N O T I C E

THIS DOCUMENT HAS BEEN REPRODUCED FROM MICROFICHE. ALTHOUGH IT IS RECOGNIZED THAT CERTAIN PORTIONS ARE ILLEGIBLE, IT IS BEING RELEASED IN THE INTEREST OF MAKING AVAILABLE AS MUCH INFORMATION AS POSSIBLE

OLD DOMINION UNIVERSITY RESEARCH FOUNDATION

DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS
SCHOOL OF ENGINEERING
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA

(NASA-CR-164036) A CLASS OF UNSTEADY, THREE-DIMENSIONAL FLOW STRUCTURES IN TURBULENT BOUNDARY LAYERS Final Report, 1 Sep. 1979 - 31 Aug. 1980 (Old Dominion Univ., Norfolk, Va.) 12 p HC A02/MF A01 CSCL 20D

N81-19414

Unclas
G3/34 41666

A CLASS OF UNSTEADY, THREE-DIMENSIONAL FLOW STRUCTURES IN TURBULENT BOUNDARY LAYERS

By

Robert L. Ash, Principal Investigator

Final Report
For the period September 1, 1979 - August 31, 1980

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia

Under
Research Grant NSG 1634
J.N. Hefner, Technical Monitor
High Speed Aerodynamics Division

March 1981

DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS
SCHOOL OF ENGINEERING
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA

A CLASS OF UNSTEADY, THREE-DIMENSIONAL FLOW STRUCTURES IN TURBULENT BOUNDARY LAYERS

By

Robert L. Ash, Principal Investigator

Final Report
For the period September 1, 1979 - August 31, 1980

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia 23665

Under
Research Grant NSG 1634
J.N. Hefner, Technical Monitor
High Speed Aerodynamics Division

Submitted by the
Old Dominion University Research Foundation
P.O. Box 6369
Norfolk, Virginia 23508

March 1981

A CLASS OF UNSTEADY, THREE-DIMENSIONAL FLOW STRUCTURES IN TURBULENT BOUNDARY LAYERS

By

Robert L. Ash[1]

SUMMARY

A restricted class of mathematically admissible, unsteady, three-dimensional flows has been identified which may constitute part of the structure observed in turbulent boundary layers. This report presents the development of the model and discusses some general results. The resulting solution has characteristics which suggest how upwelling low-speed flow can trigger a downward jetting of irrotational high-speed fluid into the boundary layer.

SYMBOLS

$\vec{e}_1, \vec{e}_2, \vec{e}_3$	unit vectors
P	pressure
$\vec{u}$	local velocity vector
U_∞	free-stream velocity
u, v, w	velocity components
V	complex velocity function
γ	reciprocal streamwise disturbance length
Γ	ratio of disturbance to free-stream velocity
ν	kinematic viscosity
ξ	complex variable
ρ	density

[1]Professor, Department of Mechanical Engineering and Mechanics, Old Dominion University, Norfolk, Virginia 23508.

INTRODUCTION

Experimental investigations continue to show that turbulent boundary-layer flows are composed of coherent, unsteady flow structures distributed randomly in space and time (see ref. 1, for example). The mechanisms responsible for the evolution and destruction of those structures are not understood. Neither is the process responsible for the production and distribution of Reynolds stresses. In order to modify turbulent flows in a favorable or controlled manner, the physics of the flow must be better understood.

Presently, experimental flow visualization studies have shown that large, unsteady vorticular structures are a major part of the outer turbulent boundary layer (refs. 1 and 2). The actual form and size of a single, identifiable flow structure is unknown because there is no method available to separate one structure from the remnants of all the others in which it is immersed. Furthermore, Hama (ref. 3) has shown how visual observations of unsteady streaklines can lead to erroneous conclusions and to improper isolation and interpretation of structural features. The present investigation has attempted to identify mathematically allowable solutions to the unsteady-three-dimensional Navier-Stokes equations which may constitute part of a turbulent boundary layer. The ultimate objective of this work is to enable experimental identification of turbulent flow structures and develop techniques for modifying turbulent flows favorably.

Flow structures—identified loosely as vortices—are apparently embedded in turbulent flows. A variety of structures may be present, and none of them are presumed so large that they are required to satisfy inner and outer boundary conditions simultaneously. Rather, structures near the wall are presumed to satisfy wall boundary conditions while outer structures must satisfy the irrotational outer conditions. The approach employed here was to attempt to find solutions to the constant property Navier-Stokes equations which could be subjected to either the inner or outer boundary conditions (but not both). Thus far, models appropriate to outer flow conditions appear to show more promise than the near wall structures. The results of the outer structure studies follow.

GOVERNING EQUATIONS

The local velocity vector $\vec{u}$, given by

$$\vec{u} = u\,\vec{e}_1 + v\,\vec{e}_2 + w\,\vec{e}_3 \tag{1}$$

where $\vec{e}_1$, $\vec{e}_2$, and $\vec{e}_3$ are unit vectors in the x-, y- and z- directions of a Eulerian coordinate, respectively, which must satisfy the Navier-Stokes equations:

Continuity:

$$\vec{\nabla} \cdot \vec{u} = 0 \tag{2}$$

and Conservation of Momentum:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\,\vec{u} = -\frac{1}{\rho}\vec{\nabla} P + \nu \nabla^2 \vec{u} \tag{3}$$

where

$$\vec{\nabla} \equiv \vec{e}_1 \frac{\partial}{\partial x} + \vec{e}_2 \frac{\partial}{\partial y} + \vec{e}_3 \frac{\partial}{\partial z} \tag{4}$$

and P and ρ are pressure and density, respectively.

Referring to figure 1, a coordinate system is chosen which has its origin at the nominal turbulent-nonturbulent interface. The boundary is assumed to be initially parallel, and if no interfacial motion were present, the velocity distribution for positive values of y would simply be

$$\vec{u} = U_\infty\,\vec{e}_1 \tag{5}$$

In reality, the turbulent-nonturbulent interface is characterized by three-dimensional peaks and valleys which occur rather abruptly, then are obliterated slowly as they convect downstream (see ref. 1). The mathematical model which follows has attempted to couple the low-speed burst disturbances from the wall region with outer eddy structures. It is assumed that the low-speed fluid parcel ejected from the wall beneath a flat plate, turbulent boundary layer is not able to penetrate the turbulent-inviscid interface. Rather, this slow moving fluid is surrounded by fine structure turbulence; as it arrives at the upper reaches of the boundary layer, the fine structure shields the streaky fluid from the free stream. The large velocity gradients which must be present act to sustain the fine structure. In addition, the upward moving filament causes a small bulge to occur in the turbulent-non-turbulent interface region.

Consideration of pressure changes resulting from a bulge suggests from Bernoulli's equation that, if the velocity above the disturbance increases, the pressure decreases, drawing the bulge further upward. This type of disturbance would tend to pull the turbulent fluid rapidly upward into the irrotational regime as it is swept downstream. However, there appears to be no stopping mechanism to control bulge height. An alternate model would be to assume the inviscid fluid maintains a constant volume flow by producing an interfacial depression to compensate for the bulge. Under those conditions, the bulge need not produce a local velocity increase and the destabilizing pressure disturbance need not occur. Those disturbances have been studied here.

Consider the class of velocity distributions given by

$$\vec{u} = U_\infty \vec{e}_1 + v(x,y,z,t)\, \vec{e}_2 + w(x,y,z,t)\, \vec{e}_3 \tag{6}$$

where U_∞ is assumed constant. Then, the continuity equations reduce to

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{7}$$

which can be satisfied when v and w are functions of x, t and complex variable ξ, where

$$\xi = y + i\, z \tag{8}$$

Then, equation (7) becomes

$$\frac{\partial v}{\partial \xi} = -i\, \frac{\partial w}{\partial \xi} \tag{9}$$

which can be integrated to yield

$$w(x,\xi,t) = i\,v(x,\xi,t) + g(x,t) \tag{10}$$

Here, we choose $g(x,t) = 0$, since no apparent benefits result from inclusion of a function of x and t alone.

Equation (10) can be utilized in equations (3) and (6) to develop expressions for the conservation of momentum in the three-coordinate directions:

x or $\vec{e}_1$ direction:

$$\frac{\partial P}{\partial x} = 0 \tag{11}$$

y or $\vec{e}_2$ direction:

$$\frac{\partial v}{\partial t} + U_\infty \frac{\partial v}{\partial x} = -\frac{1}{\rho}\frac{\partial P}{\partial \xi} + \nu \frac{\partial^2 v}{\partial x^2} \tag{12}$$

and z or $\vec{e}_3$ direction:

$$i\left[\frac{\partial v}{\partial t} + U_\infty \frac{\partial v}{\partial x} = -\frac{1}{\rho}\frac{\partial P}{\partial \xi} + \nu \frac{\partial^2 v}{\partial x^2}\right] \tag{13}$$

Obviously, equations (12) and (13) are the same. Furthermore, the requirement that $\frac{\partial P}{\partial x} = 0$ implies that $p = p(\xi,t)$, which requires that pressure fluctuations consistent with the above equations be "global" in the x-direction. Those pressure fluctuations are not particularly interesting, and it is therefore assumed that $\frac{\partial P}{\partial \xi} = 0$. Hence, the equations of motion are satisfied by

$$\frac{\partial v}{\partial t} + U_\infty \frac{\partial v}{\partial x} = \nu \frac{\partial^2 v}{\partial x^2} \tag{14}$$

where the nonlinear terms in the momentum equations are cancelled through the relationship:

$$w = iv(x,\xi,t) \tag{15}$$

The linear partial differential equation (14) can be solved using classical techniques, and if V is defined by

$$V = v - iw \tag{16}$$

the solution to equation (14) subject to initial conditions:

$$V(x,iz,o) = v(x,iz,o) - iw(x,iz,o) \tag{17}$$

can be satisfied by

$$V = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left\{-\left[\omega\bar{z}+\beta^2\bar{t}+i\beta(\bar{x}-\bar{t})\right]\right\}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(\bar{\eta},i\bar{\zeta},o)\exp\left[i(\omega\bar{\zeta}+\beta\bar{\eta})\right]d\bar{\zeta}\,d\bar{\eta}\,d\beta\,d\omega \tag{18}$$

where the overbars on the spatial variables have been made dimensionless by

$$\overline{(\ )} = \frac{U_\infty(\ )}{\nu} \tag{19}$$

and dimensionless time is given by

$$\bar{t} = \frac{U_\infty^2 t}{\nu} \tag{20}$$

Presently, the initial velocity disturbances which are considered global through equation (18) are not as interesting as spatially periodic. That is, if the disturbance is finite with respect to the spanwise coordinate, the velocity distribution is given by

$$v = f(x,t)\exp\left[\frac{2\pi}{\lambda}(iz-y)\right] \tag{21}$$

and

$$w = if(x,t)\exp\left[\frac{2\pi}{\lambda}(iz-y)\right] \tag{22}$$

where λ is an unspecified spanwise wavelength, and $f(x,t)$ is a solution to

$$\frac{\partial F}{\partial \bar{t}} + \frac{\partial F}{\partial \bar{x}} = \frac{\partial^2 F}{\partial \bar{x}^2} \tag{23}$$

with

$$F \equiv \frac{f}{U_\infty} \tag{24}$$

The solution to F can be gotten using Fourier transforms as

$$F(\bar{x},\bar{t}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\beta^2\bar{t}}\, e^{-i\beta(\bar{x}-\bar{t})} \int_{-\infty}^{\infty} e^{i\beta\xi}\, F(\xi,o)\, d\xi d\beta \tag{25}$$

At this point, solutions to $F(\bar{x},\bar{t})$ are being studied for various types of initial disturbances in the $\bar{x}$-direction. The results of that investigation will require considerably more computational time and development of additional computer graphics capabilities in order to verify the presence of these structures in a turbulent boundary layer.

One disturbance which has produced interesting results is

$$F(\bar{x},o) = \Gamma\, e^{-\gamma^2\bar{x}^2} \tag{26}$$

There, the integration of equation (25) is possible and

$$\frac{\vec{u}}{U_\infty} = \vec{e}_1 + \frac{\Gamma \exp\left[2\pi(iz-y)(\lambda)^{-1} - (\bar{x}-\bar{t})^2(4\bar{t} + 1/\gamma^2)^{-1}\right]}{(4\gamma^2\bar{t} + 1)^{1/2}} (\vec{e}_2 - i\vec{e}_3) \tag{27}$$

Under those initial conditions (a small area of fluid is given an initial upward velocity distribution at the turbulent-nonturbulent interface), the model predicts that the bulge is accompanied by a laterally shifted downward jetting of high-speed irrotational fluid into the boundary layer. This process may approximate the inrush phase near the wall, which triggers new bursts.

CONCLUSIONS

A mathematical model has been developed which predicts deformation of the interface between a turbulent boundary layer and the outer, irrotational flow. Although solutions can be developed which describe the full x-y plane initial velocity conditions, those solutions do not appear to yield results which describe physics within the turbulent boundary layer. On the other hand, a variety of restricted solutions can be developed which represent surface deformations resulting from initial velocity disturbances in finite regions of the x-y plane. Those calculations have shown that bulges at the interface can produce jetting of irrotational fluid toward the wall which may describe some turbulent features. Investigation of the finite area disturbances should be pursued to permit either isolation or elimination of the proposed structures as important features in a turbulent boundary layer.

REFERENCES

1. Smith, C.; and Abbott, D.E., eds.: Workshop on Coherent Structure of Turbulent Boundary Layers. AFOSR/Lehigh Univ., 1978.
2. Falco, R.E.: Coherent Motions in the Outer Region of Turbulent Boundary Layers. Phys. Fluids, Vol. 20, 1977, pp. S124-S132.
3. Hama, F.R.: Streaklines in a Perturbed Shear Flow. Phys. Fluids, Vol. 5, 1962, pp. 644-650.

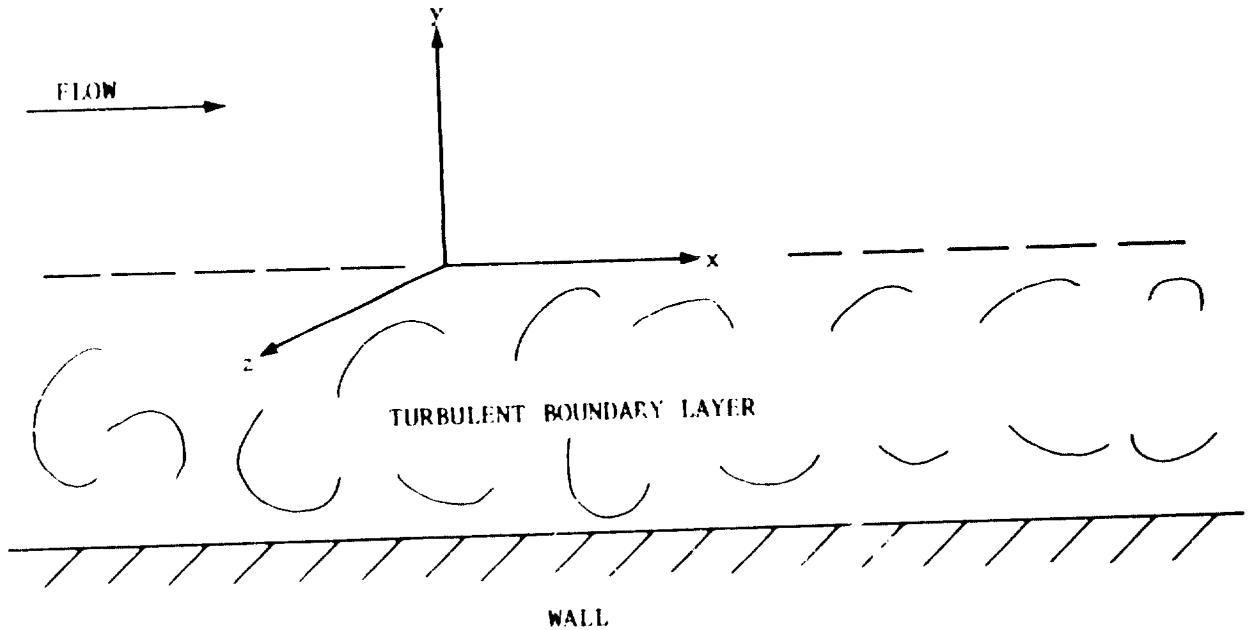

Figure 1. Schematic of coordinate system.