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# PROBIEM OF GAS ACCRETION ON A GRAVITATIONAL CENTER 

i. A. Ladygin

# Translation of "K zadache ob akkretsii gaza na gravitiruyushchiy tsentr", Academy of Sciences USSR, Institute of Space Research, Moscow, Report Pr-442, 1978, pp. 1-14 

(NaSA-TH-76202) PROBLEM OF GAS ACCRETION UN


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|  | 10. Work Unil No. |
| 9. Puflotming Orgonination Name and Addost <br> Leo Kanner Associates Redwood City, California 94063 | 11. Contract a Grant No. NASw-3199 |
|  | 13. Trpe ol Repoll eni Poriod Covered |
| 12. Sponsoring Agency Hame and Addioss <br> National. Aeronautics and Space Administration, Washington, D. C. 20546 | Translation |
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## 16. Abstract

The study deals with a method of the approximated solution of the problem of accretion on a rapidly moving gravitational center. This solution is obtained in the vicinity of the axis of symmetivy in the region of the potential flow.

| 17. Key Words (Selected by Author(s)) | 18. Distribution Sistoment |
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PROBLEM OF GAS ACCRETION ON A GRAVITATIONAL CENTER

V. A. Ladygin

Examined in the study is a method of approximated solution of the problem of gas accretion on a rapidly moving gravitational center. The solution is obtained in some vicinity of the axis of symmetry in the region of potential flow. Galculations on a computer showed the effectiveness of the given method.

## Introduction

The solution of the problem of stationary asas accretion on a moving gravitational center simulates the movement of a substance in interstellar space in the vicinity of a black hole. A detailed picture of gas accretion on a black hole is of interest in connection with the problem of observation of black holes.

The qualitative study of guch accretion, as well as twodimensional numetical calculations of this problem, are available in studies [1] - [4]. The s'budy of self-modeling solutions, which may represent asymptotics of the flow of gas near a gravitational center, was carried out ir. itudy [5].

In the present study, the system of equations of twodimensional gas dynamics, which describes gas accretion, in contrast to studies [2] - [4], is solved in an approximate manner, by means of expansion into a linear series, according to one of the indeperdent variables (angle $\theta$ ), and by means of "abridging" of the obtained infinite system of common differential equations.

[^0]Although orly the region of potential flow of the gas is studied in the present study, this method may be used also for a nonpotential flow behind a shock wave.

1. Formulation of the Problem and Basic Equations

Studied herein is the steady-state axisymmetrical flow of an ideal polytropic iras, devoid of viscosity and thermal conductivity in a gravitational field of a material point of mass M.

At infinity, the approach stream is assumed to be homogeneous and supersonic.

$$
\begin{aligned}
& \rho \rightarrow \rho_{m} \text { mconst, } P \rightarrow P_{m}=\text { cons, } \\
& V_{s}-V_{w} \cos \theta, \quad V_{\theta} \rightarrow-V_{\infty} \sin \theta, \\
& V_{\infty}=\text { const>0, } \quad r \rightarrow \infty
\end{aligned}
$$

With these boundary conditions, the flow prior to the shock wave is potential and isentropic, and, in a spherical system of coordinates, it is described by the system or equations:

$$
\begin{aligned}
& \frac{V_{1}^{2}+V_{i}^{*}-G M}{2}+\gamma K \rho^{\gamma}-\Psi \quad 12,2! \\
& \text {-energy integral, } \\
& \frac{\partial}{\partial z}\left(\tau V_{0}\right)-\frac{\partial V}{\partial \theta}=0 \quad / 2.3 / 2 \\
& \text {-condition of potentiality, } \\
& \frac{1}{7} \frac{\partial}{\partial \tau}\left(\rho r^{*} V_{i}\right)+{ }_{r} \frac{\sin \sin \theta}{\partial \theta} \frac{\partial}{\partial \theta}\left(\rho V_{\theta} \sin \theta\right)=0 \quad / 2.4 / \\
& \text {-equation of continuity, } \\
& P=(\gamma-i) K \rho^{i} \quad 12.5 / \\
& \text {-equation of state, }
\end{aligned}
$$

where $V_{k}, V_{0}, \rho$, and $P$ are the radial and angular components of velocity, the density, and the pressure of the gas,
$G$ is the gravitational constant, $\gamma$ is the indicator of the polytropic curve.

$$
\text { The constants } \mathrm{K} \text { and } \boldsymbol{\Psi} \text { are determined with the boundary }
$$

conditions (2.1).

Through transformation of the analog
 parameter problem (2.1)-(2.6) (parameters $\gamma, G \cdot M, P_{\infty}, \rho_{\infty}, V_{0}$, ) is reduced to a two-parameter problem (parameters: $\gamma$ and $M_{0}=$ $\frac{V_{\infty}}{C_{\infty}}$ is the Mach speed at infinity).

Therefore, without bounding the generality, one can assume

On the strength of (2.4), the expression

$$
-c \rho \sin \theta V_{\theta} d z+z_{\rho}^{x} \rho \sin \theta V_{r} d \alpha
$$

is a complete differential of some function $S(\tau, \theta)$ of the current lines, and, consequently,
where $d=\frac{1}{\rho}$ is the specific volume.

## 2. Description of the Method of Solution of the Problem

We will derive the formulas for the approximated solution
of the system (2.8)-(2.11). Since the flow is axisymmetrical, then

$$
\begin{align*}
& d(\tau, \theta)=d(\tau,-\theta), \quad \because(z, \theta)=K_{k}(\tau,-\theta), \\
& V_{0}(\tau, \theta)=-V_{\theta}(\tau,-\theta), S(\tau, \theta)=S(\tau, \cdot \theta)
\end{align*}
$$

We will assume that the functions $d, V_{\tau}, V_{0}, S$ are analytic according to $\theta$ in some vicinity of $\theta=0$. Then, $d$, $V_{F}$, $S$ are exp led into an exponential series according to even powers of $\theta$, and $V_{0}$, according to uneven powers of $\theta$, i.e.,

One can think that $S(\gamma)=0$, since the function $S(\tau, \theta)$ is determined with an accuracy up to the additive constant, and is constant along the trajectory, while the axis of symmetry $\theta=0$ is the trajectory of the particles.

We will substitute the expansion (3.2) into the system (2.8)-(2.11), and group the terms with identical powers of $\theta$. We will obtain the system of relationships

$$
\begin{aligned}
& \frac{V_{2}}{2}+\frac{h_{1}}{4}+\cdots \\
& 2,11 /=\frac{f}{d z}\left(\tau_{\mu_{x-1}}\right)-2 k q_{k} \\
& \text { 18.6/ } \\
& \text { /3.7/ } \\
& \text { / } \mathrm{K}=1,2, \ldots{ }^{\prime} \text {, }
\end{aligned}
$$

where $a_{i}, V_{i}, h_{i}$ are the coefficients of the expansion into an exponential series, respectively, of the functions $\sin \theta$.
$V_{t}^{a}+V_{0}^{a}, d^{\prime-*}$

$$
\begin{aligned}
& d^{r} r_{-h}=\sum_{i=1}^{*} h_{1 k} \theta^{2 k}
\end{aligned}
$$

We will expand the equality

$$
d h_{4}-\mathrm{a}-\mathrm{r} m \mathrm{~A}
$$

into an exponential series according to $\theta$.

We will obtain a system of relationships between the coefficients $\left\{h_{i}\right\}$ and $\left\{d_{i}\right\}$.

$$
\begin{aligned}
& \text { } 1 . \mathrm{s}^{2} \text { ? }
\end{aligned}
$$

$$
\begin{aligned}
& \text { 13.10/ }
\end{aligned}
$$

We will transform the system (3.3)-(3.10). In place of the coefficients $S_{2 k}$, we will examine the functions

$$
F_{i x}=\operatorname{cin}_{\mathrm{z}^{2}} \quad \quad n, \ldots, \ldots /
$$

which have finite limits with $2 \rightarrow+\infty$. Using (3.5) and (3.6). we preclude the coefficients $h_{i}$ in (3.10). We will introduce the new independent variable

$$
u=\ln 2 \quad \text { a.12 }
$$

and also the functions

$$
\mu_{t N}=d_{1 k} / d_{0} \quad / k=1,2, \ldots /
$$

We will obtain an infinite system of common differential equations relative to the functions $\rho_{2 k-1}, F_{2 k}$

The funations $\left[T_{X K}\right]_{k=1}^{\infty},\left\{q_{*}\right\}_{k=0}^{\infty}, C_{0}^{2}$ (square of the speed of sound on the exis of symmetry $\theta=0$ ), which are part of the righthand portion of system (3.13), as wall as the functions $\left\{V_{2 k}\right\}_{k=0}^{\infty}$, $\left\{\mu_{2 k}\right\}_{k=1 \prime}^{\infty}$, are sequentially determined through the relationships:

$$
\begin{array}{rl}
C_{0}^{2} n(\gamma-1)\left(\psi-e^{-H}-2 F_{2}^{*}\right), & 13.14 / \\
q_{0}=2 F_{1}, V_{0}=4 F_{1}^{2}, T_{4}=-P_{1} & 3.15 /
\end{array}
$$

Then, if the coefficients $\mu_{\alpha_{i}}, V_{x i}, q_{x i}(i<K)$ and $\Psi_{x j}(j<K+1)$ hav already been calculated, then $\mu_{m}, V_{a k}, q_{k k}$ are determined from the system of three linear equations:
and the function $T_{2(k+1)}$ is calculated according to the formula

$$
\begin{equation*}
T_{2 k+11}=-\sum_{1+1+1+i} \prod_{1} \mu_{j} \sum_{T+1, i} p_{j} \tag{4}
\end{equation*}
$$

From (2.1), (2.9), and (3.2), we will obtain the boundary conditions for system (3.13)

$$
p_{n},-\cdots \frac{\left(\theta^{2}+n,\right.}{t n},
$$

with $l i \rightarrow+\infty$

$$
F_{u x} \rightarrow{ }_{2 x}+\cdots
$$

$$
13.24
$$

$$
/ K o n, 2, \cdots /
$$

If the system (3.13)-(3.20) is solved, then the coefficients $d_{2 k}$ and $S_{2 k}$ are determined by the formulas

$$
\begin{aligned}
& d_{0}=c_{0}^{h^{2}}, \quad d_{2 x}=d_{0} \mu_{2 k}, \\
& S_{3 k}=e^{t 4} F_{1 k} / d_{5}
\end{aligned}
$$

$$
/ 3.2 \mathrm{I} /
$$

$$
/ \mathrm{kNI}, 2, \ldots / /
$$

The cinlculitions fording to formulas (3.13)-(3.19), with small values of 7 , requires large outlays of computer time, since the right-hand portions of the system (3.13) are unbounded with $r \rightarrow 0$. Therefore, in the region of $u<0$, it is advisable to make a substitution of the variables:

In this case, only equations (3.23) and (3.14) change. taking on the from:
/3.23/
73.24

For approximated calculation of the first $N$ coefficients of expansion (3.2), we will examine system (3.13) for $k=1,2, \ldots N$. Formulas (3.14)-(3.19) make it possible, in the right-hand portion of (3.13), to express $q_{2 i}$ : $\mu_{\alpha i}, V_{\alpha i}(i<N)$ and $T_{2 i}(, /<N+1)$ in the form of functions from $u, P_{1}, \ldots, p_{2 n-1}, F_{2}, \ldots, F_{w n}$ For a similar recording of the coefficients $q_{2}$ and $V_{Z N}$, re will make use of formulas (3.16) and (3.17) for $k=N$, with $\mu_{\omega}=0$ being assumed in (3.17). This additional assumption does not contradict the boundary conditions

$$
\mu_{1} \cdot-0 \text { with } u \ldots \infty
$$

$$
\mathrm{k}=1,2, \ldots \mathrm{~d} .
$$

## 2. Discussion of Results

The obtained closed system 2 N of common differential equations relative to the functions $p_{1}, \ldots, p_{2 N-1}, F_{2}, \ldots, F_{2 N}$

$$
\begin{align*}
& q_{1}^{*}=e^{* k} q_{1 s} \quad T_{n=}^{*}=e^{* M} T_{n N} . \\
& C_{0}^{*}=e^{* \pi} \text {. . } \quad V_{1 *}^{*}=e^{k} V_{v}
\end{align*}
$$

was intagrated numerioally on a computer by the Runge-Kutt method in the interval $-4 \ln 10 \times u<10 \ln 10 \quad\left(1.0 ., 10^{-4}<t<10^{6}\right)$ for the indioator of the polytropic ourve $\gamma=5 / 3$ and the Mach speed $M=2.4$. In this case, the boundary conditions with $\gamma++\infty$ were marsposed to the point $i=10^{\circ 0}$ Bondi radi.

The caloulation was carried ou'c for N=1,2,...,10,15,20.

For $N \geqslant 2$, within the limite of accuracy of integration $\varepsilon=10^{-5}$ of the system (3.13), the values of the density $\rho(\tau)$, Mach speed $M(\gamma)$, and modulus of speed $V(\gamma)$ on the axis of symmetry $\theta=0$ practically do not depend on the selection of $N$, and are represented in the table. The density, speed, and Mach speed approach infinity monotonousily with $\gamma \rightarrow 0$.

The solution of system (3.13) may be continued into the randomly small vicinity of the point $\gamma=0$, which indicates the absence of a departed shock wave in front of the gravidational center for a gas with $\gamma=5 / 3$. The density $\rho(\tau, \theta)$, modulus of speed $V(\gamma, \theta)$, and function of the current lines $S(\gamma, \theta)$ were calculated approximately according to the formulas

$$
\begin{array}{ll}
\rho m\left(d_{1}+d_{1} \theta^{2}+\ldots+d_{N w-1} \theta^{2 N-2}\right)^{-1} \\
V=\left(V_{2}+V_{1} \theta^{2}+\ldots+V_{1 *} \theta^{2 N}\right)^{H}, & 14.1 / \\
S \sim S_{1} \theta^{2}+\ldots+S_{1 N} \theta^{2 k} & 14.2 f \\
& 1.3 /
\end{array}
$$

For $N=10$, the level lines $\rho, V$, and $S$ are given in figures 1, 2, and 3, respectively. Here, the x-axis is the axis of symmetry. The gravitational center is located at the origin of the coordirates. Plotted along the axes of the coordinates are the distances in Bondi radii ( $R_{B}=G \cdot M / C_{\infty}^{a}$ ). The gas flows into the center from the right.

The calculation of the level lines of $\rho, V, S$ for $N=5,10$, 15, and 20 shows that, in the range $0<\theta<\pi / 2$, the values of $\rho, V$, and $S$ practically do not depend on $N$, i.e., there occurs
converence of the approximated solution to the potential flow.
For $\pi / 2<\theta<\pi$, and especially olose to the axis of symmetry $\theta=\pi_{f}$ the approximated values of $\rho, V$, and a depend strongly on $N$. In this region, thore is no converganea to the point solution, since the flow is nonpotential.

| $\because$ | $p(2)$ | M( x ) | $V(x)$ |
| :---: | :---: | :---: | :---: |
| \% | 1. *ut | $\therefore 3412$ | 2.4113 |
| t. | t.toviz | 2.4ity | 2.1884 |
| 4. | 1,0005 | 2.5014 | 2.5018 |
| 2. | 1.0018 | 2.3578 | 2.5007 |
| 1. | 1.2061 | 2.7779 | 2.7835 |
| c. 5 | 1.0186 | 3.00k | 3.1181 |
| 0.1 | 1.1:28 | 4.8124 | 5.0460 |
| U.cr | $\therefore .0644$ | 11,214 | 14.279 |
| 0.001 | 8.3078 | 25.635 | 44.717 |
| $\therefore 0_{01}$ | 13.733 | 56,124 | 141.34 |

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Figure 3


[^0]:    *Numbers in the margin indicate pagination in the foreign text.

