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Optical-Model Abrasion Cross Sections for High-Energy Heavy Ions

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and Space Administration

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INTRODUCTION

High-energy heavy-ion projectile fragmentation has been described and analyzed with an abrasion-ablation collision model (refs. 1 to 4). In the model, the projectile nuclei, moving at relativistic speeds, collide with stationary target nuclei. At small impact parameters, portions of their nuclear volumes overlap and are sheared away in the collision. This is the abrasion process. The remaining piece of projectile matter, sometimes called a prefragment, continues its trajectory with essentially its precollision velocity. Because of the nuclear dynamics of the abrasion process, the projectile prefragment is in an excited state after the collision. This excess energy is removed in the ablation process by the emission of gamma radiation and/or the evaporation of one or more nuclear particles (e.g., nucleons and composites). The remaining isotope is the nuclear fragment species which is experimentally detected and whose cross section is measured.

The abrasion part of the collision process is typically analyzed from classical geometric considerations (ref. 3) or by utilizing high-energy Glauber theory (refs. 1, 2, and 4). In the classical geometric theory, the number of abraded nucleons, and the abrasion cross section, is calculated from the geometric overlap of the two colliding nuclear volumes. Although conceptually simple, this approach has no real physical basis. The inherent assumption that nuclear matter is completely opaque to itself is in error; nuclear matter generally displays a marked degree of transparency (ref. 5). Hence, the predicted cross sections are only qualitatively accurate. High-energy Glauber theory, since it is developed from formal quantum scattering theory, is more physically realistic. Convergence of the Glauber approximation to the multiple-scattering series, however, is very slow except in the optical limit. Predictive accuracy is significantly improved in the optical limit for heavy nuclei, but is extremely poor for nuclei lighter than oxygen (ref. 4).

In a nuclear optical model, the target nucleus appears to be partially transparent to the incoming high-energy projectile since the collision mean free path is comparable to the nuclear radius. By analogy with optics, the nucleus is then considered to possess optical properties such that scattering and absorption are characterized by an "effective" complex index of refraction. Wilson and Costner (refs. 5 to 7) have developed an optical-model potential approximation to the nucleus-nucleus multiple-scattering series which converges much more rapidly than the Glauber approximation and is valid even for very light nuclei. As developed, the optical-model potential approximation, when utilized within the context of eikonal scattering theory (ref. 8, ch. 9), accurately predicts nucleus-nucleus total and absorption cross sections (ref. 7), but does not predict fragmentation cross sections.

In the present work, the first step toward developing a comprehensive fragmentation theory is undertaken by incorporating the optical-model potential approximation into an abrasion-ablation collision formalism. Within the context of eikonal scattering theory, expressions for projectile abrasion cross

sections are determined and numerical predictions obtained. In addition, the predictions are compared with experimental results and with other theoretical analyses using Glauber theory.

ABRASION THEORY

Abrasion theories developed in recent years have relied on Glauber theory as the basic formalism for the evaluation of probabilistic collision factors. Consequently, the inherent restrictions of Glauber theory are also limitations in these models. With the more powerful theoretical methods now available, it appears appropriate to develop a new abrasion theory based on these more general theoretical methods. Such a derivation follows, after a brief review of major results from current abrasion theories. The symbols used in this paper are defined on pages 12 to 13.

In the abrasion-ablation collision model, projectile fragmentation is a three-step process. In the first step, abrasion, m nucleons are knocked out of the projectile nucleus of mass number A_P , leaving an excited prefragment nucleus of mass number

$$A_F = A_P - m \tag{1}$$

In the next step, the prefragment ablates by gamma emission, particle emission (usually nucleons or alpha particles), or a combination of the two. The third and final phase involves interactions between the particles in the final state. These final-state interactions, although not unique to this collision formalism, nevertheless are significant experimentally and must be included in any complete theory.

Abrasion Cross Section

From reference 4, the cross section for abrading m projectile nucleons is

$$\sigma_m = \binom{A_P}{m} 2\pi \int \left[1 - P(b) \right]^m P(b) A_F b \, db \tag{2}$$

where $\binom{A_P}{m}$ is the binomial coefficient which reflects the number of possible combinations of m nucleons taken from an ensemble of A_P identical nucleons. The total absorption cross section,

$$\sigma_{abs} = 2\pi \int \left[1 - P(b) \right]^{A_P} b \, db \tag{3}$$

is obtained by summing over all values of m according to

$$\sigma_{\text{abs}} = \sum_{m=1}^{A_P} \sigma_m \quad (4)$$

In equations (2) and (3), $P(\vec{b})$ is the probability as a function of impact parameter for not removing a single projectile nucleon in the abrasion process. Hence, $1 - P(\vec{b})$ is the probability for removal of a nucleon.

The probability in Glauber theory is given by (ref. 4)

$$P(\vec{b}) = 2\pi \int D_P(\vec{s}) \exp\left[-A_T \sigma_{NN} D_T(\vec{s}+\vec{b})\right] s \, ds \quad (5)$$

where A_T is the mass number of the target and the $D(\vec{s})$ are the single-particle densities summed along the beam direction

$$D(\vec{s}) = \int_{-\infty}^{\infty} \rho(\vec{s}+\vec{z}) \, dz \quad (6)$$

The abrasion theory is now extended to a more general collision theory which does not exhibit the convergence problems inherent with Glauber theory. An added feature of the extended abrasion theory, which gives symmetry to the final result, is that the projectile and target nuclei are treated on an equal basis.

Generalized Abrasion Theory

From an alternate optical model derived in references 5 and 6, the absorption cross section is expressed using the eikonal approximation (ref. 8, ch. 9):

$$\sigma_{\text{abs}} = 2\pi \int_0^{\infty} \left\{ 1 - \exp\left[-2 \operatorname{Im} \chi(\vec{b})\right] \right\} b \, db \quad (7)$$

where the eikonal phase function $\chi(\vec{b})$, with the optical-model potential approximation from reference 7 incorporated, is written

$$\chi(\vec{b}) = \frac{1}{2} A_P A_T \sigma(e) [\alpha(e) + i] I(\vec{b}) \quad (8)$$

where

$$I(\vec{b}) = [2\pi B(e)]^{-3/2} \int d\vec{z} \int d^3\xi_T \rho_T(\xi_T) \int d^3\vec{y} \rho_P(\vec{b}+\vec{z}+\vec{y}+\xi_T) \exp\left[\frac{-\vec{y}^2}{2 B(e)}\right] \quad (9)$$

In equations (8) and (9), $\sigma(e)$ is the energy-dependent nucleon-nucleon cross section, $\alpha(e)$ is the energy-dependent ratio of the real part to the imaginary part of the scattering amplitudes, $B(e)$ is the energy-dependent slope parameter, and ρ_P and ρ_T are the projectile and target single-particle nuclear densities. Comparison of equations (3) and (7) implies that

$$P(\vec{b})^{A_P} = \exp\left[-2 \operatorname{Im} \chi(\vec{b})\right] \quad (10)$$

Substitution of equation (8) into equation (10) yields

$$P(\vec{b}) = \exp\left[-A_T \sigma(e) I(\vec{b})\right] \quad (11)$$

Finally, the cross section for abrading any m nucleons (eq. (2)) is written as

$$\sigma_m = \binom{A_P}{m} 2\pi \int \left\{ 1 - \exp\left[-A_T \sigma(e) I(\vec{b})\right] \right\}^m \exp\left[-A_T A_F \sigma(e) I(\vec{b})\right] b db \quad (12)$$

In evaluating equation (12), values for $\sigma(e)$ and $B(e)$ were taken from the compilations in references 9 and 10. The nuclear single-particle densities in equation (9) were extracted from the charge density data in reference 11 using the detailed procedure of reference 7.

Isotope Production Cross Section

Up to this point, all nucleons have been treated as identical objects. In order to differentiate between protons and neutrons, equation (5) is replaced by (ref. 1)

$$\sigma_{nz} = \binom{N}{n} \binom{Z}{z} 2\pi \int \left[1 - P(\vec{b}) \right]^{n+z} P(\vec{b})^{A_P - n - z} b db \quad (13)$$

where $P(\vec{b})$ is again given by equation (11). In equation (13), σ_{nz} is the cross section for abrading n out of N neutrons and z out of Z protons from the projectile nucleus. Implicit in this expression is the assumption that the neutron and proton distributions in the projectile nucleus are completely

uncorrelated. This oversimplification of the actual complex nature of nucleon correlations in nuclei provides an analytically simple and convenient starting point for computing cross sections for specific fragment species.

RESULTS

Figure 1 displays results obtained from equation (12) for ^{16}O projectile nuclei colliding with various stationary target nuclei. The incident kinetic

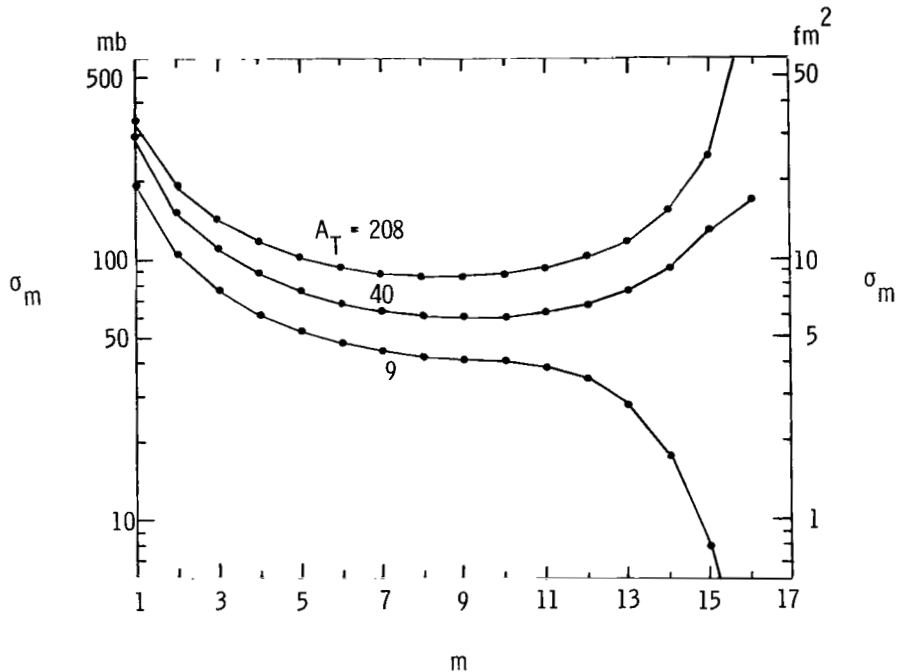


Figure 1.- Oxygen-target abrasion cross sections as a function of the number of abraded nucleons. The lines are merely to guide the eye. Incident kinetic energy is 2.1 GeV/nucleon.

energy is 2.1 GeV/nucleon. The shapes of the curves are largely determined by the $2\pi b$ factor and the effect of the spatial variations of ρ_T and ρ_P on $P(\vec{b})$ in the integrand of equation (12). The comparatively large cross sections for abrading one or two nucleons are indicative of the dominance at large impact parameters of the $2\pi b$ factor. Were it not for the large degree of nuclear matter transparency in this very low density region, these cross sections would be even larger in magnitude. Physically, these theoretical results are not unexpected. In peripheral interactions, the nucleons near the surface are least tightly bound and are more easily removed than those in the nuclear interior. Because of the short finite range of the nuclear force, abrasion is possible even if the projectile and target densities do not physically overlap. As the number of abraded nucleons increases, overlap between the projectile and target must occur. This increases the overlapping densities which do not, however, offset the initial decrease in the impact parameter. As a result, the cross

sections initially decrease with increasing values of m . Between $m = 5$ and $m = 11$, the cross section curves flatten as the increasing nuclear densities tend to balance the decrease in the $2\pi b$ factor. For $m \geq 11$, the curves display a marked dependence on the size of the target nucleus. The rapid decrease in σ_m for the ${}^9\text{Be}$ target indicates that abrasion of all, or nearly all, of the projectile nucleons by the smaller target is likely to occur only for very small impact parameters. If the target is pure hydrogen (curve not shown), the cross section for abrading all projectile nucleons in one collision, from equation (12), is less than 5 nanobarns¹ - approximately a million times smaller than for the Be target. As target size increases, the abrasion cross sections increase as m increases. This results from the larger geometric area for which the projectile and target volumes completely overlap.

Figure 2 displays abrasion cross sections obtained from the optical-model potential approximation of this work and the Glauber approximation of reference 4. Also displayed are experimental values estimated from the isotope

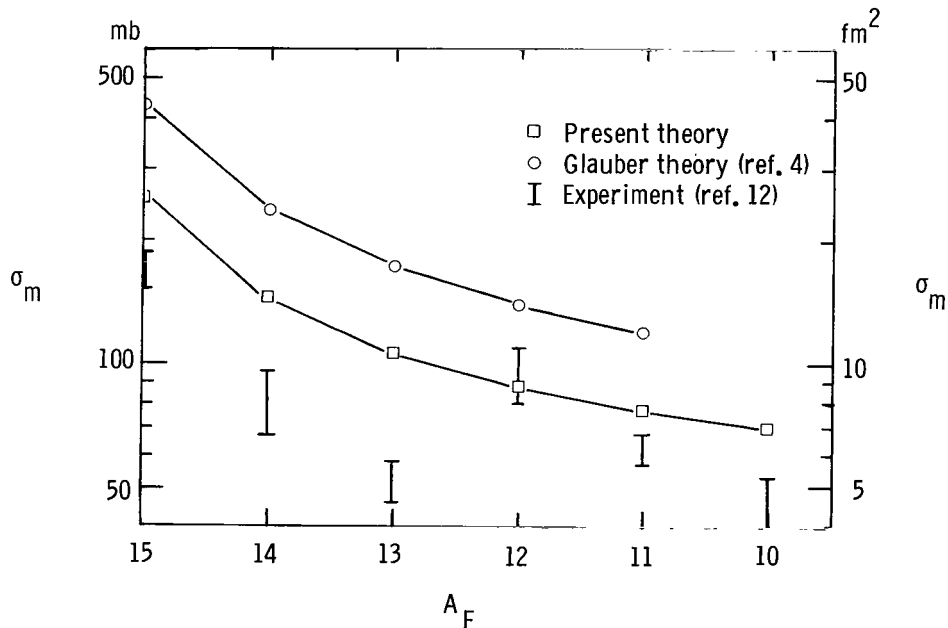


Figure 2.- Oxygen-copper abrasion cross sections versus prefragment mass number. Incident kinetic energy is 2.1 GeV/nucleon.

production cross sections of reference 12 by summing over all contributing proton numbers Z for a given mass number A according to

$$\sigma_F(A) = \sum_Z \sigma_F(Z, A) \quad (14)$$

¹ 1 nanobarn = 10^{-7} fm^2 .

The projectile nucleus is ^{16}O at an incident kinetic energy of 2.1 GeV/nucleon. The target nucleus is ^{64}Cu . Although the shapes of the theoretical curves are very similar, the optical-model abrasion cross sections of this work are markedly smaller and nearer the experimental fragmentation results. Incorporating ablation effects into the theory is expected to change the magnitudes of the theoretical curves and to significantly alter their shapes.

The tendency for theories incorporating the Glauber approximation to overestimate cross sections is apparent in figure 3. The theoretical curves displayed were obtained from equations (4) and (12) of this work and from the

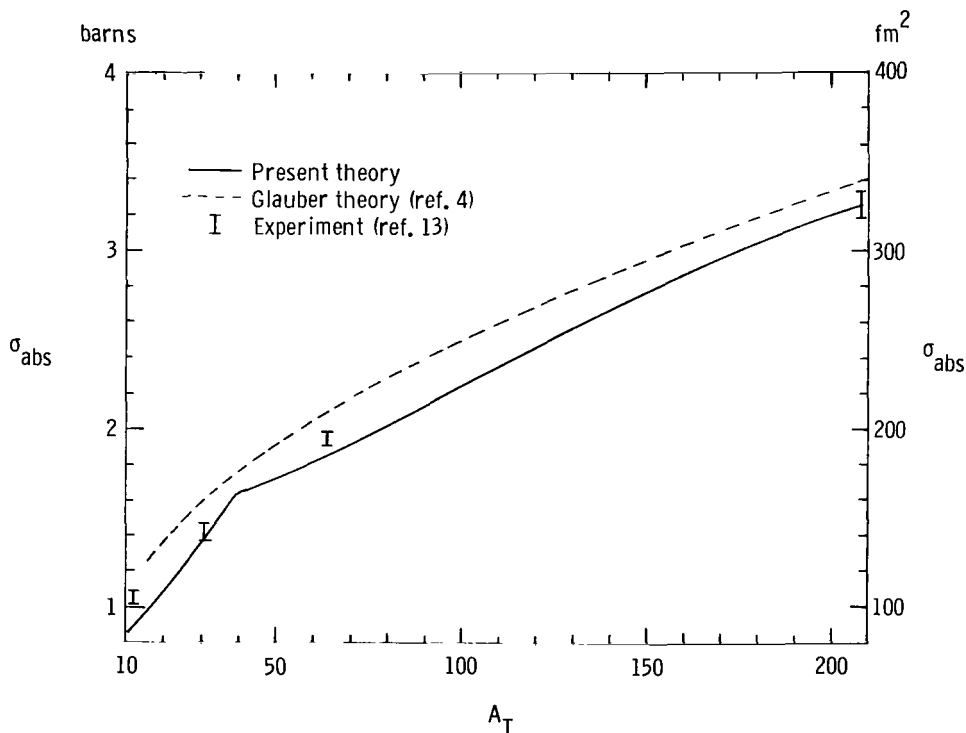


Figure 3.- Oxygen-nucleus absorption cross sections as a function of target mass number. Incident kinetic energy is 2.1 GeV/nucleon.

values listed in table I of reference 4. The experimental data are due to Heckman et al. at the Lawrence Berkeley Laboratory (ref. 13). The curve using Glauber theory does not extend below $A_T = 16$ because of the extremely poor convergence of the Glauber approximation to $P(b)$ in equation (2) for this mass region (ref. 4). The optical-model potential approximation, however, rapidly converges for any target mass number.

Table I lists representative results obtained from equation (13) for producing various isotopes by projectile abrasion in the reaction



where A_Z is the specific isotope produced and X represents anything else produced in the reaction. The incident kinetic energy is 2.1 GeV/nucleon. Also listed for comparison are abrasion results obtained using the Glauber approximation (ref. 1) and experimental isotope production cross sections from the Lawrence Berkeley Laboratory (ref. 12). In general, the cross sections determined in this work are smaller than those obtained with Glauber theory. This was also noted in the results shown in figures 2 and 3.

Since ablation has not yet been incorporated into the theory, most of the predicted cross sections overestimate the comparable experimental results. Theoretical inaccuracies are also attributable to the simplifying assumption that no correlation exists between the distributions of protons and neutrons. Important final-state interaction effects must also be identified and included in any accurate and complete theory. The capabilities of the theory for projectiles other than oxygen and energies other than 2.1 GeV/nucleons are demonstrated in table II where results from equation (13) are obtained for the reaction



at an incident kinetic energy of 1.05 GeV/nucleon. For comparison, experimental values from reference 12 are also shown.

CONCLUDING REMARKS

An optical-model potential approximation to the nucleus-nucleus multiple-scattering series, previously shown to accurately describe total and absorption cross sections for collisions between relativistic heavy ions, has been successfully incorporated into an abrasion-ablation collision model and used to describe projectile-nucleus fragmentation by abrasion. Cross sections for abrading any number of projectile nucleons, as well as for producing specific isotopic species, were calculated. Comparisons with experimental fragmentation results and with predictions from Glauber theory indicate that the model described herein yielded abrasion cross sections which were typically smaller and closer to experimental results than those predicted by Glauber theory. Unlike the optical limit of Glauber theory, which cannot be used for very light nuclei, the abrasion formalism of this work is valid for any projectile-target combination and for any incident kinetic energy at which eikonal scattering theory can be utilized.

The usually wide disparities between predicted and experimental results indicated the importance of ablation and final-state interactions to the fragmentation process and emphasized the need to incorporate them into the formalism.

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SYMBOLS

A	nuclear mass number, dimensionless
B(e)	average slope parameter of nucleon-nucleon scattering amplitude, fm ²
\vec{b}	projectile impact parameter vector, fm
e	two-nucleon kinetic energy in their center of mass frame, GeV
I(\vec{b})	defined in equation (9)
Im $\chi(\vec{b})$	imaginary part of eikonal phase shift function, dimensionless
m	number of abraded nucleons, dimensionless
N	total number of projectile nucleus neutrons, dimensionless
n	number of abraded neutrons, dimensionless
P(\vec{b})	probability for not removing a single nucleon by abrasion, dimensionless
X	all products of nuclear collision except the prefragment/fragment, dimensionless
\vec{y}	two-nucleon relative position vector, fm
Z	total number of projectile-nucleus protons, dimensionless
A _Z	particular nuclear isotope with proton number Z and mass number A, dimensionless
z	number of abraded protons, dimensionless
\vec{z}	position vector of projectile along beam direction, fm
$\binom{D}{d}$	binomial coefficient, dimensionless
$\alpha(e)$	average ratio of real part to imaginary part of nucleon-nucleon scattering amplitude, dimensionless
$\vec{\xi}_T$	collection of constituent relative coordinates for target, fm
ρ	nuclear single-particle density, fm ⁻³
$\sigma(e)$	average nucleon-nucleon total cross section, fm ² or mb
σ_{abs}	heavy-ion absorption cross section, fm ² or mb

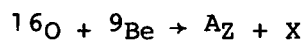
σ_{exp} experimental heavy-ion cross section, fm² or mb
 σ_{F} heavy-ion fragmentation cross section, fm² or mb
 σ_{m} cross section for abrading m nucleons, fm² or mb
 σ_{NN} nucleon-nucleon cross section, fm² or mb
 σ_{nz} cross section for abrading n neutrons and z protons, fm² or mb
 $\chi(\vec{b})$ eikonal phase shift function, dimensionless

Subscripts:

F prefragment
P projectile
T target

Arrows over symbols indicate vectors.

TABLE I.- OPTICAL MODEL ABRASION CROSS SECTIONS FOR THE REACTION

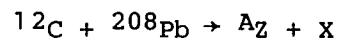


[Incident kinetic energy is 2.1 GeV/nucleon]

Species, A_Z	σ_{nZ} , mb	Cross sections from Glauber theory (ref. 1), mb	σ_{exp} (ref. 12), mb
	(a)	(a)	(a)
^{15}O	96.3	131	43.0 \pm 2.1
^{14}O	24.7	33	1.6 \pm .1
^{13}O	7.7	10	.32 \pm .04
^{15}N	96.3	131	54.1 \pm 2.7
^{14}N	56.4	75	49.5 \pm 4.0
^{13}N	30.7	40	8.0 \pm .4
^{12}N	15.3	19	.66 \pm .06
^{14}C	24.7	33	5.2 \pm 0.3
^{13}C	30.7	40	28.6 \pm 1.4
^{12}C	26.8	33	60.8 \pm 4.9
^{11}C	19.3	21	21.0 \pm 1.0

a) millibarn (mb) = 0.1 square femtometer (fm^2).

TABLE II.- OPTICAL MODEL ABRASION CROSS SECTIONS FOR



[Incident kinetic energy is 1.05 GeV/nucleon]

Species, A_Z	σ_{nz} , mb (a)	σ_{exp} (ref. 12) mb (a)
^{11}C	178.0	128.0 ± 22
^{10}C	45.8	10.9 ± 1.7
^{11}B	178.0	149.0 ± 25
^{10}B	110.0	50.9 ± 18.2
^{10}Be	45.8	10.9 ± 1.8
^9Be	61.7	22.2 ± 3.7
^9Li	13.7	1.76 ± 0.81

^{a1} millibarn (mb) = 0.1 square femtometer (fm^2).

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