

## NOTICE

THIS DOCUMENT HAS BEEN REPRODUCED FROM  
MICROFICHE. ALTHOUGH IT IS RECOGNIZED THAT  
CERTAIN PORTIONS ARE ILLEGIBLE, IT IS BEING RELEASED  
IN THE INTEREST OF MAKING AVAILABLE AS MUCH  
INFORMATION AS POSSIBLE

(NASA-TM-82135) ON THE POLARITY OF  
CYCLOSTROPHIC FLOW IN PLANETARY ATMOSPHERES  
(NASA) 9 p HC A02/MF A01 CSCL 03B

N81-23990

Unclas  
G3/91 24303



## Technical Memorandum 82136

# On The Polarity of Cyclostrophic Flow in Planetary Atmospheres

H. G. Mayr, B. J. Conrath and I. Harris

MAY 1981

National Aeronautics and  
Space Administration

**Goddard Space Flight Center**  
Greenbelt, Maryland 20771



ON THE POLARITY OF CYCLOSTROPHIC FLOW  
IN PLANETARY ATMOSPHERES

BY

H. G. MAYR, B. J. CONRATH AND I. HARRIS  
NASA/GODDARD SPACE FLIGHT CENTER  
GREENBELT, MARYLAND 20771

MAY 1981

SUBMITTED TO THE JOURNAL OF THE ATMOSPHERIC SCIENCES

## ABSTRACT

Fluids which are completely inviscid in the mathematical sense do not exist. Thus, the concepts of gradient flow and cyclostrophic balance must be interpreted as approximate solutions of a boundary value problem for small but finite viscosity. Large scale phenomena such as the superrotation on Venus and cyclones are effectively bounded by the rigidly rotating planetary surface. This polarizes the circulation and excludes so-called anomalous motions from the flow regime. With small scale phenomena such as dust devils, both directions are observed which is attributed to the stochastic nature of wind systems surrounding the disturbance.

Leovy (1973) introduced the concept of cyclostrophic balance to describe superrotation in the Venusian atmosphere but pointed out an ambiguity about the direction of the zonal wind velocities which is apparently related to the anomalous motions discussed with gradient flow (e.g., Hess, 1959; Holton, 1979). Later on in his paper Leovy (1973) argued that surface friction would tend to accelerate the atmosphere into the direction of the planet, and Hess (1959) and Holton (1979) considered the role of geostrophic winds in suggesting that anomalous motions are unlikely to occur. Our interpretation is consistent with these interpretations but emphasizes in concise form the subtle importance of viscous interaction which might further help to clarify the problem.

We consider the zonally symmetric component of the atmosphere and assume the rate of change is sufficiently slow so that time derivatives can be neglected. With a given pressure gradient, or any other meridional force, the simplified equations governing horizontal momentum conservation have the form

$$2\omega w \sin\Theta U + \frac{U^2}{r} \operatorname{tg}\Theta + \eta \frac{\partial^2 V}{\partial r^2} = - \frac{1}{r\rho} \frac{\partial p}{\partial \Theta} \quad (1)$$

$$2\omega s \sin\Theta V + \frac{VU}{r} \operatorname{tg}\Theta + \frac{V}{r \cos\Theta} \frac{\partial U}{\partial \Theta} - \eta \frac{\partial^2 U}{\partial r^2} = 0, \quad (2)$$

where vertical winds are ignored and the meridional velocities  $V$  are assumed to be small compared to the zonal velocities  $U$ , and

$\rho$ , mass density

$p$ , pressure

$\Theta$ , latitude

$\omega$ , rotation rate of the planet

$\eta$ , kinematic viscosity.

For an inviscid ( $\nu=0$ ) atmosphere, the meridional and zonal force balances are formally decoupled and (1) yields

$$U = -w r \cos \theta \pm \sqrt{\omega^2 r^2 \cos^2 \theta - \frac{\text{ctg} \theta}{\rho} \frac{\partial p}{\partial \theta}}, \quad (3)$$

both roots being allowed. The expression inside the radical is assumed to be greater than zero.

However, an atmosphere entirely without viscosity does not exist. In fact, the concept of viscosity is inherent in our fluid dynamic formulation relative to a rotating reference frame. If the viscosity were truly zero, that atmosphere would not "know" about the rotation rate of the planet, except for the planet's shape (geoid), and the concepts of Coriolis force and geostrophic approximation would be meaningless.

Obviously, the correct physical interpretation is that expression (3) must be viewed as an approximate boundary value solution of the coupled equations (1) and (2) for an atmosphere with small but finite viscosity, bounded by a rigidly rotating planetary surface. It follows that in the limit of vanishing pressure gradient the viscous interaction causes the atmosphere to corotate with the planet

$$\frac{\partial p}{\partial \theta} \rightarrow 0; U \rightarrow 0. \quad (4)$$

We require the solution to contain (4) and be continuous in the forcing function. The physical branch is then uniquely determined yielding single value expressions for the general flow regime

$$U = -w r \cos \theta + \frac{\omega}{|\omega|} \sqrt{\omega^2 r^2 \cos^2 \theta - \frac{\text{ctg} \theta}{\rho} \frac{\partial p}{\partial \theta}} \quad (5)$$

and for cyclostrophic balance

$$U = + \frac{\omega}{|\omega|} \sqrt{\frac{\text{ctg} \theta}{\rho} \frac{\partial p}{\partial \theta}} \quad (6)$$

The atmosphere of Venus is indeed observed to superrotate in the direction of the planet.

In terrestrial meteorology natural coordinates are used to describe large scale disturbances such as cyclones. With  $\omega > 0$  and  $R$  being the radius of flow curvature in the direction of unit vector  $\vec{n}$  (Holton, 1979), the expression for gradient flow, corresponding to (5), takes the form

$$U = -\omega R \sin \theta + \frac{R}{|R|} \sqrt{\omega^2 R^2 \sin^2 \theta - \frac{R}{\rho} \frac{\partial p}{\partial n}} \quad (7)$$

The limit for cyclostrophic balance

$$U = + \frac{R}{|R|} \sqrt{-\frac{R}{\rho} \frac{\partial p}{\partial n}} \quad (8)$$

is valid for small intense hurricanes.

Due to viscous interaction the negative branch cannot exist which is consistent with the fact that anomalous motions are never observed in cyclones (Holton, 1979). We emphasize that (7) and (8) can only describe long term effects.

However, anomalous motions do exist in the smaller scale vortices of dust devils, where the negative branch is observed as often as the positive one (Sinclair, 1965). The explanation for this is that we deal with a non-stationary condition, similar to turbulence where horizontal viscous shear and momentum transport are important. In the limit of cyclostrophic balance, the

boundary value solution is then no longer polarized by the rotation of the planet, as is the case for large scale phenomena such as superrotation and cyclones. Instead, the stochastic nature of wind systems bordering the disturbance prevails, so as to initiate motions in either direction.

Acknowledgement

The authors are indebted to Dr. M. A. Geller and to an anonymous referee for valuable suggestions.



## REFERENCES

Hess, S. C., Introduction to Theoretical Meteorology, Holt, New York.

Holton, J. R., An Introduction to Dynamic Meteorology, Academic Press, 1979.

Leovy, C. B., Rotation of the upper atmosphere of Venus, J. Atmos. Sci., 35,  
1218, 1973.

Sinclair, P. C., On the rotation of dust devils, Bull. Amer. Meteorol. Soc.,  
46, 388, 1965.