(NASA-CR-164189) DESIGN OF MULTIVARIABLE FEEDBACK CONTROL SYSTEMS VIA SPECTRAL ASSIGNMENT Progress Report, 1 Aug. 1980 - 28 Feb. 1981 (Old Dominion Univ., Norfolk, Va.) 20 p HC A02/MF A01 CSCL 12B G3/66

N81-21864

Unclas 42069

OLD DOMINION UNIVERSITY RESEARCH FOUNDATION

DEPARTMENT OF ELECTRICAL ENGINEERING
SCHOOL OF ENGINEERING
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA

DESIGN OF MULTIVARIABLE FEEDBACK CONTROL SYSTEMS VIA SPECTRAL ASSIGNMENT

By

Stanley R. Liberty, Principal Investigator
Roland R. Mielke

and

Leonard J. Tung

Progress Report
For the period August 1, 1980 - February 28, 1981

Prepared for the
National Aeronautics and Space Administration
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Hampton, Virginia

Under
NASA Grant NSG 1650
Martin T. Moul, Technical Monitor
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DESIGN OF MULTIVARIABLE FEEDBACK CONTROL SYSTEMS VIA SPECTRAL ASSIGNMENT

By

Stanley R. Liberty[1], Roland R. Mielke[2], and Leonard J. Tung[3]

ABSTRACT

Applied research in the area of spectral assignment in multivariable systems is reported. A new frequency domain technique for determining the set of all stabilizing controllers for a single feedback loop multivariable system is described. It is shown that decoupling and tracking are achievable using this procedure. The technique is illustrated with a simple example.

INTRODUCTION

This report summarizes the progress of applied research being conducted by the authors and colleagues under the support of NASA Grant NSG 1650. The objective of this work is to investigate the applicability of modern control theory to the design of multivariable feedback control systems, particularly as applied to aircraft flight-control problems.

Previous research efforts under NASA Grants NSG 1519 and NSG 1650 have included an investigation of eigenvalue/eigenvector assignment procedures (refs. 1-3). Early studies focused on an algorithmic formulation of the spectral assignment problem by Srinathkumar (ref. 4), while later studies included a geometric formulation of the same problem by Moore (ref. 5), Kimura (ref. 6), and Davison and Wang (ref. 7). Based on these theories, design procedures have been developed for achieving desired mode mixing (ref. 8), reducing eigensystem sensitivity to variations in plant parameters

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(ref. 9), and reducing the effects of actuator noise an system performance (ref. 10). In addition, these procedures have been successfully applied to the design of a jet engine controller (ref. 11) and an insensitive servo control system (refs. 12, 13). A detailed survey of eignevalue/eigenvector assignment design techniques was recently presented by the authors in a 21-hour short course entitled "A Geometric Approach to Control System Synthesis" at the NASA/Langley Research Center (LaRC). The course was extremely well received and demonstrated that eigenvalue/eigenvector assignment techniques are accessible to practicing control engineers.

Research efforts during the past seven months have focused on frequency domain theories utilizing spectral factorization techniques of the Wiener-Hopf class. The Wiener-Hopf theories have been studied by several investigators including Whitbeck (refs. 14-16), and were developed in the most general form by Youla (refs. 17, 18). The Youla work deals with the design of optimal controllers for multi-input-output plants imbedded in a single multivariable feedback loop configuration. Two significant contributions were made in this work. First, a procedure was given for characterizing the set of all controllers of a particular class which stabilize the overall system; second, a method for selecting an optimal controller from the set of all stabilizing controllers was presented. The optimization procedure was based on the standard Wiener-Hopf filtering problem. Youla's stabilization procedure has been further studied and generalized by Desoer (ref. 19). Based on this work, a frequency domain design procedure for constructing a stabilizing controller has been developed. This procedure is explained and illustrated with an academic example in the next section.

DESIGN OF A STABILIZING CONTROLLER

Consider the single feedback loop multivariable control system shown in figure 1. In this figure, $P(s)$ is a proper rational matrix transfer function representing the system plant, while $C(s)$ is a proper rational transfer function matrix representing the controller. It is desired to first characterize the set of all transfer function matrices $C(s)$, such that the closed-loop system is stable. Then, a search will be conducted

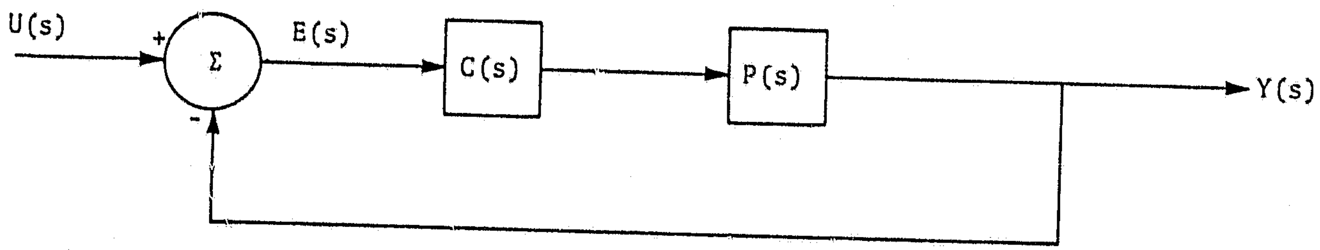

Figure 1. Multivariable feedback control system.

over this set of stabilizing controllers to locate the particular controller which also achieves desired mode decoupling and/or system tracking. The following definitions are essential to this development:

Definition 1: A proper rational matrix is said to be exponentially stable if all poles of each matrix element are located in the left half of the complex plane.

Definition 2: Given a proper rational matrix $P(s)$, the expression $P(s) = N_r(s)\, D_r^{-1}(s)$ is said to be a right exponentially stable rational fraction description (right ESRFD) of $P(s)$ if both $N_r(s)$ and $D_r(s)$ are proper and exponentially stable. In a similar manner, the expression $P(s) = D_\ell^{-1}(s)\, N_\ell(s)$ is said to be a left exponentially stable rational fraction description (left ESRFD) of $P(s)$ if both $N_\ell(s)$ and $D_\ell(s)$ are proper and exponentially stable.

Definition 3: A right ESRFD of $P(s)$, $P(s) = N_r(s)\, D_r^{-1}(s)$, is said to be coprime if there exist matrices $U_r(s)$ and $V_r(s)$, both proper and exponentially stable, such that $U_r(s)\, N_r(s) + V_r(s)\, D_r(s) = 1$. Similarly, a left ESRFD of $P(s)$, $P(s) = D_\ell^{-1}(s)\, N_\ell(s)$, is said to be coprime if there exist matrices $U_\ell(s)$ and $V_\ell(s)$, both proper and exponentially stable, such that $N_\ell(s)\, U_\ell(s) + D_\ell(s)\, V_\ell(s) = 1$.

The above definitions make possible the following characterization of the set of all stabilizing controllers for the system of figure 1. A proof of this theorem is given in reference 19.

Theorem 1: Let $P(s) = N_r(s)\, D_r^{-1}(s)$ be a right coprime ESRFD of $P(s)$ with $U_r(s)\, N_r(s) + V_r(s)\, D_r(s) = 1$, and let $P(s) = D_\ell^{-1}(s)\, N_\ell(s)$ be a left coprime ESRFD of $P(s)$ with $N_\ell(s)\, U_\ell(s) + D_\ell(s)\, V_\ell(s) = 1$. Then, the closed-loop system is stable if $C(s)$ is chosen as

$$C(s) = [W(s)\, N_\ell(s) + V_r(s)]^{-1}\, [-W(s)\, D_\ell(s) + U_r(s)]$$

where $W(s)$ is any proper exponentially stable matrix such that $|W(s)\, N_\ell(s) + V_r(s)| \neq 0$. Moreover, $C(s)$ is a proper rational matrix. The transfer function matrix relating output to input, $T_y(s)$, is given by

$$T_y(s) = N_r(s)\, [-W(s)\, D_\ell(s) + U_r(s)]$$

and the transfer function relating the error signal to input, $T_e(s)$, is given by

$$T_e(s) = 1 + T_y(s).$$

The freedom available in choosing a stabilizing controller $C(s)$ is thus characterized by the freedom available to choose $W(s)$. If the only design consideration is to stabilize the overall system, then any proper exponentially stable matrix $W(s)$ will yield an appropriate controller. If, in addition, output decoupling is desired, then a proper exponentially stable $W(s)$ such that $T_y(s)$ is diagonal is chosen. Similarly, if it is desired to track step inputs, a proper exponentially stable $W(s)$ for which $\lim_{s \to 0} \frac{1}{s} T_y(s)$ exists in chosen.

Problems in which decoupling and tracking are desired are simplified when $P(s)$ is exponentially stable. In this situation, $N_\ell(s) = N_r(s) = P(s)$, $D_\ell(s) = 1$, $D_r(s) = 1$, $U_r(s) = 0$, $U_\ell(s) = 0$, $V_\ell(s) = 1$, and $V_r(s) = 1$. Then,

$$T_y(s) = -P(s)\, W(s),$$

$$T_e(s) = 1 + P(s)\, W(s)$$

and

$$C(s) = -W(s)\, [1 + P(s)\, W(s)]^{-1}$$

An example will now be presented to illustrate this technique. Consider a two input, two output plant described by the transfer matrix

$$P(s) = \begin{bmatrix} p_{11}(s) & p_{12}(s) \\ p_{21}(s) & p_{22}(s) \end{bmatrix}$$

$$= \frac{1}{(s + 2)\,(s^2 + 4s + 2)} \begin{bmatrix} (s + 1)\,(s + 3) & 1 \\ 1 & (s + 1)\,(s + 3) \end{bmatrix}$$

Since $P(s)$ is exponentially stable, stability of the closed-loop system is assured if the controller transfer matrix is chosen as $C(s) = -W(s) \times [1 + P(s)\, W(s)]^{-1}$, where $W(s)$ is any exponentially stable rational matrix having appropriate dimension. With this controller, the output transfer function is $T_y(s) = -P(s)\, W(s)$ and the error transfer function is $T_e(s) = 1 + P(s)\, W(s)$.

If decoupling is desired, $W(s)$ must be chosen such that $T_y(s) = -P(s)\, W(s)$ is a diagonal matrix. With $W(s)$ given by

$$W(s) = \begin{bmatrix} W_{11}(s) & W_{12}(s) \\ W_{21}(s) & W_{22}(s) \end{bmatrix}$$

the closed-loop transfer function is

$$T_y(s) = -P(s)\, W(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} W_{11}(s) & W_{12}(s) \\ S_{21}(s) & W_{22}(s) \end{bmatrix}.$$

$$= \begin{bmatrix} P_{11}(s)\, W_{11}(s) + P_{12}(s)\, W_{21}(s) & P_{11}(s)\, W_{12}(s) + P_{12}(s)\, W_{22}(s) \\ P_{21}(s)\, W_{11}(s) + P_{22}(s)\, W_{12}(s) & P_{21}(s)\, W_{12}(s) + P_{22}(s)\, W_{22}(s) \end{bmatrix}$$

Thus, for decoupling,

$$W_{12}(s) = \frac{-P_{12}(s)}{P_{11}(s)} W_{22}(s) = \frac{1}{(s+1)\,(s+3)} W_{22}(s)$$

and

$$W_{21}(s) = \frac{-P_{21}(s)}{P_{22}(s)} W_{11} = \frac{1}{(s+1)\,(s+3)} W_{22}(s).$$

With this constraint on $W(s)$,

$$T_y(s) = -\begin{bmatrix} \frac{|P(s)|}{P_{22}(s)} W_{11}(s) & 0 \\ 0 & \frac{|P(s)|}{P_{11}(s)} W_{12}(s) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{(s+2)}{(s+1)(s+3)} W_{11}(s) & 0 \\ 0 & \frac{(s+2)}{(s+1)(s+3)} W_{22}(s) \end{bmatrix}.$$

If, in addition to decoupling, it is desired that the system track step inputs, then every element of the error transfer function $T_e(s)$ must contain a factor s in the numerator. For the previous selection of $W(s)$,

$$T_e(s) = \begin{bmatrix} 1 + \frac{(s+2)}{(s+1)(s+3)} W_{11}(s) & 0 \\ 0 & 1 + \frac{(s+2)}{(s+1)(s+3)} W_{22}(s) \end{bmatrix}.$$

Thus, there are many choices of $W(s)$ which will result in the desired tracking property. A particularly simple choice is $W_{11}(s) = W_{22}(s) = -\frac{3}{2}$. Then,

$$W(s) = - \begin{bmatrix} \frac{3}{2} & \frac{3}{2(s+1)(s+3)} \\ \frac{3}{2(s+1)(s+3)} & \frac{3}{2} \end{bmatrix}$$

and the controller transfer matrix becomes

$$C(s) = \begin{bmatrix} \frac{3(s+1)(s+3)}{2s\left(s+\frac{5}{2}\right)} & \frac{3}{2s\left(s+\frac{5}{2}\right)} \\ \frac{3}{2s\left(s+\frac{5}{2}\right)} & \frac{3(s+1)(s+3)}{2s\left(s+\frac{5}{2}\right)} \end{bmatrix}.$$

The system transfer functions are then given by

$$T_y(s) = \begin{bmatrix} \dfrac{3(s+2)}{2(s+1)(s+3)} & 0 \\ 0 & \dfrac{3(s+2)}{2(s+1)(s+3)} \end{bmatrix}$$

and

$$T_e(s) = \begin{bmatrix} \dfrac{s\left(s+\frac{5}{2}\right)}{(s+1)(s+3)} & 0 \\ 0 & \dfrac{s\left(s+\frac{5}{2}\right)}{(s+1)(s+3)} \end{bmatrix}.$$

The frequency domain technique demonstrated above appears to be an attractive method for control system design and is worthy of further study. Investigation of two areas in particular appear promising. First, when the above techniques are applied, the set of all stabilizing controllers for a given system configuration is obtained. The freedom available to the designer is to choose a particular controller from this set by specifying a proper exponentially stable, but otherwise arbitrary, matrix $W(s)$, which is selected to achieve a desired system characteristic such as mode decoupling, reference tracking, or disturbance rejection. However, a systematic procedure for generating a $W(s)$ to achieve given design specifications has not been thoroughly developed. Second, the stabilization techniques have been developed only for the single multivariable feedback loop configuration. Preliminary studies, however, have indicated that a similar formulation may be possible for other system topologies. In addition, several classes of problems have been identified in which the use of other topologies would greatly enhance the freedom available to the designer.

In addition to being a possibly attractive design procedure, the generality of the above approach holds promise of a tie with the spectral assignment techniques previously investigated. Investigation of these relationships should significantly enhance the applicability of the stabilization techniques and improve the spectral assignment procedures.

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EIGENVECTOR ASSIGNMENT FOR NOISE SUPPRESSION

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ABSTRACT

A new method of selecting a multivariable state feedback controller is presented. The resulting controller simultaneously realizes arbitrary distinct closed-loop eigenvalues, approximates specified modal behavior, and reduces the effects on the system of actuator noise. The method characterizes the subspace associated with each closed-loop eigenvalue in which the corresponding eignevector must lie. The vector that is to be approximated by this eigenvector is projected onto the subspace. When all the eigenvectors have been obtained in this way, the steady-state mean-squared error of the state variable of the closed-loop system is reduced local to this set of eigenvectors using a gradient search technique.

I. INTRODUCTION

Consider a linear, time-invariant, multivariable system described by $\dot{x} = Ax + Bu + \xi$, where $x, \xi \in R^n$, $u \in R^m$, and the pair (A, B) is controllable. In this paper consideration is given to the eigenvalue/eigenvector assignment problem of selecting a stationary feedback matrix F in the control law $u = Fx$ such that the eigenvalues of the closed-loop system matrix $\hat{A} = A + BF$ are arbitrarily placed. Throughout this paper the closed-loop eigenvalues are assumed to be distinct. Once the eigenvalues are assigned, freedom remaining in selecting the eigenvectors of $\hat{A}$ is utilized to approximate specified modal brhavior and to reduce the effects on the closed-loop system of the actuator noise, ξ.

It has been shown by Moore [1] that the eigenvalues of $\hat{A}$ can be arbitrarily assigned, but that each eigenvector of $\hat{A}$ must lie in a subspace of R^n determined by the corresponding eigenvalue. However, despite the restrictions imposed on the eigenvectors, there remains some freedom in selecting the eigenvectors. This freedom can be used to affect mode mixing [2, 3], reduce system sensitivity to plant parameter variations [4], and as is shown in this paper, reduce the effects of actuator noise.

An initial eigenvector assignment is made by successively projecting each vector from a specified set onto the corresponding subspace. This yields a set of eigenvectors each of which is the best approximation in a least-squares sense to the corresponding specified vector. The eigenvectors obtained are expressed as linear combinations of basis vectors for the subspaces. The steady-state mean-squared error of the state variable is expressed as a function of the scalar coefficients in these linear combinations, and is reduced local to the initially assigned eigenvectors by a gradient search procedure over the scalar coefficients.

II. INITIAL EIGENSYSTEM ASSIGNMENT

Let $\{\lambda_1, \ldots, \lambda_n\}$ be a self-conjugate set of distinct complex numbers. Then it is known that there exists a real-valued matrix F such that

$$(A + BF)\, v_i = \lambda_i v_i, \qquad i = 1, \ldots, n$$

if and only if the following three conditions are met for $i = 1, \ldots, n$:

(i) Vectors v_i are linearly independent in C^n;

(ii) $v_i = v_j^*$ whenever $\lambda_i = \lambda_j^*$;

(iii) $v_i \varepsilon$ span $\{N_{\lambda_i}\}$, where the columns of $K_{\lambda_i} = \begin{bmatrix} N_{\lambda_i} \\ \hline M_{\lambda_i} \end{bmatrix}$ form a basis for

$\ker[\lambda_i I - A \,\vdots\, B]$. Here, K_{λ_i} is partitioned compatibly with $[\lambda_i I - A \,\vdots\, B]$.

If $\{p_1, \ldots, p_n\}$ is a set of vectors, and it is desired that the ith eigenvector approximate p_i , $i = 1, \ldots, n$, then the closest approximation in a least-squares sense is obtained by projecting p_i onto span $\{N_{\lambda_i}\}$. This projection is expressed as

$$v_i = \sum_{j=1}^{n_i} \alpha_{ij} b_j^i \quad , \tag{1}$$

where b_j^i is the jth column of N_{λ_i} and $\{\alpha_{i1}, \ldots, \alpha_{in_i}\}$ is a set of scalars,

or equivalently,

$$v_i = N_{\lambda_i} \alpha_i \tag{2}$$

where $\alpha_i = \{\alpha_{i1} \ldots \alpha_{in_i}\}^T$. The p_i vectors are projected successively

and when all have been projected, F is calculated from

$$F = -[M_{\lambda_1} \alpha_1 \quad \cdots \quad M_{\lambda_n} \alpha_n] V^{-1} \tag{3}$$

where $V = [v_1 \quad \cdots \quad v_n]$.

III. NOISE REDUCTION

Consider a system with actuator noise described by

$$\dot{x} = Ax + Bu + \xi \quad , \tag{4}$$

where ξ is assumed to be zero mean white noise characterized by the following equations

$$E\{\xi(t)\} = 0 \ , \tag{5}$$

$$E\{\xi(t)\xi^T(\tau)\} = \Xi \ \delta(t-\tau), \tag{6}$$

$$E\{x(o)\} = x_o \ , \tag{7}$$

$$E\{[x(o) - x_o][x(o) - x_o]^T\} = P_o, \tag{8}$$

and

$$E\{[x(o) - x_o]\xi^T(t)\} = E\{\xi(t)[x(o) - x_o]^T\} = 0. \tag{9}$$

The effect of actuator noise on the response of the system can be reduced by proper choice of the feedback gain matrix F. Noise reduction is made in the sense that the steady-state mean-squared error of the state vector x(t) is reduced; that is,

$$\lim_{t\to\infty} E\{||x(t) - \bar{x}(t)||^2\}$$

is reduced where $\bar{x}(t) = E\{x(t)\}$.

To see how this is done, it is helpful to note that the mean-squared error is equal to the trace of the state covariance matrix, W(t). Thus,

$$\lim_{t\to\infty} E\{||x(t) - \bar{x}(t)||^2\} = \mathrm{tr}\, W_{ss} \tag{10}$$

where $W_{ss} = \lim_{t\to\infty} W(t)$. Moreover, if $\Phi(t)$ is the state transition matrix for the closed-loop system, then

$$W(t) = \int_0^t \Phi(t-\tau)\Xi\Phi^T(t-\tau)d_\tau + \Phi(t)P_o\Phi^T(t). \tag{11}$$

Differentiating (11) with respect to t yields

$$\dot{W}(t) = \hat{A}\, W(t) + W(t)\, \hat{A}^T + \Xi. \tag{12}$$

If $\hat{A}$ has eigenvalues in the left-hand half-plane, then $\lim_{t\to\infty} W(t)$ exists and therefore $\lim_{t\to\infty} \dot{W}(t) = 0$. Thus, equation (12) reduces to the Lyqpunov stability equation

$$\hat{A}\, W_{ss} + W_{ss}\hat{A}^T + \Xi = 0 \tag{13}$$

whose solution is obtained from

$$\mathrm{vec}\, W_{ss} = -[I\otimes\hat{A} + \hat{A}\otimes I]^{-1}\mathrm{vec}\, \Xi \tag{14}$$

where

$$\mathrm{vec}\,[a_1 \;\cdots\; a_n] = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$$

and

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B \\ a_{21}B & a_{22}B & \cdots & a_{2n}B \\ \vdots & & & \vdots \\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B \end{bmatrix} .$$

for $A = [a_{ij}]_{mxn}$.

Since W_{ss} is a function of $\hat{A}$, it is a function of the eigenvectors and, therefore, of the scalars in (1). Thus, $\lim_{t\to\infty} E\{\|x(t) - \bar{x}(t)\|^2\} = tr[W_{ss}]$ can be reduced local to the initial eigenvector assignment through a gradient search over these scalars. The gradient, $\nabla_W = \nabla(trW_{ss})$, is a vector with components of the form

$$\frac{\partial\, tr[W_{ss}]}{\partial\, x_{ij}} = tr\left[vec^{-1}\left\{a^{-1}(I \otimes B_{ij}V^{-1} + B_{ij}V^{-1} \otimes I)a^{-1}\, vec\, \Xi\right\}\right] \quad (15)$$

where $a = I \otimes \hat{A} + \hat{A} \otimes I$ and B_{ij} is a matrix whose only nonzero column is the i^{th} column, which is $(\lambda_i I - \hat{A})\, b_j^i$. The symbol "$vec^{-1}$" indicates the operation of "unstacking" the n^2-component vector into an $n \times n$ matrix. The steady-state mean-squared error is reduced interatively until an acceptable trade-off is reached between obtaining eigenvectors that closely approximate the desired vectors and obtaining a small steady-state error.

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