N O T I C E

DISCREPANCIES BETWEEN THEORETICAL AND EMPIRICAL MODELS OF THE FLARING SOLAR CHROMOSPHERE AND THEIR POSSIBLE RESOLUTION

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INSTITUTE FOR PLASMA RESEARCH
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ABSTRACT

Possible sources of the marked dicrepancy between empirical and theoretical (paticularly electron-heated) models of the solar chromosphere during flares are discussed. A major source of uncertainty in empirical models is the inhomogeneity of the spectral data on which they are based. Theoretical models involve several possible sources of error of which probably the most important is neglect of the radiative coupling of upper and lower chromospheric regions. In addition, both types of model are limited by their assumption of plane-parallel, and frequently static, structures.

The variety of deconvolutions involved in the construction and inter-comparison of empirical and theoretical models is another source of difficulty in model testing. A new procedure for studying flare energy input is suggested whereby the required input is derived from the empirical model chromosphere; this procedure is applied to the electron-heated case, and it is found that the integral equation defining the flare energy deposition rate (erg g^{-1} s^{-1}) can be analytically inverted to yield the injected electron flux energy spectrum from knowledge of the energy balance in the empirical atmosphere. We analyze recent empirical model results in this manner and compare the calculated injected electron flux spectrum with that needed for hard X-ray bursts in moderately large flares. This technique betters the usual procedure of predicting a model chromosphere from the hard X-ray electron flux in that it yields a measure of significance and uniqueness of the heating model parameters.

Subject Headings: hydromagnetics - Sun: chromosphere - Sun: flares - Sun: X-rays

I INTRODUCTION

A considerable effort in flare physics has recently gone into the study of the secondary response of the solar atmosphere to the primary flare energy release (see, e.g., Canfield et al. 1980 for a review). Essentially this field comprises the comparison of flare model atmospheres constructed in two distinct ways, viz. theoretically and empirically. A theoretical model is an atmospheric structure predicted by solution of the equations (e.g. transport of energy, momentum and radiation) describing the response of the preflare solar plasma to a hypothetical flare energy input. An empirical model, on the other hand, has a structure derived from observations of the flaring atmosphere, by trial and error fitting of the computed model spectrum and subsequent model adjustment. The theoretical model may then be compared with the empirical or (which is not entirely equivalent - see §III) be used to predict a spectrum for direct comparison with observations. Recent work (Machado and Emslie 1979; Lites and Cook 1979; Machado et al. 1980--hereafter MAVN) on empirical models has established marked discrepancies between these and theoretical models, particularly those in which the hypothetical energy input is particle beam collisions (e.g. Brown 1973, Brown, Canfield and Robertson 1978--hereafter BCR; LaBonte 1978; Machado, Emslie, and Brown 1978). Not only is there a quantitative discrepancy in the parameters required to describe the theoretical energy input (e.g. electron flux) and the observations (e.g. Hα brightness) of part of a particular flare (see BCR) but also qualitative differences between the general form of empirical and theoretical structures for any input parameters. In this paper we discuss possible sources of these discrepancies with a view to clarifying the most important areas for future work on this problem. Our discussion is concerned mainly with models of the chromosphere, and specifically electron-heated models.

The problem of the optically thin corona and transition region has been discussed recently by Machado and Emslie (1979) and that of the temperature minimum region by Machado, Emslie, and Brown (1978). However, many of our remarks on the sources of discrepancy in the chromosphere are equally applicable to these regions and to models heated by other forms of energy input, such as soft X-ray irradiation (Somov 1975; Machado 1978).

Figure 1

In Figure 1 (from MAVN) we compare the empirical models derived from Skylab ATM data with the electron heated (BCR) theoretical models. The main discrepancies are:

(i) The BCR models have higher temperatures in the mid-chromosphere ($T \gtrsim 10^4$K, $h \geq 1100$ km) and much more material with $10^4 \lesssim T \lesssim 2 \times 10^4$K .

(ii) The transition region in the BCR models occurs considerably higher in the atmosphere than in the empirical models. Indeed for small electron fluxes the BCR transition region occurs higher in the flaring atmosphere than in the quiet sun (Machado and Emslie 1979).

In the region $h \approx 800$-1100 km ($T \approx 8 \times 10^3 - 10^4$K) the BCR and empirical models have similar temperatures; to that extent the BCR models are locally plausible, reproducing observed Hα profiles when a moderate amount of horizontal inhomogeneity (see §IIc), consistent with the filling factor of the Hα spectrograph slit, is included. (Note however that the electron flux needed to obtain this agreement is $\lesssim$ 1/10 of that required for thick target interpretation of hard X-rays from the same flare - BCR.) Nevertheless such a spatially localized regime as that of Hα formation clearly cannot be expected to provide an adequate empirical test of a model energy input. Furthermore (see §IIb) a localized treatment is not satisfactory theoretically either. It is therefore essential to take a global view in modeling of the chromosphere, and indeed of the whole atmosphere.

In §II we consider the possible sources of the discrepancies outlined above and discuss the global radiative transfer problem more fully. In

§III we discuss the nature of the intercomparison process itself and how it renders estimation of uncertainties difficult. We then suggest a new intercomparison technique which allows estimation of these uncertainties; this technique involves computing the required energy input for a given empirical atmosphere structure, rather than constructing a theoretical model atmosphere based on a prescribed hypothetical flare energy input. In §IV we apply this technique to MAVN empirical flare data and discuss the results obtained.

II. SOURCES OF DISCREPANCY

a) General

There are several features in common to both empirical and theoretical models which render them generally unsatisfactory. First, in order to make the analyses tractable, most chromospheric modeling has thus far been based on the assumption of a steady state plane-parallel structure in hydrostatic equilibrium. These assumptions eli· ate time-dependence from the equations and permit representation of the models purely in terms of their temperature versus height structure T(h) as used in Figure 1, together with a scaling factor defining pressure or density at a single point. However, they result in omission of energy transport and spectroscopic effects due to mass motion, time dependent conduction (see Craig and McClymont 1976) and radiative transport (see Dubov 1963), and inhomogeneity. For example, if large velocity gradients were present in the atmosphere, then it is possible that they would radically alter the interpretation of spectral diagnostics in terms of a T(h) structure. (There is, however, little evidence for such velocity gradients in the layers of the flaring atmosphere under consideration [see Lites and Cook 1979].) Second, chromospheric modeling of both types involves the complex numerical solution of the relevant radiative transfer equations, the techniques of which are still under development (see Vernazza, Avrett, and Loeser [VAL] 1973, 1976, 1980) and thus require cautious treatment.

b) Deficiencies in Theoretical Models

A number of factors in the electron beam energy <u>input</u> used by BCR warrant further study. First, the input is purely collisional (see Brown 1973; Emslie 1978) taking no account of the effects of plasma wave generation (Brown and Melrose 1977; Vlahos and Papadopoulos 1979) nor of reverse current ohmic losses (e.g. Emslie 1980) on the height distribution of beam energy deposition. These are both most important highest in the atmosphere and will tend to steepen the BCR T(h) profiles toward the empirical forms, both directly and by enhancing the role of conduction (see below). Second, the effect on beam dynamics of converging magnetic field lines is neglected; its inclusion would enhance the beam deposition at higher altitudes. Third, the BCR models were computed only for electron spectral index $\delta = 4$. Use of the higher δ's typical of most small flares will also tend to steepen T(h) by depositing energy preferentially higher in the atmosphere (see Emslie 1978).

The BCR models included no thermal conduction on the grounds that conductive heating (or cooling) is negligible at the low temperatures and temperature gradients where Hα forms (see Brown 1974, 1977). We have checked this assumption directly for both the theoretical and empirical models and find (see §III) that for $T \lesssim 12000$ K the conductive term

$$\frac{d}{dh}\left(\kappa(T)\,\frac{dT}{dh}\right)$$

is totally negligible compared to the radiative losses there so that an external input is needed. This is contrary to claims by Švestka (1970), Shmeleva and Syrovatskii (1973), and Davis, Kepple, and Strickland (1977) that local conductive deposition can balance chromospheric radiative losses during the post-flash phase of the flare. At first sight this also appears contrary to the statement by Machado and Emslie (1979) that "thermal conduction alone is sufficient to produce the observed flare enhancements at all atmospheric levels, in particular in the chromosphere". However, it turns out that in the proper, global, approach to the theoretical modeling, it is essential to include conduction in the hotter layers (upper chromosphere) in order for the correct structure in the lower (Hα) chromosphere to be obtained. The reason is that these two regions are radiatively coupled, particularly by Lα which constitutes an important energy loss from the upper chromosphere but a significant input into the lower chromosphere -- Machado and Emslie (1979); MAVN. (The possibility of radiative heating was noted by BCR but not incorporated in their models.) The importance of this Lα "backwarming" stems from the fact that the energy in the Lα radiation comes from dissipation of thermal conductive flux, where the steep transition zone temperature gradient is suddenly reduced. In short, the Machado and Emslie (1979) statement refers to the fact that a thermal conductive flux in the transition zone can, through Lα backwarming, supply the energy for a significant fraction of the radiative losses in the upper chromosphere (i.e. down to about the region where Hα forms).

The excessively high BCR transition region is attributable substantially to two factors. First BCR take the quiet sun model of VAL 1973 as their preflare state in establishing the density scale and the ambient energy input in their models (see Brown 1973). Use of a plage preflare model would have resulted in deeper transition regions, particularly in low flux models. The residual discrepancy, and in particular the fact that the BCR transition zone for low electron fluxes lies above its quiet sun level, is a numerical artifact. Since BCR were attempting to model the Hα formation region, and since they did not recognize the importance of the radiative coupling of upper and lower chromospheric regions, they used only a very broad grid representation of the upper chromosphere. In fact the grid step used there was of the order of the discrepancy in transition zone height noted by Machado and Emslie (1979).

c) Deficiencies in Empirical Models

Aside from the points raised in §IIa, and the problems of model uniqueness discussed below (§III), the main deficiencies in empirical models lie in the inhomogeneity of the data used. This takes several forms. First, the evident inhomogeneity of the horizontal structure of the flare within the instrumental field means that the empirical models represent some horizontally averaged flare structure. It is therefore not particularly meaningful to compare these with theoretical models representing the flare-heated areas (as MAVN do in Figure 1). Rather we should compare them with a theoretical model diluted by the ambient atmosphere with an appropriate filling factor (which itself generally depends on height h according to the variation of flare area and of instrumental field with h) . The latter is the procedure followed by BCR in their Hα analysis.

Second, even the best observational efforts to date (MAVN) have resulted in spectra based on very inconsistent coverage in both space and time. In some cases the data used in different spectral bands were not even from a single flare, and at best were from different parts of a single flare at different times. Furthermore much of the ATM flare coverage was in the post-flash phase, so putting in doubt the relevance of the empirical models in testing theoretical models heated by electron beams which are early-stage phenomena.

Third, the flare set analyzed by MAVN comprised five subflares and one Class 1, while some of the theoretical model analyses (e.g. BCR) have been aimed at very large events.

III. CONVOLUTIONS AND STABILITY IN MODEL BUILDING

An unsatisfactory aspect of current empirical models is that they give no error bars on T(h) so that any comment on their compatibility with theoretical models is only qualitative. The question we really want to ask is: what manifold of empirical model atmospheres is capable of fitting the spectral data within their error bars, and what theoretical models fall within this manifold? The first part of this question is hard to answer due to the very large computational effort involved in iteration to obtain even one acceptable empirical model. Furthermore error bars on T(h) so obtained are almost certain to be rendered large and hard to interpret meaningfully because of the inhomogeneity of data used. Also, in comparing theoretical and empirical models quantitatively, we should develop an objective criterion of acceptability, presumably based on some statistic such as the root mean square residual on the energy balance parameters (rather than simply on temperature) relative to the probable error in the observed (empirical) values.

Even given a good set of homogeneous data with error bars, there are still fundamental limitations on the extent to which empirical models can test theoretical models because of the number of deconvolution processes involved (see Craig and Brown 1976). Specifically, in testing electron-heated chromospheric models, the usual procedure is (a) derive an electron spectrum from a noisy observed hard X-ray spectrum by deconvolution (Brown 1971; Craig 1979); (b) predict a theoretical model atmosphere heated by this electron input by solving the integro-differential equation of radiative transport; (c) derive an empirical model atmosphere structure by deconvolution of the integro-differential equation governing the formation of the observed spectrum; (d) compare (c) with (b). Given the problems of error magnification in deconvoluting from and of nonuniqueness in model-fitting to, integral equations (Craig and Brown 1976 and references therein), we should clearly be circumspect in rejecting or accepting theoretical model chromospheres heated by electrons, since their computation depends intrinsically on the solution of <u>three</u> integral equations.

Here we suggest the following alternative to the usual sequence above, which enables us to quantify uncertainties in derived quantities; the usual process of forward fitting of models to data does not permit us to do this.

Instead of predicting an optical/UV spectrum from a theoretical model flare chromosphere with the energy input function inferred from independent observations and comparing this prediction with optical and UV observations, we suggest reversing the procedure. That is, we derive the energy input function which is consistent with the heating rates derived from the optical/UV spectral data and compare this with other independent estimates of this input function. (See Machado, Emslie, and Brown 1978 for similar discussion of heating at the temperature minimum.) Specifically we propose the following procedure:

(a) Obtain empirically that atmospheric temperature structure $T(h)$ which best fits the spectral data, or better that range of $T(h)$ which fit the data within its errors.

(b) Evaluate the <u>net</u> radiative energy loss from this (these) empirical model(s) (see VAL 1980) and so deduce the <u>total</u> specific energy input $I_o(h)$ (erg g^{-1} s^{-1}) required for a steady state. ($I_o[h]$ comprises a gross radiative loss $I_{tot}[h]$ less a radiative input $I_r[h]$ from backwarming.)

(c) Subtract from $I_o(h)$ the specific conductive input $I_c = \left[\frac{1}{\rho}\frac{d}{dh}(\kappa[T]\frac{dT}{dh})\right]$ (ρ = gas density) calculated from the $T(h)$ found in (a), and also an estimate of the ambient energy input $I_a(h)$. The latter requires some assumption of how $I_a(h)$ changes between preflare and flare conditions—e.g. constant I_a per gram (see Brown 1973).

(d) This leaves the energy input $I_f(h)$ to be supplied by the flare release processes, which can then be compared with theoretical heating models. The acceptability of the fit as the various parameters of the theoretical model are varied gives an indication of the uncertainties in the values of these parameters, quantities which are difficult to obtain in the conventional comparison procedure (see BCR). It should also be noted that this procedure should enable a direct comparison of I_f with I_c, I_r and I_a at each height h, indicating unambiguously the relative role of each process (as a function of height), which up to now has remained controversial.

We illustrate the technique by considering chromospheric models heated by collisional degradation of a beam of non-thermal electrons, injected vertically along uniform field lines. Expressing I_f as a function of the Lagrangian column depth variable $N = \int_h^\infty n(h')dh'$ rather than h, then the energy input arising from an injected number flux spectrum $F_o(E_o)$ (electrons $cm^{-2}s^{-1}$ per unit E_o) is (Brown 1973; Emslie 1978)

$$I_f(N) = \frac{K}{m_H}\int_{\sqrt{3KN}}^{\infty} \frac{F_o(E_o)dE_o}{E_o(1-3\ KN/E_o^2)^{2/3}} \quad , \qquad (1)$$

where $K = 2\pi e^4 \Lambda$ in the usual notation and m_H is the mass of a hydrogen atom. Setting $x = E_o^2$, $\nu = 3KN$, $G(x) = \frac{K}{2m_H}\frac{F_o(\sqrt{x})}{x^{2/3}}$ and $J(\nu) = I_f(N)$ then

$$J(\nu) = \int_\nu^\infty \frac{G(x)\ dx}{(x-\nu)^{2/3}} \quad , \qquad (2)$$

which is an Abel integral equation with solution (assuming $G(x \pm \infty) = 0$ for all reasonable $F_o(E_o)$)

$$G(x) = -\frac{\sin\left(\frac{2\pi}{3}\right)}{\pi x}\int_x^\infty \frac{\left[\frac{2}{3} J(\nu) + \nu J'(\nu)\right] d\nu}{(\nu - x)^{1/3}} \qquad (3)$$

(Courant and Hilbert 1953, pp. 158-159).

In terms of the original variables,

$$F_o(E_o) = -\frac{2\sqrt{3}\, m_H}{\pi E_o^2}\int_{E_o^2/3K}^\infty \frac{\left[(I_f(N) + \frac{3}{2} N\, I_f'(N)\right] dN}{\left(\frac{3KN}{E_o^2} - 1\right)^{1/3}} , \qquad (4)$$

which determines the form of the injected electron flux spectrum $F_o(E_o)$ from knowledge of the behavior of I_f with N, obtained via steps (a) to (c) above. In the next Section we shall apply this method of analysis to empirical flare data from MAVN.

IV APPLICATION TO OBSERVATIONS

In Figure 2 we show the behavior of I_o, I_{tot}, I_r and I_c obtained by the procedure of VAL (1980) for model MAVN2 (see Figure 1). We use this model (as opposed to model MAVN1) because it corresponds to bright flare regions and so is more comparable with an atmosphere undergoing excitation by non-thermal particles, though it is still small compared to a BCR-sized flare. One immediately notes the following points from the Figure:

Figure 2

(i) the local condutive input I_c is less than I_o by at least nine orders of magnitude and so is totally negligible (see remarks in §II).

(ii) the ratio I_r/I_{tot} (the contribution of backwarming to the radiative energy budget) approaches unity (to within a factor of $\simeq 2$; see remarks in §IIb) at around $N \simeq 3\times10^{21}$ cm^{-2}, but falls off rapidly with depth thereafter (so that $I_{tot} \simeq I_o$ for depths greater than this).

To derive $I_f(N)$ we further need to apply process (c), i.e. subtract the ambient (preflare) input $I_a(N)$ and the local conductive input $I_c(N)$. As noted in (i) above, the latter of these is negligible. I_a can be obtained by applying the VAL (1980) method of calculating net radiative losses to a plage model atmosphere, preferably an empirical one. The only available empirical plage model is that due to Basri et al. (1979); however, this model suffers from serious errors in the upper chromospheric region, due to the assumption of local thermodynamic equilibrium for the Carbon atom (E. H. Avrett, private communication). Since this region is precisely the region of greatest interest to our present study, we do not consider it appropriate to use the Basri et al. (1979) model to obtain $I_a(N)$. We therefore have no alternative but to neglect $I_a(N)$ and restrict our attention to regions of the atmosphere in which I_a may be reasonably taken to be negligible compared to I_o ; hopefully future work on empirical plage model atmospheres may enable us to remove this restrictive condition. Figure 2 strongly suggests that I_a becomes important around $N \approx 10^{22}$ cm^{-2} (note the upturn in the graph of $I_o(N)$ at this point); this belief is somewhat strengthened by application of the VAL (1980) radiative loss calculation to the Basri et al. (1979) model, which shows that radiative losses due to Mg and Ca lines start to become very important at just this depth.

Figure 3

In Figure 3 we show the result of applying equation (4) to the I_o ($\equiv I_f$, since we are neglecting I_a) values of Figure 2. To avoid the singularity at the lower integral limit in our computations we first transformed (4) to the equivalent expression

$$F_o(E_o) = - \frac{\sqrt{3}}{2\pi} \frac{m_H}{K^2} E_o^2 \int_o^\infty I_f' \left(\frac{E_o^2}{3K} [1+x^{3/2}] \right) dx \qquad (5)$$

(see Brown 1971). In the evaluation of this integral there is a truncation error at the upper limit, since we only have data up to $x_{max} = (3KN_{max}/E_o^2-1)^{2/3}$, where N_{max} is the largest column density for which data is available. This truncation error will only seriously affect the determination of $F_o(E_o)$ near $E_o \simeq (3KN_{max})^{1/2}$, which is well outside the range of E_o values for which a spectrum is to be determined (see below); thus it does not affect the results to follow. Note the hardening in the electron spectrum above around 300 keV, corresponding to $N \simeq 10^{22}$ cm^{-2} -- cf. the lower integral limit in equation (4). This is attributable in part to our neglect of I_a, as noted above, but is certainly also in part representative of the strong heating observed near the temperature minimum which cannot be explained by non-thermal electron bombardment (Machado, Emslie, and Brown 1978). Shown in the Figure is the best-fit power-law $F_o(E_o) = F_{oo}E_o^{-\delta}$ to the (E_o,F_o) points, where only E_o values to the left of the spectral bend have been used (see remarks above on the limits of applicability of our method). This best-fit line has the form

$$F_o(E_o) = 10^{(26.91 \pm 0.53)} E_o^{-(6.21 \pm 0.24)} \text{electrons cm}^{-2}\text{s}^{-1}\text{keV}^{-1}, \qquad (6)$$

with a regression coefficient $r = 0.997$.

For comparison with hard X-ray burst analysis it is more convenient to express F_{oo} in terms of the total injected energy flux above 20 keV (say), viz.

$$F_{20} \ (\text{erg cm}^{-2}\text{s}^{-1}) = 1.6 \times 10^{-9} \int_{20}^{\infty} F_o(E_o) \, E_o \, dE_o \ . \qquad (7)$$

Using equation (6) in equation (7) we obtain, for the injected electron flux parameters:

$$\begin{aligned} \log F_{20} &= 12.01 \pm 0.53 \\ \delta &= 6.21 \pm 0.24 \end{aligned} \qquad (8)$$

From these results and their uncertainties we conclude that heating by an electron beam with a power law injection spectrum fits the observations extremely well over the atmospheric domain concerned; however, the scatter in the few data points used prevents us from deriving the required electron flux F_{20} with an accuracy better than about half an order of magnitude. We also note that the value of δ in eq. (8) is much larger than the $\delta = 4$ used by BCR. This explains the higher temperatures (compared to the MAVN models) found by BCR in the Hα formation region (§I); see comments in §II on the effect of different δ on the variation of the energy deposition rate with depth. With this in mind, we note, however, that the electron injection parameters (8) are entirely consistent with those required for hard X-ray bursts in large events (Hoyng, Brown, and van Beek 1976), viz. $F_{20} \simeq 3 \times 10^{11}$ erg cm^{-2} s^{-1} (for plausible injection areas $A \simeq 10^{18} - 10^{19}$ cm^2) and $\delta \simeq 4 - 6$. This is encouraging, although it must be noted that the above illustrative analysis is based on a very limited range of points, viz. a factor of ~ 5 in the column density N and so a factor of ~ 2 in the electron energy E_o . In addition, the error on F_{20} quoted in eq. (8) is based on the residuals of the (E_o, F_o) points about the best fit line (Figure 3) and might be substantially larger if the uncertainties in the individual points exceed these residuals; these uncertainties are, however, very difficult to estimate (E.H. Avrett, private communication). It is to be hoped that more work on preflare structures, and the acquisition of more extensive data sets, will enable the present method to be used as a better flare diagnostic technique.

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FIGURE CAPTIONS

Figure 1: Temperature versus height structures in empirical model chromospheres for small (MAVN 1) and larger (MAVN 2) flares constructed from Skylab data by Machado et al. (1980), compared with the structures predicted by Brown, Canfield, and Robertson (1978) for chromospheres heated by electron beams with power law spectral index 4 and energy fluxes (above 20 keV) of 10^9 (BCR 9) and 10^{10} (BCR 10) ergs cm^{-2} s^{-1} respectively. The column density values at the top of the Figure correspond to model MAVN 2 (cf. Figure 2).

Figure 2: Energy sources and sinks as functions of column density N inferred from the empirical MAVN2 flare structure. $I_o(N)$ is the net radiative loss (ergs g^{-1} s^{-1}), comprising a gross radiative loss term I_{tot} and a (negative) backwarming term I_r. $I_c(N)$ is the conductive deposition inferred from the temperature gradients in the atmosphere. Note the importance of backwarming effects for $N \lesssim 10^{22}$ cm^{-2} and the negligible amount of local conductive heating.

Figure 3: Injected electron flux spectrum obtained by inversion of the $I_o(N)$ values in Figure 2 (see equation [4]). Beyond $E_o \approx 300$ keV (corresponding--see equation [4]--to $N \approx 10^{22}$ cm^{-2}) the spectrum changes form dramatically; this is partly real and partly due to the neglect of preflare heating terms in the flare energy equation--see discussion in text (§III). Below this break point, the spectrum is well fit by a power law with the parameters shown; these can be compared to independently derived electron flux parameters, obtained from analysis of hard X-ray spectra (see equation [8]).

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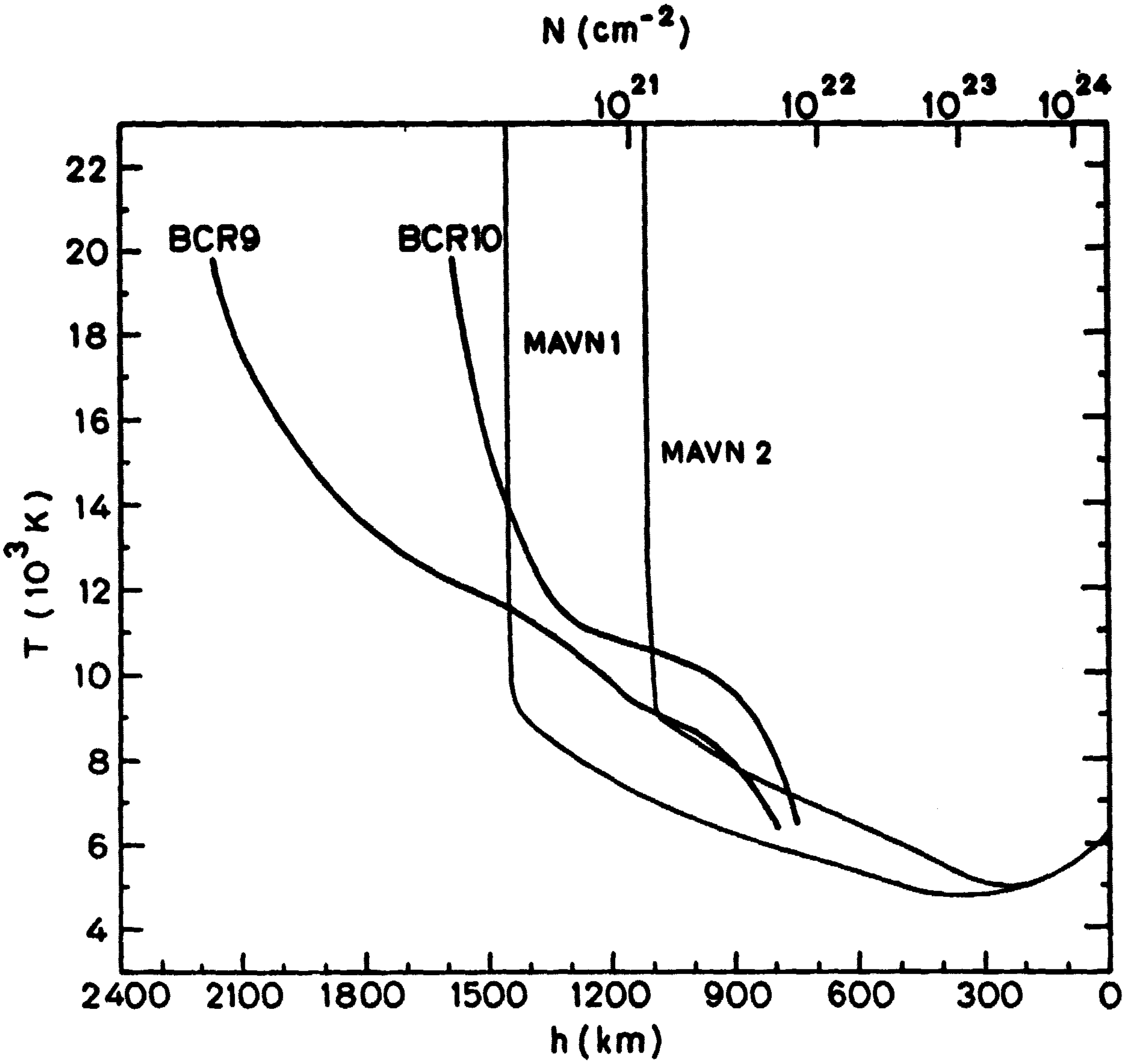
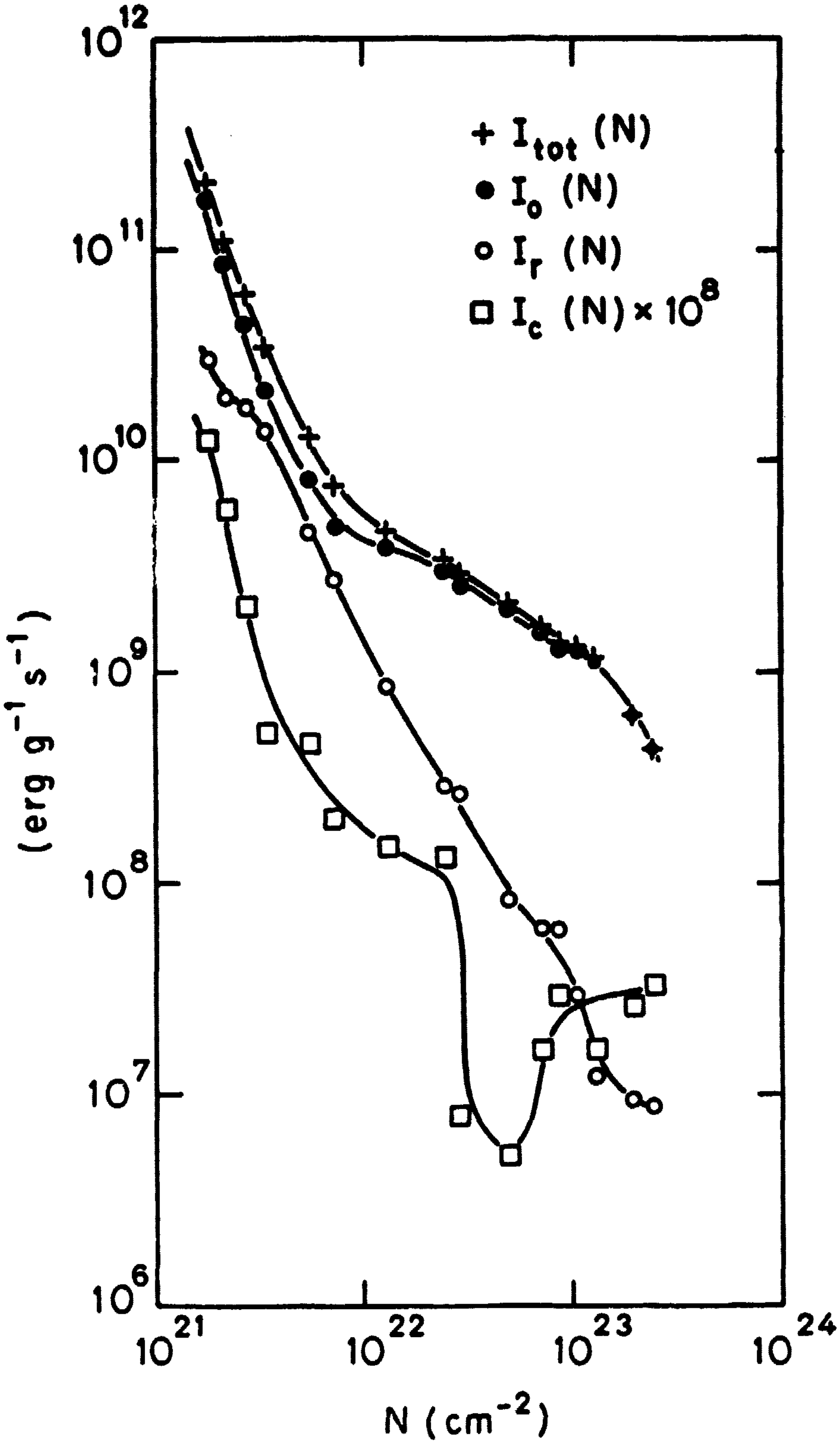
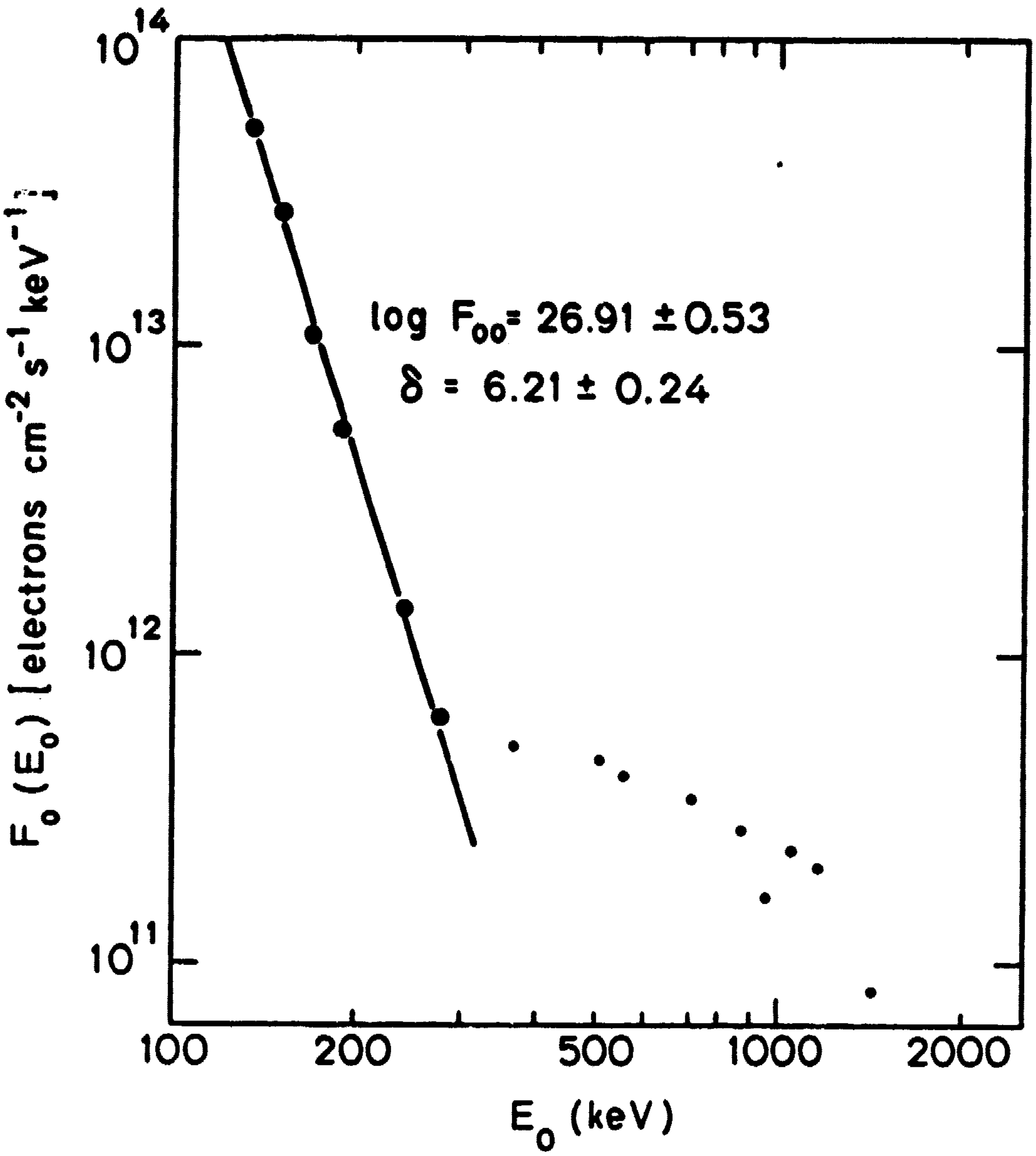

N (cm⁻²)
10^21
10^22
10^23
10^24
22
20
18
16
14
12
10
8
6
4
T (10^3 K)
BCR9
BCR10
MAVN 1
MAVN 2
2400
2100
1800
1500
1200
900
600
300
0
h (km)

Figure 1.



10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
+ I_{tot} (N)
● I_0 (N)
○ I_r (N)
□ I_c (N) × 10^8
(erg g^{-1} s^{-1})
10^{21}
10^{22}
10^{23}
10^{24}
N (cm^{-2})

Figure 2



10^{14}
10^{13}
10^{12}
10^{11}
$F_o(E_o)$ [electrons cm^{-2} s^{-1} keV^{-1}]
log F_{oo} = 26.91 ± 0.53
δ = 6.21 ± 0.24
100
200
500
1000
2000
E_o (keV)

Figure 3