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PROSPECTS FOR TLRS BASELINE ACCURACIES

IN THE WESTERN USA USING LAGEOS

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ABSTRACT

One of the main goals of the LAGEOS satellite mission is the detection of regional geotectonic movements. A parametric study with the intention to obtain the optimal baseline precision from dynamic solutions of laser ranging to LAGEOS is presented. The varied parameters are: length of reduced arc, number of tracking stations, data noise and rate, data biases, refraction errors, system efficiency, gravity model errors and errors in the value of GM. The baseline precisions are 1-10 cm depending upon the set of parameters adopted. General principles obtained from the study are also presented.

CONTENTS

1.0 INTRODUCTION

Space techniques provide unique capabilities for measuring important parameters related to earth dynamics. They include laser ranging to satellites which is a precise tool for the measurement of tectonic plate motions and crustal deformations over distances of hundreds to thousands of kilometers. The measurement and modelling of regional crustal deformations is an important subprogram of NASA's Crustal Dynamics Plan (NASA, 1979). The objectives of this program is to determine the physical mechanisms responsible for regional tectonic and geologic processes and to develop an improved understanding of how regional phenomena fit into the framework of global tectonics.

The highly stable orbit of the LAGEOS satellite allows accurate estimation of the absolute geocentric positions of the stations of the laser tracking network (Smith et al., 1979a). The relative movement of sites, however, which is an important quantity for studies of regional crustal deformation, can also be determined through the estimation of relative station positions and intersite distances (baselines).

Mobile laser systems with numerous site occupancies (the Mobile and/or Transportable Laser Systems) will play a very important role in the Regional Crustal Deformation Measurement Program (NASA, 1979). The Crustal Dynamics Project Plan contains a detailed schedule for deploying the MOBLAS and TLRS facilities for regional studies, with western North America as the highest priority area.

The plate velocities in areas selected for study in this program are of the order of a few centimeters per year, so the regional-scale deformational velocity measurements must have an accuracy of a few millimeters per year. Such accuracies will permit comparisons of the regional motions with global ones, and will permit regional differences to be detected. Such accuracies are attainable with few-centimeter accuracy in the measurement of intersite distances, within about a decade of the beginning of the measurement program. For example, a baseline accuracy of 5cm from measurements made one year apart over a period of 10 years, gives rise to a velocity uncertainty of 0.5cm/yr. As the velocity uncertainties are directly proportional to the baseline accuracies, much attention has been devoted to increasing the accuracy of the baseline determinations.

The objectives of the present simulation study are:

a. to obtain the general principles and improve our understanding of the dynamical reduction of satellite laser data for the determination of baselines of moderate length (200-500km),

b. to identify the dominant sources of error and recommend the optimal reduction method(s) that would minimize their effect on the baselines, and

c. to obtain a qualitative assessment of baseline accuracies which can be expected given our current and anticipated knowledge of the satellite's dynamic forces, data quality and geodetics of the reference system.

The LAGEOS spacecraft was specifically designed for laser tracking and is a completely passive satellite target. Orbiting at nearly 6000km altitude, LAGEOS is much less affected by the less well-known short wavelength features in the gravity field than lower geodetic laser satellites. Due to the spacecraft's spherical shape (60cm diameter) and its heavy weight (411 kg) LAGEOS is almost unaffected by the forces of the solar radiation pressure, earth albedo and air-drag. As a result, the orbit of LAGEOS may be determined with high precision for time spans of several months. This characteristic of the LAGEOS's orbit plays a very important role in the overall improvement of the baseline accuracies. Typical Kepler elements and rates for LAGEOS are given in Table 1.

Table 1. Orbital Elements For LAGEOS

Semi-major axis	12270 km
Eccentricity	.004
Inclination	109.84°
Node rate ($\dot{\Omega}$)	0.342°/day
Argument of perigee rate ($\dot{\omega}$)	-0.24°/day
Period	225.5 min.

2.0 LASER DATA ANALYSIS TECHNIQUES

The laser systems are highly accurate ranging instruments, which, while providing the most accurate means of tracking a near earth satellite, are not "all-weather systems." It is therefore quite difficult to schedule and subsequently achieve ideal tracking for numerous sites separated by great distances. It is of equal concern, in practice, to be able to recognize the ideal tracking configurations, and likewise, find the optimal usage for the data which are actually obtained. While the latter is a far more common problem, the concept of ideal tracking configurations warrants some discussion. The reduction of laser tracking of orbiting objects normally presents a dynamical modeling problem with numerous contributing forces (gravitational, radiative, etc.), all of which are known to some level of imperfection. Consequently, the elimination of the reliance of force modeling altogether for the baseline solutions is a worthwhile goal. Such solutions, which rely solely on the geometry of the tracking and are taken simultaneously from the participating sites in sufficient number, provide unique relative positioning solutions for the respective sites which are no longer in any way dependent on accurate knowledge of the orbital position of the satellite. It is sufficient to know that all sites observe a common object simultaneously, irrespective of where the object is located. The data requirements, station configurations and other constraints of this geometric method are discussed in (Escobal et al., 1973). The geometric approach to the recovery of interstation distances has certain drawbacks. Firstly, the data requirements are severe and rarely, if ever, satisfied under the present deployment schedules for existing laser systems. Secondly, since the range data themselves exclusively define the solution to the station positioning, there is no external control (such as orbital dynamics) preventing the propagation of all measurement errors into the resulting solution. While in practice, the laser systems perform quite well (Vonbun, 1977), leaving this latter concern of diminished importance, it still cannot be disregarded if 1 to 3 cm baseline accuracies are desired.

The objective of our simulations is the design and execution of an optimal solution using the available data to enhance the recoverability of the experimental objectives (precision baseline determination between the laser network sites, or subsets therein). Since, in all likelihood, dynamical methods will be required, a general discussion of this method within the context of precision station positioning is important.

Even for LAGEOS, orbital determination at the 10 cm level requires extensive knowledge of several hundred terms in the geopotential, the atmospheric density, ballistic characteristics of the spacecraft, cross-sectional solar aspect area, lunar and solar masses and ephemerides, relativistic effects and others. Although satellite ephemeris error is not synonymous with station positioning error, as a general rule, ephemeris error during the tracking of the satellite is a significant error source for station positioning which uses the satellite's orbit as a reference.

Generally, over the years of experimentation with laser ranging, it has been deduced that by employing shorter arc lengths to process subsets of the data, there is less propagation of the uncertainties of force model effects as errors in the calculated ephemeris of the spacecraft (Dunn et al., 1979). This can logically be extended to the geometrical case, where simultaneous sets of data points are processed individually. However, there are times when truly short arcs are inadequate for use due to poor data availability and the resulting inability to define the orbital reference system.

Force model errors will be present, to varying degrees, in all dynamical experiments. To minimize force model contamination, one can wait for improved force models, such as an improved harmonic coefficient model of the earth's geopotential, but such models are difficult to develop and may still prove to be of insufficient accuracy. Certainly, the design of any experiment should have as an objective the minimization of the effect of dynamical errors in the measured quantities.

The San Andreas Fault Experiment (SAFE) is a case in point (Smith et al., 1979b). The baseline between San Diego and Mount Quincy has been successively measured numerous times within the past ten years. Since the measure of large scale plate motion was sought along the San Andreas

Fault, the experiment was successfully performed through an acceptance of the presence of dynamical errors in each individual baseline measurement, but efforts were made to make this error constant for all subsequent bi-yearly recoveries. Therefore, groups of three successive passes of laser data on BE-C were used in each separate bi-yearly analysis. Geometry of the passes during the subsequent occupations was selected so that it was virtually identical to the original and the same force models were utilized. In this way, all errors which are functions of geodetic position of the spacecraft manifest themselves in a very similar way for the 1972, 1974, 1976 and 1979 subset solutions. While the determined individual baselines between San Diego and Quincy are therefore biased, the relative fault motion obtained through the differentiation of these results is a highly accurate measurement, for the bias, year to year, has been made nearly constant.

3.0 TLRS BASELINE PRECISION FROM LAGEOS: A SIMULATION

The particular focus of this error analysis is to assess the utility of the Transportable Laser Ranging System (TLRS) for monitoring long term tectonic activity in the Western United States. The TLRS has now completed its test and validation activities, and is actively deployed as part of NASA's Crustal Dynamics Program. It is a low energy laser operating using single photoelectron detection techniques (Silverberg, 1981). The system was built for NASA by the University of Texas McDonald Observatory. The high mobility of the TLRS lies in its total containment within a single RV chassis allowing it to use crudely prepared sites with setup time requirements which seldom exceed a few hours. The analysis performed in this study is also applicable to other highly mobile laser ranging systems, such as the Compact Laser Ranging System (CLRS/TLRS-II) which is undergoing development at Goddard Space Flight Center (Johnson, 1981).

Our analysis has centered on estimating baseline accuracies for various TLRS deployment schemes in California. Figure 1 presents a map indicating the three TLRS sites we have considered. They are separated by distances of between 200 and 500 km. Baselines in both North-South (in Figure 1: the baseline from Site 1 to Site 2) and East-West (Site 1 to Site 3) directions were investigated. The MOBLAS systems, like those deployed at San Diego (SANDIE) and QUINCY have scheduled site occupancies of several months. They are therefore usually treated as fixed sites within our analysis. We have tried to address most of the practical problems related to TLRS site occupancies including length of stay requirements for the TLRS, the requirements of the global laser network to support the TLRS activities, and various procedures for minimizing the effect of dynamic errors in the reduction of the laser data for baseline estimation. It is important to reiterate that the determination of intersite distances and not the geocentric station coordinates were the experimental objectives. Consequently, baseline errors due to dynamic sources could be minimized although systematic errors in the station coordinates were present. Station position errors were studied, however, to provide some insight into the characteristics of dynamical error propagation.

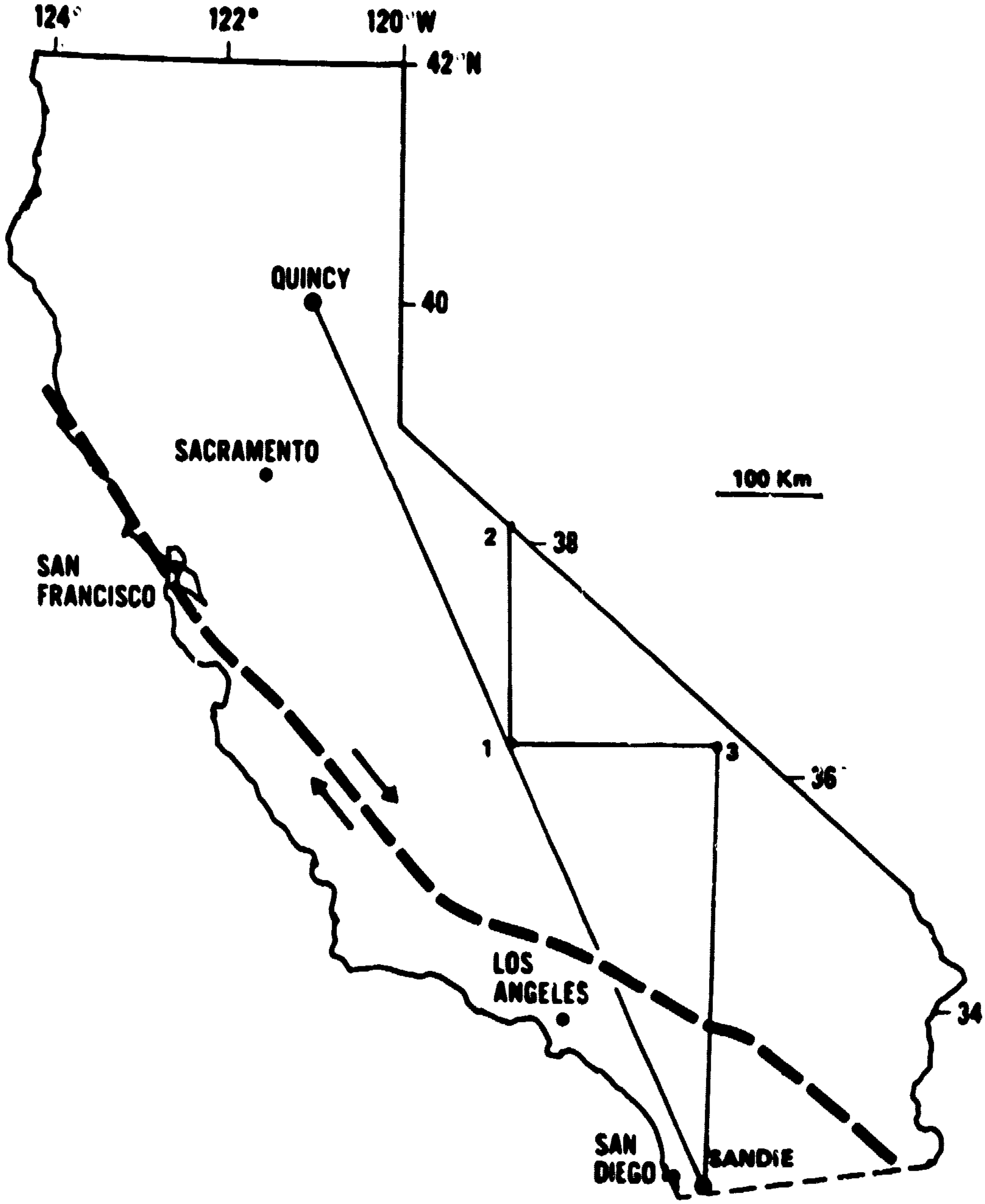

FIGURE 1. TLRS AND MOBLAS TRACKING STATIONS

In most of our simulations we have assumed that the laser systems, be they TLRS or fixed instruments, have an efficiency of 50%. By this we mean that only half of all the visible Lageos passages, complete and randomly selected, are successfully acquired. This is in fair agreement with the history of system performances on the west coast of the United States.

3.1 ORAN (ORBITAL ANALYSIS) PROGRAM

The simulation study was accomplished through the use of a modified version of the ORAN program (Martin and Roy, 1972). This computer program simulates a Bayesian least squares data reduction for orbital and geodetic parameters. It does not process actual data. Through the generation of accurate normal equations, it has the capability of computing the accuracy of the results if measurements of a given accuracy are available and processed in a least squares data reduction program. ORAN is designed to consider a data reduction process in which a number of satellite data spans are reduced individually as well as simultaneously.

The term arc refers to a specific data period over which a satellite's orbit is integrated; this data solely defines the basis by which the satellite's position is adjusted. The effect of model uncertainties and/or measurement errors are propagated into the set of basic parameters and into the estimated orbit over the arc interval. The effects of all error sources are then statistically combined to produce a measure of the total resulting accuracy for both the orbit and the other adjusted parameters. ORAN was modified to accommodate the effect of a full variance/covariance matrix of a gravity field on the adjusted parameters and to propagate the various errors into the baseline statistics directly. These modifications were necessary given the high correlation among certain coefficients of the low degree geopotential field as well as the need to thoroughly assess cancellation of errors into adjusted intersite distances from stations which are in reasonable close proximity to one another.

The error sources we have considered in our error analysis are summarized in Table 2.

Table 2. Summary of TLRS Baseline Error Analysis Parameters

<u>Adjusted Parameters:</u>

$X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}$	state vector of LAGEOS at epoch
ϕ_1, λ_1, h_1	for TLRS Site 1
ϕ_2, λ_2, h_2	for TLRS Site 2
(Implied Baseline)	

<u>Modeled Parameters:</u>

• Dynamic Errors:	
Gravity Field	100% of V/C matrix of GEM 9 to 10 x 10
GM	1 part in 10^7
Earth and Ocean Tides	1% error in k_2
Ocean Loading	100% of effect
Station Tidal Displacement	10% error in h_2 and ℓ_2
Solar Radiation Pressure	0.5% of effect
• Measurement Errors and Precision:	
System Efficiency	50% (complete passes randomly selected)
Measurement Interval	1 pt./sec.
Range Noise	10 cm
Range Bias	10 cm
Tropospheric Retraction	1% of effect
Elevation Cutoff Angle	20°
• Reference System Errors:	
Fixed Station Positions	25 cm in each coordinate
Pole ΔX, ΔY	10 milliarc seconds

3.2 LAGEOS BASELINE SIMULATION RESULTS

3.2.1 General Principles

The Lageos baseline simulations were performed in an evolutionary process. Initially, certain global properties of the baseline recovery problem were investigated. Subsequent simulations made use of the conclusions and principles of this earlier global analysis permitting details of the TLRS baseline recovery to then be studied.

The initial objectives of the study were to assess the minimum tracking requirements for the fixed laser stations of the global tracking network to support TLRS activities. A comparison was made between baseline recoveries for TLRS sites in which error sources were assumed to be constant. The tracking configuration was allowed to vary, however, so that:

- In the first case, only the fixed west coast sites of Mount Quincy and San Diego were used, while

- In the second case, six globally deployed sites (Figure 2) supported the orbit determination of LAGEOS.

Table 3 summarizes the obtained results and shows that the estimated accuracy of the TLRS baselines was not dependent on global tracking support. Both cases yielded very similiar baseline error estimates. In this simulation a 200 km (N-S) baseline was estimated between two TLRS sites within a 5 day arc length where all laser system were assumed to have 100% tracking efficiency. This level of efficiency, although unrealistic, was used so that the principle of global geometry requirements alone could be tested. Again, this simulation showed that global tracking was not required. Therefore, for all subsequent simulations we adopted a minimum requirement having only the west coast fixed mobile laser sites supporting the TLRS.

FIGURE 2
THE GLOBAL LASER NETWORK

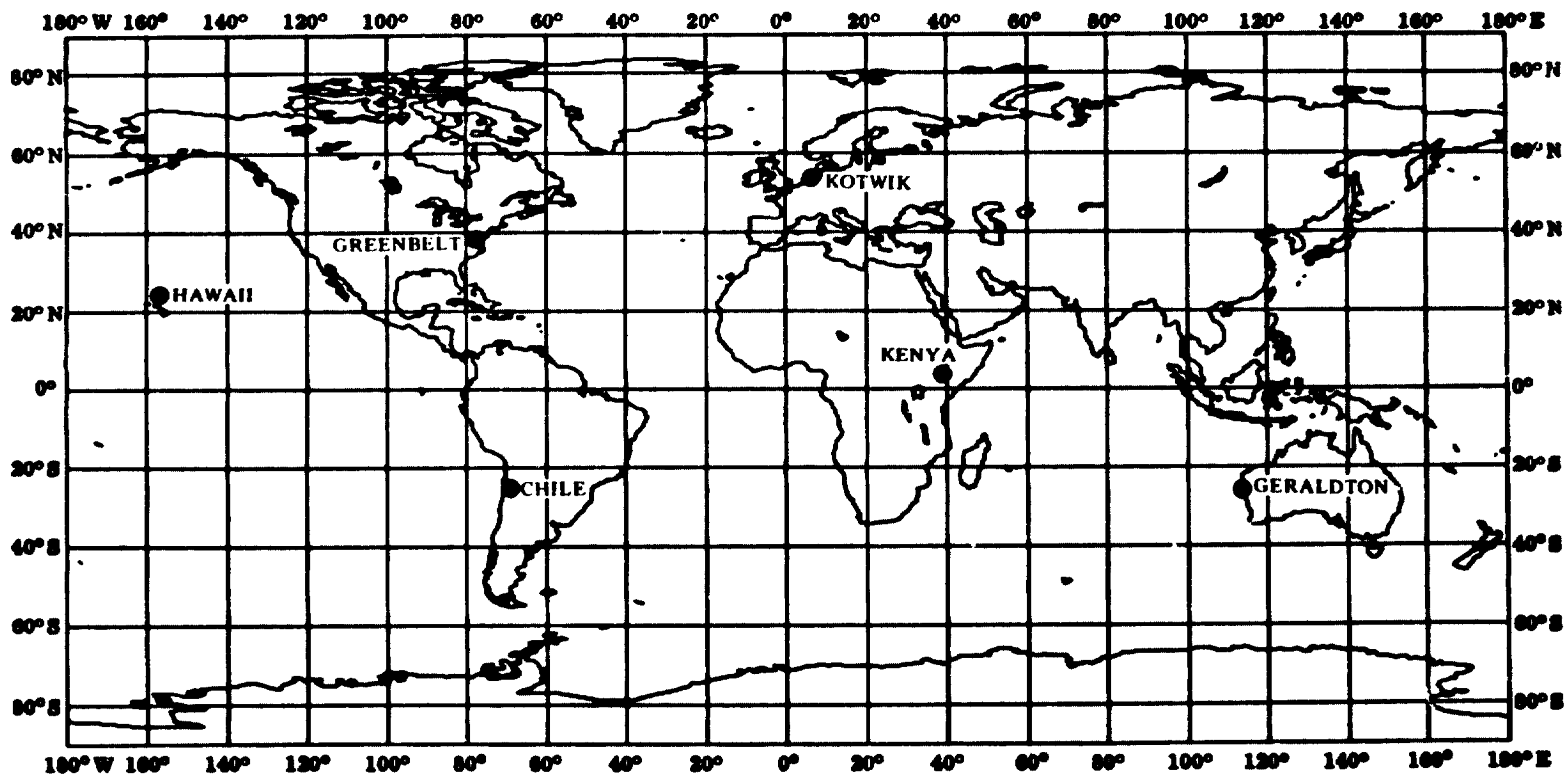

Table 3. (N-S) Baseline Accuracies (cm)
Two TLRSs: 100% Efficiency (5 Day Arc)

ERROR SOURCES	Case 1 SANDIE & QUINCY FIXED	Case 2 6 GLOBAL STATIONS FIXED
Positions	0.1	0.1
Biases	1.9	2.0
Refraction	1.5	1.5
GM	1.5	1.4
Gravity	0.3	0.3
Noise	0.1	0.1
Total (RSS)	2.9	2.9

The second conclusion from this initial simulation is summarized in Table 4. The estimated accuracy (worst case) of the geodetic coordinates of the TLRSs are compared to their baseline accuracy estimate. The estimated accuracy of the determined baseline is approximately an order of magnitude better than the coordinates themselves. The dynamic errors, therefore, are shown to manifest themselves similarly in positional errors of nearby sites making highly accurate baseline estimations a realistic goal. This is similar to the conclusions of the SAFE experiment.

We next sought an estimate of the effect of reduced tracking efficiency on the baseline error estimates. Table 5 presents the results of this phase of our analysis. Two problems were considered. Firstly, what was the impact of severe degradation of the tracking efficiency to 20% for the fixed sites along with the TLRS, (case I-b). In a second case, only the TLRS system was degraded (case I-c). It is apparent from these results that the accuracy of the baseline is highly sensitive to the efficiency of the TLRSs and much less so to the efficiency of the fixed stations. The changing magnitude of the baseline errors due to geopotential uncertainties is responsible for this effect. However, as discussed later, this property of the problem has undergone extensive evaluation.

Therefore, the general conclusions of this phase of our analysis are:

a. global laser tracking is unnecessary for TLRS support if local tracking is available,

b. the station coordinate errors are systematic, and while they are in the decimeter level, baseline precision is approximately an order of magnitude better, and

c. when the TLRS efficiency is degraded below 100%, special consideration of geopotential errors is necessary.

Table 4. TLRS Position and Baseline Accuracies (cm)
Two TLRSs: 100% Efficiency, SANDIE AND QUINCY Fixed
(5 Day Arc, Case 1)

ERROR SOURCES	LATITUDE	LONGITUDE	HEIGHT	BASELINE
Positions	16.2	17.8	7.3	0.1
Biases	6.4	0.2	15.4	1.9
Refraction	2.9	0.1	5.9	1.5
GM	27.6	0.6	41.8	1.5
Gravity	7.4	0.3	10.0	0.3
Noise	0.1	0.1	0.1	0.1
Total (RSS)	33.6	17.8	46.6	2.9

Table 5. (N-S) Baseline Accuracies (cm)
Two TLRSs: SANDIE, QUINCY Fixed (5 Day Arc)
Variable Efficiency

ERROR SOURCES	Case 1 100% All Stations	Case 1-b 20% All Stations	Case 1-c 20% TLRSs 100% Fixed
Positions	0.1	1.0	0.6
Biases	1.9	2.5	2.4
Refraction	1.5	2.0	1.8
GM	1.5	3.7	2.1
Gravity	0.3	16.7	17.0
Noise	0.1	0.2	0.2
Total (RSS)	2.9	17.4	17.4

3.2.2 Optimization of Baseline Determination for a Realistic TLRS Deployment Environment

Under the assumption of realistic tracking characteristics, the next phase of our analysis addressed the following questions:

a. what is the best reduction scheme for data from a single TLRS having consecutive site occupancies? and

b. what is the best scheme assumming two TLRSs are providing laser ranging over the same time period?

In this series of investigations we provided for a 50% tracking efficiency for all west coast sites participating in the tracking of LAGEOS. The length of stay for the TLRS was varied, as was the reduction methodology.

3.2.2.1 One TLRS

For the situation in which a single TLRS is occupying successive sites, and the determination of the baseline between these sites is the objective, numerous data reduction methods and varying lengths of stay for the TLRS are possible. We have attempted to evaluate the extremes of the suggested deployment schedule by varying the length of stay from five days (as in Bender and Goad, 1979) to thirty days per site. Figure 3 is a pictorial synopsis of the configurations and data reduction methods which were evaluated.

In the cases where the length of the reduced arc was longer than 5 days J_{11} and J_{12} were adjusted in order to accommodate long period errors in the zonals of the geopotential. This results in poorly determined individual coefficients, but the resulting ephemeris will be relatively free of long period zonal biasing. Since LAGEOS has a high inclination we adjusted zonal

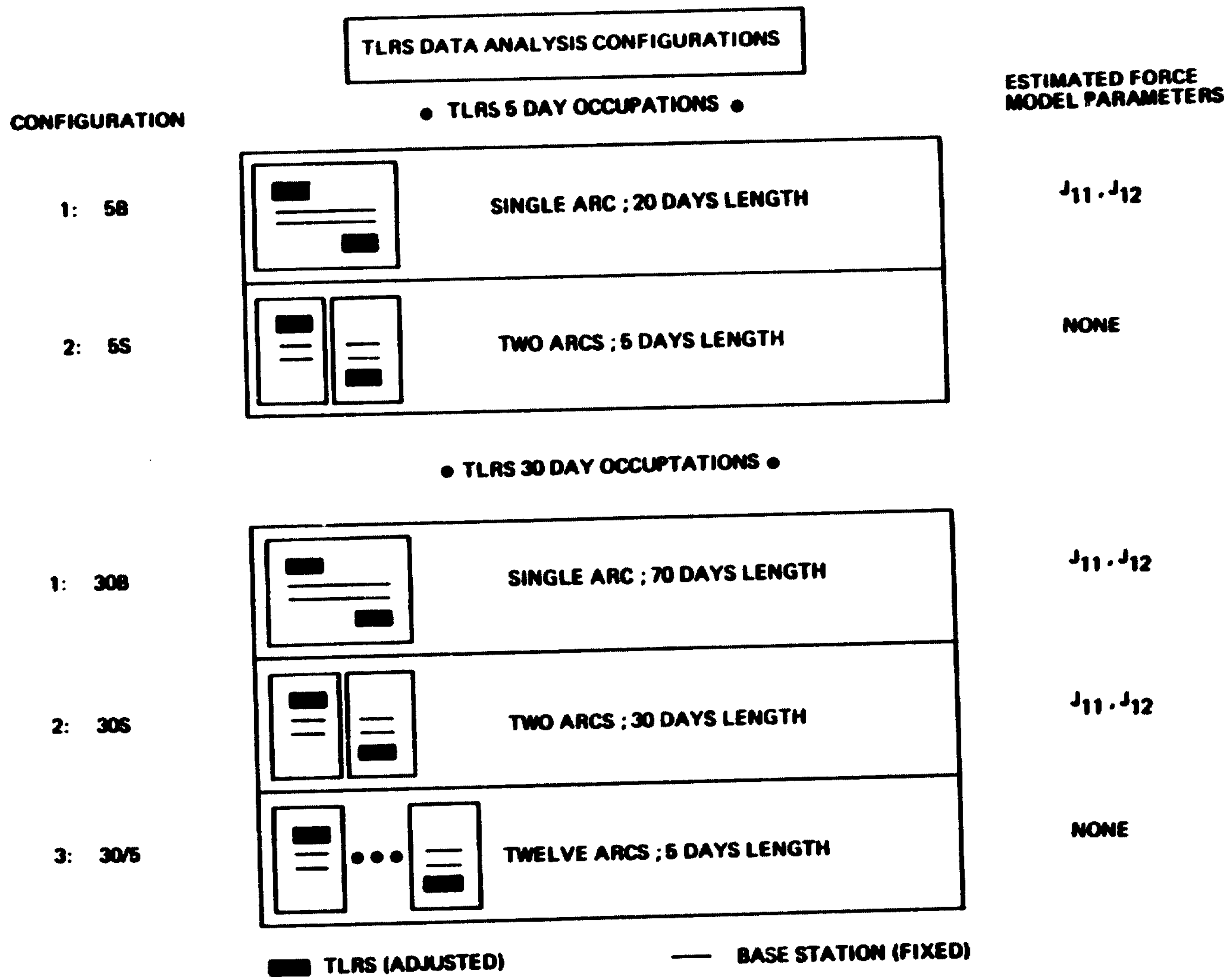
TLRS DATA ANALYSIS CONFIGURATIONS
CONFIGURATION
ESTIMATED FORCE MODEL PARAMETERS
• TLRS 5 DAY OCCUPATIONS •
1: 5B
SINGLE ARC ; 20 DAYS LENGTH
J11 , J12
2: 5S
TWO ARCS ; 5 DAYS LENGTH
NONE
• TLRS 30 DAY OCCUPTATIONS •
1: 30B
SINGLE ARC ; 70 DAYS LENGTH
J11 , J12
2: 30S
TWO ARCS ; 30 DAYS LENGTH
J11 , J12
3: 30/5
TWELVE ARCS ; 5 DAYS LENGTH
NONE
TLRS (ADJUSTED)
BASE STATION (FIXED)

FIGURE 3

coefficients of higher degree due to their insignificant short period gravitational perturbations which is not the case for the low degree terms. Short period effects will be somewhat altered nevertheless. For arcs of length shorter than 5 days, the adjustment of zonals is normally unnecessary due to the extensive ability of the orbital state vector adjustment to absorb these zonal errors.

The data reduction schemes were evaluated for multiple short arcs versus arcs of longer length. Each configuration in Figure 3 is labeled (e.g., 5B, 5S, etc.) for reference in the subsequent summary tables. Table 6 presents the breakdown of the contributing error sources in 200 km baselines (both N-S and E-W) for consecutive site occupancies of the TLRS. The baseline accuracies are computed by combining the largest errors from each of the error sources as obtained from four similar runs spanning a 2 1/2 year period. They can be interpreted as approximately 2σ accuracy estimates.

Table 6 also presents a case in which a tracking efficiency of 100% has been utilized (solution 5S/100%) for a five day arc length. This solution, consequently, has the maximum tracking geometry, the minimum errors from data noise, and is useful for comparison purposes. It provides insight into the relative degradation in baseline accuracy from sampling sources arising from poorer tracking efficiency.

A discussion of the propagation of these error sources into the baseline accuracies is useful. It is important to understand that within a dynamical system, most error sources manage to corrupt the along track position of the calculated spacecraft ephemerides. However, the timing of the laser observations, and the minimization of the range observations itself at the point of closest approach to the station, provides a means for estimating an accurate period of the orbit notwithstanding the presence of these error sources. In other words, there is a strong dynamical control for the characteristics of the along track error (e.g., in the presence of tracking data, they cannot degrade secularly). When tracking both from the orbital ascent and decent is available, along track errors tend to ultimately cancel. Systematic errors arising out-of-plane in the orbital ephemeris are less subject to dynamical control, and tend to have poorer properties for their cancellation when estimating inter-station distances. Therefore, longer arc lengths generally are required for cancellation of cross track effects because the distribution of data longitudinally must achieve a balance. For the high inclination of the LAGEOS orbit, cross track is primarily in the longitudinal direction. The propagation of errors shown in Table 6 is discussed below.

The fixed station position errors are assumed to be uniformly 25 cm in each coordinate. However, one can deduce from the result shown in Table 6, that the resulting along track errors which are in the direction of the N-S estimated baseline are always smaller than their corresponding manifestations for the E-W baseline, which is in the out-of-plane direction. Through an increase in the arc length, more error cancellation becomes the case, and the errors overall in both directions are reduced. However, there is always more tracking symmetry north to south about the TLRSs than east to west.

For data passes which are not directly overhead, the best least squares accommodation of a range bias, if the pass is symmetrical from the northern to southern horizons, is an adjustment in the TLRS longitude. As a result, as shown in Table 6, adjusted E-W baselines are more sensitive to bias effects in shorter arc lengths which have a poorer data sampling in the longitudinal direction. However, for the thirty day arcs, good geometry is achieved and the N-S and E-W sensitivities become quite similar.

A refraction error is similar to that of the range bias in that there is a non-zero mean error in range over the tracking interval. Like the range bias, this mean error can best be accommodated through an erroneous adjustment of the TLRS longitude. As seen in Table 6, for the shorter arc lengths, again, there is more sensitivity to this error source for baselines in the out-of-plane (E-W) direction. And for longer arc lengths, cancellation of this error source in both directions is achieved.

BASELINE ACCURACIES (CM)

ONE TLRS: CONSECUTIVE SITE OCCUPATIONS

ERROR SOURCES	NORTH–SOUTH 100%	NORTH–SOUTH 50%					EAST–WEST 100%	EAST–WEST 50%				
	5S	5S	5B*	30S*	30/5	30B*	5S	5S	5B*	30S*	30/5	30B*
POSITIONS	0.2	9.6	1.6	1.3	1.8	1.2	0.7	12.9	4.1	3.3	2.1	3.3
BIASES	2.1	3.9	2.9	2.0	2.5	2.0	0.7	8.0	3.5	1.8	1.3	1.2
REFRACTION	1.7	2.2	2.3	1.6	1.9	1.7	0.5	3.9	1.8	0.7	0.9	0.6
GM	1.7	3.5	5.2	1.4	2.5	1.1	2.4	18.7	14.5	5.0	1.8	1.1
GRAVITY	1.3	14.1	10.3	5.3	5.4	4.1	1.2	13.3	6.5	3.7	4.1	2.1
NOISE	0.1	0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.1	0.1	0.1
TOTAL(RSS)	3.5	18.0	12.2	6.2	7.0	5.1	2.9	27.9	16.9	7.3	5.2	4.3
GRAVITY FIELD TERMS IN ORDER OF IMPORTANCE	S_{77}	S_{76}	C_{76}	C_{76}	S_{76}	C_{76}	C_{33}	S_{77}	C_{76}	C_{77}	C_{77}	C_{76}
	C_{77}	C_{76}	S_{22}	S_{76}	S_{77}	S_{76}	C_{76}	C_{76}	S_{76}	S_{77}	S_{76}	S_{76}
	S_{97}	C_{77}	S_{76}	C_{66}	C_{76}	C_{44}	C_{22}	S_{76}	S_{22}	S_{76}	C_{30}	C_{77}
	C_{97}	C_{22}	C_{77}	S_{22}	C_{66}	S_{77}	C_{66}	C_{66}	C_{33}	S_{44}	C_{33}	S_{22}
	C_{20}	C_{44}	C_{66}	C_{77}	S_{66}	S_{22}	S_{76}	C_{30}	C_{66}	C_{22}	S_{22}	S_{41}
	C_{40}	S_{22}	C_{44}	S_{66}	S_{97}	S_{44}	S_{22}	S_{97}	S_{21}	S_{31}	C_{76}	S_{66}
	S_{66}	S_{77}	C_{21}	S_{77}	C_{77}	S_{66}	C_{20}	C_{20}	S_{31}	C_{76}	C_{66}	S_{77}
	C_{76}	S_{44}	S_{21}	S_{86}	S_{22}	C_{66}	C_{40}	C_{40}	C_{21}	C_{43}	C_{22}	C_{21}
	C_{65}	C_{97}	C_{41}	C_{44}	C_{22}	C_{77}	S_{21}	S_{31}	S_{66}	C_{66}	C_{44}	S_{31}
	C_{22}	S_{43}	C_{97}	C_{33}	C_{86}	C_{21}	S_{66}	S_{21}	C_{77}	S_{97}	S_{77}	C_{22}
	C_{66}	S_{97}	C_{33}	C_{65}	S_{43}	C_{60}	C_{53}	C_{77}	S_{77}	C_{31}	C_{97}	C_{97}
	S_{76}	C_{43}	S_{31}	C_{97}	S_{44}	C_{20}	C_{30}	S_{41}	S_{41}	S_{33}	S_{66}	S_{21}

*J_{11}, J_{12} adjusted to account for secular effects.

TABLE 6

The situation for errors in GM is more complicated. As alluded to earlier, the orbital period (mean motion) can be inferred directly from the laser system timing, the point of closest approach and the adjustment of the semi-major axis of the satellite. GM is assumed to be held constant in our simulations, and as a consequence of Kepler's third law, the fractional error in GM is approximately equal to one third of the error in the adjusted semi-major axis. This in turn, scales the size of orbit to the adopted (erroneous) value of GM. Therefore, since the range data has not undergone a similar scaling, the apparent errors in range are again similar to those of the range bias. As seen in Table 6, the same principles as those applicable to range biases apply for the propagation of the errors due to GM. Systematic longitudinal errors result and require longer arc lengths for cancellation.

The dynamical error sources arising from the uncertainty of the geopotential field dominate the degradation of the estimated baselines in Table 6. A breakdown of the individual spherical harmonic coefficients within the field reveals that orbital resonance with m=6 terms are the largest single error source (see Appendix A). However, terms having large m-daily effects are also significant (see Appendix A). The adjustment of the zonal harmonics has greatly reduced the impact of zonal harmonic uncertainty for the longer arcs. The gravity model error results in a complex degradation of the along track ephemerides accuracy of LAGEOS which is three to five times worse than the corresponding cross track errors. However, these errors are not symmetrical in either north to south or east to west directions. Given the magnitude of the along track vs. cross track effects, gravity has more severe consequence for N-S baseline adjustments than those E-W. However, with improved sampling through a lengthening of the arc, cancellation of the gravity error is apparent, but residual error is still quite substantial.

The influence of data noise is directly proportional to the square root of the number of observations. Therefore, long arc lengths, with more data, show improved noise-only baseline accuracy estimates.

Table 7. Baseline Error (cm)

One TLRS: Consecutive Site Occupations;
50% Efficiency (70 Day Arc)

ERROR SOURCE	ERROR MAGNITUDE	BASELINE ERROR
Solar Radiation	0.5% in C_R	(15.00)*
Earth Tide	1.0% in k_2	0.5
Geometric Tide	10% in h_2 and ℓ_2	0.2
Pole	0".01 in X and Y	1.4
Oceanic Loading	100% of Effect	0.0

*Requires Adjustment

It is apparent from this phase of our analysis that longer arc lengths yield improved baseline accuracies for a single TLRS having multiple consecutive site occupancies. Therefore, based solely on the above analysis, we recommend TLRS site occupancies of at least thirty days if a single system is employed and baselines between TLRS locations are the experimental objectives.

The other error sources (not mentioned in Table 6) need to be considered also for these longer arc lengths. Table 7 presents this information as the worst error, in either the E-W or N-S directions, arising from these as yet unmentioned error sources. As is evident from this table, a solar radiation pressure coefficient needs to be adjusted for arcs of 70 days length. However, all other error sources are relatively insignificant. From further simulations we have found that an adjustment of a solar radiation pressure coefficient has no influence on the baseline errors arising from any other source.

3.2.2.2 Two TLRSs

Our simulations have also dealt with a tracking configuration where there are two TLRSs available. If both systems are deployed at the same time and scheduled to track LAGEOS during the same working hours, then a new data reduction methodology can be attempted. One can limit analysis of the TLRS data sets to those observations which are simultaneously available from both sites. This is graphically displayed within Figure 4 where the simultaneous data is taken over the interval defined between points 2 and 3.

This definition of simultaneity is strict, and by this we mean that:

a. observations must be temporarily matched to within reasonable limits, and

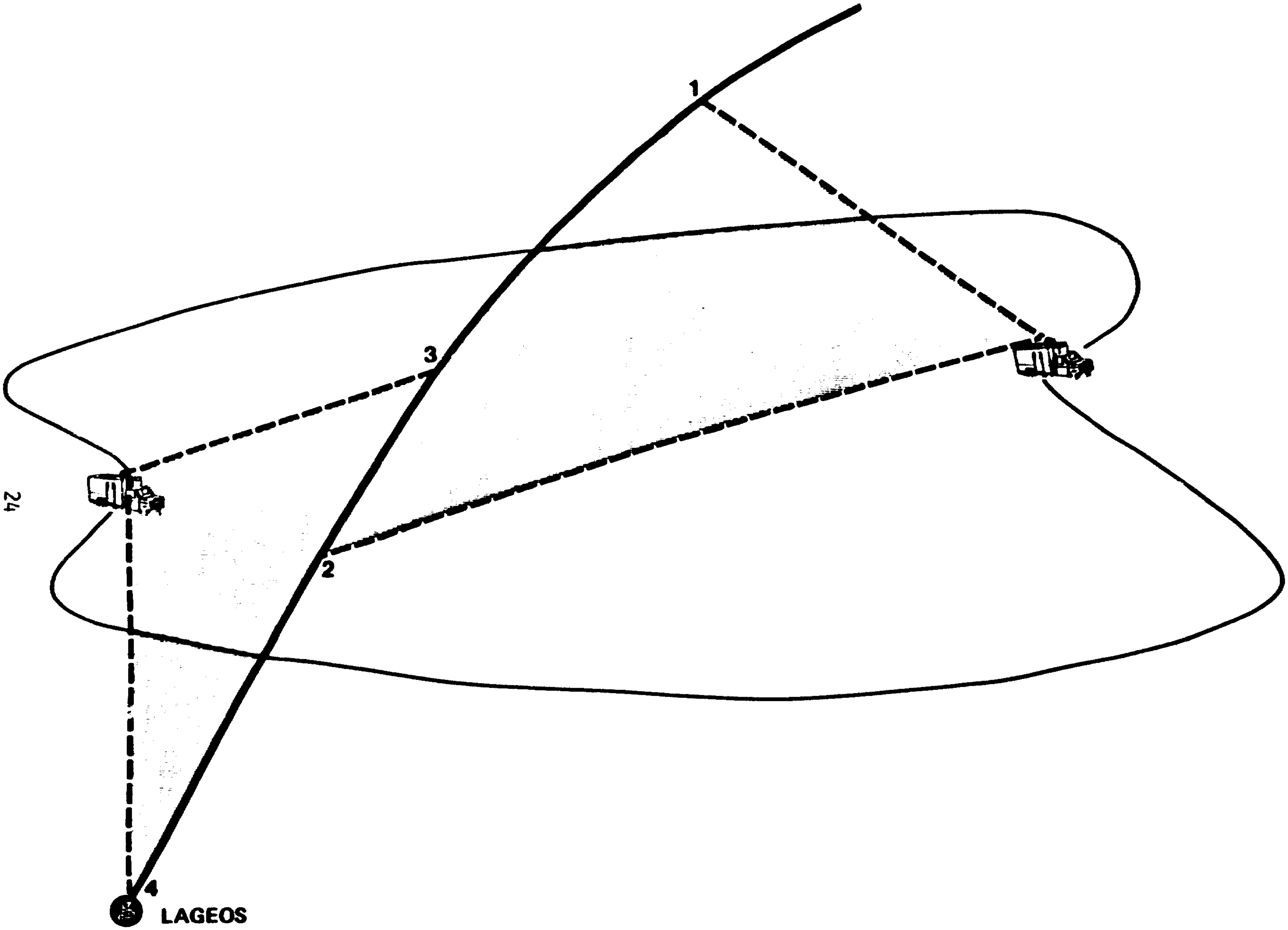

FIGURE 4. SIMULTANEOUS TRACKING FROM 2 TLRS LOCATIONS

b. the simultaneous passes in the arc must contain approximately the same number of observations (allowance can be made for a small number of segments of passes to have fewer points).

The data from the fixed stations are not subject to these restrictions and all of their available data is used in all cases.

Our interest in this approach lies in our belief that the fixed stations can sufficiently define the orbit in the vicinity of the TLRSs. Further, by limiting the TLRS data set to times of strict simultaneity, we ensure maximum cancellation of the errors in the estimated 200 km TLRS baseline.

In this series of simulations we have again assumed a tracking efficiency from all systems of 50%. Consequently, one would expect strict simultaneity to occur some fraction of this time. We have further degraded the efficiency of the systems by assuming that only 50% of the observations within the pass itself are successfully obtained. Therefore, the passes are now incomplete, and simultaneity needed to be assessed on a point by point basis.

Obviously, if this approach was shown to be optimal, long arc lengths are not required to provide cancellation o´ error source. This is the result indicated through a simulation of a five day arc length.

Table 8 intercompares the baseline accuracy estimates obtained from a configuration having tracking from two TLRSs for five day arc lengths. In cases 2-a_N and 2-a_E (the N denoting a N-S baseline, E for E-W), a tracking efficiency of 100% was used. As before, this test case was used as a standard by which degradation (or in our case, possible improvement) could be assessed. Cases 2b_N and 2b_E used all data provided by the algorithm of 50% tracking efficiency discussed earlier where both passes and observations are removed. Cases 2c_N and 2c_E use subsets of the data from 2b, but for the TLRS only those observations adhering to the simultaneity requirements are used.

BASELINE ACCURACIES (CM)

TWO TLRS'S: SIMULTANEOUS DATA CONFIGURATIONS (5 DAY ARC)

ERROR SOURCES	NORTH–SOUTH			EAST–WEST		
		50% of points; Incomplete passes			50% of points; Incomplete passes	
	All Possible Passes	All Passes	Base Stations All Passes TLRS's Simult. only (9 Passes)	All Possible Passes	All Passes	Base Stations All Passes TLRS's Simult. only (12 Passes)
	$2a_N$	$2b_N$	$2c_N$	$2a_E$	$2b_E$	$2c_E$
POSITIONS	0.1	0.6	0.1	0.2	0.3	0.1
BIASES	1.9	1.9	0.2	0.3	0.9	0.3
REFRACTION	1.5	1.5	0.4	0.1	0.6	0.5
GM	1.5	1.5	0.5	2.1	5.0	0.7
GRAVITY	0.3	10.9	0.8	0.7	3.6	0.3
NOISE	0.1	0.2	0.3	0.1	0.1	0.2
TOTAL (RSS)	2.9	11.3	1.1	2.2	6.2	1.0
GRAVITY FIELD TERMS IN ORDER OF IMPORTANCE	S_{76}	S_{76}	C_{76}	C_{33}	C_{22}	S_{22}
	S_{77}	C_{76}	C_{77}	C_{76}	S_{76}	C_{76}
	C_{32}	S_{44}	S_{77}	C_{66}	S_{66}	C_{20}
	C_{22}	S_{77}	C_{66}	C_{22}	C_{77}	C_{40}
	C_{77}	C_{44}	C_{97}	S_{22}	C_{33}	C_{22}
	S_{31}	C_{22}	C_{22}	C_{53}	S_{33}	S_{21}
	S_{32}	C_{65}	C_{20}	C_{30}	C_{65}	S_{41}
	C_{76}	C_{21}	C_{40}	S_{31}	C_{43}	S_{76}

TABLE 8

One can see that in all cases, a five day arc containing two TLRS sites yields baseline accuracies which are superior to those obtained from two successive five day solutions (solution 5S in Table 6) from a single TLRS occupying consecutive sites. However, when the data set of cases 2b are employed in their entirety, the cancellation of errors in the baseline is not very successful. This is primarily due to the along track errors (again principally resonance) arising from the uncertainty in the geopotential.

The results for the solutions employing strictly simultaneous data sets (cases $2c_N$ and $2c_E$) are extremely encouraging. The expected cancellation of unmodeled errors in the determination of the baseline has occurred to a very high level. Gravity errors are now reduced to a single centimeter error in baseline. Although orbital resonance error is still dominant, its effect on the determined baselines is now reduced by an order of magnitude.

Table 9 presents the estimated effects of all other considered parameters in the simultaneous data solutions: $2c_N$ and $2c_E$. Obviously, none of these remaining errors are significant beyond the 1 cm accuracy level. Again, Table 9 presents the worst incidence of error, in either the N-S or E-W baseline. We therefore conclude that based upon these results, the simultaneity method is optimal for analysis of data from two TLRSs. Baseline accuracies of 1 to 2 cm are possible through the implementation of this data reduction technique for those baselines of 200-500 km length.

3.2.2.3 <u>One TLRS: Application of the Simultaneity Principle</u>

The principle of employing strictly simultaneous data between adjusted stations has some application for a situation in which only a single TLRS is available. However, to do so, one of the previously fixed MOBLAS sites must now be treated as though it was the second TLRS. MOBLAS must be adjusted and only its data which is strictly simultaneous with that of the TLRS are used. The baseline of interest in this case is now the line between the

Table 9. Baseline Error (cm)

Two TLRSs: Simultaneous Data Configurations
(5 Day Arc)

ERROR SOURCE	ERROR MAGNITUDE	BASELINE ERROR
Solar Radiation	0.5% in C_R	0.0
Earth Tide	1.0% in k_2	0.0
Geometric Tide	10% in h_2 and ℓ_2	0.2
Pole	$0\overset{''}{.}01$ in X,Y	0.2
Oceanic Loading	100% of Effect	0.0

MOBLAS and the TLRS, and not those lines between successive TLRS site occupancies. To minimize the errors between TLRS baselines exclusively, the methodology detailed in Section 3.2.2.1 is still superior.

Accurate baseline determination between the TLRS and the MOBLAS is possible through the employment of the simultaneity principle. This is presented in Table 10, where SANDIE (San Diego) is now treated as an adjusting station along with the TLRS at site 3 (from Figure 1). The baseline we are now discussing is 413 km in length, over twice the length of any previously considered line. Again, as in cases 2c (in Section 3.2.2.2) the tracking efficiency level employed for all stations is 50% for both passes and individual points within the pass. There is now only a single fixed station, QUINCY, for the first case considered, which is case 3a on Table 10. All data from QUINCY allowed by the algorithm are used while SANDIE and the TLRS supply strictly simultaneous data.

In case 3a the orientation of the baseline in three dimensional space is inferior to the previous situations using two fixed stations. This distortion increases the error propagation into the baseline from cross track effects such as those arising from range biases and refraction errors. However, these error sources have been unduly pessimistically modeled throughout our simulations, and are subject to improvement. What is more significant, however, is the relative insensitivity of this approach to gravity error sources even though resonance error dominates.

In case 3b, a second fixed site located in Mexico, is introduced. The orbital plane again becomes more stabilized, and the cross track error sources diminish in the baseline statistics. We therefore conclude that the simultaneity principle within the framework of a dynamical data analysis warrants consideration if the baselines between a MOBLAS and the TLRS sites are in the direction of tectonic interest. If this is so, baselines of 2-3 cm accuracy between the TLRS and the MOBLAS sites are achievable.

BASELINE ACCURACIES (CM)

ONE TLRS: SIMULTANEOUS DATA CONFIGURATIONS (5 DAY ARC)

ERROR SOURCES	SANDIE – SITE NO. 3 (413 Km) 50% of points; Incomplete passes: Quincy All Passes SANDIE & TLRS Simultaneous Only (9 Passes) 3a	Quincy & Mexico All Passes, SANDIE & TLRS Simultaneous Only (9 Passes) 3b
POSITIONS	0.2	0.2
BIASES	3.7	1.7
REFRACTION	3.1	1.4
GM	0.9	1.1
GRAVITY	1.0	0.7
NOISE	0.3	0.2
TOTAL (RSS)	5.0	2.6
GRAVITY FIELD TERMS IN ORDER OF IMPORTANCE	S_{77}, C_{22}, C_{66}, C_{76}, C_{30}, S_{97}, S_{21}, S_{22}	C_{76}, C_{66}, S_{76}, S_{77}, S_{66}, S_{44}, C_{32}, C_{86}

TABLE 10

4.0 CONCLUSIONS

In the present study we elaborated on the expected TLRS baseline accuracies in the Western United States using the LAGEOS satellite. The conclusions we could draw from this error analysis regarding the accuracy of baselines of moderate length (e.g., 200-500 km) are given below.

a. Global laser tracking is unnecessary for TLRS support if local tracking is available.

b. The station coordinate errors are systematic, and while they are in the decimeter level, baseline precision is approximately an order of magnitude better.

c. When the TLRS efficiency is degraded below 100%, special consideration of geopotential errors is necessary.

d. For baselines between consecutive TLRS site occupations:

   - 30 days on site is necessary as baseline accuracy is directly dependent upon efficiency of TLRS,

   - Above requirement may be lessened if the gravity field is improved, and

   - Long arc reductions slightly favored over multiarc reductions in which case the 2σ accuracy of the baseline is better than 5 cm.

e. For baselines between base station and TLRS:

- If both adjusted and only simultaneous passes allowed in the solution the 2σ accuracy of the baseline is better than 1 part in $2*10^7$. The reduced arc can be as short as 5 days and the number of strictly simultaneous passes could be 6-8 (not necessarily complete).

It must be kept in mind that some of the above conclusions could possibly be radically revised after the first couple of years of observing, as a result of improved knowledge of previously poorly known quantities.

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APPENDIX A

Resonance

Resonance is a common effect experienced by near-earth orbiting objects. Basically, resonance is a short period longitudinal-dependent term of the geopotential which manifests itself as an excessively large long period perturbation on the orbit due to the commensurability of the satellite's motion with the earth's rotation.

For most orbital arc lengths which exceed even a few revolution lengths, resonance presents itself as a significant problem. However, the terms in the field which resonate with the orbit generally have a single significant contribution to the orbital motion. LAGEOS, which has a mean motion of nearly six revolutions per day, is resonant with m=6 terms of the geopotential, C,S(7,6) being dominant. The contribution of all m=6 terms, however, is almost entirely due to these resonance effects. Table A shows all significant perturbations arising from C,S(7,6) on the LAGEOS orbit. It is evident that the resonance effect arising from these terms is several orders of magnitude larger than any resulting short period or m-daily perturbation. Errors resulting from imperfect knowledge of the resonance harmonics can be accommodated through the adjustment of a select set of resonance terms. While the individual coefficients obtained through this adjustment may suffer from severe aliasing, the resonance error is minimized in the orbital trajectory.

M-Daily and Short Period Terms of the Geopotential

The dominant source of low degree and order information ($\ell \leq 8$) in the geopotential comes from the m-daily perturbations which manifest themselves on all of the orbits used in gravitational model recovery. The m-daily terms arise from short period perturbations which are independent of (i.e., averaged over) the mean anomaly. However, the accommodation of errors

TABLE A

ESTIMATED ALONG TRACK PERTURBATIONS

FROM THE DOMINANT RESONANCE TERMS ON THE LAGEOS ORBIT

HARMONIC CONSTITUENT	FREQUENCY (cyc/day)	ESTIMATED ALONG TRACK PERTURBATION (m)	GRAVITATIONAL FAMILY
$\underline{\ell}$ $\underline{m}$ $\underline{p}$ $\underline{q}$			
7 6 3 0	0.37 (2.66 day period)	28.28	shallow resonance
7 6 2 0	13.15	0.13	short period
7 6 2 1	19.53	0.06	short period
7 6 4 -1	18.78	0.08	short period
7 6 4 0	12.40	0.26	short period
7 6 3 1	6.76	0.30	short period
7 6 5 0	25.17	0.06	short period
7 6 6 -1	44.33	0.10	short period
7 6 6 0	37.94	0.12	short period
7 6 7 0	50.71	0.06	short period
7 6 3 -1	6.01	0.18	m - daily
7 6 4 1	6.01	0.18	m - daily

Based on
Normalized value:

$$J_{7,6} = \left(C_{7,6}^{\,2} + S_{7,6}^{\,2}\right)^{1/2} = 3.864 \times 10^{-7}\ *$$

*From GEM9 (Lerch et al, 1977)

from imperfect knowledge of the m-daily effects is made very complex due to the presence of a rich spectrum of other short period perturbations arising from the same low degree and order harmonic coefficients. As an illustration, Table B presents all of the significant orbital perturbations on LAGEOS produced by C,S(4,4) and C,S(6,4). The amplitude of each perturbative frequency consists of a linear combination of the effects of terms of the same order (m) having consistently either odd or even degree as shown in Table B. All of the frequencies of the C,S(4,4) terms are found in terms of C,S(6,4) although the reverse is not true. These very same frequencies will also be found in every higher even degree, 4th order term. However, the relative importance of each frequency (e.g., percent of dominant constituent in Table B) is a function of the orbital elements of the satellite and also a function of the harmonic's degree for a given order. Although C,S(4,4) has only two harmonic constituents producing nearly its total perturbation (ℓmpq=(4,4,2,0) and (4,4,2,1)) on LAGEOS, C,S(6,4) finds these two frequencies having much less importance. As shown, C,S(6,4) manifests itself on LAGEOS in five constituents which are at least 40% of its dominant effect. With the eccentricity being small, terms of odd degree will have frequencies very similar to those arising from the even degree. This further complicates the gravity modeling problem. The same type of behavior is exhibited by the entire low degree and order field.

As a consequence of these numerous frequencies and the large number of the resulting linear combinations of the harmonic coefficients a simple adjustment of a select set of coefficients presents a difficult problem. This is especially true given the nature of laser tracking, with the data acquisition problems alluded to in Section 2.0. Laser systems are not all-weather instruments, and consequently, the data which is obtained can vary dramatically from site to site, and even day to day for a given site. Therefore, there are significant problems which arise from incomplete sampling when one is attempting to uncouple a large number of mismodeled short period orbital effects from station coordinate errors if the geopotential coefficients are adjusted.

TABLE B

SIGNIFICANT GRAVITATIONAL PERTURBATIONS FROM C,S(4,4) AND C,S(6,4) ON THE LAGEOS ORBIT

HARMONIC CONSTITUENT ℓ m p q	(cyc/day)	ALONG TRACK PERT (m)	% OF DOMINANT CONSTITUENT	HARMONIC CONSTITUENT ℓ m p q	(cyc/day)	ALONG TRACK PERT (m)	% OF DOMINANT CONSTITUENT
4 4 0 0	21.54	.04	.005	6 4 1 0	21.54	.09	.223
4 4 0 1	27.92	.04	.005	6 4 1 1	27.92	.08	.170
4 4 1 -1	2.38	.32	.039	6 4 2 -1	2.38	.35	.795
4 4 1 0	8.76	.69	.085	6 4 2 0	8.76	.44	1.000
4 4 1 1	15.15	.46	.056	6 4 2 1	15.15	.20	.454
4 4 2 -1	-10.39	1.10	.136	6 4 3 -1	-10.39	.04	.079
4 4 2 0	-4.01	8.10	1.000	6 4 3 0	-4.01	.25	.568
4 4 2 1	2.38	4.90	.601	6 4 3 1	2.38	.14	.318
4 4 3 -1	-23.17	1.20	.148	6 4 4 -1	-23.17	.12	.273
4 4 3 0	-16.78	2.00	.247	6 4 4 0	-16.78	.25	.568
4 4 3 1	-10.39	0.29	.035	6 4 4 1	-10.39	.07	.159
4 4 4 -1	-35.94	0.56	.069	6 4 5 -1	-35.94	.06	.140
4 4 4 0	-29.55	0.55	.067	6 4 5 0	-29.55	.08	.179
4 4 4 1	-23.16	0.21	.025	6 4 5 1	-23.16	.00	.001
No Term	-42.32	—	—	6 4 6 0	-42.32	.14	.318
No Term	-35.94	—	—	6 4 6 1	-35.94	.05	.114

A phenomenon similar to resonance can arise between the tracking from a given station and certain frequencies of geopotential error. For example, the LAGEOS orbit is visible to a given mid-latitude site at approximately 12 hour intervals (i.e., one sees the satellite's ascent, the earth rotates beneath the orbital plane, and 12 hours later, the satellite's descent is viewed). In our LAGEOS simulations one finds a large latitude error resulting from m=1 geopotential errors. Since m=1 terms have m-daily effects which have a one cycle/day frequency, the net effect is the along track position is leading at a given time to be followed by a corresponding lag 12 hours later due to the errors in these m-daily m=1 terms. Considering the high inclination of LAGEOS, along track effects are nearly totally in the latitudinal direction. This orbital error therefore, can be directly absorbed (i.e., the range errors can be minimized) by moving the station's latitude an appropriate amount. When the data sampling is incomplete, the manifestations of the mismodeled low degree short period effects, becomes largely a function of the sampling itself, and becomes unique to each specific arc. This problem is greatly magnified and the interpretation of simulated results becomes far less uncertain, when the low degree part of the geopotential is allowed to adjust. The quality of the geopotential adjustment and the net resulting baselines can vary significantly if differing random tracking configurations are employed. The determination of a complete (albeit truncated) geopotential requires tracking from numerous satellites taken over long periods of time. Only through the analysis of this type of data set can one uncouple the linear combination of gravitational effects into well determined individual harmonic coefficients.* Data spanning a full apsidal period is normally required to separate odd from even degree effects.

*Tailored fields have the correct linear combinations to produce the correct amplitudes and phases on a given orbit but not necessarily the correct values for the individual gravitational coefficients.