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PLASMA WAVES IN A RELATIVISTIC, STRONGLY ANISOTROPIC PLASMA PROPAGATED ALONG A STRONG MAGNETIC FIELD.

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16. Abstract A study is made of the dispersion properties of plasma waves in a relativistic homogeneous plasma propagated along a strong magnetic field. It is shown that the non-damping plasma waves exist in the frequency range $\omega_p \leq \omega \leq \omega_*$. The values of ω_p and ω_* are calculated for an arbitrary homogeneous relativistic function of the particle distribution. In the case of a power ultrarelativistic distribution, it is shown that, if the ultrarelativistic "tail" of the distribution drops very rapidly, slightly damping plasma waves are possible with the phase velocity $\frac{\omega}{k} < c$

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Introduction

It follows from the general concept regarding the nature of pulsars that in the vicinity of a neutron star there may be an ultrarelativistic plasma in a strong magnetic field. Therefore, there is great interest in the problem of the generation and propagation of electromagnetic waves in a relativistic plasma occurring in a strong magnetic field [1], [2], [3], [4].

The electromagnetic properties of a relativistic plasma depend greatly on the particle distribution function (in the relativistic plasma, the role of three-dimensional dispersion is great). Kaplan and Tsytovich [1], [2], noted that close to a neutron star, there may be two processes, as a result of which an arbitrary distribution function of relativistic particles becomes almost homogeneous with respect to impulse--it is extended along the magnetic field $\bar{B}_0$. The first process is the monomerization of the particle distribution function in terms of impulse as a result of losses to synchrotron radiation, and the second process is the monomerization of the particle distribution function according to the conservation of the adiabatic invariant $p_\perp^2/B = const$, where $\bar{p}_\perp$ is the transverse component of the particle impulse when the particle moves in a slowly decreasing magnetic field. We assume the following as the specific particle distribution

*Numbers in margins indicate foreign pagination.

function in terms of energy [1], [2]

$$F_\alpha(E) = \frac{(\gamma-1)E_{*\alpha}^{\gamma-1}}{(E+E_{*\alpha})^{\gamma}} \cdot \begin{cases} 1, \text{при } E<E_{max\,\alpha}, E_{max\,\alpha} \gg E_{*\alpha} \gg m_\alpha c^2 \\ 0, \text{при } E>E_{max\,\alpha} \end{cases} \qquad (I.I)$$

where m_α is the mass of a particle of the α-type; c- speed of light. At $\gamma > 2$ the energy $E_{*\alpha}$ plays the role of the temperature, the average energy of the particles $\langle E \rangle E_{*\alpha}/($. /4

The one-dimensional distribution according to impulse, $\Phi_\alpha(p)$ which corresponds to this energy distribution, has the form

$$\Phi_\alpha(p) = (\gamma-1)\frac{P_{*\alpha}^{\gamma-1}\cdot p}{\sqrt{m_\alpha^2 c^2 + p^2}} \cdot \left(\sqrt{m_\alpha^2 c^2 + p^2} + P_{*\alpha}\right)^{-\gamma} \begin{cases} 1, \text{при } p < P_{max\,\alpha} = \frac{E_{max\,\alpha}}{c} \\ 0, \text{при } p > P_{max\,\alpha} \end{cases} \qquad (I.2)$$

In the region $p < m_\alpha c \sqrt[\gamma]{\frac{P_{*\alpha}}{m_\alpha c}}$ the distribution of (I.2), as was noted in [3], is an increasing function of p and consequently will be unstable with respect to longitudinal oscillations with $K_z c > \omega$, where $\bar{K}_z$ is the wave vector component along the magnetic field $\bar{B}$, ω - oscillation frequency. As a result of the quasilinear relaxation in the distribution (I.2) a plateau is formed in the region $p < m_\alpha c \sqrt[\gamma]{\frac{P_{*\alpha}}{m_\alpha c}}$.

The following function [3] is used by Suvorov and Chugunov as the specific distribution function:

$$\Phi_\alpha(p) = m_\alpha^2 c^2 (m_\alpha^2 c^2 + p^2)^{-\frac{3}{2}}. \qquad (I.3)$$

In the distribution (I.3), the average particle energy is $\langle E \rangle_\alpha = \frac{\pi}{2} m_\alpha c^2$. The particles are ultrarelativistic only in the distribution tail with the power index $\gamma = 3$. . As was shown in [5], if the radiation cooling occurs in a strong magnetic field of $B > 10^6$ followed by G, then the first process of monomerization of an arbitrary isotropic ultrarelativistic electron distribution is more effective than the second process. In this case, a one-dimensional distribution is established (I.3) in the time $t \sim m_\alpha c / e_\alpha B$; e_α - charge of particles of the α- type.

Observations, particularly observations of the radiation of a pulsar in the Crab nebula, provide a basis for assuming [6]that the region responsible for radiation in the optical and x-ray frequency ranges (in this region there is radiation cooling) monomerization of the electron distribution function due to synchrotron radiation) lies close to the light cylinder, where $B \sim 10^6$ G. It may be assumed that in the lower layers/5 of the pulsar magnetosphere the particles are accelerated up to ultrarelativistic energies by the longitudinal electric fields E_z and plasma heating.

This article studies the longitudinal (plasma) waves in a relativistic plasma consisting of particles with arbitrary one-dimensional distribution functions $\Phi_\alpha(p)$. The waves are propagated along the magnetic field B . The results are compared with the results of [2], [3]. The ions may not be relativistic with an isotropic distribution function. Not only ions may be particles with a positive charge, but also positrons. The case is studied in greater detail when $\Phi_\alpha(p)$ in the ultrarelativistic region decreases according to the power law $\Phi_\alpha(p) \sim p^{-\gamma_\alpha}$ (the corresponding energy distribution $f_\alpha(E) \sim E^{-\gamma_\alpha}$), and in the nonrelativistic region $\Phi_\alpha(p)$ is continued so that $\Phi_\alpha(p)$ is not an increasing function everywhere, and the average energy $\langle E \rangle_\alpha \gg m_\alpha c^2$. The following expression is most suitable as the function having these properties

$$\Phi_\alpha(p) = A_\alpha \cdot \frac{1}{\sqrt{m_\alpha^2 c^2 + p^2}} \cdot (p + p_{*\alpha})^{-\gamma_\alpha + 1} \begin{cases} 1, \text{ при } p < p_{max\,\alpha},\ p_{max\,\alpha} \gg p_{*\alpha} \gg m_\alpha c \\ 0, \text{ при } p > p_{max\,\alpha} \end{cases} \quad (I.4)$$

where $\alpha = 1,2,3,4...$ The distribution $\Phi_\alpha(p)$ is normed to unity i.e.. $\int \Phi_\alpha(p)\,dp = 1$. At $\gamma_\alpha > 1$ the coefficient $A_\alpha \simeq \dfrac{p_{*\alpha}^{\gamma_\alpha - 1}}{\ln 2 \frac{p_{*\alpha}}{m_\alpha c}}$.
If $\gamma_\alpha > 2$, then

for $\gamma_\alpha = 2$, $\dfrac{\langle E \rangle}{m_\alpha c^2} \simeq \dfrac{p_{*\alpha}}{m_\alpha c}\left(\ln 2\dfrac{p_{*\alpha}}{m_\alpha c}\right)^{-1} \dfrac{1}{\gamma_\alpha - 2}$, $\dfrac{\langle E \rangle}{m_\alpha c^2} \simeq \dfrac{p_{*\alpha}}{m_\alpha c}\left(\ln 2\dfrac{p_{*\alpha}}{m_\alpha c}\right)^{-1} \ln \dfrac{p_{max\,\alpha}}{p_{*\alpha}}$

I. Dispersion equation of plasma waves propagated along the external magnetic field in a one-dimensional relativistic plasma.

We shall assume that the distribution function Ψ_α of particles of the α-type is one-dimensional in the basic state, i.e.

$$\Psi_\alpha(P,\varphi) = \frac{N_\alpha}{2\pi}\cdot\frac{\Phi_\alpha(P)}{P^2}\cdot\frac{1}{\sin\varphi}\cdot\delta(\sin\varphi) \tag{2.1}$$

where N_α is the concentration; P - impulse; φ - pitch angle (angle between $\vec{P}$ and the magnetic field $\vec{B}$); $\delta(x)$ - delta-function; $\Phi_\alpha(P_z)$ - one dimensional distribution of particles in terms of longitudinal impulses. The distribution (2.1) may be regarded as the limit of distribution with the transverse temperature $T_\perp \to 0$. .

The dispersion equation for plasma waves propagated along an external magnetic field ($K = K_z$, $K_\perp = 0$) in a one-dimensional plasma has the form [2], [3] $\eta(\omega, k_z) = 0$, where

$$\eta(\omega, K_z) = 1 + \sum_\alpha \delta\eta_\alpha,$$

$$\delta\eta_\alpha = \frac{\omega_{L\alpha}^2}{\omega}\cdot m_\alpha \int_{-\infty}^{+\infty} V_z\, \zeta(\omega - k_z V_z)\cdot \Phi_\alpha'(P_z)\,dP_z, \tag{2.2}$$

where $\omega_{L\alpha}^2 = \frac{4\pi N_\alpha e_\alpha^2}{m_\alpha}$, V_z - particle velocity along the magnetic field and

$$\zeta(x) \equiv \frac{2\pi}{i}\delta_+(x),\ \delta_+(x) = \frac{1}{2\pi}\int_0^\infty e^{ixt}dt = \frac{1}{2}\delta(x) + \mathcal{P}\frac{i}{2\pi x} \tag{2.3}$$

The term $\mathcal{P}$ means that a singularity for X = 0 must be assumed to have the meaning of the basic value. $\eta(\omega, k_z)$ - component of the tensor of the complex dielectric constant $\varepsilon_{zz}(\omega, k)$.

If we use a one-dimensional velocity distribution $f(v)$, then we obtain

$$\delta\eta_\alpha = \frac{\omega_{L\alpha}^2}{\omega}\int_{-c}^{+c} v_z \,\delta(\omega - k_z v_z)\frac{d}{dv_z}\left[\left(1-\frac{v_z^2}{c^2}\right)^{3/2} f_\alpha(v_z)\right] dv_z \qquad (2.4)$$

We shall omit the index z for k_z, v_z, p_z below.

2. Long-wave plasma oscillations and plasma waves with the phase velocity, $v_\phi = \frac{\omega}{k} = c$ in a one-dimensional relativistic plasma.

Let us derive the dispersion equation of long-wave plasma oscillations. Using (2.2), we may calculate the Hermitian part η for a one-dimensional relativistic plasma with an arbitrary particle distribution function under the approximation of long waves:

$$k^2\langle v^2\rangle \ll \omega^2. \qquad (3.1)$$

If the plasma is ultrarelativistic $(\langle v^2\rangle = c^2)$, then the approximation of long waves will have the form /7
$k^2c^2 \ll \omega^2$. The anti-Hermitian part η and the Landau damping decrement of the plasma oscillations equal zero. In the approximation considered (3.1) the Hermitian part η has the form

$$\eta^{g}(\omega,k) = 1 - \sum_\alpha \frac{\omega_{L\alpha}^2}{\omega^2}\left[\left\langle\left(\frac{m_\alpha c^2}{E_\alpha}\right)^3\right\rangle_\alpha + 3\frac{k^2}{c^2}\left\langle v^2\left(\frac{m_\alpha c^2}{E_\alpha}\right)^3\right\rangle_\alpha\right], \qquad (3.2)$$

where $E_\alpha = c\sqrt{m_\alpha^2c^2 + p^2}$. The brackets $\langle\ \rangle_\alpha$ mean averaging over the one-dimensional distribution function $\varphi_\alpha(p)$, i.e. $\langle\varphi(p)\rangle = \int_{-\infty}^{\infty}\varphi(p)\varphi_\alpha(p)\,dp$.

In analogy with the non-relativistic plasma, we use the term plasma frequency for the frequency of long-wave $(k \to 0)$ plasma oscillations

$$\omega_p^2 = \sum \omega_{p\alpha}^2, \quad \omega_{p\alpha}^2 = \omega_{L\alpha}^2 \left\langle \left(\frac{m_\alpha c^2}{E_\alpha}\right)^3 \right\rangle_\alpha . \tag{3.3}$$

The dispersion equation of the plasma oscillations in the approximation considered

$$\omega^2 = \omega_p^2 \left(1 + 3\frac{K^2}{\omega^2} \cdot \frac{\sum\limits_\alpha \left\langle v^2 \left(\frac{m_\alpha c^2}{E_\alpha}\right)^3 \right\rangle_\alpha}{\sum\limits_\alpha \left\langle \left(\frac{m_\alpha c^2}{E_\alpha}\right)^3 \right\rangle_\alpha}\right) \tag{3.4}$$

Expression (3.2) in the zero approximation with respect to K ($k = 0$) and consequently the expression for the plasma frequency (3.3) may be obtained from elementary considerations, see Appendix I.

Using

$$\left\langle v^2 \left(\frac{m_\alpha c^2}{E_\alpha}\right)^3 \right\rangle_\alpha = c^2 \left[\left\langle \left(\frac{m_\alpha c^2}{E_\alpha}\right)^3 \right\rangle_\alpha - \left\langle \left(\frac{m_\alpha c^2}{E_\alpha}\right)^5 \right\rangle \right] \tag{3.5}$$

we may write (3.4) in the following form:

$$\omega^2 = \omega_p^2 + 3\delta K^2 c^2, \tag{3.6}$$

where

$$\delta = 1 - \frac{\sum\limits_\alpha \left\langle \left(\frac{m_\alpha c^2}{E_\alpha}\right)^5 \right\rangle_\alpha}{\sum\limits_\alpha \left\langle \left(\frac{m_\alpha c^2}{E_\alpha}\right)^3 \right\rangle_\alpha} . \tag{3.7}$$

If the plasma is not relativistic ($E_\alpha \simeq m_\alpha c^2$), then from (3.4), (3.6) we have the well-known Vlasov dispersion equation and /8
$\delta = \frac{\langle v^2 \rangle}{c^2} \ll 1$.. In the case of the relativistic distribution δ is determined basically by the contribution by the particles to the non-relativistic and slightly relativistic region, and does not depend on the ultrarelativistic "tail" of the distribution. Substituting the distributions (I.2), (I.3), (I.4) into (3.3), we obtain

$$\omega_{p\alpha 1}^2 = \frac{\gamma - 1}{2} \cdot \frac{m_\alpha c^2}{E_{0\alpha}} \cdot \omega_{L\alpha}^2 , \tag{3.8}$$

$$\omega_{p\alpha 2}^2 = \frac{3\pi}{16}\cdot\omega_{L\alpha}^2 \; ; \tag{3.9}$$

$$\omega_{p\alpha 3}^2 = \frac{\pi}{4}\cdot\frac{1}{\ln 2\frac{p_{*\alpha}}{m_\alpha c}}\cdot\omega_{L\alpha}^2 \text{, at } \gamma_\alpha > 1, \tag{3.10}$$

if $\gamma_\alpha = 1$, then

$$\omega_{p\alpha 3}^2 = \frac{\pi}{4}\cdot\frac{1}{\ln 2\frac{p_{max\,\alpha}}{m_\alpha c}}\cdot\omega_{L\alpha}^2 \tag{3.11}$$

The coefficient δ from the dispersion equation (3.6) will thus assume the following values: $\delta_1 = \frac{1}{2}$, $\delta_2 = \frac{1}{6}$, $\delta_3 = \frac{1}{4}$. In [1], [2], instead of the coefficient $\delta = \frac{1}{2}$ in the disper-sion equation of plasma waves the coefficient $\delta = \frac{1}{4}$ is written incorrectly in the long-wave approximation.

Let us examine plasma waves with the phase velocity $v_\varphi = \frac{\omega}{k} = c$. In this case η will have the form:

$$\eta = 1 - \frac{\omega_e^2}{\omega^2}, \quad \omega_e^2 = \sum_\alpha \omega_{\ell\alpha}^2 \tag{3.12}$$

where

$$\omega_{\ell\alpha}^2 = \omega_{L\alpha}^2\left[2\left\langle\frac{E_\alpha}{m_\alpha c^2}\right\rangle_\alpha - \left\langle\frac{m_\alpha c^2}{E_\alpha}\right\rangle_\alpha - 2\frac{p_{max\,\alpha}}{m_\alpha c}\,\Phi_\alpha(p_{max\,\alpha})\right] \tag{3.13}$$

If $\gamma_\alpha > 2$, then we may disregard the last term in the brackets of (3.13). In the case of an ultrarelativistic plasma, we may disregard the second term in the brackets as compared with the first term. /9

Substituting the distributions (I.2), (I.3) (I.4) into (3.13), we obtain the following expressions for

$$\omega_{\ell\alpha 1}^2 = \omega_{L\alpha}^2\cdot\frac{E_{*\alpha}}{m_\alpha c^2}\cdot\frac{2}{\gamma-2} = \frac{4}{(\gamma-1)(\gamma-2)}\,\omega_{p\alpha 1}^2\cdot\left(\frac{E_{*\alpha}}{m_\alpha c^2}\right)^2 \text{ at } \gamma > 3 \tag{3.14}$$

$$\omega_{\ell\alpha 2}^2 = \frac{3\pi}{4}\,\omega_{L\alpha}^2 = 4\,\omega_{p\alpha 2}^2 \tag{3.15}$$

$$\omega_{\ell\alpha 3}^2 = \omega_{L\alpha}^2\cdot\frac{p_{*\alpha}}{m_\alpha c}\cdot\frac{1}{\ln 2\frac{p_{*\alpha}}{m_\alpha c}}\cdot\frac{2}{\gamma_\alpha - 2} = \omega_{p\alpha 3}^2\cdot\frac{p_{*\alpha}}{m_\alpha c}\cdot\frac{8}{\pi(\gamma_\alpha-2)}, \; \gamma_\alpha > 3. \tag{3.16}$$

If $\gamma_\alpha = 2$ in the distribution (I.4), then

$$\omega_{e\alpha 3}^2 = \omega_{L\alpha}^2 \cdot \frac{p_{0\alpha}}{m_\alpha c} \cdot \frac{1}{\ln 2\frac{p_{0\alpha}}{m_\alpha c}} \left[2\ln\frac{p_{max\alpha}}{p_{0\alpha}} - 1\right] = \omega_{p\alpha 3}^2 \cdot \frac{p_{0\alpha}}{m_\alpha c} \cdot \frac{4}{\pi}\left[\ln\frac{p_{max\alpha}}{p_{0\alpha}} - 1\right] \quad (3.17)$$

if $\gamma_\alpha = 1$, then

$$\omega_{e\alpha 3}^2 = -\frac{\pi}{2}\,\omega_{L\alpha}^2 \frac{1}{\ln\frac{p_{max\alpha}}{m_\alpha c}} \quad (3.18)$$

i.e., if $\gamma_\alpha = 1$ for all types of particles, then there are no plasma waves with $\frac{\omega}{k} = c$ in such a plasma, and the dispersion curve $\omega(k)$ does not intersect the straight line $\frac{\omega}{k} = c$.

The expressions for $\omega_{p\alpha 1,2}^2$ and $\omega_{e\alpha 1,2}^2$ coincide with the corresponding expressions in [2], [3].

We should note that if we substitute the one-dimensional non-relativistic distribution into (3.3) and (3.13), then we obtain

$$\omega_{p\alpha\gamma}^2 = \omega_{L\alpha}^2 \left(1 - \frac{3}{2}\frac{\langle p^2\rangle_\alpha}{m_\alpha^2 c^2}\right),$$
$$\omega_{e\alpha\gamma}^2 = \omega_{L\alpha}^2 \left(1 + \frac{3}{2}\frac{\langle p^2\rangle_\alpha}{m_\alpha^2 c^2}\right). \quad (3.19)$$

Here $\frac{\langle p^2\rangle_\alpha}{m_\alpha^2 c^2} \ll 1$.

Thus in a one-dimensional relativistic plasma non-damping waves with the frequency ω are possible

$$\omega_p \leq \omega \leq \omega_e \quad (3.20)$$ /10

The greater is the proportion of relativistic (ultrarelativistic) particles in the distribution, the wider is the frequency range of the non-damping plasma waves. The frequency ω_e separates the non-damping $(\omega < \omega_e,\ \omega/k > c)$ and the damping longitudinal waves $(\omega > \omega_e,\ \omega/k <$, if such waves exist, see [2]. The specific form of the dispersion equation for plasma waves in the frequency region $\omega_p < \omega < \omega_e$, and the answer to the problem of the existence of plasma waves with $\omega > \omega_L$ ($\omega/k < c$) require a more detailed examination with the specific particle distribution function. As was shown in [3], there are no longitudinal waves with the frequency $\omega > \omega_L (\omega/k < c)$ in a plasma with the distribution function (I.3).

3. Plasma waves in a one-dimensional relativistic Maxwell plasma.

Relativistic particles cannot obey the Maxwell distribution function, since collisions of such particles are extremely rare. However, due to the fact that the "tail" of the Maxwell distribution rapidly decreases, the integral in (2.2) has a simple asymptotic expansion and this single-parametric distribution may be advantageously regarded as an illustration.

Let us substitute the one-dimensional relativistic Maxwell distribution into (2.2):

$$\Phi_\alpha(p) = \frac{1}{m_\alpha c \, K_1\left(\frac{m_\alpha c^2}{T_\alpha}\right)} \cdot \exp\left(-\frac{c\sqrt{m_\alpha^2 c^2 + p^2}}{T_\alpha}\right) \tag{4.1}$$

where $K_1(x)$ is the McDonald function. As a result of elementary transformations, we obtain the following expression for η_l :

$$\eta^l(\omega,k) = 1 - \sum_\alpha \omega_{L\alpha}^2 \cdot K_1^{-1}\left(\frac{m_\alpha c^2}{T_\alpha}\right) \cdot \int_1^\infty \frac{t\sqrt{t^2-1}}{k^2c^2-(k^2c^2-\omega^2)t^2} \cdot \exp\left(-\frac{m_\alpha c^2}{T_\alpha} t\right) dt \tag{4.2}$$

Let us consider the case when the plasma is ultrarelativistic, i.e., $\frac{m_\alpha c^2}{T_\alpha} \equiv z \ll 1$. Let us use the result of the asymptotic expansion of the integral (4.2) with respect to the parameter $z \ll 1$ for waves with $\omega > kc$, see appendix 2. We use /11 the fact that $K_1(z) \sim z^{-1}$ at $z \ll 1$. As a result we obtain

$$\eta(\omega,k) = 1 - \frac{\sum \omega_{L\alpha}^2 \frac{m_\alpha c^2}{T_\alpha}}{\omega^2 - k^2c^2} \tag{4.3}$$

Thus the dispersion equation of plasma waves

$$\omega^2 = \omega_p^2 + k^2c^2, \quad \omega_p^2 = \sum \omega_{L\alpha}^2 \frac{m_\alpha c^2}{T_\alpha} \tag{4.4}$$

The plasma frequency is determined by particles with a lower temperature T_α. The dispersion equation (4.4) is similar to the dispersion equation for transverse electromagnetic waves in an isotropic non-relativistic plasma when there is no external magnetic field, but (4.3), (4.4) cease to be valid at $\omega \gg \omega_p, kc \approx \omega$.

We should note that if there is no external magnetic field, and the electrons have an ultrarelativistic isotropic distribution, then the frequency of longitudinal oscillations of such an ultra-relativistic electron plasma [7] $(k \to 0)$

$$\omega^2 = \omega_{pe}^2, \quad \omega_{pe}^2 = \frac{1}{3}\omega_{Le}^2 \frac{m_e c^2}{T_e}, \qquad (T_e \ll T_i) \tag{4.5}$$

For waves with the phase velocity $v_\varphi = c$ we obtain

$$\eta = 1 - \frac{\omega_e^2}{\omega^2}, \quad \omega_e^2 = \sum_\alpha \omega_{e\alpha}^2, \quad \omega_{e\alpha}^2 = 2\omega_{L\alpha}^2 \frac{T_\alpha}{m_\alpha c^2} = 2\omega_{p\alpha}^2 \cdot \left(\frac{T_\alpha}{m_\alpha c^2}\right)^2. \tag{4.6}$$

Thus there are non-damping longitudinal waves in the frequency range $\omega_p \leq \omega \leq \omega_e$, in this plasma, where ω_p and ω_e are determined in (4.4), (4.6). The dispersion equation of such waves at $\omega_p \leq \omega \leq \omega_e$ is (4.4).

4. Plasma waves in a one-dimensional ultrarelativistic plasma with a power function of particle distribution.

Let us assume the distribution function of particles of the α-type has the form (I.4). Let us consider plasma waves for which the following condition is satisfied

$$\lambda_\alpha = \frac{\omega}{\sqrt{|k^2c^2-\omega^2|}} \cdot \frac{m_\alpha c}{p_{0\alpha}} \ll 1 \qquad (5.1)$$

at $\gamma_\alpha > I$, and at $\gamma_\alpha = I$ we have /12

$$\lambda_{1\alpha} = \frac{\omega}{\sqrt{|k^2c^2-\omega^2|}} \cdot \frac{m_\alpha c}{p_{max\,\alpha}} \ll 1 \qquad (5.2)$$

The conditions (5.1), (5.2) are satisfied in a very wide interval of a change in w and k, since $\frac{m_\alpha c}{p_{0\alpha}} \ll I$ and $\frac{m_\alpha c}{p_{max\,\alpha}} \ll I$. We should note that at $kc \gg \omega$ the value of $\lambda_\alpha, \lambda_{1\alpha}$ has a simple physical meaning $\lambda_\alpha = \frac{p_r}{p_{0\alpha}}, \lambda_{1\alpha} = \frac{p_r}{p_{max\,\alpha}}$, where p_r is the impulse of a particle in resonance with the wave $(\omega = k v_r)$. As a result of asymptotic expansions of the integral in (2.2) with the distribution function (1.4) with respect to the small parameter λ_α (or $\lambda_{1\alpha}$ at $\gamma_\alpha = I$), we obtain

$$\delta\eta_\alpha = -\frac{\omega_{p\alpha}^2}{k^2c^2}\, 2 \cdot \left\{ -1 + \left(\begin{array}{ll} \dfrac{1}{\sqrt{1-c^2k^2/\omega^2}}, & \text{при } ck/\omega < 1 \\ \dfrac{1}{i\sqrt{c^2k^2/\omega^2-1}}, & \text{при } ck/\omega > 1 \end{array} \right) \right\} \qquad (5.3)$$

where

$$\omega_{p\alpha}^2 = \omega_{L\alpha}^2 \frac{\pi}{4} \cdot \frac{1}{\ln 2\frac{p_{0\alpha}}{m_\alpha c}} \text{ at } \gamma_\alpha > I, \quad \omega_{p\alpha}^2 = \omega_{L\alpha}^2 \frac{\pi}{4} \frac{1}{\ln 2\frac{p_{max\,\alpha}}{m_\alpha c}} \text{ at } \gamma_\alpha = I \qquad (5.4)$$

If (5.1) (or (5.2) at $\gamma_\alpha = I$) is satisfied for all types of particles, then

$$\eta(\omega,k) = 1 - \frac{\omega_p^2}{k^2c^2} \cdot 2 \cdot \left\{ 1 + \left(\begin{array}{ll} \dfrac{1}{\sqrt{1-c^2k^2/\omega^2}}, & \text{at } ck/\omega < I \\ \dfrac{1}{i\sqrt{c^2k^2/\omega^2-1}}, & \text{at } ck/\omega > I, \end{array} \right) \right\}, \qquad (5.5)$$

where $\omega_p^2 = \sum \omega_{p\alpha}^2$. It may be seen from (5.5) that in the approximation considered (5.1)((5.2) at $\gamma_\alpha = I$) the imaginary part in the expression for η is comparable in terms of the modulus with the real part, just as in the case with the distribution (I.3) [3]. The dispersion equation $\eta(\omega, k) =$ $= 0$ in the region examined ω, k has a solution only at $ck < \omega$

$$\frac{k^2c^2}{\omega^2} = \frac{1 - 4v + \sqrt{1 + 8v}}{2}, \qquad (5.6)$$

where $v = \frac{\omega_p^2}{\omega^2}$. It may be seen from (5.6) that in the plasma considered non-damping longitudinal waves are possible at $0 < v \leq 1$, i.e. waves with a frequency $\omega \geq \omega_p$ are possible. In the approximation $\frac{kc}{\omega} \to 0$ from (5.6) we may obtain the dispersion equation $\omega^2 = \omega_p^2 + \frac{2}{3}k^2c^2$, as follows from (3.6) with $\delta_3 = \frac{1}{4}$ from (3.11). For waves with the frequency $\omega \gg \omega_p$ and $(v \ll 1)$ the following dispersion law follows from (5.6)

$$\frac{k^2c^2}{\omega^2} = 1 - \frac{1}{2}v^2. \qquad (5.7)$$

Since the condition (5.1) must be satisfied, then (5.7) holds for the frequencies

$$\frac{\omega_p^4}{\omega^4} \gg 2\max\left(\frac{m_\alpha^2c^2}{p_{\alpha}^2}\right) \qquad (5.8)$$

If the plasma consists of "hot" (with the distribution function (1.4) electrons and cold ions (the ions may have a distribution function which is isotropic with respect to impulse), then the dispersion equation of plasma waves will have the form of (5.6), where $v = \frac{\omega_{pe}^2}{\omega^2 - \omega_{pi}^2}$. In this case the plasma waves are possible with the frequency $\omega \geq \omega_{pe}$.

Let us consider plasma waves with the dispersion equation

$$kc = \omega(1 + \Delta), \quad |\Delta| \ll 1 \qquad (5.9)$$

and let us assume the following for particles of the α - type with $\gamma_\alpha > 3$

$$\lambda_\alpha \gg 1. \tag{5.10}$$

For waves with the phase velocity $\frac{\omega}{k} < c$ condition (5.10) means that the resonance particles are found in the tail of the distribution ($p_r \gg p_{T\alpha}$). The contribution of particles of the α - type to the Hermitian part η will be

$$\delta\eta^2 = -\frac{\omega_{e\alpha}^2}{\omega^2} \tag{5.11}$$

The expression for $\omega_{e\alpha}^2$ can be seen in (3.16). If (5.10) is satisfied for all types of particles, then

$$\eta^2 = 1 - \frac{\omega_e^2}{\omega^2}, \quad \omega_e^2 = \sum \omega_{e\alpha}^2 \tag{5.12}$$

The dispersion equation of plasma waves

$$kc \approx \omega = \omega_e \tag{5.13}$$

If $\gamma_\alpha < 3$, then in order that (5.11) hold, it is necessary that the following condition be satisfied to a greater extent than (5.10) /13

$$\lambda_{1\alpha} \gg 1. \tag{5.14}$$

When the condition (5.14) is satisfied for waves with $\frac{\omega}{k} < c$ there are no resonance particles $p_r \gg p_{max\,\alpha}$), and consequently there is no Landau damping.

Let us consider in greater detail the most interesting case of plasma waves with $\frac{\omega}{k} < c$. If these waves exist, then they will undergo Landau damping (if $\lambda_{1\alpha} < 1$). In this plasma "Chernekov" generation of longitudinal waves by beams of electrons is possible.

Let us assume that the condition (5.10) and $\gamma_\alpha > 4$ is satisfied for particles of the α-type. Let us use asymptotic estimations of the integral in (2.2) with respect to the parameter λ_α^{-1} $(\lambda_{1\alpha} < I)$, and we obtain

$$\delta\eta_\alpha = -\frac{\omega_{e\alpha}^2}{\omega^2} - \frac{\omega_{e\alpha}^2}{\omega^2}\cdot\frac{4\gamma_\alpha-9}{2(\gamma_\alpha-3)}\cdot\lambda_\alpha^{-2} + i\gamma_\alpha(\gamma_\alpha-2)\frac{\omega_{e\alpha}^2}{\omega^2}\cdot\lambda_\alpha^{-\gamma_\alpha+2} \qquad (5.15)$$

If (5.10) is satisfied for all types of particles and $\gamma_\alpha > 4$, then

$$\eta = 1 - \frac{\omega_e^2}{\omega^2} - \frac{\sum \omega_{e\alpha}^2 \frac{4\gamma_\alpha-9}{2(\gamma_\alpha-3)}\cdot\lambda_\alpha^{-2}}{\omega^2} + i\sum\gamma_\alpha(\gamma_\alpha-2)\frac{\omega_{e\alpha}^2}{\omega^2}\lambda_\alpha^{-\gamma_\alpha+2}. \qquad (5.16)$$

Thus the dispersion equation of plasma waves

$$\frac{k^2c^2}{\omega^2} - 1 = \frac{\omega^2-\omega_e^2}{\sum\omega_{e\alpha}^2\frac{4\gamma_\alpha-9}{2(\gamma_\alpha-3)}\frac{R_\alpha^2}{m_\alpha^2c^2}} \qquad (5.17)$$

The ratio of the increment of the plasma oscillations γ_l $(\mathrm{Im}\,\omega \equiv \gamma_l)$ to the frequency of the oscillations is

$$\frac{\gamma_l}{\omega} = -\frac{\sum\omega_{e\alpha}^2\gamma_\alpha(\gamma_\alpha-2)\left(\frac{R_\alpha}{m_\alpha c}\right)^{\gamma_\alpha-2}\left[\frac{\omega^2-\omega_e^2}{\sum\omega_{e\alpha}^2\frac{4\gamma_\alpha-9}{2(\gamma_\alpha-3)}\frac{R_\alpha^2}{m_\alpha^2c^2}}\right]^{\frac{\gamma_\alpha-2}{2}}}{2\omega_e^2} \ll 1. \qquad (5.18)$$

If the plasma consists of electrons and positrons with identical distribution functions $\Phi_\alpha(p)$ $(R_\alpha = R, \gamma_\alpha = \gamma)$, then the dispersion equation of plasma waves will be

$$\frac{k^2c^2}{\omega^2} - 1 = \frac{\omega^2-\omega_e^2}{\omega_e^2}\cdot\frac{2(\gamma-3)}{4\gamma-9}\cdot\frac{m^2c^2}{R^2} \qquad (5.19)$$

and the ratio of the oscillation increment to the frequency

$$\frac{\gamma_l}{\omega} = -\frac{\gamma(\gamma-2)}{2}\cdot\left(\frac{2(\gamma-3)}{4\gamma-9}\right)^{\frac{\gamma-2}{2}}\cdot\left(\frac{\omega^2-\omega_e^2}{\omega_e^2}\right)^{\frac{\gamma-2}{2}} \qquad (5.20)$$

since according to condition (5.10) $\lambda_\alpha \gg I$, we have

$$\frac{\omega^2 - \omega_k^2}{\omega_e^2} \ll 1 \qquad (5.21)$$

If the plasma consists of "hot" electrons with a power distribution function (I.4) with $\gamma_e > 4$ and "cold" ions (the ion distribution function may be isotropic), then in the approximation $\lambda_e \gg I$ and $\lambda_{1e} < I$ (in the frequency region $\omega \ll \omega_{ke}$) we may disregard the contribution of ions to the Hermetian and anti-Hermetian part η (since $\omega_{ke} \gg \omega_{ki}$). The dispersion equation of plasma oscillations and the expressions for the oscillation increment have the form (5.19), (5.20) and (5.21) if we set $\omega_k^2 = \omega_e^2, \gamma = \gamma_e$. In the frequency region ω such that $\lambda_e \ll 1, \lambda_i \gg 1$ (if the ions are colder than the electrons) $\delta\eta_e$ has the form (5.3); $\delta\eta_i$ - (5.15). In this plasma, in the approximation considered only waves with $\omega > kc$ are possible. The dispersion equation will have the form (5.6), where $v = \frac{\omega_{pe}^2}{\omega^2 - \omega_{ke}^2}$.

Let us conside: the case when $\gamma_\alpha = 3$ and $\lambda_\alpha \gg I$ ($\lambda_{1\alpha} < I$). Then the contribution of particles of the α-type to η will have the following form:

$$\delta\eta_\alpha = -\frac{\omega_{e\alpha}^2}{\omega^2} + \frac{3}{2}\cdot\frac{\omega_{e\alpha}^2 \lambda_\alpha^{-2} \ln \lambda_\alpha^{-2}}{\omega^2} + i\,3\,\frac{\omega_{e\alpha}^2}{\omega^2}\cdot\frac{p_{0\alpha}}{m_\alpha c}\sqrt{\frac{k^2c^2}{\omega^2} - 1} \qquad (5.22)$$

If $\gamma_\alpha = 3$ and $\lambda_\alpha \gg I$ ($\lambda_{1\alpha} < I$ for all types of particles, or if the electrons "are hot" with $\gamma_e = 3$, $\lambda_e \gg I$ ($\lambda_{1e} < I$), and the ions are "cold", then the dispersion equation $1 + \sum \delta\eta_\alpha$ in the frequency region examined does not have a solution. The situation is the same if $\gamma_\alpha < 3$.

Thus in an ultrarelativistic one-dimensional plasma with a power distribution function for the particles with respect to impulse (1.4) (or in a plasma where the electrons are "hot" with the distribution function (1.4) and the ions are cold) plasma waves are possible with the frequency $\omega \gg \omega_{ke}$ and the dispersion law (5.6). The phase velocity of these waves is greater than the speed of light and consequently, they do not undergo Landau damping. If the ultrarelativistic "tail" of the particle power distribution decreases rather rapidly, and the exponent $\gamma_\alpha > 3$ ($\gamma_e > 3$), then slightly damping plasma waves are possible with the phase velocity $v_\varphi < c$. The frequency of these waves is $\omega = \omega_e(1+\Delta)$, where $0<\Delta \ll 1$. If the exponent $\gamma < 3$, then there will be no plasma waves with the phase velocity $v_\varphi < c$ and the frequency $\omega > \omega_e$.

Appendix I

Let us consider one-dimensional oscillations of a relativistic plasma along the z-axis. We shall assume the plasma is one-dimensional (thermal motion only along the z axis), uniform and stationary. We shall disregard the collisions. We shall assume that the electric field and the velocity of particles of x-type change in time according to an harmonic law, i.e., $\tilde{v}_{z\alpha}, \tilde{E}_z \sim e^{-i\omega t}$ and $|\tilde{v}| \ll c$. We shall consider the oscillations in the approximations of long waves $K \to 0$. The equation of motion for particles of the α-type

$$\frac{d}{dt} P_{z\alpha} = e_\alpha \tilde{E}_z, \tag{II.I.I}$$

where p - impulse; e_α - charge of particles of the α-type. Since the oscillations are considered in the linear approximation, then $|\tilde{v}_{z\alpha}| \ll c$ and the particle velocity $v_\alpha = v_{T\alpha} + \tilde{v}_{z\alpha}$, where $v_{T\alpha}$ is the velocity of "thermal" motion. We obtain the following from (II.1.1) and the condition $K \to 0$

$$m_\alpha \frac{\partial}{\partial t} \frac{v_{T\alpha} + \tilde{v}_{z\alpha}}{\sqrt{1 - \frac{(v_{T\alpha} + \tilde{v}_{z\alpha})^2}{c^2}}} = e_\alpha \tilde{E}_z \tag{II.I.2}$$

From this we have

$$\frac{m_\alpha \tilde{v}_{z\alpha}}{\left(\sqrt{1 - \frac{(v_{T\alpha} + \tilde{v}_{z\alpha})^2}{c^2}}\right)^3} = e_\alpha \tilde{E}_z . \tag{II.I.3}$$

Since $|\tilde{v}_{z\alpha}| \ll c$, we may assume that the particle energy remains constant, i.e.

$$\mathcal{E}_\alpha = \frac{m_\alpha c^2}{\sqrt{1-\frac{(v_{0\alpha}+\tilde{v}_{z\alpha})^2}{c^2}}} \simeq \frac{m_\alpha c^2}{\sqrt{1-\frac{v_{0\alpha}^2}{c^2}}} \qquad \text{(II.I.4)}$$

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We obtain an expression for the average velocity of particles of α-type from (II.1.3) and (II.1.4)

$$\langle \tilde{v}_{z\alpha}\rangle_\alpha = i\frac{e_\alpha}{m_\alpha \omega}\left\langle\left(\frac{m_\alpha c^2}{\mathcal{E}_\alpha}\right)^3\right\rangle_\alpha \tilde{E}_z \qquad \text{(II.I.5)}$$

From this we have the current density

$$\tilde{J}_z = \sum_\alpha e_\alpha N_\alpha \langle \tilde{v}_{z\alpha}\rangle_\alpha = i\sum_\alpha \frac{e_\alpha^2 N_\alpha}{m_\alpha \omega}\left\langle\left(\frac{m_\alpha c^2}{\mathcal{E}_\alpha}\right)^3\right\rangle \tilde{E}_z, \qquad \text{(II.I.6)}$$

i.e., the conductivity

$$\sigma = i\sum_\alpha \frac{e_\alpha^2 N_\alpha}{m_\alpha \omega}\left\langle\left(\frac{m_\alpha c^2}{\mathcal{E}_\alpha}\right)^3\right\rangle. \qquad \text{(II.I.7)}$$

Thus

$$\eta = 1 + i\frac{4\pi}{\omega}\sigma = 1 - \sum_\alpha \frac{4\pi N_\alpha e_\alpha^2}{\omega^2 m_\alpha}\left\langle\left(\frac{m_\alpha c^2}{\mathcal{E}_\alpha}\right)^3\right\rangle_\alpha. \qquad \text{(II.I.8)}$$

If in analogy with a cold plasma we write $\eta = 1 - \frac{\omega_p^2}{\omega^2}$, then we obtain the following expression for the plasma frequency

$$\omega_p^2 = \sum_\alpha \omega_{L\alpha}^2 \left\langle\left(\frac{m_\alpha c^2}{\mathcal{E}_\alpha}\right)^3\right\rangle_\alpha, \qquad \text{(II.I.9)}$$

where $\omega_{L\alpha}$ is the Langmuir frequency of particles of the α-type.

Appendix 2

Let us consider the approximation $\gamma^{-1} \equiv x \gg 1$ for waves with $\omega > kc$. In this case the integral in (4.2) does not have singularities and is calculated in the regular sense. The integral in (4.2) may be represented in the following form

$$J(x) = J_1(x) + J_2(x). \qquad \text{(II.2.I)}$$

where

$$J_1(x) = e^{-\frac{1}{x}} \int_0^\infty \frac{t\sqrt{t^2+2t}}{k^2c^2+(\omega^2-k^2c^2)(t+1)^2} e^{-\frac{t}{x}} dt, \qquad \text{(II.2.2)}$$

$$J_2(x) = e^{-\frac{1}{x}} \int_0^\infty \frac{\sqrt{t^2+2t}}{k^2c^2+(\omega^2-k^2c^2)(t+1)^2} \cdot e^{-\frac{t}{x}} dt \, . \qquad \text{(II.2.3)}$$

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To expand the integrals $J_1(x)$ and $J_2(x)$ in an asymptotic series with respect to the small parameter x^{-1}, we use the method of successive approximation [8]. As a result we obtain the following at $x \to \infty$

$$J_1(x) \sim e^{-\frac{1}{x}} \cdot \frac{x}{\omega^2-k^2c^2} \qquad \text{(II.2.4)}$$

The notation $f(z) \sim a \cdot g(z)$ at $z \to z_0$ is equivalent to $f(z) = a g(z) + O(g(z))$. Similarly

$$J_2(x) \sim \frac{\ln x}{\omega^2-k^2c^2} - \frac{C}{\omega^2-k^2c^2} + A_0, \qquad \text{(II.2.5)}$$

where C is the Euler constant, ($C \approx 0,577$), and

$$A_0 = \int_0^1 \frac{\sqrt{t^2+2t}}{k^2c^2+(\omega^2-k^2c^2)(t+1)^2} + \int_1^\infty \left[\frac{\sqrt{t^2+2t}}{k^2c^2+(\omega^2-k^2c^2)(t+1)^2} - \frac{1}{\omega^2-k^2c^2} \frac{1}{t} \right] dt \qquad \text{(II.2.6)}$$

Thus at $x \to \infty$

$$J(x) \sim e^{-\frac{1}{x}} \frac{1}{\omega^2-k^2c^2} (x + \ln x) \sim \frac{x}{\omega^2-k^2c^2} \, . \qquad \text{(II.2.7)}$$

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