SUBPROGRAMS FOR INTEGRATING THE EQUATIONS OF MOTION OF SATELLITES. FORTRAN FOUR.

V. I. PROKHORENKO

Translation of "Podprogrammy integrirovaniya uravneniy dvizheniya iskusstvennykh sputnikov zemli. Fortran IV., Academy of Sciences USSR, Institute of Space Research, Moscow, Report Pr-302, 1977, pp 1-73

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16. Abstract In this work is contained a description of the subprograms intended for the formation of the right members of the equations of motion of artificial earth satellites (ISZ), integration of systems of differential equations by Adams' method, and the calculation of the values of various functions from the AES parameters of motion.

These subprograms are written in the Fortran IV language and constitute an essential part of the package of applied programs for the calculation of navigational parameters AES.

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PREFACE

The present preprint is a continuation of the description, begun in (5), of the **description of the** library of subprograms, arising in the process of creation of a packet of applied programs for the calculation of navigational parameters of artificial **Earth satellites of the** Earth (AES), wherein are contained the subprograms having indexes from F to I. In the subprograms for the calculation of the right members of the AES equations of motion, presented in Chapter I, the subprograms described in (5) are utilized. Here there operate also the principles of scaling the dimensional quantities, determined in (5). The constants and scale factors for the subprograms of chapters 1, 3, and 4 are routed to the domain COMMON by conversion to the subprogram CONST. The system of coordinates determined in (5) is employed.

The subprogram for integration of the system of differential equations by the method of Adams, described in Chapter 2, on the other hand, is sufficiently autonomous and may be used for integration of any system of ordinary differential equations.

In Chapter 3 are presented subprograms ensuring the fixation of the attainment in the process of integration of the assigned values of different functions from the solution of systems of differential equations; the minimum and maximum of an arbitrary continuous function from the solution as a function of an independent variable; the exit of the AES at the ascending node of the orbit; the minimal and maximal altitudes of the AES above the surface of the terrestrial ellipsoid.

Chapter 4 contains subprograms for the computation of the values of various functions from the parameters of motion of the AES.

The subprogram ROOT4(I11), used for the computation of the moments of entry of the AES into the umbra of the earth, is intended for the calculation of the roots of the algebraic equations of the fourth, third and second degree, has a significantly independent character and may be used for the solution of other problems.

The subprogram FA GRAV (F 03) was written by E. E. Ryazanova. For the computation of the geomagnetic parameters B, L the subprograms BL, INVAR, LINES. STAR, CARMEL, INTEG, NEWMAG, ASIN (I 04) are utilized, submitted through the courtesy of Yu. N. Gal'perin and V. M. Sinitsyn. The author of the remaining subprograms is the author of the preprint.

The author wishes to express his thanks to E. A. Chistyakova for assistance in the editing of the texts of the subprograms for publication, to L. V. Zaytseva and V. F. Smirnova for help in the preparation of the manuscript.

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SUBPROGRAMS FOR INTEGRATING THE EQUATIONS OF MOTION OF SATELLITES. FORTRAN FOUR.

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Chapter I. Right members of the system of equations of motion of the AES (index F)

I.1. Equation of motion of the AES. Various models.

We will write the differential equations of motion of the AES in an absolute system of coordinates in the general case in the following form: /5*

$$\begin{aligned}
&\dot X = V_X, \quad \dot Y = V_Y, \quad \dot Z = V_Z, \\
&\dot V_X = \Delta_N \dot V_X + \Delta_A \dot V_X + \Delta_G \dot V_X + \Delta_S \dot V_X + \Delta_L \dot V_X, \\
&\dot V_Y = \Delta_N \dot V_Y + \Delta_A \dot V_Y + \Delta_G \dot V_Y + \Delta_S \dot V_Y + \Delta_L \dot V_Y, \\
&\dot V_Z = \Delta_N \dot V_Z + \Delta_A \dot V_Z + \Delta_G \dot V_Z + \Delta_S \dot V_Z + \Delta_L \dot V_Z.
\end{aligned} \tag{1.1}$$

In the Greenwich relative system of coordinates these equations have the form

$$\begin{aligned}
&\dot x = v_x, \quad \dot y = v_y, \quad \dot z = v_z, \\
&\dot v_x = \omega_S^2 x + 2\omega_S v_y + \Delta_N \dot v_x + \Delta_A \dot v_x + \Delta_G \dot v_x + \Delta_S \dot v_x + \Delta_L \dot v_x, \\
&\dot v_y = \omega_S^2 y - 2\omega_S v_x + \Delta_N \dot v_y + \Delta_A \dot v_y + \Delta_G \dot v_y + \Delta_S \dot v_y + \Delta_L \dot v_y, \\
&\dot v_z = \Delta_N \dot v_z + \Delta_A \dot v_z + \Delta_G \dot v_z + \Delta_S \dot v_z + \Delta_L \dot v_z,
\end{aligned} \tag{1.2}$$

where the projections of acceleration, determined by the influence of the corresponding forces, are:

*Numbers in the margin indicate pagination in the foreign text."

$\Delta_N \dot{v}_x, \Delta_N \dot{v}_y, \Delta_N \dot{v}_z$ --normal gravitational field of the earth

$\Delta_A \dot{v}_x, \Delta_A \dot{v}_y, \Delta_A \dot{v}_z$ --resistance of the earth's atmosphere

$\Delta_G \dot{v}_x, \Delta_G \dot{v}_y, \Delta_G \dot{v}_z$ --gravitational anomalies

$\Delta_S \dot{v}_x, \Delta_S \dot{v}_y, \Delta_S \dot{v}_z$ --gravitational perturbations of the moon and sun

$\Delta_L \dot{v}_x, \Delta_L \dot{v}_y, \Delta_L \dot{v}_z$ --pressure of light

$\omega_3^2 x, \omega_3^2 y$ --centrifugal force

$2\omega_3 v_y, 2\omega_3 v_x$ --force of Coriolis

ω_3 --angular velocity of the earth's rotation

For the calculation of the right members of the system of equations of motion of AES one has the collection of subprograms: FNGRAV, FAGRAV, FATM, FGRSM, FLIGHT, each of which allows for its component in the right members of the equations of motion of the AES. /6

These subprograms have a subsidiary nature; from them it is possible to construct the subprogram for calculation of the right-hand sides for a system of equations, with this or that degree of completion for the described motion of the AES. In (4) were introduced the indexes KC, KG, KA, KS, KL and table 2.1, permitting the regulation of the variants of the system of forces, acting on the AES (and of the system of coordinates, in which the motion of the AES is analyzed).

Table 1.1 is a repetition of table 2.1 from work (4).

We recall also the designation of indexes.

The index KS characterizes the system of coordinates (possible values 1.2).

To each of the forces acting is attached its index:

KG--force of the earth's attraction (possible values 0,1,2,3,4);
KA--atmospheric resistance (possible values 0,1,2,3,4);
KS--gravitational perturbation by the moon and sun (possible values 0,1,2,3);
KL--pressure of light (possible values 0,1,2).

Giving to each of the above enumerated indexes the value determined, we give the determined model of forces, characterized by the five-valued index, consisting of the values of the indexes

KC, KG, KA, KS, KL.

There is a subprogram FORCE, realizing all possible variants of the models of force, specified by table 1.1. However for concrete models it is possible to recommend to the user the creation of "truncated" subprograms, which may be obtained from the subprogram FORCE by means of discarding conversions to those subprograms, which are not utilized in the given model.

Table 1.1 System of forces, considered in the equations of motion of the AES. Indexes KC, KG, KA, KS, KL /7

	Systems of Coord.	Grav. Field	Atmo-sphere	Grav. Perturba-tion	Pressure of Light
	KC	KO	KA	KS	KL
	1	2	3	4	5
0	oscula-ting elts.	central	not con-sidered	not con-sidered	not considered
1	Greenwich relative	normal	without calc. var.	from moon	without calculation of the earth's umbra
2	absolute	zonal harmo-nics	with calc. of long period var.	from sun	with calculation of the earth's umbra
3		full anomalies	with calc. of long and short period var.	from moon and sun	
4		harmo-nics 22,30,40	CIRA-72		

The subprogram FC11ØØ(F06), may serve as an example of such a subprogram; in this subprogram is realized the model of the motion of the AES, corresponding to the following values of the indexes: KC equals 1 or 2, KG equals 0 or 1, KA equals 1, KS equals 0, KL equals 0. /8

In order that the designations of concrete subprograms of the right-hand sides, created by the user, reflect the model, realized in them, of the motion of the AES, it is suggested that one construct identifiers of these subprograms according to this sort of principle: the first letter--F, after it a five-valued index, corresponding to the chosen variant of model from table 1.1 (as this was done in the case of FC11ØØ).

1.2 Calculation of the normal gravitational field of the earth and (for the case of the Greenwich system of coordinates) translational and coriolis accelerations (F01-FNGRAV).

1. Designation. The terms $\Delta_N \dot{V}_X, \Delta_N \dot{V}_Y, \Delta \dot{V}_Z$ are determined; they are dependent on the influence of the normal gravitational field of the earth in the right members of the equations of motion of the AES (1.1) in the absolute system of coordinates. If the motion of the AES is calculated in Greenwich system of coordinates (system of equations 1.2), then these sums are determined:

$$\Delta_N \dot{v}_x + \omega_3^2 x + 2\omega_3 v_y, \quad \Delta_N \dot{v}_y + \omega_3^2 y - 2\omega_3 v_x, \quad \Delta_N \dot{v}_z$$

2. Structure. The subprogram FNGRAV. General units: $/CA00/_2$, $/CRZ/_1$, $/CAE/_2$, $/CAEL/_1$, $/COmZP/_2$.

3. Conversion: CALL FNGRAV (KC,KG, Y, HC, RC, F)

4. Initial data: KC--index, characterizing the system of coordinates (KC = 1 for Greenwich system of coordinates, KC=2 for absolute system of coordinates:;

KG--index, characterizing the gravitational field of the earth (for KG equal to zero there is considered only the central field)

Y_6--main part, containing X, Y, Z, V_X, V_Y, V_Z or x, y, z, v_x, v_y, v_z.

5. Results: /9

HC--elevation of AES above the surface of the earth;
RC--modulus of the radius-vector of the AES;
F_6--main part of the right members of the equation of motion of the AES, containing $\dot{X}$, $\dot{Y}$, $\dot{Z}$, $\dot{V}_X$, $\dot{V}_Y$, $\dot{V}_Z$ or $\dot{x}$, $\dot{y}$, $\dot{z}$, $\dot{v}_x$, $\dot{v}_y$, $\dot{v}_z$.

6. Utilization of the groups COMMON: The constants of the groups are utilized: $/CAOO/_2$, $/CRZ/_1$, $/CAE/_2$, $/CAEL/_1$, $/COMZP/_2$ (see in §(3)) Nos. 2, 10, 11, 12, 15 table 2.1)

7. Algorithm:
a) absolute system of coordinates;

$$\dot{X} = V_X, \quad \dot{Y} = V_Y, \quad \dot{Z} = V_Z,$$
$$\dot{V}_X = \Delta_N \dot{V}_X = -\alpha_{00} RX/r^3 - C(D-1)RX/r^3,$$
$$\dot{V}_Y = \Delta_N \dot{V}_Y = -\alpha_{00} RY/r^3 - C(D-1)RY/r^3,$$
$$\dot{V}_Z = \Delta_N \dot{V}_Z = -\alpha_{00} RZ/r^3 - C(D-3)RZ/r^3,$$

where $C=3/2\ \alpha_{20}(R/r)^2$, $D=5Z^2/r^2$, $r=(X^2+Y^2+Z^2)^{1/2}$, R--average radius of the earth, alpha$_{00}$, alpha$_{20}$ --parameters of the normal gravitational field of the earth; for alpha$_{20}$=0 there is considered only the influence of the central gravitational field of the earth;

b) Greenwich system of coordinates:

$$\dot{x} = v_x, \quad \dot{y} = v_y, \quad \dot{z} = v_z,$$
$$\dot{v}_x = \Delta_N \dot{v}_x + \omega_3^2 x + 2\omega_3 v_y,$$
$$\dot{v}_y = \Delta_N \dot{v}_y + \omega_3^2 y - 2\omega_3 v_x,$$
$$\dot{v}_z = \Delta_N \dot{v}_z,$$

c) elevation h_c of the AES above the earth's surface ellipsoid is computed according to the formula $h_c = r - (a_e - \text{alpha}_e z^2/r^2)$, where a_e, alpha--semimajor axis and constriction of the general terrestrial ellipsoid.

8. Text

/10

```
      SUBROUTINE FNGRAV(KC,KG,V,HC,R,F)
      DIMENSION V(6),F(6)
      COMMON /CA00/A00,A20
      COMMON /CAE/AE,AL
      COMMON/CAEL/AEL
      COMMON/CRZ/RZ
      COMMON /COMZP/OMZ2,OMZK
      W=V(3)*V(3)
      R2=W+V(1)*V(1)+V(2)*V(2)
      W=W/R2
      R=SQRT(R2)
      HC=R-AE+AEL*W
      W1=RZ/R
      B=W1/R2
      A=B*A00
      BC2=0
      IF(KG)1,2,1
    1 BC2=3.*A20*W1*W1*B
      A=A+BC2*(2.5*W-.5)
    2 DO 3 J=1,3
      F(J+3)=-A*V(J)
    3 F(J)=V(J+3)
      F(6)=F(6)+BC2*V(3)
      GOTO (4,5),KC
    4 F(4)=F(4)+OMZK*V(1)+OMZ2*V(5)
      F(5)=F(5)+OMZK*V(2)-OMZ2*V(4)
    5 RETURN
      END
```

1.3 Calculation of the influence of atmospheric resistance (FO2-FATM).

1. Designation. The terms $\Delta_A\dot{V}_X, \Delta_A\dot{V}_Y, \Delta_A\dot{V}_Z$. are determined, depending on the influence of the atmospheric resistance in the equations of (1.1) or (1.2).

2. Structure. Subprogram FATM.

General groups: $/BSB/_1$, $/COMS/_1$.

3. Conversion: CALL FATM (KC, Y, Γ, F).

4. Initial data: KC--index, characterizing the system of coordinates; Y_6--main part, containing X, Y, Z, V_X, V_Y, V_Z or x, y, z, v_x, v_y, v_z; Γ--density of the atmosphere; F_6--main part of right members of equations of motion of ABS $\dot{X}, \dot{Y}, \dot{Z}, \dot{V}_X, \dot{V}_Y, \dot{V}_Z$ or $\dot{x}, \dot{y}, \dot{z}, \dot{v}_x, \dot{v}_y, \dot{v}_z$ with the computation of the influence of atmospheric resistance.

6. Use of the domain COMMON. Before conversion to the subprogram FATM in the group COMMON/BSB/SB it is necessary to address the value of the ballistic coefficient in the system of units, obtained from the calculations, for this it is sufficient that the value of the ballistic coefficient, given originally in the system of units kg, m, s, be divided by the scale factor from the group COMMON/CESB/ESB (see in (5) No. 12 table 2.3). /11

From the group COMMON/COMZ/OMZ there is utilized the constant (see in (5) No. 14 table 2.1)

7. Algorithm

$$\begin{cases} \Delta_A \dot{V}_X = -SB \cdot \rho V^{omn} \cdot V_X^{omn}, \\ \Delta_A \dot{V}_Y = -SB \cdot \rho V^{omn} \cdot V_Y^{omn}, \\ \Delta_A \dot{V}_Z = -SB \cdot \rho V^{omn} \cdot V_Z^{omn}; \end{cases}$$

$$\dot{V}_X = F_X + \Delta_A \dot{V}_X, \quad \dot{V}_Y = F_Y + \Delta_A \dot{V}_Y, \quad \dot{V}_Z = F_Z + \Delta_A \dot{V}_Z,$$

where SB--ballistic coefficient $SB=C_x \cdot F_M/2m$

C_x--dimensionless coefficient of air resistance

F_M--square of the mid section, m--mass of AES,

ρ--air density

V_X^{omn}, V_Y^{omn}, V_Z^{omn} --components of the vector of flight speed relative to the air $V^{omn} = \left(V_X^{omn\,2} + V_Y^{omn\,2} + V_Z^{omn\,2}\right)^{1/2}$.

In the Greenwich system of coordinates $v_x^{omn} = v_x, v_y^{omn} = v_y, v_z^{omn} = v_z$.

In the absolute system of coordinates $V_X^{omn} = V_X + \omega_3 Y, V_Y^{omn} = V_Y - \omega_3 X, \quad V_Z^{omn} = V_Z$.

F_X, F_Y, F_Z --other terms in the right members of equations (1.1) or (1.2).

8. Text

```
      SUBROUTINE FATM(KC,Y,P,F)
      COMMON /BSB/SB
      COMMON /COMZ/OMZ
      DIMENSION Y(6),F(6),V(3)
      DO 1 J=1,3
    1 V(J)=Y(J+3)
      GOTO(2,3),KC
    3 V(1)=V(1)+OMZ*V(2)
      V(2)=V(2)-OMZ*V(1)
    2 W=0
      DO 4 J=1,3
    4 W=W+V(J)*V(J)
      W=SB*P*SQRT(W)
      DO 5 J=1,3
    5 F(J+3)=F(J+3)-W*V(J)
      RETURN
      END
```

1.4. Calculation of anomalies of gravitational field of the earth (FO3-FAGRAV)

1. Designation. The terms $\text{delta}_G \dot{V}_X$, $\text{delta}_G \dot{V}_Y$, $\text{delta}_G \dot{V}_Z$ in the right members of the system of equations (1.1) or (1.2), derived from the influence of anomalies of the earth's gravitational field.

2. Structure. Subprogram FAGRAV.

There are utilized the exterior subprograms: DEG2, DEG3, DEG4 (E01) General group: /RAD/$_2$.

3. Conversion: CALL FAGRAV (Y, XG, F, KG, NM).

4. Initial data.

The main part Y_6, containing X, Y, Z, V_X, V_Y, V_Z or x, y, z, v_x, v_y, v_z; the main part XG_3, containing x, y, z;

F_6--main part of right members of equations (1.1) or (1.2), containing the terms, determined by the influence of other factors;

KG--index, determining the nature of the considered anomalies of the gravitational field of the earth (possible values: 2, 3, 4);

For KG=2 one considers only the zonal harmonics in the breakdown of the gravitational field of the earth, for KG=3 there are considered the zonal, tesseral, and sectorial harmonics, for KG=4 one considers only the harmonics 22, 30, 40;
NM--number of considered harmonics (NM less than 22).

Note 1. For KG=2 or 3 it is necessary first to turn to the subprogram CONGR(A02) for the addresses in the group COMMON/BCONGR/$_{536}$ coefficients of the gravitational field of the earth. For KG=4 one uses the coefficients from the block COMMON/CA22/4 (see (5) No. 4 table 2.1), value NM is not used here.

Note 2. The main part XG is used only for KG=3 or 4.

5. Results: Main part F_6 containing new values $\dot{X}$, $\dot{Y}$, $\dot{Z}$, $\dot{V}_X$, $\dot{V}_Y$, $\dot{V}_Z$ or $\dot{x}$, $\dot{y}$, $\dot{z}$, $\dot{v}_x$, $\dot{v}_y$, $\dot{v}_z$ with calculation of the influence of the anomalies of the earth's gravitational field.

6. Use of the domain COMMON. In the block COMMON/RAD/RC,R1 one addresses the values RC=r and R1=r_1.

7. Algorithm:

$$\begin{cases} \Delta_0 \dot{V}_X = -\Delta g_r X/r - \Delta g_m ZX/rr_1 - \Delta g_\ell Y/r_1, \\ \Delta_0 \dot{V}_Y = -\Delta g_r Y/r - \Delta g_m ZY/rr_1 + \Delta g_\ell X/r_1, \\ \Delta_0 \dot{V}_Z = -\Delta g_r Z/r + \Delta g_m r_1/r, \end{cases}$$

$\dot{V}_X = F_X + \mathrm{delta}_G \dot{V}_X$, $\dot{V}_Y = F_Y + \mathrm{delta}_G \dot{V}_Y$, $\dot{V}_Z = F_Z + \mathrm{delta}_G \dot{V}_Z$, where $r = (X^2+Y^2+Z^2)^{\frac{1}{2}}$, /13
$r_1 = (X^2+Y^2)^{\frac{1}{2}}$, F_X, F_Y, F_Z--components of other terms in the right members of equations (1.1) or (1.2),
delta g_r--radial component of vector delta g, acceleration due to the influence of anomalies of the earth's gravitational field,
delta g_m--meridional component of the vector delta g,
delta g_1--projection of the vector delta g on the normal to the plane of the meridian.

Note. The components of the vector delta g, for some or some other recommended anomalies of the earth's gravitational field, respectively, are calculated with the subprograms with index E01: DEG 2, DEG 3 or DEG 4 see in (5) p. 6.1 (there is introduced also the algorithm for the computation of the components of vector delta g).

8. Text.

```
      SUBROUTINE FAGRAV(X,XG,F,JV,NM)
      DIMENSION X(6),F(6),DG(3)
      DIMENSION XG(3)
      COMMON/RAD/R1,R
      X2=X(1)*X(1)
      Y2=X(2)*X(2)
      Z2=X(3)*X(3)
      R12=X2+Y2
      R1=SQRT(R12)
      R2=R12+Z2
      R=SQRT(R2)
      NV=JV-1
      W=R*R1
      GO TO (1,2,3),NV
    1 CALL DEG2(X,DG,NM)
      GO TO 4
    2 CALL DEG3(XG,DG,NM)
      GO TO 4
    3 CALL DEG4(XG,DG)
    4 F(4)=F(4)-(DG(1)*X(1))/R-(DG(2)*X(1)*X(3))/W-(DG(3)*X(2))/R1
      F(5)=F(5)-(DG(1)*X(2))/R-(DG(2)*X(2)*X(3))/W+(DG(3)*X(1))/R1
      F(6)=F(6)-(DG(1)*X(3))/R+(DG(2)*R1)/R
      RETURN
      END
```

1.5 Calculation of the gravitational perturbations connected with the influence of the moon or sun (F04-FGRSS). /14

1. Designation. The terms $delta_S\dot{V}_X$, $delta_S\dot{V}_Y$, $delta_S\dot{V}_Z$ in the right members of the equations of motion (1.1) or (1.2), dependent on the gravitational perturbation of the moon or sun.

2. Structure. Subprogram FGRSS.

3. Conversion: CALL FGRSS (GDR3, Y, XS, RS, F).

4. Initial data: $GDR3 = mu_s/R_s^2$, where mu_s--product of the gravitational constant and the mass of the moon (or sun), R_s--modulus of the radius vector of the moon (or sun).

The main part Y_6, containing the values X, Y, Z, V_X, V_Y, V_Z or x, y, z, v_x, v_y, v_z; main part XS_3, containing the values X_s^o, Y_s^o, Z_s^o or x_s^o, y_s^o, z_s^o -- directed cosines of the radius-vector of the moon (or sun); main part F_6, containing the values $\dot{X}$, $\dot{Y}$, $\dot{Z}$, $\dot{V}_X$, $\dot{V}_Y$, $\dot{V}_Z$ or $\dot{x}$, $\dot{y}$, $\dot{z}$, $\dot{v}_x$, $\dot{v}_y$, $\dot{v}_z$, considering the other terms in the right members of equations (1.1) or (1.2).

5. Results. In the main part F_6 the new values of the right members of the equations of motion of the AES are addressed with the computation of the influence of gravitational perturbations, produced by the moon (or sun).

6. Algorithm:

$$\Delta_S\dot{V}_X = \mu_S/r_S^2\left(\left(X_S^o - X/r_S\right)r_S^3/|r_S - r|^3 - X_S^c\right),$$
$$\Delta_S\dot{V}_Y = \mu_S/r_S^2\left(\left(Y_S^o - Y/r_S\right)r_S^3/|r_S - r|^3 - Y_S^o\right),$$
$$\Delta_S\dot{V}_Z = \mu_S/r_S^2\left(\left(Z_S^o - Z/r_S\right)r_S^3/|r_S - r|^3 - Z_S^c\right),$$
$$\dot{V}_X = F_X + \Delta_S\dot{V}_X, \quad \dot{V}_Y = F_Y + \Delta_S\dot{V}_Y, \quad \dot{V}_Z = F_Z + \Delta_S\dot{V}_Z,$$
$$|r_S - r|/r_S = \left(\left(X_S^c - X/r_S\right)^2 + \left(Y_S^c - Y/r_S\right)^2 + \left(Z_S^o - Z/r_S\right)^2\right)^{1/2},$$

r_s--modulus of the radius vector of the moon (or sun), F_X, F_Y, F_Z --components of other terms in the right members of equations (1.1) and (1.2).

7. Text /15

```
      SUBROUTINE FORSS(GDRS,V,XP,RP,F)
      DIMENSION V(6),XP(3),F(6),XPR(3),W(3)
      R=0
      DO 1 J=1,3
      XPR(J)=V(J)/RP
      W(J)=XP(J)-XPR(J)
    1 R=W(J)*W(J)+R
      R=1./R/SQRT(R)
      DO 2 J=1,3
    2 F(J+3)=F(J+3)+GDRS*(W(J)*R-XP(J))
      RETURN
      END
```

1.6 Computation of the influence of the pressure of light (F05-FLIGHT)

1. Designation. The terms $\text{delta}_L\dot{V}_X$, $\text{delta}_L\dot{V}_Y$, $\text{delta}_L\dot{V}_Z$, are determined, considering the influence of light pressure on the right members of equations (1.1) or (1.2).

2. Structure. Subprogram FLIGHT.
General blocks: $/BB/_1$, $/CRZ/_1$.

3. Conversion. CALL FLIGHT (KL, Y, RC, XS, RS, F).

4. Initial data: KL--index, governing the computation of the umbra of the earth (for KL=1 is produced the calculation of the influence of light pressure independently of the shadow, for KL=2 the shadow is considered);
The main part Y_6, containing the values X, Y, Z, V_X, V_Y, V_Z or x, y, z, v_x, v_y, v_z; main part XS_3, containing the directed cosines of the radius-vector of the sun: $X^o_\odot$, $Y^o_\odot$, $Z^o_\odot$ or $x^o_\odot$, $y^o_\odot$, $z^o_\odot$; considering the other terms in the right members of equations (1.1) or (1.2).

5. Results. In main part F_6 are addressed the new values of right members of equations of motion of the AES with calculation of the influence of the pressure of light.

6. Use of the domain COMMON. Before conversion to the

subprogram FLIGHT in the block COMMON/BB/B it is necessary to address /16
the value of the coefficient B (see par. 7). Starting from this, that in the kg, m, s system of units this coefficient has the dimensions m/s^2, it is necessary to make it agree with the system of units used in the computations, dividing by the scale factor from the block COMMON/CELB/ELB (see (5), table 2.3, No. 13).

From the block COMMON/CPZ/ one useds the constant (No. 10 table 2.1 (5)).

7. Algorithm:

$$\dot{V}_X = -B(X^0_\bullet - X/r_\bullet)\, r_\bullet^3/|r_\bullet - r|^3 + F_X ,$$

$$\dot{V}_Y = -B(Y^0_\bullet - Y/r_\bullet)\, r_\bullet^3/|r_\bullet - r|^3 + F_Y ,$$

$$\dot{V}_Z = -B(Z^0_\bullet - Z/r_\bullet)\, r_\bullet^3/|r_\bullet - r|^3 + F_Z ,$$

где $|r_\odot - r|/r_\odot = \left((X^0_\bullet - X/r_\odot)^2 + (Y^0_\bullet - Y/r_\odot)^2 + (Z^0_\odot - Z/r_\odot)^2\right)^{1/2}$,

$$B = \frac{F_M}{m} k q_0$$

where F_M--square of the mid section, m--mass of the satellite, $q_0 = 4.5\ 10^{-7} kg/m^2$--pressure of light in the region of the earth's orbit, k=1 for full optical reflection, k=1.44 for full diffused reflection.

In the case of calculation of the influence of light pressure with calculation of the shadow of the earth the entry condition of the AES into the earth's shadow is verified:

$$\cos\gamma < 0 \quad и \quad r \sin\gamma < R ,$$

где $\cos\gamma = (XX^0_\odot + YY^0_\odot + ZZ^0_\odot)/r$, $r = (X^2 + Y^2 + Z^2)^{1/2}$,

R--average radius of the earth.

8. Text

```
      SUBROUTINE FLIGHT(KL,W,R,XS,RS,F)
      COMMON /DB/D
      DIMENSION Y(6),XS(3),W(3),F(6)
      COMMON/CRZ/RZ
      GOTO(3,6),KL
6     CN=(W(1)*XS(1)+W(2)*XS(2)+W(3)*XS(3))/R
      IF(CN)1,3,3
1     IF((R*SQRT(1.-CN*CN)-RZ)2,3,3
3     R1=R
      DO 4 J=1,3
      W(J)=XS(J)-Y(J)/RS
4     R1=W(J)*W(J)+R1
      R2=SQRT(R1)
      R2=R1*R2
      DO 5 J=1,3
5     F(J+3)=F(J+3)-D*W(J)/R2
2     RETURN
      END
```

1.7 Right members of the system of equations of motion of the AES, considering the normal gravitational field of the earth and standard five-layered atmosphere (F06-FC11ØØ)

1. Designation. The subprogram FC 11ØØ is intended for use in the capacity of a subprogram for the right members for integration of a system of differential equations of motion of the AES for models, characterized by the indexes 11100, 21100, 10100, 20100 (see table 1.1).

2. Structure. Subprogram FC11ØØ
External subprograms: FNGRAV (FO1), RO(DO1), FATM(FO2).
General blocks: $/BHC/_1$, $/BK/_5$, $/BSB/_1$.

3. Conversion: CALL FC11ØØ (T, Y, F).

4. Initial data: T--independent variable (time), Y_6--main part, containing X, Y, Z, V_X, V_Y, V_Z or x, y, z, v_x, v_y, v_z

5. Results: F_6 --main part, containing $\dot{X}$, $\dot{Y}$, $\dot{Z}$, $\dot{V}_X$, $\dot{V}_Y$, $\dot{V}_Z$ or $\dot{x}$, $\dot{y}$, $\dot{z}$, $\dot{v}_x$, $\dot{v}_y$, $\dot{v}_z$.

6. use of the domain COMMON. In the block COMMON /BSB/SB is is necessary to give the value of the ballistic coefficient, relative to the scale factor from the block COMMON/CESB/ESB (see (5)), table 2.3, No. 12).

In the block COMMON/BK/KC, KG,KA, KS, KL there must be given the values of the indexes of the model of forces (see par. 1.1). In the given case for the index KC the possible values are 1, 2; for KG --0, 1; the values of the remaining indexes are not used. /18

In the block COMMON/BHC/HC , as a result of the work of the subprogram FC11ØØ is addressed the value HC--elevation of the AES above the surface of the terrestrial ellipsoid.

7. Text

```
SUBROUTINE FC1108(X,V,F)
DIMENSION V(6),F(6)
COMMON/BHC/HC
COMMON/BK/KC,KG,KA,KS,KL
CALL FNGRAV(KC,KG,V,HC,R,F)
CALL RO(HC,P)
CALL FATM(KC,V,P,F)
RETURN
END
```

1.3. Universal subprogram for calculation of right members of equations of motion of AES with the use of various models of forces (F07-FORCE, RODENS, LOGMOD)

1. Designation. Subprogram FORCE is intended for use as a subprogram of right-hand sides for integration of the system of equations of motion of the AES for any models of forces, described in table 1.1. The subprogram LOGMOD for the given model of forces determines the value of the logical parameters, governing the computation of sidereal time, the positions of the moon and sun.

2. Structure. The package of subprograms.

Inputs .. for the users: FORCE, LOGMOD.

Internal inputs RODENS

Utilized external subprograms:

FNGRAV(F01), FATM(F02), FAGRAV(F03), FORSS(F04), FLIGHT(F05), AGIGAC(B06), RO(D01), DENS(D02), SELENA (C01), STT(B05), OCLTLN(B10), SUN(C02), ADEN, AMBAR, GRAV, TLOCAL(D03), DEG2, DEG3, DEG4(E01).

General blocks Общие блоки: /BK/$_5$, /BLOO/$_4$, /BSB/$_1$, /BB/$_1$, /BNM/$_1$, /BRO/$_5$, /BDT/$_1$, /BSO/$_3$, /BDYEAR/$_1$, /BCONGR/$_{506}$, /QKOEF/$_{50}$, /SYEAR/$_{38}$, /CAED/$_1$, /CA00/$_2$, /CA22/$_4$, /CRE/$_1$, /CRZ/$_1$, /CAE/$_2$, /CORL/$_1$, /CGRS/$_2$, /CDSJS/$_1$, /COMZ/$_4$, /COMZP/$_2$, /CAEL/$_2$,

/CGM/₁, /CSDAY/₁, /CT3/₁, /CPI/₃, /CE3/₁, /BEM/₂, /CERO/₁, /CHA/₂₄, /BHC/₁, /BFT/₁₄, /BIECL/₈, /RAD/₂.

3. Conversion CALL FORCE (T,Y,F) /19
4. Initial data: T--independent variable (time), Y_6--main part of the functions sought: X ,Y, Z, V_X, V_Y, V_Z or x, y, z, v_x, v_y, v_z.
5. Results: F_6--main part of functions derived from those sought: $\dot{X}$, $\dot{Y}$, $\dot{Z}$, $\dot{V}_X$, $\dot{V}_Y$, $\dot{V}_Z$ or $\dot{x}$, $\dot{y}$, $\dot{z}$ $\dot{v}_x$, $\dot{v}_y$, $\dot{v}_z$-- right members of the system of differential equations (1.1) or (1.2).
6. Use of the domain COMMON. Before conversion to the subprogram FORCE it is necessary to ensure in the domain COMMON the values of the parameters utilized. In the block /BK/KC, KA, KG, KS, KL it is necessary to address the values of the indexes of the chosen model of forces (see table 1.1). In the block COMMON/BLOG/LSUN, LSUNS, LSEL, LST by means of conversion to the subprogram LOGMOD (described below, in par. 8).it is necessary to address the values of the logical variables, used for regulation of the computation of sidereal time (for LST=TRUE), positions of the moon (for LSEL=TRUE), and sun (for LSUN=TRUE).

The remaining blocks COMMON are used only for the determined values of the indexes of the model of forces, for which in each case there is an indication. For KA greater than or equal to 1 in the bloc. COMMON/BSB/SB it is necessary to address the value of the ballistic coefficient, relative to the scale factor from the block COMMON/CESB/₁ (see (5), table 2.3, No. 12).

For KL greater than 0 (computation of the pressure of light) in the block /BB/B it is necessary to replace the value of the coefficient B₁(see par. 1.6) relative to the scale multiplier from block /CELB/₁ (see (5)), table 2.3, No. 13) For KG=2 or 3 it is necessary in the block COMMON/BNM/₁ to address the value of the number of harmonics considered in the gravitational field of the earth, in the block COMMON/BCONGR/₉₄₆ --values of the coefficients of the breakdown of the gravitational field of the earth (by conversion to the subprogram CONGR(A02) (5)). For KG=4 in the block COMMON/CAØØ/₂ it is necessary to transfer the values of the corresponding variables from the block /CAØØA/₂ (see (5), table 2.1, Nos. 2, 3). /20

For KA greater than 1 or KS greater than 0 or KL greater than 0 inthe block COMMON/BDT/₁ it is necessary to address the value of the date of tying the time in the mode RUD (relative to the Julian date). For KC=1, if KA is greater than 1 or KS is greater than 0 or KL greater than O, and for KC=2, if KG is greater than , in the block COMMON/B3C/ SC, TS, NS it is necessary to address the value SO --sidereal time at midnight Greenwich of the date DT, and TS and NS to set equal to zero.

For KA=2 or 3 it is necessary in the blocks COMMON/SKOEF/$_{50}$ and /SYEAR/$_{30}$ to address the values of the coefficients of the model of the atmosphere and set of numerical correctiens for the semi-annual effect, which is accomplished by conversion to the subprogram VKMA(DO2); in the block /BDYEAR/$_1$ one addres es the date and time in the form of the number of 24-hr. periods from the beginning of the year; in the block COMMON/BRO/F107, FO, AP, D, I one ad res es the values F107 of the intensity of solar radio-radiation F_{107} with computation of the time of retardation (for KA=2 one sets F107 less less than 0.5), FO--of the average level of solar radio-radiation (possible values: 75, 100, 125, 150), AP trihourly index of geomagnetic perturbation with computation of the time lag (for KA=2 set AP less than 0.5), D--parameter, regulating the calculation of the semi-annual effect (for D less than O the semiannual effect is not considered), I--parameter, regulating the calculation of the diurnal effect (CE) (for I<0 CE is considered without the term with the the coefficient C_y, for I=0 CE is not considered, for I greater than 0 CE is considered fully) For KA=4 (atmospher CIRA-72) in the block COMMON/BRO/F107, FO, AKP, D, I it is sufficient to address the values F107--index $F_{10.7}$ (time of retardation 1.17 days), FO--values F_{107}, averaged for four solar rotations, AKP--geomagnetic index K_p, considering 3^-=2.667, 3^o=3.000, 3^+=3.333 and so on (time of retardation --0.279 days), values of D and I without importance.

In the block COMMON BHC/HC/RAD/RC, R1/BIECL/CE, SE/BFT/TM, ST, RS, AS, BS, RL, XS(3), XL(3), GS, GL in the process of operation of the subprogram FORCE one addres es the values HC--elevation of the AES above the surface of the terrestrial ellipsoid, RC, R1--radius- /21 vector of AES and projections of the radius vector of the AES on the plane of the earth's equator (in the subprogram AGRAV); CE, SE--cosine and sine of the angle of inclination of the plane of the earth's equator to the plane of the ecliptic (in the subprogram SUN); TM=T--current time (Moscow); ST--sidereal time; AS, BS--right ascension and declination of the sun; RS, XS(3), RL, XL(3)--are, respectively, for the modules and directed cosines (in the system of coordinates, determined by the index KC) radius vector of the sun and moon; GS, GL--$mu_{\odot} / R_S^2$ and mu_{moon} R_L^2. In the remaining blocks COMMON, enumerated in para. 2, it is necessary to address the values of the constants and sale factors in conjunction with the tables 2.1-2.4 (5) by means of conversion to the subprogram CONST(AO1).

7. The subsidiary subprogram RODENS, intended for switching into the subprogram FORCE of the subprograms of computation ofthe density of the earth's atmosphere according to various models. For KG=2 or 3 one uses the model, realized in the subprogram DENS(DO2), for KG=4 one uses the model CIRA-72, subprogram ADEN (DO3).

Structure. Subprogram RODENS.

External subprograms AGIGAC(B06), GCLTLN(B10), DENS(DO2), ADEN, AMBAR, GRAV, TLOCAL(DO3).
General blocks: $/BK/_5$, $/BDT/_1$, $/BRO/_5$, $/CKOEF/_0$, $/SYEAR/_{38}$, $/BDYEAR/_1$, $/CGM/_1$, $/CT3/_1$, $/CSDAY/_1$, $/CE3/_1$, $/BEM/_2$, $/CERO/_1$.

Conversion: CALL RODENS (T, Y, HC, RC, ST, AS, BS, P).
Initial data: T--time; Y_c--X, Y, Z, V_X, V_Y, V_Z OR x, y, z, v_x, v_y, v_z;RC, HC--modulus of radius vector and elevation of AES above the surface of the terrestrial ellipsoid; ST- sidereal time; AS, BS--right ascension and declination of the sun.

Result: P--density of the atmosphere (in the system of units used for the computation). Concerning the use of blocks COMMON see para. 6.

8. Subprogram LOGMOD, starting from the values of the indexes of the model of forces, given in the block COMMON/BK/KC, KG, KA, KS, KL, addresses in the domain COMMON/BLOG/LSUN, LSUNS, LSEL, LST the values /22 of the logical parameters, which may be used for the regulation of the calculation of sidereal time (LST) and positions of the moon (LSEL) and sun (LSUN). Below are introduced the conditions, under which each of these parameters takes on the value TRUE.

LSUN=TRUE for KG greater than 1 or KL greater than 0 or KA greater than 1;
LSUNS=TRUE for KG greater than 1 or KL greater than 0
LSEL=TRUE for KS=1 or KS=3;
LST=TRUE for KA greater than 1 or KS greater than 0 or KL greater than 0, if KC=1 and for KG greater than 2, if KC =2.

Conversion: CALL LOGMOD.

9. Text

```
      SUBROUTINE FORCE(X,V,F)
      DIMENSION V(6),F(6),VG(3)
      LOGICAL LSUN,LSUNS,LSEL,LST
      COMMON/BFT/T,ST,RS,AS,BS,RL,XS(3),XL(3),GS,GL
      COMMON/BLOG/LSUN,LSUNS,LSEL,LST
      COMMON/BK/KC,KG,KA,KS,KL
      COMMON /CGRS/ GRS,DGRS
      COMMON /BHC/HC
      COMMON/BNM/NMG
      COMMON/CGRL/GRL
      COMMON /BDT/DT
      IF(T.EQ.X)
     -           GOTO 6
      T=X
      IF(LST)
     -      . ST=STT(X)
      IF(.NOT.LSUN)
     -                 GOTO 10
      CALL SUN(DT,X,RS,AS,BS,XS)
      GS=GRS/RS/RS*DGRS
      GOTO(20,10),KC
   20 IF(LSUNS)
     -               CALL AGIGAC(ST,XS,1,XS)
   10 IF(.NOT.LSEL)
     -                    GOTO 6
      CALL SELENA(DT,X,XL,RL)
      GL=GRL/RL/RL
      GOTO(5,6),KC
    5 CALL AGIGAC(ST,XL,1,XL)
    6 CONTINUE
       CALL FNGRAV(KC,KG,V,HC,R,F)
      IF(KG-2) 101,24,1
    1 DO 23 J=1,3
   23 VG(J)=V(J)
      GOTO(24,25),KC
   25 CALL AGIGAC(ST,VG,1,VG)
   24 CALL FAGRAV(V,VG,F,KG,NMG)
  101 IF(KA-1)102,11,12
   12 CALL RODENS(X,V,HC,R,ST,AS,BS,P)
      GOTO 3
   11 CALL RO(HC,P)
    3 CALL FATM(KC,V,P,F)
  102 IF(KL)105,105,13
   13 CALL FLIGHT(KL,V,R,XS,RS,F)
  105 IF(KS-1)104,21,22
   22 CALL FGRSS(GS,V,XS,RS,F)
      GOTO(21,104,21),KS
   21 CALL FGRSS(GL,V,XL,RL,F)
  104 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RODENS(T,V,HC,R,ST,AS,BS,P)
      DIMENSION V(6),RO(6),SU(2),SAT(3),X(3),TEMP(2),ALION(6)
      COMMON/BK/KC,KG,KA,KS,KL
      COMMON /BRO/GEO(3),D,I
      COMMON /BDT/DT
      COMMON/BDYEAR/DTNG
      COMMON/CGM/GM
      COMMON /CT3/T3
      COMMON/CSDAV/SDAV
      COMMON/CE3/E3
      COMMON/BEM/EM,ESEC
      COMMON /CERO/ERO
      DO 1 J=1,3
    1 X(J)=V(J)/R
      GOTO(3,2),KC
    3 CALL AGIGAC(ST,X,2,X)
    2 SU(1)=AS
      SU(2)=BS
      SAT(3)=HC/E3*EM
      IF(KA.EQ.4)
     *                 GOTO 7
    6 IF(D)4,5,5
    5 D=DTNG+T/SDAV
    4 CALL DENS(SAT(3),X(1),X(2),X(3),SU,GEO(3),GEO(1),D,I,RO)
      P=RO(6)*ERO
      GOTO 10
    7 CALLGCLTLN(X,SAT(2),SAT(1))
      AMJD=DT+(T-T3)/SDAV+15019.5
      CALL ADEN(AMJD,SU,SAT,GEO,TEMP,ALION,AMMW,P)
      P=P/GM*ERO
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE LOGMOD
      COMMON/BLOG/LSUN,LSUNS,LSEL,LST
      LOGICAL LSUN,LSUNS,LSEL,LST
      COMMON/BK/KC,KG,KA,KS,KL
      LOGICAL LSUNA,LG,LS,LKG
      COMMON/BFT/T,ST(13)
      T=-100.
      LSUNA=KA.GT.1
      LSUNS=KS.GT.1.OR.KL.GT.U
      LSEL=KS.EQ.1.OR.KS.EQ.3
      LG=KG.GT.2
      LSUN=LSUNA.OR.LSUNS
      LS=LSUN.OR.LSEL
      LKG=KC.EQ.1
      LST=LS.AND.LKG.OR.LG.AND..NOT.LKG
      RETURN
      END
```

Chapter 2. Integration of systems of differential equations (index G). /24

2.1. Integration of a system of differential equations by the method of Adams (GC1-ADAMSP)

1. Designation. In the subprogram the method of Adams is realized with the use of 8 differences, with automatic choice of step for integration of a system of differential equations:

$$\dot{Y}=F(t,Y)$$

with the initial conditions $Y(t_0)=Y_0$, where Y, $\dot{Y}$, Y_0, F-n-dimensional vectors, t--independent variable. The start is produced by the Runge-Kutta method of the fourth order.

2. Structure. Subprogram ADAMSP.
One uses the external subprograms: FRP and BKK--composed by the user.

3. Conversion: CALL ADAMSP (KPI, H, T, Y, FRP, BKK, NKY, MKY, E1, E2, N, AF, F8, F4, T1, Y1, Y2, Y3).

4. Initial data: KPI--variable, ensuring the possibility of integrating with constant pace or with automatic choice of pace (in the first case it is necessary to take KPI=1, in the second KPI=2); H--initial step of integration; T--first value of the independent variable;
Y--unidimensional block of dimension N, containing the initial values of the desired functions;
FRP--name of the subprogram, computing the right members of equations, set up by the user;
BKK--name of the subprogram, prepared by the user, intended for control of the end of integration. In this subprogram there occurs conversion from the subprogram ADAMSP after each step of the integration;
NKY, MKY--number of the first and last function in the block Y, respectively, according to which proceeds the control of the accuracy of the integration with automatic choice of step; /25
E1, E2-blocks of dimension N, containing lower and upper bounds of the permissible errors of the controlled functions;
These values are used: E1(NKYO, E2(NKY), E1(NKY+1), E2(NKY=1)... E1(MKY), E2(MKY);
N=n dimension of the system of equations
AF, Y2, Y3--auxiliary blocks of dimension N;
F8--two-dimensional block of dimension (N,8)
F4--twodimensional block of dimension (N,4).

Note 1. The designations of subprograms FRP and BKK must be written in EXTERNAL subprogram, used for subprogram ADAMSP.

Subprogram FRP must have the form SUBROUTINE FRP(T,Y,F), where T--independent variable
Y--block of current values of the desired functions
F--block of right members, length of blocks Y and F equal to N.

Subprogram BKK: SUBROUTINE BKK(T, Y, F8,IBKK, H),
where T--current value of independent variable
Y--current value of desired functions, block of dimension N, H--step of integration,
F8--block of dimension (N,8), containing values of the right members

in 8 current points of integration.
IBKK--integral variable, which, within BKK, must be ascribed the value 2 for fulfillment of the conditions, according to which is determined the moment of exit from the integration (before the start of integration the subprogram ADAMSI ascribes to IBKK the value 1).

In subprograms IRI and BKK the values T, Y, F, H are established by subprogram ADAMSI in the process of integration.

Note 2. Some times in the starting portion there arises a need to break down the step, not connected with ensuring the prescribed accuracy of integration. Ascribing to index IBKK the value 3 leads to interruption of the process of starting that was begun, disconnecting the influence of KII on the choice of step, return to the starting point and to the start with a halved step. 126

Giving the index KBKK the value 4 leads to a new interruption of the process of integration, establishment of the initial step, reestablishment of the influence of KII on the choice of step, return to the initial point and to the transfer to the standard process of integration.

The subprogram AFSI3(HC4) may serve as an illustration of the application of this possibility see par. 3.4.

5. Results: TI and the block Y_1--values of the independent variable and the desired functions at the final point of the integration.

6. Method. We consider the system (2.1). According to the known values Y_k in the Kth point and the approximate values $Y_{k=1}$ produced at the Kth and 7 preceeding points, one determines by the extrapolation formula of Adams $Y_{k+1}=Y_k+ h_t \sum_{i=0}^{7} \alpha_i F_{k-i}$

where h_t--step of integration.

Interpolation formula of Adams $Y_{k+1}=Y_k+h_t \sum_{i=0}^{7} \beta_i F_{k+1-i}$

allows the obtained values of Y_1 to be made more precise.

The coefficients of the interpolation and extrapolation formulas of Adams have the following values:

α_0 = 3,58995535,
α_1 = -9,52520668,
α_2 = I8,0545387,
α_3 = -22,027753,
α_4 = I7,3796544,
α_5 = -8,6I2I2797,
α_6 = 2,445I6369
α_7 = -0,304224537

β_0 = 0,304224537,
β_1 = I,I56I5906,
β_2 = -I,0069I964,
β_3 = I,0I79646I,
β_4 = -0,732035384,
β_5 = 0,343080357,
β_6 = -0,0938409392,
β_7 = 0,0II8673942.

For automatic choice of step of integration one computes the differences delta $Y_{k+1} = Y^{*}_{k+1} -- Y_{k+1}$.

We give the allowable bounds of error for integration: 0 less than or equal to epsilon$_{1,j}$ less than or equal to epsilon$_{2,j}$. If there are fulfilled the conditions epsilon$_{1,j}$ les than or equal to absolute value of delta $Y_{j,k+1}$ less than or equal to epsilon$_{2,j}$, $j=N_1$, N_1+1, ...N_2, where N_1, N_2--numbers of the first and last controlled functions, then the step of integration h_t does not change.

If the absolute value of delta $Y_{j,k+1}$ is less than epsilon$_{1,j}$ for all $j=N_1,\ldots.N_2$ then there results a doubling of the step. For this integration continues with step h_t until the accumulation of the necessary number of points, for which after an interval of time, the multiple $2h_t$ the known values of the function F, after which the step is doubled.

If the absolute value of delta$Y_{j,k+1}$ is greater than epsilon$_{2,j}$ even if for one value of j: N_1 less than or equal to j less than or equal to N_2, then a breakdown of the step is produced. For this it is necessary to have the values of F in the seven preceding points with the interval $h_t/2$. It is possible to obtain them by means of the interpolation according to the formula of Lagrange for known values of F with step h_t.

We introduce the variable xi = $(t_k--t)/h_t$, then the interpolation formula of Lagrange for the determination of F(t) is written in the form:

$$F(t)=F_{k-7}(\xi^7-21\xi^6+175\xi^5-735\xi^4+1624\xi^3-1764\xi^2+720\xi)/5040 -$$

$$-F_{k-6}(\xi^7-22\xi^6+190\xi^5-820\xi^4+1849\xi^3-2038\xi^2+840\xi)/720 +$$

$$+F_{k-5}(\xi^7-23\xi^6+270\xi^5-925\xi^4+2144\xi^3-2412\xi^2+1008\xi)/240 -$$

$$-F_{k-4}(\xi^7-24\xi^6+226\xi^5-1056\xi^4+2545\xi^3-2952\xi^2+1260\xi)/144 +$$

$$+F_{k-3}(\xi^7-25\xi^6+247\xi^5-1219\xi^4+3112\xi^3-3796\xi^2+1680\xi)/144 -$$

$$-F_{k-2}(\xi^7-26\xi^6+270\xi^5-1420\xi^4+3929\xi^3-5274\xi^2+2520\xi)/240+$$

$$+F_{k-1}(\xi^7-27\xi^6+295\xi^5-1665\xi^4+5104\xi^3-8028\xi^2+5040\xi)/720-$$

$$-F_k(\xi^7-28\xi^6+322\xi^5-1960\xi^4+6769\xi^3-13132\xi^2+13068\xi-5040)/5040.$$

To obtain the values of F in the first 7 points one uses the Runge-Kutta method of the fourth order. /28

$$Y_{k+1} = Y_k + \frac{1}{6}\left(\ell_1 + 2\ell_2 + 2\ell_3 + \ell_4\right),$$

$$\ell_1 = h_t F\left(t_k, Y_k\right),$$

$$\ell_2 = h_t F\left(t_k + \frac{1}{2}h_t,\ Y_k + \frac{1}{2}\ell_1\right),$$

$$\ell_3 = h_t F\left(t_k + \frac{1}{2}h_t,\ Y_k + \frac{1}{2}\ell_2\right),$$

$$\ell = h_t F\left(t_k + h_t,\ Y_k + \ell_3\right).$$

Literature: (2), p. 390.

7. Text.

```
      SUBROUTINE ADAMSP(KP1,H,X,V,PRP,BKK,NKU,MAU,E1,E2,
     *                  N,AF,F,F1,X1,V1,V2,V3)
      DIMENSION V(N),AF(N),F(N,8),F1(N,4),E1(N),E2(N),
     *          V1(N),V2(N),V3(N),A(8),B(8),R(8)
      HN=H
 1002 IA=KP1
      H=HN
 1004 IBKK=1
      DO 1001 J=1,8
      DO 1001 I=1,N
 1001 F(I,J)=0
C RUNGE-KUTTA
  101 Z=X+6.7*H
  100 DO 2 J=1,N
    2 V1(J)=V(J)
      X1=X
      A(1)=.5*H
      A(2)=A(1)
      A(3)=H
      A(4)=H
      A(5)=A(1)
    3 X2=X1
      DO4 J=1,N
      V2(J)=V1(J)
    4 V3(J)=V1(J)
      DO 15 I=1,4
      CALL PRP(X1,V2,AF)
      IF(I-1)6,6,7
    6 GOTO(10,7,7),IA
   10 DO 12 J=1,N
   12 F(J,8)=AF(J)
      IF(IA-1)8,5,8
    5 CALL BKK(X1,V1,F,IBKK,N)
      GOTO(8,1000,1003,1002),IBKK
 1003 IA=1
      H=H*.5
      GOTO 1004
    8 DO 9 K=1,7
      DO 9 J=1,N
```

```
    9 F(J,K)=F(J,K+1)
    7 X1=X2+A(I)
      DO 15 J=1,N
      V1(J)=V1(J)+A(I+1)*AF(J)/3.
   15 V2(J)=V3(J)+A(I)*AF(J)
      IF((X1-Z)*SIGN(1.,H))3,16,16
   16 GOTO(102,18,19),IA
   18 IA=IA+1
      DO 20 J=1,N
   20 F1(J,1)=V1(J)
      H=H*.5
      GOTO 100
   19 DO 17 J=NKV,MKV
      IF (ABS(V1(J)-F1(J,1))-E2(J))17,17,21
   17 CONTINUE
      H=H*2.
      IA=1
      GOTO 101
   21 IA=IA-1
      GOTO 101
  102 IA=0
C ADAMS
  103 A(1)=-.304224537*H
      A(2)=2.44516369*H
      A(3)=-8.61212797*H
      A(4)=17.3796544*H
      A(5)=-22.027753*H
      A(6)=18.0545387*H
      A(7)=-9.52520668*H
      A(8)=3.58995535*H
      B(1)=.0113673942*H
      B(2)=-.0938409392*H
      B(3)=.343080357*H
      B(4)=-.732035384*H
      B(5)=1.01796461*H
      B(6)=-1.00691964*H
      B(7)=1.15615906*H
      B(8)=.304224537*H
  104 CALL PRP(X1,V1,AF)
      DO 23 J=1,N
      V3(J)=V1(J)
      V2(J)=V1(J)
   23 F(J,8)=AF(J)
      CALL BKK(X1,V1,F,IBKK,H)
      GOTO(105,1000,1003,1002),IBKK
  105 IF(IA-7)118,34,118
   34 H=H*2.
      DO 35 K=1,3
      L=8-K
      M=L-K
      DO 35 J=1,N
   35 F(J,L)=F(J,M)
      DO 36 K=1,4
      DO 36 J=1,N
   36 F(J,K)=F1(J,K)
      DO 50 J=1,8
      A(J)=A(J)*2.
   50 B(J)=B(J)*2.
  119 IA=0
  118 X1=X1+H
      DO 24 K=1,8
```

```
      DO 24 J=1,N
   24 V2(J)=V2(J)+A(K)*F(J,K)
      CALL PRP(X1,V2,AF)
      DO 26 K=1,7
      DO 26 J=1,N
   26 V1(J)=V1(J)+B(K)*F(J,K+1)
      DO 25 J=1,N
   25 V1(J)=V1(J)+B(8)*AF(J)
      GOTO(106,107),KPI
  107 DO 28 J=NKV,MKV
      IF(ABS(V1(J)-V2(J))-E2(J))28,28,108
   28 CONTINUE
      DO 29 J=NKV,MKV
      IF(ABS(V1(J)-V2(J))-E1(J))29,29,116
  116 IA=0
      GOTO 106
   29 CONTINUE
C H=H*2
      IA=IA+1
      IF(IA-1)31,31,106
   31 DO 33 K=1,4
      L=2*K-1
      DO 33 J=1,N
   33 F1(J,K)=F(J,L)
C H=H
  106 DO 30 K=1,7
      DO 30 J=1,N
   30 F(J,K)=F(J,K+1)
      GOTO 104
C H=H/2
  108 X1=X1-H
      Z=3.5
      DO 37 K=1,4
      R(8)=-((((((Z-28.)*Z+322.)*Z-1960.)*Z+6769.)*Z
     *     -13132.)*Z+13068.)/5040.*Z+1.
      R(7)=((((((Z-27.)*Z+295.)*Z-1665.)*Z+5104.)*Z
     *     -8028.)*Z+5040.)/720.*Z
      R(6)=-((((((Z-26.)*Z+270.)*Z-1420.)*Z+3929.)*Z
     *     -5274.)*Z+2520.)/240.*Z
      R(5)=((((((Z-25.)*Z+247.)*Z-1219.)*Z+3112.)*Z
     *     -3796.)*Z+1680.)/144.*Z
      R(4)=-((((((Z-24.)*Z+226.)*Z-1036.)*Z+2545.)*Z
     *     -2952.)*Z+1260.)/144.*Z
      R(3)=((((((Z-23.)*Z+207.)*Z-925.)*Z+2144.)*Z
     *     -2412.)*Z+1008.)/240.*Z
      R(2)=-((((((Z-22.)*Z+190.)*Z-820.)*Z+1849.)*Z
     *     -2038.)*Z+840.)/720.*Z
      R(1)=((((((Z-21.)*Z+175)*Z-735.)*Z+1624.)*Z
     *     -1764.)*Z+720.)/5040.*Z
      DO 38 J=1,N
      F1(J,K)=0
      DO 38L=1,8
   38 F1(J,K)=F1(J,K)+R(L)*F(J,L)
   37 Z=Z-1.
      DO 39 K=5,7
      L=K+K-8
      DO 39 J=1,N
   39 F(J,L)=F(J,K)
      DO 40 K=1,4
      L=2*K-1
   40 F(J,L)=F1(J,K)
      DO 41 J=1,N
      V2(J)=V3(J)
   41 V1(J)=V3(J)
      DO 51 J=1,8
      A(J)=A(J)*.5
   51 B(J)=B(J)*.5
      H=H*.5
      GOTO 119
 1000 RETURN
      END
```

2.2. Interpolation according to Adams (GO2-ADINT). /31

1. Designation. The subprogram allows in the process of integration by the method of Adams of the system of differential equations $\dot{Y}=F(t,Y)$, where Y, $\dot{Y}$, F--N-dimensional vectors, the computation of the values of the desired functions at the moments of time t_{nu}, not multiples for the step of integration.

2. Structure. Subprogram ADINT.

3. Conversion: CALL ADINT (Z, F8, Y, YR, M, H, N).

4. Initial data:

Z--value of the variable $xi=(t_k--t_{nu})/h_t$, where t_{nu}--given moment of time, h_t--step of integration, t_k--current value of the variable of integration, corresponding to the condition: t_k--h_t less than t_{nu} less than t_k;

F8--block of dimension (N, 8), containing those produced from the desired functions in the eight last points: t_{k-7}, t_{k-6}, ...t_k;

Y--block of dimension N, containing the values of the desired functions at the point t_k;

M--number of interpolated functions (M less than or =N);

H--step of integration (h_t);

N--order of the system of equations.

5. Results: YR--block of dimension N, containing $y_j(t_{nu})$, j=1,...,M.

6. Algorithm: We introduce the variable $xi=(t_k--t_{nu})/h_t$, (t_{nu}, h_t and t_k determined above).

The value of the desired functions at the point t_{nu} is computed by the formula $y_j(t_{nu})=y_j(t_k)--h_t \text{ sigma}_{i=k-7}^{k} f_{i,j} \text{ psi}_i/120960$,

where j=1,..., M less than or =N; $f_{i,j}$ --values of the right members on the points t_i (i=k-7,...,k); $y_j(t_k)$--values of the desired functions at the point t_k;

$$\psi_{k-7} = (3\xi^8 - 72\xi^7 + 700\xi^6 - 3528\xi^5 + 9744\xi^4 - 14112\xi^3 + 8640\xi^2),$$

$$\psi_{k-6} = -(21\xi^8 - 528\xi^7 + 5320\xi^6 - 27552\xi^5 + 77658\xi^4 - 114128\xi^3 + 70560\xi^2),$$

$$\psi_{k-5} = (63\xi^8 - 1656\xi^7 + 17388\xi^6 - 93240\xi^5 + 270144\xi^4 - 405216\xi^3 + 254016\xi^2),$$

$$\psi_{k-4} = -(105\xi^8 - 2880\xi^7 + 31640\xi^6 - 177408\xi^5 + 534450\xi^4 - 826560\xi^3 + 529200\xi^2),$$

$$\psi_{k-3} = (105\xi^8 - 3000\xi^7 + 34580\xi^6 - 204792\xi^5 + 653520\xi^4 - 1062880\xi^3 + 705600\xi^2),$$

$$\psi_{k-2} = -(63\xi^8 - 1872\xi^7 + 22680\xi^6 - 143136\xi^5 + 495054\xi^4 - 886032\xi^3 + 635040\xi^2),$$

$$\psi_{k-1} = (21\xi^8 - 648\xi^7 + 8260\xi^6 - 55944\xi^5 + 214368\xi^4 - 449568\xi^3 + 423360\xi^2),$$

$$\psi_{k} = -(3\xi^8 - 96\xi^7 + 1288\xi^6 - 9408\xi^5 + 40614\xi^4 - 105056\xi^3 + 156816\xi^2 - 120960\xi).$$

Литература: [2] .

```
      SUBROUTINE ADINT(Z,F,V,VR,M,H,N)
      DIMENSION F(N,8),V(N),VR(N),R(8)
      R(1)=((((((3.*Z-72.)*Z+700.)*Z-3528.)*Z
     +     +9744.)*Z-14112.)*Z+8640.)*Z
      R(2)=((((((-21.*Z+528.)*Z-5320.)*Z+27552.)*Z
     -     -77658.)*Z+114128.)*Z-70560.)*Z
      R(3)=((((((63.*Z-1656.)*Z+17388.)*Z-93240.)*Z
     *     +270144.)*Z-405216.)*Z+254016.)*Z
      R(4)=((((((-105.*Z+2880.)*Z-31640.)*Z+177408.)*Z
     -     -534450.)*Z+826560.)*Z-529200.)*Z
      R(5)=((((((105.*Z-3000.)*Z+34580.)*Z-204792.)*Z
     +     +653520.)*Z-1062880.)*Z+705600.)*Z
      R(6)=((((((-63.*Z+1872.)*Z-22680.)*Z+143136.)*Z
     -     -495054.)*Z+886032.)*Z-635040.)*Z
      R(7)=((((((21.*Z-648.)*Z+8260.)*Z-55944.)*Z
     +     +214368.)*Z-449568.)*Z+423360.)*Z
      R(8)=((((((-3.*Z+96.)*Z-1288.)*Z+9408.)*Z-40614.)*Z
     +     +105056.)*Z-156816.)*Z+120960.
      DO 1 J=1,M
    1 VR(J)=V(J)
      DO 2 K=1,8
      R(K)=R(K)*H*Z/120960.
      DO 2 J=1,M
    2 VR(J)=VR(J)-F(J,K)*R(K)
      RETURN
      END
```

Chapter 3. SUBPROGRAMS, ENSURING IN THE PROCESS OF INTEGRATION OF THE SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS THE ATTAINING OF THE PRESCRIBED VALUES OF THE FUNCTIONS FROM THE SOLUTION (INDEX H)* /33

3.1. Arriving at the prescribed value of an arbitrary function from the solution (HOI-REACH) in the process of integration.

1. Designation. In the process of integration of the system of differential equations (2.1) the subprogram REACH allows one to determine the value of the independent variable TR, corresponding to reaching the given value FR1 of an arbitrary continuous function phi(T)=FREACH(T,Y), Y--N-dimensional vector.

2. Structure. Subprogram REACH. Utilized external subprograms: ADINT(GO2), subprogram--function, FREACH (composed by the user).

3. Conversion:
CALL REACH (T, Y, F8, H, E, FR, FR1,FRK, TR, FREACH, N, YR).

4. Initial data: T--independent variable;
Y_N--block of current values of the vector Y;

F8--two-dimensional block (N, 8)--values of right members in 8 points with step H;

H--value of the step of integration;

FREACH--designation of the subprogram--function, computed

phi(T)--
FR--current value of the function FREACH;

*For the subprograms of this chapter the conversion must take place in the process of integration of the system of differential equations, i.e. from the subprograms BKK, to which is transferred the regulation from the subprogram ADAMSF (see par. 2.1) after each step of integration.

FRK--value of FREACH at the previous step of integration (at the point T--H); /34
FR1--given value of the function FREACH;

N- order of system of equations;
E--allowable relative error, ensuring the choice of value of the independent variable TR, satisfying the relationship

$$\left| \frac{FR1 - FREACH(TR, YR)}{\Delta F} \right| \leq E$$

where delta F--increment in the function FREACH from the step of integration.

Note 1. Subprogram --function FREACH, composed by the user, must have the form: FUNCTION FREACH(T,Y), where T--independent variable;
Y_N--block of current values of the vector Y.

Note 2. Function FREACH must be described in the EXTERNAL subprogram, using the subprogram REACH.

Note 3. Before use of subprogram REACH it is necessary to find that moment of time in the process of integration, for which is satisfiend one of the relationships
FRK less than or =FR1 less than or =FR or FRK greater then or = FR1 greater than or =FR.

5. Results: TR- value of independent variable, corresponding to reaching the function FREACH of the value FR1;
YR_N--block of values of the desired functions Y at the point TR.

6. Algorithm. Let us consider, in the process of integration of the system of equations (2.1) a continuous function phi(T)=FREACH (T,YO, where Y--N-dimensional vector, taking on the values FR at the current moment of time T, FRK--at the moment of time T_k=T--H,
where H--step of integration.

It is necessary to find the moment of time, for which the function phi(T) takes on the value FR1. /35

Under the condition, that there is fulfilled one of the inequalities RK les than or equal to FR1 less than or equal to FR or FRK greater than or =FR1 greater than or =FR, we determine F=FR--FRK, xi_1=(FR--FR1)/delta F.

We set some epsilon greater than 0. If the absolute value of xi_1 is less than or =epsilon, that moment of time TR=T--xi_1H gives the solution of the problem proposed. In the opposite case, using the subprogram of interpolation by Adams' method (par. 2.2), we determine the value YR at the moment of time TR and the value P=phi (TR)=FREACH(TR, YR).

For the new iteration we determine xi_2+ (P-FR1)/delta F, we verify the condition absolute value of xi_2 less than or =epsilon. If this condition is satisfied, then TR=T--xi_1H--xi_2H gives the solution to the proposed problem, otherwise we go on to a new iteration and so on.

7. Text.

```
      SUBROUTINE REACH(T,V,F,H,E,FR,FR1,FRK,TR,FREACH,N,VR)
      DIMENSION V(N),F(N,8),VR(N)
      TR=T
      IF(ABS(F(1,1)+F(2,1)+F(3,1))-1.E-15)1,1,4
    4 DP=FR-FR1
      DZ=FR-FRK
      W=0.
    3 W1=DP/DZ
      W=W+W1
      TR=T-W*H
      IF(ABS(W1)-E)1,1,2
    2 CALL ADINT(W,F,V,VR,H,H,N)
      P=FREACH(TR,VR)
      DP=P-FR1
      GOTO 3
    1 RETURN
      END
```

3.2 Determination, in the process of integration, of the minimum and maximum of an arbitrary continuous function from the solution as functions of an independent variable (HO2-EXTRM, PARAB) /36

1. Designation. Subprogram EXTRM determines in the process of integration of the system of equations (2.1) the minimum and maximum of an arbitrary continuous function phi(T)=FREACH(T,Y), where Y--N-dimensional vector, and the value of the independent variable, corresponding to the minimal and maximal values of this function.
2. Structure. Subprogram ESTRM.
Internal inputs: PARAB.
Utilized external subprograms: ADINT(GO2), FREACH (composed by the user).
General blocks: $/BMIN/_2$, $/BFC/_1$, $/BSM/_1$.
3. Conversion:
CALL EXTRM (T,Y,F,H,P1,P2, FMIN,TMIN,FMAX,TMAX, FREACH, N, YR)

4. Initial data: T--independent variable;
Y_N--block of current values of vector Y;
H--step of integration;
F--twodimensional block (N, 8), containing the values of the right-hand sides at 8 points with step H;
P1, P2--permissible relative errors, ensuring the choice of extremum, satisfying the relationships

$$\left|\frac{FIE-\varphi(TE)}{\Delta F}\right|<P1 \quad \text{OR} \quad \left|\frac{TIE-TE}{H}\right|<P2,$$

where FIE, TIE--value of the function and of the argument corresponding to the actual extremum; phi(TE, TE--approximate values of the extremum of the function and the corresponding argument; deltaF--increment in the function phi(T) for the step of integration;
N--order of the system of equations; /37
FREACH--designation of the subprogram--function, computed phi(T). This subprogram is composed by the user and has the following construction:
FUNCTION FREACH (T,Y)
where T--independent variable;
Y_N--block of values of the vector Y.

Note. The name of the subprogram--function FREACH must be described in EXTERNAL subprogram, used for subprogram EXTRM.

5. Results: FMIN--last local minimum of function FREACH for the interval of time from the beginning of integration to the current moment; TMIN--moment of time, corresponding to the value FREACH=FMIN; FMAX--last local maximum of function FREACH in the same time interval, TMAX--moment of time, corresponding to the value FREACH=FMAX; YR_N--block of values of vector Y at the point of the extremum.

6. Use of the domain COMMON: After the beginning of integration

in the block COMMON/BMIN/H1, H2 one must substitute these values: H1=0, H2=0.

In the block COMMON/BRC/$_1$ at each step of integration it is necessary to address the current value of the function FREACH.

In the block COMMON/BSM/$_1$ the subprogram EXTRM addresses the value of the step of integration.

7. Algorithm.

Let HC, H2, H1 be the values of the function FREACH at three consecutive moments of time t_i: $t_1=t$, $t_2=t-h$, $t_3=t-h-h_r$ where h is the step of integration, $h_1=h$ for integration with constant step (for integration with automatic choice of step h_1 may be different from h).

If the inequality (H2-H1)(HC-H2) less than 0 is satisfied, then the extremum (for H2-H1 greater than 0--maximum, for H2-H1 less than 0--minimum) is found in the time interval (t_3, t_1). We will term such a situation extremal. It remains to determine more precisely the moment of time, corresponding to the extremum, and the value of the extremum. /38

We proceed from the moments of time t_i to the moments tau_i $=(t-t_i)/h$; then $tau_1=0$, $tau_2=1$, $tau_3=1+h_1/h$ correspond to the values of the function HC, H2, H1.

For convenience in further discussions, we shall rename the moments of time and the corresponding values of the function, starting from the following considerations.

IF $|HC-H2|<|H2-H1|$ THEN , $B_1=H2$, $x_1=\tau_2=1$;

$B_2=HC$, $x_2=\tau_1=0$; $B_3=H1$, $x_3=1+h_1/h$.

IF $|HC-H2|>|H2-H1|$ THEN , $B_1=H2$, $x_1=\tau_2=1$;

$B_2=H1$, $x_2=\tau_3=1+h_1/h$; $B_3=HC$, $x_3=\tau_1=0$.

Thus, we have 3 values of the function B_i for three values of the arguments x_i. We construct a parabola, passing through these points, and find the value of W, the argument of the extremum guaranteed by this parabola.

Turning to the subprogram of interpolation following Adams (par. 2.2) with the obtained value of W, we determine the value YR of the vector Y at the moment in time TR=t-Wh, then the value HR of the function FREACH at this moment intime.

If the absolute value of$(HR-B_1)$/delta F is less than $epsilon_1$, or the absolute value of $W-x_1$ is less than $epsilon_2$, where $epsilon_1$, $epsilon_2$--given values of the permissible errors, deltaF =HC-H2, then the problem is solved, otherwise we go on over to a new iteration, using the values $B_3=B_2$, $B_2=B_1$, $B_1=HR$ at the moments in time $x_3=x_2$, $x_2=x_1$, $x_1=W$ and so forth.

8. Text.


```
      SUBROUTINE EXTRM(T,V,F,H,P1,P2,FMIN,TMIN,FMAX,TMAX,
     *                 FREACH,N,VR)
      DIMENSION V(N),F(N,8),VR(N)
      DIMENSION X(3),B(3)
      COMMON /BSM/S2
      COMMON /BMIN/H1,H2
      COMMON /BFC/HC
      IF(H2)1,2,1
    1 IF(H1)3,5,3
    3 W1=H2-H1
      W2=HC-H2
      IF(W1*W2)4,5,5
    4 IF(F(1,1)+F(2,1)+F(3,1))30,5,30
   30 K=SIGN(1.,W1)
      B(1)=H2
      X(1)=1.
      X(3)=1.+S2/H
      IF((HC-H1)*K)9,8,8
    8 B(2)=HC
      X(2)=0.
      B(3)=H1
      GOTO 10
    9 B(2)=H1
      X(2)=X(3)
      B(3)=HC
      X(3)=0
   10 CALL PARAB(X,B,XM)
      W=XM
      CALL ACINT(XM,F,V,VR,N,H,N)
      TR=T-W*H
      HR=FREACH(TR,VR)
      IF((HR-B(1))*K)25,21,20
   20 IF(ABS((HR-B(1))/W2)-P1)21,21,23
   23 IF(ABS(XM-X(1))-P2)21,21,24
   24 DO 26 J=1,2
      L=4-J
      X(L)=X(L-1)
   26 B(L)=B(L-1)
      B(1)=HR
      X(1)=XM
      GOTO 10
   25 HR=B(1)
   21 IF(K)32,33,33
   32 FMIN=HR
      TMIN=TR
      GOTO 5
   33 FMAX=HR
      TMAX=TR
    5 H1=H2
    2 H2=HC
      S2=H
      RETURN
      END
```

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9. Tho subsidiary subprogram FARAB, intended for the determination of the extremum of the parabola, passing through three points: phi(x_1), phi(x_2), phi(x_3) of some function phi(x). /40

Conversion: CALL FARAB (X, B, XM).

Initial data: X_3--block of values of the argument, B_3--block of values of the function.

Results: XM--value of the argument, giveing the extremum of the parabola, passing through the given 3 points.

Algorithm. Let the values B_i of some function phi(x) be known for three values of the argument x_i: B_i==phi(x_i).

It is neccesary to determine XM--the value of the orgument, giving the extremum of the parabola, passing through the indicated values B_i.

$$XM = \frac{1}{2}\,\frac{(B_2-B_1)(x_3^2-x_1^2)-(B_3-B_1)(x_2^2-x_1^2)}{(B_2-B_1)(x_3-x_1)-(B_3-B_1)(x_2-x_1)}\,.$$

Tekct.

```
      SUBROUTINE PARAB(X,B,XM)
      DIMENSION X(3),B(3),W(2),T(2),T2(2)
      V=X(1)*X(1)
      DO 1 J=2,3
      W(J-1)=B(J)-B(1)
      T(J-1)=X(J)-X(1)
    1 T2(J-1)=X(J)*X(J)-V
      XM=.5*(W(1)*T2(2)-W(2)*T2(1))/(W(1)*T(2)-W(2)*T(1))
      RETURN
      END
```

3.3. Exit of the AES at the ascending node of the orbit (NO3-NODE) /41

1. Designation. The subprogram permits one to determine in the process of integration of the systems of equations (1.1) or (1.2) the moment of time, corresponding to the AES crossing the ascending node, and also the coordinates and components of the velocity vector of the AES at this moment in time.

2. Structure. Subprogram NODE.
Utilized external subprograms : ADINT(300)
General domain: /BOK/$_2$, /BIN/$_1$.

3. Conversion: CALL NODE(T,Y,H,F8, TV, YV, NB).

4. Initial data: T- time;
Y--block of current values X, Y, Z, V_X, V_Y, V_Z ;

H--step of integration;
F8--two-dimensional block of dimension (6,8), containing the values of $\dot{X}$, $\dot{Y}$, $\dot{Z}$, $\dot{V}_X$, $\dot{V}_Y$, $\dot{V}_Z$ produced at 8 points: T-7H, ..., T-H, T;

NB--loop number;

5. Results: If TV is greater than O, then TV, YV_6--moment of time and block of values X_V, Y_V, Z_V, V_{X_V}, V_{Y_V}, V_{Z_V} corresponding to the AES crossing the ascending node.

Note. If the condition of exit of AES at the ascending node is fulfilled at the take-off stage by the method of Runge-Kutta (i.e. initial conditions are given in the vicinity of the ascending node), then the procedure of determining more precisely the moment of exit at the node is disconnected, and the loop number (NB) is increased by 1.

6. Use of the domain COMMON.
Before the beginning of integration in the block COMMON/BOK/ZK, ZKN it is necessary to address the value Z(t_0) to the place ZK and ZKN; in the block COMMON/BIN/INOD one sets INOD=1.

7. Algorithm.
In the process of integration of equations of motion of the AES one determines the moment of time t, for which is fulfilled the condition: Z(t-h) Z(t) less than O, Z(t) greater than O.

Starting from the value of the parameter xi$_1$=Z(t)/delta Z, where delta Z=Z(t)--Z(t-h), using the subprogram of interpolation according to Adams (par. 2.2), we determine the values X_V, Y_V, Z_V /42

V_{X_V}, V_{Y_V}, V_{Z_V} at the moment of time t_{nu} =t-xih, where h--step of integration.

If the absolute value of Z_V/delta Z is less than epsilon as given, then t_V, X_V, Y_V, Z_V, V_{X_V}, V_{Y_V}, V_{Z_V} give the solution to the proposed problem. If the condition is not fulfilled, then a new iteration is carried out, starting from the value xi$_2$=xi$_1$ +

Z_V/delta Z and so forth.

Note. The chosen value epsilon=10^{-7} (variable EN, value of which is given in the operator DATA) ensures exit at the ascending node with an error, not exceeding 0.01 m. for the AES with apogee on the order of 500 km.

6. Text.

```
      SUBROUTINE NODE(X,V,H,F,TV,VV,NB)
      DIMENSION V(6),VV(6),F(6,8)
      DATA EN/1.E-7/
      COMMON/BZK/ZK,ZKN
      COMMON/BIN/INOD
       TV=-100.
      K=SIGN(1.,H)
      W=V(3)*ZK
   20 IF(W)1,3,3
    1 IF(ABS(F(1,1)+F(2,1)+F(3,1))-1E-16)10,10,8
   10 GOTO (16,3),INOD
   16 INOD=2
      NB=NB+1
      GOTO 3
    8 IF(V(3)*K)3,7,7
    7 DZ=V(3)-ZK
      W=V(3)/DZ
    9 CALL ADINT(W,F,V,VV,6,H,6)
      W1=VV(3)/DZ
      W=W+W1
      IF(ABS(W1)-EN)11,11,9
   11 TV=X-W*H
    3 ZK=V(3)
      RETURN
      END
```

5.4. Determination in the process of integration of the minimal and maximal elevation of the AES above the surface of the terrestrial ellipsoid (H04--APSIS). /43

1. Purpose. The subprogram APSIS allows the determination of the minimal and maximal value of the ele vation of the AES above the surface of the terrestrial ellipsoid in the process of integration of the equations of motion of the AES (1.1) or (1.2).

2. Structure. Subprogram APSIS.

External subprograms employed: HEIGHT(B10) ADINT(G02), PARAB(H02).

Common blocks: /BZK/$_2$, /BAIS/$_3$, /BNC/$_1$, /BS2/$_1$.

3. Conversion: CALL AFSIS (T, Y, H, F8, HMIN, HMAX, IBKK).

4. Initial data: T--time;
Y--block of current values of X, Y, Z, V_X, V_Y, V_Z ;
H--step of integration;
F8--two-dimensional block (6, 8), containing the values of $\dot{X}$, $\dot{Y}$, $\dot{Z}$, $\dot{V}_X$, $\dot{V}_Y$, $\dot{V}_Z$ derived at the moments of time: T-7H, T-6H, ...,T;
BKK--variable, controlling the exit from the integration by the subprogram ADAMSP (par. 2.1)

5. Results:
HMIN, HMAX--minimal and maximal values of the elevation of the **AES** above the surface of the earth's ellipsoid in the time interval from the beginning of integration (or from the start of the loop) to the current moment of time.

Remark 1. In order that HMIN, HMAX be determined as minimal and maximal values of the elevations not on the whole interval of integration, but on each loop separately, it is necessary to address the values HC--of the current altitude of **AES**--at the moments of exit of the **AES** at the orbital node for HMIN, HMAX.

Remark 2. If the extremal situation arises in the starting portion, then the variable IBKK receives the value 3.

In this case the program of integration ADAMSP automatically reduces the step in half, switches out the inflnence of KPI on the choice of step and begins the start from the initial point. After HMIN or HMAX is found, IBKK receives the value 4, the initial step of integration and the influcence of KPI on choice of step are automatically renewed, and a new start will begin from the initial point and the integration enters the standard regime.

6. Use of the domain COMMON. Before the beginning of integration in the blocks COMMON/BZK/ZK, ZKN/BAIS/1HM, H1, H2 it is necessary to give the value $Z(t_0)$ to the variables ZK, ZKN, to set 1HM=1, HI=0, H2=0.

In the block COMMON/BHC/HC at each step of integration it is necessary to address the value HC, current elevations of **AES** above the surface of the earth.

In the block COMMON/BS2/$_1$, the subprogram AFSIS addresses the value of the step of integration. In the block COMMON/BTAIS/ TMIN/TMAX are addressed the moments of time, corresponding to HMIN, HMAX.

7. The Algorithm is completely analogous to the algorithm, used in the subprogram EXTRM(HC2), only in place of the function FREACH one uses the altitude of **AES** above the surface of the earth's ellipsoid.

For the permissible errors are chosen the values epsilon_1= 0.0001, epsilon_2=0.01 (variables P1, P2, values are given to these by the operator DATA).

8. Text.

```
      SUBROUTINE APSIS(T,V,H,F,HMIN,HMAX,IBKK)
      DIMENSION X(3),B(3),VR(6),F(6,8)
      DIMENSION V(6)
      DATA P1,P2/.0001,.01/
      COMMON/BS2/S2
      COMMON /BAPS/IHM,H1,H2
      COMMON/BZK/ZK,ZKN
      COMMON/BHC/HC
      COMMON/BTAPS/TMIN,TMAX
      IF(H2)1,11,1
   11 HMIN=HC
      HMAX=HC
      GOTO 2
    1 IF(H1)3,5,3
    3 W1=H2-H1
      W2=HC-H2
      IF(W1*W2)4,5,5
    4 IF(F(1,1)*F(2,1)*F(3,1))30,31,30
   31 GOTO(51,51,5),IHM
   51 IHM=2
      IBKK=3
   16 H1=0
      H2=0
      ZK=ZKN
      RETURN
   30 K=SIGN(1.,W1)
      B(1)=H2
      X(1)=1.
      X(3)=1.+S2/H
      IF((HC-H1)*K)9,8,8
    8 B(2)=HC
      X(2)=0.
      B(3)=H1
      GOTO 10
    9 B(2)=H1
      X(2)=X(3)
      B(3)=HC
      X(3)=0
   10 CALL PARAB(X,B,XM)
      CALL APINT(XM,F,V,VR,6,H,6)
      CALL HEIGHT(VR,HR)
      IF((HR-B(1))*K)25,21,20
   20 IF(ABS((HR-B(1))/W2)-P1)21,21,23
   23 IF(ABS(XM-X(1))-P2)21,21,24
   24 DO 26 J=1,2
      L=4-J
      X(L)=X(L-1)
   26 B(L)=B(L-1)
      B(1)=HR
      X(1)=XM
      GOTO 10
   25 HR=B(1)
   21 IF(K)27,28,28
   27 IF(HR-HMIN)32,29,29
   32 HMIN=HR
      TMIN=T-XM*H
      GOTO 29
   28 IF(HR-HMAX)29,29,33
   33 HMAX=HR
      TMAX=T-XM*H
   29 GOTO(5,53,5),IHM
   53 IHM=3
      IBKK=4
      GOTO 16
    5 H1=H2
    2 H2=HC
      S2=H
      RETURN
      END
```

CHAPTER 4. SUBPROGRAMS FOR COMPUTATION OF THE VALUES OF DIFFERENT FUNCTIONS FROM THE PARAMETERS OF MOTION OF THE AES (index 1).

4.1. Right ascension of the AES, local time and zenith distance of the sun (IO1--RIGHTA, TLOC, ZDSUNL, ZDSUNC).

1. Purpose. The subprogram TLOC for known geographical longitude (lambda) of AES and sidereal time (ST) determines the right ascension of the AES.

The subprogram TLOC for known geographical longitude of the point (lambda), sidereal time (ST), and right ascension of the sun ($alpha_{\odot}$) determines local time.

The subprogram ZDSUNL for known geographical latitude (phi) and longitude (lambda) of a point and for the directed cosine of the radius vector of the Sun ($x^o_{\odot}$, $y^o_{\odot}$, $z^o_{\odot}$) in the Greenwich system of coordinates determines the zenith distance of the sun.

The subprogram ZDSUNC determines the zenith distance of the sun for known directed cosines of the normal to the surface of the earth's ellipsoid (x^o_N, y^o_N, z^o_N) and for directed cosines of the radius-vector of the sun ($x^o_{\odot}$, $y^o_{\odot}$, $z^o_{\odot}$) in the Greenwich system of coordinates.

2. Structure. The package of independent subprogram-functions: RIGHTA, TLOC, ZDSUNL, ZDSUNC.
Common domain: $/CPI/_3$, $/CDEGR/_1$, $/CHRAD/_1$.

Remark 1. Subprogram--function ZDSUNC refers to the subprogram-function ZDSUNL.

3. Conversion: A=RIGHTA (ST, ALN),
TL=TLOC (ST, ALN, AS),
ZS=ZDSUNL (ALT, ALN, XSG),
ZS=ZDSUNC (XNG, XSG).

4. Initial data: ST--sidereal time in radians; ALT, ALN--geographical latitude and longitude of the point in radians; AS--right ascension of the sun in radians.

Block XNG_3 --directed cosines of the normal to the surface of the earth's ellipsoid at the point underneath the satellite;

Block XSG_3--directed cosines of the radius vector of the sun.

5. Results: A--right ascension of the AES in degrees; TL--local time in hours; ZS--zenith distance of the sun in degrees.

6. Use of the domain COMMON. The constants from these blocks are used: COMMON$/CPI/_3$, $/CDEGR/_1$, $/CHRAD/_1$ (see 950, No. 1, 2, 3 table 2.3).

7. Algorithm. Right ascension of the AES--lambda*, local time --$t_{\odot}$ and the zenith distance of the sun --$ZS_{\odot}$ are determined according to the following formulas:

$$\lambda^{s} = \lambda + ST ,$$
$$t_{\odot} = \lambda^{s} - \alpha_{\odot} + \pi ,$$
$$\cos ZS_{\odot} = x_N^{\circ} x_{\odot}^{\circ} + y_N^{\circ} y_{\odot}^{\circ} + z_N^{\circ} z_{\odot}^{\circ} .$$

8. Texts. /47

```
      FUNCTION  RIGHTA(ST,ALN)
      COMMON /CDEGR/DEGR
      COMMON /CPI/PI,PID2,PI2
      T=ST+ALN
      K=T/PI2
      T=T-K*PI2
      RIGHTA=T*DEGR
      RETURN
      END
```

```
      FUNCTION TLOC(ST,ALN,AS)
      COMMON/CPI/PI,PID2,PI2
      COMMON /CHRAD/HRAD
      T=ST+ALN-AS+PI
    6 IF(T)4,5,5
    4 T=T+PI2
      GOTO 6
    5 K=T/PI2
      T=T-K*PI2
      TLOC=T*HRAD
      RETURN
      END
```

```
      FUNCTION ZDSUNC(XE,XSG)
      DIMENSION XE(3),XSG(3)
      COMMON/CPI/PI,PID2,PI2
      COMMON /CDEGR/DEGR
      C=0
      DO 3 J=1,3
    3 C=XE(J)*XSG(J)+C
      S=SQRT(1-C*C)
      IF(C)7,8,7
    8 ZS=PID2
      GOTO 11
    7 ZS=ATAN(S/C)
      IF(ZS)10,11,11
   10 ZS=ZS+PI
   11 ZDSUNC=ZS*DEGR
      RETURN
      END
```

```
      FUNCTION ZDSUNL(ALT,ALN,XSG)
      DIMENSION XSG(3),XR(3)
      CB=COS(ALT)
      XR(1)=COS(ALN)*CB
      XR(2)=SIN(ALN)*CB
      XR(3)=SIN(ALT)
      ZDSUNL=ZDSUNC(XR,XSG)
      RETURN
      END
```

4.2. Geomagnetic coordinates of the AES, geomagnetic local time (IO2--GMLL, FGEOM). /48

1. Purpose. For the point, given by directed cosines (x_N, y_N, z_N) in the Greenwich system of coordinates for the normalto the earth's ellipsoid, one determines the geomagnetic latitude and longitude (subprogram GMLLO and geomagnetic time (subprogram TGEOM).

2. Structure. Subprogram GMLL and subprogram-function TGEOM. General domain: $/CPI/_3$, $/CDEGR/_1$, $/BGLL/_2$, $/BGS/_2$.

3. Conversion: CALL GMLL (XNG, GLT, GLN),
TM=TGEOM (XNG, XSG).

4. Initial data: Block XNG_3--directed cosines of the normal to the surface of the earth's ellipsoid in the Greenwich system of coordinates; the block XSG_3--directed cosines of the radius vector of the sun in the Greenwich system of coordinates.

5. Results: GLT, GLN--geomagnetic latitude and longitude in degrees;
TM--local geomagnetic time in hours.

6. Use of the domain COMMON. From the blocks $COMMON/CPI/_3$, $/CDEGR/_1$, the constants are used (see (5), Nos. 1, 2, table 2.3)

In the block COMMON/BGLL/GLT, GLN the subprogram TGEOM gives the value of the geomagnetic latitude and longitude of the point, in the block COMMON/BGS/SLT, SLN--the values of the geomagnetic latitude and longitude of the sun (in degrees). /49

7. Algorithm. The geomagnetic latitude phi_m and longitude lambda_m are computed by the formulas:

$$\sin\varphi_m = x_N^0 x_G^0 + y_N^0 y_G^0 + z_N^0 z_G^0 ,$$

$$\mathrm{tg}\,\lambda_m = \frac{y_N^0 x_G^0 - x_N^0 y_G^0}{z_G^0 \sin\varphi_m - z_N^0} ,$$

$$x_G^0 = 0{,}71447078,\quad y_G^0 = -0{,}186126001,\; z_G^0 = 0{,}979924705 -$$

directed cosines (in the Greenwich system of coordinates) of the radius vector of the geomagnetic pole (phi_G=78.5°, lambda_G=-69°).

Geomagnetic time TM=lambda_m---$\mathrm{lambda}_{m\odot}$, where $\mathrm{lambda}_{m\odot}$ --geomagnetic longitude of the sun, calculated by the same formulas, as lambda_m, if instead of x_N^0, y_N^0, z_N^0 one substitutes the values $x_\odot^0$, $y_\odot^0$, $z_\odot^0$, directed cosines of the radius vector of the sun inthe Greenwich system of coordinates.

8. Texts.

```
      SUBROUTINE GMLL(XE,GLT,GLN)
      DIMENSION XE(3)
      COMMON/CDEGR/DEGR
      COMMON/CPI/PI,PID2,PI2
      SF=XE(1)*.071447078-.186126881*XE(2)+.979924785*XE(3)
      CF=SQRT(1-SF*SF)
      IF(CF)1,2,1
    2 GLT=SIGN(PID2,SF)*DEGR
      GOTO 5
    1 GLT=ATAN(SF/CF)*DEGR
    5 W=SF*.979924785-XE(3)
      CF=XE(2)*.071447078+.186126881*XE(1)
      IF(W)3,4,3
    4 GLN=SIGN(PID2,CF)
      GOTO 9
    3 GLN=ATAN(CF/W)
      IF(W)7,8,8
    7 GLN=GLN+PI
      GOTO 6
    8 IF(GLN)9,6,6
    9 GLN=GLN+PI2
    6 GLN=GLN*DEGR
      RETURN
      END
```

```
      FUNCTION TGEOM(XE,XSG)
      COMMON /BGS/SLT,SLN
      COMMON/BGLL/GLT,GLN
      DIMENSION XE(3),XSG(3)
      CALL GMLL(XE,GLT,GLN)
      CALL GMLL(XSG,SLT,SLN)
      W=(GLN-SLN)/15.+12.
      IF(W)1,2,2
    1 W=24.+W
    2 TGEOM=W
      RETURN
      END
```

4.3. Auroral longitude, auroral time (IO3--AULONG, TAUR)

1. Purpose. For the point, given by directed cosines of the normal to the surface of the earth's ellipsoid (x_N^o, y_N^o, z_N^o) in the Greenwich system of coordinates, the auroral longitude (subprogram AULONG) and auroral time (subprogeam TAUR) are determined.

2. Structure. Subprogram--functions AULONG, TAUR. Common blocks: $/CPI/_3$, $/CHRAD/_1$, /BAU/1.

3. Conversion: AU=AULONG (XNG),
TA=TAUR (XNG, XSG).

Remark 1. Subprogram--TAUR refers to the subprogram AULONG.

4. Initial data: The block XNG_3--directed cosines of the normal to the surface of the earth's ellipsoid in the Greenwich system of coordinates; block XSG_3--directed cosines of the radius vector of the sun inthe Greenwich system of coordinates.

5. Results: AU--auroral longitude inhours; TA--auroral time in hours.

6. Use of the domain COMMON. From the blocks $COMMON/CPI/_3$, $/CHRAD/_1$ one uses the constants (see (5), Nos. 1, 3 table 2.3).

In the block COMMON/BAU/AU the subprogram TAUR addresses the value of the auroral longitude.

7. Algorithm. The auroral longitude lambda$_{alpha}$ according to /51
(6) is computed by the formulas: $\lambda_\alpha = \mathrm{arctg}(A/B) - 56^\circ,61$,

$$= 0,930414\, x_\alpha - 0,331032\, y_\alpha - 0,157062\, z_\alpha,$$

$$= -0,353629\, x_\alpha - 0,923585\, y_\alpha - 0,147963\, z_\alpha,$$

$$_\alpha = x_N^o + 0,062724,$$

$$_\alpha = y_N^o - 0,022319,$$

$$_\alpha = z_N^o - 0,010589.$$

Auroral time T$_{alpha}$ =lambda$_{alpha}$--lambda$_{alpha\odot}$, lambda$_{alpha\odot}$ computed by the same formulas as lambda$_{alpha}$, if instead of x_N^o, y_N^o, z_N^o one substitutes $x_\odot^o$, $y_\odot^o$, $z_\odot^o$, directed cosines of the radius vector of the sun in the Greenwich system of coordinates.

8. Texts.

```
      FUNCTION AULONG(XE)
      COMMON/CPI/PI,PID2,PI2
      COMMON/CHRAD/HRAD
      DIMENSION XE(3)
      X1=XE(1)+.062724
      X2=XE(2)-.022319
      X3=XE(3)-.010589
      A=.930414*X1-.331082*X2-.157062*X3
      B=-.353629*X1-.923585*X2-.147963*X3
      IF(B)1,2,1
    2 W=SIGN(PID2,A)
      GOTO 5
    1 W=ATAN(A/B)
      IF(B)4,5,5
    4 W=W+PI
      GOTC 3
    5 IF(W)6,3,3
    6 W=W+PI2
    3 AULONG=(W-.98803089)*HRAD
      RETURN
      END
```

```
      FUNCTION  TAUR(XE,XSG)
      DIMENSION XE(3),XSG(3)
      COMMON/BAU/AU
      AU=AULONG(XE)
      AUS=AULONG(XSG)
      W=AU-AUS+12.
      IF(W)1,2,2
    1 W=24.+W
    2 TAUR=W
      RETURN
      END
```

4.4. Geomagnetic parameters B, L (104- BL, INVAR, LINES, STAR, CARMEL, NEWMAG, INTEG, ASIN). /52

1. Purpose. Subprograms intended for the calculation of B, L, the parameters of the magnetic field of theearth.

2. Structure. Package of subprograms. Input for user: BL. Internal entries: INVAR, LINES, STAR, CARMEL, NEWMAG, INTEG, ASIN. Common blocks: $/P1/_{1}$, $/MAG/_{1204}$, $/CEOM/_{2}$, $/COP/_{3}$.

3. Conversion: CALL BL (XC, YC, ZC, NG1, TE1, B3, FL).

4. Initial data: XC, YC, ZC--Greenwich coordinates of the point in meters, NG1--number of harmonics considered (NG1 less than or = 9), TE1--period, for which are computed the coefficients of the expansion for the magnetic field of the earth, minus 1960 (for example, if the period 1965 is chosen, then TEI=5).

5. Results B3--block, containing four actual magnitudes: B_{phi}, B_{theta}, B_r, B, coordinates of the vector B in the geographical

system of coordinates and the modulus of the vector B in degre s, FL--value of L in radii of the earth.

6. Use of domain COMMON.

In theblock COMMON/COP/$_3$ are addressed H_C, phi_C, $lambda_C$ -- geographic coordinates of the magnetic--conjugate point (H_C in km, phi_C, $lambda_C$--in degrees).

7. There is a more detailed description in (6).

4.5. Invariant geomagnetic latitude.
(105--AINLAT, OINLAT)

1. Purpose. The subprogram AINLAT determines the invariant geomagnetic latitude for known values B and L, of the geomagnetic parameters. The subprogram OINLAT determines the invariant latitude for the basis of the line of force. /53

2. Structure. Subprograms--functions AINLAT, OINLAT.
Common block /CDEGR/$_1$.

3. Conversions: ALI=AINLAT (B, FL),
ALO=OINLAT (FL).

4. Initial data: B, FL--B, L--parameters of the magnetic field of the earth.

5. Results: ALI--invariant geomagnetic latitude indegrees, ALO--invariant latitude of the basis of the line of force in degrees.

6. Use of the domain COMMON: One uses the constant from the block /CDEGR/$_1$ (see (5) No. 2 table 2.3)

7. Algorithm. The invariant geomagnetic longitude Lambda, according to (6), is computed according to the formula:

$$\Lambda = \operatorname{arctg}\left(\sqrt{1/B_S - 1}\right),$$

где $B_S = \sum_{n=0}^{7} b_n a^n$, $\quad a = \frac{(0{,}311653/B)^{1/3}}{L}$;

$b_0 = 1{,}259921$, $\quad b_4 = -0{,}00308824$,

$b_1 = -0{,}1984259$, $\quad b_5 = 0{,}00082777$,

$b_2 = -0{,}04686632$, $\quad b_6 = -0{,}00105877$,

$b_3 = -0{,}01314096$, $\quad b_7 = 0{,}00183142$.

Инвариантная широта у основания силовой линии,

$$\Lambda_0 = \operatorname{arctg}\left(\sqrt{L-1}\right),$$

где B, L — геомагнитные параметры.

8. Texts.

```
      FUNCTION OINLAT(FL)
      COMMON /CDEGR/DEGR
      OINLAT=0
      TN=FL-1.
      IF(TN)2,2,1
    1 OINLAT=ATAN(SQRT(TN))*DEGR
    2 RETURN
      END
```

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```
      FUNCTION AINLAT(B,FL)
      COMMON /CDEGR/DEGR
      A=(.311653/B)**(.333333333)/FL
      BS=A*(((((((A*.00183142-.00105877)*A+.00082777)*A-.00308826)
     *  *A-.01314096)*A-.04686632)*A-.1984259)*A+1.259921)
      AINLAT=0.
      TN=1./BS-1.
      IF(TN)2,2,1
    1 AINLAT=ATAN(SQRT(TN))*DEGR
    2 RETURN
      END
```

4.6. Geographical coordinates of T--point--points of intersection with the plane of the terminator of the line, connecting the AES with the sun (106--PTERM) /54

1. Purpose. From the known Greenwich coordinates of the AES (x, y, z) and the directed cosines of the radius vector of the sun ($x^{0}_{\odot}$, $y^{0}_{\odot}$, $z^{0}_{\odot}$) one determines the geographical coordinates of T--the point.

2. Structure: Subprogram PTERM. One uses the external subprogram GEOGRC (B10).

3. Conversion: CALL PTERM (XG, XSG, HT, TLT, TLN).

4. Initial data: Block XG_3--Greenwich coordinates of AES; block XSG--directed cosines of radius vector of the sun in Greenwich system of coordinates.

5. Results: HT, TLT, TLN--elevation above the surface of the

earth, latitude and longitude of T--point (angles in radians).

6. Algorithm: x_T, y_T, z_T-- Greenwich coordinates of the T-point, points of intersection with the plane of the terminator of the line, connecting the AES with the sun (or continuation of this line), are computed with the formulas:

$$x_T = x - a_s x^{\circ}_{\bullet}, \quad y_T = y - a_s y^{\circ}_{\bullet}, \quad z_T = z - a_s z^{\circ}_{\bullet},$$
$$a_s = x x^{\circ}_{\bullet} + y y^{\circ}_{\bullet} + z z^{\circ}_{\bullet},$$

The geographical coordinates of T--point a [illegible] the algorithm par. 3.10 (b).

7. Text.

```
      SUBROUTINE PT  (XG,XSG,[illegible]N)
      DIMENSION XG(3),XSG(3),XT(3)
      C=0
      DO 3 J=1,3
    3 C=C+XG(J)*XSG(J)
      DO 4 J=1,3
    4 XT(J)=XG(J)-C*XSG(J)
      CALL GEOGRC(XT,HT,TLT,TLN,XT)
      RETURN
      END
```

4.7. Determination of the Greenwich coordinates of the point, given geographical latitude, longitude and elevation above the surface of the earth's ellipsoid, and matrices of the transformation from the Greenwich to the point system of coordinates, connected with this point (I 07--POINT).

1. Purpose. The Greenwich coordinates (x_{IG}, y_{IG}, z_{IG})are determined for the point I, prescribed geographical latitude, (phi$_I$) longitude (lambda$_I$) and elevation (h_I) above the surface of the earth's ellipsoid.
The point topocentric system of coordinates, connected with the point I, is determined in the following manner.

The origin of the coordinates coincides with the point I, the axis Ix_I is directed to the north pole of the earth by the tangent to the meridian of the point I, the axis Iy_I--by the external normal to the surface of the earth's ellipsoid, the axis Iz_I completes the system up to the right side.

The subprogram POINT determines the matrix of the transformation from the Greenwich system of coordinates to the point system.

2. Structure. Subprogram POINT.
Common domain: /CAE/2.

3. Conversion: CALL [illegible] (HP, ILT, ILN, XPG, GP).

4. Initial data: HP, ILT, ILN--elevation, geographical latitude, longitude of the point P.

5. Results: Block XPG_3--Greenwich coordinates of the point P; /56
GP- twodimensional block (3, 3) matrix of the transformation from Greenwich system of coordinates to the point system.

6. Use of the domain COMMON: one uses the constants from the block /CAF/[illegible] (see ([illegible]), No. 11, table 2.1).

7. Algorithm: Coordinates of point P in the Greenwich system of coordinates are determined with the formulas:

$$x_{PO} = (a_e/A + h_p)\cos\varphi_p \cos\lambda_p,$$
$$y_{PO} = (a_e/A + h_p)\cos\varphi_p \sin\lambda_p,$$
$$z_{PO} = (a_e(1-\alpha)^2/A + h_p)\sin\varphi_p,$$
$$A = (\cos^2\varphi_p + (1-\alpha)^2\sin^2\varphi_p)^{1/2},$$

alpha$_e$, alpha--semimajor axis and coefficient of compression of the earth's ellipsoid.

Matrix of the transformation (GP_{ij}) from the Greenwich system of coordinates to the point system (determined in par. 1), which has the following form:

$$[GP_{ij}] = \begin{bmatrix} -\sin\varphi_p\cos\lambda_p & -\sin\varphi_p\sin\lambda_p & \cos\varphi_p \\ \cos\varphi_p\cos\lambda_p & \cos\varphi_p\sin\lambda_p & \sin\varphi_p \\ -\sin\lambda_p & \cos\lambda_p & 0 \end{bmatrix}$$

6. Text:

```
SUBROUTINE POINT(HP,ALTP,ALNP,XP,GP)
DIMENSION XP(3),GP(3,3)
COMMON/CAE/AE,AL
CL=COS(ALTP)
SL=SIN(ALTP)
CLN=COS(ALNP)
SLN=SIN(ALNP)
ALK=1.-AL
ALK=ALK*ALK
AN=SQRT(CL*CL+ALK*SL*SL)
A=AE/AN
GP(1,1)=-SL*CLN
GP(1,2)=-SL*SLN
GP(1,3)=CL
GP(2,1)=CL*CLN
GP(2,2)=CL*SLN
GP(2,3)=SL
GP(3,1)=-SLN
GP(3,2)=CLN
GP(3,3)=0.
XP(3)=(A*ALK+HP)*SL
A=A+HP
XP(1)=A*GP(2,1)
XP(2)=A*GP(2,2)
RETURN
END
```

4.8. Change from the Greenwich system of coordinates to the point topocentric form, determination of ranging, angle of location, azimuth (108--GRIX, TURN). 27

1. Purpose. Subprogram GRIX for known Greenwich coordinates of the AES (x, y, z) and Greenwich coordinates of the point P (x_{PG}, y_{PG}, z_{PG}) determines the coordinates of AES (x_P, y_P, z_P) in point-topocentric system of coordinates; DAL, inclined range (distance from AES to point P); AM, angle of location--angle between radius vector of AES and the plane, perpendicular to the axis P_{y_P} of the point topocentric system of coordinates; AZ azimuth--angle between direction for the sun (angle P_{x_P}) and the projection of the radius vector of the AES on the plane perpendicular to the axis P_{y_P} (opposite the hand of the clock). The components of the vector of velocity in the point system of coordinates may be determined, using the subprogram TURN, intended for the multiplication of the matrix of dimension (N, N) by the N-dimensional vector.

2. Structure. The subprograms GRIX, TURN.

External subprograms utilized: GCLTLN (B10)

3. Conversion to GRIX:

CALL GRIX (XG, XGP, GP, XP, DAL, AM, AZ).

Initial data: Block XG_3--Greenwich coordinates of AES; block XGP_3--Greenwich coordinates of point P; GP--twodimensional block (3,3)--matrix of the transformation from the Greenwich to the point system of coordinates.

Results: block XP_3--coordinates of AES in topocentric point system of coordinates, DAL, AM, AZ, determined above.

4. Conversion to subprogram TURN.

CALL TURN (X, A, Y, N).

Initial data: X--block of dimension N; A--block of dimension (N, N); N--dimension.

Results: Y--block of dimension N (Y=AX). /58

6. Algorithm

$$\begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = [GP_{ij}] \begin{bmatrix} x - x_{PG} \\ y - y_{PG} \\ z - z_{PG} \end{bmatrix}; \quad \begin{bmatrix} v_{x_p} \\ v_{y_p} \\ v_{z_p} \end{bmatrix} = [GP_{ij}] \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix};$$

$$DAL = \left((x - x_{PG})^2 + (y - y_{PG})^2 + (z - z_{PG})^2\right)^{1/2};$$

$$AM = arctg\left(z_P/(x_P^2 + y_P^2)^{1/2}\right);$$

$$AZ = Arctg\,(y_P/x_P),$$

(G_{ij})--matrix of transformation from Greenwich system of coordinates to the point form;

7. Text:

```
      SUBROUTINE GRPX(XG,XGP,GP,XP,DAL,AM,AZ)
      DIMENSION XG(3),XGP(3),GP(3,3),XP(3),X(3)
      DAL=0.
      DO 2 J=1,3
      X(J)=XG(J)-XGP(J)
    2 DAL=DAL+X(J)*X(J)
      DAL=SQRT(DAL)
      CALL TURN(X,GP,XP,3)
      X(1)=XP(1)
      X(2)=XP(3)
      X(3)=XP(2)
      CALL GCLTLN(X,AM,AZ)
      RETURN
      END
```

```
      SUBROUTINE TURN(XG,GP,XP,N)
      DIMENSION XG(N),GP(N,N),XP(N)
      DO 1 I=1,N
      XP(I)=0.
      DO 1 J=1,N
    1 XP(I)=XP(I)+GP(I,J)*XG(J)
      RETURN
      END
```

4.9. Transformation from the absolute system of coordinates to the solar-ecliptic (109--ABSECL).

1. Purpose. For known coordinates of AES (X, Y, Z) in the absolute system of coordinates one determines the coordinates of the AES in the solar-ecliptic system of coordinates, determined by the following means: the center of the system O coincides with the center of the earth, axes CX_{ce}, CY_{ce} lie in the plane of the ecliptic, the axis CX_{ce} directed towards the sun, axis OZ_{ce} perpendicular to the plane of the ecliptic and forming an acute angle with the axis of rotation of the earth directed toward the north. /52

2. Structure. Subprogram ABSECL.
Subprograms utilized: GCLTLN (B10). Common block $/BIECL/_2$.

3. Conversion: CALL ABSECL (XA, XAS, XSE, CLTS, CLNS).

4. Initial data: Block XA_3--absolute coordinates of ISZ; Block XAS_3--directed cosines of radius vector of the sun in the absolute system of coordinates.

5. Results: Block XSE_3--coordinates of AES ; CLTS, CLNS--geocentric latitude and longitude of AES in solar-ecliptic system of coordinates.

6. Use of the domain COMMON. From the block COMMON $/BIECL/_2$ one uses cos epsilon and sin epsilon, computed in the subprogram SUN(COZ), where epsilon--angle of inclination of the plane of the earth's equator to the plane of the ecliptic.

7. Algorithm:

$$\begin{bmatrix} X_{SE} \\ Y_{SE} \\ Z_{SE} \end{bmatrix} = \begin{bmatrix} X^{\circ}_{\odot} & Y^{\circ}_{\odot} & Z^{\circ}_{\odot} \\ -r_1 \operatorname{sign} Y^{\circ}_{\odot} & \dfrac{X^{\circ}_{\odot} Y^{\circ}_{\odot}}{r_1} \operatorname{sign} Y^{\circ}_{\odot} & \dfrac{X^{\circ}_{\odot} Z^{\circ}_{\odot}}{r_1} \operatorname{sign} Y^{\circ}_{\odot} \\ 0 & \dfrac{Z^{\circ}_{\odot}}{r_1} \operatorname{sign} Y^{\circ}_{\odot} & \dfrac{Y^{\circ}_{\odot}}{r_1} \operatorname{sign} Y^{\circ}_{\odot} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix},$$

где $r_1 = (Y^{\circ 2}_{\odot} + Z^{\circ 2}_{\odot})^{1/2}$.

8. Text:

8. Text:

```
      SUBROUTINE ABSECL(V,XS,XEC,AEC,BEC)
      COMMON /BIECL/CEP,SEP
      DIMENSION V(3),XS(3),XEC(3)
      R1=SQRT(XS(2)*XS(2)+XS(3)*XS(3))
      IF(R1)2,3,2
    3 W=CEP
      W1=SEP
      GOTO 4
    2 W=XS(2)/R1
      W1=XS(3)/R1
    4 XEC(1)=0.
      DO 1 J=1,3
    1 XEC(1)=XEC(1)+V(J)*XS(J)
      [illegible])-W*V(1)-W1*V(1)+V(3)*W1*XS(1)
      XEC(3)=V(3)*W-V(2)*W1
      W=SIGN(1.,XS(2))
      XEC(2)=XEC(2)*W
      XEC(3)=XEC(3)*W
      CALL GCLTLN(XEC,AEC,BEC)
      RETURN
      END
```

4.10. Mome[illegible]s of time of the entry of AES into the earth's shadow and exit from the shadow (I10--[illegible])

1. [illegible]rpose. For known elements of the orbit: a, e, i, Omega, omega, for position in the orbit (u) at a given moment in time t one determines the moments of time of entry of the ISZ into the shadow of the earth and exit from the shadow.

2. Structure: Subprogram SHADAW. One uses external subprograms ROOT 4 (I11), SUN (CO2). Common blocks: $/CGR/_1$, $/CRE/_1$, $/CPI/_3$.

3. Conversion: CALL SHADOW (DT, T, A, T1, T2).

4. Initial data: Block A_6, containing elements of the orbit and the argument of latitude: a, e, i, Omega, omega, u; DT--data in the form RJD, T--Moscow time, corresponding to the argument of latitude u.

5. Results: T1, T2--are, respectively, the moments of entry of the AES into the shadow of the earth and exit from theshadow (in units given by the scale factor EJEC). If AES does not get into the shadow, then T1=T2=0.

6. Use of the domain COMMON: One uses the constants from the blocks COMMON/CGR/$_1$, $/CRE/_1$, $/CPI/_3$, (see (5), Nos. 5, 9, table 2.', No. 1 table 2.3).

7. Algorithm: Moments of time (T1) of entry of AES into the shadow of the earth and (T2) of exit from the shadow are determined without computation of the effect of the penumbra; with the assumption that the earth has no compression and is not displaced orbitally. (7). These moments of time correspond to actual anomalies of the AES, satisfying the following relationship:

$$\cos\psi = \frac{R_\odot r}{R_\odot r} = -\frac{(r^2 - a_e^2)^{1/2}}{r} \qquad (1)$$

where R_{theta}--radius vector of the sun, r--radius vector of the AES, a_e-- equatorial radius of the earth, psi -angle between radius-vector of AES and radius vector of the sun. The relationship (1) leads to the equation of the fourth degree with respect to /61 the cosines of the actual anomaly:

$$S^{\circ} = A_0\cos^4 v + A_1\cos^3 v + A_2\cos^2 v + A_1\cos v + A_4 , \qquad (2)$$

where
где $A_0 = \left(\frac{a_e}{p}\right)^4 e^4 - 2\left(\frac{a_e}{p}\right)^2(\xi^2-\beta^2)e^2 + (\beta^2+\xi^2)^2$,

$$A_1 = 4\left(\frac{a_e}{p}\right)^4 e^3 - 4\left(\frac{a_e}{p}\right)^2(\xi^2-\beta^2)e,$$

$$A_2 = 6\left(\frac{a_e}{p}\right)^4 e^2 - 2\left(\frac{a_e}{p}\right)^2(\xi^2-\beta^2) - 2\left(\frac{a_e}{p}\right)^2(1-\xi^2)e^2 + $$
$$+ 2(\xi^2-\beta^2)(1-\xi^2) - 4\beta^2\xi^2,$$

$$A_3 = 4\left(\frac{a_e}{p}\right)^4 e - 4\left(\frac{a_e}{p}\right)^2(1-\xi^2)e ,$$

$$A_4 = \left(\frac{a_e}{p}\right)^4 - 2\left(\frac{a_e}{p}\right)^2(1-\xi^2) + (1-\xi^2)^2;$$

p — parameter of orbit
параметр орбиты $= a(1-e^2)$,

$$\beta = X^{\circ}_{\odot}P_X + Y^{\circ}_{\odot}P_Y + Z^{\circ}_{\odot}P_Z ,$$

$$\xi = X^{\circ}_{\odot}Q_X + Y^{\circ}_{\odot}Q_Y + Z^{\circ}_{\odot}Q_Z ,$$

$X^{\circ}_{\odot}, Y^{\circ}_{\odot}, Z^{\circ}_{\odot}$ — направляющие косинусы радиуса-вектора Солнца, $R_{\odot}$,
directed cosines of radius vector of the sun

$$P_X = \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i ,$$

$$P_Y = \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i ,$$

$$P_Z = \sin\omega\sin i ,$$

$$Q_X = -\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i ,$$

$$Q_Y = -\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i ,$$

$$Q_Z = \cos\omega\sin i .$$

a The solutions to equations (2), satisfying the conditions:

beta cos v + xi sin v less than 0,
$S = (1 + e\cos v)^2 + (p/a_e)^2$ (beta cos v + xi sin v)$^2 - (p/a_e)^2 = 0$.

are assumed for further consideration.
The condition:

$$S' = P^2(\beta\cos v + \xi\sin v)(\xi\cos v - \beta\sin v) - a_e^2 e(1 + e\cos v)\sin v > 0$$

determines the value of the actual anomaly v_1 in the entry of theISZ into the shadow of the earth, S' less than zero correspoinds to the value v_2, determining the exit of theISZ from theshadow.

With the value of t, time, for corresponding given position of AES. in orbit, we determine tau, the time of passage of the AES. through perigee: /62

$$\tau = t - (E_0 - e\sin E_0)/\lambda ,$$

$$\text{где } \lambda = \sqrt{\mu/a^3} , \quad \operatorname{tg} E_0/2 = ((1-e)/(1+e))^{1/2} \operatorname{tg} v_0/2 , \quad v_0 = u - \omega ,$$

mu--produce of gravitational constant times the mass of the earth. Moments of time T_i ($i=1, 2$) are determined by the known values t and v_i:

$$T_i = \tau + (E_i - e\sin E_i)/\lambda ,$$

$$\operatorname{tg} E_i/2 = ((1-e)/(1+e))^{1/2} \operatorname{tg} v_i/2 .$$

8. Text:

```
      SUBROUTINE SHADOW(DT,T,A,T1,T2)
      DIMENSION A(6),P(9),Q(5),Y(6),R(3)
      DIMENSION XS(3)
      COMMON/CGR/GR
      COMMON/CPI/PI,PID2,PI2
      COMMON /CAE/AE,U
      T1=0.
      T2=0.
      DO 1 J=1,3
      K=(J-1)*3+1
      P(K)=COS(A(J+2))
    1 P(K+1)=SIN(A(J+2))
      EK=SQRT((1.-A(2))/(1.+A(2)))
      AL=SQRT(A(1)/GR)*A(1)
      V=(A(6)-A(5))
      W2=SIN(V)
      W1=COS(V)+1.
      IF(W1.GT.1.E-15)
     -                    V=ATAN(W2*EK/W1)*2.
      IF(V.LT.0.)
     -             V=V+PI2
      TR=V-A(2)*SIN(V)
      CALL SUM(DT,T,W1,W2,W3,XS)
      IF(P(1).EQ.0)
     -                GOTO 36
      W2=-XS(2)*P(4)+XS(1)*P(5)
      W1=-XS(1)*P(4)-XS(2)*P(5)
      W3=ATAN2(W2,W1*P(1))
      V=W3-A(5)
      W2=SIN(V)
      W1=COS(V)+1.
      IF(W1.GT.1.E-15)
     -                    V=ATAN(W2*EK/W1)*2.
      IF(V.LT.0.)
     -             V=V+PI2
      T3=T+(V-A(2)*SIN(V)-TR)*AL
```

```
      CALL SUN(DT,TS,W1,W2,W3,XS)
   36 CONTINUE
      AP=P(1)*P(8)
      AQ=P(1)*P(7)
      P(3)=P(2)*P(8)
      P(6)=-P(4)
      P(9)=P(2)*P(7)
      DO 2 J=1,2
      P(J)=P(7)*P(J+3)-AP*P(J+6)
    2 P(J+3)=-P(8)*P(J+3)-AQ*P(J+6)
      P(6)=P(9)
      B=0
      S=0
      DO 3J=1,3
      B=XS(J)*P(J)+B
    3 S=XS(J)*P(J+3)+S
      W1=A(2)*A(2)
      PK=A(1)*(1.-W1)/AE
      PK=PK*PK
      W2=1./PK
      W1=W2*W2
      AP=B*B
      AQ=S*S
      W3=AQ-AP
      W4=1.-AQ
      P(6)=2.
      P(5)=4.*A(2)
      P(4)=2.*A(2)*A(2)
      P(3)=2.*P(4)
      P(2)=P(3)*A(2)
      P(1)=P(2)*A(2)/4.
      Q(1)=AP+AQ
      Q(1)=Q(1)*Q(1)
      Q(2)=0
      Q(3)=2.*W3*W4-4.*AP*AQ
      Q(4)=0
      Q(5)=W4*W4-W1
      DO 5 J=1,3
      Q(J)=Q(J)+W1*P(J)-W2*P(J+3)*W3
    5 Q(J+2)=Q(J+2)+W1*P(J+3)-W2*P(J+3)*W4
      CALL ROOT4(Q,V,IR)
      N=4
      K=1
      IF(IR(1))8,8,6
    6 K=3
    8 IF(IR(2))9,9,11
   11 N=2
    9 IF(K-N)12,12,7
   12 DO 17 J=K,N
      IF(ABS(V(J))-1.)18,18,17
   18 W3=SQRT(1.-V(J)*V(J))
      W1=B*V(J)
      W5=1.+A(2)*V(J)
      W6=W5*W5-PK
      DO 30 I=1,2
      W4=S*W3
      W4=W1+W4
      V=W6+PK*W4*W4
      IF(ABS(V)-1.E-3)27,30,30
   27 IF(W4)19,30,30
   19 V=PI
      W2=1.+V(J)
      IF(W2.GT.1.E-15)
     -                        V=ATAN(W3*EK/W2)*2.
      IF(V.LT.0.)
     -                 V=V+PI2
      V=T+(V-A(2)*SIN(V)-TP)*AL
   23 W2=PK*W4*(S*V(J)-B*W3)-W5*A(2)*W3
      IF(W2)24,25,25
   25 T1=V
      GOTO 30
   24 T2=V
   30 W3=-W3
   17 CONTINUE
    7 RETURN
      END
```

4.11. Calculation of the roots of algebraic equations of the fourth, third and second degree (I11-ROOT4) /64

1. Purpose. Determination of the roots of the algebraic equation

$$A_0x^4 + A_1x^3 + A_2x^2 + A_3x + A_4 = 0,$$

of the fourth $A_0 \neq 0$, third ($A_0=0$) and second ($A_0=0$, $A_1=0$) degree with real coefficients.

2. Structure: Subprogram ROOT 4.

3. Conversion: CALL ROOT4 (A, Y, IR).

4. Initial data: Block A--coefficients of equation (1) in order: A_0, A_1, A_2, A_3, A_4.

5. Results: Block Y_4--roots of equation (1);

Block IR_2 --characteristics of the roots:

IR (1)=
- 0, if Y(1), Y(2)--real roots
- 4, if Y(1), Y(2) real and imaginary part of a complex root (for complex-conjugate root Y(2) is taken with the opposite sign)
- 1, if $A_0= A_1= A_2=0$ (equation of first degree)

IR (2) =
- 0, if Y(3), Y(4)--real roots
- 4, if Y(3), Y(4)--real and imaginary part of complex root (for complex conjugate root Y(4) taken with opposite sign)
- 3, if $A_0=0$ (equation of third degree)
- 2, if $A_0=A_1=0$ (equation of second degree)
- 1, if $A_0=A_1=A_2=0$ (equation of first degree).

Remark. For IR (1) =I Y(1)--root of equation of the first degree. For IR (2) =3 Y(3)--real root of an equation of the third degree.

6. Algorithm. For the solution of equation (1) one uses the algorithm, described in (7) (p. 446):

a) Equation of 4 degree is solved by the method of Descartes: Dividing equation (1) by A_0 (if $A_0=0$, then we proceed to part delta), we obtain

$$y^4 + B_1y^3 + B_2y^2 + B_3y^3 + B_4 = 0.$$

We transform thisequation, excluding from it the term of the third degree:

$$x^4 + Px^2 + Qx + R = 0,$$

$$P = 6h^2 + 3B_1h + B_2,$$

$$Q = 4h^3 + 3B_1h^2 + 2B_2h + B_3,$$

$$R = h^4 + B_1h^3 + B_2h^2 + B_3h + B_4,$$

$$h = -B_1/4, \quad y = x + h.$$

For Q=0 we obtain the biquadratic equation, for the solution of which we use the formulas of p rt v. For $Q \neq 0$ we consider the cubic resolvent

$$
\begin{aligned}
&t^3 + c_2 t^2 + c_3 t + c_4 = 0, \\
&c_2 = 2P, \quad c_3 = P^2 - 4R, \quad c_4 = -Q^2.
\end{aligned} \qquad (3)
$$

We transform equation (3) into the form:

$$
\begin{aligned}
&z^3 + az + b = 0, \\
&a/3 = c_3/3 - s^2, \quad b/2 = +s^3 - c_3 s/2 + c_4/2, \\
&s = c_2/3, \quad z = t + s, \\
&\Delta = \frac{a^3}{27} + \frac{b^2}{4}.
\end{aligned} \qquad (4)
$$

4

If Delta is greater than 0, then the solution of equation (4) is found […] by the formula of Kaplan:

$$z_3 = (-b/2 + (\Delta)^{1/2})^{1/3} + (-b/2 - (\Delta)^{1/2})^{1/3}$$

z_1, z_2 – комплексные корни.

z_1, z_2--complex roots.

If Delta =0, then the ro ts of equation (4) are determined by the formula:

и Δ =0, то корни уравнения (4) определяем по (

$$z_1 = 2(-b/2)^{1/3} \qquad z_2 = z_3 = (+b/2)^{1/3}$$

If Delta is less than 0, then we use the representation of the roots as resolvents in trigonometric form,

$$z_1 = E_0 \cos\varphi/3, \quad z_2 = E_0\cos(\varphi/3 + 120^\circ), \quad z_3 = E_0\cos(\varphi/3 + 240^\circ),$$

где $E_0 = 2\left(-\frac{a}{3}\right)^{1/2}, \quad \cos\varphi = -\frac{b}{2}\Big/\left(-\frac{a^3}{27}\right)^{1/2}, \quad 0 < \varphi < \pi.$

We select a critical root of equation (3):

$$R' = \max(z_1 - s,\ z_2 - s,\ z_3 - s),$$

where z_1-s--real root of equation (3).

Knowing R', it is possible to break down equation (2) with the multipliers:

$$
\begin{aligned}
&(x^2 + x\sqrt{R'} + \xi)(x^2 - x\sqrt{R'} + \beta) = 0, \\
&\xi = 1/2(P + R' - Q/\sqrt{R'}), \qquad \beta = 1/2(P + R' + Q/\sqrt{R'})
\end{aligned}
$$

Using therelationships of division V, we obtain the solutions x_i $(i=1,4)$ of two quadratic equations (5), and consequently--the roots of equation (1): $y_i=x_i+h$.

6. Cubic equation we write in the form

$$A_1 y^3 + A_2 y^2 + A_3 y + A_4 = 0.$$

Dividing the left side by A_1 (if $A_1=0$, we go on to division b), we obtain

$$y^3 + c_2 y^2 + c_3 y + c_4 = 0.$$

This equation coincides with equation (3), the solution of which is described in division a.

Remark For Delta greater than zero the complex roots y_1, y_2 of equation (6) are obtained by the solution of a quadratic equation

$$y^2 + (c_2 + z_3 - s)y + c_3 + (c_2 + z_3 - s)(z_3 - s),$$

where z_3--real root of equation (4), $s=c_2/3$.

b) Dividing the left side of the quadratic equation $A_2 y^2 + A_3 y + A_4 = 0$ by A_x (if $A_2=0$, then we go on to division c), we obtain:

$$y^2 + B_3 y + B_4 = 0.$$

The roots of thisequation

$$y_{1,2} = \frac{-B_3 \pm \sqrt{B_3^2 - 4B_4}}{2}$$

c) Equation of the first degree $A_3 y + A_4 = 0$.

Solution $y = -A_4/A_3$.

7. Text:

```
      SUBROUTINE ROOT4(A,V,IR)
      DIMENSION A(5),V(4),IR(2),IN(3),B(4)
      DATA PI,PID2,PI2/3.14159265,1.57079632,6.28318531/
      DATA IN/1,0,2/
      K=2
      IR(1)=0
      IR(2)=1
      IF(A(1))1,100,1
    1 K=1
      DO 2 J=1,4
      V(J)=0.
    2 B(J)=A(J+1)/A(1)
      H=-B(1)/4.
      V(2)=1.
      DO 3 J=1,4
    3 V(2)=V(2)*H+B(J)
      Q=4.
      W=3.
      DO 4 J=1,3
      Q=Q*H+B(J)*W
    4 W=W-1.
      P=(2.*H+B(1))*3.*H+B(2)
      V(1)=P
      N=1
      IF(ABS(Q)-1.E-16)103,103,100
  103 DO 7 J=1,N,2
      I=IN(J)
      D=V(J)*V(J)-4.*V(J+1)
      V(J)=-V(J)/2.
      IF(D)5,6,6
    6 D=SQRT(D)/2.
      V(J+1)=V(J)-D
      V(J)=V(J)+D
      IR(I)=0
      GOTO 7
    5 V(J+1)=SQRT(-D)/2.
      IR(I)=4
    7 CONTINUE
      GOTO(104,105,107),K
  104 IF(IR(1))10,11,10
   11 V(4)=-V(1)
      V(2)=-V(2)
      V(3)=0.
      V(1)=0.
      N=3
      K=3
      GOTO 103
   10 W1=V(1)*V(1)
      W=W1+V(2)*V(2)
      R1=SQRT(W)
      W=SQRT(W+W1)
      R1=SQRT(R1)*2.
      V(1)=V(2)/R1
      V(2)=W/R1
      V(3)=H-V(1)
      V(1)=V(1)+H
      V(4)=V(2)
      IR(2)=4
```

```
100 IF(A(2))15,101,15
 15 DO 16 J=2,4
 16 B(J)=A(J+1)/A(2)
    IR(2)=3
    GOTO 109
108 B(2)=V(1)*2.
    B(3)=V(1)*V(1)-4.*V(2)
    B(4)=-Q*Q
109 S=B(2)/3.
    W=S*S
    AR=B(3)/3.-W
    BR=S*B(3)/2.-S*W-B(4)/2.
    D=AR*AR*AR+BR*BR
    IF(D)17,18,19
 17 W=2.*SQRT(-AR)
    IF(BR)20,21,20
 21 W1=PID2
    GOTO23
 20 W1=SQRT(-D)
    IF (BR)51,50,51
 50 W1=PID2
    GOTO 23
 51 W1=ATAN(W1/BR)
    IF(BR)22,23,23
 22 W1=PI+W1
 23 DO 24 J=1,3
    V(J)=W*COS(W1/3.)
 24 W1=W1+PI2
    GOTO 110
 19 W=SQRT(D)
    V(1)=BR+W
    V(2)=BR-W
    DO 25J=1,2
    W=ABS(V(J))
    W1=ALOG(W)
    W1=EXP(W1/3.)
 25 V(J)=SIGN(W1,V(J))
    V(3)=V(1)+V(2)-S
    V(1)=0.
    V(2)=0.
    K=IR(2)
    R1=V(3)
    GOTO (111,205,205),K
 18 W=ABS(BR)
    W1=ALOG(W)
    W1=EXP(W1/3.)
    V(1)=SIGN(W1,BR)
    V(2)=-V(1)
    V(1)=2*V(1)
    V(3)=V(2)
110 DO26J=1,3
 26 V(J)=V(J)-S
    GOTO (106,105,205),K
106 R1=AMAX1(V(1),V(2),V(3))
111 V(1)=SQRT(R1)
    V(3)=-V(1)
    W=P+R1
    W1=Q/V(1)
    V(2)=(W-W1)/2.
    V(4)=(W+W1)/2.
    N=3
    K=3
    GOTO 103
107 DO 30 J=1,N,2
    I=IN(J)
    V(J)=V(J)+H
    IF(IR(I))30,31,30
 31 V(J+1)=V(J+1)+H
 30 CONTINUE
    GOTO 105
205 V(1)=B(2)+V(3)
    V(2)=V(1)*V(3)+B(3)
    N=1
    K=2
    GOTO 103
101 IF(A(3))36,102,36
 36 V(1)=A(4)/A(3)
    V(2)=A(5)/A(3)
    IR(2)=2
    N=1
    GOTO 103
102 IF(A(4))37,105,37
 37 V(1)=-A(5)/A(4)
    GOTO 105
    END
```

LIST OF SUBPROGRAMS BY NAME