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# APPLICATION OF CONSTRAINED OPTIMIZATION TO ACTIVE CONTROL OF AEROELASTIC RESPONSE

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# APPLICATION OF CONSTRAINED OPTIMIZATION TO ACTIVE CONTROL OF AEROELASTIC RESPONSE

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## ABSTRACT

Active control of aeroelastic response is a complex problem in which the designer usually tries to satisfy many criteria which are often conflicting. To further complicate the design problem, the state-space equations describing this type of control problem are usually of high order, involving a large number of states to represent the flexible structure and unsteady aerodynamics. Control laws based on the standard Linear-Quadratic-Gaussian (LQG) method are of the same high order as the aeroelastic plant. To overcome this disadvantage of the LQG method, a new approach was developed for designing low-order optimal control laws (Ref. 1). In this approach, a nonlinear programming algorithm is used to search for the values of the control law variables that minimize a composite performance index. The purpose of this paper is to extend this approach to the constrained optimization problem. The method involves searching for the values of the control law variables that minimize a basic performance index while satisfying several inequality constraints that describe the design criteria. The method is applied to gust load alleviation of a drone aircraft.

| | |
|---|---|
| ● OBJECTIVE: | DESIGN A LOW ORDER OPTIMIZED CONTROL LAW FOR A HIGH ORDER SYSTEM TO MEET SEVERAL DESIGN OBJECTIVES |
| ● METHOD: | SEARCH FOR CONTROL LAW DESIGN VARIABLES THAT MINIMIZE A BASIC PERFORMANCE INDEX WHILE SATISFYING SEVERAL DESIGN CONSTRAINTS |
| ● APPLICATION: | GUST LOAD ALLEVIATION |

## OUTLINE

This paper is divided into three major areas. The first area describes the differences between the usual design philosophy and the present philosophy. The second area describes an implementation of the present design philosophy into a control law synthesis procedure. Finally, numerical results of applying the procedure to a gust load alleviation example is given.

- DESIGN PHILOSOPHIES
- IMPLEMENTATION OF PHILOSOPHY
- NUMERICAL RESULTS

## DESIGN PHILOSOPHIES

This is the first of two slides in the "design philosophies" section of the paper.

In an optimal control problem, when several design criteria need to be satisfied, two design philosophies can be implemented.

The usual approach involves optimizing a single composite performance index (J), in which all design objectives are lumped together. The relative importance of individual responses $Y_{D_i}$ and control inputs $U_i$ is decided by weighting matrices $Q_1$ and $Q_2$. A problem with this approach is the selection of the relative values of the weighting matrices. In one selection method, often called "Bryson's Rule," the $Q_1$ and $Q_2$ matrices are chosen to be diagonal, with the diagonal elements being the inverses of the maximum-response-squared and the maximum-control-squared (Ref. 2). The designer then uses his experience and intuition to adjust the weighting matrices to achieve satisfaction of all design criteria. In this approach, it is often difficult to exert direct control over individual responses.

In a complementary design approach, it has been suggested to incorporate most of the design objectives into a set of inequality constraint equations, instead of lumping all of them in J (Ref. 3). Through these constraints, the designer has a choice of which responses he wishes to directly control through the constraints. This method has the disadvantage of requiring the use of nonlinear programing techniques to solve a constrained optimization problem.

### USUAL APPROACH

- UNCONSTRAINED OPTIMIZATION

$$J = Y_D^T Q_1 Y_D + U^T Q_2 U$$

- PROBLEM: $Q_1$ AND $Q_2$
- BRYSON'S RULE

$$Q_1 = \begin{bmatrix} \ddots & & \\ & \frac{1}{Y_{D_{max}}^2} & \\ & & \ddots \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} \ddots & & \\ & \frac{1}{U_{max}^2} & \\ & & \ddots \end{bmatrix}$$

ADJUST $Q_1$ AND $Q_2$ UNTIL
DESIGN OBJECTIVES ARE MET

### PRESENT APPROACH

- CONSTRAINED OPTIMIZATION

$$J = Y_D^2 \text{ OR } U^2$$

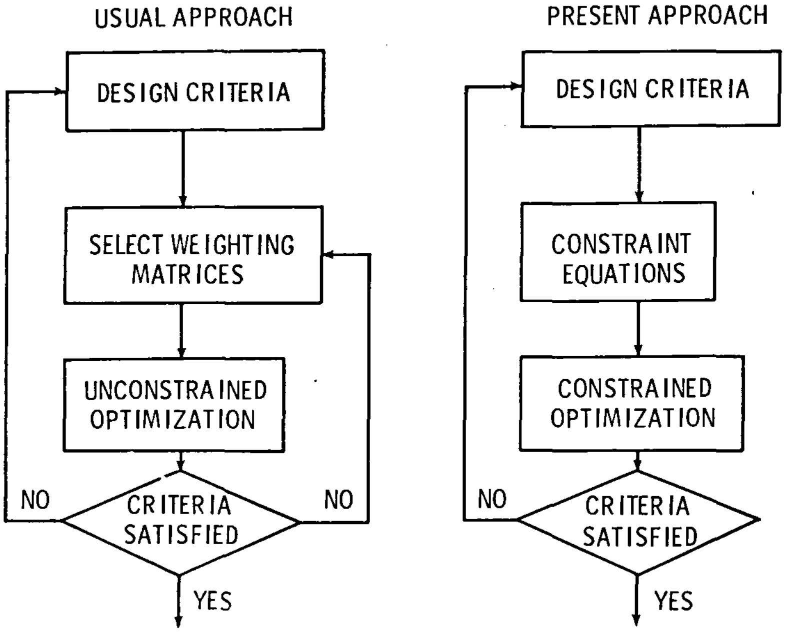
$$g_i = \left( \frac{Y_{D_i}}{Y_{D_{i_{max}}}} \right)^2 - 1 \leq 0$$

- DIRECT CONTROL OVER SPECIFIC RESPONSES
- REQUIRES NONLINEAR PROGRAMMING

# BLOCK DIAGRAMS

Block diagrams which illustrate the differences between the two approaches previously discussed are given on this slide. Both approaches have a common starting point of establishing a set of design criteria. Examples of design criteria include things such as specific reductions in responses, maximum control inputs, stability margins, etc. When employing the usual approach, a set of weighting matrices is selected as inputs to the unconstrained optimization. After the unconstrained optimization is completed, the individual design responses are compared against the design criteria. If the design criteria are not met, two possible paths are available for re-optimizing in an effort to meet the design criteria. The typical first choice is the path to the right, in which a new set of weighting matrices is selected and the optimization is performed again. Selecting new weighting matrices does not afford direct control over the responses, and consequently, there is no assurance that the new selection will result in satisfaction of the criteria. If, after several iterations through the right-hand path, the criteria are still not satisfied, the path to the left is taken, in which the design criteria are changed, and the process is repeated.

When employing the present approach, constraint equations are formulated which describe the design criteria. A constrained optimization is then performed which attempts to satisfy all of the constraints. The design criteria will be met if all of the constraints are satisfied. If the constraints are not satisfied, then the design criteria must be changed and the optmization performed again. The major difference between the present approach and the usual approach is elimination of the iterative loop on the weighting matrices.



# AEROELASTIC SYSTEM

## State-Space Representation

This is the first of four slides in the "implementation" section of the paper. Developments of the state-space equations describing an aeroelastic system are given in several references (Refs. 4-6). In the plant, the matrices F, $G_u$, and $G_w$ are composed of structural mass, damping, stiffness, and aerodynamic matrices and are functions of Mach number and dynamic pressure. The $X_s$ vector is usually composed of modal coordinates, aerodynamic and actuator states. The vector U is the actuator command. w is a white noise input representing atmospheric turbulence. The matrix H contains the modal amplitudes at the sensor locations. In the measurement equation, sensor noise is represented by the white noise v. The matrix $H_D$ contains coefficients for each of the modes that represent quantities such as bending moments, shears, torques, and accelerations. The form of the control law is also chosen to be in state-space form. It is also assumed that the order of the control law is much smaller than the order of the plant. The actual synthesis of the control law (A, B, and C) will be discussed next.

- PLANT $\dot{X}_s = FX_s + G_uU + G_ww$
- MEASUREMENT $Y = HX_s + v$
- DESIGN RESPONSE $Y_D = H_DX_s$
- CONTROL LAW $\dot{X}_c = AX_c + BY$

  $U = CX_c$

## DESIGN OBJECTIVE

The basic objective of the control law synthesis is to find the values of A, B, and C which minimize a performance index and satisfy a set of inequality constraints. A problem with this approach is the selection of a set of design variables for the controller. There are several possible ways for this selection. In a cononical form of the control law, the coefficients of numerator and denominator polynomials can be selected. Poles and zeros of the control law could also be chosen. There are also several other ways to select design variables. In the method described in reference 1, the elements of B and C are chosen as design variables. The matrix A is a function of B and C such that the controller is analogous to the LQG estimator. This same selection of design variables is used in the present constrained optimization problem.

- FIND ELEMENTS OF A, B, AND C WHICH MINIMIZE A PERFORMANCE INDEX AND SATISFY THE CONSTRAINTS
- DESIGN VARIABLES
  - COEFFICIENTS OF TRANSFER FUNCTION
  - POLES AND ZEROS
- CHOOSE B AND C AS DESIGN VARIABLES
  - SET A AS A FUNCTION OF B AND C SUCH THAT THE CONTROLLER IS ANALOGOUS TO LQG ESTIMATOR

## SYNTHESIS PROCEDURE

The synthesis procedure begins with constructing the augmented state equations (closing the loop). It is assumed that we are dealing with stochastic processes and that all responses are defined by their covariance matrices. This leads to solving Lyapunov equations for these quantities. It is shown in reference 1 that the performance index and its gradients require solutions of dual Lyapunov equations. It can also be shown that the constraint functions and their gradients require solutions to Lyapunov equations.

- CONSTRUCT AUGMENTED STATE EQUATIONS

$$\begin{Bmatrix} \dot{X}_s \\ \dot{X}_c \end{Bmatrix} = \begin{bmatrix} F & G_u C \\ BH & A \end{bmatrix} \begin{Bmatrix} X_s \\ X_c \end{Bmatrix} + \begin{bmatrix} G_w & 0 \\ 0 & B \end{bmatrix} \begin{Bmatrix} w \\ v \end{Bmatrix}$$

- STOCHASTIC PROCESSES (w and v WHITE NOISE)

- SOLVE LYAPUNOV EQUATIONS

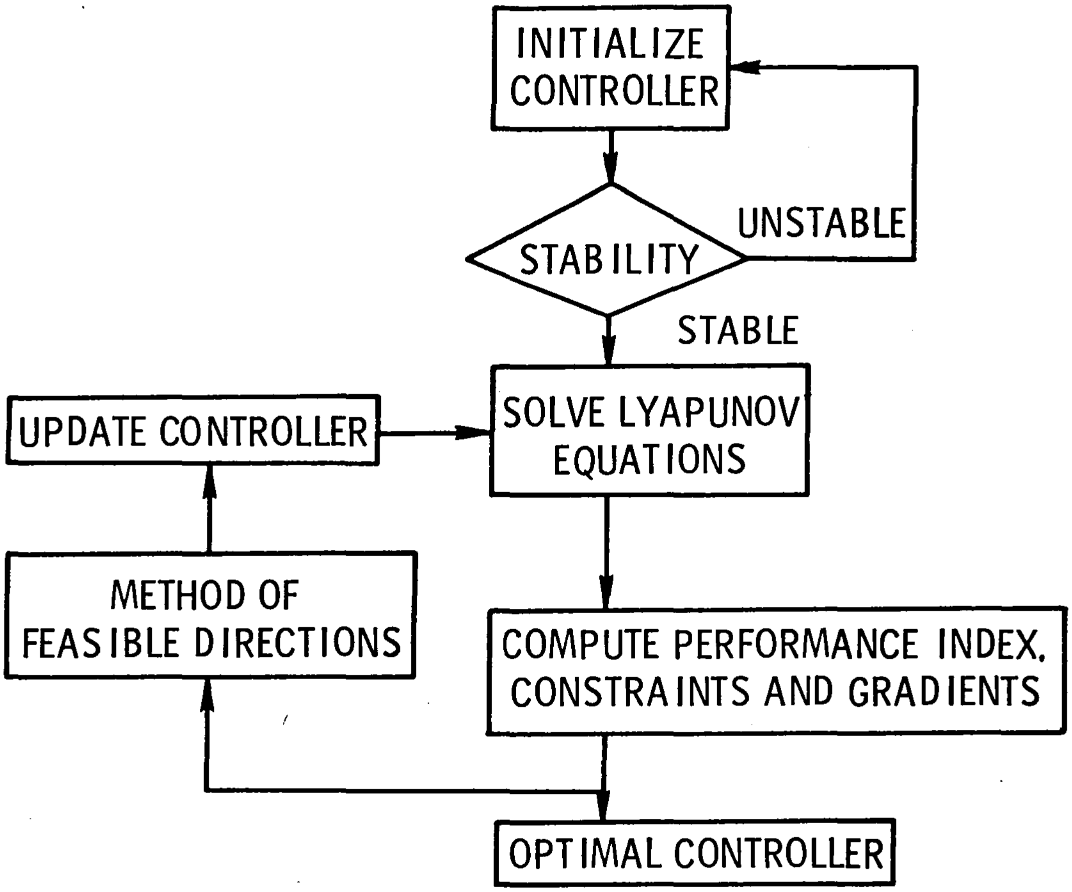
$$X_a F_a^T + F_a X_a = Q$$

$$\Downarrow$$

1) PERFORMANCE INDEX
2) CONSTRAINT VALUES
3) GRADIENTS OF PERFORMANCE INDEX AND CONSTRAINTS

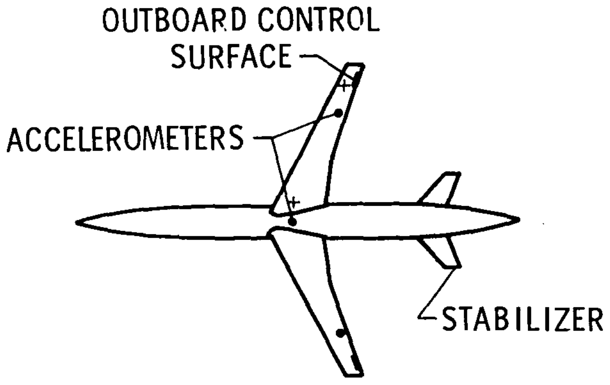
## BLOCK DIAGRAM OF DESIGN ALGORITHM

A block diagram of the design algorithm is shown on this slide. This entire block diagram represents the block titled constrained optimization in slide 4. The method used to initialize the controller matrices is described in reference 1. The method basically involves selecting a few "key states" to be estimated and using a subset of the full-state and Kalman gain matrices as initial values of B and C. Since there is no guarantee on stability at this point, an eigenvalue analysis must be performed to insure stability before the optimization process begins. After an initial controller has been found that stabilizes the system, the Lyapunov equations are solved to obtain values of the performance index, constraints, and their gradients. The method of feasible directions (Ref. 7) is employed to perform the constrained optimization by updating the values of B and C. This iterative loop is continued until the performance index is minimized and the constraints are satisfied.



## EXAMPLE PROBLEM DESCRIPTION

This is the first of three slides in the "numerical results" section of the paper. The vehicle shown in the figure is used to illustrate the present control law synthesis procedure. The vehicle is an unmanned drone airplane used in NASA's Drones for Aerodynamic and Structural Testing (DAST) program (Ref. 8). The objective is to design a gust load alleviation system. Both an outboard control surface and a horizontal stabilizer are used as the control surfaces. Two accelerometers, one in the wing tip and one in the fuselage near the airplane center-of-gravity, are used as feedback sensors. A 30th order model was generated that included two rigid body modes, three flexible modes, and actuator dynamics. A 2nd order filter that approximates the Dryden gust spectrum and driven by white noise was added to give a 32nd order system. The order of the controller was selected as four. Design responses were selected as wing root bending moment, wing root shear, outboard bending moment and torsion, accelerations, and control surface rates and deflections. The performance index was selected as outboard control input. Constraints were formulated to decrease wing root bending moment and shear by 50% below their open loop values while not increasing outboard bending moment and torsion above their open loop values.

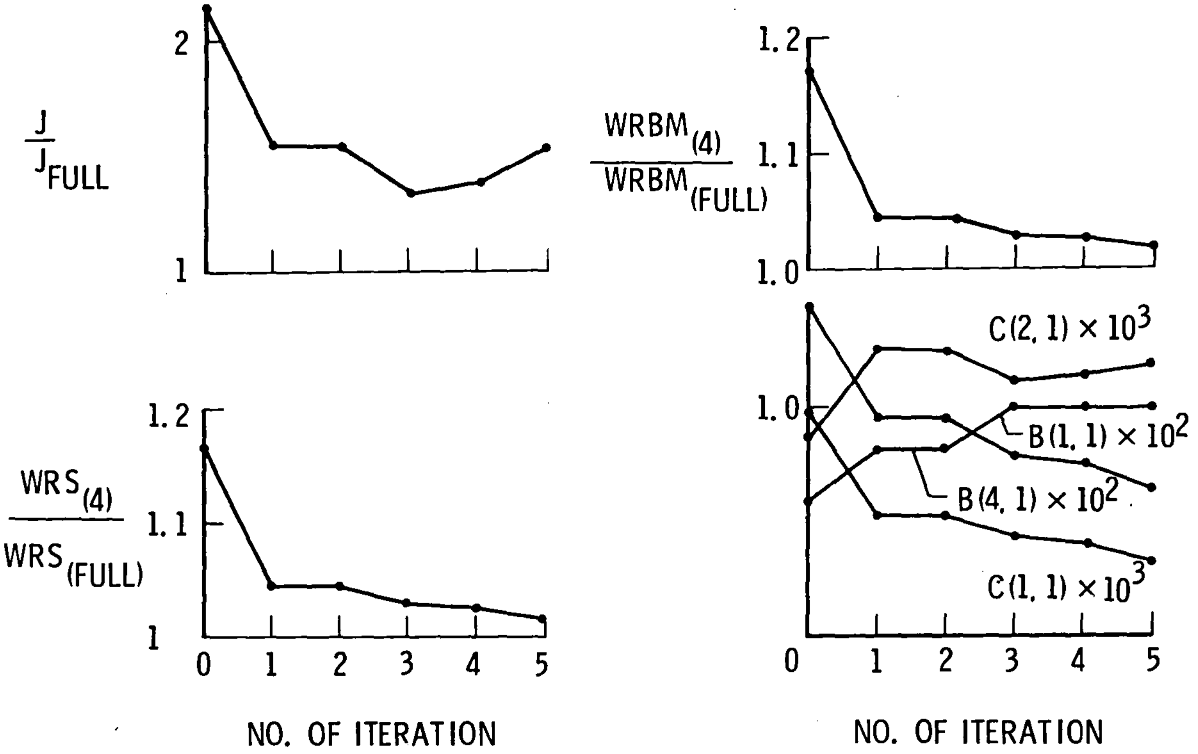


- 32ND ORDER PLANT
- 4TH ORDER CONTROLLER

- DESIGN RESPONSES
  1) LOADS (+)
  2) ACCELERATIONS (•)
  3) CONTROL ACTIVITY
- PERFORMANCE INDEX
  - OUTBOARD CONTROL INPUT - $U_2$
- CONSTRAINTS
  - 50 % REDUCTION IN ROOT BENDING MOMENT AND SHEAR
  - NO INCREASE IN OUTBOARD BENDING MOMENT AND TORSION

## CONVERGENCE PATTERN

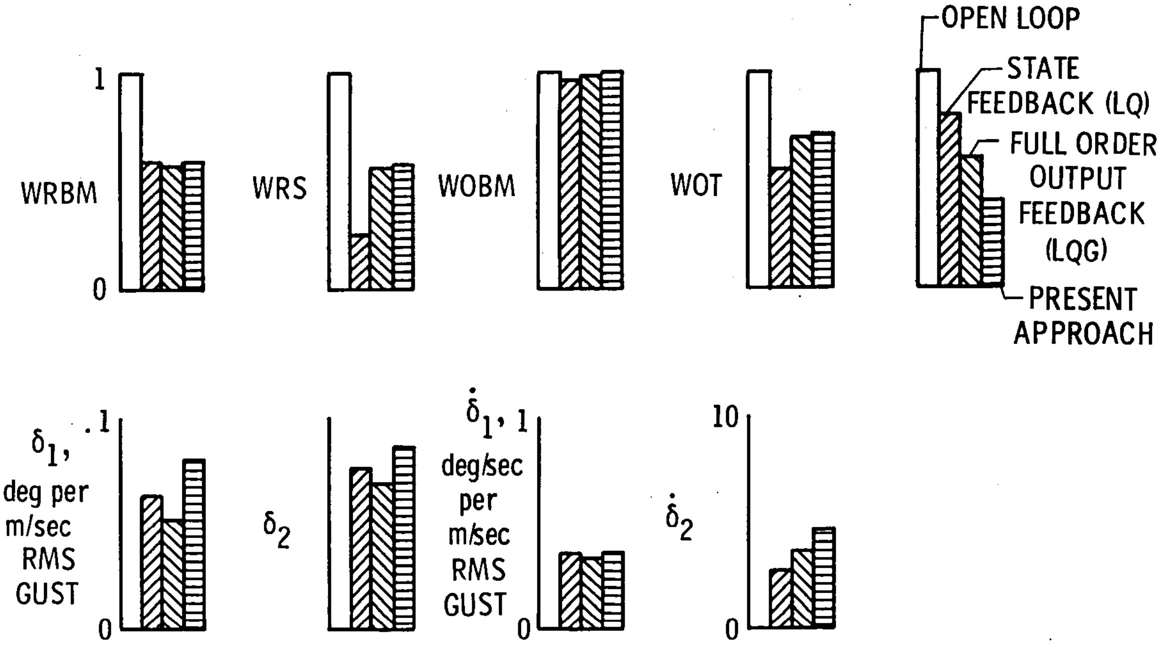
This slide shows the variation of the performance index (J), wing root bending moment (WRBM), wing root shear (WRS), and several of the design variables during five iterations of the constrained optimization. Four key states were estimated which were the deflection and rate of both pitch and the first flexible mode. The performance index, WRBM, and WRS are normalized by the corresponding values with a LQG controller used for initialization. The LQG controller was designed with $Q_1$ and $Q_2$ selected using Bryson's rule. The WRBM and WRS are reduced to within 2 percent of the LQG values without increasing the outboard bending moment (WOBM) or torsion (WOT). It is noted that during the fourth and fifth iterations, WRBM and WRS are reduced at the cost of increased outboard control activity. Some comparative results of the responses are shown in the next slide.



## COMPARISON OF RESPONSES

This bar chart compares the wing loadings and control surface activity using full-state feedback (LQ), full-order output feedback (LQG), and the present approach (4th order controller) respectively. In the first row of figures, the loadings are normalized by the corresponding open loop values, shown by the first unit bar. The second bar is the LQ value which is the ideal case since all states are assumed measured. The third bar is the LQG value. The last bar is the result using the present approach. Both the LQG and 4th order controller reduce the WRBM and WRS approximately 40%. The WOBM does not change appreciably. The wing outboard torsion (WOT) is reduced by about 30% using the present approach.

The second row of figures compares the closed loop stabilizer and outboard control surface activities. The 4th order controller always requires higher control surface activity. The maximum stabilizer deflection is about half the allowable limit of 0.157 degrees. The stabilizer's rate and those of the outboard control are well within allowable limits.



# CONCLUSIONS

- METHOD DEVELOPED TO DESIGN OPTIMAL LOW-ORDER CONTROLLERS TO MEET SEVERAL DESIGN OBJECTIVES
- CONSTRAINED OPTIMIZATION TECHNIQUE USED TO EXERT DIRECT CONTROL OVER SPECIFIC RESPONSES
- GUST LOAD ALLEVIATION EXAMPLE
  - 4TH ORDER CONTROLLER FOR 32ND ORDER PLANT
  - 40% REDUCTION IN WRBM AND WRS
  - 30% REDUCTION IN WOT
  - CONTROL SURFACE ACTIVITY WELL WITHIN LIMITS

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16. Abstract

Active control of aeroelastic response is a complex problem in which the designer usually tries to satisfy many criteria which are often conflicting. To further complicate the design problem, the state-space equations describing this type of control problem are usually of high order, involving a large number of states to represent the flexible structure and unsteady aerodynamics. Control laws based on the standard Linear-Quadratic-Gaussian (LQG) method are of the same high order as the aeroelastic plant. To overcome this disadvantage of the LQG method, a new approach was developed for designing low-order optimal control laws (Ref. 1). In this approach, a nonlinear programing algorithm is used to search for the values of the control law variables that minimize a composite performance index. The purpose of this paper is to extend this approach to the constrained optimization problem. The method involves searching for the values of the control law variables that minimize a basic performance index while satisfying several inequality constraints that describe the design criteria. The method is applied to gust load alleviation of a drone aircraft.

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