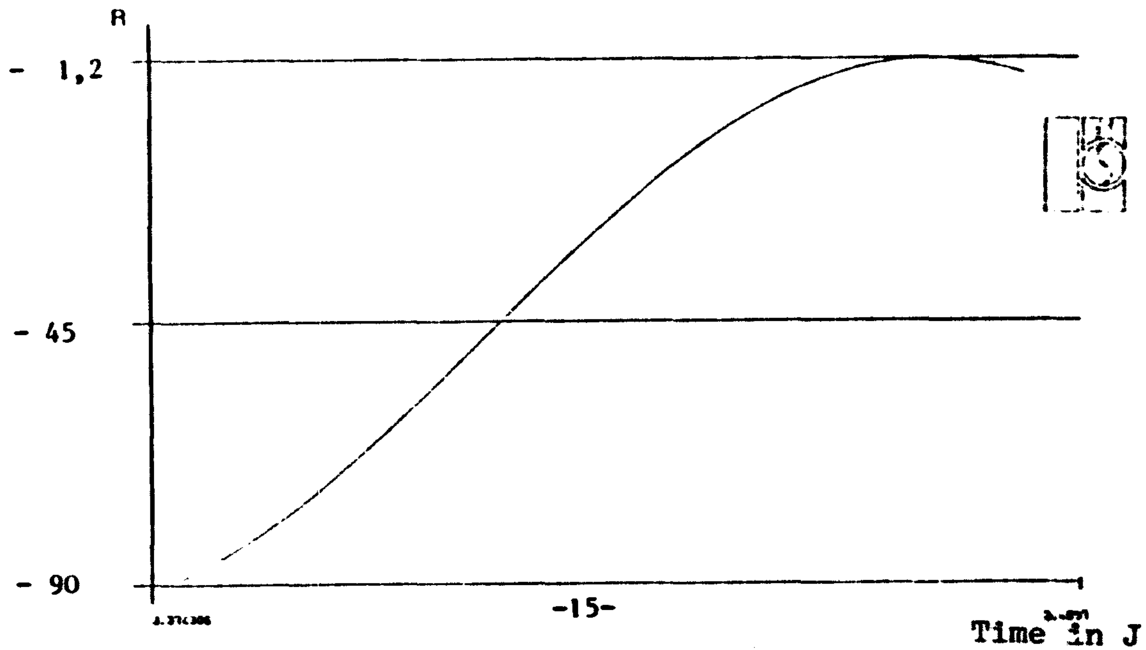
The mixing of errors in measurements and coordinates of stations does not give combinations of the distortions which coul d be translated into short period anomalies.

Distance difference (Earth center/ satellite) between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 650 km
- *Initial parameters of the orbit at 7763.374305556
- *Sigedi measurements with noise
- *Systematic air in the stations of the P.S. is -0.00001 degrees



-14-

Distance difference (Earth center/ satellite) between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case i.e., satellite at 650 km
- *Initial parameters of the orbit at 7763.374305556
- *Sigedi measurements with noise
- *Systematic air in the stations of the P.S. is -0.00001 degrees
- Station coordinates with noise 0.2m



-15-

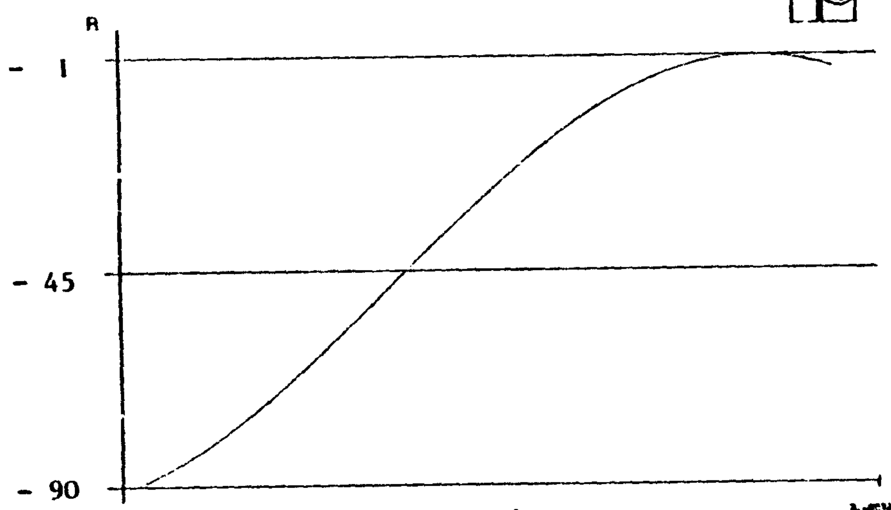
/57

16-17-18 - SIMULATION OF MEASUREMENTS WITH NOISE
- TREATMENT WITH THE MODEL GEM 10B
- NOISE IN THE STATION COORDINATES
- BIAS IN THE SOUTHERN STATION COORDINATES

-17 . ONLY VELOCITY MEASUREMENTS
-18 . ONLY DISTANCE MEASUREMENTS

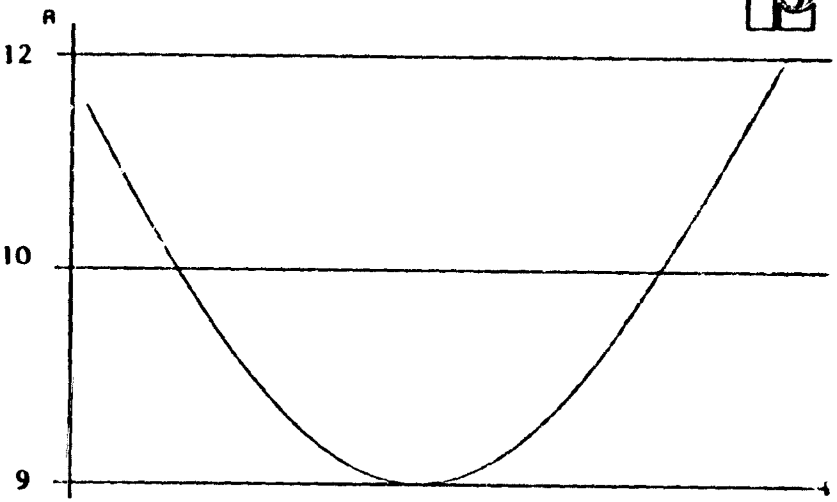
Here we can see the predominant role of distance measurements: We find a substantial improvement between curves 17 (Doppler alone) and 18 (distance alone). One could believe that by comparing 16 and 17 that the improvement due to distance is perturbed by the Doppler measurement and, therefore, the distance would not be valuable. This is a concrete illustration of the problem of mixing different kinds of measurement types where the algorithm to be used must be carefully selected. In particular, one has to consider the weighting problems which were not into account here.

Distance difference (Earth center / satellite) between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 650 km
- *initial parameters of the orbit at 7763.374305556
-*Sigedi measurements with noise
-*systematic air in the stations of the P.S. of -0.00001 degrees
-*station coordinate with noise 0.2m



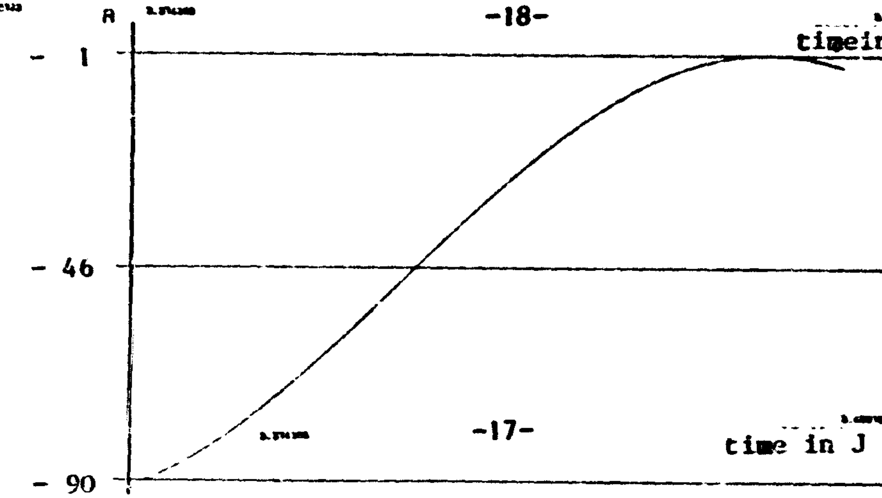
-16-

Distance difference (Earth center/ satellite) between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 650 km
- *initial parameters of the orbit at 7763.374305556
-*Sigedi measurements with noise
-*systematic air in the stations of the P.S. of -0.00001 degrees
-*station coordinate with noise 0.2m



-18-

Distance difference (Earth center/ satellite) between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 650 km
- *initial parameters of the orbit at 7763.374305556
- *Sigedi measurements with noise
- *systematic air in the stations of the P.S. of -0.00001 degrees
- station coordinate with noise 0.2m



-17-

/59

/60

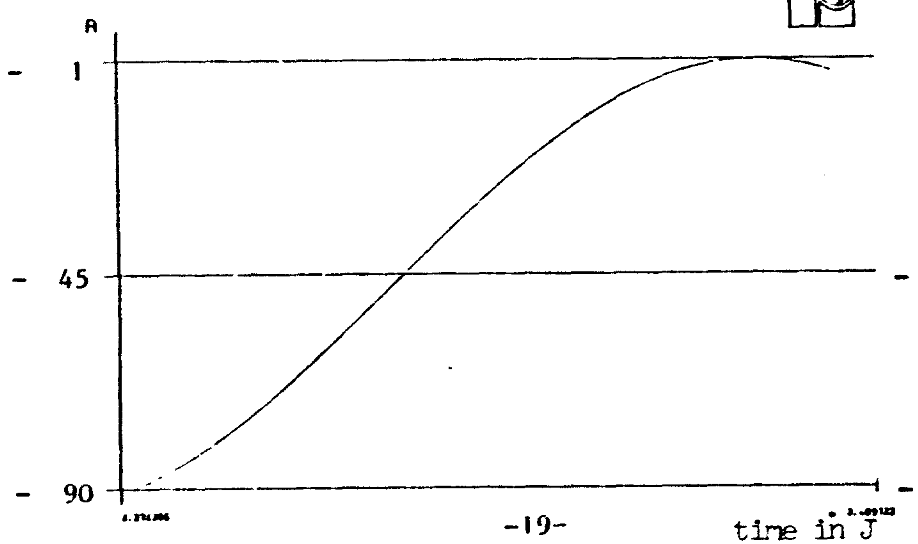
19-20-21-22 - SIMULATION OF MEASUREMENTS WITH NOISE
- TREATMENT WITH THE MODEL GEM 10B
- NOISE IN THE STATION COORDINATES
- BIAS IN THE SOUTHERN STATION COORDINATES

- 20- . ARC GOES TO THE LEFT; VISIBLE ONLY BY 7 STATIONS
- 21- . ARC GOES TO THE LEFT (7 STATIONS VISIBLE)
  . ONLY DISTANCE MEASUREMENTS ARE CONSIDERED
- 22- . ARC GOES TO THE LEFT (7 STATIONS VISIBLE)
  . ONLY VELOCITY MEASUREMENTS ARE CONSIDERED

Influence of the geometry of the path: The path corresponding to curve 19 is the one shown by a solid line which is centered with respect to the stations. The one for curve 20 is shown by dotted lines on the map. We find a slight degradation.

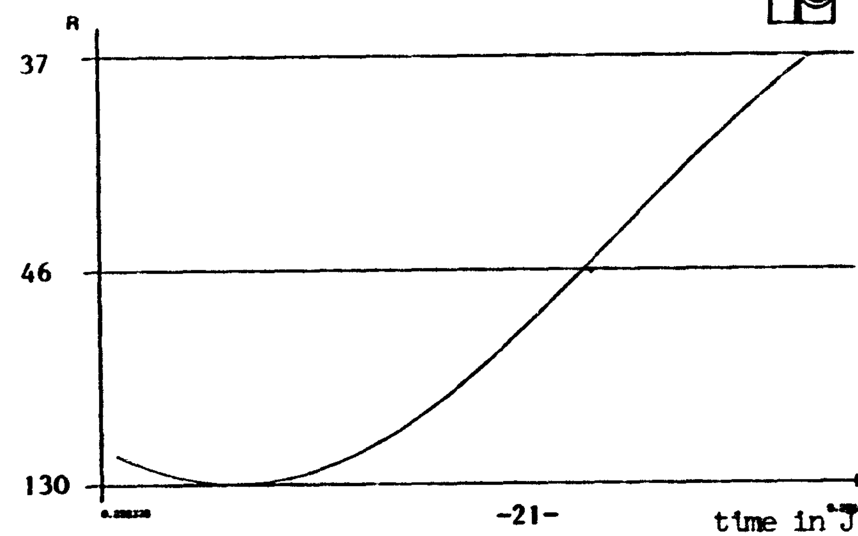
In contast to curves 17 and 18, the predominance of one type of measurements with respect to the other is not clear, which illustrates the geometric factor of these studies very well.

Distance difference (Earth center/satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 650 km
- *initial parameters of the orbit at 7763.374305556
- *Sigedi measurements with noise
- *systematic air in the stations of the P.S. of
  -0.00001 degrees
- *station coordinate with noise 0.2m



-19-

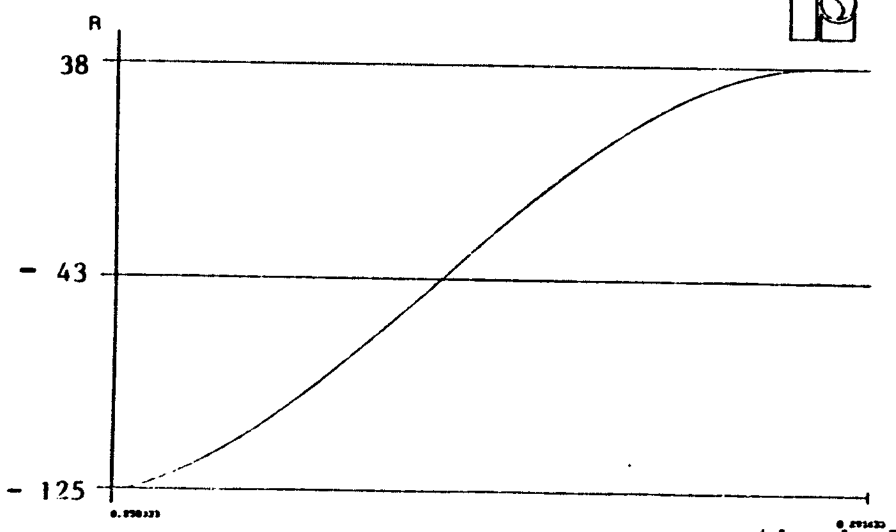
Distance difference (Earth center/satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 650 km
-*initial parameters of the orbit at 7760.258333333
-*Sigedi measurements with noise
-*systematic air in the stations of the P.S. of
  -0.00001 degrees
-*station coordinate with noise 0.2m



-21-

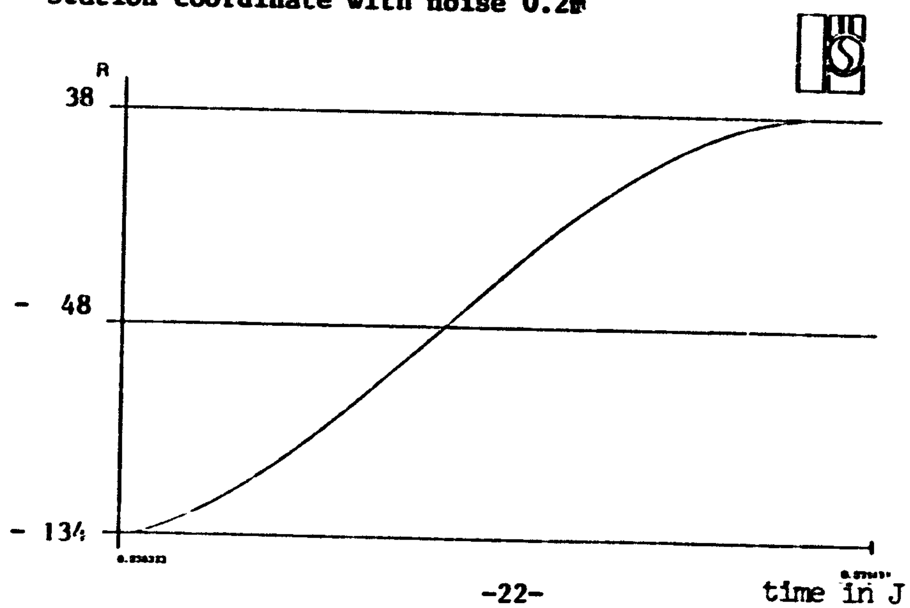
/61

Distance difference (Earth center/satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 650 km
-*initial parameters of the orbit at 7760.458333333
-*Sigedi measurements with noise
-*systematic air in the stations of the P.S. of
  -0.00001 degrees
-*station coordinate with noise 0.2m



-20-

Distance difference (Earth center/satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 650 km
-*initial parameters of the orbit at 7760.258333333
-*Sigedi measurements with noise
-*systematic air in the stations of the P.S. of
-0.00001 degrees
-*station coordinate with noise 0.2m



-22-

/61

23-24 - SIMULATION OF MEASUREMENTS WITH NOISE /62

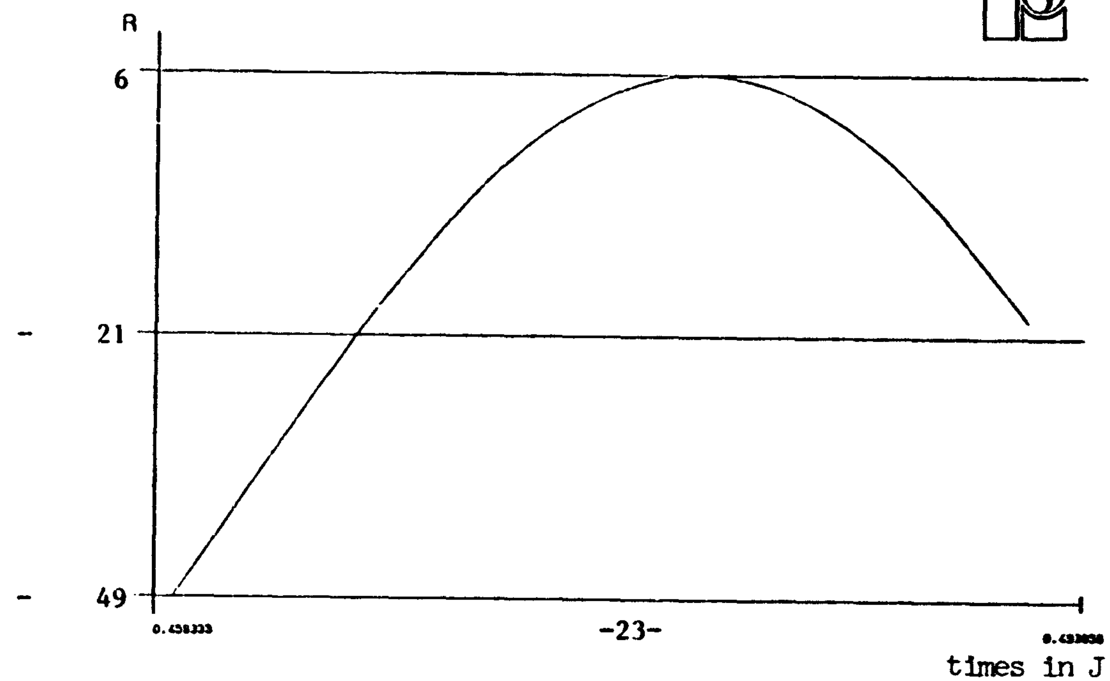
- NOISE IN THE STATION COORDINATES
- BIAS IN THE SOUTHERN STATION COORDINATES
- ARC GOES TO THE RIGHT, VISIBLE ONLY BY 9 STATIONS

-23- . TREATMENT WITH THE MODEL GEM 10B

-24- . TREATMENT WITH THE MODEL GEM 8

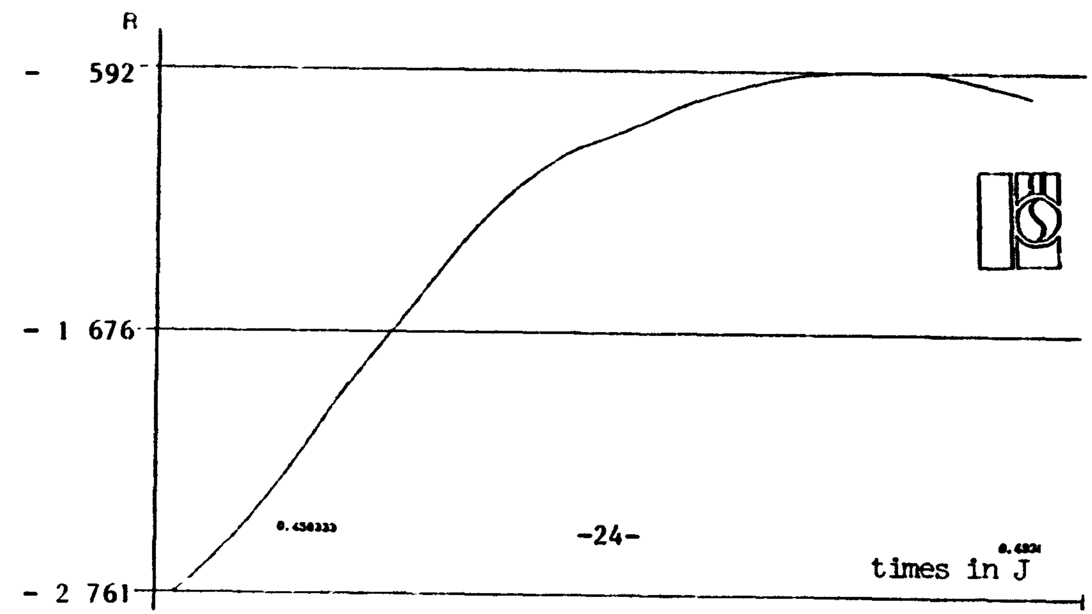
A first estimate of the influence of the Earth potential illustrated by curves 23 and 24. We can immediately see its important role. The general appearance of the curves up to here was regular. The change in the potential model perturbs these regular features and shows a kind of undulation around the generally regular curves. The evaluation of the very short period perturbations (5 to 15 minutes) presages such a phenomena which has now been verified.

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 650 km
-*initial parameters of the orbit at
  7760.458333333
-*Sigedi measurements with noise
-*systematic air in the stations of the
  P.S. of -0.00001 degrees
-*station coordinate with noise 0.2m



Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 650 km
-*initial parameters of the orbit at
  7760.458333333
-*Sigedi measurements with noise *Tragedi,GEM8
-*systematic air in the stations of the
  P.S. of -0.00001 degrees
-*station coordinate with noise 0.2m

/63



25-26 - SIMULATION OF MEASUREMENTS WITH NOISE
- NOISE IN THE STATION COORDINATES
- BIAS IN THE SOUTHERN STATION COORDINATES
- TREATMENT WITH MODEL GEM 8

-25- . ARC GOES TO THE RIGHT, VISIBLE ONLY BY 9 STATIONS

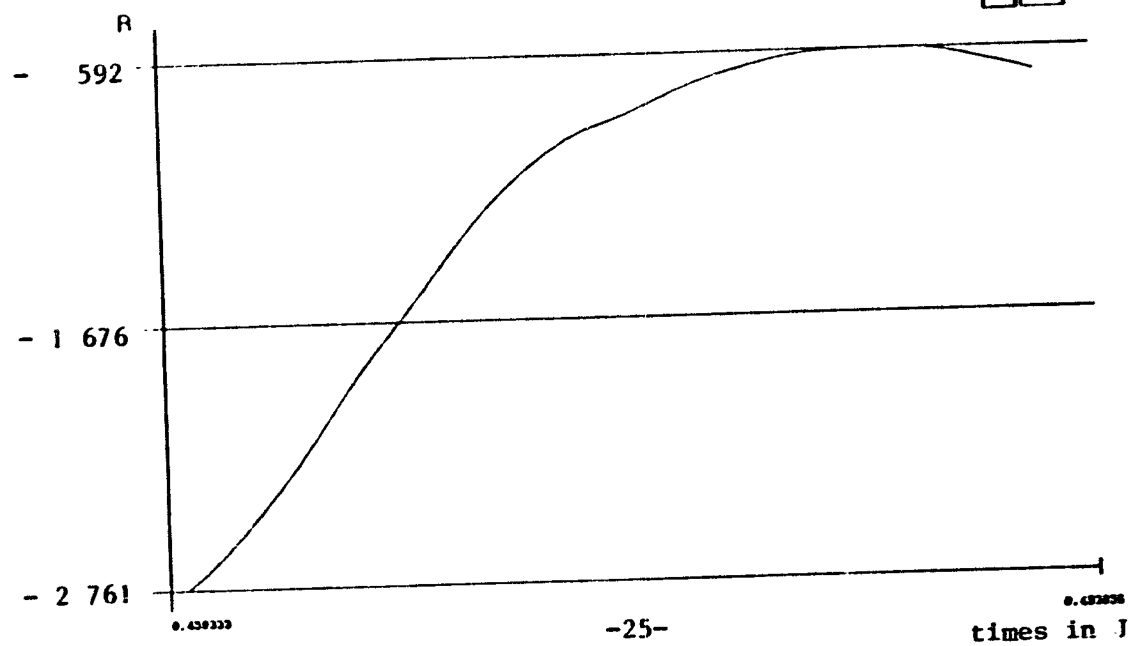
-26- . ARC GOES TO THE LEFT, VISIBLE ONLY BY 7 STATIONS

Other illustrations of the influence of potential on other arcs.

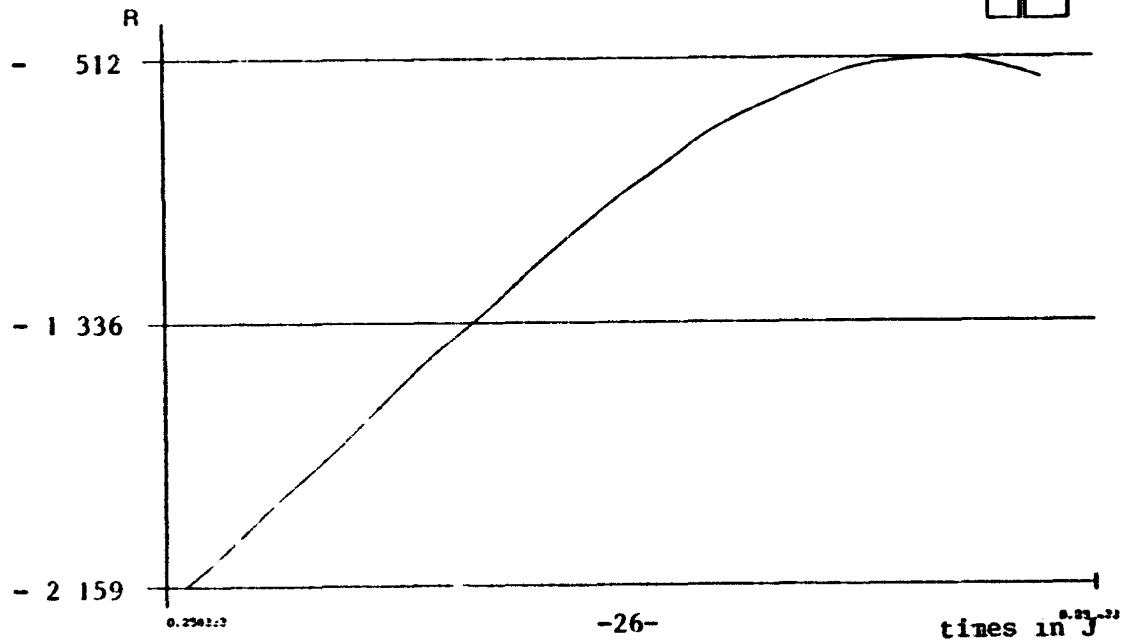
The study of the influence of the potential is difficult because we have no access to reality. The potential models are not adequately accurate.

In order to simulate this imperfection, we selected the change in the model (GEM 10B and GEM 8) which is no less unrealistic than to introduce noise in a given model. All of the coefficients are highly correlated in determining the model.

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation,
- orbit tabulated in an imperfect case,
  i.e., satellite at 650 km
-*initial parameters of the orbit at
  7760.458333333
-*Sigedi measurements with noise *Tragedi,GEM8
-*systematic air in the stations of the
  P.S. of -0.00001 degrees
-*station coordinate with noise 0.2m



Distance difference (Earth center/ satellite)between
- orbit tabulated during simulation,
- orbit tabulated in an imperfect case, i.e., satellite at 650 km
-*initial parameters of the orbit at 7760.458333333
- *Sigedi measurements with noise *Tragedi, GEM8
- *systematic air in the stations of the P.S. of -0.00001 degrees
- *station coordinate with noise 0.2m



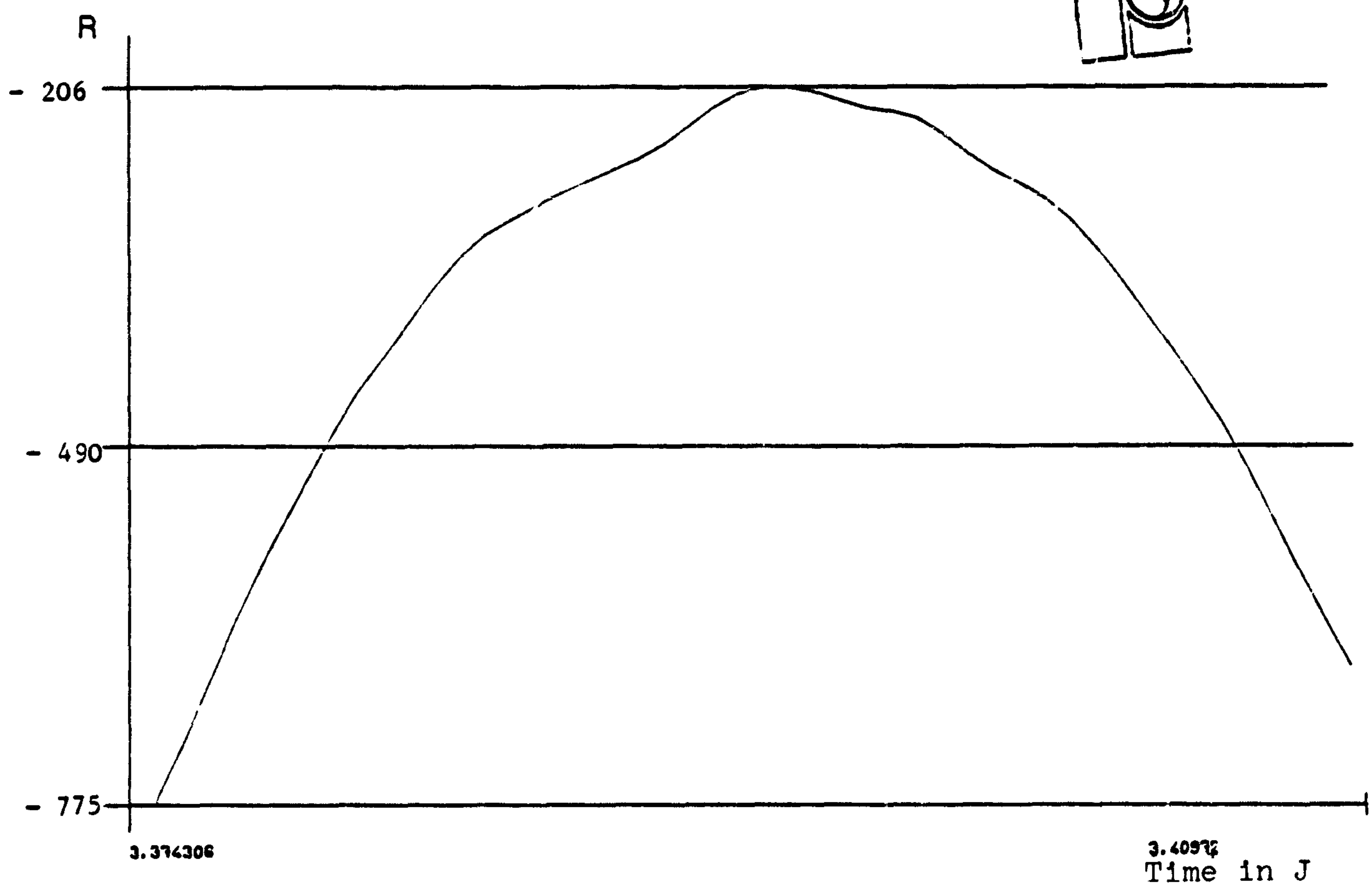
/65

/66

-27 - SIMULATION OF PERFECT MEASUREMENTS
- PROCESSING WITH THE MODEL GEM 8

Here, everything is strictly identical between the simulation and the processing except for a change in the potential. Therefore, one can see that it is impossible, if one wishes to remain at the 5 cm level, to operate using a classical orbiting method. It is therefore necessary to at least carry out a partial geometric determination of the orbit.

Distance difference (Earth center/
satellite between
- orbit tabulated during simulation,
- orbit tabulated in an imperfect case,
  i.e., satellite at 650 km
- *initial parameters of the orbit at 7763.374305556
- *Tragedi in the perfect case * treatment with GEM 8

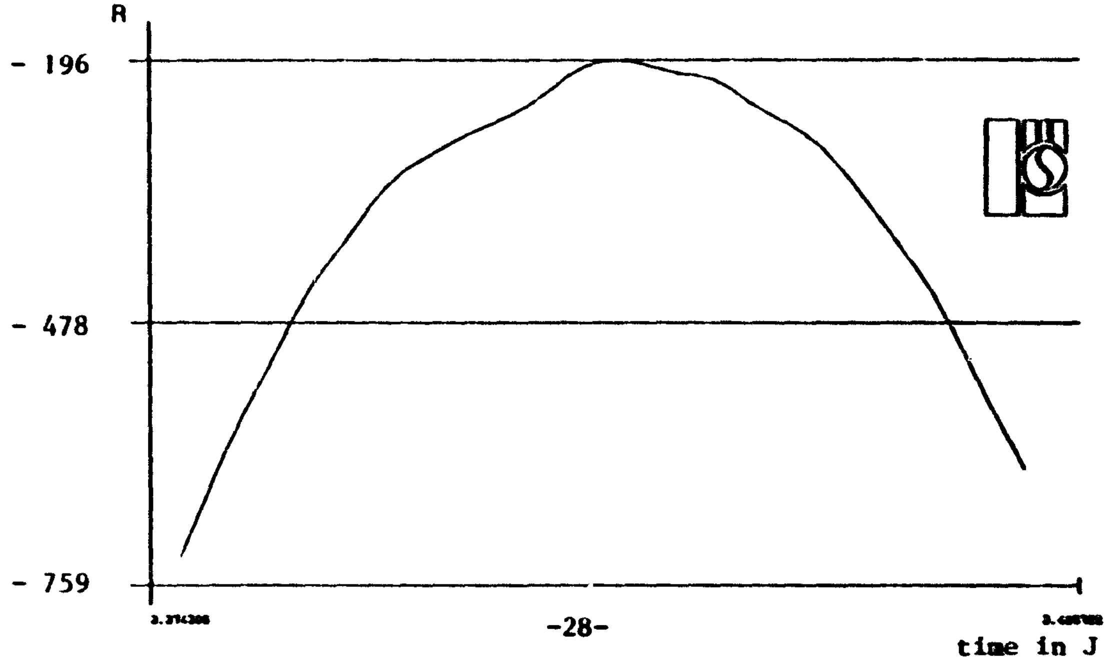


28-29 - SIMULATION OF PERFECT MEASUREMENTS
- TREATMENT WITH THE MODEL GEM 8

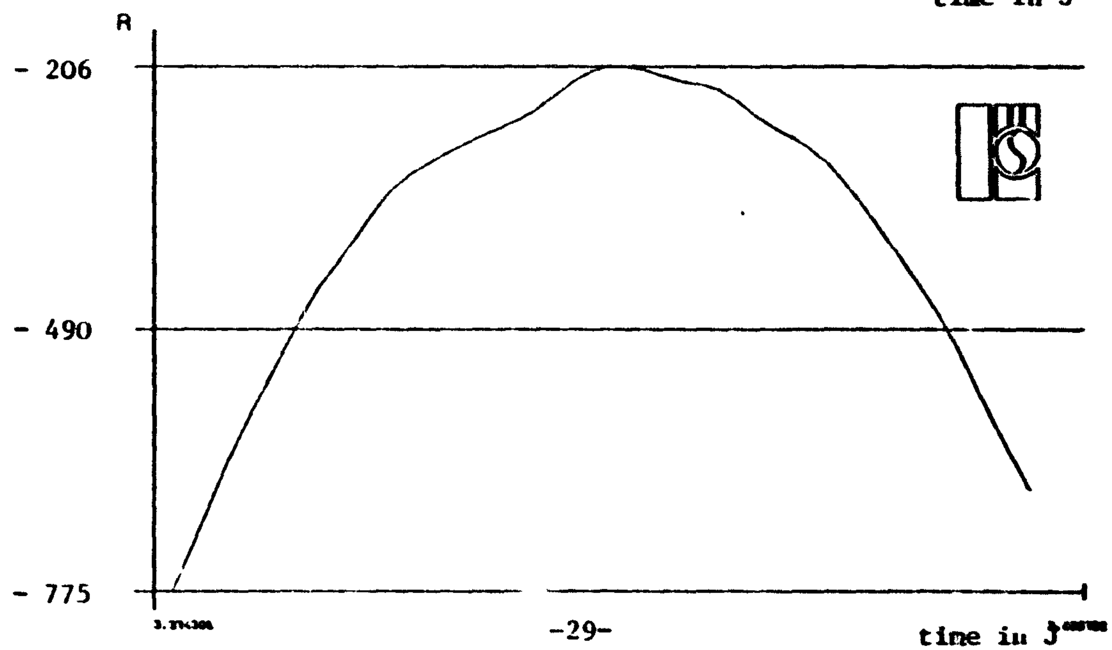
-28 . distance measurements only
-29 . velocity measurements only

Illustration of the preponderant role of the potential: The measurement type is not significant.

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 650 km
-*initial parameters of the orbit at
  7763.374305556
-*Tragedi in the perfect case *treatment
  with GEM 8



Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation,
- orbit tabulated in an imperfect case,
  i.e., satellite at 650 km
- *initial parameters of the orbit at
  7763.374305556
- *Tragedi in the perfect case *treatment
  with GEM 8



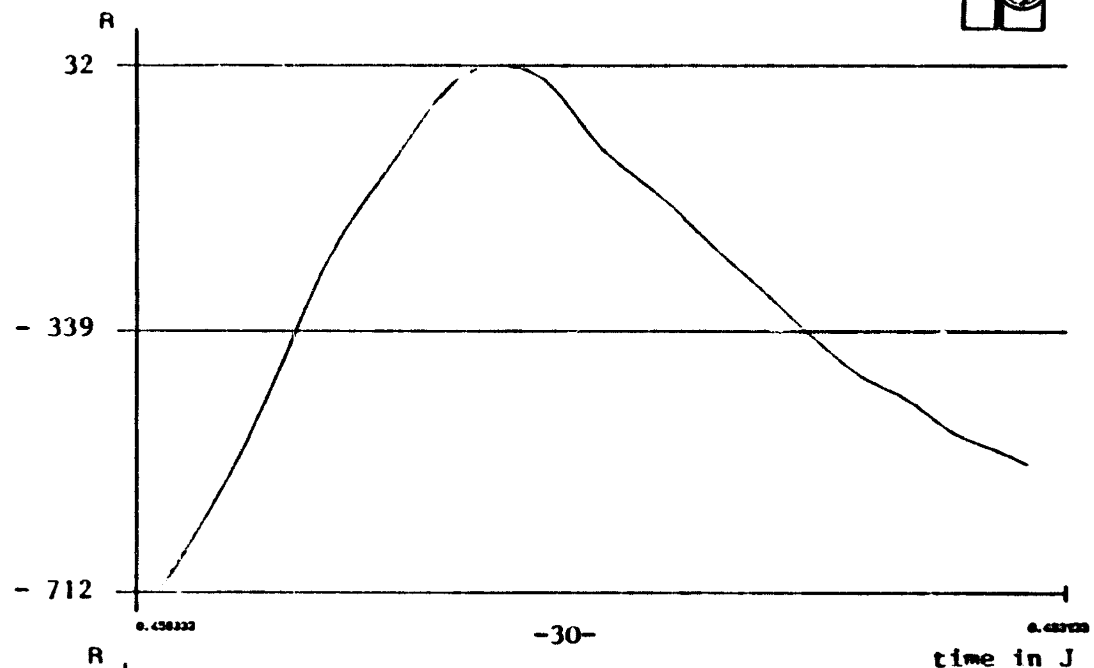
/69

30-31 - SIMULATION OF PERFECT MEASUREMENTS
- TREATMENT WITH THE MODEL GEM 8
- ARC GOES TO THE RIGHT (9 STATIONS VISIBLE)

-31- . DISTANCE MEASUREMENTS ONLY

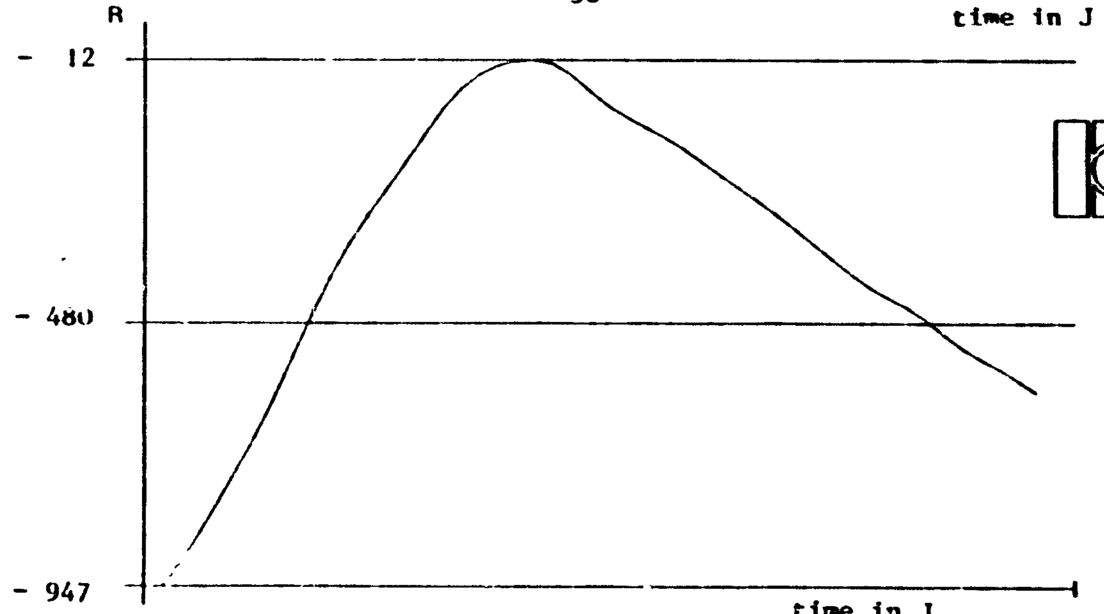
The influence of the potential is not completely predictable for two different arcs.

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 650 km
-*initial parameters of the crbit at
  7760.456333333
- *Tragedi in the perfect case *treatment
  with GEM 8



-30-

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation,
- orbit tabulated in an imperfect c ase,
  i.e., satellite at 650 km
-*initial parameters of the orbit at
  7760.458333333
-*Tragedi in the perfect case *treatment
  with GEM 8



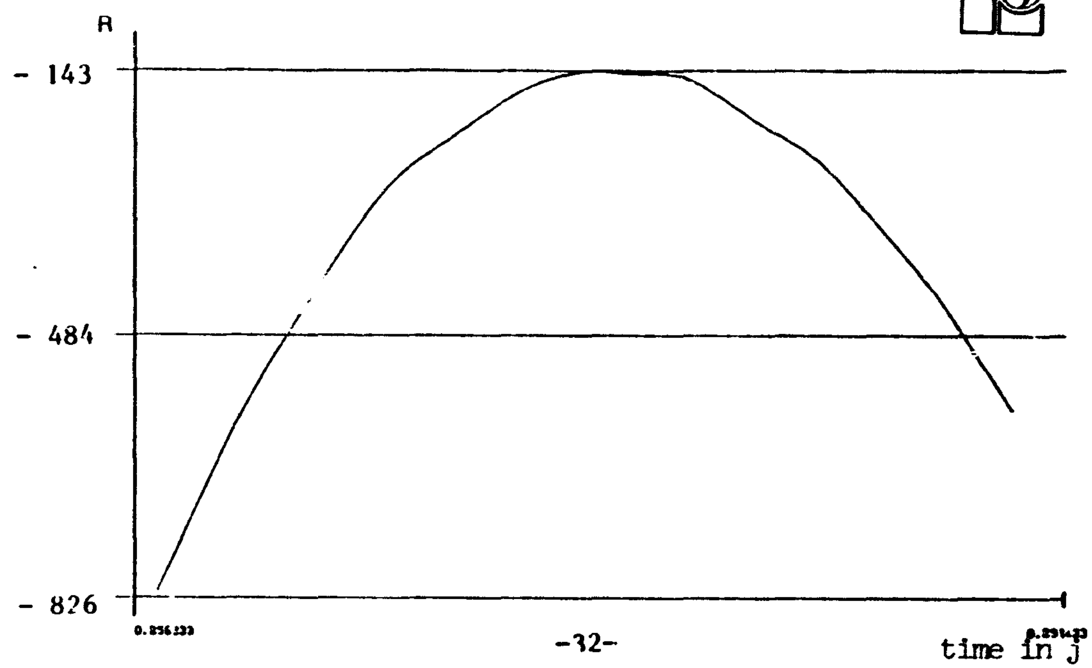
-31-

/71

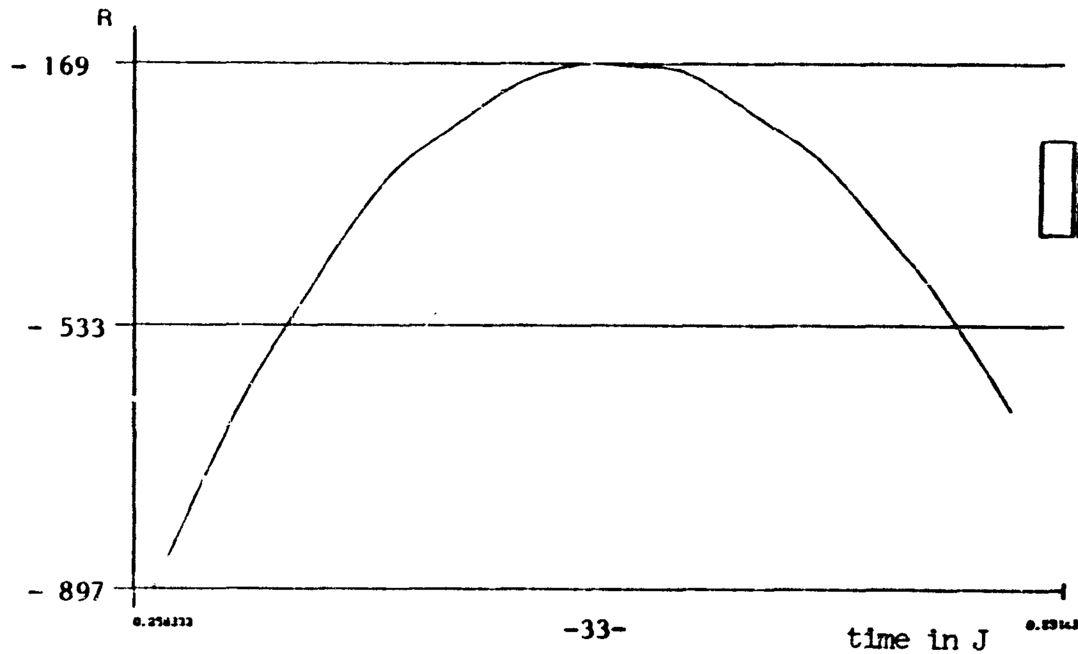
32-33 - SIMULATION OF PERFECT MEASUREMENTS
- TREATMENT WITH THE MODEL GEM 8
- ARC GOES TO THE LEFT (7 STATIONS VISIBLE)

-33- . DISTANCE MEASUREMENTS ONLY

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 650 km
- *initial parameters of the orbit at
  7760.258333333
- *Tragedi in the perfect case *treatment
  with GEM 8



Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 650 km
- *initial parameters of the orbit at
  7760.258333333
- *Tragedi in the perfect case *treatment
  with GEM 8

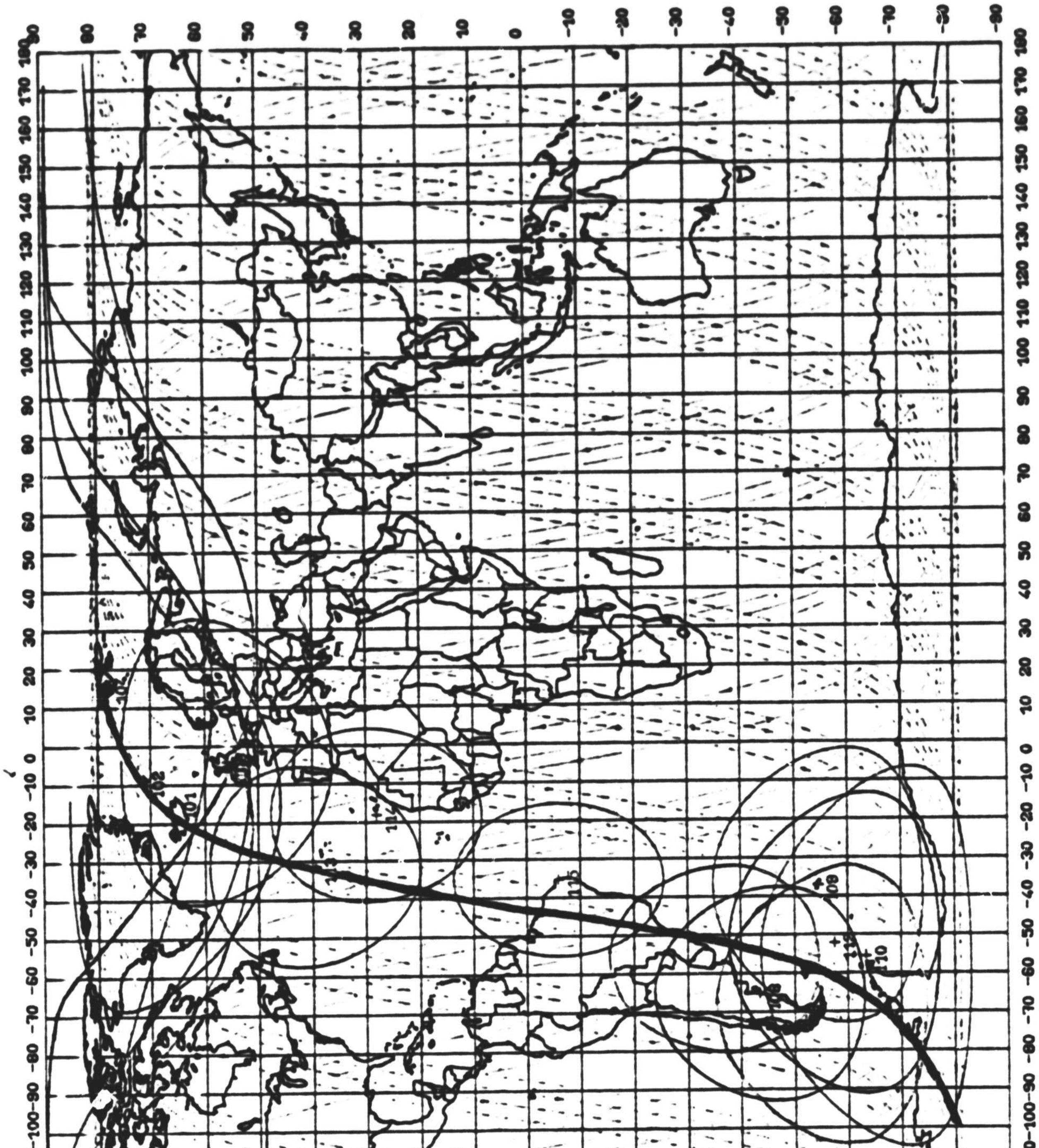


/73

RESULTS FOR AN ALTITUDE

OF 850 KM

Remark: Most of the commentary made for the 650 km altitude are applicable for 850 km and will not be repeated.



solid line: arc studied

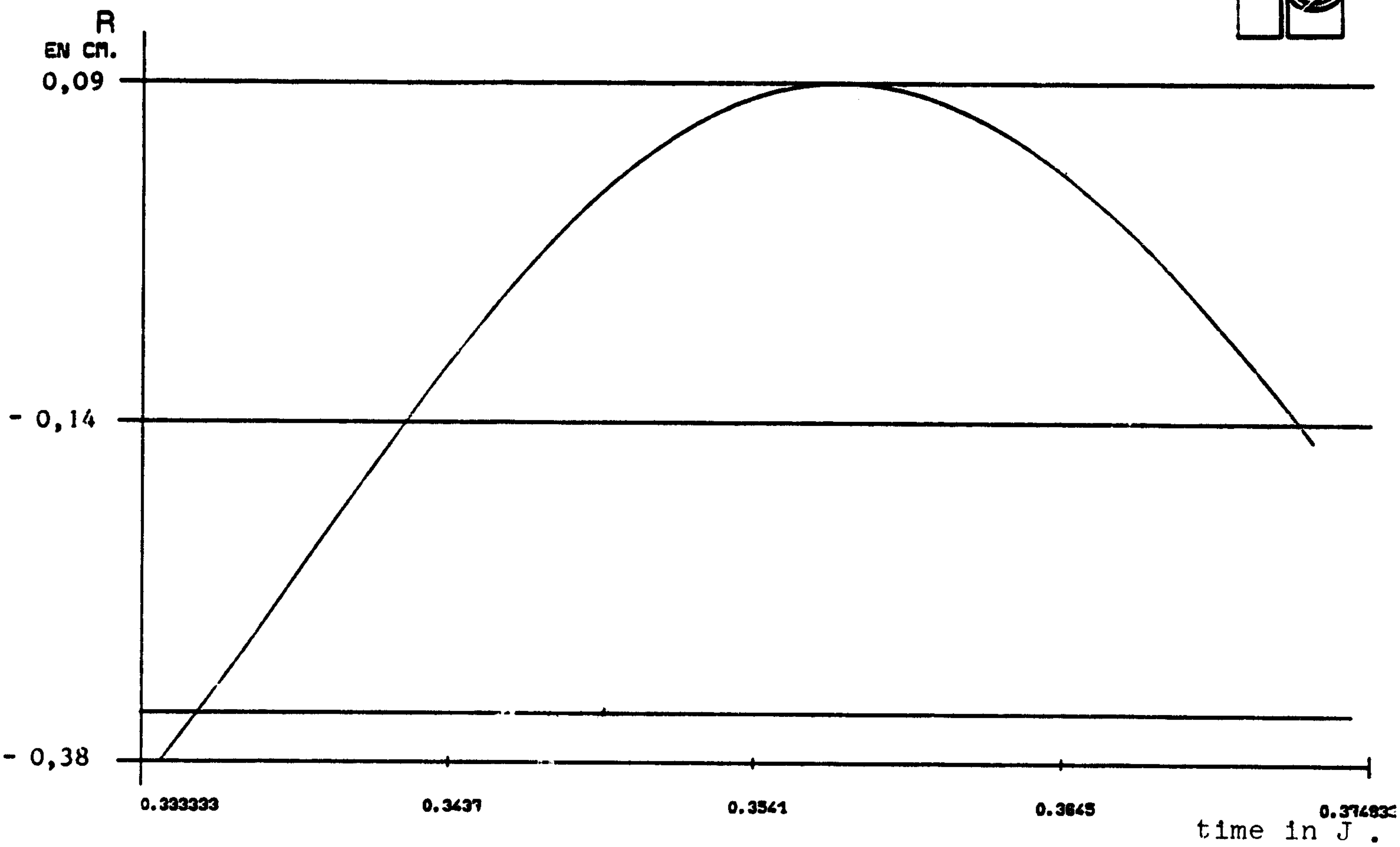
satellite altitude: 850 km

visibility circles 10°

-34 - SIMULATION OF MEASUREMENTS WITH NOISE

- TREATMENT WITH THE MODEL GEM 10B

Distance difference (Earth center/satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 650 km
-*initial parameters of the orbit 7760.333333333
-*Tragedi in the perfect case



35-36-37 - SIMULATION OF PERFECT MEASUREMENTS
- TREATMENT WITH THE MODEL GEM 10B

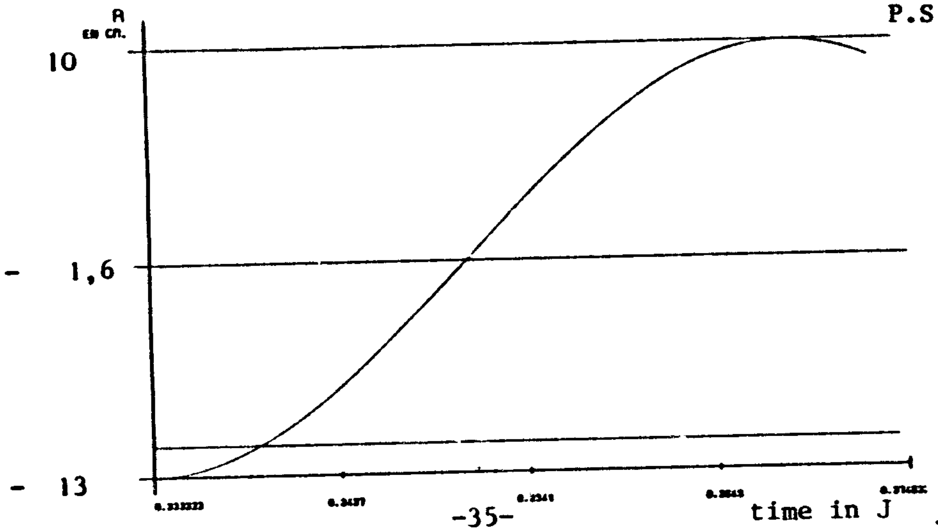
-35- . NOISE IN STATION COORDINATES

-36- . NOISE IN THE STATION COORDINATES

. BIAS IN THE STATION COORDINATES OF THE SOUTH AND NATAL

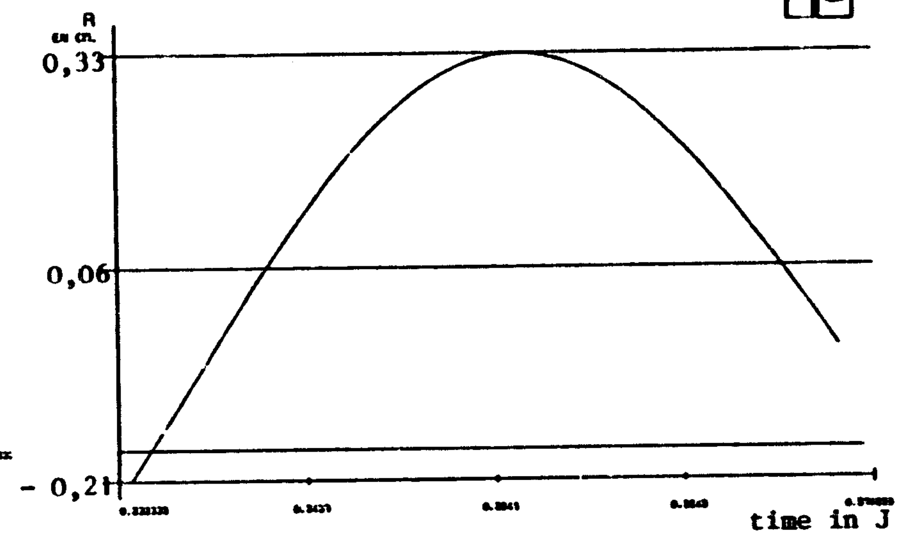
. BIAS IN THE STATION COORDINATES OF THE SOUTH AND NATAL

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 850 km
-*initial parameters of the orbit at
  7760.333333333
-*station coordinate with noise 0.2m



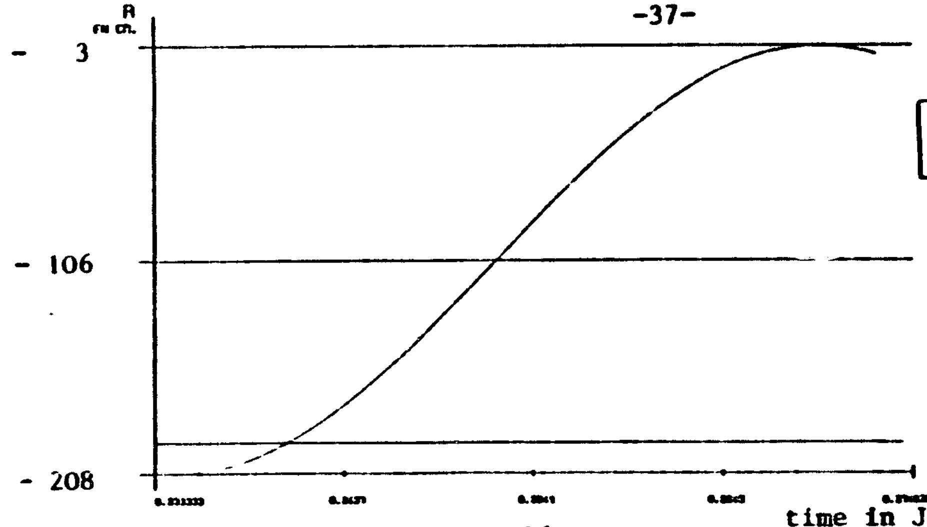
-35-

Distance difference (Earth center/
satellite)between
-orbit tabulated during simulation
-orbit tabulated in an imperfect case,
 i.e., satellite at 650 km
-*initial parameters of the orbit at
  7760.333333333
-*systematic air in the stations of the
  P.S. of -0.00001 degrees



-37-

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 850 km
-*initial parameters of the orbit at
  7760.333333333
-*station coordinate with noise 0.2m
-*systematic air in the stations of the
 P.S. of -0.00001 degrees. Free of the
 parameters of the A.S.



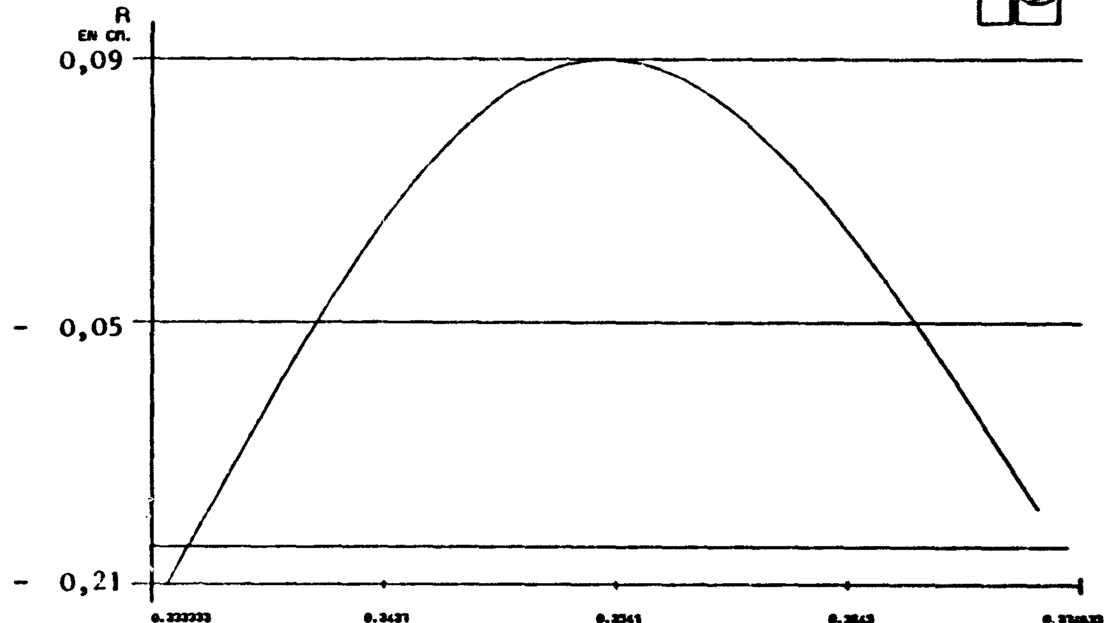
-36-

/79

38-39 - SIMULATION OF PERFECT MEASUREMENTS
- TREATMENT WITH THE MODEL GEM 10B BY ELIMINATING THE NATAL STATION MEASUREMENTS

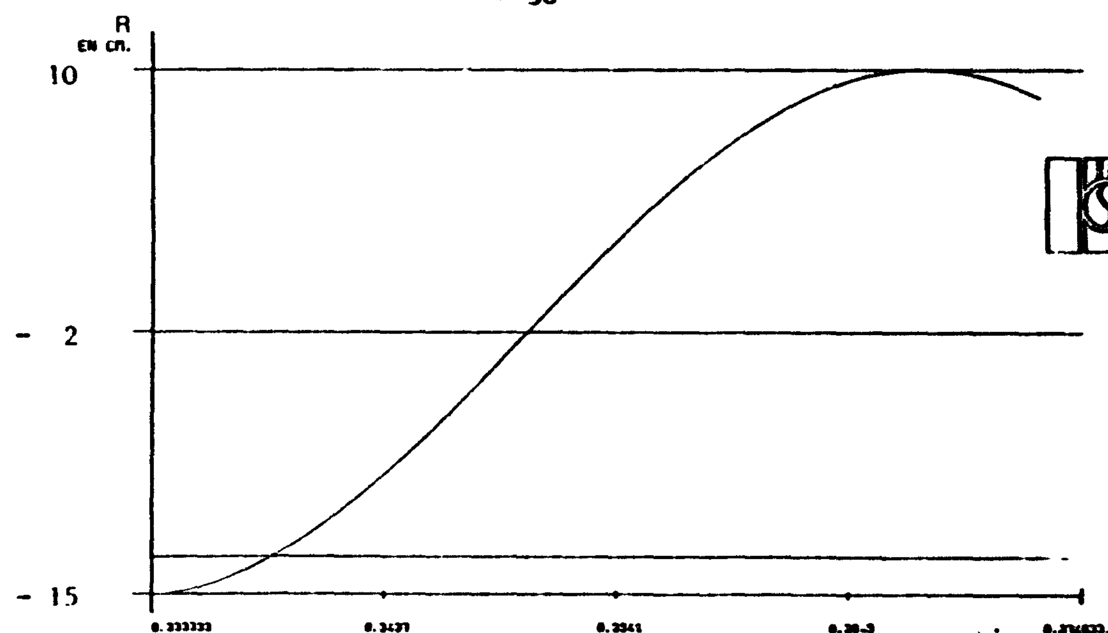
-39- . NOISE IN THE STATION COORDINATES

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation,
- orbit tabulated in an imperfect case,
  i.e., satellite at 850 km
-*initial marameters of the orbit at
  7760.333333333
-*Tragedi in the perfect case
-*Natal station eliminated



-38-

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation,
- orbit tabulated in an imperfect case,
  i.e., satellite at 850 km
-*initial parameters of the orbit at
  7760.333333333
-*station coordinates with noise 0.2M (fixed)
-*Natal station eliminated



-39-

/81

40-41-42 - SIMULATION OF PERFECT MEASUREMENTS

- ELIMINATION OF THE FOLLOWING STATIONS:
- . NATAL . CANARIES . FLORES
- TREATMENT WITH THE MODEL GEM 10B

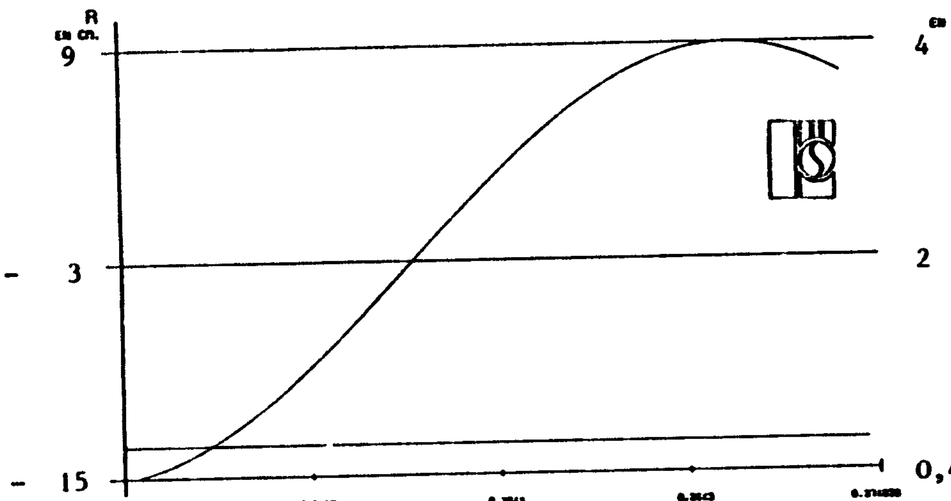
-40- . NOISE IN THE STATION COORDINATES

-41- . NOISE IN THE SOUTHERN STATION COORDINATES

. BIAS IN THE SOUTHERN STATION COORDINATES

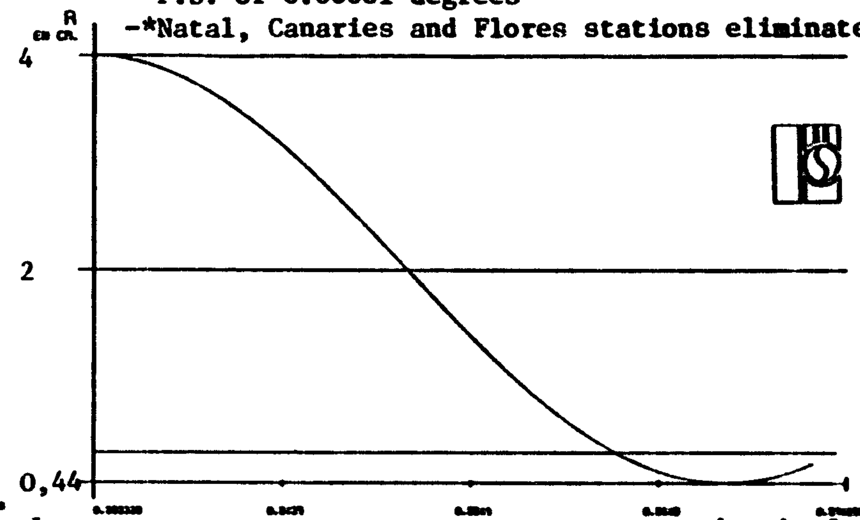
-42- . BIAS IN THE SOUTHERN STATION COORDINATES

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 850 km
-*initial parameters of the orbit at
  7760.333333333
-*station coordinate with nosie 0.2 (fixed
-*Natal, Canaries and Flores stations eliminated



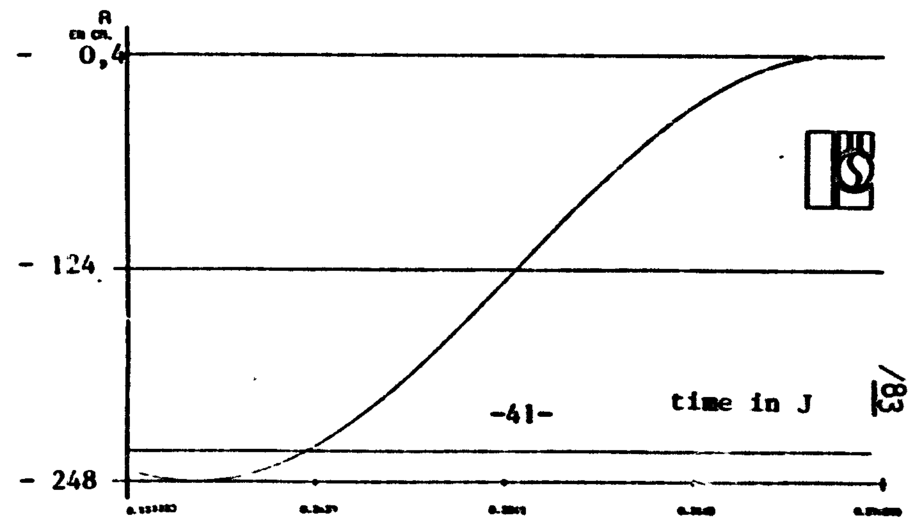
-40-

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 850 km
-*initial parameters of the orbit at
  7760.333333333
-*systematic air in the stations of the
  P.S. of-0.00001 degrees
-*Natal, Canaries and Flores stations eliminated



-42-

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation,
- orbit tabulated in an imperfect case,
  i.e., satellite at 850 km
-*initial parameters of the orbit at
  7760.333333333
-*station coordinate with noise 0.2m (fixed)
-*systematic air in the stations of the P.S.
  of -0.00001 degrees
-*Natal, Canaries and Flores stations eliminated



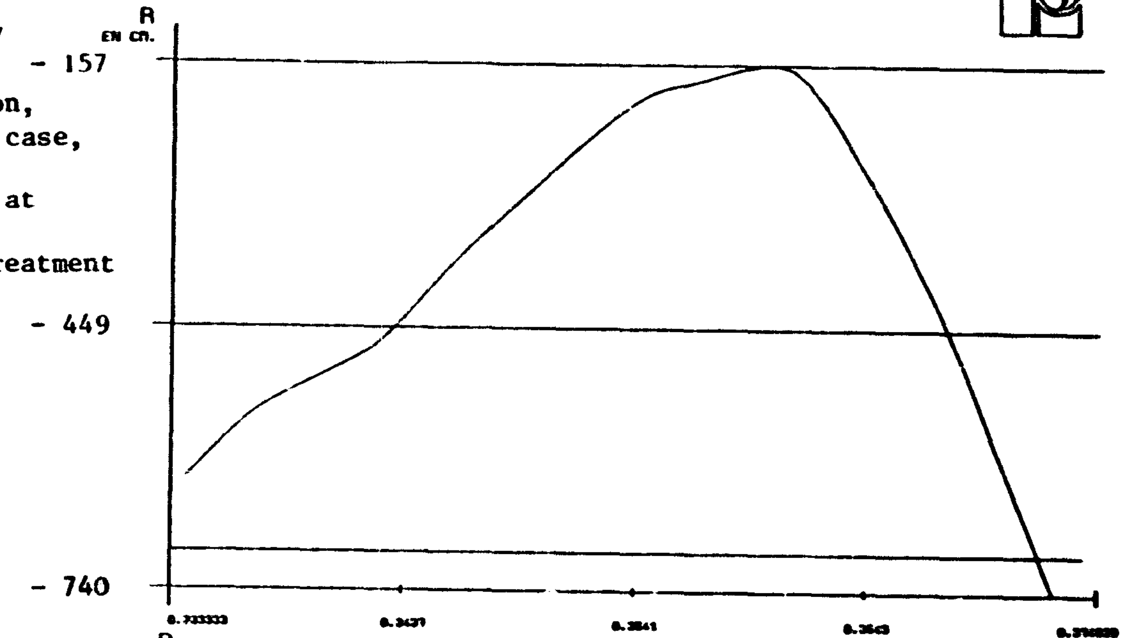
-41-

/83

43-44 - SIMULATION OF PERFECT MEASUREMENTS
- TREATMENT WITH THE MODEL GEM 8

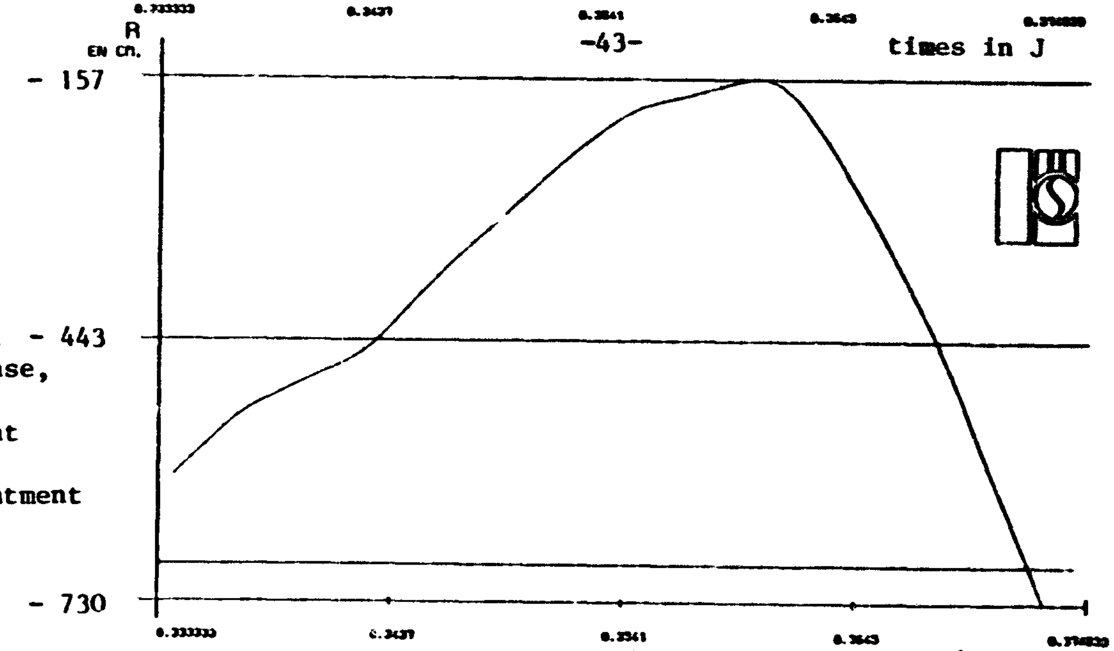
-43- . ELIMINATION OF THE NATAL STATION MEASUREMENTS

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation,
- orbit tabulated in an imperfect case,
  i.e., satellite at 850 km
-*initial parameters of the orbit at
  7760,333333333
-*Tragedi in the perfect case * treatment
  with GEM8
-*Natal station eliminated



-43-

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation,
- orbit tabulated in an imperfect case,
  i.e., satellite at 850 km
- *initial parameters of the orbit at
  7760.333333333
-*Tragedi in the perfect case * treatment
  with GEM8



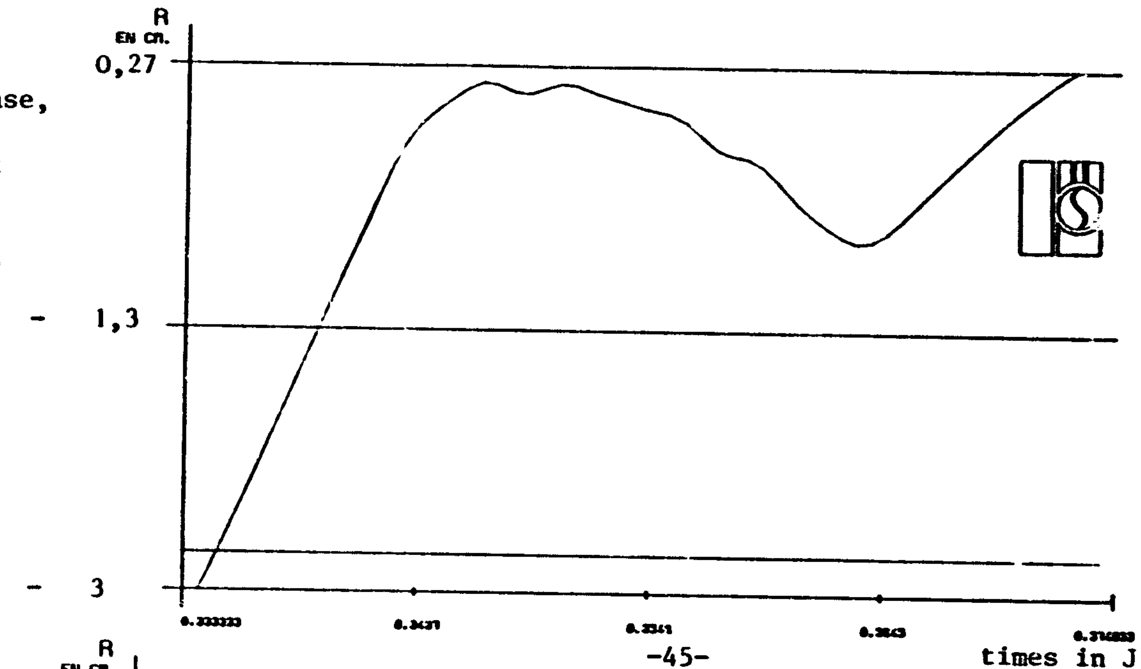
-44-

/85

45-46 - SIMULATION OF PERFECT MEASUREMENTS
- ELIMINATION OF STATION MEASUREMENTS:
NATAL CANARIES FLORES

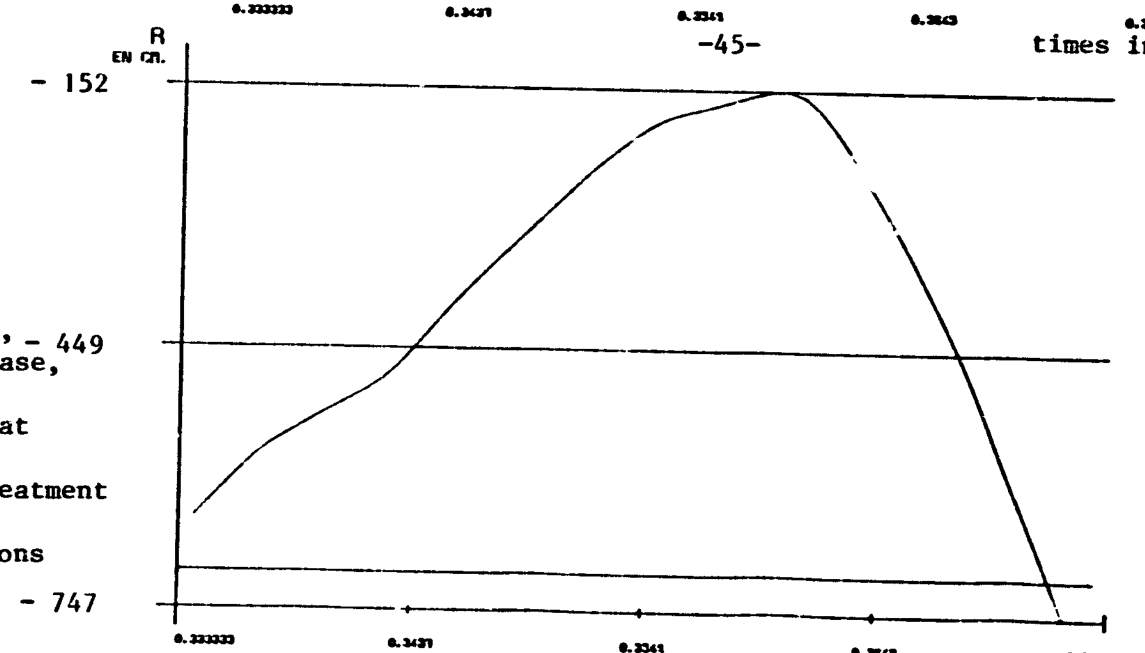
-45- . TREATMENT WITH THE MODEL GEM 10B
-46- . TREATMENT WITH THE MODEL GEM 8

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 850 km
-*initial parameters of the orbit at
  7760.333333333
-*Tragedi in the perfect case
-*Natal, Canaries and Flores stations
  eliminated



-45-

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation,
- orbit tabulated in an imperfect case,
  i.e., satellite at 850 km
- *initial parameters of the orbit at
  7760.333333333
- *Tragedi in the perfect case * treatment
  with GEM8
- *Natal, Canaries and Flores stations
  eliminated



-46-

/87

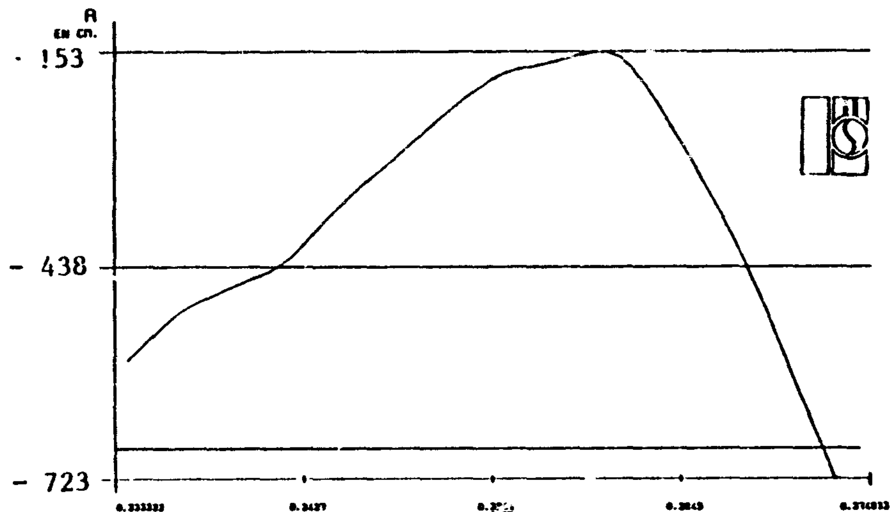
47-48-49 - SIMULATION OF PERFECT MEASUREMENTS
- TREATMENT WITH THE MODEL GEM 8

-47- . DISTANCE MEASUREMENTS ONLY
-48- . VELOCITY MEASUREMENTS ONLY

Distance difference (Earth center/ satellite)between
- orbit tabulated during simulation,
- orbit tabulated in an imperfect case, i.e., satellite at 850 km. Distance measurements only.

-*initial parameters of the orbit at 7760.333333333

-*Tragedi in the perfect case * treatment with GEM 8



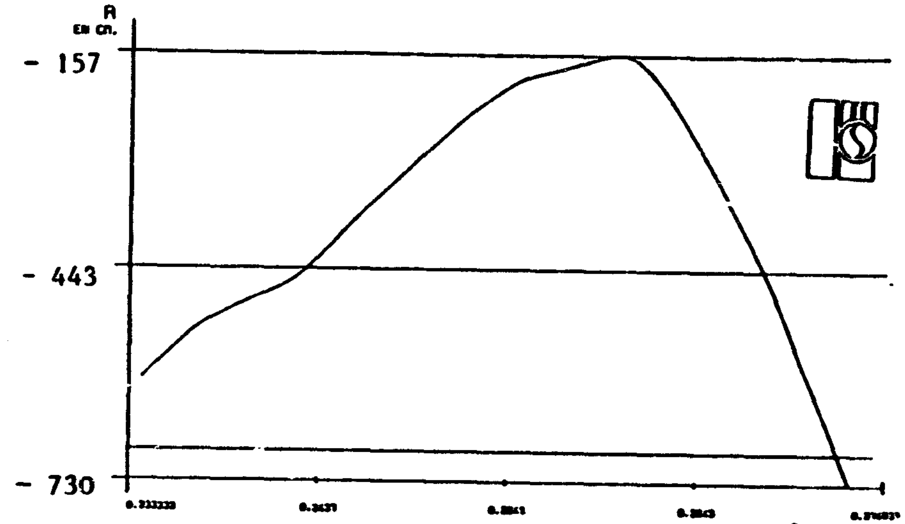
-47-

time in J

Distance difference (Earth center/ satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 850 km.

-*Initial parameters of the orbit at7760.333333333

-*Tragedi in the perfect case * treatment with GEM 8



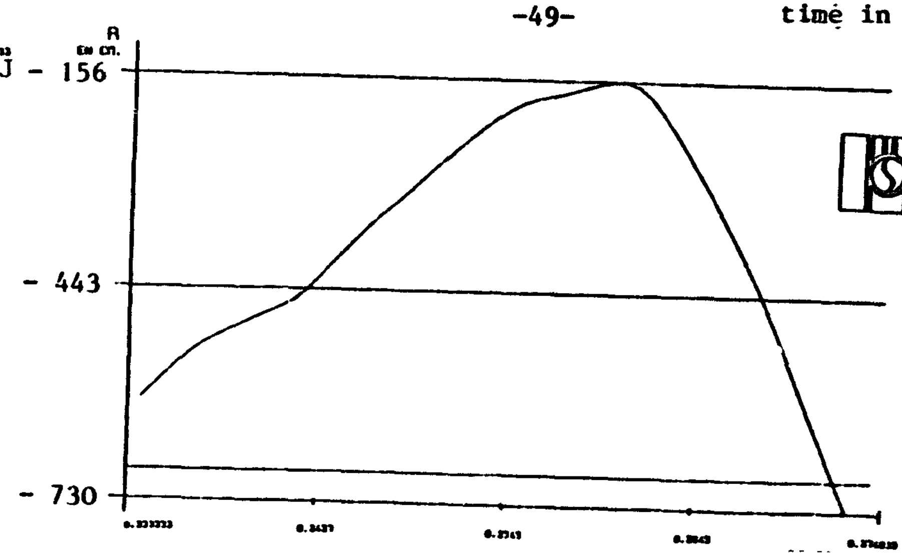
-49-

time in J

Distance difference (Earth center/ satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 850 km. Velocity measurements only.

-*initial parameters of the orbit at 7760.333333333

-*Tragedi in the perfect case * treatment with GEM 8



-48-

time in J

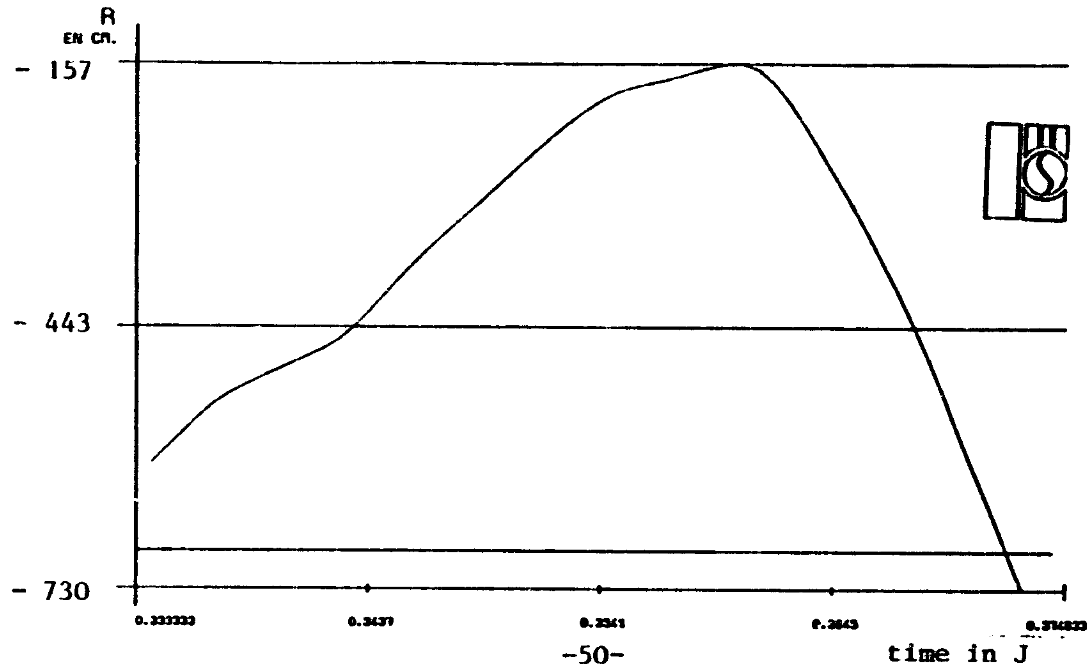
50-51 - SIMULATION OF PERFECT MEASUREMENTS

-50- . TREATMENT WITH THE MODEL GEM 8

-51- . TREATMENT WITH THE MODEL GEM 8, BUT THIS MODEL TAKES INTO ACCOUNT THE CONSTANTS OF MODEL GEM 10B.

The potential models are determined in a global fashion from studying the satellite trajectory, except for GM sometimes which comes from planetary probe studies. This is also true for the illipsoid which is used as a reference for the station coordinates. One could imagine that this disparity between GEM 10B which was used to simulate the measurements and GEM 8 with which processing was performed would be the cause of this nonregular form (curve 50). Therefore, we gave the same numerical values as GEM 10B to the fundamental constants of GEM 8 (curve 51).

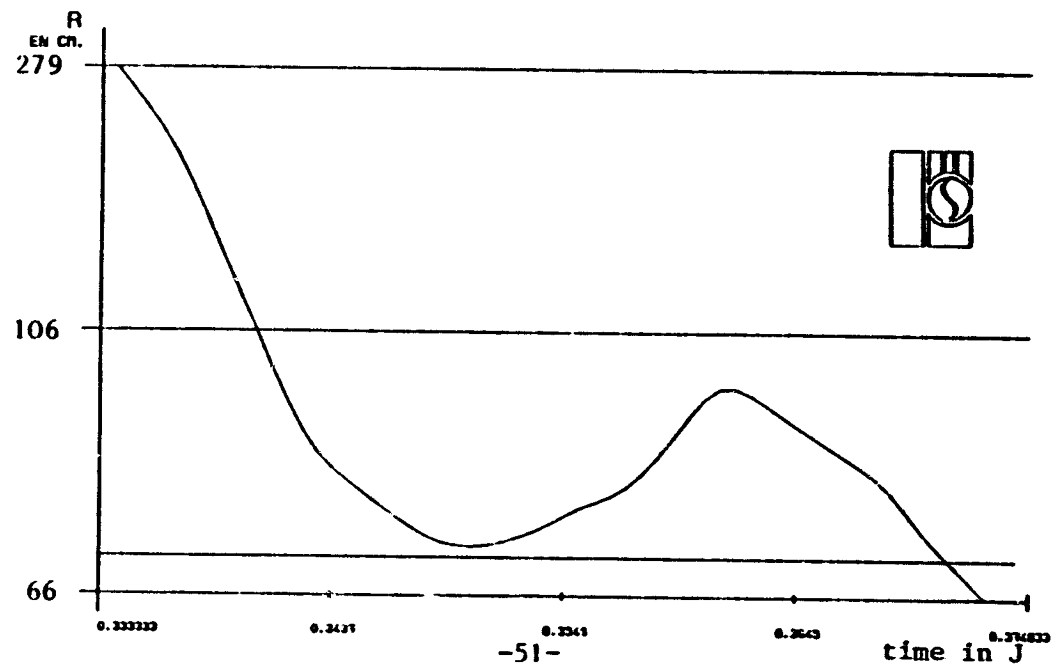
Distance difference (Earth center/ satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 850 km
-* initial paramet.rs of the orbit at 7760.333333333
-* Tragedi in the perfect case * treatment with GEM 8



-50-

Distance difference (Earth center/ satellite)between
-orbit tabulated during simulation
-orbit tabulated in an imperfect case, i.e., satellite at 850 km
-* initial parameters of the orbit at 7760.333333333
-* Tragedi in the perfect case * treatment with GEM 8 (constants of GEM 10B)

/91



-51-

52-53

-52- . SIMULATION OF PERFECT MEASUREMENTS
. SIMULATION OF PERFECT MEASUREMENTS TAKING FRICTION INTO ACCOUNT
COMPARISON OF TWO RADIUS VECTORS

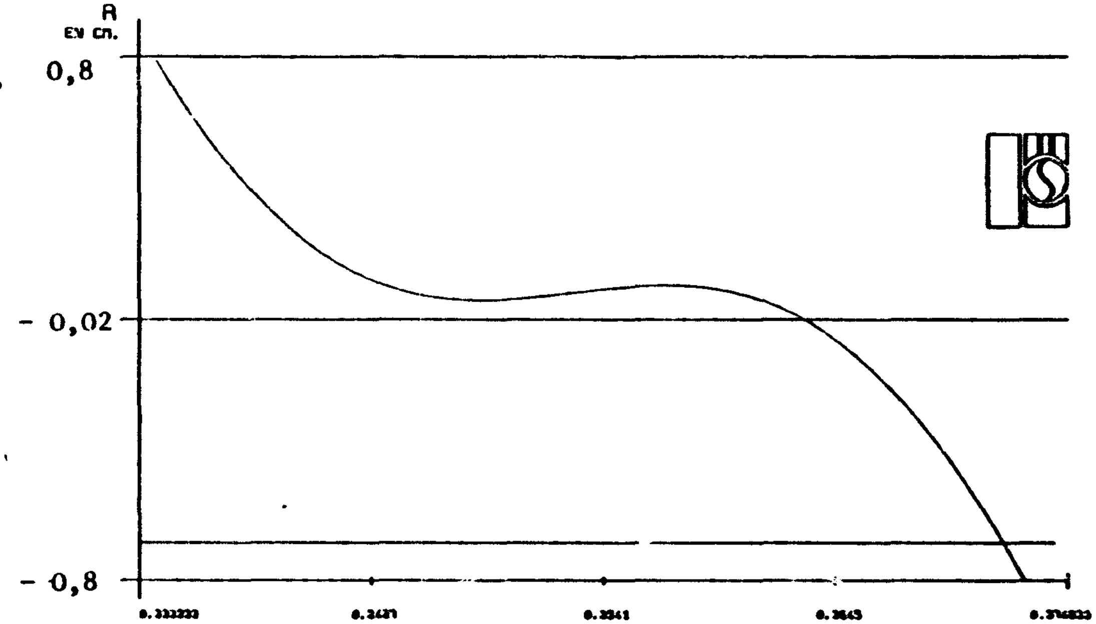
-53- . SIMULATION OF PERFECT MEASUREMENTS TAKING INTO ACCOUNT
. TREATMENT WITH THE MODEL GEM 10B, TAKING INTO ACCOUNT THE FRICTION MODEL

In the HASP method, we find that the nongravitational accelerations are negligible compared with the effects of the Earth's potential. This point was tested with atmospheric friction.

Curve 53 shows the difference between the ρ calculated without atmospheric friction and ρ calculated with friction. The global effect varies between 7 and 9 centimeters over the arc length.

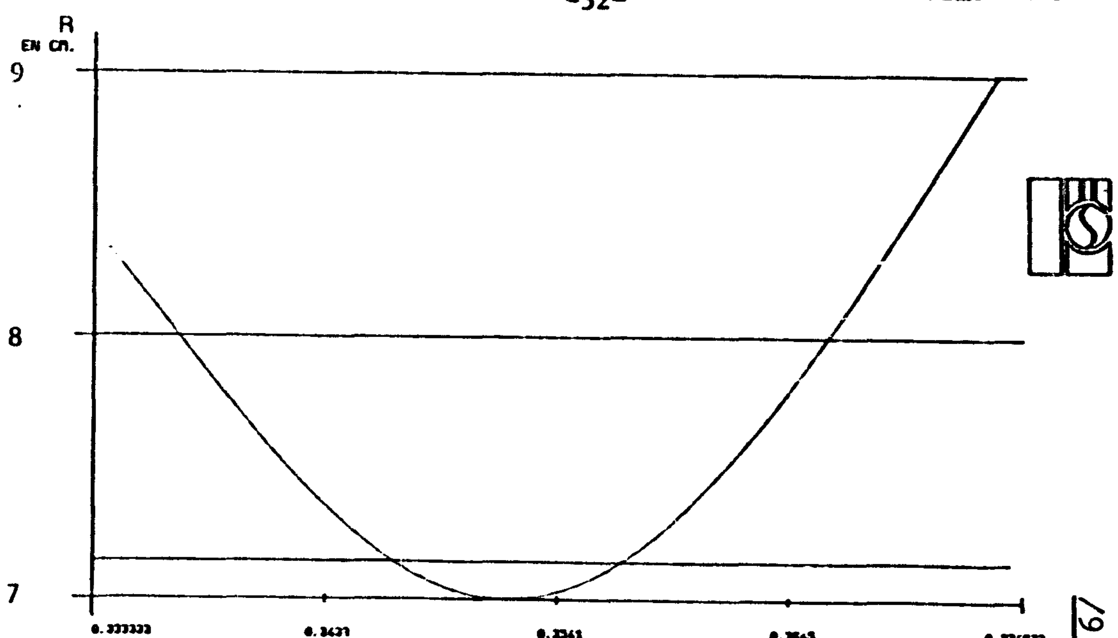
In curve 52, one can see that this effect can be absorbed by a coefficient estimated for the past itself, and, therefore, the residual effect is less than 1 centimeter.

Distance difference (Earth center/ satellite) between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 850 km
-*Sigedi friction/Tragedi friction



-52-

Distance difference (Earth center/ satellite) between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 850 km
-* Sigedi perfect/Sigedi friction



-53-

/93

54-55-56 - SIMULATION OF PERFECT MEASUREMENTS
- TREATMENT WITH THE MODEL GEM 8

-54- . TABULATION OF THE ORBIT DURING THE FIRST TREATMENT ITERATION

-55- . TABULATION OF THE ORBIT DURING THE FIRST TREATMENT ITERATION

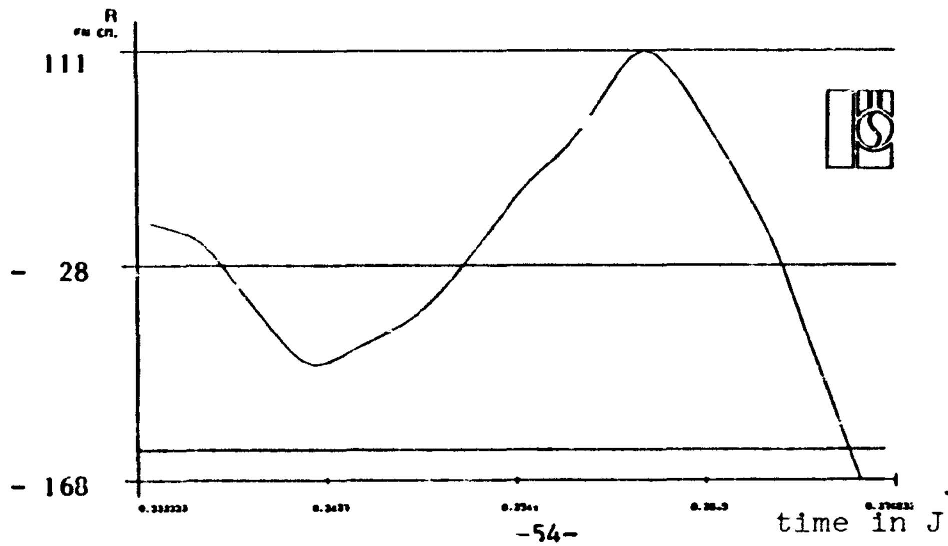
. MODEL GEM 8 TAKES INTO ACCOUNT THE CONSTANTS OF MODEL GEM 10B

-56- . MODEL GEM 8 TAKES INTO ACCOUNT THE CONSTANTS OF THE MODEL GEM 10B

When one tabulates the orbit after the first iteration, we can directly see the effect of changing the potential of the Earth center-satellite vector (curves 54 or 55).

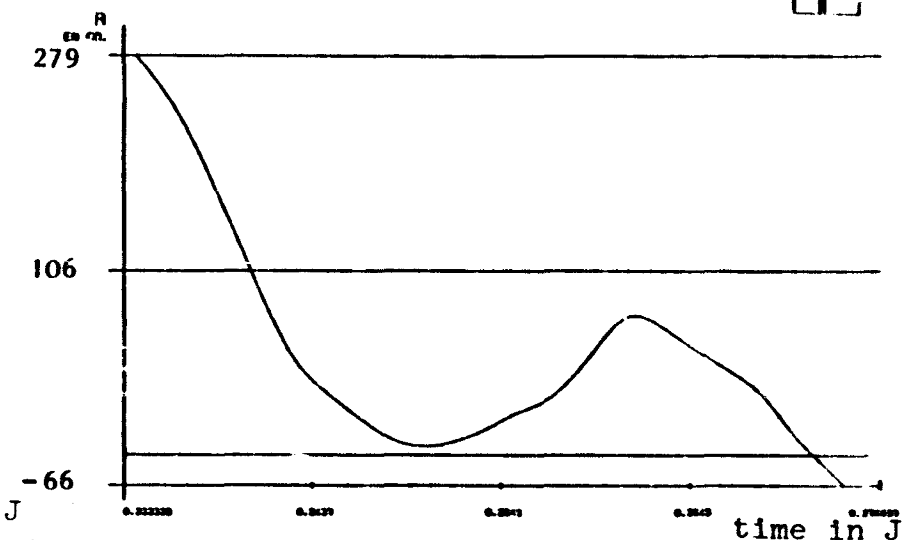
These effects cannot be absorbed by an adjustment of the initial orbital conditions: Comparison between curves 55 and 56.

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 850 km
- Tragedi in the perfect case * treatment
  with GEM8
- orbit at the 1st iteration



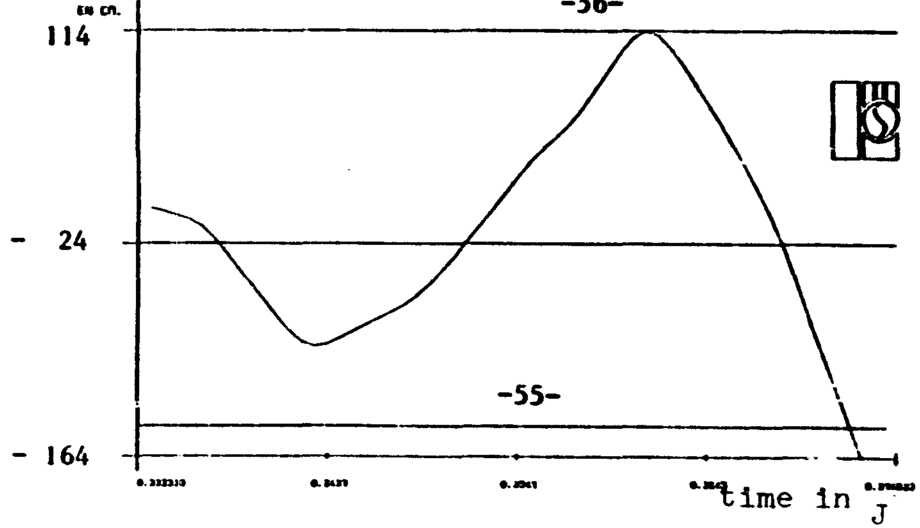
-54-

Distance difference (Earth center/
satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 850 km
-*initial parameters of the orbit at
  7760.333333333
-*Tragedi in the perfect case * treatment with
  GEM 8(Constants of GEM10B)



-56-

Distance difference (Earth center
satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case,
  i.e., satellite at 850 km
-*Tragedi in the perfect case * treatment
  with GEM8 (constants of GEM10B)
- orbit after 1st iteration



-55-

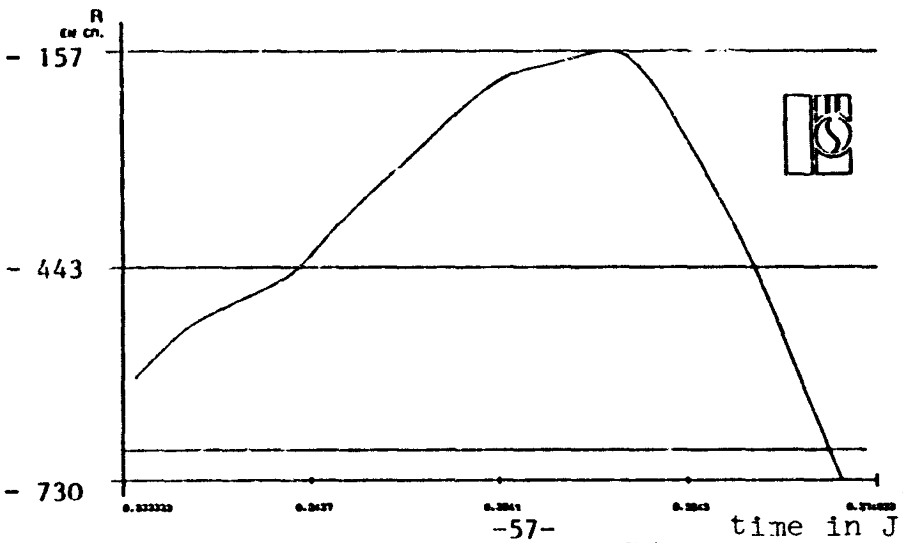
/95

57-58-59-60

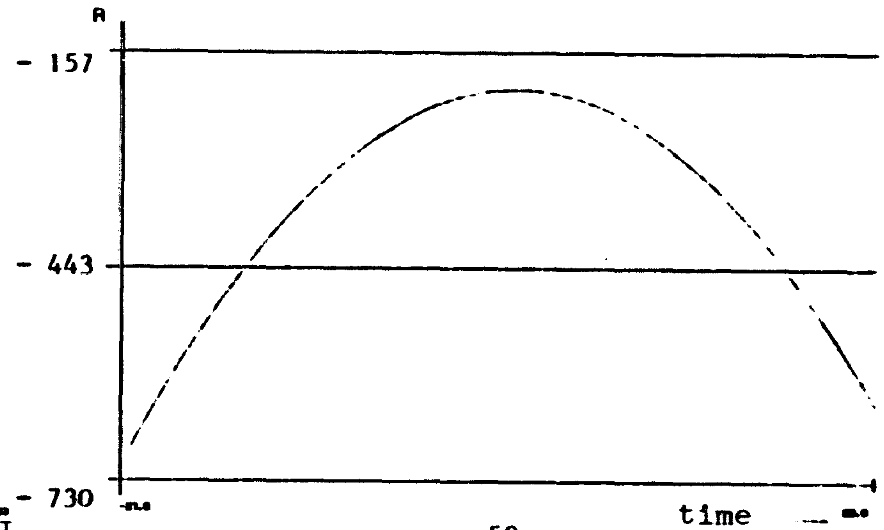
- -57- . SIMULATION OF PERFECT MEASUREMENTS
  - . TREATMENT WITH THE MODEL GEM 8
- -58- . SIMULATION OF PERFECT MEASUREMENTS
  - . TREATMENT WITH THE MODEL GEM 8
  - . TABULATION OF THE ORBIT AFTER FIRST PROCESSING ITERATION
- -59- . SMOOTHING OF CURVE 57
- -60- . RESIDUES BETWEEN CURVES 57 and 59

Between 57 (after adjustment of tne processing orbit) and 58 (only effect of changing the Earth potential), we can see the inefficiency of the algorithms collected. One could conclude that the effects found are from the processing and not from the potential. This is why we smooth curve 57 by curve 59 and the differences are plotted in figure 60. This shows the short period perturbations due to the Earth potential.

Distance difference (Earth center/satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 850 km
-*Tragedi in the perfect case * treatment with GEM 8

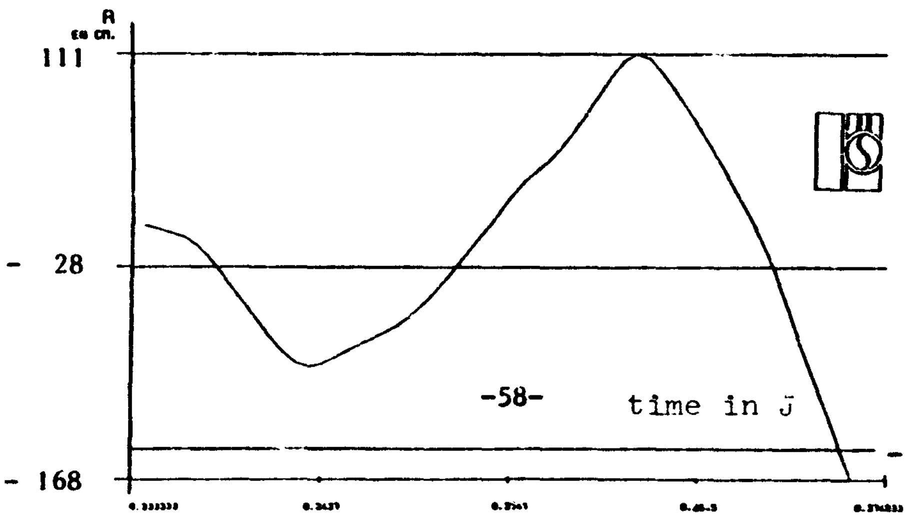


-57-

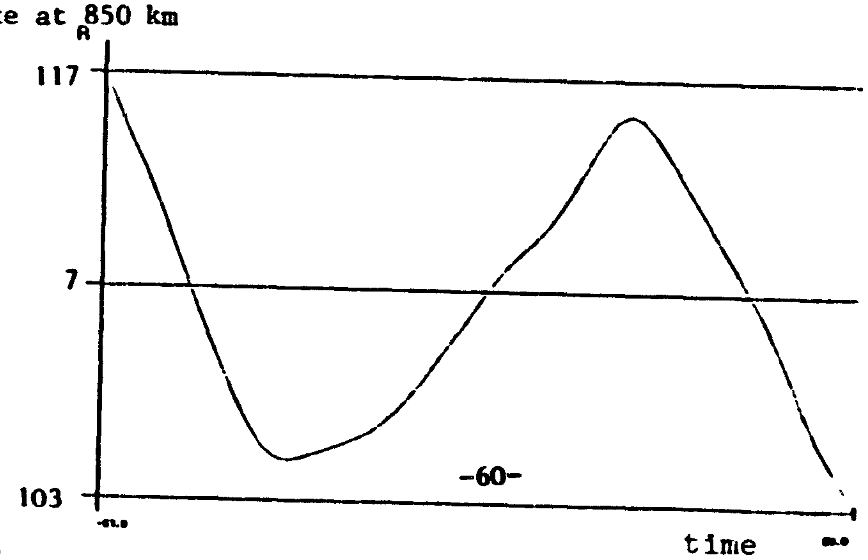


-59-

Distance difference (Earth center/satellite)between
-orbit tabuiated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 850 km



-*Tragedi in the perfect case* treatment with GEM 8
- orbit at the first iteration



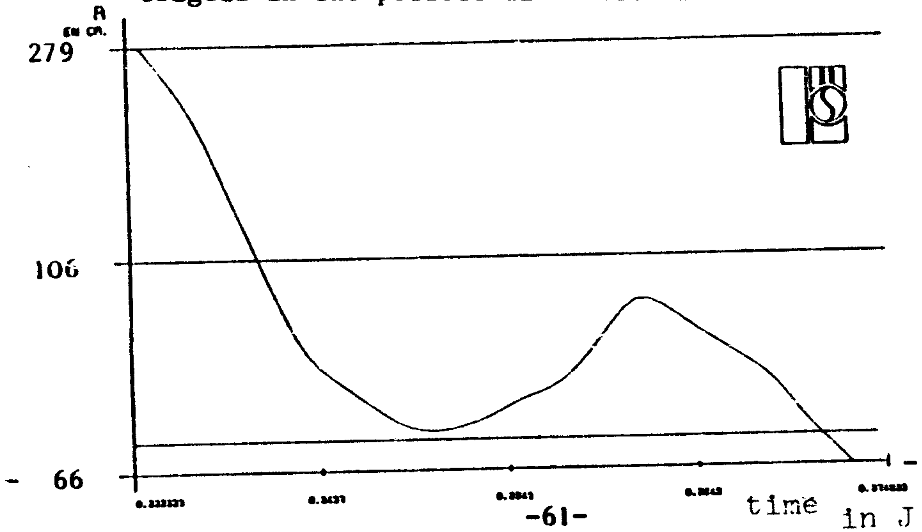
/97

61-62-63-64

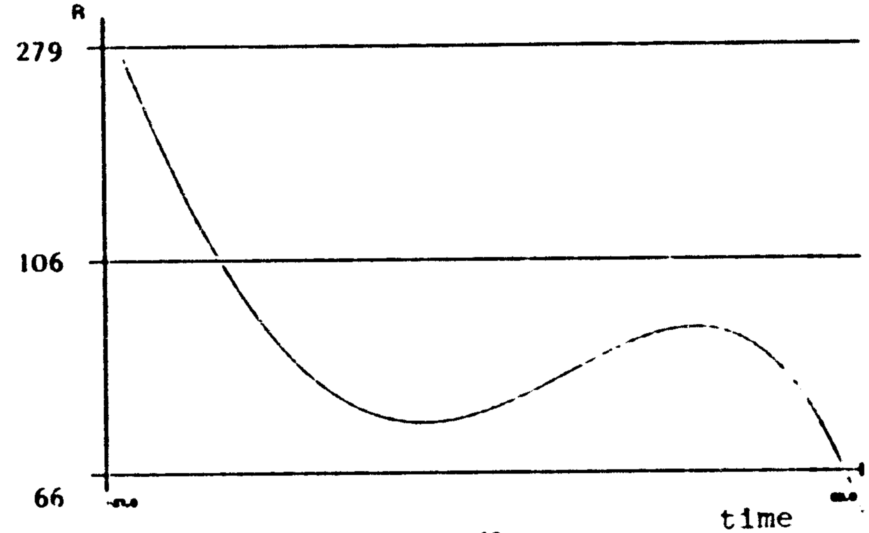
-61-62-	- SIMULATION OF PERFECT MEASUREMENTS	
	- TREATMENT WITH THE MODEL GEM 8, MODEL CONSIDERING THE CONSTANTS OF GEM 10B	
-62-	- TABULATION OF THE ORBIT AFTER FIRST TREATMENT ITERATION	
-63-	- SMOOTHING OF CURVE 61	
- 64-	- RESIDUES BETWEEN CURVES 61 and 63	

Results are similar to the preceding case with a simple change of the potential. Here, again the short period perturbations are quite evident in curve 64.

Distance difference (Earth center/satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 850 km
-*Tragedi in the perfect case *treatment with GEM 8 (constants of GEM 10B)

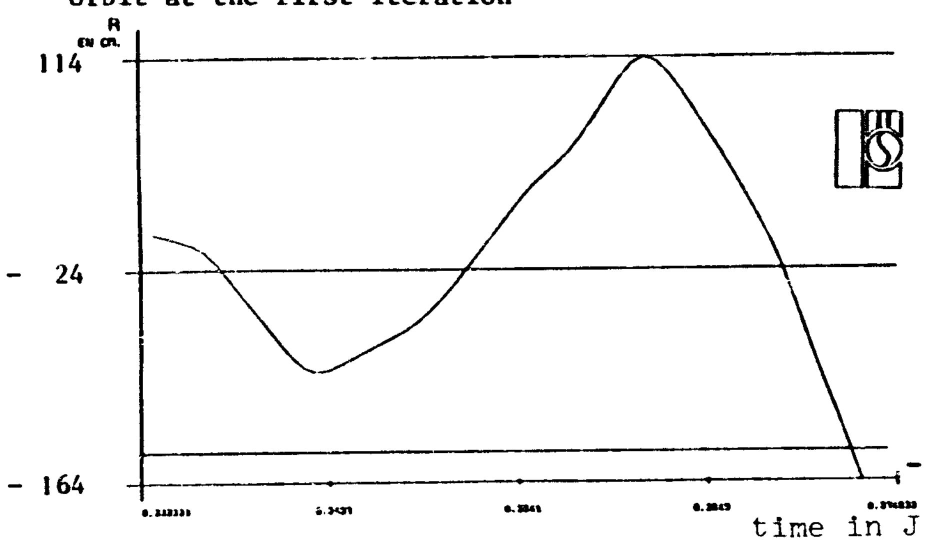


-61-

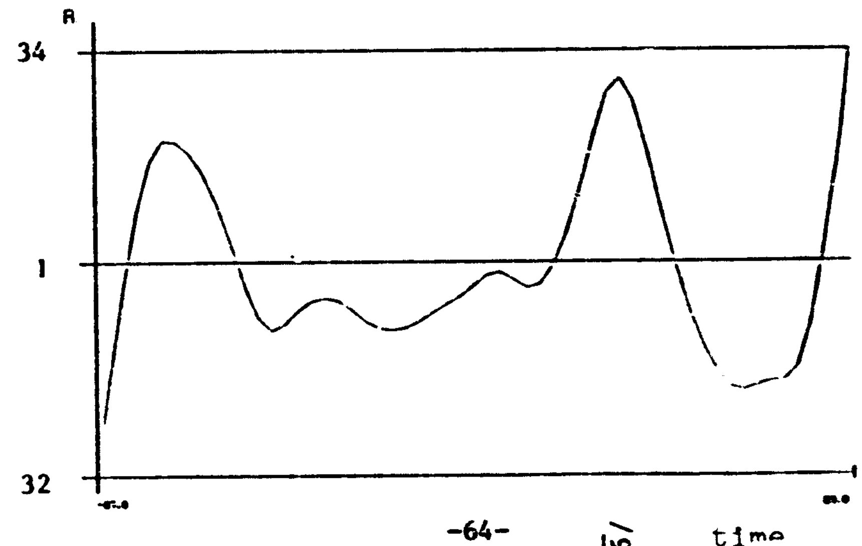


-63-

Distance difference (Earth center/satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 850 km
-*Tragedi in the perfect case * treatment with GEM 8 (constants of GEM 10B)
-orbit at the first iteration



-62-



-64-

65-66-67-68 - SIMULATION OF PERFECT MEASUREMENTS AND BIAS IN THESE MEASUREMENTS
- TREATMENT WITH THE MODEL GEM 10B
- NOISE IN STATION COORDINATES OF 2 CM
- BIAS IN THE SOUTHERN STATION COORDINATES OF $1.10^{-5} * \cos(\phi)$; ϕ = LATITUDE

-65- . 6 NORTHERN STATIONS,
6 SOUTHERN STATIONS,
3 CENTER STATIONS

-66- . 4 NORTHERN STATIONS,
4 SOUTHERN STATIONS,
3 CENTER STATIONS

-67- . 6 NORTHERN STATIONS,
6 SOUTHERN STATIONS,

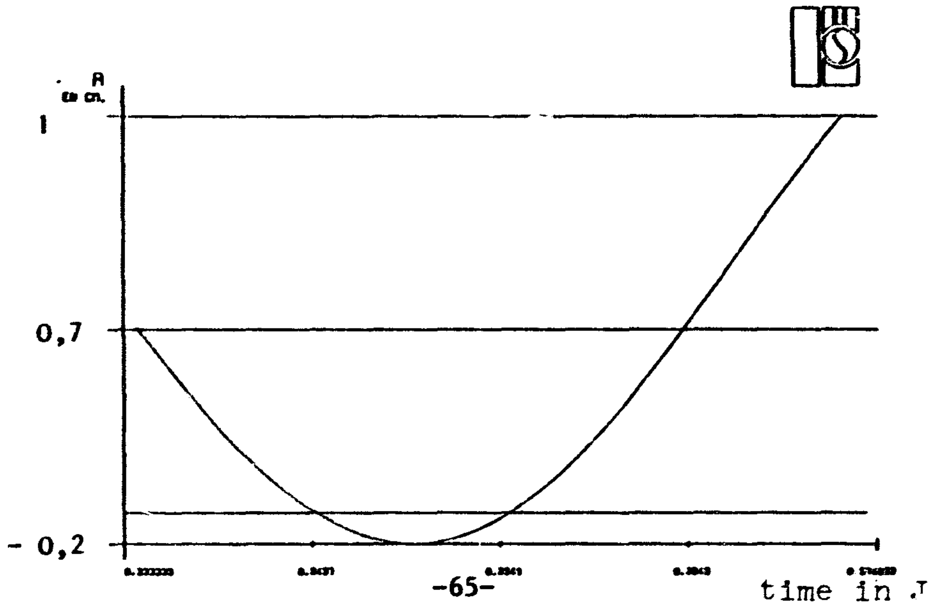
-68- . 4 NORTHERN STATIONS,
4 SOUTHERN STATIONS

Here we show the degradation of results when one reduces the number of stations (without introducing errors in the Earth potential). The suppression of the center stations which provide complete coverage of the trajectory arc has a substantial effect.

Also note in this case that we introduced a bias in the distance measurement and the Doppler measurements.

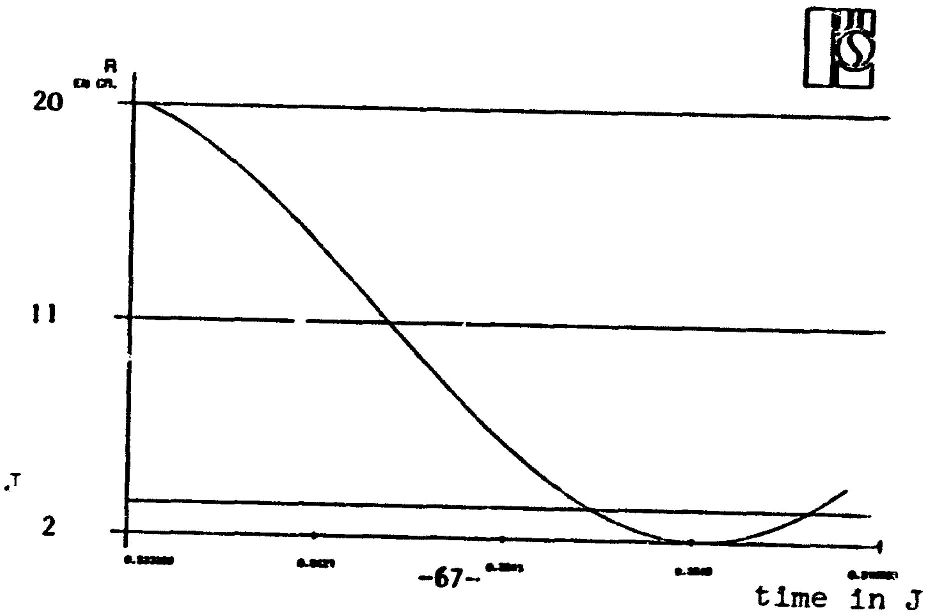
C - 2

Distance difference (Earth center/ satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 850 km
-*Measurements of Sigedi with noise but bias in the measurements
-*Coordinates of stations with noise 0.02 m (fixed)
-*systematic air in the stations of the P.S. of -0.00001 degrees



-65-

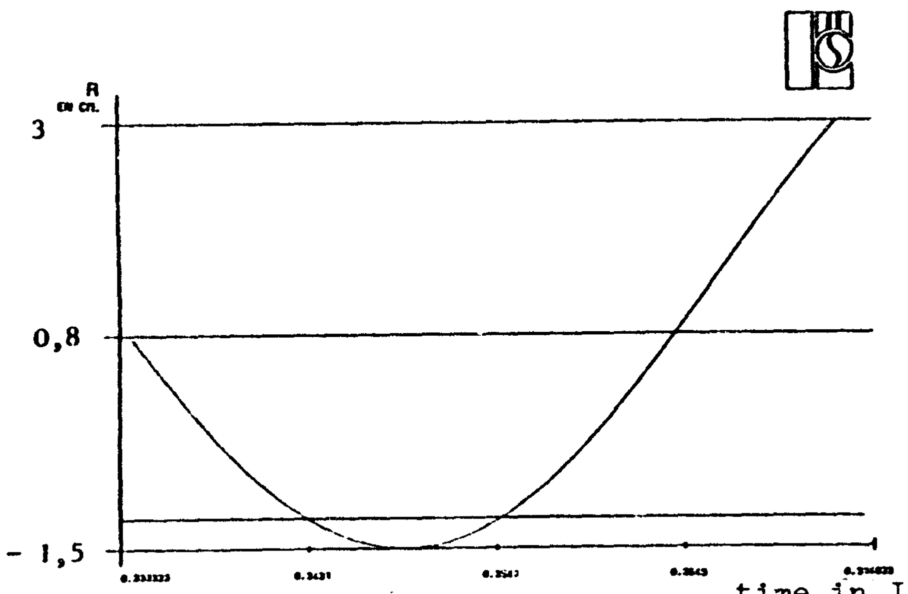
Distance difference (Earth center/ satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 850 km
-*Measurements of Sigedi with noise but bias in the measurements
-*coordinates of stations with noise 0.02 m (fixed)
-*systematic air in the stations of the P.S. of -0.00001 degrees
- 6 stations in the north
  6 stations in the south



-67-

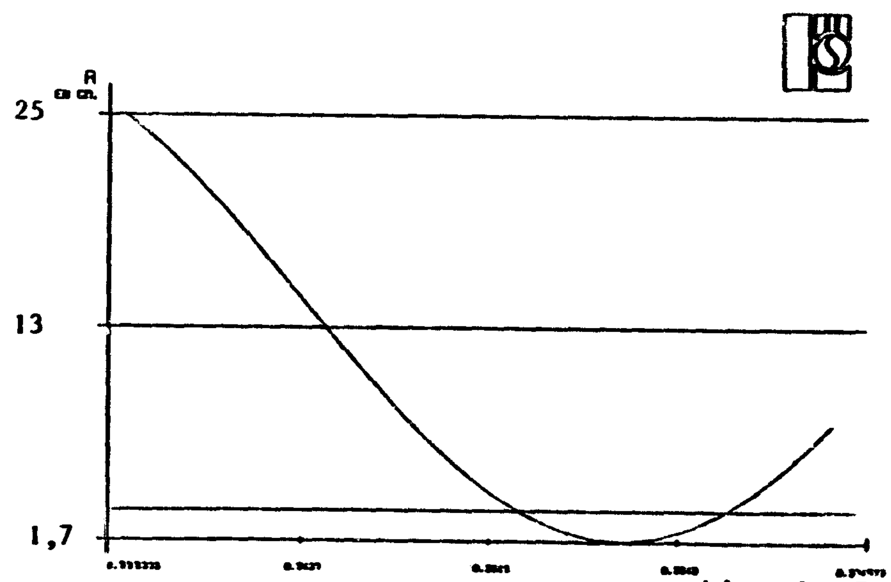
/101

Distance difference (Earth center/ satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 850 km
-*measurements of Sigedi with noise but bias in the measurements
-*coordinates of stations with noise 0.02m (fixed)
-*systematic air in the stations of the P.S. of -0.00001 degrees
  4 stations in the north, 4 stations in the south, 3 stations in the center



-66-

Distance difference (Earth center/ satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 850 km
-*measurements of Sigedi with noise but bias in the measurements
-*coordinates of stations with noise 0.02m (fixed)
-*systematic air in the stations of the P.S. of -0.00001 degrees
  4 stations in the north 4 stations in the south



-68-

COMPARISON OF THE INFLUENCE OF THE POTENTIAL BETWEEN ALTITUDES 650 KM AND 850 KM

We find no substantial difference between the two cases, except for the general appearance of each curve.

On the other hand, the type of measurements utilized, i.e.,

- Doppler and distance
- distance alone
- Doppler alone

does not play any important role at this level.

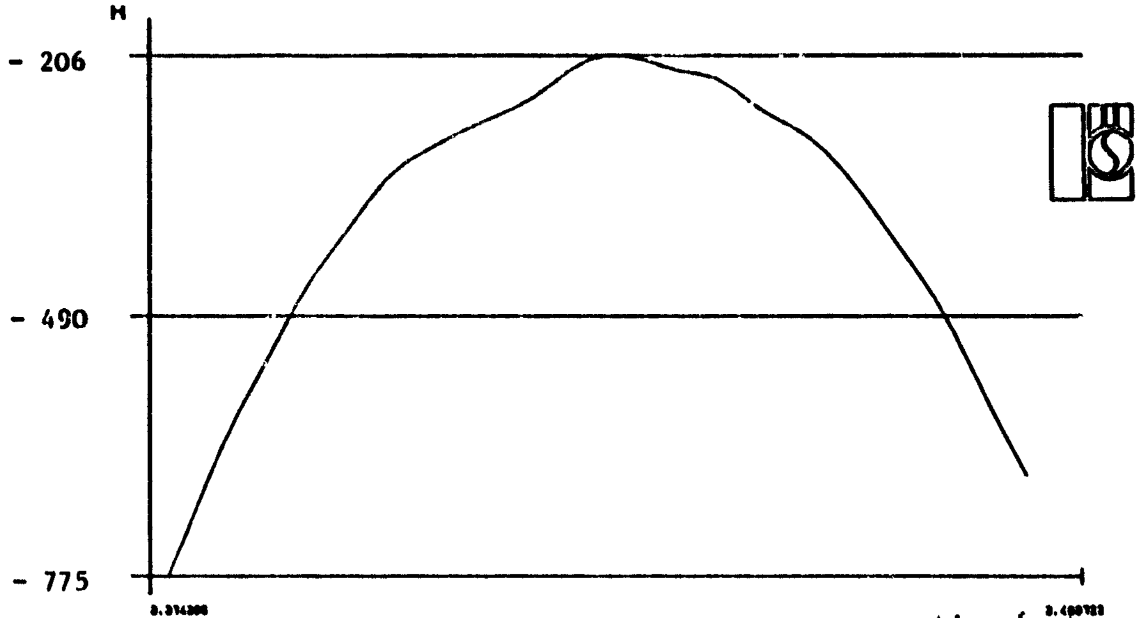
69-70 - SIMULATION OF PERFECT MEASUREMENTS

- TREATMENT WITH THE MODEL GEM 8

-69- . 650 KM

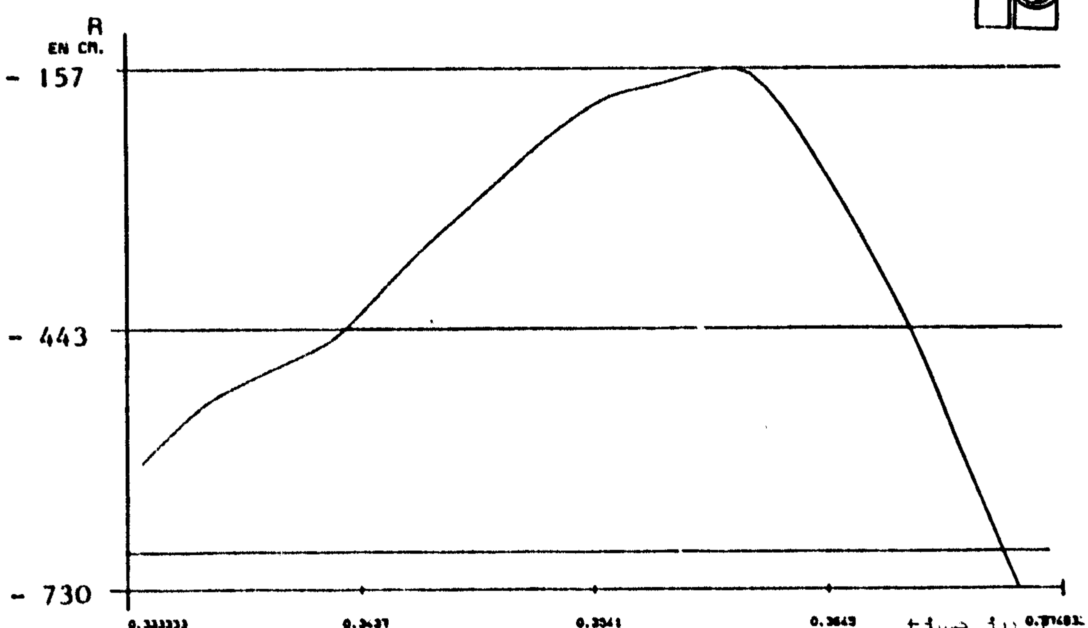
-70- . 850 KM

Distance difference (Earth center/satellite) between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 650 km
-*initial parameters of the orbit 7763.374305556
-*Tragedi in the perfect case *treatment with GEM8



-69-

Distance difference (Earth center/satellite) between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 850 km
-*initial parameters of the orbit 7760.333333333
-*Tragedi in the perfect case *treatment with GEM8



-70-

71-72 - SIMULATION OF PERFECT MEASUREMENTS

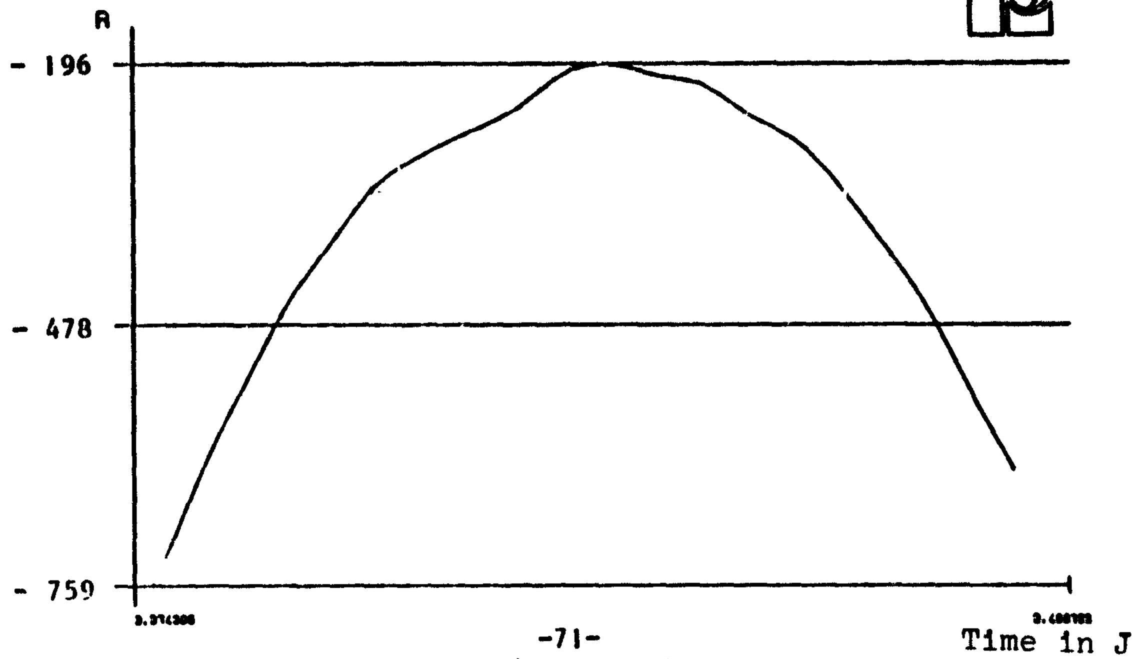
- TREATMEN. WITH THE MODEL GEM 8
- ONLY DISTANCE MEASUREMENTS

-71- . 650 KM

-72- . 850 KM

Distance difference (Earth center/satellite)between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 650 km distance measurements only

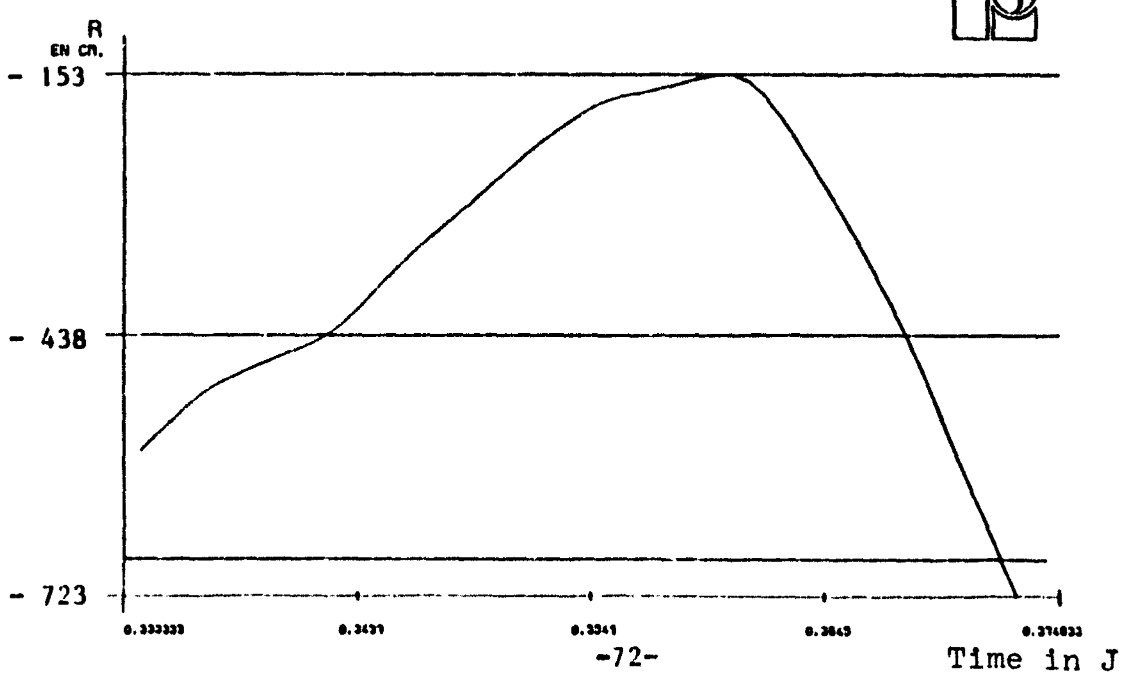
-*initial parameters of the orbit 7763.374305556
-*Tragedi in the perfect case *treatment with GEM 8



-71-

Distance difference (Earth center/satellite) between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 850 km distance measurements only

-*initial parameters of the orbit 7760.333333333
-*Tragedi in the perfect case *treatment with GEM 8



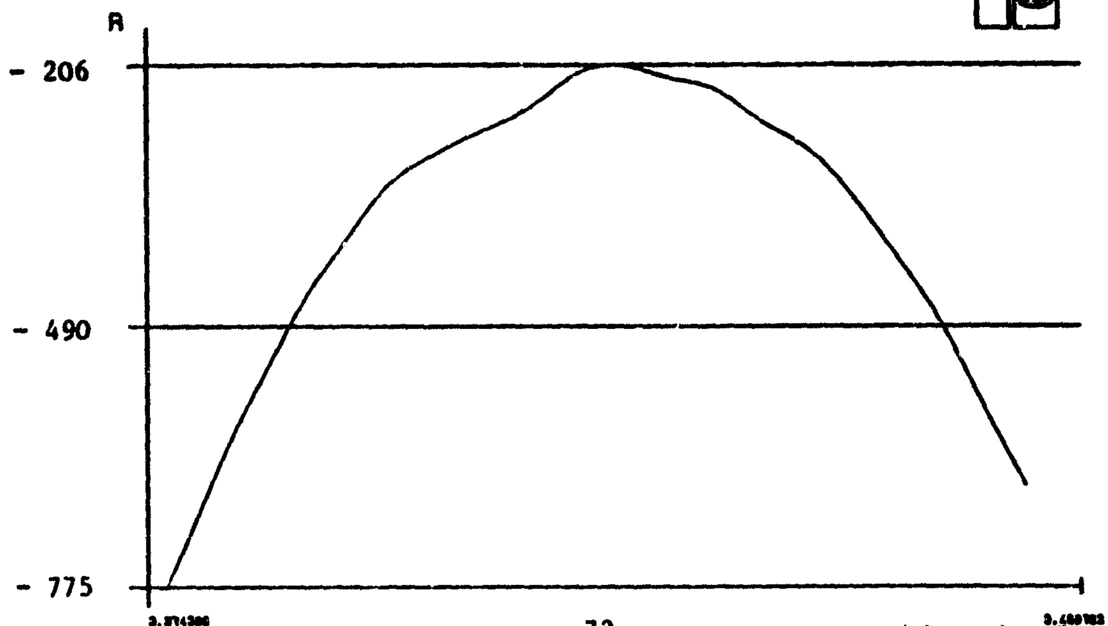
-72-

73-74 - SIMULATION OF PERFECT MEASUREMENTS
- TREATMENT WITH THE MODEL GEM 8

-73- . 650 KM

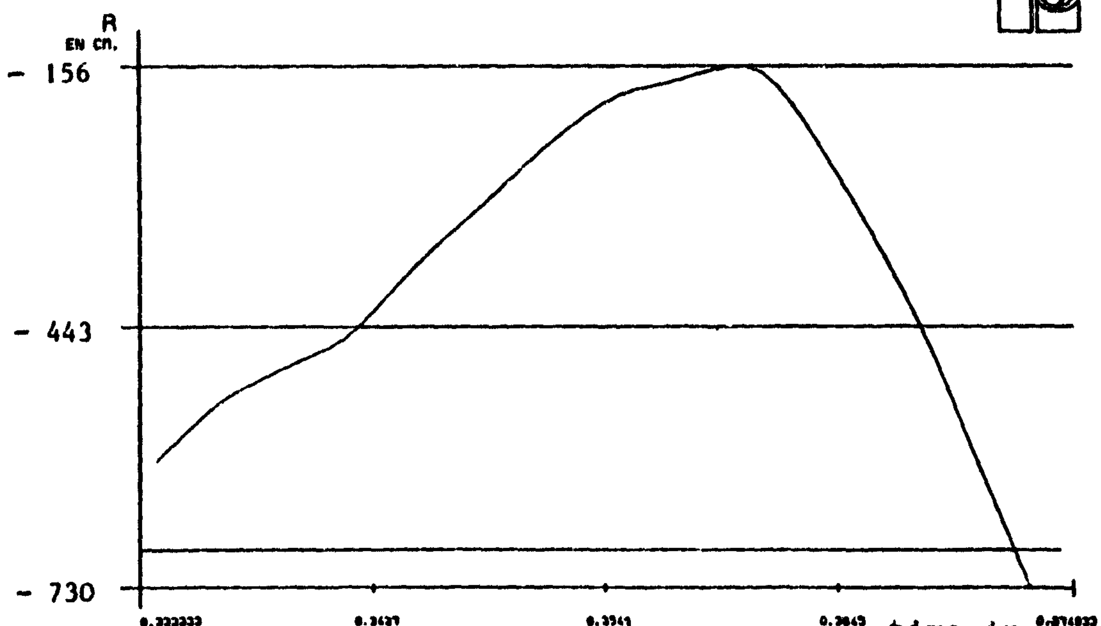
-74- . 850 KM

Distance difference (Earth center/satellite) between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 650 km
  velocity measurements only
-*initial parameters of the orbit at 7763.374305556
-*Tragedi in the perfect case *treatment with GEM 8



-73-

Distance difference (Earth center/satellite) between
- orbit tabulated during simulation
- orbit tabulated in an imperfect case, i.e., satellite at 850 km
  velocity measurements only
-*Initial parameters of the orbit at 7760.333333333
-*Tragedi in the perfect case *treatment with GEM 8



-74-

C O N C L U S I O N

/110

The experience acquired in precise trajectory determination allows the formulation of several ideas for a position determination system at the level of a few centimeters.

With the reservation of technical feasibility which has not been confirmed, but which nevertheless seems possible, it is necessary to define a collection of equipment for achieving this object. For example, it is necessary not only to understand the instrument, but also the calculation methods must remain compatible with the instrument accuracy.

The tracking system includes distance and Doppler measurements both ways. The measurements are made on board the satellite and have only a single time and frequency reference for the entire tracking complex. This facilitates the correction and transmission of measurements to the computation center. This is a concept which allows operations with almost complete certainty for any atmospheric conditions.

From the point of view of calculation methods, it is not possible to determine an orbit with an accuracy of a few centimeters with a period of several months, using classical techniques. This involves a global reference system for all of the stations and an inertial reference system which has to be related to the former. This, of course, leads to the development of a method based on steadying short arcs (on the order of 1/2 a revolution) in which the entire satellite trajectory and the station network is determined with respect to one another. The station network can be defined intrinsically in an independent manner, but will be the only reference with respect to which the orbital arcs are calculated.

/111

Since there is no force model which at the present time allows positioning of the satellite with an accuracy of a few centimeters and on the other hand, it seems rather unlikely that a global potential model, for example, using spherical harmonics, could take into account perturbations of a few centimeters, we decided to determine the trajectory arc in a geometric fashion. This leads to an increase in the number of observation stations: This is why they are to be designed as simple as possible.

This geometric solution can be simplified because the present analysis shows that it is not necessary to observe the entire satellite trajectory point-by-point, but instead, part of the trajectory arc can be observed either in a partial manner, or to its end. In the latter case, the limit is on the order of 2000 to 3000 kilometers. This is only due to the short period perturbations of the Earth potential because we have shown that all of the other error sources (station position, accuracy in measurements, nongravitational forces) could not introduce short period perturbations. Thus, one can adapt to even the ocean geography.

The concept of the HASP method has been defined and has been justified in broad terms. We believe that its complete analysis has not yet been finished. If one really wishes to exploit more in depth studies, one will have to determine the arc limits to be geometrically observed. Also, algorithms which are best suited to this trajectory determination will have to be developed.

If these conditions are satisfied for an ocean or part of one, the problems for calculating the complete satellite trajectory in real time with an accuracy on the order of one or two meters will be simply resolved by adding a few stations (on the order of 10 at the most) to the network of tracking stations covering the region under study.

/112

Finally, the simulations made for two satellite altitudes 650 km and 850 km have not shown any distinct differences, except that there is a slight degradation for the lowest altitudes when there is a substantial number of observation stations. We, therefore, prefer an 850 km orbit but this selection is not determining for the success of the HASP system.

APPENDIX

FORMALISM OF KAULA

Development of the perturbation function in osculating elements. /114

We will start with the expression

$$\mathcal{R} = \frac{\mu}{r} \sum_{l=2}^{+\infty} \sum_{m=-l}^{l} \left(\frac{R}{r}\right)^{l} K_{lm} P_{lm} (\sin \phi)\, e^{im\lambda}$$

Kaula developed the following perturbation function

$$\mathcal{R} = \frac{\mu}{a} \sum_{l=2}^{+\infty} \left(\frac{R}{a}\right)^{l} \sum_{m=0}^{l} \sum_{p=0}^{l} F_{lmp} (I) \sum_{q=-\infty}^{+\infty} G_{lpq} (e)\, S_{lmpq}$$

with

$$S_{lmpq} = \begin{pmatrix} C_{lm} \\ -S_{lm} \end{pmatrix}_{l-m \text{ impair}}^{l-m \text{ pair}} \cos \psi_{lmpq} + \begin{pmatrix} S_{lm} \\ C_{lm} \end{pmatrix}_{l-m \text{ impair}}^{l-m \text{ pair}} \sin \psi_{lmpq}$$

and

$$\psi_{lmpq} = (l-2p+q)\, M + (l-2p)\, \omega + m\, (\Omega-\theta)$$

Integration of the LaGrange equations

One integrates the LaGrange equations by replacing the osculating elements $a, e, I, \omega, \Omega, M$ in the second terms by the average elements $\bar{a}, \bar{e}, \bar{I}, \bar{\Omega}, \bar{\omega}, \bar{M}$ where a, e, I are constants and $\bar{\Omega}, \bar{\omega}, \bar{M}$ are linear functions of time.

Thus, the total variation of an osculating element E^j is equal to the superposition of the variations ΔE^j, to the first order produced by each of the terms R_{lmpq} of the perturbation function ($\mathcal{R} = \Sigma\, \mathcal{R}^{lmpq}_{lmpq}$. For example, the zonal coefficient C_{30} gives the following perturbation to the argument ω:

$$\Delta e = -\frac{3\mu}{2(1-\bar{e}^{2})^{2}\, n\bar{a}^{3}} \left(\frac{R}{\bar{a}}\right)^{3} C_{30} \sin \bar{I} \left(\frac{5}{4} \sin^{2} \bar{I} - 1\right) \frac{\sin \bar{\omega}}{\dot{\omega}}$$

In a general manner, the integration of a term R_{lmpq} gives:

$$\Delta a_{lmpq} = \mu R^l \frac{2F_{lmp} G_{lpq} (l-2p+q) S_{lmpq}}{n a^{l+2}\left[(l-2p)\dot{\omega} + (l-2p+q)\dot{M} + m(\dot{\Omega}-\dot{\theta})\right]}$$

$$\Delta e_{lmpq} = \mu R^l \frac{F_{lmp} G_{lpq} (1-e^2)^{1/2}\left[(1-e^2)^{1/2}(l-2p+q) - (l-2p)\right] S_{lmpq}}{n a^{l+3} e \left[(l-2p)\dot{\omega} + (l-2p+q)\dot{M} + m(\dot{\Omega}-\dot{\theta})\right]}$$

$$\Delta \omega_{lmpq} = \mu R^l \frac{\left[(1-e^2)^{1/2} e^{-1} F_{lmp} (\partial G_{lpq}/\partial e) - \cot I (1-e^2)^{-1/2} (\partial F_{lmp}/\partial I) G_{lpq}\right] \bar{S}_{lmpq}}{n a^{l+3}\left[(l-2p)\dot{\omega} + (l-2p+q)\dot{M} + m(\dot{\Omega}-\dot{\theta})\right]}$$

$$\Delta I_{lmpq} = \mu R^l \frac{F_{lmp} G_{lpq} \left[(l-2p)\cos I - m\right] S_{lmpq}}{n a^{l+3} (1-e^2)^{1/2} \sin I \left[(l-2p)\dot{\omega} + (l-2p+q)\dot{M} + m(\dot{\Omega}-\dot{\theta})\right]}$$

/115

$$\Delta \Omega_{lmpq} = \mu R^l \frac{(\partial F_{lmp}/\partial I) G_{lpq} \bar{S}_{lmpq}}{n a^{l+3} (1-e^2)^{1/2} \sin I \left[(l-2p)\dot{\omega} + (l-2p+q)\dot{M} + m(\dot{\Omega}-\dot{\theta})\right]}$$

$$\Delta M_{lmpq} = \mu R^l \frac{\left[-(1-e^2)\, e^{-1}\, (\partial G_{lpq}/\partial e) + 2(l+1) G_{lpq}\right] F_{lmp} \bar{S}_{lmpq}}{n a^{l+3}\left[(l-2p)\dot{\omega} + (l-2p+q)\dot{M} + m(\dot{\Omega}-\dot{\theta})\right]}$$

where $\bar{S}_{lmpq}$ is the integral of S_{lmpq} with respect to its argument.