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FINAL REPORT

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PINHOLE-CORONOGRAPH TRACKING CONTROL

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From

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Introduction

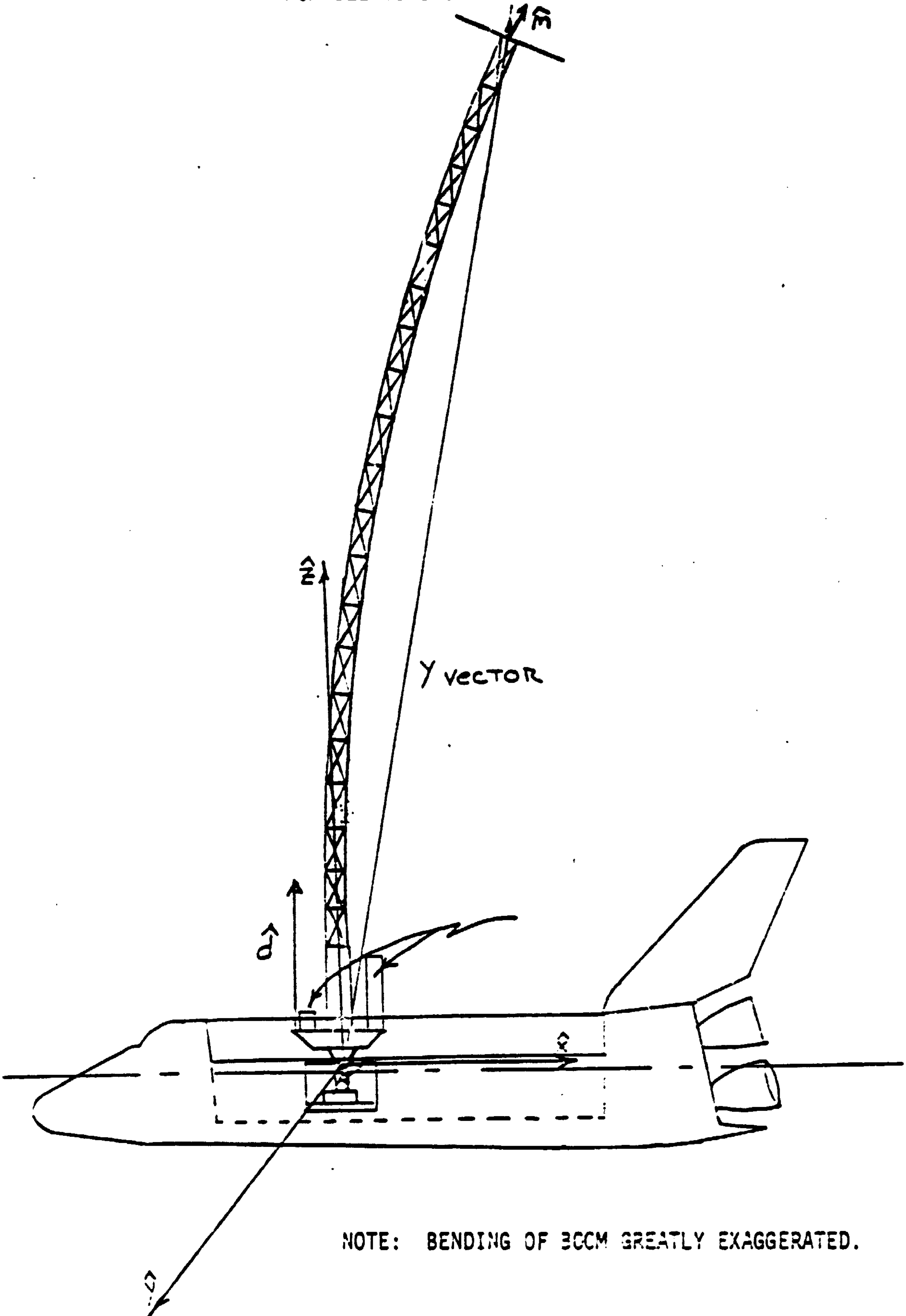
The pinhole-occulter system is a Space Shuttle based experiment for the production of hard X-ray images taken primarily from the Sun. The system is basically a pinhole camera utilizing a deployable 50-m flexible boom for separating the pinhole from the recording devices located in the Shuttle as seen in Figure 1. At the distal end of the boom from the Shuttle is a 50 kg mask containing pinholes and coronograph shields. At the proximal end the detectors are located and mounted, along with the boom, to a gimbal pointing system (either IPS or AGS) aligned with the target along vector $\hat{d}$.

The mask must be pointed at the X-ray source, along vector $\hat{m}$ with a high degree of pointing stability to align the axes of the detectors with the pinholes and shields. Failure to do so will result in a blurring of the images on the detectors and a loss of resolution. Being a Shuttle based experiment, the system will be subjected to the disturbances of the Shuttle. The worst of these is thruster firing for orbit correction; the Shuttle uses a bang-bang thruster control system to maintain orbit to within $\pm$ 0.1°. Other disturbances include man motion, motion induced by other systems, and gravity gradient torques.

The control system of the pointing mount can sense both position and velocity of the mask tip and uses these to accurately estimate the flexible body modes of the system. An optimal control/suppression scheme is then used to control these modes. Disturbances are detected by sensors and are used as commands to drive the system. The AGS, with perfect sensors was utilized in this three axis control system study. The analysis of sensor/drive errors as well as a free body analysis will be the subjects of a later report.

FIGURE 1

PINHOLE CORONOGRAPH COORDINATE AXES



NOTE: BENDING OF BOOM GREATLY EXAGGERATED.

Background

The basis of the dynamic model was to use standard modal analysis. On any flexible structure, modes (eigenvalues) are set up and damp out. Each mode (a pair of complex conjugate eigenvalues) has two complex conjugate eigenvectors, associated with it. By using a finite element model of the beam, there is a mass associated with each node (connection points of the elements). The eigenvectors along with these nodal masses can be used to calculate the deflection of the beam at any node due to any mode. Motions of the base excite the modes and induce deflections of the beam tip.

Given these flexible body modes, the state equations for the system can easily be written as: [1,2]

$$\dot{x} = Ax + B\ddot{u} \tag{1}$$

$$y = Cx + Du \tag{2}$$

The inputs to the system are accelerations in each of the three axes while displacements feed forward to the output. Partitioning the states of the system into controlled states, x_c and suppressed states, x_s, we have:

$$\begin{bmatrix} \dot{x}_c \\ \dot{x}_s \end{bmatrix} = \begin{bmatrix} A_c & 0 \\ 0 & A_s \end{bmatrix} \begin{bmatrix} x_c \\ x_s \end{bmatrix} + \begin{bmatrix} B_c \\ B_s \end{bmatrix} \ddot{u} \tag{3}$$

$$\begin{bmatrix} y \end{bmatrix} = \begin{bmatrix} C_c & \vdots & C_s \end{bmatrix} \begin{bmatrix} x_c \\ x_s \end{bmatrix} + D u \, . \tag{4}$$

By noticing that the double integration of the acceleration inputs are part of the outputs, we can define six (6) new states, x_R, which are the rigid body modes of the boom.[3] This fact introduced into Eqns. 3 and 4 yields the equations of state for the system:

$$\begin{bmatrix} \dot{x}_c \\ \dot{x}_s \\ \dot{x}_R \end{bmatrix} = \begin{bmatrix} A_c & 0 & 0 \\ 0 & A_s & 0 \\ 0 & 0 & A_R \end{bmatrix} \begin{bmatrix} x_c \\ x_s \\ x_R \end{bmatrix} + \begin{bmatrix} B_c \\ B_s \\ B_R \end{bmatrix} \ddot{u} \qquad (5)$$

$$\begin{bmatrix} y \end{bmatrix} = \begin{bmatrix} C_c & \vdots & C_s & \vdots & C_R \end{bmatrix} \begin{bmatrix} x_c \\ x_s \\ x_R \end{bmatrix} \qquad (6)$$

Where the control vector $\ddot{u}$ and output y are three dimensional vectors.

This partioning of the state vectors reflects the objectives of the control system design. The controlled states, x_c, must be controlled to achieve satisfactory system performance while the suppressed states, x_s, are known but not critical to the control design [4,5].

What we would like to do is effect changes in x_c and no changes in either x_s or x_R. The effect of the controller on the rigid modes or suppressed modes is called control spillover. We wish on one hand to optimize the system for the controlled states and limit the control spillover: the controlled states are controlled while the suppressed states are not excited by the controller and the rigid states are unaffected by it.

Let us proceed as follows: suppose we have an optimum controller

$$\ddot{u}^0 = -Kx_c \tag{7}$$

determined by using an index of performance

$$PI = \int_0^\infty (x_c^T Q_c x_c + \ddot{u}^T R_c \ddot{u})\, dt \quad . \tag{8}$$

We would then have in the control equations:

$$\begin{bmatrix} \dot{x}_c \\ \dot{x}_s \\ \dot{x}_R \end{bmatrix} = \begin{bmatrix} A_c - B_c K, & 0, & 0 \\ -B_s K, & A_s, & 0 \\ -B_R K, & 0, & A_R \end{bmatrix} \begin{bmatrix} x_c \\ x_s \\ x_R \end{bmatrix} \tag{9}$$

Clearly, we would like to have for no control spillover:

$$B_s K = 0 \tag{10}$$

$$B_R K = 0 \tag{11}$$

However, equation 11 implies K = 0 because of the form of B_R[3]. Equation 10 implies that B_s and K are orthogonal [5,6,7]. We can modify equations 10 and 11 so that

$$B_1 K = \begin{bmatrix} B_s \\ \\ B_R \end{bmatrix} K = 0 \quad , \tag{12}$$

which merely implies the orthogonality of B_1 and K.

By modifying the performance index to include spillover terms ($B_1\ddot{u}$) we can simultaneously minimize the PI and the product $B_1 K$. Including these spillover terms in the PI, we have:

$$PI = \int_0^\infty [x_c^T \, Q_c \, x_c + \ddot{u}^T R_c \, \ddot{u} + (B_1 \ddot{u})^T R_s \, (B_1 \ddot{u})] \, dt \tag{13}$$

$$= \int_0^\infty [x_c^T \, Q_c \, x_c + \ddot{u}^T (R_c + B_1^T \, R_s \, B_1) \, \ddot{u}] \, dt \ . \tag{14}$$

Now by heavily penalizing the PI to suppressed state spillover ($R_s >>> R_c$), we can force the maximization of the orthogonality of B_1 and K. The feedback coefficient matrix can now be solved from the familiar matrix riccati equation of Appendix 2.

The control spillover into the rigid body states cannot be avoided, but represents only steady state following errors. These errors can be lessened by the proper selection of controller gains for command inputs. By the design of an optimal tracker, orthoganal to the suppressed states, the steady state following errors can be minimized.

Once the optimal controller is determined, the boom tip responses and control inputs ($-Kx_c$) can be calculated using the system equations 5, 6 and 9. Taking the LaPlace transform of equation 9 substituted into equation 5 we have:

$$sX_c(s) = (A_c - B_cK) \, X_c(s) + s^2 \, B_c \, U(s) \tag{15}$$

$$sX_s(s) = (A_s) \, X_s(s) - B_sK \, X_c(s) + s^2 \, B_s \, U(s) \tag{16}$$

$$sX_R(s) = - B_RK \, X_c(s) + A_R \, X_R(s) + s^2 \, B_R \, U(s) \ . \tag{17}$$

Rearranging the terms of equations 15-17 and defining

$$\phi_o(s) = (SI - A_c + B_cK)^{-1}$$

$$\phi_s(s) = (SI - A_s)^{-1}$$

$$\phi_R(s) = (SI - A_R)^{-1} \ , \tag{18}$$

we have:

$$X_c(s) = s^2 \Phi_0(s) B_c U(s) \tag{19}$$

$$X_s(s) = s^2 \Phi_s(s) B_s [I_3 - K \Phi_0(s) B_c] U(s) \tag{20}$$

$$X_R(s) = s^2 \Phi_R(s) B_R [I_3 - K \Phi_0(s) B_c] U(s). \tag{21}$$

The term, $K \Phi_0(s) B_c$, in equations 20 and 21 represent the control spill-over in the suppressed states and rigid body states respectively. This term represents the activation of the suppressed states in equation 20 and a reduction of the magnitude of the response of the controlled states. In equation 21, this term represents a further reduction in the magnitude of the controlled states and the introduction of steady state following errors.

Using the output equation 6 for the system, we can find the boom tip responses as:

$$Y(s) = C_c X_c(s) + C_s X_s(s) + C_R X_R(s)$$

$$= (s^2 C_c \Phi_0 B_c + s^2 C_s \Phi_s B_s [I_3 - K\Phi_0 B_c] + s^2 C_R \Phi_R B_R [I_3 - K\Phi_0 B_c])U(s). \tag{22}$$

But

$s^2 C_R \Phi_R B_R = I_3$ since the positional input is fed forward to the output in eqn. 2.

Therefore,

$$Y(s) = [s^2 C_c \Phi_0 B_c + s^2 C_s \Phi_s B_s (I_3 - K\Phi_0 B_c) + I_3 - K\Phi_0 B_c]U(s) , \tag{23}$$

which are the controlled modes, suppressed modes with spillover and rigid body modes with spillover.

The Tracking Problem

The tracking capability of the system represents the ability of the system to follow steady state command signals. In the case of the POF boom it is desired to track second (2nd) order polynomials. [3] The system output (tip angle of the boom) is given by equation 23 but the only steady state terms are given by the rigid body modes with spillover. Namely,

$$Y_{ss}(s) = (I_3 - K\Phi_0 B_c)U(s) \tag{24}$$

where U(s) is the command signal.

It is desired to track a system of constant equation R, such that

$$\tilde{Y}(s) = \frac{R}{s^3} \tag{25}$$

with a command signal

$$U(s) = \frac{\alpha_3 s^2 + \alpha_2 s + \alpha_1}{s^3} \tag{26}$$

which yields the tracking error

$$E(s) = Y_{ss} - \tilde{Y}(s) = (I_3 - K\Phi_0 B_c)\left(\frac{\alpha_3 s^2 + \alpha_2 s + \alpha_1}{s^3}\right) - \frac{R}{s^3}\ . \tag{27}$$

The first term of equation 27 reduces to a ratio of polynomials in s, whose denominator is composed of the optimum controlled modes, D(s). The total tracking error is then the ratio of two polynomials in s;

$$E(s) = \frac{N(s)}{s^3 D(s)} = \frac{K_0}{s^3} + \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{N_1(s)}{D(s)} \tag{28}$$

where the right half equation is the partial fraction expansion of E(s). Notice that the steady state tracking errors are given by the first three terms while the term, $\frac{N_1(s)}{D(s)}$, represents the excited controlled modes which rapidly damp out.

The approach taken in this work was to varying the α_1, α_2 and α_3 matrices to obtain zero steady state error coefficient; namely $K_0 = K_1 = K_2 = 0$. A complete derivation of this approach is shown in appendix one.

Results

The matrix riccati equation was solved using the PI of equation 14 and no control spillover occurred in the $\hat{Z}$ axis. This yielded an equation 24.

$$Y_{ss}(s) = \begin{bmatrix} 1 - \frac{1.765+2.5s}{s^2+2.69s+2.99} - \frac{4.3+1.2s}{s^2+2.05s+56.3} & \emptyset & \emptyset \\ \emptyset & 1 - \frac{1.765+2.66s}{s^2+2.69s+2.99} - \frac{4.3+1.3s}{s^2+2.05s+56.3} & \emptyset \\ \emptyset & \emptyset & 1 \end{bmatrix} U(s) \quad (29)$$

Using the approach above it was found the tracking error could be minimized by:

$$U_{\hat{x}}(s) = \frac{1.44s^2+2.91s+3}{s^3} R_{\hat{x}o} \quad (30)$$

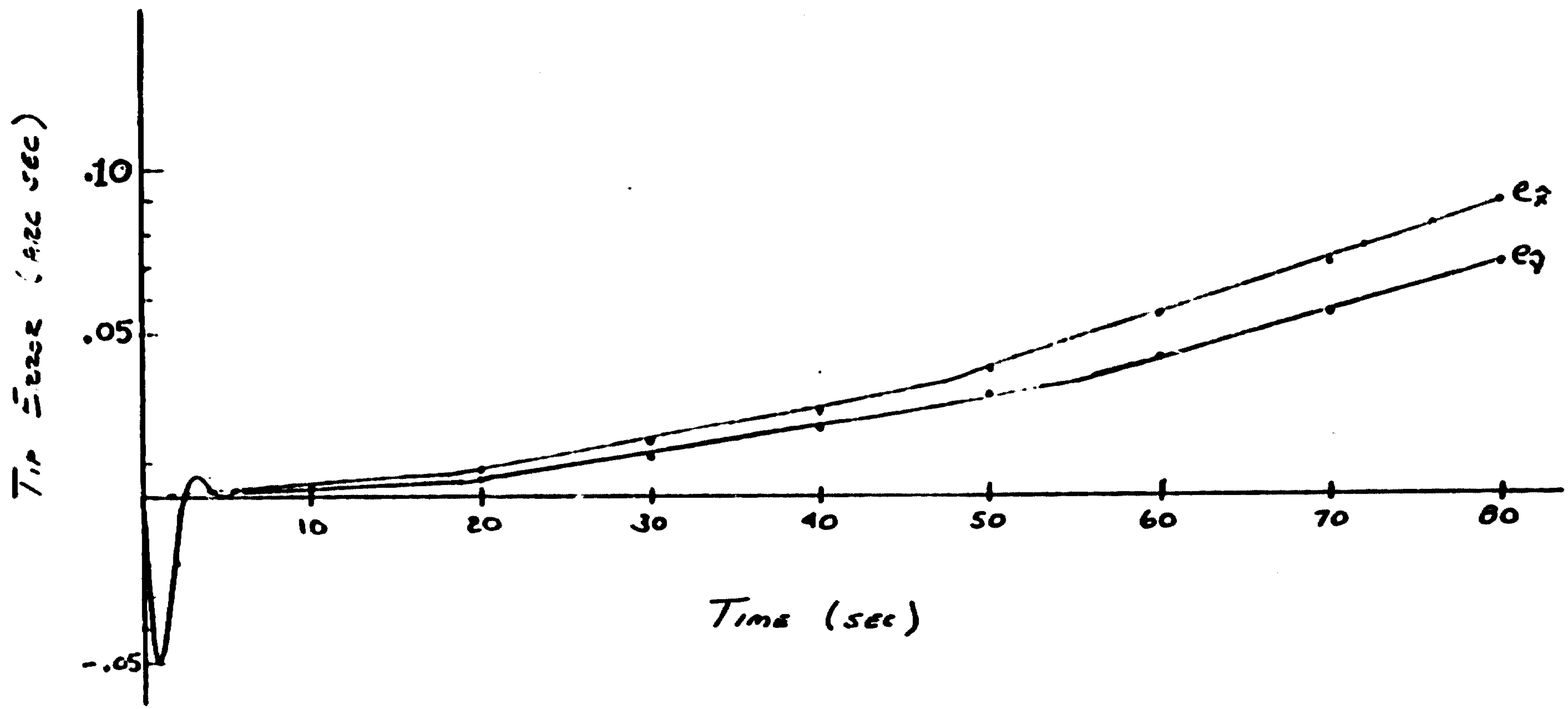
$$U_{\hat{y}}(s) = \frac{.828s^2+3.408s+3}{s^3} R_{\hat{y}o} \quad (31)$$

and,

$$U_{\hat{z}}(s) = \frac{R_{\hat{z}o}}{s^3}. \quad (32)$$

Plots of $e_{\hat{x}}(t)$, and $e_{\hat{y}}(t)$ are shown in Figure Two. Following errors start at zero as the shuttle starts drifting through its deadband and gradually increase to about .1 arc sec after 100 sec. During a typical shuttle deadband drift lasting 50 secs the maximum tracking error in $\hat{x}$ is .04 arc sec and .03 arc sec in $\hat{y}$.

FIGURE 2: Tip Following Errors During Gravity Gradient Drifting



Conclusions

The gimbal driven POF boom has been shown to be capable of tracking during shuttle gravity gradient drifting to within a few hundredths of an arc sec. The regulator system was reoptimized to eliminate $\hat{z}$ axis tracking errors entirely. This reoptimization did not change the characteristic eigenvalues of the system.

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APPENDIX I: DERIVATION OF TRACKING CONTROLLER

TRACKING CONTROLLER.

TO DERIVE THE OPTIMAL TRACKING CONTROLLER WE START WITH THE STEADY STATE TIP MOTION EQUATION IN LA PLACE FORM, NAMELY:

$$Y(s)_{ss} = \begin{bmatrix} 1 - \frac{1.765+2.5s}{s^2+2.69s+2.99} - \frac{4.3+1.2s}{s^2+2.05s+56.3}, & \phi, & \phi \\ \phi, & 1 - \frac{1.765+2.66s}{s^2+2.69s+2.99} - \frac{4.3+1.3s}{s^2+2.05s+56.3}, & \phi \\ \phi, & \phi, & 1 \end{bmatrix} U(s) \qquad (1)$$

$$= \begin{bmatrix} 1 - \frac{3.7s^3+14.4s^2+159.5s+112.2}{s^4+4.74s^3+64.8s^2+157.6s+168.3}, & \phi, & \phi \\ \phi, & 1 - \frac{3.96s^3+15s^2+168.8s+112.2}{s^4+4.74s^3+64.8s^2+157.6s+168.3}, & \phi \\ \phi, & \phi, & 1 \end{bmatrix} U(s) \qquad (2)$$

THE DESIRED TIP ANGLES ARE

$$\tilde{Y}(s) = \frac{1}{s^3}\begin{bmatrix} R_{\gamma 0} \\ R_{\eta 0} \\ R_{\xi 0} \end{bmatrix}, \qquad (3)$$

AND THE AVAILABLE CONTROLS ARE

$$U(s) = \frac{1}{s^3}\begin{bmatrix} \alpha_3 s^2 + \alpha_2 s + \alpha_1 \\ \beta_3 s^2 + \beta_2 s + \beta_1 \\ R_{\xi 0} \end{bmatrix}. \qquad (4)$$

THE TIP TRACKING ERRORS ARE THEN GIVEN BY

$$E(s) = Y(s)_{ss} - \hat{Y}(s) \quad . \qquad (5)$$

IMMEDIATELY WE SEE $E_{\hat{z}}(s) = 0$ BUT, IN GENERAL, $E_{\hat{x}}(s) \neq 0 \neq E_{\hat{y}}(s)$.

THE APPROACH TAKEN WAS TO PERFORM PARTIAL FRACTION EXPANSIONS ON $E_{\hat{x}}(s)$ AND $E_{\hat{y}}(s)$ AND CALCULATE $K_1, K_2, K_3, B_1, B_2, B_3$ FOR ZERO STEADY STATE ERROR TERMS. THE DAMPED SINUSOIDAL TERMS OF THE PARTIAL FRACTION EXPANSION DO NOT AFFECT STEADY TRACKING AND ARE IGNORED. IN GENERAL,

$$E(s) = \frac{N(s)}{s^3 D(s)} = \frac{K_0}{s^3} + \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{N_1(s)}{D(s)} \qquad (6)$$

$$K_0 = s^3 E(s)\Big|_{s=0} \qquad (7)$$

$$K_1 = \frac{d}{ds} s^3 E(s) = \frac{d}{ds} \frac{N(s)}{D(s)} = \frac{N'D - ND'}{D^2} \qquad (8)$$

AND,

$$K_2 = \frac{d^2}{ds^2} s^3 E(s) = \frac{d^2}{ds^2} \frac{N}{D} = \frac{N''D^2 - ND''D - 2N'DD' + 2N(D')^2}{D^3}. \qquad (9)$$

USING THESE FORMULAS (5-9) WE PROCEED TO FORM $E_{\hat{x}}(s)$, $E_{\hat{y}}(s)$ ETC.

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OF POOR QUALITY

$$E_{\hat{x}}(s) \cong -\frac{(3.7s^3+14.4s^2+159.5s+112.2)(\alpha_3 s^2+\alpha_2 s+\alpha_1)}{(s^4+4.74s^3+64.8s^2+157.6s+168.3)\,s^3} + \frac{\alpha_3 s^2+\alpha_2 s+\alpha_1-R_{x0}}{s^3}$$

Let $\alpha_4 = \alpha_1 - R_{x0}$

$$\begin{array}{l}
s^4+4.74s^3+64.8s^2+157.6s+168.3 \\
\underline{\qquad \alpha_3 s^2 + \alpha_2 s + \alpha_4} \\
\alpha_3 s^6 + 4.74\alpha_3 s^5 + 64.8\alpha_3 s^4 + 157.6\alpha_3 s^3 + 168.3\alpha_3 s^2 \\
\qquad \alpha_2 s^5 + 4.74\alpha_2 s^4 + 64.3\alpha_2 s^3 + 157.6\alpha_2 s^2 + 168.3\alpha_2 s \\
\underline{\qquad\qquad \alpha_4 s^4 + 4.74\alpha_4 s^3 + 64.3\alpha_4 s^2 + 157.6\alpha_4 s + 168.3\alpha_4} \\
\alpha_3 s^6 + (4.7\alpha_3+\alpha_2)s^5 + (64.8\alpha_3+4.7\alpha_2+\alpha_4)s^4 + (157.6\alpha_3+64.8\alpha_2+4.7\alpha_4)s^3 + \\
\qquad (168.3\alpha_3+157.6\alpha_2+64.8\alpha_4)s^2 + (168.3\alpha_2+157.6\alpha_4)s + 168.3\alpha_4
\end{array}$$

$$\begin{array}{l}
3.7s^3+14.4s^2+159.5s+112.2 \\
\underline{\qquad \alpha_3 s^2 + \alpha_2 s + \alpha_1} \\
3.7\alpha_3 s^5 + 14.4\alpha_3 s^4 + 159.5\alpha_3 s^3 + 112.2\alpha_3 s^2 \\
\qquad 3.7\alpha_2 s^4 + 14.4\alpha_2 s^3 + 159.5\alpha_2 s^2 + 112.2\alpha_2 s \\
\underline{\qquad\qquad 3.7\alpha_1 s^3 + 14.4\alpha_1 s^2 + 159.5\alpha_1 s + 112.2\alpha_1} \\
3.7\alpha_3 s^5 + (14.4\alpha_3+3.7\alpha_2)s^4 + (159.5\alpha_3+14.4\alpha_2+3.7\alpha_1)s^3 + (112.2\alpha_3+159.5\alpha_2+14.4\alpha_1)s^2 \\
\qquad + (112.2\alpha_2+159.5\alpha_1)s + 112.2\alpha_1
\end{array}$$

$$E_{\hat{x}}(s) = \frac{\begin{array}{l}\alpha_3 s^6 + (\alpha_2+\alpha_3)s^5 + (49.9\alpha_3+\alpha_2+\alpha_4)s^4 + (-1.9\alpha_3+49.9\alpha_2+\alpha_1-4.7R_{x0})s^3 \\ \quad + (56.1\alpha_3-1.9\alpha_2+50.4\alpha_1-64.8R_{x0})s^2 + (56.1\alpha_2-1.9\alpha_1-157.6R_{x0})s \\ \quad -56.1\alpha_1-168.3R_{x0}\end{array}}{s^3(s^4+4.74s^3+64.8s^2+157.6s+168.3)}$$

$$= \overbrace{\frac{56.1\alpha_1-168.3R_{x0}}{s^3}}^{K_{0\hat{x}}} + \frac{K_{1\hat{x}}}{s^2} + \frac{K_{2\hat{x}}}{s} + \underbrace{\cdots}_{\text{DAMPED TERMS}}$$

STEADY STATE

$$K_{1\hat{x}} = \left.\frac{d}{ds}\,s^3 E_{\hat{x}}(s)\right|_{s=0} = \frac{168.3(56.1\alpha_2-1.9\alpha_1-157.6R_{x0}) - 157.6(56.1\alpha_1-168.3R_{x0})}{168.3^2}$$

$$= \frac{56.1\alpha_2 - 54.4\alpha_1}{168.3}$$

$$K_{2\hat{x}} = \frac{d^2}{ds^2} s^3 E_{\hat{x}}(s)$$

$$= \frac{2(168.3)^2(56.1\alpha_3 - 1.9\alpha_2 + 56.4\alpha_1 - 64.8R_{\hat{x}0}) - (2)(168.3)(64.8)(56.1\alpha_1 - 168.3R_{\hat{x}0})}{168.3^3}$$

$$- \frac{2(168.3)(157.6)(56.1\alpha_2 - 1.9\alpha_1 - 157.6R_{\hat{x}0}) - (157.6)^2(2)(56.1\alpha_1 - 168.3R_{\hat{x}0})}{168.3^3}$$

$$= \frac{+112.2\alpha_3 - 108.8\alpha_2 + 159.54\alpha_1}{168.3}$$

LET

$$K_0 = 0 = 56.1\alpha_1 - 168.3R_{\hat{x}0}$$

$$\alpha_1 = \frac{168.3}{56.1}R_{x0} \cong 3 \quad R_{\hat{x}0}$$

$$K_1 = 0 = \frac{56.1\alpha_2 - 56.4\alpha_1}{168.3}$$

$$\alpha_2 = \frac{56.4}{56.1}\alpha_1 = 2.91 \quad R_{\hat{x}0}$$

$$K_2 = 0 = \frac{-108.8\alpha_2 + 159.54\alpha_1 + 112.2\alpha_3}{168.3}$$

$$\alpha_3 = \frac{108.8\alpha_2 - 159.54\alpha_1}{112.2} = \frac{108.8(2.91) - 159.54(3)}{112.2}R_{x0}$$

$$\alpha_3 = -1.44R_{\hat{x}0}$$

$$\therefore \quad U_{\hat{x}}(s) = \frac{(-1.44s^2 + 2.91s + 3)}{s^3} \qquad R_{\hat{x}0}$$

$$u_{\hat{x}}(t) = (-1.44 + 2.91t + 1.5t^2)R_{\hat{x}0} \qquad t \geq 0$$

$$E_{\hat{y}}(s) = \frac{\beta_3 s^2 + \beta_2 s + \beta_1 - R_{\hat{y}0}}{s^3} - \frac{(3.96s^3 + 15s^2 + 168.8s + 112.2)(\beta_3 s^2 + \beta_2 s + \beta_1)}{s^3(s^4 + 4.74s^3 + 64.3s^2 + 156.6s + 140.2)}$$

LET $\beta_4 = \beta_1 - R_{\hat{y}0}$

$$\begin{array}{r} 3.96s^3 + 15s^2 + 168.8s + 112.2 \\ \beta_3 s^2 + \beta_2 s + \beta_1 \\ \hline 3.96\beta_3 s^5 + 15\beta_3 s^4 + 168.8\beta_3 s^3 + 112.2\beta_3 s^2 \\ 3.96\beta_2 s^4 + 15\beta_2 s^3 + 168.8\beta_2 s^2 + 112.2\beta_2 s \\ 3.96\beta_1 s^3 + 15\beta_1 s^2 + 168.8\beta_1 s + 112.2\beta_1 \\ \hline 3.96\beta_3 s^5 + (15\beta_3 + 3.96\beta_2)s^4 + (168.8\beta_3 + 15\beta_2 + 3.96\beta_1)s^3 + (112.2\beta_3 + 168.8\beta_2 + 15\beta_1)s^2 \\ + (112.2\beta_2 + 168.8\beta_1)s + 112.2\beta_1 \end{array}$$

$$E_{\hat{y}}(s) = \frac{\begin{array}{c} \beta_3 s^6 + (.74\beta_3 + \beta_2)s^5 + (49.8\beta_3 + .74\beta_2 + \beta_4)s^4 + (-11.2\beta_3 + 49.8\beta_2 + .74\beta_1 - 4.7R_{y0})s^3 \\ + (56.1\beta_3 - 11.2\beta_2 + 49.8\beta_1 - 64.8R_{y0})s^2 + (56.1\beta_2 - 11.2\beta_1 - 157.6R_{y0})s \\ + (56.1\beta_1 - 168.3R_{y0}) \end{array}}{s^3(s^4 + 4.74s^3 + 64.8s^2 + 157.6s + 168.3)}$$

$$= \frac{K_{0\hat{y}}}{s^3} + \frac{K_{1\hat{y}}}{s^2} + \frac{K_{2\hat{y}}}{s} + \underbrace{\cdots}_{\text{DAMPED TERMS}}$$

$$K_{0\hat{y}} = \frac{56.1\beta_1 - 168.3R_{y0}}{168.3}$$

$$K_{1\hat{y}} = \frac{d}{ds} s^3 E_{\hat{y}}(s)\Big|_{s=0} = \frac{168.3(56.1\beta_2 - 11.2\beta_1 - 157.6R_{y0}) - 157.6(56.1\beta_1 - 168.3R_{y0})}{168.3^2}$$

$$K_{1\hat{y}} = \frac{56.1\beta_2 - 63.73\beta_1}{168.3}$$

$$K_{2\hat{y}} = \frac{d^2}{ds^2} s^3 E_{\hat{y}}(s)\Big|_{s=0}$$

$$= \frac{2(168.3)^2(56.1\beta_3 - 11.2\beta_2 + 49.8\beta_1 - 64.8R_{y0}) - (2)(64.8)(168.3)(56.1\beta_1 - 168.3R_{y0})}{168.3^3}$$

$$- \frac{(2)(168.3)(157.6)(56.1\beta_2 - 11.2\beta_1 - 157.6R_{y0}) - (2)(157.6)^2(56.1\beta_1 - 168.3R_{y0})}{168.3^3}$$

$$K_{\ddot{\delta}} = \frac{(3)(168.3)^2(56.1\beta_3 - 11.2\beta_2 + 49.8\beta_1 - 64.5R_{\gamma 0}) - (2)(64.5)(168.3)(56.1\beta_1 - 168.3R_{\gamma 0})}{168.3^3}$$

$$- \frac{2(168.3)(153.6)(56.1\beta_2 - 11.2\beta_1 - 153.6R_{\gamma 0}) - 2(153.6)^2(56.1\beta_1 - 168.3R_{\gamma 0})}{168.3^3}$$

$$= \frac{112.2\beta_3 - 127.47\beta_2 + 175.76\beta_1}{168.3}$$

LET $K_{\dot{\delta}} = K_{\ddot{\delta}} = K_{\dddot{\delta}} = 0$

$K_{\dot{\delta}} = 0 \Rightarrow \quad 56.1\beta_1 = 168.3 R_{\gamma 0}$

$\beta_1 = 3. \; R_{\gamma 0}$

$K_{\ddot{\delta}} = 0 \Rightarrow \quad 56.1\beta_2 = 63.73\beta_1$

$\beta_2 = 3.408 R_{\gamma 0}$

$K_{\dddot{\delta}} = 0 \Rightarrow \quad \beta_3 = \frac{127.47\beta_2 - 175.76\beta_1}{112.2} = \frac{127.46(3.408) - 175.76(3)}{112.2}$

$$\beta_3 = -.828 R_{\gamma 0}$$

$$U_{\gamma}(s) = \frac{-.828s^2 + 7.774s + 5.007}{s^3} R_{\gamma 0}$$

$$u_{\gamma}(t) = (-.828 + 7.774t + 2.5t^2) R_{\gamma 0} \qquad t \geq 0$$

APPENDIX II: FEEDBACK MATRICES

----------- FEEDBACK K MATRIX --------------

.5151-003	.7772-003	-.1298-002	-.1958-002	-.1487-001	-.4259-002	.1719-001	.4925-002
-.1298-002	-.1958-002	-.5151-003	-.7772-003	.1719-001	.4925-002	.1487-001	.4259-002
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

---------- BC*K MATRIX ------------

.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.1765+001	.2663+001	-.1490-007	.0000	-.2714+002	-.7778+001	-.9453+001	-.2708+001
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.2980-007	.7451-008	.1765+001	.2663+001	.9453+001	.2708+001	-.2714+002	-.7778+001
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.2889+000	-.3756+000	.8671-001	.1308+000	.4292+001	.1229+001	-.8941-007	-.1490-007
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.8671-001	-.1308+000	-.2889+000	-.3756+000	.2980-007	-.2980-007	.4292+001	.1229+001

---------- B1*K MATRIX --------------

.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.3250-001	-.4906-001	.9143-001	.1379+000	.9893+000	.2834+000	-.1232+001	-.3528+000
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.9143-001	-.1379+000	-.3250-001	-.4906-001	.1232+001	.3528+000	.9893+000	.2834+000
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.4258-001	-.6425-001	-.2472-001	-.3733-001	.5227+000	.1496+000	.6081+000	.1742+000
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.2472-001	.3730-001	-.4258-001	-.6425-001	-.6081+000	-.1742+000	.5223+000	.1496+000
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.5151-003	.7772-003	-.1298-002	-.1958-002	-.1487-001	-.4259-002	.1719-001	.4925-002
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.1298-002	-.1958-002	-.5151-003	-.7772-003	.1719-001	.4925-002	.1487-001	.4259-002
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

$B_R K$ (bracketing the last six rows)

RICCATI EQN: $-\dot{P} = Q_c + PA + A^T P - P B_c R^{-1} B_c^T P$

WHERE $R = R_c + B_1^T R_1 B_1$

& $K = R^{-1} B_c^T P$

ORIGINAL PAGE IS
OF POOR QUALITY