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THE REDUCED ORDER MODEL PROBLEM IN DISTRIBUTED PARAMETER SYSTEMS ADAPTIVE IDENTIFICATION AND CONTROL

PROGRESS REPORT II

for NASA Grant NAG-I-7

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I. Recent Progress Summary

This progress report summarizes the recent and projected efforts in investigating the reduced order model problem in distributed parameter systems adaptive identification and control under NASA Grant NAG-I-7 sponsorship. A lengthy simulation study [1] of the reduced order problem in scalar adaptive control of lumped-parameter systems, projected in a previous interim report, has only recently been completed. A comprehensive examination (compiled over the past several months) of real-time centralized adaptive control options for flexible spacecraft is provided in the remainder of this section of this progress report. (The real-time objective, as used here, excludes the possibility of separating identification and control in time as suggested in [2].) This overview prompts the departure from the anticipated narrow focus on the NASA Langley beam control experiment and a shift to development of an original, general approach to this problem as projected in section II. Section III lists the references cited in the first two sections. Sections IV and V provide a listing of recent presentations and publications of work sponsored by NASA Grant NAG-I-7. The final section is a cumulative list of sponsored papers, which have appeared in the open literature.

1. Introduction to Four Approaches to Adaptive Control of Flexible Spacecraft

The four approaches to adaptive control of flexible spacecraft discussed in the next four subsections are:

(i) Assume that sufficiently accurate eigenshapes are provided a priori and perform simultaneous adaptive modal identification and control as, e.g., in [3] and [4].□

(ii) Assume that eigenshapes can be accurately approximated by a finite dimensional, linear combination of preselected orthogonal spatial functions (as suggested in [2]) and simultaneously estimate the parameters forming the eigenshapes and the parameters in their dynamic amplitude behavior. Combine this identification with on-line solution of the decoupled control problems as in [3] and [4].□

(iii) Treat the actuator/distributed parameter system (DPS)/sensor combination as a multi-input, multi-output (MIMO) system with finite (but large) "state" dimension. Adaptively observe [5] [6] or identify [7] this MIMO system and solve the coupled MIMO control problem on-line using these parameter (and state) estimates in a state feedback [8, sect. 6.3] [5] or transfer function configuration [8, sect. 7.3] [9].□

(iv) Select an adequately dimensioned feedback control structure for the MIMO actuator/DPS/sensor system and directly update the controller parameters to asymptotically achieve pole placement (as proposed in the scalar case in [10] and [11]). □

These four approaches (especially the last three, which remain open development issues) are discussed in sufficient detail to pinpoint their respective limitations requiring further study.

2. Simultaneous Modal Identification and Control

In [3] and [4] a modal adaptive control strategy for distributed parameter systems is developed, which relies on a priori specification of the decoupling, spatial eigenbasis. As is commonly acknowledged, the most accurate modal model synthesis procedures yield eigenshape predictions with possibly high inaccuracy increasing with the spatial frequency of the eigenshapes. It is expected that in certain cases such a priori eigenshape errors in the strategy of [3] and [4] would lead to instability. The unanswered question is the problem dependent on: How inaccurate can these prespecified eigenshapes be before such unacceptable behavior results? This does not even consider the problems introduced by the reduced-order effects of modal expansion truncation as noted in [4] and [12]. The next three subsections are aimed at circumventing the requirement of exact eigenshape prespecification. The study of reduced order effects would follow as noted in Section II. See the appendix in subsection 6 for a brief summary of the separated variable technique of modeling distributed parameter systems used in [3] and [4].

3. Simultaneous Eigenshape and Modal Dynamics Estimation

Consider the distributed parameter system output described, as in [3] and [4], by the sum of products of modal eigenshape and decoupled amplitude dynamics

$$y(x_q,k) = \sum_{i=1}^{N} \phi_i(x_q) y_i(k) \tag{3-1}$$

where $y(x_q,k)$ represents the output measured at the qth sensor location x_q at time k, $\phi_i(x_q)$ is the magnitude of the ith eigenshape at location x_q, and $y_i(k)$ is the amplitude of the ith mode at time k. Since the modal dynamics are uncoupled

$$y_i(k) = \sum_{\ell=1}^{n} [a_{i\ell} y_i(k-\ell) + b_{i\ell} u_i(k-\ell)], \tag{3-2}$$

where the order of the dynamics n is typically 2 for the linearized small amplitude motion of flexible structures and $u_i(k-\ell)$ the ith modal input at time $k-\ell$

$$u_i(k) = \sum_{j=1}^{C} \phi_i(x_j) u(x_j,k), \tag{3-3}$$

where $u(x_j,k)$ is the actual force applied through the actuator located at x_j. Assume, as suggested in [2], that the eigenshapes ϕ_i can be formed as a finite-dimensional, linear combination of prespecified independent spatial functions $f_s(x)$ as

$$\phi_i(x) = \sum_{s=1}^{P} c_{is} f_s(x). \tag{3-4}$$

The simultaneous eigenshape and modal dynamics estimation problem is, given the "structural" indices of (3-1)-(3-4), i.e. the prespecified basis functions $f_s(x)$, the number of significant modes N, the order of the modal dynamics n, the number of actuators C, and the number of sensors Q, to apply an actual input sequence at each actuator $\{u(x_j,k)\}$, measure the resulting output at each sensor $\{y(x_q,k)\}$, and recursively estimate the eigenshape parameters c_{is} and the modal dynamic parameters $a_{i\ell}$ and $b_{i\ell}$ (i.e. $N(P+2n)$ parameters) to minimize the prediction errors

$$e(x_q,k) = y(x_q,k) - \hat{y}(x_q,k). \tag{3-5}$$

These predicted outputs can be formed from (3-4) with the parameters c_{is} replaced by their current estimates $\hat{c}_{is}(k)$ in

$$\hat{\phi}_i(x,k) = \sum_{s=1}^{P} \hat{c}_{is}(k) f_s(x) \tag{3-6}$$

providing the estimated mode shapes $\hat{\phi}_i$ used in

$$\hat{u}_i(k) = \sum_{j=1}^{C} \hat{\phi}_i(x_j,k) u(x_j,k) \tag{3-7}$$

to provide estimated modal inputs $\hat{u}_i(k)$ to

$$\hat{y}_i(k) = \sum_{\ell=1}^{n} [\hat{a}_{i\ell}(k)\hat{y}_i(k-\ell) + \hat{b}_{i\ell}(k)\hat{u}_i(k-\ell)] \tag{3-8}$$

to provide modal output estimates to

$$\hat{y}(x_q,k) = \sum_{i=1}^{N} \hat{\phi}_i(x_q,k)\hat{y}_i(k). \tag{3-9}$$

The approach of [3] and [4], as noted in the preceding section, is to assume that the ϕ_i are known and from measurements of $y(x_q,k-\ell)$ to solve (3-1) for the $y_i(k-\ell)$ to be used in

$$\hat{y}_i(k) = \sum_{\ell=1}^{n} [\hat{a}_{i\ell}(k)y_i(k-\ell) + \hat{b}_{i\ell}(k)u_i(k-\ell)] \tag{3-10}$$

for comparison with each $y_i(k)$ from solution of (3-1) given $y(x_q,k)$. In order for (3-1) to be uniquely solvable the number of sensors Q must equal the number of modes N to solve

$$\begin{bmatrix} y(x_1,k) \\ y(x_2,k) \\ \vdots \\ y(x_Q,k) \end{bmatrix} = \begin{bmatrix} \phi_1(x_1) & \phi_2(x_1) & \cdots & \phi_N(x_1) \\ \phi_1(x_2) & \phi_2(x_2) & \cdots & \phi_N(x_2) \\ \vdots & \vdots & & \vdots \\ \phi_1(x_Q) & \phi_2(x_Q) & \cdots & \phi_N(x_Q) \end{bmatrix} \begin{bmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_N(k) \end{bmatrix} \tag{3-11}$$

by matrix inversion or its equivalent. The question is whether or not such a technique can (or should) be incorporated to alter (3-6)-(3-9) and avoid the propagation of $\hat{y}_i$ in (3-8). One problem is the need for as many sensors as modes. If this is not feasible, it seems that the left-side of (3-11) could be augmented with further measurements $y(x_q,k+\ell)$. However, this would require inclusion of the plant dynamics on the right hand side resulting in essentially a multivariable modal observer configuration, which does not address the same problem as (3-11) but reverts to the full problem of (3-6)-(3-9). (See the next section for consideration of the multivariable adaptive observer problem, without the modal structure.) Assuming that $Q = N$ is feasible the problem remains of how to simultaneously provide a correction term to (3-6) if the $\hat{\phi}_i$ are used in (3-11). That is, since (3-11) with $\phi_i(\cdot)$ replaced by $\hat{\phi}_i(\cdot,k)$ could be used to provide the $\hat{y}_i(k-\ell)$ in (3-8), would $y-\hat{y}$ using (3-9) provide useful information regarding the error in the $\hat{c}_{is}$?

Since $Q = N$ is itself unattractive, consider the approach of (3-6)-(3-9). First, combine (3-2) and (3-3) to form

$$y_i(k) = \sum_{\ell=1}^{n} [a_{i\ell} y_i(k-\ell) + b_{i\ell} \sum_{j=1}^{C} \phi_i(x_j) u(x_j, k-\ell)]. \tag{3-12}$$

Substituting (3-12) into (3-1) yields

$$y(x_q,k) = \sum_{i=1}^{N} \phi_i(x_q) \{ \sum_{\ell=1}^{n} [a_{i\ell} y_i(k-\ell) + b_{i\ell} \sum_{j=1}^{C} \phi_i(x_j) u(x_j,k-\ell)] \}. \tag{3-13}$$

Then using (3-4) in (3-13) yields

$$y(x_q,k) = \sum_{i=1}^{N} [\sum_{s=1}^{P} c_{is} f_s(x_q)][\sum_{\ell=1}^{n} a_{i\ell} y_i(k-\ell)]$$

$$+ \sum_{i=1}^{N} [\sum_{s=1}^{P} c_{is} f_s(x_q)][\sum_{\ell=1}^{n} b_{i\ell} \sum_{j=1}^{C} \{ \sum_{m=1}^{P} c_{im} f_m(x_j) \} u(x_j,k-\ell)] \tag{3-14}$$

or

$$y(x_q,k) = \sum_{i=1}^{N} \sum_{s=1}^{P} \sum_{\ell=1}^{n} a_{i\ell} c_{is} f_s(x_q) y_i(k-\ell)$$

$$+ \sum_{i=1}^{N} \sum_{s=1}^{P} \sum_{\ell=1}^{n} \sum_{j=1}^{C} \sum_{m=1}^{P} c_{is} c_{im} b_{i\ell} f_s(x_q) f_m(x_j) u(x_j,k-\ell). \tag{3-15}$$

Note that even if the measurement point x_q is constant over all k, since

$$\sum_{i=1}^{N} \phi_i(x_q) \sum_{\ell=1}^{n} a_{i\ell} y_i(k-\ell) \neq \sum_{\ell=1}^{n} \sum_{i=1}^{N} a_{i\ell} \sum_{i=1}^{N} \phi_i(x_q) y_i(k-\ell)$$

$$= \sum_{\ell=1}^{n} \sum_{i=1}^{N} a_{i\ell} y(x_q,k-\ell), \tag{3-16}$$

(3-15) is not directly transferable to a multi-input ($u(x_j,\cdot)$ over j), single-output ($y(x_q,\cdot)$ for one q) ARMA process. This is due to spatial coupling, i.e. the output $y(x_1,\cdot)$ at sensor location x_1 is dependent on the past outputs $y(x_q,\cdot)$ over all q not just those for q = 1. Therefore even the problem of estimating the $N(P+P^2)n$ different parameter products in (3-15) (rather than the desired $N(P+2n)$ parameters $a_{i\ell}$, $b_{i\ell}$, c_{is}) cannot be phrased as an equation error parameter estimation [13] problem, because the y_i are not available. (Note $c_{is} = c_{im}$ for $s = m$). Due to the lack of a single-output ARMA form an output error formulation [7] [14] is also impossible. One temptation is to relate the form of (3-8) to that for output error estimation. The measurement of y and not y_i requires (3-9) for the prediction error and causes a combination of the y_i as in a parallel filter implementation, as noted in [15]. The difficulty of (3-6)-(3-9) in addition to this structural peculiarity noted in [15] is that the effective parameters in this y_i combiner are also now unknown, which leads to products of unknown parameters, as is apparent from (3-15). This product form will be termed (as in [16]) a bilinear-in-the-parameters estimation problem, for which there is no known globally stable recursive solution.

Do not confuse the difficulty in establishing a multi-input, single-output ARMA model from (3-15) with an inability to do so in general. Clearly if a linear multi-input, multi-output ARMA form exists of the form

$$X(k) = \sum_{i=1}^{n} [A_i X(k-i) + B_i U(k-i)] \tag{3-17}$$

then

$$X(z) = [z^n I - \sum_{i=1}^{n} A_i z^{n-i}]^{-1} [\sum_{i=1}^{n} B_i z^{n-i}] U(z) \tag{3-18}$$

or

$$X(z)\{\det(z^n I - \sum_{i=1}^{n} A_i z^{n-i})\} = \{\mathrm{Adj}[z^n I - \sum_{i=1}^{n} A_i z^{n-i}]\}[\sum_{i=1}^{n} B_i z^{n-i}] U(z) . \tag{3-19}$$

Taking the inverse z-transform yields

$$X(k) = \sum_{i=1}^{t} a_i X(k-i) + \sum_{i=1}^{t} \bar{B}_i U(k-i), \tag{3-20}$$

where t = (n)·(dimension of X). So, a degree change and loss of internal information is required to use a model of the form of (3-20) if it is used in place of (3-17). That a coupled model, such as (3-17), exists will be the premise for the next two sections.

Returning to the computation of the prediction error in (3-5) further emphasizes the bilinear-in-the-parameters form of the underlying parameter estimation problem. Using (3-13) and the similar form arising from (3-6)-(3-9), (3-5) becomes

$$e(x_q,k) = \sum_{i=1}^{N} \sum_{s=1}^{P} [c_{is} - \hat{c}_{is}(k)] f_s(x_q) y_i(k)$$

$$+ \sum_{i=1}^{N} [\sum_{s=1}^{P} \hat{c}_{is}(k) f_s(x_q)][y_i(k) - \hat{y}_i(k)]. \tag{3-21}$$

Using (3-2) and (3-8) converts (3-21) to

$$e(x_q,k) = \sum_{i=1}^{N} \sum_{s=1}^{P} [c_{is} - \hat{c}_{is}(k)] f_s(x_q) y_i(k)$$

$$+ \sum_{i=1}^{N} [\sum_{s=1}^{P} \hat{c}_{is}(k) f_s(x_q)][\sum_{\ell=1}^{n} \{a_{i\ell} - \hat{a}_{i\ell}(k)\} \hat{y}_i(k-\ell)$$

$$+ \{b_{i\ell} - \hat{b}_{i\ell}(k)\} \hat{u}_i(k-\ell)]$$

$$+ \sum_{i=1}^{N} [\sum_{s=1}^{P} \hat{c}_{is}(k) f_s(x_q)][\sum_{\ell=1}^{n} a_{i\ell} \{y_i(k-\ell) - \hat{y}_i(k-\ell)\}$$

$$+ b_{i\ell} \{u_i(k-\ell) - \hat{u}_i(k-\ell)\}]. \tag{3-22}$$

Then from (3-3), (3-4), (3-6), and (3-7), (3-22) becomes

$$e(x_q,k) = \sum_{i=1}^{N} \sum_{s=1}^{P} [c_{is} - \hat{c}_{is}(k)][f_s(x_q) y_i(k)]$$

$$+ \sum_{i=1}^{N} \sum_{\ell=1}^{n} [a_{i\ell} - \hat{a}_{i\ell}(k)][\sum_{s=1}^{P} \hat{c}_{is}(k) f_s(x_q) \hat{y}_i(k-\ell)]$$

$$+ \sum_{i=1}^{N} \sum_{\ell=1}^{n} [b_{i\ell} - \hat{b}_{i\ell}(k)][\sum_{s=1}^{P} \hat{c}_{is}(k) f_s(x_q) \hat{u}_i(k-\ell)]$$

$$+ \sum_{i=1}^{N} \sum_{m=1}^{P} [c_{im} - \hat{c}_{im}(k)][\sum_{\ell=1}^{n} \sum_{j=1}^{c} b_{i\ell} \{\sum_{s=1}^{P} \hat{c}_{is}(k) f_s(x_q)\} f_s(x_j) u(x_j,k-\ell)]$$

$$+ \sum_{i=1}^{N} \sum_{\ell=1}^{n} [y_i(k-\ell) - \hat{y}_i(k-\ell)][a_{i\ell}\{\sum_{s=1}^{P} \hat{c}_{is}(k) f_s(x_q)\}]. \qquad (3\text{-}23)$$

Due to (3-16) the last term in (3-23) is not a regression of $e(x_q,\cdot)$ that could be moved to the left side of (3-23) as in an output error formulation [14]. Also $b_{i\ell}$ is unknown in the next to the last term (as in y_i in the first term) in (3-23) where it is needed to form the "input" to the $\tilde{c}_{im}(\overset{\Delta}{=} c_{im} - \hat{c}_{im})$ segment of the weighted parameter error combination. This latter problem can be solved by approximating $b_{i\ell}$ with $\hat{b}_{i\ell}$ (and y_i with $\hat{y}_i$) as is done in [16] and [17] for a different bilinear-in-the-parameters estimation problem. Since $c_{is} = c_{im}$ for $s = m$ the two terms in $\tilde{c}$ present a nontraditional problem.) This clearly limits any subsequent estimation scheme, based on this approximation, to local convergence. The structure of the last term in (3-23) is the more bothersome issue. Assuming that $y_i \simeq \hat{y}_i$ is an unacceptable method of ignoring this last term. To assume that $a_{i\ell}$ is constant over i is also absurd (unless a breakdown similar to (3-23) is achievable for the plant form in (3-20), which presently seems possible only by losing the decoupled structure of (3-2)). Another improbable situation is that one set of constant prediction error smoothing coefficients would make each of the i forward dynamics in an output error identifier error system strictly positive real [18]. The form of (3-23) is enticingly close to a standard output error formulation but the problem noted in (3-16) alleviated only by effective solution of (3-11) hinders further consideration of this distributed parameter system identification technique.

The justification for developing a simultaneous eigenshape and amplitude dynamics estimator is apparent from [3], [4], and the preceding section, i.e. the recursive estimation of the $a_{i\ell}$, $b_{i\ell}$, c_{is}, and c_{im} permits real-time solution of decoupled, scalar pole placement problems. This contrasts with the large computational effort involved in solution of the pole placement problem for a coupled matrix ARMA description as would result from an arbitrary fixed choice for the "modal" shape "basis" c_{is}. Coupling would result in (3-2), i.e. each modal output y_i would be dependent on past values of all modal outputs and modal inputs, not just its own as in (3-2). Note that "pulse" forms for the $c_{is}f_s(x)$ products, for example, would result in a measured, input $(u(x_j,\cdot)$ - output $(y(x_q,\cdot))$ matrix ARMA description, thereby effectively bypassing the modal coordinate transformations. As this approach makes the control problem solution more involved, the parameter estimation problem becomes solvable. Such an approach is taken in the next two sections.

4. Simultaneous Multivariable Identification and Control

As an alternative to the modal decomposition representation for DPS, consider a coupled multi-input multi-output system description. In the time domain, this becomes a state space representation with multiple inputs and outputs and suitable state variables. The state space dimension will depend on the characteristic behavior of the system in the space and time dimensions and the modeling accuracy required. Since DPS require an infinite number of modes in their modal description, this corresponds to the need for an infinite dimensional state space for a correspondingly complete description. In practice, however, only a finite set of modes and therefore finite state space will be assumed for analysis. (See the appendix (sec. 6) for a more detailed discussion of this modeling issue.) Even though a DPS is accurately modeled by a finite state space description, this dimension may be too large to manipulate in any reasonable real-time control application. The required additional reduction in system state dimension results in the reduced order control problem, and subsequent ill effects caused by modeling inaccuracy spillover, etc. [12].

This discussion though, will be limited to the use of simultaneous identification and control (indirect adaptive control) on multivariable systems without considering these reduced-order modeling effects. The time domain approach of simultaneous parameter identification and state observation for use in state variable feedback, will be based on the parameterized scalar adaptive observer developed by G. Kreiselmeier [5][19]. The alternate approach to the indirect adaptive control problem will be based on the frequency domain representation of the multivariable system. This is commonly expressed by either a transfer function matrix or matrix fraction description relating plant inputs to plant outputs. Here, the discussion will focus on the frequency domain version of a Luenberger observer and full state feedback to accomplish the desired closed loop control objective.

4.1. Time Domain Approach

Multivariable extension of Kreisselmeier's adaptive observer [19] can be approached by first looking at the major steps in the development for single-input single-output systems. If the plant is known to be observable and of state dimension n, i.e.

$$\dot{x} = Ax + bu \qquad A: n\times n, \qquad b: n\times 1, \qquad x: n\times 1$$
$$y = c^T x \qquad c: n\times 1 \tag{4-1}$$

where (A,c) is observable, an n dimensional observer can be constructed to assymptotically estimate the plant states [20]

$$\dot{\hat{x}} = F\hat{x} + g\, y + h\, u \qquad F: n\times n,\ g: n\times 1,\ h: n\times 1. \tag{4-2}$$

For the state estimate error $\tilde{x} = x - \hat{x}$ to approach zero, it is required that

$$F = A - gc^T \qquad \text{and } b = h \tag{4-3}$$

where F has eigenvalues strictly in the left half complex plane. This observer has the structure in Fig. 4.1. The solution for the state estimator is

$$\hat{x} = e^{(A-gc)t}\hat{x}_0 + \int_0^t e^{(A-gc)(t-\tau)}(g\, y + h\, u)d\tau . \tag{4-4}$$

However, if the parameters of the plant are unknown i.e. A, B, C are known only in dimension, then the observer parameters g and h must be estimated such that the state estimate $\hat{x}$ does indeed converge to the plant state x. Kreisselmeier's estimation methods [19] rely on being able to separate the observer dynamics and the observer parameters g and h. For scalar input and output systems, this is easily accomplished by simply commuting terms in the integrand in (4-4)

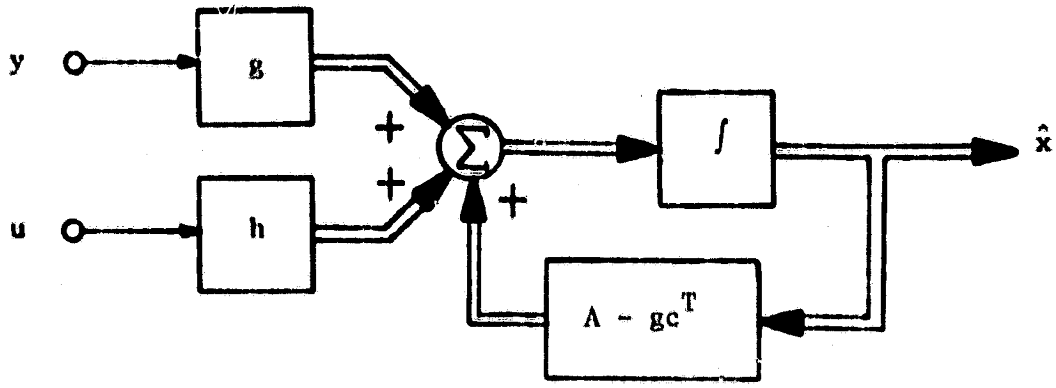

Fig. 4.1 : Luenberger Observer

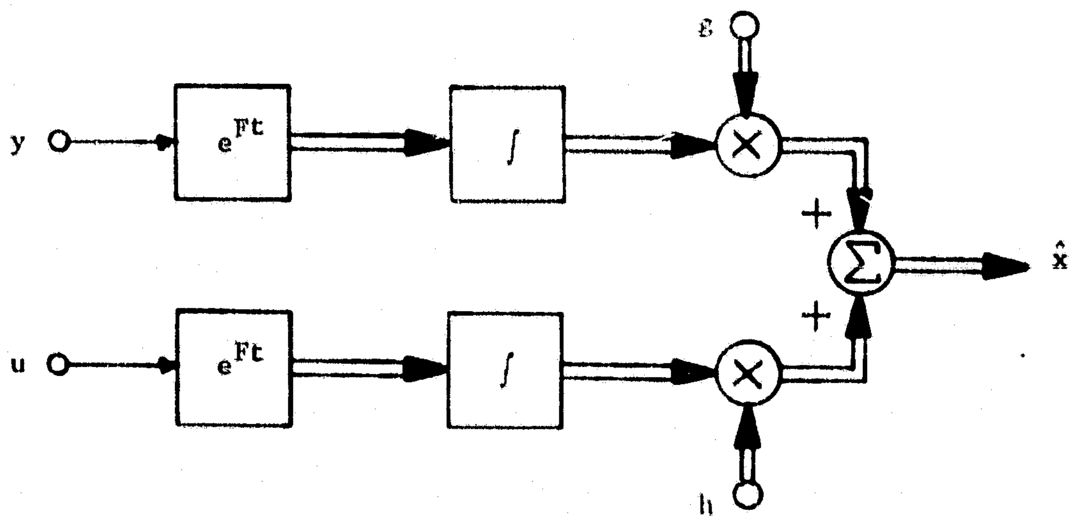

Fig. 4.2 : Kreisselmeier's Scalar Parameterized Observer

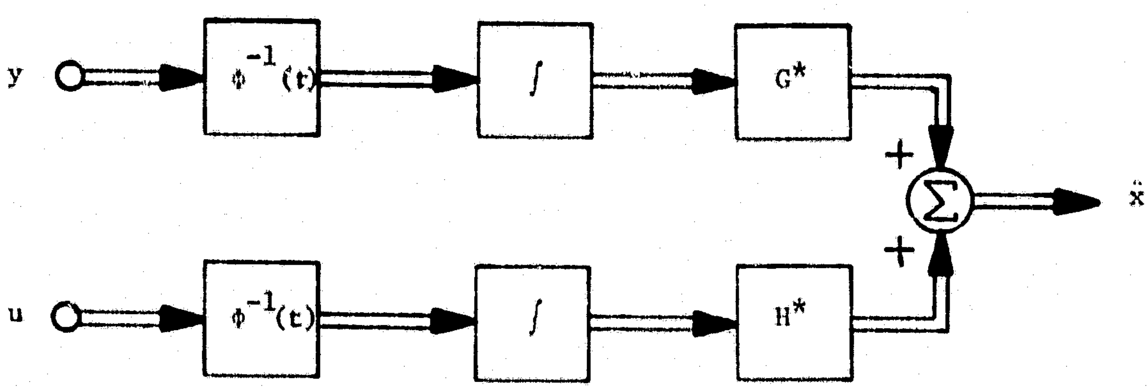

Fig. 4.3 : Multivariable Parameterized Observer

$$\int_0^t e^{(A-gc)(t-\tau)}(g\,y + h\,u)d\tau$$

$$= \left[\int_0^t e^{(A-gc)(t-\tau)} y\; d\;\tau\right] g + \left[\int_0^t e^{(A-gc)(t-\tau)} u\; d\;\tau\right] h \; . \qquad (4\text{-}5)$$

The system can now be represented by the structure in Fig. 4-2.

If this approach is now used for the multi-input multi-output case, the plant state description becomes

$$\dot{x} = Ax + Bu \qquad A: nxn \quad B: nxm, \quad u: mx1, \quad x: nx1$$
$$y = Cx \qquad C: pxn, \quad y: px1 \qquad (4\text{-}6)$$

where there are m inputs and p outputs. The system is still assumed observable. The observer is described by

$$\dot{\hat{x}} = Fx + Gy + Hu \qquad F: nxn \quad G: nxp, \quad H: nxm \qquad (4\text{-}7)$$

with F having strictly left half plane eigenvalues and

$$F = A - GC \quad , \qquad B = H \qquad (4\text{-}8)$$

to cause assymptotic state observation. This system has the same structure as the scalar system shown in fig. 4-1, except that y and u are now vectors and g and h are now matrices. The solution for the state estimate $\hat{x}$ (corresponding to (4-4)) is

$$\hat{x} = e^{(A-GC)t}\hat{x}_0 + \int_0^t e^{(A-GC)(t-\tau)}(Gy + Hu)d\tau \qquad (4\text{-}9)$$

Notice that the outputs y and inputs u will not, in general, commute with G and H, respectively.

To separate the observer dynamics from the observer parameters contained in G and H, a unique solution to

$$Gy = y^*G^* \quad \text{and} \quad Hu = u^*H^* \qquad (4\text{-}10)$$

<u>OR</u>

$$e^{(A-GC)t}\; G = G^*\; e^{(A-GC)^*t}$$
$$\text{and} \quad e^{(A-GC)t}\; H = H^*\; e^{(A-GC)^*t} \qquad (4\text{-}11)$$

must exist for some set of $(\cdot)^*$quantities. Notice that a unique solution to equation (4-10) will not exist unless G and y span equivalently dimensioned subspaces, i.e. G and y are square, invertible matrices of the same dimension. The same is true for H and u. The only possibility is when y and u are scalars, which reduces the problem to the SISO case discussed earlier.

Equation (4-11) has similar restrictions. Here, however, if G and H are square, invertible, and of the same dimension as $e^{(A-GC)t}$, then unique solutions for G* and H* will exist. For convenience, let $e^{(A-GC)t} = \Phi(t)$, then

$$G^* = \Phi(t)G\Phi^{-1}(t) \qquad G: n\times n, \text{ invertible}$$
$$H^* = \Phi(t)H\Phi^{-1}(t) \qquad H: n\times n, \text{ invertible} \tag{4-12}$$

The solution for the state estimate becomes

$$\hat{x} = \Phi(t)\hat{x}_0 + G^* \int_0^t \Phi(t)^{-1} y\, d\tau + H^* \int_0^t \Phi(t)^{-1} u\, d\tau \ . \tag{4-13}$$

This system has the structure in Fig. 4.3, which is similar to Fig. 4.2. The required restrictions for this result, however, are severe:

(i) The G* and H* estimates must be assymptotically invertible for $\hat{x}$ to converge to x .

(ii) The number of inputs and outputs must be the same as the number of states.

(iii) The minimum number of states used to describe the system behavior must not be overestimated, or G* and H* will never be assymptotically invertible and the state observer may never converge to the true plant states. Also, a non-minimal state description implies that some states are either unobservable or uncontrollable or both. If an observed state is uncontrollable, the feedback law may require unbounded control inputs in an effort to effect such a state. This may drive the system out of the region of linear operation, and is clearly to be avoided.

The specialization of the plant model for flexible spacecraft may lessen the severity of the restriction in point (ii), since it may be possible to add sensors and actuators to satisfy this point. Restrictions (i) and (iii) still remain, though, with their inherent numerical problems.

4.2. Frequency Domain Approach

Consider the plant having m inputs, p outputs and a pxm proper transfer function matrix T(z). Represent T(z) in a left matrix fraction description (MFD), not necessarily minimal [8] (i.e. irreducible, relatively left prime [20])

$$T(z) = P^{-1}(z)L(z) \tag{4-14}$$

where P(z) and L(z) are polynomial matrices. The elements of P and L are polynomials in z whose coefficients are unknown. The system output y(z) and input u(z) are then related by

$$y(z) = P^{-1}(z)L(z)u(z) \tag{4-15}$$

and

$$P(z)y(z) = L(z)u(z) \tag{4-16}$$

Rewrite P(z) and L(z) as sums of products of constant coefficient matrices, P_i and L_i, and powers of z:

$$\left[\sum_{i=0}^{n} P_i z^i\right] y(z) = \left[\sum_{i=0}^{q} L_i z^i\right] u(z) \tag{4-17}$$

where n is the largest power of z in P(z) and q is the largest power of z in L(z), where $q \leq n$ due to the properness of T(z). Now if P(z) is row proper [8], P_0 will be invertible and the matrix ARMA difference equation for y(k) can be given by

$$y(k) = P_0^{-1}\left[-\sum_{i=1}^{n} P_i y(k-i) + \sum_{i=0}^{q} L_i u(k-n+q-i)\right] \tag{4-18}$$

Based on this matrix ARMA, some estimation procedure, e.g. [7], can then be used to estimate the ARMA coefficients, thereby providing the plant parameter estimates for the left MFD.

A curious quirk in multivariable systems not found in scalar systems is that an equivalent right matrix fraction description for T(z)

$$T(z) = R(z)Q^{-1}(z) \quad (4\text{-}19)$$

does not lead, in general, to an ARMA difference equation for y(k). Notice that the dual to equation (4-15)

$$y(z) = R(z)Q^{-1}(z)u(z) \quad (4\text{-}20)$$

cannot be separated to a dual form of equation (4-16) since R(z) is not square unless p = m and even then not necessarily invertible. If such a special case holds, then a result similar to (4-16) is

$$\det[Q(z)]\text{adj}[R(z)]y(z) = \det[R(z)]\text{adj}[Q(z)]u(z) \quad (4\text{-}21)$$

If Q(z) and R(z) contains only common unimodular [21] right factors (Q(z) and R(z) relatively right prime (r.r.p.) [8][21]) then the highest power of z on each side of (4-21) will be $\leq 2n$. Here n is the order of the plant, being the number of shifts in a difference equation needed to describe the plant. Since an n^{th} order system has the order of det[Q(z)] equal to n (Q(z), R(z) r.r.p.) or greater than n (Q(z),R(z) not r.r.p.) [8,p.173] and the order of adj[R(z)] in z is one or greater, the minimum order on both sides of (4-21) is n. This minimum order of n occurs when R(z) contains only constant elements in which case the system has no transmission zeros [8,p.189]. Thus, a minimal right MFD can result directly from a matrix ARMA difference equation only in the special case where p = m and R(z) is unimodular. For any particular plant in an adaptive control structure, the system order n must also be known for an estimation procedure to eventually converge to a minimal right MFD for the plant. <u>However, if $p \neq m$ then a right MFD can never result from a matrix ARMA difference</u>

equation. Contrast this to the existence of the left MFD based on a matrix ARMA. A left MFD, so derived, always exists, although it will be minimal only if the orders n and q are known for the plant. Therefore, how the estimates of the plant parameters embodied in the left MFD are used in feedback control, as well as the role minimality plays in the control effort calculation will now be discussed.

Using the frequency domain representation of a Luenberger observer [8,p.238] provides full plant "state" information for feedback to provide arbitrary pole placement. This "transfer function compensation" scheme has the structure found in Fig. 4.4. If K(z) and H(z) can be found to satisfy the well-known Bezout identity [21]

$$K(z)Q(z) + H(z)R(z) = I \tag{4-22}$$

then the partial state v can be recreated by measurements of the plant inputs u and outputs y, i.e.

$$K(z)Q(z)v + H(z)R(z)v = v \tag{4-23}$$

The new plant input is then

$$u = r - F(z)v \tag{4-24}$$

and since

$$u = Q(z)v \tag{4-25}$$

$$r = [Q(z) + F(z)]v \tag{4-26}$$

and with

$$y = R(z)v \tag{4-27}$$

$$y = R(z)[Q(z) + F(z)]^{-1}r \tag{4-28}$$

in which F(z) is chosen such that

$$Q(z) + F(z) = P_d(z) \tag{4-29}$$

where $P_d(z)$ is the desired closed-loop denominator matrix. At each iteration of an adaptive control algorithm F(z) could be found, based on the

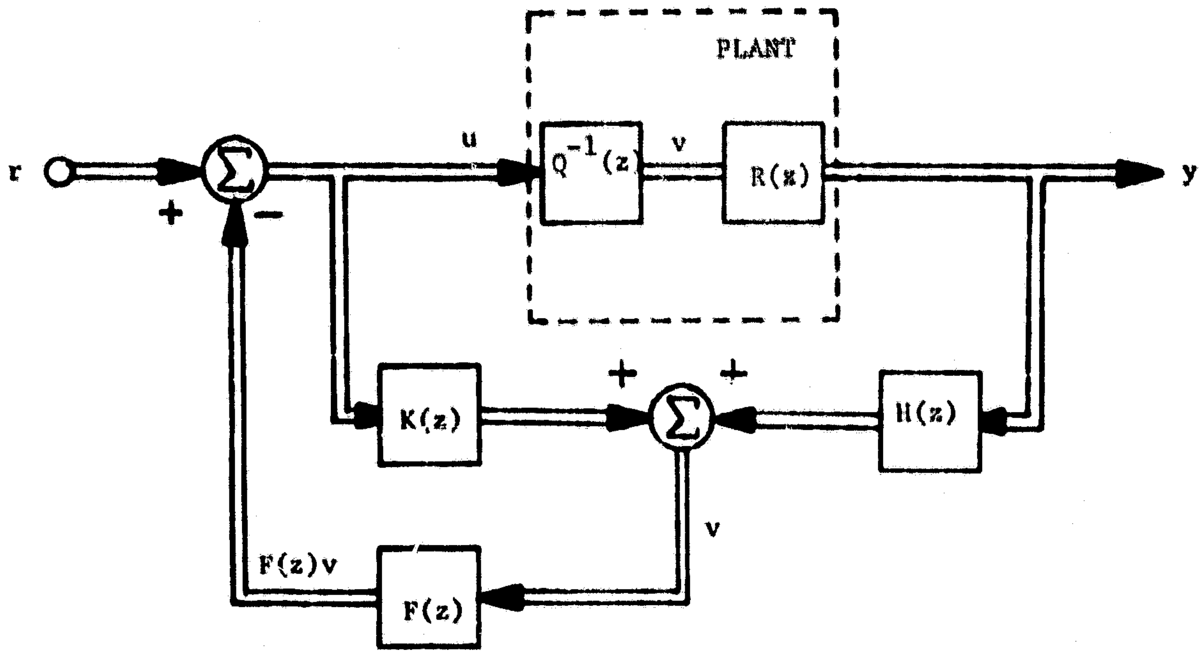

Fig. 4.4 : Frequency Domain Multivariable Pole Placement

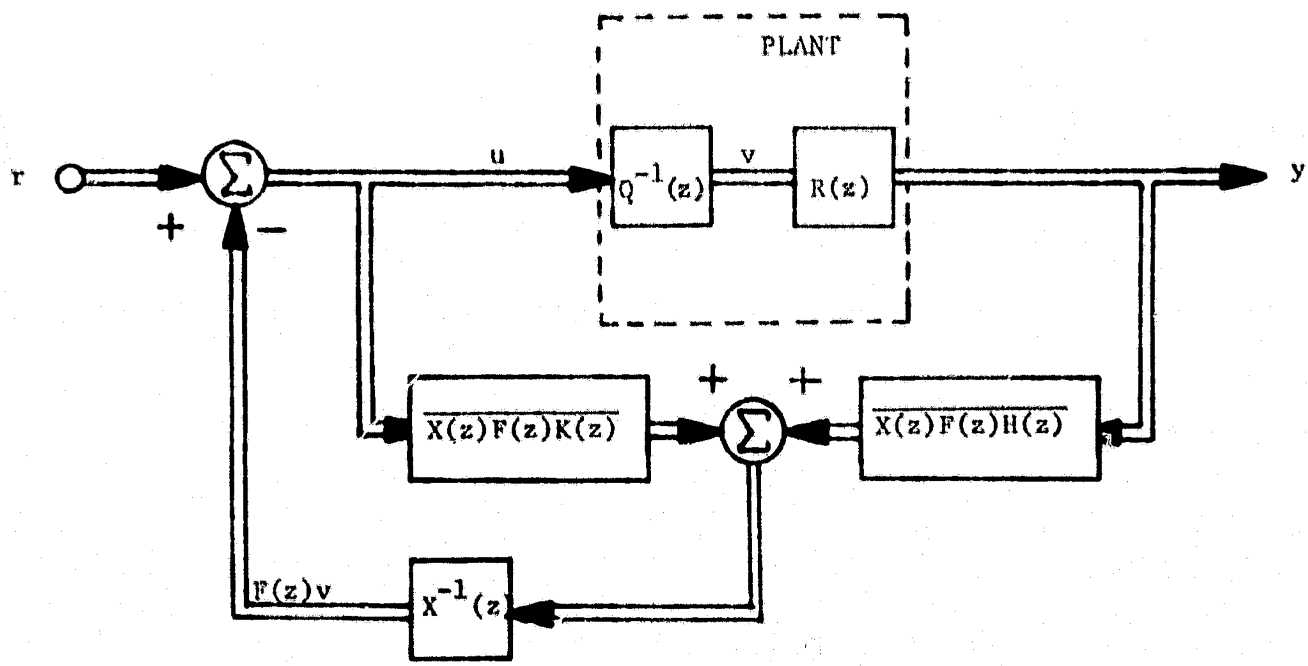

Fig. 4.5 : Proper Frequency Domain Pole Placement Compensation

current estimate of the plant matrix Q(z), to satisfy (4-29). Note however, that Q(z) is the denominator matrix of a right MFD, and the estimation of the plant parameters produces a left MFD. Moreover, the right MFD factors Q(z) and R(z) must be relatively right prime for (4-22) to be guaranteed a solution for K(z) and H(z) [21]. A required step, then, will be to find a minimal right MFD from a (not necessarily minimal) left MFD at each iteration. That this can always be done (but is computationally involved) will now be shown.

Following the procedure in [8][21] for obtaining a greatest common left divisor (g.c.l.d.) for the left MFD $P^{-1}(z)L(z)$, find a unimodular left multiplier U(z) for the pair P(z) and L(z) to reduce the construction to lower left triangular form

$$[P(z), L(z)]U(z) = [M(z), \underline{0}]. \qquad (4\text{-}30)$$

The g.c.l.d. M(z) is lower triangular and has the same rank as that of [P(z),L(z)]. If [P(z),L(z)] has full row rank m for all z, then M(z) has rank m for all z, and is therefore unimodular. In this case, since a g.c.l.d. of the pair is unimodular, that pair is relatively left prime. If the pair [P(z),L(z)] has rank m for almost, but not all z, then M(z) will have similar rank, and M(z) will be invertible. In this case the pair is not r.l.p. but can be made so by eliminating the common non-unimodular factor M(z). Multiplying both sides by $M^{-1}(z)$ yields

$$M^{-1}(z)[P(z), L(z)]U(z) = [I_m, \underline{0}] \qquad (4\text{-}31)$$

or equivalently (I_m is an mxm identity matrix)

$$[P^*(z), L^*(z)]U(z) = [I_m, \underline{0}] \qquad (4\text{-}32)$$

where P*(z) and L*(z) are r.l.p. factors of a left MFD for T(z). Now a r.r.p. MFD for T(z) can be obtained by partitioning U(z) as

$$[P^*(z), L^*(z)] \underbrace{\begin{bmatrix} K^*(z) & R(z) \\ H^*(z) & -Q(z) \end{bmatrix}}_{U(z)} = [I_m, \underline{0}] \quad . \tag{4-33}$$

Since $U(z)$ is unimodular, $R(z)$ and $Q(z)$ are r.r.p. [22] and

$$P^*(z)R(z) - L^*(z)Q(z) = 0 \tag{4-34}$$

giving

$$R(z)Q^{-1}(z) = P^{*-1}(z)L^*(z) = T(z) \tag{4-35}$$

making $R(z)Q^{-1}(z)$ a minimal right MFD for $T(z)$. Now since $K^*(z)$ and $H^*(z)$ are also r.r.p., there exist a $K(z)$ and $H(z)$ that satisfy

$$H(z)K^*(z) + K(z)H^*(z) = I_m \tag{4-36}$$

and if $K(z)$ and $H(z)$ are r.l.p. then

$$H(z)R(z) - K(z)Q(z) = 0 \tag{4-37}$$

resulting in the construction [21]

$$\begin{bmatrix} P^*(z) & L^*(z) \\ H(z) & K(z) \end{bmatrix} \underbrace{\begin{bmatrix} K^*(z) & R(z) \\ H^*(z) & -Q(z) \end{bmatrix}}_{U(z)} \quad \begin{bmatrix} I_m & 0 \\ 0 & I_p \end{bmatrix} \tag{4-38}$$

where $U(z)$ is unimodular, and therefore invertible, so the solution for $H(z)$ and $K(z)$ can be given by

$$[H(z), K(z)] = \begin{bmatrix} \underline{0} & \underline{0} \\ \underline{0} & I_p \end{bmatrix} U^{-1}(z) \quad . \tag{4-39}$$

This $H(z)$ and $K(z)$ are a set of polynomial matrices that satisfy equation (4-22) for the re-creation of the partial state v. It should be pointed out that the above procedure for finding these matrices, while always possible, is almost never a trivial matter. The key difficulty is in finding the unimodular matrix $U(z)$, which must be done at every iteration of the adaptive algorithm on each new estimate of the P(z) and L(z) matrices.

After the required H(z) and K(z) matrices have been found, a problem still exists in the implementation of the feedback control: H(z), K(z), and F(z) are polynomial matrices so that feedback paths are non-proper, hence the system is not realizable in real time applications. To overcome this difficulty, introduce a stable, invertible matrix X(z) into (4-22) along with the feedback F(z)

$$X(z)F(z)K(z)Q(z) + X(z)F(z)H(z)R(z) = X(z)F(z). \tag{4-40}$$

For any X(z) and F(z), the above calculated H(z) and K(z) are solutions to (4-40), and since the r.l.p. pair P*(z) and L*(z) form a left prime basis [22] for Q(z) and R(z), the general solution for (4-40) is

$$\begin{aligned} &[\ \overline{X(z)F(z)K(z)},\ \overline{X(z)F(z)G(z)}\ ] \\ &\quad = X(z)F(z)[K(z),H(z)] + W(z)[P^*(z),L^*(z)] \end{aligned} \tag{4-41}$$

where W(z) is any polynomial matrix. In this solution, X(z)F(z)[K(z),H(z)] is the particular solution and W(z)[P*(z),L*(z)] is the homogeneous solution. Now choose W(z) such that the general solution $[\ \overline{X(z)F(z)K(z)},\ \overline{X(z)F(z)H(z)}\ ]$ has row degree less than X(z) so that $X(z)^{-1}\ \overline{X(z)F(z)K(z)}$ and $X(z)^{-1}\ \overline{X(z)F(z)H(z)}$ are proper matrix fractions. The system now takes the form shown in Fig. 4-5. The selection of W(z) depends on the particular choice of X(z) and the F(z),L(z) and P(z) at each iteration of the adaptive algorithm and is not a trivial problem. For example, the procedure in [8] requires the inversion of an "eliminant matrix" to solve for the $\overline{X(z)F(z)K(z)}$ and $\overline{X(z)F(z)H(z)}$ (in their notation K(s) and H(s) respectively, which is typically a 10x10 matrix for p=m=3. Note also that the matrix manipulations involved in any solution technique are complicated by the polynomial form of the matrix elements, particularly if a machine-calculated solution is desired.

Minimality of the system MFD is important in two respects. The first is that the estimation procedure, used to provide the coefficients of the initial left MFD, may require that a minimal structure be known a priori for the plant estimation to converge to some meaningful characterization of the plant [7]. This required a priori information includes knowledge of the degree n of the plant and effective foreknowledge of the controllability or observability indices [8], as well as other structural information [23]-[25], such as the relative system degree n-q which is related to high frequency behavior. The second reason for minimality occurs in the solution for the feedback dynamics based on some estimated plant MFD. Here, the general solution does not require the initial left MFD estimate to be minimal, since the required minimal right MFD is found in the course of the solution regardless of the minimality of the left MFD. However, the solution process is simplified if a minimal left MFD is available.

The key problems with this frequency domain approach are:

(i) Depending on the particular plant parameter estimation schemes and control effort calculation techniques used, minimal system descriptions may be required. (This is analogous to the result obtained in the discussion of time domain indirect adaptive control.)

(ii) The necessary calculations (left to right MFD conversion and compensator parameter calculation) are excessive for performance at each step in a real-time adaptive algorithm.

The next section will discuss direct adaptive control, which does not require explicit identification of the plant parameters, as a possible alternative to this scheme and its associated problems.

5. Direct Adaptive Multivariable Pole Placement

Consider the multi-input, multi-output plant described by the partial state description

$$A(q^{-1})z(k) = u(k) \tag{5-1}$$

$$B(q^{-1})z(k) = y(k), \tag{5-2}$$

where the nxn polynomial matrix $A(q^{-1})$ in the time delay operator q^{-1} is invertible such that (5-1) and (5-2) results in the right matrix fraction description (MFD) [21,chpt.6]

$$y(k) = B(q^{-1})A^{-1}(q^{-1})u(k), \tag{5-3}$$

where y and u are appropriately dimensioned output and input vectors. Note that a left MFD results from a matrix ARMA model as shown in [7] and discussed in the preceding section. It turns out that left MFDs are best suited for parameter estimation and state observation but right MFDs are assumed for feedback control design. As shown in [8] and [9], in order to achieve pole placement via the control law

$$C(q^{-1})u(k) = r(k) + D(q^{-1})y(k), \tag{5-4}$$

where C and D are appropriately dimensioned polynomial matrices, C and D must be chosen to satisfy

$$F(q^{-1}) = C(q^{-1})A(q^{-1}) - D(q^{-1})B(q^{-1}), \tag{5-5}$$

where $F(q^{-1})$ is the desired denominator polynomial matrix. This is substantiated by substituting (5-1) and (5-2) into (5-4) for

$$C(q^{-1})A(q^{-1})z(k) = r(k) + D(q^{-1})B(q^{-1})z(k) \tag{5-6}$$

or

$$[C(q^{-1})A(q^{-1}) - D(q^{-1})B(q^{-1})]z(k) = r(k). \tag{5-7}$$

Using (5-5) in (5-7) and assuming F is invertible yields

$$z(k) = F^{-1}(q^{-1})r(k). \tag{5-8}$$

Use of (5-2) in (5-8) results in the right MFD

$$y(k) = B(q^{-1})F^{-1}(q^{-1})r(k), \tag{5-9}$$

which in comparison with (5-3) shows that the poles of (5-3) have been shifted but that the transmission zeros of (5-3) are unchanged if (5-3) and (5-9) are minimal. A critical question is the a priori structural information required to structure C and D and F such that a solution exists to (5-5).

Following the scalar discrete-time strategy [11] (based on the continuous-time strategy in [10]) for adaptively parameterizing (5-4) without a priori specification of A and B in (5-3), a discrete-time multivariable, adaptive pole placer will be proposed. Use (5-5) to operate on z yielding

$$F(q^{-1})z(k) = C(q^{-1})A(q^{-1}) - D(q^{-1})B(q^{-1})z(k) \tag{5-10}$$

If (5-3) is minimal, according to the Bezout identity [21,p.379] $G(q^{-1})$ and $H(q^{-1})$ exist such that

$$G(q^{-1})A(q^{-1}) + H(q^{-1})B(q^{-1}) = I. \tag{5-11}$$

Inserting (5-11) into (5-10) yields

$$F(q^{-1})G(q^{-1})A(q^{-1})z(k) + F(q^{-1})H(q^{-1})B(q^{-1})z(k)$$
$$= C(q^{-1})A(q^{-1})z(k) - D(q^{-1})B(q^{-1})z(k). \tag{5-12}$$

Using (5-1) and (5-2) in (5-12) yields

$$F(q^{-1})G(q^{-1})u(k) + F(q^{-1})H(q^{-1})y(k)$$
$$= C(q^{-1})u(k) - D(q^{-1})y(k) \tag{5-13}$$

Assuming that F and G (and H) are interchangeable yields

$$G(q^{-1})\{F(q^{-1})u(k)\} + H(q^{-1})\{F(q^{-1})y(k)\}$$
$$- C(q^{-1})\{u(k)\} + D(q^{-1})\{y(k)\} = 0. \tag{5-14}$$

As in [10] and [11], estimating G, H, C, and D results in

$$e(k) = \tilde{G}(q^{-1},k)\{F(q^{-1})u(k)\} + \tilde{H}(q^{-1},k)\{F(q^{-1})y(k)\}$$
$$- \tilde{C}(q^{-1},k)\{u(k)\} + \tilde{D}(q^{-1},k)\{y(k)\}$$

$$= \hat{C}(q^{-1},k)\{u(k)\} - \hat{D}(q^{-1},k)\{y(k)\}$$
$$- \hat{G}(q^{-1},k)\{F(q^{-1})u(k)\} - \hat{H}(q^{-1},k)\{F(q^{-1})y(k)\}, \qquad (5\text{-}15)$$

where, e.g., $\tilde{G} = G - \hat{G}$. The error vector e in (5-15) is recognizable as an equation error formulation [13],[26], which suggests a recursive solution of the form

$$\theta(k+1) = \theta(k) + P(k)X(k)e(k) \qquad (5\text{-}16)$$

where

$$\theta(k) = [\hat{C}(q^{-1},k)\ \hat{D}(q^{-1},k)\ \hat{G}(q^{-1},k)\ \hat{H}(q^{-1},k)],$$

$$X(k) = \begin{bmatrix} u(k) \\ y(k) \\ F(q^{-1})u(k) \\ F(q^{-1})y(k) \end{bmatrix} \qquad (5\text{-}18)$$

and P is a suitable chosen step-size matrix. Note that this recursion could be performed line by line with each of the entries in the equation error vector e, which permits parallel processing thereby reducing the computation time per iteration. The number of terms in each of these parallel problems increases linearly with the degree of the system in (5-3) thereby requiring an increase in the order of the entries in (5-5).

As noted in [10] and [11] the stability problem even for the scalar case is unresolved. If the G and H of (5-11) are known exactly and not updated in (5-16) then, at least in the scalar case [10][11], stability can be assured by the technical device of [27] due the stably invertible transfer function from u (and y) to e. Comparing (5-5) and (5-11) reveals that foreknowledge of G and H is equivalent to foreknowledge of the solution to the decoupling, inverse control problem, which need not be internally stable. Clearly knowledge of this solution corresponds to knowledge of the plant parameters. However this encourages the expectation of local stability if the G and H are approximately correct initially. In order to retain the similarity of growth rates of the input or output and the equation error, [10] suggests bounds on the $\hat{G}$ and $\hat{H}$. How this is to be achieved with limited a priori plant information is uncertain; though a priori ranges for A and B may translate into acceptable G and H. Possibly as uncertainty in A and B increases the acceptable range for G and H narrows to the solution of (5-11).

Peculiar to the multivariable case is the structural information required for C and D to provide a solution to (5-5), especially if F is selected in order to form (5-14) from (5-13). In the scalar case, this structural information is limited to plant order and bulk delay (or relative degree). The extra complications in the multivariable case, just for inverse or model-following control, require foreknowledge of the interactor matrix [23][24] or the Hermite form [25]. For this pole placement case, structural constraints may be different.

The one possibility of this direct adaptive implementation of pole placement, versus indirect schemes, is the seeming possibility of order overspecification in the scalar case [11]. This is not possible in the indirect case due to the uncontrollable pole-zero cancellation required in the identified model for zero identification error. This uncontrollability would result in a request for infinite controller gains leading to adaptive controller instability or requiring further logic for avoidance of this difficulty. As described in [11] e in the overspecified scalar version of (5-15) can be zero with the disappearance of some poles in the overall transfer function. This cancellation is stable due to the stability of F and therefore does not destabilize the adaptive controller. This possibility of overspecification in the scalar case raises the hope of overspecified structural indices in the multivariable case, which could reduce the severity of the restrictions mentioned in the preceding paragraph.

6. Appendix: Separated Variable Modeling

Background

A plausability argument will be made for the discrete state space representation of a distributed parameter system as an approximation to the partial differential equation (P.D.E.) representation, subject to a limited number of sensor and actuator point locations on the system. The argument rests heavily on the validity of the separated variable solution technique for the P.D.E.. A solution composed of a factor dependent only on time and a factor dependent only on the spatial variables can be obtained, provided that the system possesses at least cylindrical symmetry about the t-axis in the space spanned by the spatial coordinates and the t-coordinate. Only the class of systems for which this is the case will be considered here. Also, the system is assumed linear.

P.D.E. Representation and Solution

A linear N^{th} order P.D.E. in R^K can be represented in general by the equation

$$\sum_{i=0}^{I} \sum_{j=0}^{J} \sum_{k=1}^{K} \alpha_{i,j,k}(\underline{x},t) \frac{\partial^{i+j}}{\partial x_k^i \partial t^j} [\underline{u}(\underline{x},t)] = \underline{f}(\underline{x},t). \tag{6-1}$$

$$I + J = N$$

In this equation $\underline{u}(x,t)$ is a vector of the out-of-equilibrium deflections of the system in the spatial coordinates indexed by k. $\underline{u}$ is a function of the spatial position vector $\underline{x}$ and time. $\underline{f}(\underline{x},t)$ is a vector forcing function, also a function of the position vector $\underline{x}$ and time. The $\alpha_{i,j,k}(\underline{x},t)$ terms are the coefficients of the various partial derivatives of $\underline{u}$. The solution of this equation for $\underline{u}(\underline{x},t)$ is required to satisfy the P.D.E. and be uniquely determined by the boundary and initial conditions on some domain Ω in $\underline{x}$ and t throughout which the P.D.E. representation is valid.

If the assumed separated variable solution

$$\underline{u}(\underline{x},t) = X(x)T(t) \tag{6-2}$$

is substituted into the P.D.E., an integrating factor can be found such that the equation can be arranged having sums of terms, each dependent only on $\underline{x}$ or on t. Those terms that depend on $\underline{x}$ alone sum to a constant that is the negative of the sum of the t-dependent terms. This separation constant then appears in the separate solutions for X(x) and T(t), and will be seen to play an important role in the connection between the spatial and temporal system solutions. Since the P.D.E. equation is linear, the solution can be expressed as the sum of a part due to the natural response to initial conditions (homogeneous solution) and a part due to the system forcing function (particular solution). The homogeneous solution $\underline{u}_H(\underline{x},t)$ will be considered first.

The separate homogeneous equations for X(x) and T(t) take the general forms

$$\sum_{j=0}^{J} \beta_j(T(t),t) \frac{d^j}{dt^j} [T(t)] = 0 \tag{6-3}$$

$$\sum_{i=0}^{I} \sum_{k=1}^{K} \gamma_{i,k}(\underline{X}(\underline{x}),\underline{x}) \frac{\partial^i}{\partial x_k^i} [\underline{X}(\underline{x})] = 0 \tag{6-4}$$

where the separation constant σ is buried in the β_j and $\gamma_{i,k}$ coefficients. J is the order of the O.D.E. in time, and I is the order of the P.D.E. in space. Under certain conditions, the solutions to the above equations can be given by linear combinations of orthogonal eigenfunctions $\psi(t)$ & $\underline{\phi}(\underline{x})$

$$T(t) = \sum_{n=1}^{\infty} \Delta_n \psi_n(t) \qquad (6\text{-}5)$$

$$\underline{X}(\underline{x}) = \sum_{n=1}^{\infty} \underline{\Gamma}_n \underline{\phi}_n(\underline{x}) \qquad (6\text{-}6)$$

where the constants Δ_n depend on the initial conditions, and the constants $\underline{\Gamma}_n$ are determined by boundary conditions. The eigenfunctions (modes) are indexed by n which, along with the separation constant σ, determines the frequency of the eigenfunctions. Thus the temporal mode frequencies ω_{nt} and spatial mode frequencies ω_{nx} are related by σ.

The total solution is then

$$\underline{u}_H(\underline{x},t) = \underline{X}(x)T(t) = \sum_{n=1}^{\infty} \Delta_n \psi_n(t) \underline{\Gamma}_n \underline{\phi}_n(x). \qquad (6\text{-}7)$$

For any particular system represented on the region Ω by the above solution, the factors $\underline{\Gamma}_n, \psi_n$ and $\underline{\phi}_n$ are dependent on the physical nature of the system, and the constants Δ_n depend on the initial conditions of the system. If the former factors are known, and the Δ_n can be sensed or estimated on some manner, then the entire status or state of the system is completely known in that the output at any time and position can be predicted.

In practice, the state of the system must be sensed by some finite collection of sensors, each of which has a limited area of interaction with the system and has a limited frequency response. Therefore, some spatial as well as temporal modes will not be sensed. The deflections, velocities, etc., must then be considered as approximations to the true ones at the sensor locations. Represent the approximate deflections as made available by physical sensors as a finite sum of the eigenfunctions, known constants, and constants to be estimated:

$$\hat{\underline{u}}_H(\underline{x},t) = \sum_{n=1}^{L} \Delta_n \psi_n(t) \underline{r}_n \underline{\phi}_n(x) \tag{6-8}$$

where the lowest frequency (spatial and temporal) modes are not necessarily the ones sensed, and therefore the index n no longer refers to consecutive mode frequencies. By sensing the $\underline{u}$ at various points on the structure, it is desired that the unknown coefficients in the sum be estimated so that the system deflection at any time and at any point in space can be predicted. The next section discusses the estimation problem for the case of a single point sensor.

State Estimation - Single Sensor

If the sensor is located at some point $\underline{x}_o$ on the system, the sensed deflection at that point is represented by

$$\underline{u}_H(\underline{x}_o,t) = \sum_{n=1}^{L} \underbrace{\Delta_n \underline{\phi}_n(x_o) \underline{r}_n}_{\text{constants}} \cdot \underbrace{\psi_n(t)}_{\text{eigenfunctions}} \tag{6-9}$$

This equation is a linear combination of L solutions of the J^{th} order O.D.E. in time. Each solution to the J^{th} order O.D.E. can be represented by a linear combination of solutions of a coupled system of J first order O.D.E.s. These J solutions are represented by J state variables. The total representation for $\hat{\underline{u}}$ is then a linear combination of L sets of J state variables. Therefore the output $\hat{\underline{u}}$ can be considered a linear combination of L·J state variables. This can be represented by the following vector-matrix equation:

$$\begin{aligned} \dot{\underline{v}}(t) &= A\underline{v}(t) \;;\; \underline{v}(t_o) = \underline{v}_o \\ \hat{\underline{u}}_H(\underline{x}_o,t) &= \underline{c}^T \underline{v}(t) \end{aligned} \tag{6-10}$$

where the $\underline{\nu}$ is a vector of the state variables, A is a parameter matrix that contains the information about the natural response to initial conditions, and $\underline{c}$ is a vector that determines how the state variables combine to form the output $\hat{\underline{u}}$. Note that the $\underline{u}$ vector does not represent a vector of outputs but rather a single output on the system at a location specified by a position vector. The initial conditions are specified by the vector $\underline{\nu}_o$. It is important to notice that the expression for $\hat{\underline{u}}$ in terms of the eigenfunctions $\psi_n(t)$ in equation (6-9) has been replaced by a similar sum of more elementary eigenfunctions in equation (6-10). These elementary functions are all solutions to a first order differential equation in time and they all have the form

$$\psi_p^o(t) = q_o \exp[r_p(t-t_o)] \cdot \nu_{o_p} = \nu_p(t) \tag{6-11}$$

where the zero superscript denotes an elementary eigenfunction, the index p ranges from 1 to L·J, and the q_p and r_p are constants determined by the physical properties of the system. The $\hat{\underline{u}}$ is then given by:

$$\hat{\underline{u}}_H(x_o,t) = \sum_{p=1}^{L\cdot J} c_p \nu_p(t) \tag{6-12}$$

where the c_p are the elements of the $\underline{c}$ vector. If the physical properties of the system are known by some estimation procedure on the sensor output $\hat{\underline{u}}$, and the state variables are known at some time $t = t_o$ through some state observation procedure then the state at any time after $t = t_o$ can be found from equation (6-11) and the sensed deflection at any time after t_o can be obtained from equation (6-12). It should be pointed out here that some vibration modes may not be represented in the sensed deflection of equation (6-9) even though their spatial and temporal frequencies as well as modal amplitudes are within the detectable region of the sensor. This is due to the possibility that the sensor may be located at a point where the deflection of the body due to some modes is always too small to detect no matter what the modes' amplitude at other points may be. In this case, the sensor is located at a vibrational node of those particular spatial modes. Such modes are then unobservable in the sensed deflection given by equation (6-9). It is important then that the sensor (or sensors) be located such that this observability problem does not affect those modes of interest in the system.

The discussion so far has centered on the deflection of the system due to initial conditions only (i.e. homogeneous response). The forced (particular) response involves the additional consideration of external disturbance forces and actuator forces applied for control purposes. These forces can be included in the system model by realizing that external forces add energy to each of the characteristic modes in space and time as determined by the system's physical properties. With respect to the eigenfunction expansion description of the system, the forces on the body as function of $\underline{x}$ and time contribute toward spatial modal forces as functions of time as expressed by

$$\underline{f}(\underline{x},t) = \sum_{n=1}^{\infty} \underline{\Lambda}_n(t)\underline{\phi}_n(\underline{x}) \qquad (6\text{-}13)$$

where $\underline{f}$ is the collective force on the body and $\underline{\Lambda}_n(t)$ are the time varying coefficients of the spatial mode shapes $\phi_n(\underline{x})$. Thus the force on the body is represented by a sum of modal forces. For each mode, the deflection resulting from a corresponding modal force depends on the physical nature of the system such as modal mass, modal damping, modal stiffness, etc.

Just as in the expression for the sensed system deflection given by equation (6-9), all physical actuators are limited as to the spatial and temporal frequencies they can excite, so the sum in equation (6-13) is not infinite but limited to say Q excitable modes:

$$\hat{\underline{f}}(x,t) = \sum_{n=1}^{Q} \underline{\Lambda}_n(t)\underline{\phi}_n(\underline{x}) \quad . \tag{6-14}$$

The modal force amplitudes effectively applied to the system are the Q functions of time $\underline{\Lambda}_n(t)$. If these modal force amplitudes are introduced into the O.D.E. in time, equation (6-3), as a non-homogeneous term on the right hand side, the result is a J^{th} order non-homogeneous differential equation. If the equation is linear, each solution can be represented, as before, by a linear combination of J first order non-homogeneous O.D.E. solutions. Using such a set of J equations and corresponding state variables for each of the L solutions of equation (6-3) that can be sensed by the sensor, a non-homogeneous state space model of dimension $L \cdot J$ is obtained

$$\dot{\underline{\nu}}(t) = A\underline{\nu}(t) + B_o\hat{\underline{f}}(\underline{x},t)$$
$$\hat{\underline{u}}(\underline{x}_o,t) = \underline{c}^T\underline{\nu}(t) \tag{6-15}$$

where B is a matrix whose elements are spatial operators on $\hat{\underline{f}}(x,t)$ with respect to the elementary eigenfunctions instead of the original eigenfunctions as in equation (6-13). These elements depend on the locations of the sensors. The system now takes the form of a multi-input, single output state space model. Now the system state depends on initial conditions as well as the applied forces. Here, a similar problem exists with the location of the actuators. Depending on the relative location of the sensors and actuators, some modes may be excited that have nodes at the sensor locations, and are therefore unobservable. Also, some sensed modes may be impossible to control if the actuators are located at those mode's zeros, or nodes. The result in the latter case is uncontrollability of those system vibrations. For this discussion, it will be assumed that there are no problems with observability and controllability in the system model.

State Estimation - Multiple Sensors

If more than one sensor is used to detect the system state, an equation similar to (6-9) can be written for each sensor. The constants premultiplying the modes in the sum for each sensor will depend on the location occupied on the system by each sensor. Since the same eigenfunctions are common to all such sensor deflection representations, the same set of state variables can be used to describe the system state at any location, where a different linear combination of the state variables is used at each different location. The vector of sensor measurements is given by

$$\begin{bmatrix} \hat{\underline{u}}(\underline{x}_1,t) \\ \hat{\underline{u}}(\underline{x}_2,t) \\ \vdots \\ \hat{\underline{u}}(x_\ell,t) \end{bmatrix} = C\underline{v}(t) \tag{6-16}$$

where C is a matrix whose ℓ rows $\underline{c}^T_\ell$ reflect the particular linear combination of states at each sensor position.

These particular state variables, being solutions to a modal system representation, result in a block diagonal A matrix in equation (6-15). Each block represents the solution for each mode, which is orthogonal to any other mode, and hence each block is an independent dynamic system of dimension J.

Summary

In more or less their order of importance and development, the assumptions and results of this appendix are

- The P.D.E. representation is linear.
- This P.D.E. is separable into space and time components. Separate space and time solutions provide that the eigenfunction form in space does not depend on time and the form in time does not depend on the location in space where measurements are taken.
- The solutions of the separate homogeneous space and time equations can be represented as infinite sums of orthogonal eigenfunctions.
- The frequencies of the space and time eigenfunctions are related by the P.D.E. separation constant.
- The constants in the linear combinations of these eigenfunctions depend on boundary conditions (for the spatial equation) and on initial conditions (for the temporal equation).
- The infinite linear combinations of eigenfunctions must be considered finite for any realizable measurement or actuation due to physical limitations.
- Sensor and actuator placement is very important with respect to controllability and observability of system modes of vibration.
- A state space representation of the system for arbitrary combinations of actuators and sensors can be theoretically found if the O.D.E. in time is linear.
- The order of the state space description depends on the order of the O.D.E. in time and on the number of modes sensable by the sensor.
- System forces do not alter the eigenfunction forms, but do effect the modal amplitudes.
- Knowledge of the states in the state variable representation is

sufficient to characterize the state of the entire system within the accuracy of the sensor measurements.

- A reduced order model results when the number of states selected in the model are fewer than can specify sensor-measurable modes.

II. Projections

The last three approaches to adaptive control of multi-input, multi-output, truncated, linear models of flexible structures discussed in the preceding section (in subsections 3-5) will form the basis of our ongoing efforts. Our current emphasis is in reverse order to the order of their presentation, i.e. we intend to investigate, in the following priority

(i) Direct adaptive multivariable pole placement

(ii) Simultaneous coupled multivariable system identification and control via time or frequency domain approaches

(iii) Simultaneous eigenshape and dynamic modal parameter estimation and decoupled modal control.

In order to interrelate these approaches a modeling study is planned to develop algorithms for conversion from one model form to another, i.e. the decoupled modal, canonical state, and matrix fraction descriptions of flexible structures for vairous sensor actuator locations. Once a promising adaptive control candidate emerges it will be investigated in a reduced order setting appropriate to DPS or flexible spacecraft control [12].

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V. Recent Sponsored Publications

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