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EFFECTS OF TURBULENCE ON A  
KINETIC AURORAL ARC MODEL

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## Abstract

We extend ~~an~~ earlier plasma kinetic model of an inverted-V auroral arc structure to include <sup>which</sup> ~~in a phenomenological way~~, the effects of electrostatic turbulence <sup>is proposed</sup> with  $k_{\perp}/k_{\parallel} \ll 1$ . In the absence of turbulence, a parallel potential drop is supported by magnetic mirror forces and charge quasi-neutrality, with energetic auroral ions penetrating to low altitudes; relative to the electrons, the ions' pitch angle distribution is skewed toward smaller pitch angles. The electrons energized by the potential drop form a current which excites electrostatic turbulence; ~~we consider the specific case of the ion cyclotron mode.~~ ~~The turbulence heats the ions in  $T_{\perp}$  only, thus tending to reduce the differential pitch angle anisotropy between electrons and ions, which in turn reduces the potential drop - an effect of opposite sign to that associated with anomalous resistivity.~~ In equilibrium the plasma is marginally stable, ~~with growth rates and diffusion constants some two orders of magnitude below naive estimates.~~ The conventional anomalous resistivity contribution to the potential drop is very small, ~~because~~ anomalous resistivity processes are far too dissipative to be powered by auroral particles; ~~this is why growth rates and diffusion constants are so small.~~ Under certain circumstances equilibrium may be impossible and relaxation oscillations set in; ~~the time scale for such pulsating auroras is the ion transit time of 6-10 seconds.~~

### Abstract

We extend an earlier plasma-kinetic model of an inverted-V auroral arc structure to include, in a phenomenological way, the effects of electrostatic turbulence with  $k_{\parallel}/k_{\perp} \ll 1$ . In the absence of turbulence, a parallel potential drop is supported by magnetic mirror forces and charge quasi-neutrality, with energetic auroral ions penetrating to low altitudes; relative to the electrons, the ions' pitch-angle distribution is skewed toward smaller pitch angles. The electrons energized by the potential drop form a current which excites electrostatic turbulence; we consider the specific case of the ion cyclotron mode. The turbulence heats the ions in  $T_{\perp}$  only, thus tending to reduce the differential pitch angle anisotropy between electrons and ions, which in turn reduces the potential drop - an effect of opposite sign to that associated with anomalous resistivity. In equilibrium the plasma is marginally stable, with growth rates and diffusion constants some two orders of magnitude below naive estimates. The conventional anomalous resistivity contribution to the potential drop is very small, because anomalous-resistivity processes are far too dissipative to be powered by auroral particles; this is why growth rates and diffusion constants are so small. Under certain circumstances equilibrium may be impossible and relaxation oscillations set in; the time scale for such pulsating auroras is the ion transit time of 6-10 seconds.

## I. INTRODUCTION

Recently many authors have been concerned with plasma-kinetic mechanisms for explaining auroras associated with inverted-V's (auroral arcs, for brevity) [Swift, 1975; Kan, 1975; Hudson and Mozer, 1978; Chiu and Schulz, 1978; Fridman and Lemaire, 1980; Lyons, 1980, Chiu and Cornwall, 1980]. The fundamental objective is to set up a parallel potential drop of 1-10 keV along an auroral field line which will accelerate electrons and precipitate them into the ionosphere. To this end, three mechanisms are most widely cited: 1) charge-separation effects (Poisson's equation); 2) anomalous resistivity; 3) magnetic mirror forces coupled with differential electron-ion pitch-angle anisotropy. Because of the large ( $> 10^4$ ) transverse dielectric constant of the magnetized auroral plasma, transverse derivatives in Poisson's equation completely dominate derivatives along the field line [Chiu and Cornwall, 1980]. We will therefore not consider unmagnetized double layers (potential drop  $\sim$  keV in a distance  $\sim$  0.1 km along a field line) any further, although several authors have invoked them for auroral arcs [Block, 1975; Shawhan et al., 1978]. The remaining effect of Poisson's equation is to couple neighboring field lines on a length scale  $\geq R_L$  ( $R_L$  is the ion Larmor radius) [Swift, 1975; Chiu and Cornwall, 1980]. This sort of coupling will not be vital for our present purposes, so we will demand instead that the plasma be quasi-neutral:  $N_i = N_e$  everywhere.

Next consider anomalous resistivity. While it may be very important in some laboratory circumstances, a simple energy-balance argument severely limits its role in the aurora, at least in the scenario we adopt. In that scenario, all auroral energy comes from the kinetic energy of plasmashet particles, and is likely to be dominated by the ions. Suppose that turbulence somehow generates an anomalous collision frequency  $\nu_A$  for electrons (of mass

m), so that the current-field relation is

$$J_{\parallel} = \sigma_{\parallel} E_{\parallel} ; \sigma_{\parallel} = Ne^2/m v_A \quad (1)$$

The electron current is of order  $J_{\parallel} \sim eNv_e$ , where  $v_e$  is a typical electron velocity, so that the power dissipated on the auroral field line, integrated over the region of length  $\bar{\ell}$  where the turbulence is present is, in order of magnitude,

$$\int ds J_{\parallel}^2/\sigma_{\parallel} \sim \bar{\ell} Nm v_e^2 v_A \quad (2)$$

This must be less than the input power, which comes from fields and particles in the plasmashet. There are several estimates one can make for this, but none are plausibly larger than the ion energy flux. (Quasi-neutrality of the plasmashet limits the electron energy flux to be of the order of the ion energy flux.) For example, one might begin with the power per unit volume  $\vec{J}_{\perp} \cdot \vec{E}_{\perp}$  dissipated in MHD flow. We estimate  $J_{\perp} \sim c\bar{v}p/B$ ,  $E_{\perp} \sim 1$  mV/m, and multiply the product by a length which we take to be the same as that setting the scale of the pressure gradient. The result is substantially less than the ion power  $NMV^3$  ( $M$  is the ion mass;  $V$  is a typical ion velocity), and it is this latter quantity which we use as the input power; it is of the order of a few ergs/cm<sup>2</sup> sec. In terms of the average plasmashet electron energy  $W_e$  and plasmashet ion energy  $W$ , we have  $v^2/v_e^2 = mW/(MW_e)$ , so that requiring (2) to be less than  $NMV^3$  implies

$$v_A < (W/W_e)(V/\bar{\ell}) \quad (3)$$

For typical auroral parameters this amounts to  $\nu_A/\omega_{pe} < 10^{-5}$ , considerably less than the range  $\nu_A/\omega_{pe} \approx 10^{-4} - 10^{-2}$  found in simulations or laboratory experiments, which refer to quite different physical circumstances not found in auroras.

The point, however, is not the absolute size of  $\nu_A$ , but rather what sort of potential drop it can lead to. Anomalous resistivity is so dissipative that if turbulence and heating reach the maximum level allowed by (2), only a small potential drop is generated (a somewhat similar point was made by Fälthammar [1977]). Let us use (2) and (3) in (1), estimating the total potential drop between the ionosphere and the equator as  $\Delta\phi \approx -\ell E_{\parallel}$ . One finds

$$e\Delta\phi < W \left( \frac{m}{M} \frac{W}{W_e} \right)^{1/2} \quad (4)$$

In the plasmashet,  $W/W_e \approx 2-4$  is typical [e.g., Frank et al., 1981] so the potential drop, scaled in units of ion energy, is perhaps 0.05 - 0.1. Note that this is independent of the length  $\bar{\ell}$  of the turbulent region. In contrast to the bound (4) on  $e\Delta\phi$  maintainable by anomalous resistivity, the theory outlined below, as well as experimental observations, suggest that  $e\Delta\phi \approx W$ .

Thus the anomalous-resistivity potential drop is limited severely by the available auroral power, which is why canonical estimates  $\nu_A \approx (10^{-4} - 10^{-2}) \omega_{pe}$  are so far off the mark: they refer to circumstances where the potential drop is maintained by an external battery, which can supply the power which is necessarily dissipated by anomalous resistivity.

This is an example of a theme which will occur several times in the course of this work: microscopic wave-plasma processes are capable of trans-

ferring energy between particle species, or turning this energy into wave turbulence, at rates which are enormous compared to the input power available. The plasma processes are analogous to an enormous pipeline through which only a trickle of water flows; it is completely wrong to estimate the water flow from the diameter of the pipe. The overall balance of plasma processes is in large part governed by macroscopic, not microscopic, parameters. (This is true at least for plasma effects in which  $k_{\perp} R_L > 1$ , but not necessarily if  $k_{\perp} R_L < 1$ .)

Turn now to the third kinetic process of magnetic mirror forces. It has been considered in great detail by Chiu and Schulz [1978] (using quasi-neutrality) and Chiu and Cornwall [1980] (who added Poisson's equation and ionospheric charge and current conservation, effects whose greatest importance is to couple neighboring field lines). These authors (as well as Lyons [1980] and Fridman and Lemaire [1980]) pointed out that there is, for magnetic fields with large mirror ratios, a linear relation between parallel current and potential drop

$$-J_{\parallel} = Q\Delta\phi \quad (5)$$

(we drop a term referring to diffuse particle precipitation) with  $Q \approx Ne^2/(mV_e)$ . Again by taking  $\Delta\phi \approx -\lambda E_{\parallel}$ , one finds by comparing (5) with (1) that the effective "collision" frequency is simply the inertial frequency  $\nu_I$ :

$$\nu_I = V_e/\lambda \quad (6)$$

which is larger by roughly the factor  $(M/m)^{1/2}$  than  $\nu_A$  in (3), and of course is consistent with  $e\Delta\phi \approx MV^2$ , as observed. It is important to note that this



much larger effective collision frequency is not associated with dissipation of any sort, and equations such as (2) with  $v_I$  in place of  $v_A$  are meaningless for the magnetic mirror force process.

Somewhere between the extremes of totally dissipation-free auroral arc models (e.g., Chiu and Schulz [1978]) and maximally-dissipative models of anomalous resistivity lies a middle ground in which there is turbulence and dissipation, but in an amount limited by the auroral input power. We have already argued that this limitation implies little or no turbulence contribution to the parallel potential drop, so we must look elsewhere for the effect of turbulence. In fact, it turns out that the effect is precisely opposite to that of anomalous resistivity: electrostatic ion cyclotron turbulence tends to reduce the parallel potential drop. The argument is that such turbulence (or similar electrostatic modes with  $E_{\parallel} \ll E_{\perp}$ ) is very efficient at heating ions, thereby increasing  $T_{\perp}/T_{\parallel}$  for the ions. (At the same time electrons are slowed down and heated mostly in  $T_{\parallel}$ ). As a result, the differential pitch-angle anisotropy between ions and electrons is reduced. In the Chiu-Schulz kinetic model, precisely this differential anisotropy drives the parallel potential drop, which - as the detailed calculations presented below show - is reduced as  $T_{\perp}/T_{\parallel}$  increases.

There are now two effects tending to stabilize the turbulent wave amplitude: 1) the well known (e.g., Palmadesso et al. [1974]) increase in the electron critical drift velocity  $V_c$  with increasing  $T_{\perp}$ ; 2) a decrease in the electron drift velocity  $V_D$  because the parallel potential drop has decreased. As Palmadesso et al. [1974] show, the first effect alone acts to bring  $V_D - V_c$  to a very small value (compared to other characteristic velocities), and the ion cyclotron mode is essentially marginally stable (the growth rate  $\gamma$  is linear in  $V_D - V_c$ ). With only the first effect present, we will

show below an exponential decay to marginal stability ( $\gamma=0$ ). Adding the second effect may have more dramatic effects: with  $V_D$  decreasing at the same time  $V_C$  increases it is possible for  $\gamma$  to go negative, with consequent shut-down and decay of turbulence. Eventually fresh ions (with  $T_{\perp}/T_{\parallel} \lesssim 1$ ) are injected and restore the initially unstable situation. These ions enter the system on the ion transit time scale  $\ell/V \approx 10$  sec; since this is greater than the time scale for shutting off turbulence, the arc can pulsate at this period. In the present paper we will not take up this mechanism for pulsating arcs in any detail; however, we note that the observed characteristic period of pulsations (both patches and arcs) is very nearly the ion transit time [Royrvik and Davis, 1977].

Whether pulsation or decay to marginal stability takes place depends on initial conditions; eventually some sort of equilibrium will be reached. We can make some simple estimates of turbulence parameters for the electrostatic ion cyclotron mode, when a marginally stable equilibrium has set in. First it is necessary to identify the wave saturation mechanism. Formation of a quasi-linear plateau on the electron distribution is not favored, because old electrons are rapidly transported out of the region of instability while new ones are being carried in, and convective losses are slow because of the very low wave group velocity. It is thus plausible that ion resonance broadening [Dum and Dupree, 1970] and the rather smaller effect of geometric resonance broadening (e.g., Schulz [1972]) are the dominant saturation mechanisms. Dum and Dupree show that, in this case, the RMS wave amplitude is largely independent of the ion velocity diffusion coefficient; their formulas applied to the auroral arc problem suggest wave amplitudes of the order of tens of mV/m, which is roughly what is observed [Temerin et al., 1981]. The diffusion coefficient for ion diffusion in  $V_{\perp}$  is related not to the wave intensities,

but to the linear growth rate  $\gamma_L$ :

$$D = \frac{\Omega^2}{k_{\perp}^2} \gamma_L \quad (7)$$

where  $\Omega$  is the ion cyclotron frequency. The most unstable modes have  $k_{\perp} \approx \Omega/V$ , and  $\gamma_L$  is of the order of

$$\gamma_L \approx \pi^{1/2} \Gamma_1 \Omega (V_D - V_C)/V_{\#e} \quad (8)$$

where  $\Gamma_1 \lesssim 0.2$ ,  $V_D$  is the electron current velocity along the field line, and  $V_C$ , the critical drift velocity, is nominally of order  $10 V \approx 0.2 V_e$  [Kindel and Kennel, 1971]. Thus a nominal value for  $\gamma_L$  is of order  $0.1 \Omega$ ; with nominal auroral values of  $\Omega \approx 100 \text{ sec}^{-1}$  and  $V \approx 10^8 \text{ cm/sec}$ , we estimate  $D \approx 10^{17} \text{ cm}^3 \text{ sec}^{-1}$ .

In fact this estimate for  $D$  is too large by orders of magnitude, for essentially the same reason as the nominal estimates of anomalous resistivity were wrong: such a  $D$  yields too much dissipation to be supplied by the auroral input. As an ion travels along the field line from equator to ionosphere, its perpendicular energy increases by wave turbulence, by an amount roughly equal to  $2M \int ds D/V_{\parallel}$ . Since the ultimate source of energy for ion heating is the ions themselves (that is, heating in  $T_{\perp}$  is a less-than-perfectly efficient way of transferring energy from parallel ion motion to perpendicular ion motion, via a potential drop which induces turbulence) this quantity must certainly be less than roughly the ion kinetic energy which gives the limit

$$D < \frac{V^3}{4\ell} \approx 10^{14} \text{ cm}^3 \text{ sec}^{-1} \quad (9)$$

(estimating  $\lambda \approx 4 R_G$ ). When this upper limit as  $D$  is used in (7) to estimate  $\gamma_L$ , it is seen that  $\gamma_L$  scales as  $V/\lambda \ll 0.1 \Omega$ . This means, of course, that turbulence is generated at very near marginal stability.

In this paper we make quantitative the considerations given above, using a schematic but adequate representation of electrostatic turbulence with  $k_\perp \gg k_\parallel$ . The starting point is the Boltzmann equation for particles in an inhomogeneous magnetic field, with an applied DC potential drop, and in the presence of perpendicular velocity-space diffusion. The diffusion coefficient  $D$  is independent of velocity, and is considered as an appropriate average over the turbulent spectrum. Addition of turbulent diffusion makes the Boltzmann equation analytically insoluble, but we show that for an initially bi-Maxwellian velocity distribution function  $f$ , a good approximation is that  $f$  continues to be bi-Maxwellian, but with a temperature  $T_\perp$ , as well as an effective electrostatic potential, dependent on field-line integrals of  $D$  weighted with various functions. ( $T_\parallel$  does not change because of  $D$ .) That is, the effects of diffusion can be well modeled by leaving out the explicit diffusion operator and renormalizing  $T_\perp$  and the parallel potential drop  $\Delta\phi$ .

This is a great simplification, because it allows us to treat our problem as a simple modification of the Chiu-Schulz problem (to the extent, at least, that we ignore coupling of neighboring field lines and the ionosphere, which is permissible here). We have run the Chiu-Schulz numerical programs for various values of  $D$  (that is, various renormalized  $T_\perp$  and  $\Delta\phi$ ) and find that, as anticipated,  $\Delta\phi$  is forced to decrease as  $D$  increases. At a critical value of  $D$  the solutions either break down or  $\Delta\phi$  is essentially zero; the critical value of  $D$  found in the numerical work is somewhat smaller than given in (9). This is, of course, expected since (9) is an upper limit.

Section II contains a derivation of the approximate solution of the Boltzmann equation. Section III discusses the physics of electrostatic ion cyclotron turbulence, and Section IV contains numerical results. Section V is a brief recapitulation and statement of work for the future.

## II. BOLTZMANN EQUATION WITH TURBULENCE IN AN INHOMOGENEOUS MAGNETIC FIELD

The Boltzmann equation for ions suffering velocity-space diffusion in the component of velocity perpendicular to the magnetic field is, after cyclotron averaging,

$$0 = \left\{ \frac{\partial}{\partial t} + v_{\parallel} \frac{\partial}{\partial s} + \dot{v}_{\parallel} \frac{\partial}{\partial v_{\parallel}} + \dot{v}_{\perp} \frac{\partial}{\partial v_{\perp}} - \frac{1}{v_{\perp}} \frac{\partial}{\partial v_{\perp}} (D v_{\perp} \frac{\partial}{\partial v_{\perp}}) \right\} f \quad (10)$$

where

$$\dot{v}_{\parallel} = \frac{-v_{\perp}^2}{2B} \frac{\partial B}{\partial s} + \frac{e}{M} E_{\parallel} \quad (11a)$$

$$\dot{v}_{\perp} = \frac{v_{\parallel} v_{\perp}}{2B} \frac{\partial B}{\partial s} \quad (11b)$$

We take  $D$  to be independent of velocity and time, and concern ourselves only with static ( $\partial/\partial t = 0$ ) cases. For  $D = 0$ , the well known solution to (10) is

$$f = f_0 (e_{\parallel 0}, e_{\perp 0}) \quad (12)$$

$$e_{\parallel 0} = \frac{M v_{\parallel}^2}{2} + \frac{M v_{\perp}^2}{2} \left(1 - \frac{B_0}{B}\right) + e (\phi - \phi_0) \quad (13)$$

$$e_{\perp 0} = \frac{M v_{\perp}^2}{2} \frac{B_0}{B} \quad (14)$$

The subscript 0 labels a particular point on the field line, where  $s = 0$ ,  $B = B_0$ , and the electrostatic potential  $\phi = \phi_0$  (see Fig. 1). In the case of energetic plasmashet ions, the point 0 will be the equator of an auroral field line.

If  $D \neq 0$  it is impossible to solve (10) exactly, except in a formal sense. The formal solution is nonetheless instructive for further developments, so we give it here.

First introduce the fundamental quantity

$$U_{\parallel}(s, s', v_{\parallel}, v_{\perp}) = \left\{ v_{\parallel}^2 + v_{\perp}^2 \left( 1 - \frac{B'}{B} \right) + \frac{2e}{M} (\phi - \phi') \right\}^{1/2} \quad (15)$$

where  $B' = B(s')$ ,  $B = B(s)$  and analogously for  $\phi$  and  $\phi'$ , and  $v_{\parallel}$ ,  $v_{\perp}$  are the velocity components at  $s$ . We restrict ourselves to regions of space and velocity space such that  $U_{\parallel}$  is strictly positive, that is, no ion mirrors in the region between  $s$  and  $s'$ . The general case without this restriction requires very cumbersome notation. In our coordinate system (Fig. 1),  $U_{\parallel} > 0$  refers to downgoing ions, and in the earth's field we take  $s > s'$ ,  $B > B'$ .

Clearly,  $U_{\parallel}$  is a constant of the motion for  $D = 0$ :

$$\hat{L} U_{\parallel} \equiv \left( v_{\parallel} \frac{\partial}{\partial s} + \dot{v}_{\perp} \frac{\partial}{\partial v_{\perp}} + \dot{v}_{\parallel} \frac{\partial}{\partial v_{\parallel}} \right) U_{\parallel} \approx 0; \quad U_{\parallel}(s'=s) = v_{\parallel} \quad (16)$$

In terms of  $U_{\parallel}$ , we define the fundamental diffusion variable  $Q$ :

$$Q = \int_0^s \frac{ds' D}{U_{\parallel}} \quad (17)$$

and note that

$$\hat{L} Q = D \quad (18)$$

and in terms of  $Q$ , we define a Green's function for the diffusion operator:

$$G = \frac{1}{2Q} I_0 \left( \frac{V_{\perp} V'_{\perp}}{2Q} \right) \exp \left[ -\frac{1}{4Q} (V_{\perp}^2 + V'_{\perp}{}^2) \right] \quad (19)$$

$$G \rightarrow \frac{1}{V'_{\perp}} \delta (V_{\perp} - V'_{\perp}), \quad Q \rightarrow 0 \quad (20)$$

Here  $I_0$  is a Bessel function of imaginary argument.  $G$  obeys the Boltzmann equation (10) up to an error which is second order in  $s$ , the length of field line over which the solution applies. That is,

$$\hat{L}G - \frac{1}{V_{\perp}} \frac{\partial}{\partial V_{\perp}} \left( DV_{\perp} \frac{\partial G}{\partial V_{\perp}} \right) = 0 \left[ D \left( Q^{-1} \frac{\partial Q}{\partial V_{\perp}} \right)^2 G \right] \quad (21)$$

with the RHS of (21) failing to vanish because  $\partial Q / \partial V_{\perp} \neq 0$ . But from the definition (15) of  $U_{\parallel}$ , it is clear that, in order of magnitude, and for  $s \ll B / (\partial B / \partial s) \equiv \ell_0$ ,

$$Q^{-1} \frac{\partial Q}{\partial V_{\perp}} \sim U_{\parallel}^{-1} \frac{\partial U_{\parallel}}{\partial V_{\perp}} \sim \frac{s}{V \ell_0} \quad (22)$$

so the RHS of (21) is of order  $DG(s/V\ell_0)^2$ . A typical term on the LHS is  $O(VG/\ell_0)$ , and (as will be clarified later; see Eq. (9)) since  $D/V^2 < V/\ell_0$ , the error in the Green's function solution is  $\sim (s/\ell_0)^2$ . Choose  $s$  to make this as small as desired; then, to the chosen accuracy, the solution to the Boltzmann equation which approaches a given boundary value  $f_0$  at  $s = 0$  is

$$f = \int_0^{\infty} V'_{\perp} dV'_{\perp} G(V_{\perp}, V'_{\perp}) \times \\ \times f_0 \left[ \frac{M V_{\parallel}^2}{2} + \frac{M V'_{\perp}{}^2}{2} \left( 1 - \frac{B_0}{B} \right) + e^{(\phi - \phi_0)}, \frac{M V'_{\perp}{}^2}{2} \frac{B_0}{B} \right] \quad (23)$$



One may now simply iterate (23) going from  $s$  to  $2s$ , using  $f$  at  $s$  as the input on the RHS, and then to  $3s, \dots Ns$ . The exact solution is approached as  $s \rightarrow 0$  and  $N \rightarrow \infty$  with  $Ns$  fixed.

This formal solution is of little use in general, but one special case will draw our attention: if  $f_0$  in (23) is bi-Maxwellian, the output distribution function is also bi-Maxwellian, except for the velocity dependence of  $Q$ . It will turn out that this velocity dependence is relatively unimportant, so we consider the possibility of an approximately bi-Maxwellian solution by direct substitution in the Boltzmann equation. The solution ansatz necessarily involves an overall factor dependent on  $s$ , which can be interpreted as a sort of renormalization of the electrostatic potential. An alternative development of an approximate solution to (10) leading to similar solutions below can be obtained by the method of moments. Since this method may provide alternative insight into the solution scheme, we give a summary of it in the Appendix.

The form chosen for  $f$  is

$$f = e^{\eta(s)} e^{-\theta_{\parallel} V_{\parallel}^2 - \theta_{\perp} V_{\perp}^2 - (2e/M) \theta_{\parallel} \phi(s)} \quad (24)$$

where  $\theta_{\perp}, \theta_{\parallel}$  may depend on  $s$ , but not on  $V$ . When this is substituted in (10) (with  $\partial/\partial t = 0$ ) the following equations are required to be satisfied, in addition to  $\theta_{\parallel} = \text{constant}$ .

$$\hat{L} \theta_{\perp} + V_{\parallel} (\theta_{\perp} - \theta_{\parallel}) \frac{1}{B} \frac{\partial B}{\partial s} + 4 \theta_{\perp}^2 D = 0 \quad (25)$$

$$\hat{L} \eta + 4 \theta_{\perp} D = 0 \quad (26)$$

where  $\hat{L}$  is the differential operator defined in (16). For  $D = 0$ , the solutions are  $\eta = \text{constant}$ ,  $\theta_{\parallel} = \text{constant}$ , and

$$\theta_{\perp} = \hat{\theta}_{\perp}(s) \equiv (\theta_{\perp 0} - \theta_{\parallel}) B_0/B + \theta_{\parallel} \quad (27)$$

Of course, this simply reproduces (12) - (14) for a bi-Maxwellian.

For  $D \neq 0$ , (25) and (26) are not strictly consistent, because of the non-trivial appearance of  $V_{\parallel}$ . To achieve consistency, we will replace  $V_{\parallel}$  by an average value  $\bar{U}_{\parallel}$  which is independent of the local velocities  $V_{\parallel}$ ,  $V_{\perp}$ , but might depend on  $s$ . In order that this approximation make sense, we must separately consider downgoing ( $V_{\parallel} > 0$ ) and upgoing ions; for the moment we treat only downgoing ions by taking  $f$  to vanish for  $V_{\parallel} < 0$ . Then  $\bar{U}_{\parallel}(s)$  can be defined by averaging over this distribution function at various points along the field line, using the  $D = 0$  solution for  $f$  as given by (12) - (14).

In this approximation,  $\theta_{\parallel}$  is still constant and (25) and (26) become

$$\frac{\partial}{\partial s} \theta_{\perp} + (\theta_{\perp} - \theta_{\parallel}) \frac{1}{B} \frac{\partial B}{\partial s} + 4 \theta_{\perp}^2 D \bar{U}_{\parallel}^{-1} = 0 \quad (28)$$

$$\frac{\partial \eta}{\partial s} + 4 \theta_{\perp} D \bar{U}_{\parallel}^{-1} = 0 \quad (29)$$

These equations appear to be non-linear, but actually they can be linearized. The first step is to multiply (29) by  $\theta_{\perp}$  and subtract it from (28); the result can be rearranged to

$$\frac{\partial}{\partial s} (B \theta_{\perp} e^{-\eta}) = \theta_{\parallel} e^{-\eta} \frac{\partial B}{\partial s} \quad (30)$$

which allows us to express  $\theta_{\perp}$  in terms of  $Y \equiv e^{-\eta}$ :

$$B \theta_{\perp}(s) = Y^{-1}(s) \left[ \theta_{\parallel} \int_0^s ds' Y(s') \frac{\partial B}{\partial s'} + B_0 \theta_{\perp 0} \right] \quad (31)$$

Now (29) can be written

$$\frac{\partial Y}{\partial s} = 4 D \theta_{\perp} \bar{U}_{\parallel}^{-1} Y \quad (32)$$

and the combination of (31) and (32) gives a linear second-order differential equation for Y:

$$\frac{\partial}{\partial s} \left( \frac{B \bar{U}_{\parallel}}{4 D} \frac{\partial Y}{\partial s} \right) = \theta_{\parallel} \frac{\partial B}{\partial s} Y \quad (33)$$

This could readily be solved numerically, but the approximations made in deriving the equation do not justify anything more elaborate than saving terms of first order in D. (The reason is that we have assumed  $\theta_{\perp}$  and  $\eta$  to be independent of  $\vec{V}$ . If this assumption is dropped, (25) and (26) could be solved without the approximation  $V_{\parallel} \rightarrow \bar{U}_{\parallel}$ . But then the equations themselves would not be quite correct; there would be terms of  $O(D^2)$  coming from the action of the diffusion operator on  $\theta_{\perp}$  and  $\eta$ ).

The lowest-order solution for  $\theta_{\perp}$  is

$$\theta_{\perp}(s) - \hat{\theta}_{\perp}(s) = - \int_0^s ds' (4 D \hat{\theta}_{\perp} \bar{U}_{\parallel}^{-1})(s') \left[ \theta_{\perp 0} \frac{B_0}{B} + \theta_{\parallel} \left( \frac{B' - B_0}{B} \right) \right] \quad (34)$$

where, as before,  $B = B(s)$  and  $B' = B(s')$ . It is useful to phrase the lowest-order correction to  $\eta$  in terms of a phantom electrostatic potential  $\phi_D$  defined by

$$\phi_D = \phi - \frac{Mn}{2e\theta_{\parallel}} \quad (35)$$

We find

$$\phi_D - \phi = \frac{2M}{e\theta_{\parallel}} \int_0^s ds' (D \hat{\theta}_{\perp} \bar{U}_{\parallel}^{-1}) (s') \quad (36)$$

The RHS of (34) and (35) shows that the smallness parameter of our approximation is roughly  $\int ds' D\theta_{\perp}/\bar{U}_{\parallel}$  which is essentially the ratio of the perpendicular energy gain of an ion due to diffusion in a single pass along the field line (as noted in Sec. I) to the initial ion energy. As we have seen, this cannot be larger than one, or the ions cannot maintain the necessary differential pitch-angle anisotropy which drives the potential drop which drives the electron current which drives the turbulence. On the other hand, the smallness parameter is not necessarily much less than one, so our efforts here can be considered only semiquantitative.

In summary, the primary effect of diffusion is to leave a bi-Maxwellian a bi-Maxwellian, but with modified  $\theta_{\perp}$  and with an effective electrostatic potential  $\phi_D$  which is greater than the true potential drop. The inverse temperature  $\theta_{\perp}$  decreases as diffusion acts, that is,  $T_{\perp}$  is increased as we expect. The fact that  $\phi_D$  is greater than  $\phi$  might be interpreted as the presence of positive phantom charge which tends to keep the protons out of the ionosphere.

The above discussion also holds with but trivial changes for upgoing ions, whose  $T_{\perp}$  increases as they move toward the equator. Whether up- or downgoing,  $T_{\perp}$  heating raises the ion mirror point and diminishes the parallel potential drop; thus turbulence acts with opposite sign to anomalous resistivity, and furthermore is self-limited by global considerations, not by the usual local saturation mechanisms.

To be complete we ought to look at the Boltzmann equation for electrons. As discussed in the next section, electrons are mostly heated in  $T_{\parallel}$  as well as having their field-aligned current velocity slowed down; that is, the flow motion is thermalized. This acts also to reduce the ion-electron differential pitch-angle anisotropy and to diminish the parallel potential drop. In view of some uncertainties in the quantitative description of this process (see Section III), we do not attempt to treat electron heating and slowing down here, but it is certainly no more complicated than what we have done for the ions. We will be content to note that the effects of turbulence on electrons reinforce those acting on ions.

### III. GLOBAL PLASMA PHYSICS OF THE ELECTROSTATIC ION CYCLOTRON MODE

So far we have only discussed one-half of the problem: what effect does a given perpendicular-diffusion constant  $D$  have on the ion distribution? The other half is, of course, how much turbulence and what value of  $D$  do the distribution functions, magnetic field geometry, and boundary conditions yield?

It turns out that many of the plasma-physical numbers are largely determined by global scaling laws, having little to do with the local plasma environment. For example, the net growth rate  $\gamma$  scales as the inertial frequency  $V/\ell$  which involves  $\ell$ , the length of the field line, and  $D$  is of order  $V^3/\ell$ . The dominance of global over local effects will be an interesting challenge to experts in computer simulations of plasmas (recent work on ion cyclotron turbulence is reported by Okuda, Chang and Lee [1981]; Okuda and Ashour-Abdalla [1981]).

Let us briefly recapitulate our scenario. Incoming auroral ions have average mirror points which would be closer to the ionosphere than those of the electrons, if it were not for the electric field which this enforced charge separation would produce. The parallel potential drop causes the electrons to have a net flow along the field line, which is strong enough to trigger the electrostatic ion cyclotron mode. The resulting turbulence (for which  $E_{\parallel}/E_{\perp} \ll 1$ ) heats the ions in  $T_{\perp}$ , which raises their mirror points and reduces the potential drop, thus reducing the electron drift velocity and the growth rate. At the same time, ion heating in  $T_{\perp}$  acts directly to reduce the growth rate by increasing the threshold drift velocity (an effect invoked by Palmadesso et al. [1974]). One of two effects ensues: 1) the plasma reaches marginal stability in less than the ion inertia time  $\ell/V$ , with final values

of  $\gamma$  and  $D$  as estimated above; 2) the electron drift velocity  $V_D$  decreases and the critical drift velocity  $V_C$  increases in such a way that  $V_D - V_C$  becomes negative, the waves are decayed or convected away, and the turbulence disappears. The initial unstable state will be restored after a time of order  $\lambda/V$ , and the whole structure will undergo relaxation oscillations (observed as pulsations).

In this section, we discuss the plasma physics of the electrostatic ion cyclotron mode, including saturation mechanisms (ion resonance broadening, geometric resonance broadening); heating and slowing-down rates for ions and electrons; influence of this heating on growth rates; and dynamics of the approach either to marginal stability or to a relaxation-oscillation mode (pulsations). Let us begin with the dispersion relation for drifting bi-Maxwellian distribution functions, thus generalizing Kindel and Kennel [1971]. As in earlier sections, electron quantities are subscripted, and quantities without subscripts refer to ions, except in equations (37) - (38).

In a spatially homogeneous magnetic field the electrostatic dispersion relation is

$$k^2 = - \sum \frac{\Gamma_N(\mu)}{\lambda_{D\parallel}^2} \left[ 1 + \frac{[\bar{\omega} + N\Omega(1 - T_{\parallel} T_{\perp}^{-1})]}{k_{\parallel} \bar{v}_{\parallel}} z \left( \frac{\bar{\omega} + N\Omega}{k_{\parallel} \bar{v}_{\parallel}} \right) \right] \quad (37)$$

The sum is over all  $N$  from  $-\infty$  to  $\infty$ , and also over species. For each species, we define

$$\Gamma_N(\mu) = e^{-\mu} I_N(\mu), \quad \mu = \frac{k_{\perp}^2 \bar{v}_{\perp}^2}{2 \Omega^2}, \quad \lambda_{D\parallel}^2 = \frac{\bar{v}_{\parallel}^2}{2 \omega_p^2}, \quad (38)$$

$$\bar{v}_{\perp, \parallel} = \left[ 2 \frac{T_{\perp, \parallel}}{M} \right]^{1/2}, \quad \bar{\omega} = \omega - k_{\parallel} V_D$$

We take it that for ions there is no drift, and that  $(\omega + N\Omega) \gg k_{\parallel} \bar{V}_{\parallel}$ , while for the drifting electrons  $\bar{\omega} + N\Omega \ll k_{\parallel} V_{\parallel}$ . The fastest-growing modes have  $\mu \approx 1$  (for ions), and  $k_{\parallel}^2/k_{\perp}^2 \ll 1$ . Then the dispersion relation is

$$\frac{\omega - \Omega}{\Omega} = \frac{\Gamma_{\perp} T_{\parallel}}{T_{\perp}} (\zeta + i\lambda)^{-1} \quad (39)$$

$$\zeta = \frac{T_{\parallel}}{T_{\parallel e}} + \frac{T_{\parallel}}{T_{\perp}} [1 - \Gamma_{\perp} - \mu^{-1} (1 - \Gamma_0)] \quad (40)$$

$$\lambda = \pi^{1/2} \frac{T_{\parallel}}{T_{\parallel e}} \left( \frac{\omega - k_{\parallel} V_D}{k_{\parallel} \bar{V}_{\parallel e}} \right) + \frac{\pi^{1/2} e^{-\rho^2}}{k_{\parallel} \bar{V}_{\parallel}} \Gamma_{\perp} [\omega + \Omega(T_{\parallel} T_{\perp}^{-1} - 1)] \quad (41)$$

where  $\rho = (\omega - \Omega)/(k_{\parallel} \bar{V}_{\parallel})$ . The linear growth rate in a homogeneous field is thus

$$\gamma_L = - \frac{\lambda \Omega \Gamma_{\perp} T_{\parallel}}{T_{\perp} \zeta^2} \quad (42)$$

In an inhomogeneous field, there is damping from geometric resonance broadening, that is, ion cyclotron resonance is lost as the wave propagates along the field. A straightforward calculation shows that the damping decrement is

$$\gamma_{GRB} = - \left( \frac{\bar{V}_{\parallel}}{\pi} \frac{\partial \Omega}{\partial s} \right)^{1/2} ; \quad (43)$$

this is of order  $1 \text{ sec}^{-1}$  on auroral field lines, and is not insignificant. The net growth rate is  $\gamma = \gamma_L - \gamma_{GRB}$ .

For this mode, the group velocity is small compared to the electron drift speed, so that fresh electrons are constantly being injected into the region of instability. This inhibits quasi-linear saturation by formation of a plateau on the electron distribution function, so the most important satura-



tion mechanisms are ion resonance broadening [Dum and Dupree, 1970] and ion heating [Palmadesso et al., 1974]. According to Dum and Dupree, the damping decrement due to ion resonance broadening is

$$\gamma_{\text{IRB}} = - \frac{k_{\perp}^2 D}{\Omega^2} \quad (44)$$

and equilibrium is reached when  $\gamma = \gamma_{\text{IRB}}$ . Here  $D$  is the diffusion coefficient for ion diffusion in  $V_{\perp}$ , as used in Section II. As mentioned in Section I, estimates such as  $V_D = \omega/k_{\parallel} \approx \omega/k_{\parallel}$ , or  $\gamma \sim 0.1 \Omega$ , yield estimates of  $D$  which are much too large to be furnished by auroral ions.

In addition to ion diffusion (heating in  $T_{\perp}$ ), the electrons suffer a slowing-down of their drift velocity and heating in  $T_{\parallel e}$ . These processes are non-resonant (see the inequalities below (38)), and therefore somewhat difficult to estimate accurately, but non-resonant quasi-linear theory would suggest

$$- \frac{\partial V_D}{\partial t} < \frac{\pi e^2}{m^2 \bar{V}_{\parallel e}^2} \sum_k |\phi_k|^2 k_{\parallel} \quad (45)$$

where  $\phi_k$  is the wave potential. This has been estimated by Dum and Dupree:

$$\langle |e \phi_k|^2 \rangle^{1/2} < 0.2 T_e \quad (46)$$

With  $k_{\perp} \approx \Omega/V$ ,  $k_{\parallel}/k_{\perp} < 0.1$  [Kindel and Kennel, 1971] and  $T_e \approx 1$  keV, (46) indicates an RMS wave amplitude of tens of mV/m (as seen by auroral satellites; Temerin et al., [1981]) and (45) yields a slowing-down time of  $(1-10)\Omega^{-1}$ . This is a lower limit on the e-folding time for ion heating in  $T_{\perp}$ , which is driven by electron energy losses. It is a very generous lower limit,

not actually approached in fact; it corresponds to similar naive estimates of  $\gamma_L \approx 0.1 \Omega$  or of ion heating rates which do not consider the limitations on energy transfer processes arising from global considerations. The only point in making these estimates is to show that the rates of transfer from one particle species to another, or to waves, are far greater than needed, a point also emphasized by Palmadesso et al. [1974].

Shortly we will make estimates of the actual transfer rates on auroral field lines, getting much smaller values. In this connection it is important to recall that, with ion resonance broadening as a dominant saturation mechanism, the ion diffusion coefficient  $i$  is not strongly coupled to  $|\phi_k|^2$ ; even very small values of  $D$  are consistent with the Dum-Dupree estimates of  $\phi_k$  as in (46).

Palmadesso et al. [1974] have pointed out that ion heating reduces the growth rate  $\gamma_L$ , thus taking the system toward marginal stability, that is,  $\gamma = \gamma_{IRB}$ , but with reduced values of both  $\gamma$  and  $\gamma_{IRB}$ . Let us make a quantitative estimate of this effect. Consider a plasma which is unstable to wave growth, but in which there are no waves to begin with. The condition of instability requires  $V_D > V_C$ , where the critical drift velocity  $V_C$  is simply the minimum drift velocity (over  $k_{\parallel}, k_{\perp}$ ). Kindel and Kennel [1971] find the minimum at  $k_{\perp} \bar{v} \approx \Omega$ ,  $k_{\parallel}/k_{\perp} \approx 0.1$  for an isotropic plasma with  $T \approx T_e$ . With geometric resonance broadening added, one finds for this isotropic case

$$V_C \approx \bar{v} \left( \frac{\omega}{\omega - \Omega} \right) \left[ \ln \frac{2 T_e \bar{v}_e \Gamma_1}{T \bar{v}} \right]^{1/2} + \frac{T_e \bar{v}_e}{\Omega T \Gamma_1 \pi^{1/2}} |\gamma_{GRB}| \approx 10 \bar{v} \quad (47)$$

The geometric resonance broadening correction is normally 20% - 30% of the Kindel-Kennel term. The frequency  $\omega$  depends on  $T_{\perp}$  (see (39), (40)); inserting this dependence in (47) and differentiating yields the estimate

$$\delta V_C \approx V_C \frac{\delta T_L}{T_e} \quad (48)$$

(some unimportant terms have been omitted).

We can find the time rate of change of  $V_C$  by knowing the ion heating rate, which is roughly  $\Omega D$  per particle as one finds from the Boltzmann equation (10) by multiplying by  $1/2 M V_L^2$  and integrating. With  $D = \gamma \Omega^2 / k_L^2 \approx \gamma \bar{v}_L^2 / 2$ , the ion heating rate is

$$\dot{T}_L / T_L \approx \gamma \quad (49)$$

and from (48)

$$\dot{V}_C / V_C = \gamma T_L / T_e \approx \gamma \quad (50)$$

Using (40) - (42), we estimate (setting all temperatures equal)

$$\gamma \approx \pi^{1/2} \frac{\Omega \Gamma_1}{\bar{v}_e} (V_D - V_C) \quad (51)$$

So equations (50) - (51) lead to a differential equation for  $V_C$ :

$$\dot{V}_C = \pi^{1/2} \frac{\Omega \Gamma_1}{\bar{v}_e} V_C (V_D - V_C) \quad (52)$$

Now consider the scenarios where  $V_D$  is constant, which is appropriate to many laboratory experiments and possibly some auroral cases. Equation (52) is readily solved, showing an exponential approach to equilibrium ( $\dot{V}_C = 0$ ). The characteristic time is

$$\hat{T} = \left[ \pi^{1/2} \frac{\Omega r_1 v_D}{\bar{v}_e} \right]^{-1} \approx 20 \Omega^{-1} \quad (53)$$

where the value for  $\hat{T}$  stems from the estimate  $v_D/\bar{v}_e \approx 0.2$  given by Kindel and Kennel (1971) for  $T_e = T$ . It follows that the plasma can deviate substantially from marginal stability only for times  $\leq \hat{T} \approx 0.2 - 2$  sec, following some initial, unstable disturbance.

Actually, it is quite unrealistic to suppose that  $v_D$  remains constant as  $v_e$  changes. The same effect - ion heating - which tends to increase  $v_e$  also tends to reduce the drift velocity, as we now estimate. Begin with an oversimplified picture of the change in the quasi-neutrality condition, in which the heating of electrons in  $T_{\perp e}$  is ignored. A change  $\delta T_{\perp}$  in ion temperature must be accompanied by a change in the electrostatic potential  $\phi$ , in order that quasi-neutrality can be maintained:

$$N_i (T_{\perp} + \delta T_{\perp}, T_{\parallel}, \phi + \delta\phi) = N_e (T_e, \phi + \delta\phi) \quad (54)$$

Originally this equation was satisfied with  $\delta T_{\perp}$  and  $\delta\phi$  equal to zero. If the distribution functions are Maxwellian, and expressed in terms of constants of the motion as in (12) - (14), the first-order expansion of (54) is, very roughly,

$$- N_i \frac{\delta T_{\perp}}{T_{\perp}} - N_i \frac{\delta\phi}{T_{\perp}} = + N_e \frac{\delta\phi}{T_e} \quad (55)$$

All we hope to do here is to get the signs right; each term in (55) might be multiplied by a numerical coefficient somewhat different from unity. With  $N_i \approx N_e$ ,  $T_{\perp} \approx T_e$ , (55) yields

$$e \delta\phi \approx -\frac{1}{2} \delta T_{\perp} \quad (56)$$

The change in the electron drift velocity is directly obtained, because  $1/2 m V_{\parallel e}^2 + \mu B - e\phi$  is constant and the magnetic moment  $\mu$  is unchanged:

$$\delta V_D = \delta V_{\parallel e} = + \frac{e \delta\phi}{m V_{\parallel e}} = - \frac{\delta T_{\perp}}{2m \bar{V}_{\parallel e}} \quad (57)$$

This shows that  $V_D$  decreases with an increase in  $T_{\perp}$ , but it is not straightforward to incorporate (57) into the differential equation (52) for  $V_C$ . The reason is that information concerning the changes  $\delta\phi$ ,  $\delta T_{\perp}$  is not transmitted instantaneously to the whole auroral field line; instead it must propagate along the line at a finite velocity. Clearly the characteristic time for changing a parameter like  $T_{\perp}$  which characterizes the global ion distribution function is the inertial time  $\ell/V$ , which is several seconds and much larger than the time it takes to reach marginal equilibrium,  $\hat{T}$ . Thus in (52)  $V_D$  should not be evaluated at the same time  $t$  as is  $V_C$ , but rather at a retarded time. In spite of this technical complication, we can appreciate from (52) that it is possible for  $\dot{V}_C$  to go from positive to negative in a finite time if  $V_D$  is decreasing, rather than simply going asymptotically to zero as in the constant  $V_D$  case. If this does happen, the growth rate becomes negative, and turbulence originally present is damped away. At this point, ion heating stops, to be replaced by ion cooling at a rate  $\dot{T}_{\perp}/T_{\perp} \approx -\ell/V$  as new ions are injected. After roughly an inertial time, the ions will be cool enough in  $T_{\perp}$  to be unstable once again, and more turbulence is generated. We have, then, a relaxation oscillator to generate auroral pulsations, with a characteristic time constant  $V/\ell \approx 6 - 10$  sec.

A complete discussion of this mechanism - including damping - will be given elsewhere. Here we only comment that 1) whether oscillations actually take place or not depends sensitively on parameters and initial conditions; and auroras are not required to pulsate; 2) our mechanism and its time scale are quite different from a recent proposal for auroral flickering [Silevitch, 1980], but not necessarily incompatible with the Coroniti-Kennel [1970] pulsation picture.

We conclude this section with some estimates of plasma parameters when marginal stability has set in. It has already been observed in Section I that the ion heating in  $T_{\perp}$  during the inertial ( $\approx$  quarter-bounce) time cannot exceed the initial ion energy, since that is what drives the heating (by making a parallel potential drop and an unstable current). This led to the estimate  $D < V^3/(4\ell) \approx 10^{14} \text{ cm}^2 \text{ sec}^{-3}$ , and (via (7) or (44)) to  $\gamma \approx V/\ell$ . When  $D$  reaches this value, ion heating and cooling (by injection of more ions) are roughly in balance.

Then (51) yields

$$(V_D - V_C)/V_C \approx \bar{V}_e V / (\ell \Omega V_C \Gamma_1 \pi^{1/2}) \approx 0.02 \quad (58)$$

These estimates are crude at best, but they suggest that the drift velocity and the critical velocity are closely tied together, and a relatively small change in either one can turn off the turbulence. If, for example,  $V_C = 2 \times 10^9 \text{ cm sec}^{-1}$  then a change in  $V_D$  of 2% of this value, or  $-\delta V_D = 4 \times 10^7 \text{ cm sec}^{-1}$ , reduces wave growth to zero. Such a change might be produced by an upward fluctuation in  $D$  from its saturation value of  $\approx V^3/(4\ell)$ , producing a net ion heating of roughly

$$\delta T_{\perp} = 2M \int \frac{ds}{V_{\parallel}} D \quad (59)$$

When (59) is used in (57), one finds an upward fluctuation of  $\approx 4 \times 10^{13} \text{ cm}^2 \text{ sec}^{-3}$  will turn off wave turbulence.

In the next Section, these crude estimates are made somewhat more quantitative by computer calculations.

#### IV. DETAILED NUMERICAL ESTIMATES

To do better than the rough guesses of Section III, we need to know two quantities with some accuracy: 1) the relationship between the total potential drop  $\phi$  and the perpendicular ion temperature  $T_{\perp}$ ; 2) the relationship between a given  $D(s)$  and the change in  $T_{\perp}$  for a single pass of ions between equator and ionosphere, as given in equations (28) - (29). Because explicit diffusion is nearly equivalent to a renormalization of  $\phi$  and  $T_{\perp}$  in the ion distribution function, the first relation can be found by a straightforward modification of the Chiu-Schulz (1978) calculations; the second is found by numerical integration of the differential equations. These calculations are reported here, and they show that there is a maximum diffusion coefficient above which there is no potential drop. The value of  $D_{MAX}$  is roughly consistent with our earlier guesses.

To achieve the above two objectives, we need to obtain a direct nonlinear differential equation for  $\theta_{\perp}$  in the variable  $B$  since the Chiu-Schulz calculations are best expressed in the magnetic field variable. With a straightforward, but slightly different set of manipulations from that following (28) and (29), we obtain

$$\frac{d}{dB} (B\theta_{\perp}) = \theta_{\parallel} - \frac{4D}{\bar{U}_{\parallel}(\phi)} \left(\frac{ds}{dB}\right) B \theta_{\perp}^2 \quad (60)$$

where  $(ds/dB)$  for a dipole field can be numerically expressed as a function of  $B$  to very high accuracy. For a given  $\phi(B)$ , the heated ion temperature  $T_{\perp} \equiv (M/2 \theta_{\perp})$  can be found by solving (60) with a highly accurate Runge-Kutta routine. But  $\phi(B)$  must be obtained through the Chiu-Schulz quasi-neutrality solution, thus ideally  $\theta_{\perp}(B)$  and  $\phi(B)$  should be obtained by simultaneous solution of (60) and quasi-neutrality. However, the accuracy to which (60) is



derived (first order in  $D$ ) does not justify such an elaborate and expensive procedure. We have decoupled (60) from the quasi-neutrality calculation by assuming for  $\phi$  in  $\bar{U}_{\parallel}$  of (60)

$$\phi(B) = \Delta\phi [B-B_0]/[B_{\ell}-B_0] \quad (61)$$

where  $\Delta\phi$  is the initial potential drop. Since  $\bar{U}_{\parallel}$  is dominated by the magnetospheric ion thermal energy ( $\sim 10$  keV), the use of (61), in which  $e\Delta\phi \sim 1$  keV, is not a significant source of error. The boundary conditions of (60) for downgoing and upgoing ions must be carefully distinguished. For a downgoing ion, the plasmashet ion temperature determines the boundary value  $\theta_{\perp 0}$ , but for an upgoing ion of magnetospheric origin, the boundary temperature at  $s=\ell$  is  $\theta_{\perp \ell}$  determined in its previous downward trajectory. Theoretically, a mirroring ion can keep on heating up to higher and higher temperatures by virtue of its bounce motion; however, we note that the electrostatic potential is supported on the bounce time scale [Chiu and Cornwall, 1980] so there is no reason to assume magnetospheric ions to be trapped for more than a couple of bounces. The diffusion coefficient  $D$  in (60) is in general an unknown function of  $s$  or  $B$ . We adopt a simple model such that  $D$  is a nonzero constant throughout the field line. The constant value of  $D$  is varied to determine the effect of cyclotron turbulence on the potential  $\phi$ . Constant turbulence along the whole field line is not necessarily realistic, but it will serve for this initial inquiry. Note that the  $\theta_{\perp}^2$  term in (60) is always positive in the direction of particle motion and  $\theta_{\perp} \propto 1/T_{\perp}$  so the nature of the solution, by virtue of the negative sign preceding the  $\theta_{\perp}^2$  term, is that  $\theta_{\perp}$  decreases from a starting boundary value  $\theta_{\perp 0}$  or  $\theta_{\perp \ell}$  in the direction of motion; thus,  $T_{\perp}$  increases from the boundary value. However, the decrease in  $\theta_{\perp}$  can-

not be unbounded, for if  $D$  is too large,  $\theta_{\perp}$  can go through a zero at some  $B$ . Such a solution is unphysical and serves as an upper bound on  $D$ , although this upper bound is not yet  $D_{MAX}$  because the effects of heat diffusion upon  $\phi$  needs to be determined by solving the quasineutrality portion of the problem. For reasonable values of the diffusion constant ( $D < 10^{15} \text{ cm}^2/\text{sec}^3$ ),  $\theta_{\perp}$  usually does not go through zero; hence  $D_{MAX}$  may be determined by whether a quasineutral solution of particle and field equilibrium can be supported.

To motivate the detailed calculations of ion heating effects on the auroral acceleration potential, let us illustrate the basic dependence of magnetospheric ion density distribution as function of potential drop and temperature anisotropy. Figure 2 shows the densities  $n_{M+}$  of down-going and up-going magnetospheric ions at the midpoint of the  $L=8.4$  field line. The equi-density contours are shown as functions of the parallel potential drop and the perpendicular temperature; since the parallel temperature is held fixed at 3 keV, the variation of perpendicular temperature is equivalent to variation of anisotropy. We note that, for fixed potential drop, the higher the anisotropy ( $T_{\perp}/T_{\parallel}$ ) the lower the density of magnetospheric ions at the midpoint of the field line because the ion mirror shrinks towards the equator, as qualitatively expressed in (60). Similarly, as we would expect, magnetospheric ions are repelled towards the equator by higher auroral parallel potential drops which accelerate electrons downwards. Now quasi-neutrality requires a certain number of ions to be present to balance against the density of electrons; thus, for a given electron density distribution, heating of ions would require the potential drop to decrease accordingly in order to maintain the ion density along one of the equi-density contours of Figure 2. This is the basic effect investigated in detail below, although the results of Figure 2 is not directly applicable because the effects of a potential decrease upon

the distribution of electrons must be taken into account also. To illustrate this point further, we have investigated the dependence of the potential drop upon the ion anisotropy by obtaining solutions of a Chiu-Schulz kinetic model for  $L=8.4$  with the following fixed parameters:  $T_{\perp-} = 7$  keV,  $T_{\parallel-} = 3$  keV,  $T_{\parallel+} = 3$  keV,  $n_{M-} = 0.6$  cm<sup>-3</sup>,  $n_{O+} = 70$  cm<sup>-3</sup> and  $n_{H+} = 5$  cm<sup>-3</sup>. Figure 3 shows the result of a search of the acceptable potential drop in the above kinetic model as the magnetospheric ion anisotropy is varied by varying the constant temperature  $T_{\perp}$  while all other parameters are held fixed. We note that the potential drop maintainable by the electron temperature anisotropy does decrease as the ion anisotropy approaches the electron value. The stippled area shows the uncertainty in our solution search due to our not using a special high-accuracy computational routine, which is expensive to run.

The illustrations above are based on arbitrary variations of the constant ion perpendicular temperature; therefore, they cannot be directly related to the effects of ion cyclotron heating. To do so, we need to solve (60) to obtain the perpendicular ion temperature in the form of  $(M/2\theta_{\perp})$  as functions of the heat diffusion coefficient  $D$  and magnetic field ratio  $B/B_0$ . The solutions of (60) for  $D = 10^{14}$  cm<sup>2</sup>/sec<sup>3</sup> for downgoing (arrows pointing away from the equator,  $B/B_0 = 1$ ) and upgoing (arrows pointing towards the equator) ions are shown in Figure 4. The solutions are obtained for an initial magnetospheric ion distribution with  $T_{\perp} = 3$  keV and  $T_{\parallel} = 1.5$  keV, which yields  $(M/2\theta_{\perp})$  given by the dashed curve (with  $D=0$ ) in Figure 4. The initial state for the upgoing ions is assumed to be the heated state at  $B_{\ell}/B_0$ . Since the heating effect is prominent primarily in the upgoing magnetospheric ions, only about one half of the ion density distribution is severely affected by the potential-drop-reducing effects discussed in Figures 2 and 3. Thus, we would expect the potential drop to vary fairly slowly with increasing  $D$  until

the solution of (60) is sufficiently close to  $\theta_{\perp} = 0$ , near which no quasi-neutral solution can be obtained.

An illustration of the complete scenario described in this paper is obtained by combining the solution of (60), such as shown on Figure 4, with a simultaneous search for the consistent auroral potential drop as we vary the ion diffusion coefficient  $D$ . The initial conditions for the computation are the same as that of Figure 3 except that the initial potential drop is set at 2.20 keV (to increase the sensitivity of the quasi-neutrality solution to parameter changes). The dependence of the auroral potential drop on the ion diffusion coefficient  $D$  in the range  $(10^{12} - 3 \times 10^{14}) \text{ cm}^2/\text{sec}^3$  is shown on Figure 5. As we have expected, the potential drop decreases slowly with increasing  $D$  until  $D \sim 10^{14} \text{ cm}^2/\text{sec}^3$ , at which point the upgoing ion heating effects (Figure 4) become prominent. Above  $D = 3 \times 10^{14} \text{ cm}^2/\text{sec}^3$ , the solution to (60) breaks down; while above  $\sim 2.5 \times 10^{14} \text{ cm}^2/\text{sec}^3$  the quasineutral solution is no longer smoothly varying, so we leave it indeterminate. Thus, we estimate  $D_{\text{MAX}}$  to be  $\sim 3 \times 10^{14} \text{ cm}^2/\text{sec}^2$ . Again, as in Figure 3, we have not used the high-accuracy mode of the Chiu-Schulz model in order to save computation time; this results in some uncertainty indicated by the stippling.

By investigating the properties of the Chiu-Schulz kinetic model of the auroral potential drop, we have demonstrated that ion heating leads to a decrease of the auroral potential drop, with concomitant effects as discussed elsewhere.

## V. SUMMARY AND CONCLUSIONS

We have given a semi-quantitative picture of self-consistent ion cyclotron turbulence on auroral field lines, with the following major conclusions:

1. Unless plasmashet ions are very much more energetic than the electrons, anomalous resistivity is not a large contributor to parallel electrostatic potential drops; supporting the kind of potential drop actually observed requires too much dissipation of energy to be provided by input from the plasmashet.

2. Nonetheless, wave turbulence can be present; the ion cyclotron turbulence levels suggested by the ion-resonance-broadening saturation mechanism of Dum and Dupree are comparable to those observed on auroral field lines.

3. The diffusion coefficient  $D$  and net growth rate  $\gamma$  are very much smaller than estimates based solely on local plasma properties; instead they are scaled with global parameters:  $D \sim V^3/\ell$ ,  $\gamma \sim V/\ell$  where  $V$  is a typical ion velocity and  $\ell$  the length of the portion of the field line on which there is turbulence. Crude estimates of these effects are supported by calculations based on the Chiu-Schulz quasi-neutrality model, with  $T_{\perp}$  augmented because of diffusive heating of ions in  $V_{\perp}$ .

4. The underlying reason for this limit on  $D$  and  $\gamma$  is that heating in  $T_{\perp}$  must be supported by ion kinetic energy, so the effect is one of transfer of parallel energy to perpendicular energy (with losses to electron heating and wave generation). This necessarily inhibits formation of a potential drop as the ions gain anisotropy and mirror higher along the line; eventually the potential is too small to maintain sufficient electron current to drive the

instability. Thus turbulence decreases the auroral potential drop, which is opposite to the effect of anomalous resistivity.

5. Depending on initial and boundary conditions, an unstable auroral system may evolve in less than a few seconds to marginally stable equilibrium, with turbulence levels,  $D$ , and  $\gamma$  as estimated above. Or it may undergo relaxation oscillations as ion heating increases the critical drift velocity  $V_C$  at the same time it reduces the actual drift velocity  $V_D$  (by lowering the potential drop). There are generally two characteristic times for these relaxation oscillations, the larger of which is the ion inertial time  $\ell/V$  which is the time it takes to restore an unstable situation from one in which  $V_D - V_C$  has turned negative. This time is of order 3-10 seconds, which is observed to be the dominant period of auroral pulsations. In another work we will consider the pulsation problem in more detail, contrasting our mechanism with earlier proposals [Coroniti and Kennel, 1970; Silevitch, 1980].

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## Appendix

In this appendix we give an alternative development of an approximate solution to (10) which yields essentially the same results as in Section II. Since this development is based upon different arguments, we offer it here for alternative insight.

Substitution of the chosen form (24) into (10) yields

$$\begin{aligned} & \left\{ -V_{\parallel} \frac{\partial \gamma}{\partial s} - \frac{2eE_{\parallel}}{M} \theta_{\parallel} V_{\parallel} + 4D\theta_{\perp} \right\} V_{\perp} f \\ & + \left\{ -V_{\parallel} \frac{\partial \theta_{\perp}}{\partial s} + \frac{V_{\parallel}}{B} \frac{\partial B}{\partial s} \theta_{\parallel} - \frac{V_{\parallel}}{B} \frac{\partial B}{\partial s} \theta_{\perp} - 4D\theta_{\perp}^2 \right\} V_{\perp}^3 f = 0 \end{aligned} \quad (A1)$$

where  $\gamma(s) \equiv \eta(s) + (2e/M) \theta_{\parallel} \phi(s)$  and  $\theta_{\parallel}$  is tacitly assumed constant. Now we take the  $n^{\text{th}}$  moment of (A1) in  $V_{\perp}$ . Since  $\int dV_{\perp} V_{\perp}^{n+1} f$  and  $\int dV_{\perp} V_{\perp}^{n+3} f$  cannot maintain a unique ratio to each other for all  $n$ , we conclude that (24) satisfies (10) only if

$$\left\{ -V_{\parallel} \frac{\partial \gamma}{\partial s} - \frac{2eE_{\parallel}}{M} \theta_{\parallel} V_{\parallel} + 4D\theta_{\perp} \right\} f = 0 \quad (A2)$$

$$\left\{ -V_{\parallel} \frac{\partial \theta_{\perp}}{\partial s} + \frac{V_{\parallel}}{B} \frac{\partial B}{\partial s} (\theta_{\parallel} - \theta_{\perp}) - 4D\theta_{\perp}^2 \right\} f = 0 \quad (A3)$$

Elimination of the D-associated terms in (A2) and (A3) yields

$$\left\{ -\frac{\partial \theta_{\perp}}{\partial s} + \frac{1}{B} \frac{\partial B}{\partial s} (\theta_{\parallel} - \theta_{\perp}) + \theta_{\perp} \frac{\partial \eta}{\partial s} \right\} V_{\parallel} f = 0 \quad (A4)$$

$$\left\{ V_{\parallel} \frac{\partial \eta}{\partial s} + 4D\theta_{\perp} \right\} f = 0 \quad (A5)$$

In (A4), the spatial and velocity operators on  $f$  have been separated into product form; therefore, the curly bracket must itself be zero. The solution to (A4) is just (31). Equation (A5) is the equivalent of (29). Its solution is non-trivial because the operator mixes space and velocity space functions. The solution ansatz to (29) is therefore equivalent to replacing the velocity space variable  $V_{\parallel}$  by the spatial variable  $\bar{U}_{\parallel}$ , in which case the curly bracket in (A5) is itself zero.

### Figure Captions

Figure 1: The coordinate system and dipole geometry assumed in this paper.

Figure 2: Constant density contours of upgoing and downgoing magnetospheric ions at the midpoint of the  $L=8.4$  field line.

Figure 3: Auroral potential drop as function of ion temperature anisotropy in the Chiu-Schulz kinetic model.

Figure 4: Perpendicular ion temperatures  $M/2\theta_{\perp}$  on the  $L=8.4$  field line as indicated by the ratio of the local magnetic field  $B$  to the equatorial magnetic field  $B_0$ .

Figure 5. Auroral potential drop as function of ion diffusion coefficient.

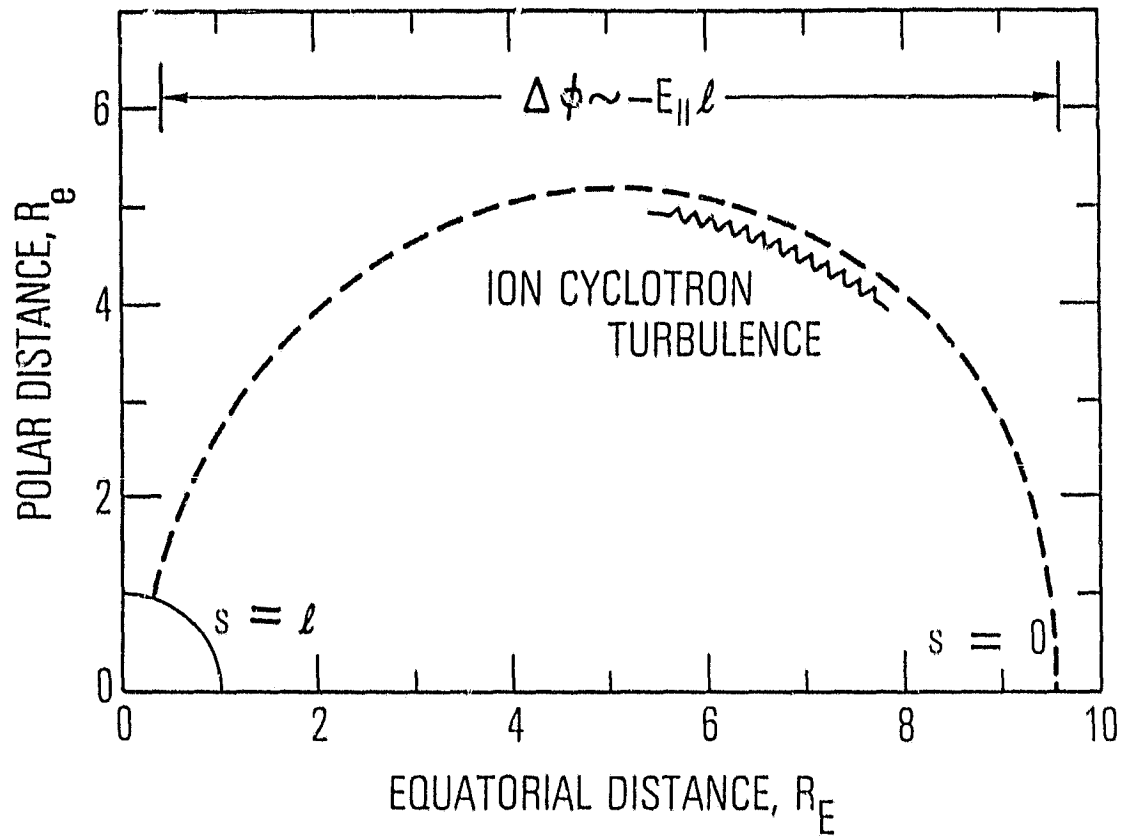


Fig.  
1

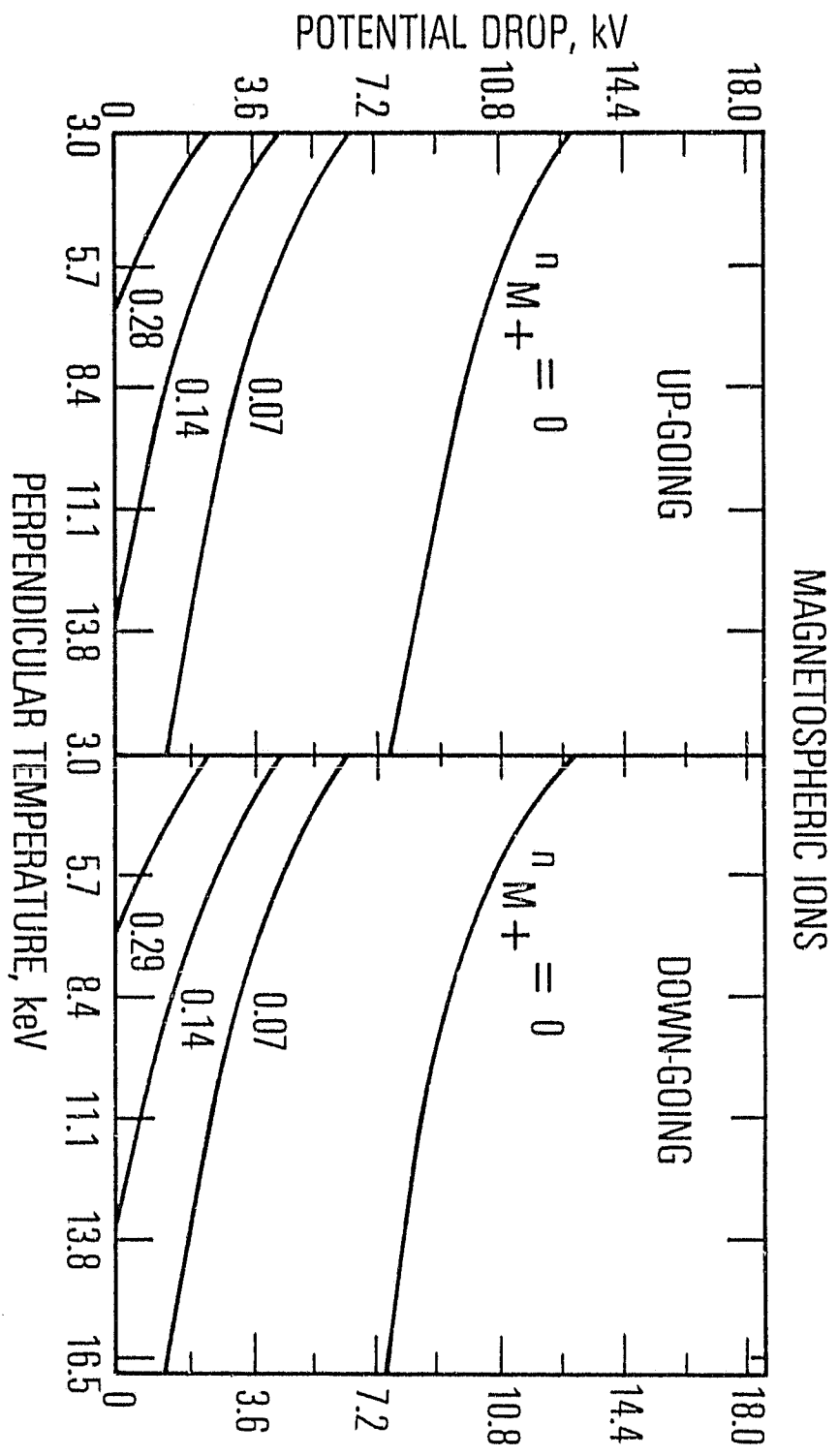
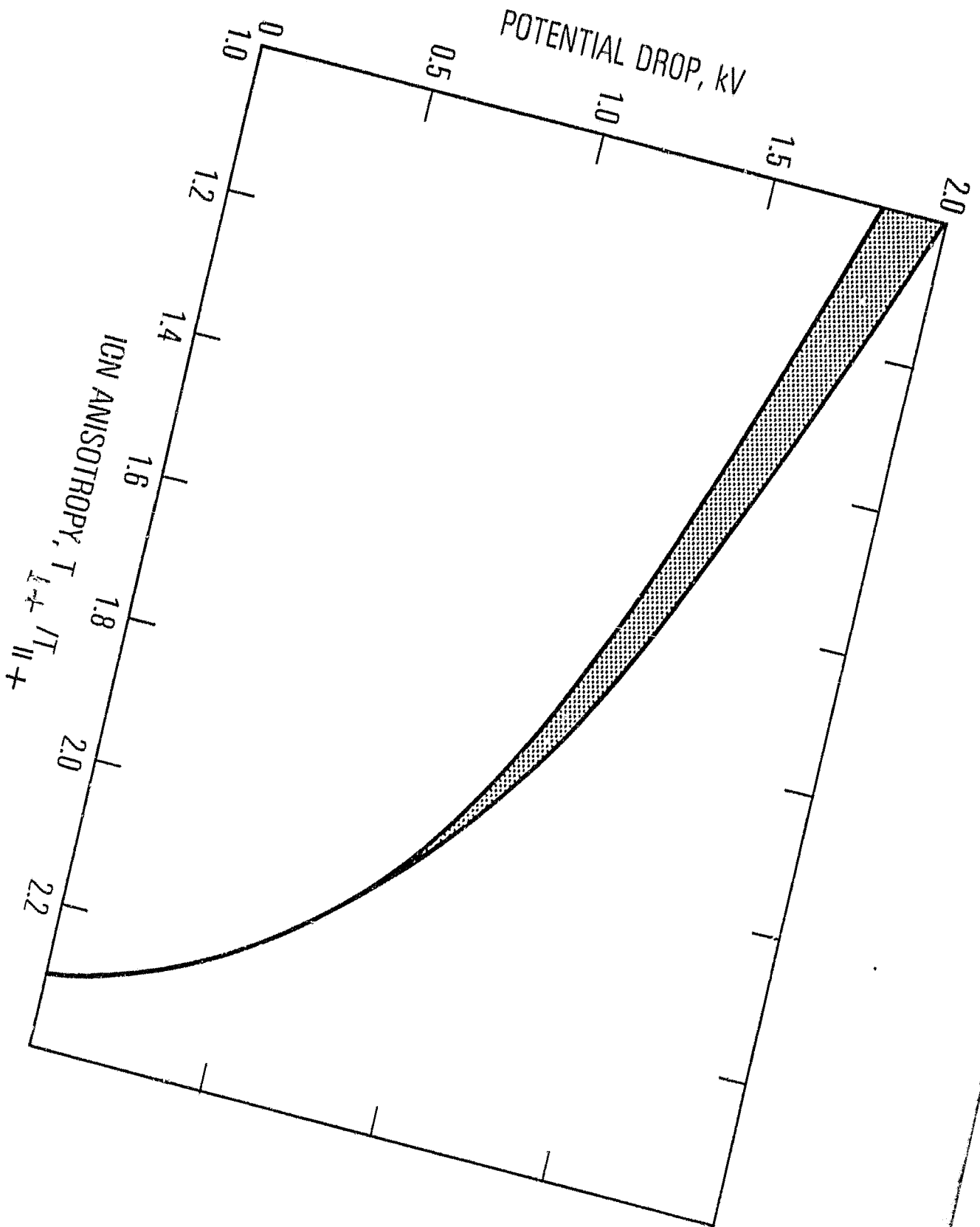


Fig.  
2



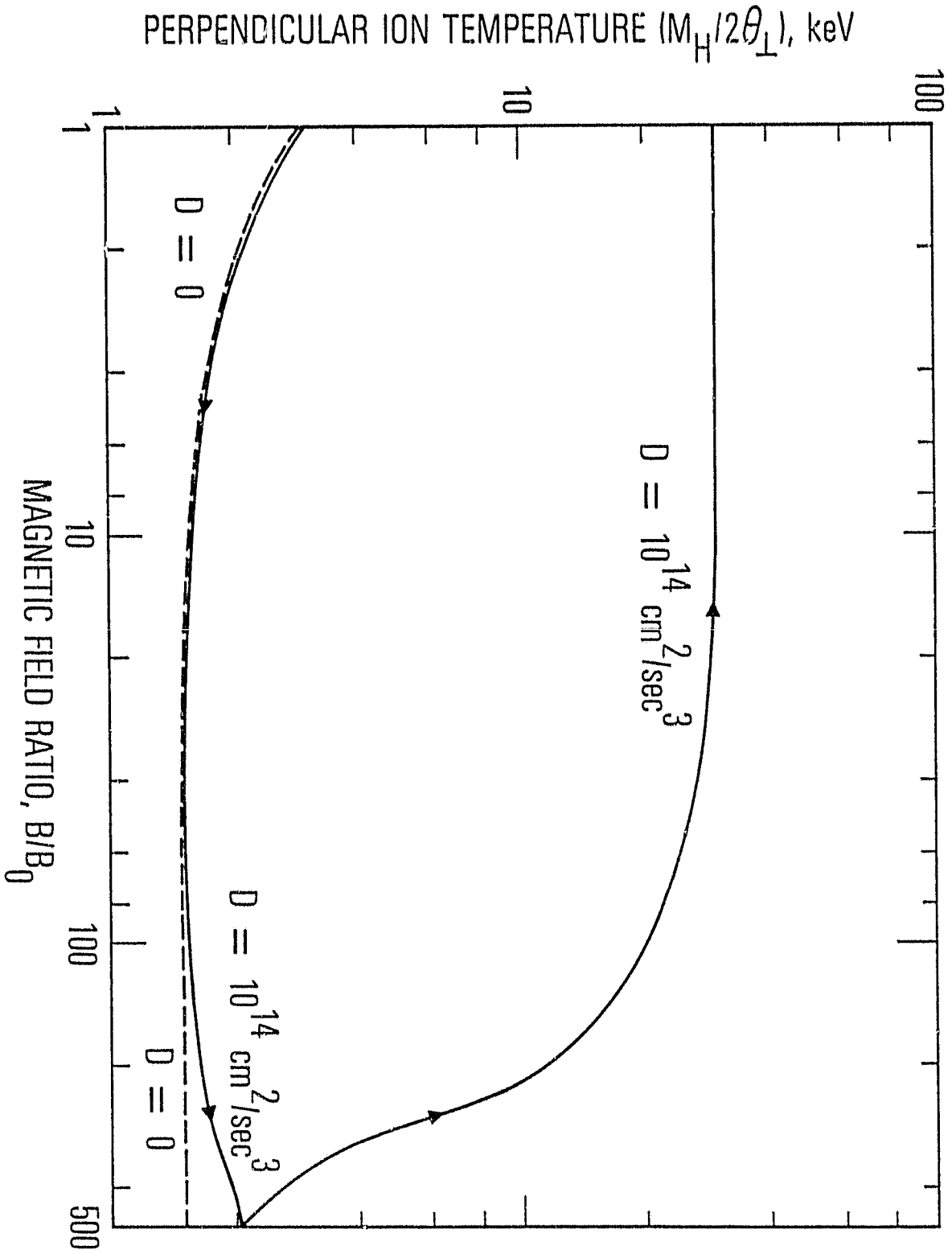


Fig.  
4



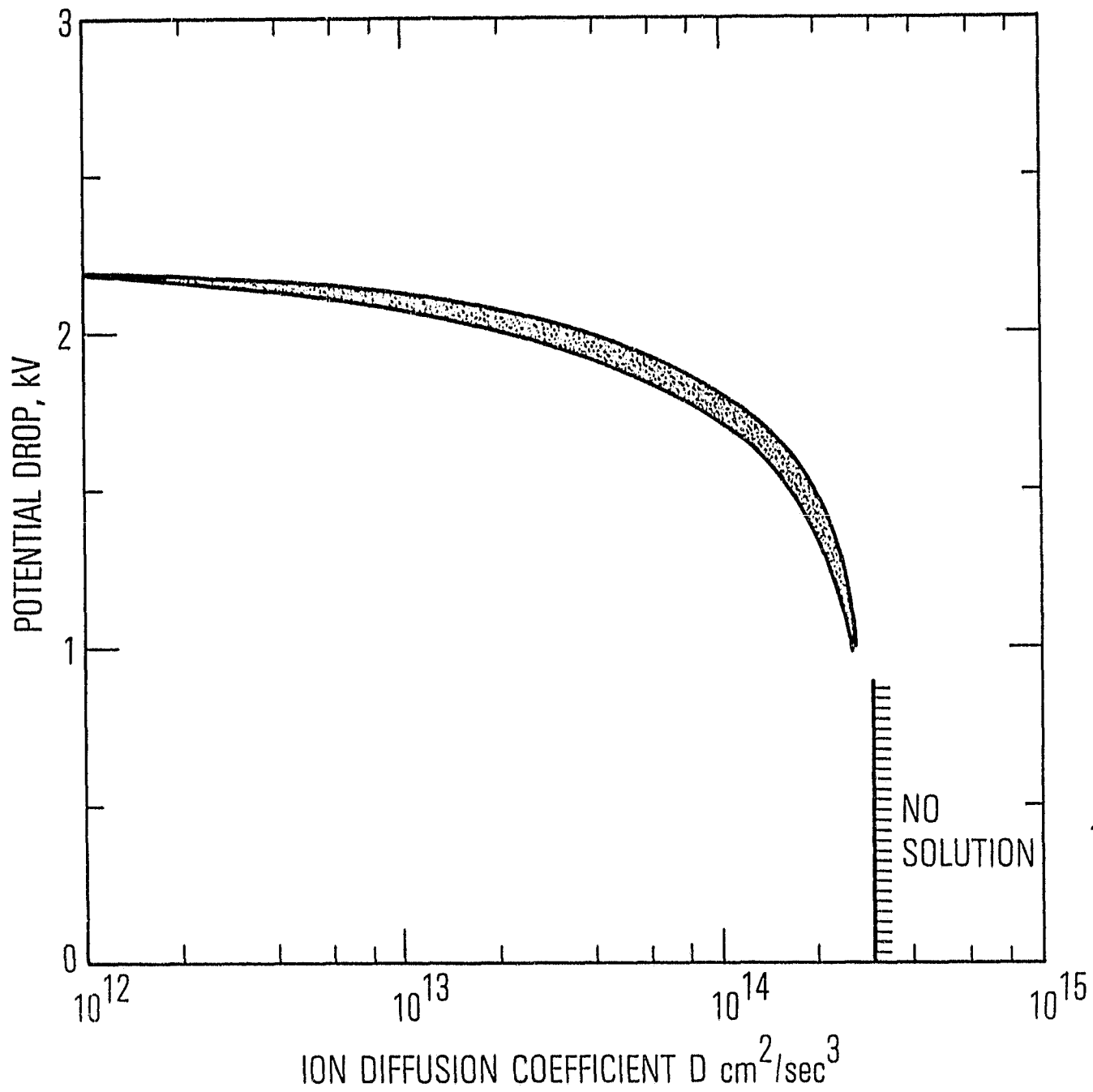


Fig 5