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A Procedure for Designing Forebodies With Constraints on Cross-Section Shape and Axial Area Distribution

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SUMMARY

A method is described for designing a forebody with cross sections which vary smoothly from an initial prescribed nose shape to a different prescribed base shape in such a way that the cross-section areas conform to a preassigned axial area distribution. It is shown that these conditions can be satisfied with a remaining degree of freedom, which can be used to accomplish a modest amount of geometric or pressure tailoring of the forebody. An example is provided which involves modifying the pressure distribution along a given meridian line of the forebody.

INTRODUCTION

The problem of designing the forebody of a supersonic airplane or missile is generally influenced by at least two factors. One of these is the axial area distribution, which, for reasonably slender configurations, governs the wave drag associated with the forebody. The other factor is the cross-section shape, which, except for axisymmetric missile bodies, generally varies from the nose to the base of the forebody. Near the nose the cross section is most often circular or nearly circular. Near the end of the forebody the cross section may acquire a canopy shape, as would be appropriate for a fighter, or perhaps an elliptic shape for certain missile configurations.

A need exists, therefore, for the capability of generating a smooth transition from one cross-section shape to another while maintaining a prescribed distribution of cross-section areas. This problem, which is the primary subject of this paper, is a purely geometric one. However, the forebody shape is not uniquely determined by these constraints. It is therefore possible to accomplish a certain amount of modification, for structural or aerodynamic purposes, while adhering to the constraints. One possibility that is discussed, with an included example, is to "tailor" the pressure distribution along a meridian. This capability might be required, for example, to maintain a nearly uniform flow ahead of an inlet.

The basic geometric theory used in the procedure is essentially exact. The pressure calculations are performed by the method of reference 1, which represents a numerical solution of the exact inviscid (Euler) equation for flow over a supersonic body.

SYMBOLS

A cross-section area

C_p pressure coefficient

E function used in defining cross-section shape (eq. (1b))

f	function describing cross-section shape (eq. (1a))
ℓ	forebody length
M	local Mach number
R	scale factor (eq. (2))
x,y,z	coordinates in axial, lateral, and vertical directions
α	parameter used in defining cross-section shape (eq. (1b))
β	$= \sqrt{M^2 - 1}$
ν	angle between local streamline direction and free-stream direction

Subscripts:

a	adjusted by scale factor R
b	forebody base
n	nondimensionalized cross-section shapes
o	nose of forebody

ANALYSIS AND EXAMPLES

Geometric Theory

The axial distribution of cross-section areas $A(x)$ is assumed to be given. It is also assumed that the cross-section area at the nose is zero:

$$A(0) = 0$$

The cross section at any x-station is taken to be symmetric about a central vertical plane. The shape, for most practical forebody designs, can be expressed as a single-valued function if the vertical coordinate z is taken to be the independent variable:

$$y = f(z)$$

For example, if the cross-section shape were as shown in figure 1(a), it would be mathematically described by the half-section as shown in figure 1(b). The function that defines in this way the cross-section shape at the nose is denoted by $f_o(z)$. The corresponding function describing the cross-section shape at the base is denoted by $f_b(z)$. At intermediate stations the shape is a transitional one, representing some combination of the initial and base shapes. The combining can be accomplished in any one of several ways. A simple linear interpolation formula that could be used is

$$f(z) = \frac{(x_b - x)}{x_b} f_o(z) + \frac{x}{x_b} f_b(z)$$

This expression, although simple, has at least one serious disadvantage. If the base shape $f_b(z)$ has a corner, or even a region of high curvature, each section will have a corresponding corner or region of high curvature. The resulting configuration would, therefore, be less desirable from an aerodynamic point of view.

A preferable expression is a kind of geometric mean:

$$f(z) = [f_o(z)]^{1-E(x)} [f_b(z)]^{E(x)} \tag{1a}$$

$$E = \left(\frac{x}{x_b}\right)^{\alpha} \qquad (\alpha > 0) \tag{1b}$$

With this formula, regions of high curvature are smoothed out. Furthermore, the parameter α provides a means of adjusting the rate at which the shape changes. If α is small, E is approximately one, except very near $x = 0$. Consequently, the factor containing $f_b(z)$ dominates except very near $x = 0$. Thus, the shape changes very rapidly from the initial $f_o(z)$ form to an approximation to $f_b(z)$. Conversely, when α is large, the shape variation is very gradual at first, and most of the change occurs near x_b.

Thus far, only the cross-section shape transition has been considered. It remains to adjust the actual sizes of the cross sections in accordance with the prescribed distribution of areas $A(x)$. This adjustment is more readily accomplished if the functions $f(z)$ describing the cross-section shapes are first nondimensionalized. It is convenient to nondimensionalize the functions $f(z)$ so that their common domain is $-1.0 \leq z \leq 1.0$. Each function is then integrated. Since $f(z)$ represents the shape of only the right-hand side of the forebody cross section, the area of the nondimensionalized shape is twice this integral:

$$A_n(x) = 2 \int_{-1}^{1} f(z)\ dz$$

Both the abscissa z and the ordinate f are then adjusted by the factor

$$R = \sqrt{\frac{A(x)}{A_n(x)}} \tag{2}$$

Thus,

$$z_a = Rz \tag{3a}$$

$$f_a = Rf \tag{3b}$$

The area under the adjusted curve $f_a(z)$ is therefore

$$2 \int_{-R}^{R} Rf\ d(Rz) = 2R^2 \int_{-1}^{1} f\ dz = R^2\ A_n(x) = A(x)$$

as required. By using this procedure, a smooth transition from the initial shape to the base shape can be accomplished while maintaining the specified area distribution.

Supplementary Design Conditions

The method discussed in the preceding section can be carried out regardless of the value of α. Therefore, α is a free parameter and can be used to satisfy some supplemental geomtric or aerodynamic condition.

Geometry tailoring.- An example of using the parameter α to control the geometric shape development is shown in figure 2. Figure 2(a) gives the required area distribution, and figure 2(b) gives the specified base shape. The initial cross section is taken to be circular. Figure 3(a) shows in perspective view the resulting distribution of cross sections for $\alpha = 0.75$. It is seen that the transition to the canopy shape is so rapid that the initial circular shape is barely apparent. The concave regions in the cross-section shape is barely apparent. The concave regions in the cross-section shape near the nose are undesirable from both aerodynamic and fabrication considerations. Consequently, the forebody shape was redesigned using $\alpha = 4.0$. The resulting development, shown in figure 3(b), retains a near-circular shape for some distance aft of the nose, and the canopy shape develops near the base. Of course, both of these forebody designs have the same axial area distribution.

Pressure tailoring.- It is also possible to tailor the pressure distribution along a specified meridian while maintaining the specified area distribution and the given nose and base shapes. This is not accomplished by varying α, but by compressing the coordinates slightly in one direction while expanding them in the perpendicular direction in such a way that the specified area distribution is retained. An example is shown in figure 4 for a forebody having elliptic cross sections of varying eccentricity. Figure 4(a) shows the specified area distribution and the side and top meridian shapes for an initial design satisfying these conditions. The nose cross section is taken to be circular. Figure 4(b) shows the pressure distribution along the side meridian of the initial design. A variation of this pressure distribution was specified, as shown in figure 4(b), with the purpose of reducing the level of the forward positive pressure peak by 10 percent.

The required increments in the pressure distribution were then calculated. The variation in the ordinates along the meridian corresponding to the pressure variation was estimated by the simple Prandtl-Meyer relation

$$\Delta C_p \approx \frac{2}{\beta} \Delta\nu \tag{4}$$

where $\Delta\nu$ is approximately equal to the local slope increment and the local value of β is used. This relation loses accuracy if the local Mach number approaches one or if the cross-section curvature becomes large in the neighborhood of the meridian line. Since the relation is only roughly applicable for problems of the type under consideration, the calculation would normally have to be iterated several times in order to obtain the required pressure distribution with a high degree of precision. (This approach to aerodynamic design problems is treated in more detail in ref. 2.)

As shown in figure 4(b), the desired reduction in the positive pressure peak for the example under consideration was obtained with a single calculation. Therefore, the procedure was not iterated.

When the new coordinates are assigned along the $z = 0$ meridian, the coordinate z_b at the base may be altered. If so, it must be restored to its original value to maintain the specified cross-section shape at the base. This is accomplished by making a linear adjustment in the coordinates along the $z = 0$ meridian, beginning at the nose. If the gap in z_b is small, this adjustment has only a slight effect on the pressure distribution since, according to equation (4), the change in C_p depends on the change in slope, which receives a small constant alteration in the course of the adjustment.

Finally, once the coordinates along the z-meridian have been altered, the other coordinates must be adjusted to maintain the specified area distribution. The simplest procedure is probably to form the ratio of the new $z = 0$ coordinate to the old one at each x-station; then multiply the original lateral dimensions by this ratio and multiply the vertical dimensions by its inverse. In this manner a certain degree of tailoring of the pressure distribution along a meridian can be accomplished while adhering to constraints on the body area distribution and cross-section shapes.

CONCLUDING REMARKS

A method has been described for calculating a distribution of cross-section shapes that vary smoothly from prescribed nose shape to a different prescribed base shape in such a way that the cross-section areas conform to a preassigned axial area distribution. It was found that these conditions can be satisfied with a remaining degree of freedom, which can be used to accomplish a modest degree of geometric or pressure tailoring of the forebody. An example was provided which involved modifying the pressure distribution along a given meridian line of the forebody.

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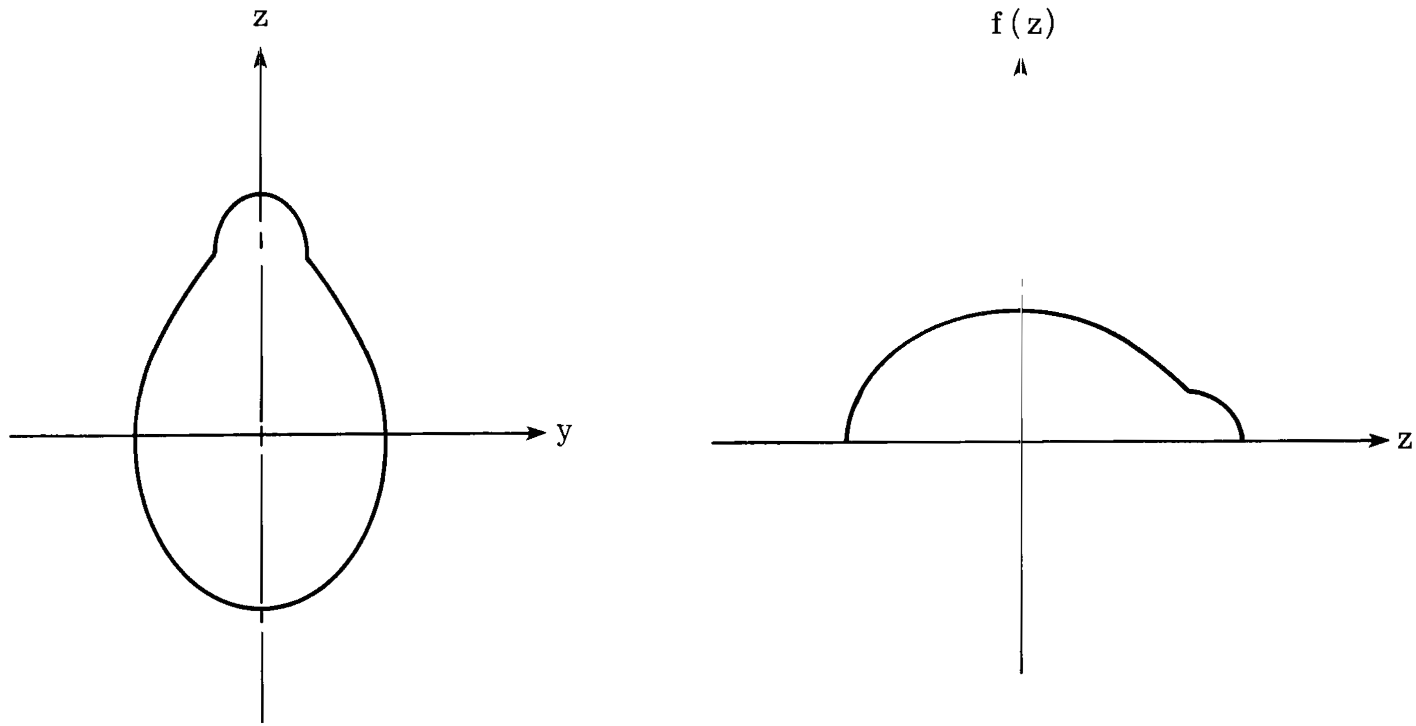
June 1, 1981

REFERENCES

1. Marconi, Frank; and Yaeger, Larry: Development of a Computer Code for Calculating the Steady Super/Hypersonic Inviscid Flow Around Real Configurations. Volume II - Code Description. NASA CR-2676, 1976.

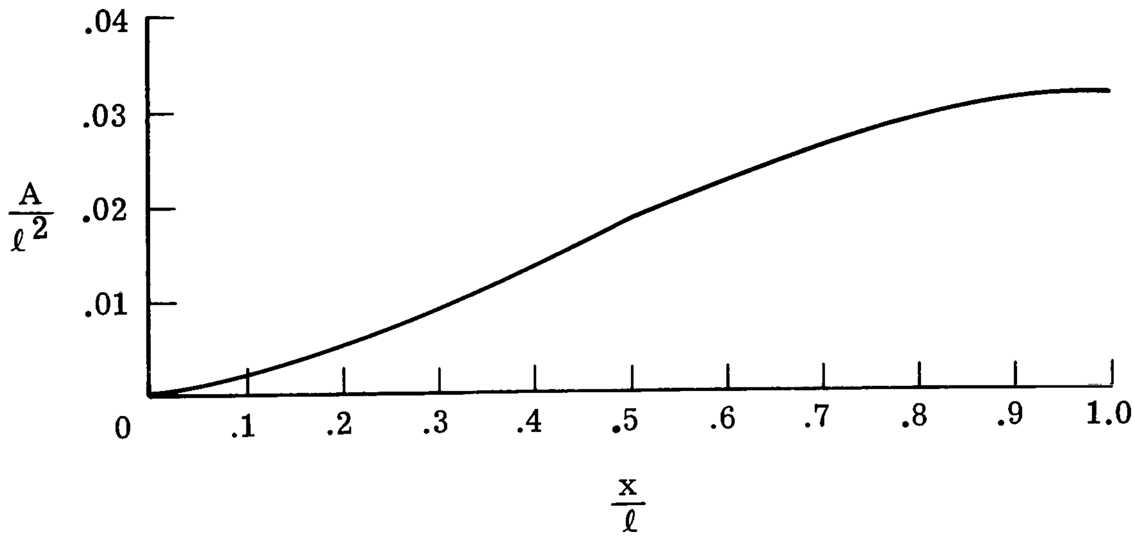
2. Davis, Warren H., Jr.: Technique for Developing Design Tools From the Analysis Methods of Computational Aerodynamics. AIAA J., vol. 18, no. 9, Sept. 1980, pp. 1080-1087.



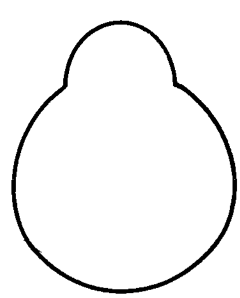
(a) Forebody cross section.

(b) Single-valued function f(z).

Figure 1.- Definition of single-valued function f(z) describing symmetric cross section.

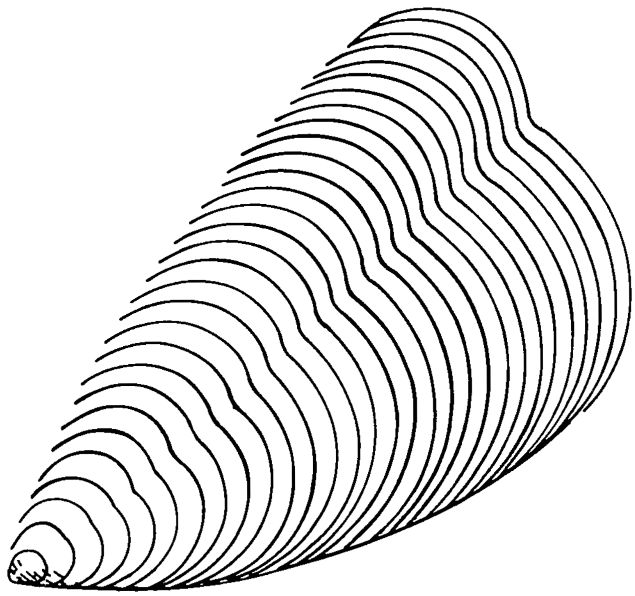


(a) Required area distribution.

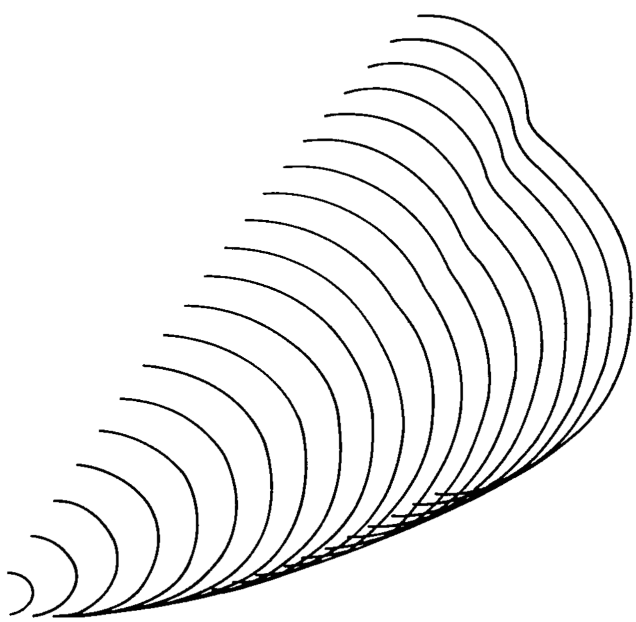


(b) Specified base shape.

Figure 2.- Specifications for forebody design.

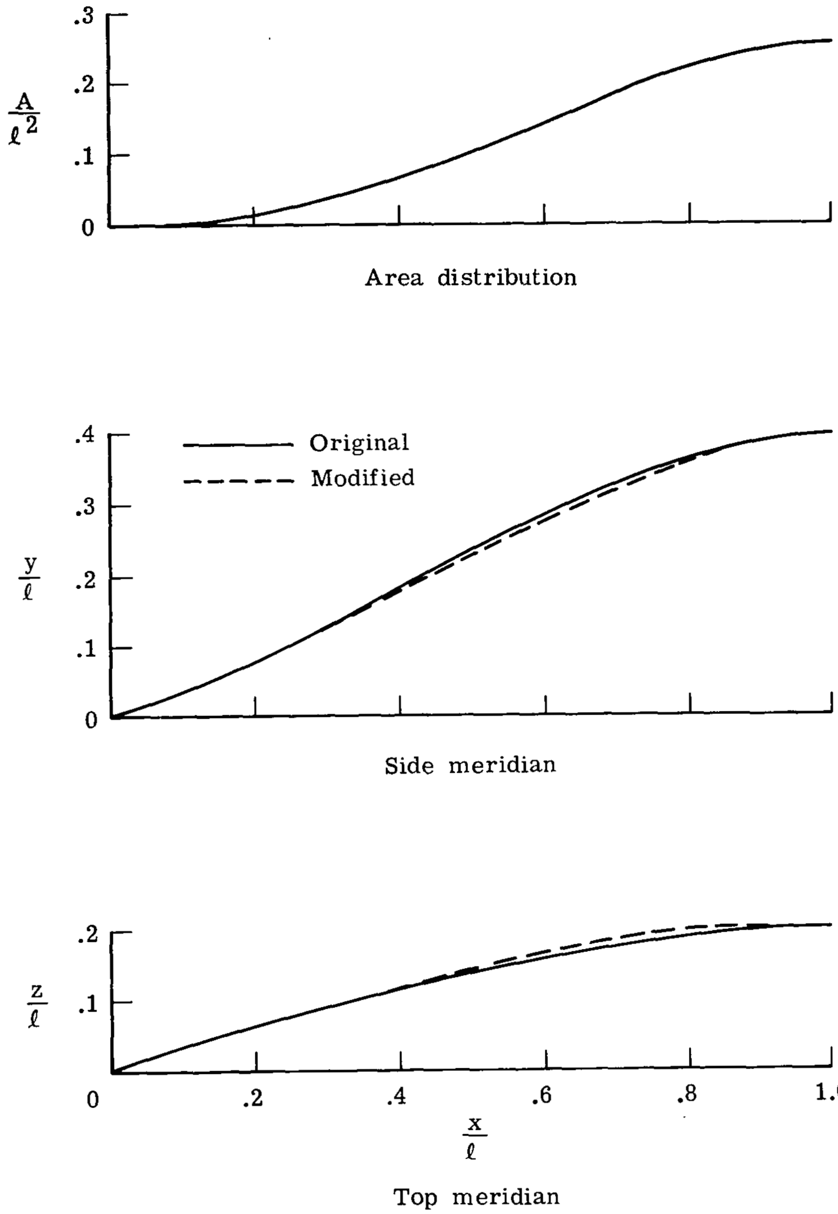


(a) Perspective view of initial design. $\alpha = 0.75$.



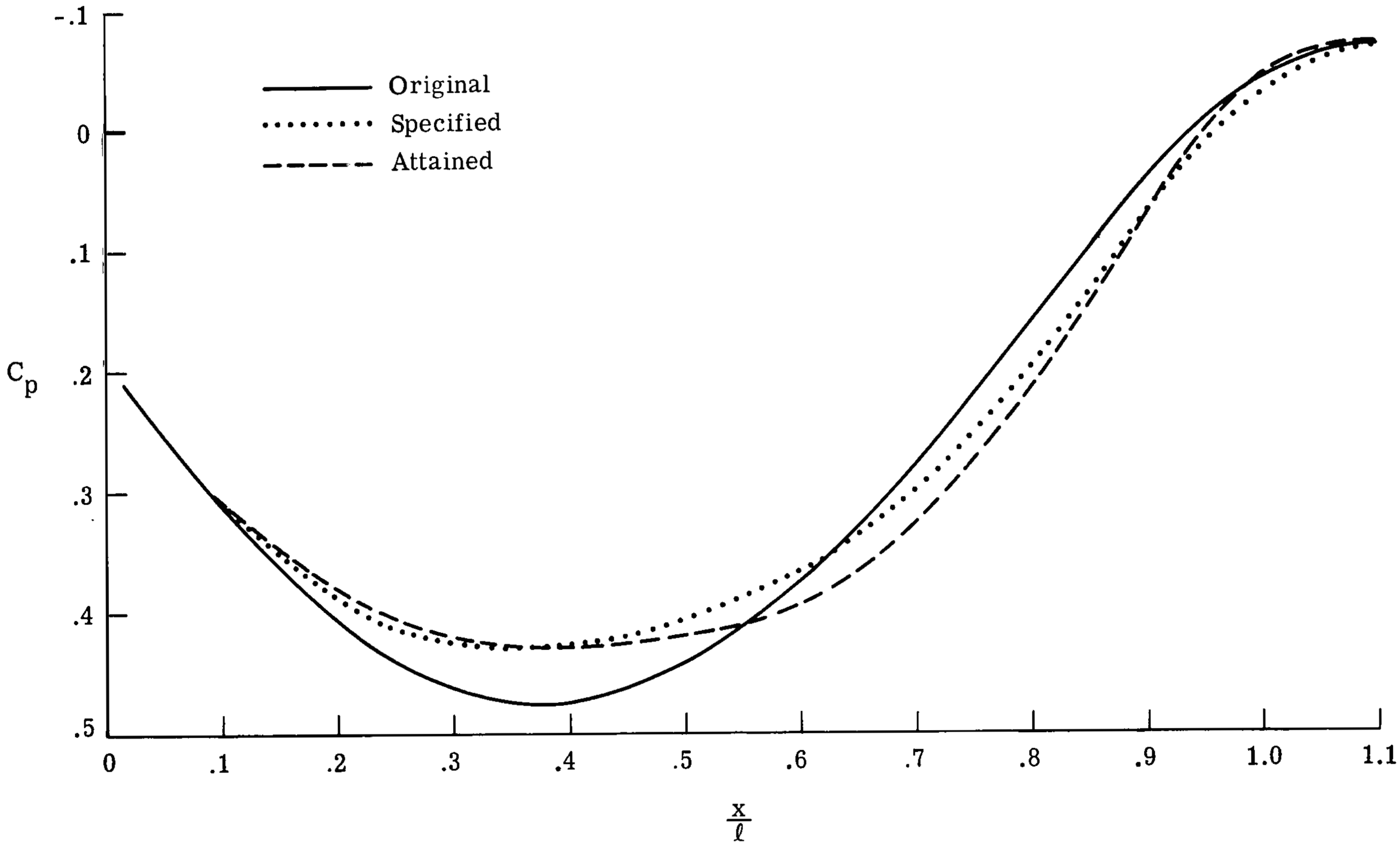
(b) Perspective view of modified design. $\alpha = 4.0$.

Figure 3.- Designs resulting from specifications of figure 2.



(a) Original and revised geometry.

Figure 4.- Modification of elliptic cross-section forebody for specified change in pressure distribution along side meridian.



(b) Original and specified pressure distribution along side meridian and pressure distribution attained with one redesign calculation.

Figure 4.- Concluded.

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16. Abstract

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